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THE ELECTRODYNAMICS OF SUBSTANCES WITH SIMULTANEOUSLY NEGATIVE VALUES OF ϵ AND μ

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1. INTRODUCTION

THE dielectric constant ϵ and the magnetic permeability μ are the fundamental characteristic quantities which determine the propagation of electromagnetic waves in matter. This is due to the fact that they are the only parameters of the substance that appear in the dispersion equation

$$\left|\frac{\omega^2}{c^2}\varepsilon_{il}\mu_{lj}-k^2\delta_{ij}+k_ik_j\right|=0, \qquad (1)$$

which gives the connection between the frequency ω of a monochromatic wave and its wave vector **k**. In the case of an isotropic substance, Eq. (1) takes a simpler form:

$$k^2=\frac{\omega^2}{c^2}n^2. \qquad (2)$$

Here n^2 is the square of the index of refraction of the substance, and is given by

$$n^2=\varepsilon\mu. \qquad (3)$$

If we do not take losses into account and regard n, ϵ, and μ as real numbers, it can be seen from (2) and (3) that a simultaneous change of the signs of ϵ and μ has no effect on these relations. This situation can be interpreted in various ways. First, we may admit that the properties of a substance are actually not affected by a simultaneous change of the signs of ϵ and μ. Second, it might be that for ϵ and μ to be simultaneously negative contradicts some fundamental laws of nature, and therefore no substance with $\epsilon < 0$ and $\mu < 0$ can exist. Finally, it could be admitted that substances with negative ϵ and μ have some properties different from those of substances with positive ϵ and μ. As we shall see in what follows, the third case is the one that is realized. It must be emphasized that there has not so far been any experiment in which a substance with $\epsilon < 0$ and $\mu < 0$ could be observed. We can, however, at once give a number of arguments as to where and how one should look for such substances. Since in our opinion the electrodynamics of substances with $\epsilon < 0$ and $\mu < 0$ is undoubtedly of interest, independently of our now having such substances available, we shall at first consider the matter purely formally. Thereafter in the second part of this article we shall consider questions connected with the physical realization of substances with $\epsilon < 0$ and $\mu < 0$.

II. THE PROPAGATION OF WAVES IN A SUBSTANCE WITH $\epsilon < 0$ AND $\mu < 0$. "RIGHT-HANDED" AND "LEFT-HANDED" SUBSTANCES

To ascertain the electromagnetic laws essentially connected with the sign of ϵ and μ, we must turn to those relations in which ϵ and μ appear separately, and not in the form of their product, as in (1)—(3). These relations are primarily the Maxwell equations and the constitutive relations

$$\left.\begin{aligned}\operatorname{rot}\mathbf{E}&=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\\ \operatorname{rot}\mathbf{H}&=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t},\end{aligned}\right\} \qquad (4)^*$$

$$\left.\begin{aligned}\mathbf{B}&=\mu\mathbf{H},\\ \mathbf{D}&=\varepsilon\mathbf{E}.\end{aligned}\right\} \qquad (4')$$

For a plane monochromatic wave, in which all quantities are proportional to $e^{i(kz-\omega t)}$, the expressions (4) and (4′) reduce to

$$\begin{aligned}[\mathbf{kE}]&=\frac{\omega}{c}\mu\mathbf{H},\\ [\mathbf{kH}]&=-\frac{\omega}{c}\varepsilon\mathbf{E}.\end{aligned} \qquad (5)^\dagger$$

It can be seen at once from these equations that if $\epsilon > 0$ and $\mu > 0$ then **E**, **H**, and **k** form a right-handed triplet of vectors, and if $\epsilon < 0$ and $\mu < 0$ they are a left-handed set.[1] If we introduce direction cosines for the vectors **E**, **H**, and **k** and denote them by α_i, β_i, and γ_i, respectively, then a wave propagated in a given medium will be characterized by the matrix [2]

$$G=\begin{pmatrix}\alpha_1&\alpha_2&\alpha_3\\ \beta_1&\beta_2&\beta_3\\ \gamma_1&\gamma_2&\gamma_3\end{pmatrix}. \qquad (6)$$

