

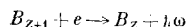
DIELECTRONIC RECOMBINATION

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INTRODUCTION

IN calculations of the ionization equilibrium in a plasma, account is usually taken of two possible recombination mechanisms: photorecombination of an ion BZ_{+1}



and nonradiative three-particle recombination



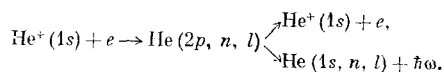
In the latter process, the second electron is needed to carry away the excess energy, since the transfer of this energy to the nucleus is incompatible with the momentum and energy conservation laws. In a low-density plasma, three-particle recombination plays practically no role. An estimate shows that photorecombination prevails at an energy density

$$N_e < \left(\frac{\alpha}{a_0}\right)^3 Z^4 \frac{kT}{Ry} \sim 10^{18} Z^4 \frac{kT}{Ry} \text{ cm}^{-3},$$

where T —plasma temperature, $Ry = 13.6 \text{ eV}$ —ionization energy of the hydrogen atom, and Z —ion charge. To be sure, this is an overestimate, since it takes no account of the stepwise three-particle recombination. Nonetheless, three-particle recombination apparently plays no role at $N_e < 10^{13} \text{ cm}^{-3}$.

The probability of radiative recombination in collisions between an electron and an ion is relatively small ($\sim \alpha^3 = (e^2/\hbar c)^3 \sim 10^{-6}$), in view of the weakness of the electromagnetic interaction.

In addition to the two indicated processes, resonant capture of an electron at a high level (n, l) is possible, with simultaneous excitation of one of the electrons of the ion BZ_{+1} . If the electron energy (within the limits of the level width) equals the excitation energy minus the binding energy at the (n, l) level, then the capture process takes place without radiation. The two-electron excited state produced thereby is unstable. It decays either via auto-ionization, or as a result of radiative transition into a stable excited state BZ , which lies below its ionization limit*. In the first case, the electron returns to the continuous spectrum. The second variant leads to a recombination of the ion BZ_{+1} —this indeed is dielectronic recombination (henceforth d.r.). Figure 1a shows the elementary scheme of the process. Figure 1b shows the same process more rigorously, as a transition in a two-electron system. An example is the reaction



*Here and below a state is defined arbitrarily as "stable" if it is not subject to auto-ionization decay, i.e., it lies below the ionization boundary. A radiative transition into the ground state is of course possible in this case.

This process has been known for quite a long time^[1], but its role was underestimated. The importance of d.r. in the establishment of ionization equilibrium in a plasma was first pointed out by Burgess^[2], who presented more accurate quantitative estimates. In^[2-4] they calculated the rates of the d.r. in a number of concrete cases. Subsequently the role of d.r. in processes occurring in certain astrophysical objects were discussed in a number of papers, for example^[5-10]. In^[5], at least, there are many inaccuracies, which in some cases lead to incorrect conclusions. It has become clear recently that the role of d.r. is not confined to the influence on the ionization equilibrium, and can be quite appreciable in the population of the high-excited levels^[8,11], thus opening up new wide possibilities in many astrophysical applications.

We also analyze in the article processes that decrease the role of d.r. under concrete astrophysical conditions as a result of ionization from high level by background radiation^[12] and by electron impact.

The auto-ionization states, which are an intermediate stage in the d.r. process, play an important role in the interpretation of the experiments on scattering of electrons by atoms and ions. They appear as sharp resonances in the energy dependence of the elastic-scattering cross section. The theory of resonant scattering has been extensively developed in recent years. The position of the resonances makes it possible to determine the position and the width of the auto-ionization levels, which are quite difficult to observe by optical means. Nonetheless, we shall not discuss here the connection between the d.r. and resonant scattering. The point is that, as will be shown later, the d.r. proceeds mainly through high auto-ionization levels with a large value of the principal quantum number n . In theory and experiments on resonant scattering, to the contrary, the main interest is focused on states with small n . This difference is fundamental. The probability of electron capture at the n -th level is proportional to $1/n^3$ and is particularly large when n is small. However, an important role is played in the d.r. phenomenon not only by the capture of the electron, but by maintaining it at a given level for a time $\sim 10^{-8} Z^{-4} \text{ sec}$, needed for the radiative transition into the stable state. Since the probability of the auto-ionization decay is also proportional to $1/n^3$, the contribution of the n -th level to the total d.r. coefficient does not depend on n at all values of $n < n_S$ ($n_S \gg 1$, see Ch. II). Thus, the role of the lower levels "becomes dissolved" in the large number of levels with large n .

We note, in conclusion, one more circumstance. Resonant scattering is possible also from neutral atoms as a result of unstable excited states of the negative ion. But the d.r. of a neutral atom with formation of a negative ion is impossible, since process should

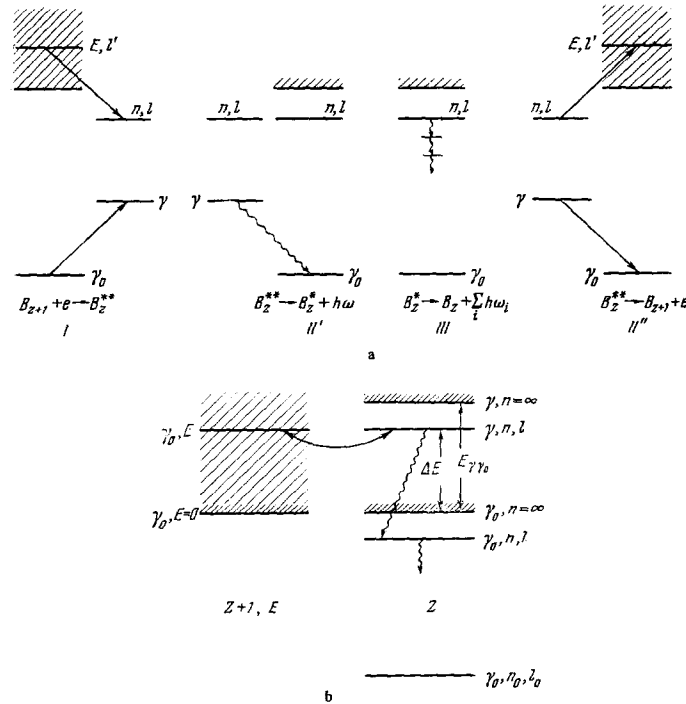


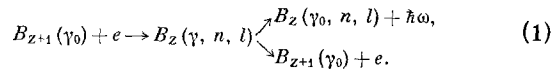
FIG. 1. Scheme of the dielectronic recombination process. a) Elementary scheme of three stages of recombination; b) more rigorous scheme in terms of a unified system $B_{Z+1} + e \rightarrow B_Z$.

terminate with an excited stable state of the negative ion, which is optically coupled to the