The determinant of this matrix is equal to +1 if the vectors **E**, **H**, and **k** are a right-handed set, and −1 if this set is left-handed. Denoting this determinant by p, we can say that p characterizes the "rightness" of the given medium. The medium is "right-handed" if p = +1 and "left-handed" if p = −1. The elements of the matrix (6) satisfy the relation

$$G_{ik}=pA_{ik}. \qquad (7)$$

Here A_{ik} is the algebraic complement of the element G_{ik}. Furthermore the elements of G are ortho-

*rot = curl.

†$[\mathbf{kE}] \equiv \mathbf{k}\times\mathbf{E}$.

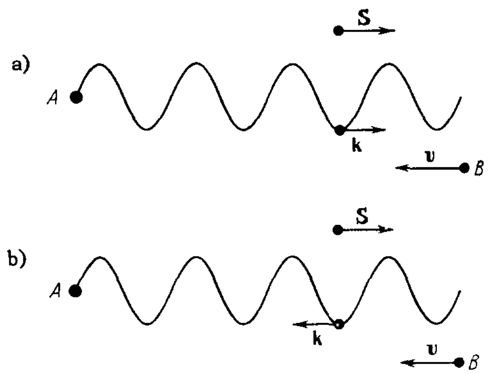

FIG. 1. a) Doppler effect in a right-handed substance; b) Doppler effect in a left-handed substance. The letter A represents the source of the radiation, the letter B the receiver.

normal. The energy flux carried by the wave is determined by the Poynting vector S, which is given by

$$\mathbf{S} = \frac{c}{4\pi}[\mathbf{EH}]. \tag{8}$$

According to (8) the vector S always forms a right-handed set with the vectors E and H. Accordingly, for right-handed substances S and **k** are in the same direction, and for left-handed substances they are in opposite directions.[3] Since the vector **k** is in the direction of the phase velocity, it is clear that left-handed substances are substances with a so-called negative group velocity, which occurs in particular in anisotropic substances or when there is spatial dispersion.[4] In what follows we shall for brevity use the term "left-handed substance," keeping in mind that this term is equivalent to the term "substance with negative group velocity." Let us now consider the consequences of the fact that in left-handed substances the phase velocity is opposite to the energy flux. First, in left-handed substances there will be a reversed Doppler effect.[1,3]

Indeed, suppose for example that a detector of radiation which is in a left-handed medium moves relative to a source which emits a frequency ω_0. In its motion the detector will pursue points of the wave which correspond to some definite phase, as is shown in Fig. 1. The frequency received by the detector will be smaller than ω_0, not larger as it would be in an ordinary (right-handed) medium. Using the quantity p for the medium in question, we can write the formula for the Doppler shift in the form

$$\omega = \omega_0\left(1 - p\frac{v}{u}\right). \tag{9}$$

Here the velocity v of the detector is regarded as positive when it is receding from the source. The velocity u of the energy flux is regarded as always positive.

The Vavilov-Cerenkov effect will also be reversed, just like the Doppler effect.[1,3] If a particle moves in a medium with speed v in a straight line (Fig. 2), it will emit according to the law $e^{i(k_z z + k_r r - \omega t)}$, and the wave vector of the radiation will be given by $k = k_z/\cos\theta$ and is in the general direction of the velocity v. The quantity k_r will be different in different media, in accordance with the expression

$$k_r = p\left|\sqrt{k^2 - k_z^2}\right|. \tag{10}$$

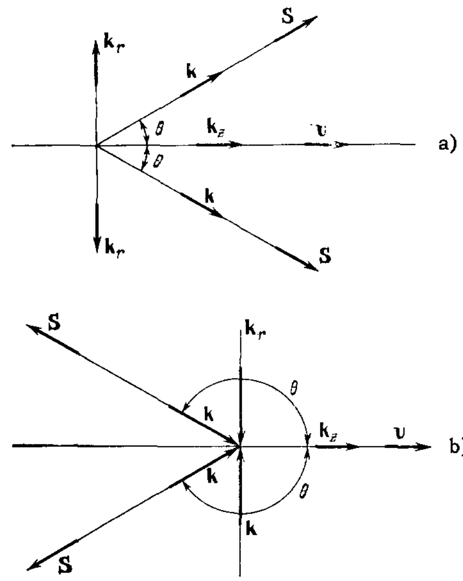

FIG. 2. a) The Vavilov-Cerenkov effect in a right-handed substance; b) The same effect in a left-handed substance.

This choice of the sign for the square root in (10) will assure that the energy moves away from the radiating particle to infinity. It is then clear that for left-handed media the vector k_r will be directed toward the trajectory of the particle, and the cone of the radiation will be directed backward relative to the motion of the particle. This corresponds to an obtuse angle θ between v and S. For a medium of either "rightness" this angle can be found from the expression

$$\cos\theta = p\left|\sqrt{\frac{c^2}{v^2 n^2}}\right|. \tag{11}$$

III. THE REFRACTION OF A RAY AT THE BOUNDARY BETWEEN TWO MEDIA WITH DIFFERENT RIGHTNESSES

In the passage of a ray of light from one medium into another the boundary conditions

$$E_{t_1} = E_{t_2}, \qquad H_{t_1} = H_{t_2}, \tag{12}$$

$$\varepsilon_1 E_{n_1} = \varepsilon_2 E_{n_2}, \qquad \mu_1 H_{n_1} = \mu_2 H_{n_2} \tag{13}$$

must be satisfied, independently of whether or not the media have the same rightness. It follows from (12) that the x and y components of the fields E and H in the refracted ray maintain their directions, inde-

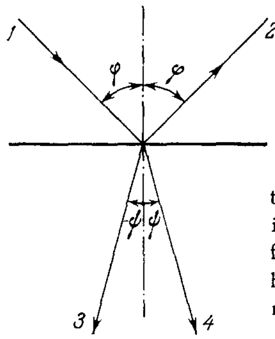

FIG. 3. Passage of a ray through the boundary between two media. 1 – incident ray; 2 – reflected ray; 3 – reflected ray if the second medium is left-handed; 4 – refracted ray if the second medium is right-handed.

pendently of the rightnesses of the two media. As for the z component, it keeps the same direction only if the two media are of the same rightness. If the rightnesses are different, the z components change sign. This corresponds to the fact that in passage into a medium of different rightness the vectors **E** and **H** not only change in magnitude owing to the difference in ϵ and μ but also undergo a reflection relative to the interface of the two media. The same thing happens to the vector **k** also. The simultaneous reflection of all three vectors corresponds precisely to a change of sign of the determinant G in (6). The path of the refracted ray produced as the result of such reflections is shown in Fig. 3. As we see, when the second medium is left-handed the refracted ray lies on the opposite side of the z axis from its position in the case of a right-handed second medium.[5] It must be noted that the direction of the reflected ray is always the same, independent of the rightnesses of the two media. It can be seen from Fig. 3 that the usual Snell's law

$$\frac{\sin\varphi}{\sin\psi} = n_{1,2} = \sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}} \tag{14}$$

has to be given a more precise form if the rightnesses of media 1 and 2 are different. The correct way to write the formula is now

$$\frac{\sin\varphi}{\sin\psi} = n_{1,2} = \frac{p_2}{p_1}\left|\sqrt{\frac{\varepsilon_2\mu_2}{\varepsilon_1\mu_1}}\right|. \tag{15}$$

Here p_1 and p_2 are the rightnesses of the first and second media. It is clear from (15) that the index of refraction of two media can be negative if the rightnesses of the media are different. In particular, the index of refraction of a left-handed medium relative to vacuum is negative.[1]

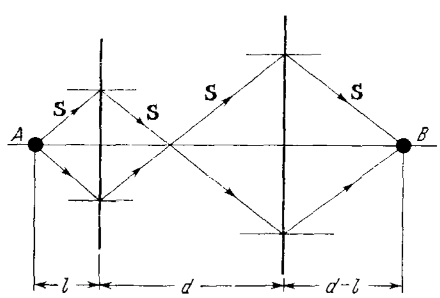

FIG. 4. Passage of rays of light through a plate of thickness d made of a left-handed substance. A – source of radiation; B – detector of radiation.

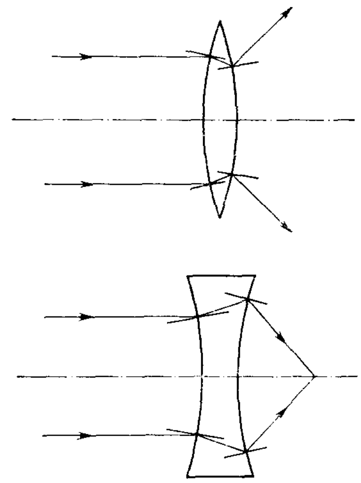

FIG. 5. Paths of rays through lenses made of left-handed substances, situated in vacuum.

Fresnel's formulas are commonly used to find the amplitudes of the reflected and refracted light.[2] These formulas involve the quantities ϵ, μ, n, φ, ψ. In order not to make mistakes one must always use the absolute values of these quantities in Fresnel's formulas.

An interesting case is that of a ray passing from a medium with $\epsilon_1 > 0$, $\mu_1 > 0$ into one with $\epsilon_2 = -\epsilon_1$, $\mu_2 = -\mu_1$. In this case the ray undergoes refraction at the interface between the two media, but there is no reflected ray. The use of left-handed substances would in principle allow the design of very unusual refracting systems. An example of such a system is a simple plate of thickness d made of a left-handed substance with n = −1 and situated in vacuum. It is shown in Fig. 4 that such a plate can focus at a point the radiation from a point source located at a distance $l < d$ from the plate. This is not a lens in the usual sense of the word, however, since it will not focus at a point a bundle of rays coming from infinity. As for actual lenses, the paths of rays through lenses made of a left-handed substance are shown in Fig. 5. It is seen that the convex and concave lenses have "changed places," since the convex lens has a diverging effect and the concave lens a converging effect.

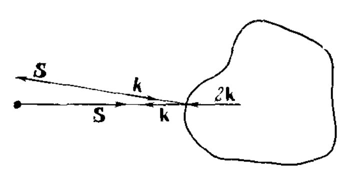

FIG. 6. Reflection of a ray propagated in a medium with $\epsilon < 0$ and $\mu < 0$ from an ideally reflecting body. The source of radiation is denoted by a heavy black point.

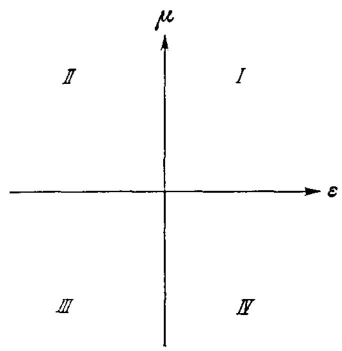

FIG. 7. $\epsilon - \mu$ diagram.

A monochromatic wave in a left-handed medium can be regarded as a stream of photons, each having a momentum $\mathbf{p} = \hbar\mathbf{k}$, with the vector $\mathbf{k}$ directed toward the source of radiation, not away from it as is the case in a right-handed medium. Therefore a beam of light propagated in a left-handed medium and incident on a reflecting body imparts to it a momentum $\mathbf{p} = 2N\hbar\mathbf{k}$ (N is the number of incident photons) directed toward the source of the radiation, as shown in Fig. 6. Owing to this the light pressure characteristic for ordinary (right-handed) substances is replaced in left-handed substances by a light tension or attraction.

These are some of the features of the electrodynamics of left-handed substances. Let us now consider the question of their physical realization. For this purpose we first examine what the values of ϵ and μ are that various substances may have.

IV. WHAT SORT OF VALUES OF ϵ AND μ ARE IN PRINCIPLE POSSIBLE?

Figure 7 shows a coordinate system in which values of ϵ and μ are marked off on the axes. We shall try to locate in it all known substances, at first confining ourselves to the case in which ϵ and μ are isotropic. Then the first quadrant contains the majority of isotropic dielectrics, for which ϵ and μ are positive. In the second quadrant ($\epsilon < 0$, $\mu > 0$) there will be plasmas, both gaseous plasmas [6] and solid-state plasmas.[7-9] In a plasma with no magnetic field the value of ϵ is given by

$$\varepsilon = 1 - \sum \frac{\omega_0^2}{\omega^2}, \tag{16}$$

where $\omega_0^2 = 4\pi Ne^2/m$, N being the concentration of the carriers, e their charge, and m their mass, and the summation is over all types of carriers. It is not hard to see that at small frequencies ϵ is smaller than zero. For $\epsilon > 0$ and $\mu > 0$ the value of n^2 given by (3) is negative, which leads to reflection of waves from such a medium. This fact is well confirmed by experiment, for example, in the ionosphere.

The third and fourth quadrants in Fig. 7 are unoccupied. So far there is not a single substance known with $\mu < 0$. As we shall see in what follows, this is not accidental.

Let us now go on to anisotropic substances. In this case the quantities ϵ and μ are tensors, and we cannot make use at once of a diagram like Fig. 7. In some substances, however, we can do this for waves propagated in particular directions. Gyrotropic substances are especially interesting in this respect. For gyrotropic substances the tensors ϵ_{ik} and μ_{ik} are of the forms

$$\varepsilon_{ik} = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \tag{17}$$

$$\mu_{ik} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{18}$$

A well known example of a gyrotropic substance is a plasma in a magnetic field, which is characterized by a tensor ϵ_{ik} of the form (17) and a scalar value of μ. If a plane circularly polarized transverse wave of the form $e^{i(kz - \omega t)}$ is propagated in such a plasma, with $\mathbf{k} \parallel z \parallel \mathbf{H}_0$, then n^2 is given by

$$n^2 = \mu(\varepsilon_1 \pm \varepsilon_2). \tag{19}$$

The sign $\pm$ corresponds to the two directions of polarization of the wave. If $|\epsilon_2| < |\epsilon_1|$ and $\epsilon_1 > 0$, two waves can be propagated in the plasma, but if $|\epsilon_2| > |\epsilon_1| = -\epsilon_1$, then only one wave is propagated, that for which $n^2 > 0$. In these cases the plasma must be placed in the first quadrant of Fig. 7 (μ is of the order of 1). As for the second wave in the case $|\epsilon_2| > |\epsilon_1| = -\epsilon_1$, it cannot be propagated because for it $\epsilon < 0$, which by (19) leads to an imaginary value of n. In this case the plasma belongs in the second quadrant in Fig. 7.

Another example of gyrotropic substances is various magnetic materials, in which, in contrast to the plasma, it is μ and not ϵ that is a tensor. For these materials the analog of (19) is

$$n^2 = \varepsilon(\mu_1 \pm \mu_2). \tag{20}$$

Here also there can in principle be a situation in which $|\mu_2| > |\mu_1| = -\mu_1$, and this case corresponds to the fourth quadrant in Fig. 7.

Quite recently there have begun to be intensive studies of gyrotropic substances in which both ϵ and μ are tensors.[10-15] Examples of such substances are pure ferromagnetic metals and semiconductors. For such substances the index of refraction of a circularly polarized wave travelling along the field is given by

$$n^2 = (\varepsilon_1 \pm \varepsilon_2)(\mu_1 \pm \mu_2), \tag{21}$$

and it is easy to see that in this case the effective electric and magnetic permeabilities can both be less than zero, while n^2 remains positive and the wave will be propagated.[3,13,15] Such substances occupy the third and last quadrant of Fig. 7. Accordingly we see that we must look for substances with $\epsilon < 0$ and

$\mu < 0$ primarily among gyrotropic media. Furthermore it is obvious thta negative values of ϵ and μ in gyrotropic substances can be realized only for those waves that are propagated along the magnetic field. For other directions of propagation ϵ and μ can no longer be regarded as scalars. Nevertheless, for a certain range of angles between H and k the vectors S and k will make an angle close to 180° and will qualitatively satisfy all of the laws which are characteristic for left-handed substances.

In concluding this section we note that simultaneous negative values of ϵ and μ can be realized only when there is frequency dispersion. In fact, it can be seen from the relation

$$W = \varepsilon E^2 + \mu H^2 \tag{22}$$

that when there is no frequency dispersion nor absorption we cannot have $\epsilon < 0$ and $\mu < 0$, since in that case the total energy would be negative. When there is frequency dispersion, however, the relation (22) must be replaced by

$$W = \frac{\partial(\varepsilon\omega)}{\partial\omega} E^2 + \frac{\partial(\mu\omega)}{\partial\omega} H^2. \tag{23}$$

In order for the energy W given by (23) to be positive it is required that

$$\frac{\partial(\varepsilon\omega)}{\partial\omega} > 0, \qquad \frac{\partial(\mu\omega)}{\partial\omega} > 0. \tag{24}$$

These inequalities do not in general mean that ϵ and μ cannot be simultaneously negative, but for them to hold it is necessary that ϵ and μ depend on the frequency.

It is appropriate to emphasize here that the conclusion that there is a light attraction in left-handed substances, which was obtained at the end of Sec. III from quantum arguments, can also be obtained in a purely classical way. To do this we must use the classical expression for the momentum of the field [17]

$$\mathbf{p} = \frac{\varepsilon\mu}{c^2}\mathbf{S} + \frac{\mathbf{k}}{2}\left(\frac{\partial\varepsilon}{\partial\omega}E^2 + \frac{\partial\mu}{\partial\omega}H^2\right), \tag{25}$$

the relations (23) and (24), and also the connection between the Poynting vector $\mathbf{S}$ and the group velocity $\mathbf{v}_g = \partial\omega/\partial\mathbf{k}$,

$$\mathbf{S} = W \cdot \mathbf{v}_{\mathrm{gr}}. \tag{26}$$

Combining the expressions (23)—(26), we get

$$\mathbf{p} = \frac{W}{v_{\mathrm{ph}}} = \frac{W}{\omega}\cdot\mathbf{k}. \tag{27}$$

It follows from this that in left-handed substances the field momentum $\mathbf{p}$ is directed opposite to the Poynting vector $\mathbf{S}$.

V. GYROTROPIC SUBSTANCES POSSESSING PLASMA AND MAGNETIC PROPERTIES

It is characteristic of gyrotropic media of this kind that, first, they contain sufficiently mobile carriers forming an electron-hole plasma, and, second, that there exists a system of interacting spins which provide a large magnetic susceptibility. This assures the simultaneous propagation of spin and plasma waves, and naturally there is an interaction between them. If this interaction is strong enough, the waves propagated in such a substance are of a mixed, spin-plasma, character. In this case the values of ϵ and μ are of the following form [15]:

$$\varepsilon = 1 - \sum \frac{\omega_0^2}{\omega(\omega \pm \Omega)}, \qquad \mu = 1 + \frac{\omega_s}{\eta k^2 + \Omega' \pm \omega}, \tag{28}$$

$$n^2 = \varepsilon\mu. \tag{29}$$

Here ω is the frequency, $\omega_0^2 = 4\pi Ne^2/m$ is the square of the plasma frequency, N is the concentration and m the mass of the carriers; the summation is taken over all types of carriers; $\Omega = eB/mc$, and

$$B = H_0 + 4\pi M_s, \quad \omega_s = \frac{ge}{2m_0c}\cdot 4\pi M_s, \quad \Omega' = \frac{ge}{2m_0c}H_0, \quad \eta = \frac{ge}{m_0}\frac{A}{M_s},$$

where H_0 is the external field, M_S is the saturation magnetization, A is the exchange-interaction constant, m_0 is the mass of the electron, and e is its charge.

In [15] a graphical analysis has been made of Eq. (29), with ϵ and μ of the form (28); it was shown that for certain relations between the parameters that appear in (28) it is possible to have wave propagation with $\epsilon < 0$ and $\mu < 0$ in conducting ferromagnetic substances. As examples of such conducting ferromagnetic substances there are in the first place ferromagnetic metals, for example nickel. There is already a communication [18] on the observation of coupled spin-plasma waves in this metal, but it is not clear just what were the values of ϵ and μ in this case, and in particular whether or not they were negative. As for semiconductors which have magnetic properties, several such compounds have recently been indicated, [19-21] in particular $CuFeS_2$, UTe_2, InSb-FeSb, and others. At present the mobility of the carriers in these materials is still very small, and does not allow the observation of weakly damped waves in them. Constant technological progress gives the hope, however, that such materials with good mobility will be produced, and then experiments with substances in which ϵ and μ are less than zero will surely become practicable. It must be noted that such experiments with gyrotropic substances can confirm only some of the properties of left-handed substances which we have expounded. For example, it will be very difficult to make experiments on the refraction of waves, since for rays propagated in left-handed substances at an angle with the external magnetic field the relation (21) is no longer valid. As has already been stated, it will be approximately correct only for small angles between the field and the vector $\mathbf{k}$, and all experiments must be arranged so as not to go beyond such angles. In view of this difficulty, it would be very desirable to have an isotropic left-handed substance. Unfortunately, as has already been said, we do not know of even a single substance which

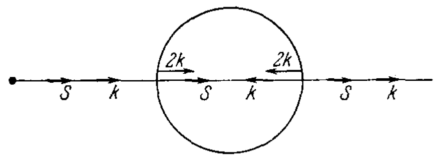

FIG. 8. Passage of a ray through a sphere with $\epsilon < 0$ and $\mu < 0$ situated in vacuum. The source of radiation is indicated with a heavy black point.

could be isotropic and have $\mu < 0$. This is true because the sources of magnetic field are not charges, but dipoles. If the magnetic field, like the electric field, could arise from charges, a gas of such charges would have a magnetic permeability given, in analogy with (16), by the formula

$$\mu = 1 - \frac{\omega_M^2}{\omega^2}. \qquad (30)$$

Here $\omega_M^2 = 1 - 4\pi N g_1^2/m_1$, where N_1 is the concentration of the charges, g_1 is their magnitude, and m_1 their mass. The hypothesis of the existence of such charges was stated by Dirac as early as 1931 [22]; there have been many papers on the possible properties of this charge (the Dirac monopole), for example a review article,[23] and also [24–26]. So far, however, attempts to observe it have given no result.[23–27] If the monopole were found, then a mixture of ordinary plasma and a gas of monopoles would have a value of μ given by (30), and the μ of the mixture would be given by

$$\varepsilon = 1 - \frac{\omega_0^2}{\omega^2}. \qquad (31)$$

At sufficiently low frequencies such a mixture would be a left-handed isotropic substance.

Let us imagine [28] that such a mixture uniformly fills a sufficiently large spherical region in space, and that outside this region there is vacuum. For simplicity suppose that inside the sphere $\omega_M = \omega_0$, and that radiation is incident from outside the sphere at a frequency such that inside the sphere $\epsilon = \mu = -1$. Then there will be no refraction of an incident ray at the points where it enters and leaves the sphere. At the same time there will be imparted to the sphere at each of these points a momentum directed toward its center, as is shown in Fig. 8. If we imagine that the sphere is irradiated from all sides, then it will experience an isotropic compression. Accordingly, if Dirac monopoles were uniformly distributed in space together with an ionized gas, under the influence of radiation this mixture would be continuously concentrated in certain local regions. This argument is of course very approximate, and does not take into account many other factors, but it may possibly help to explain the lack of success in the experimental observation of Dirac monopoles.

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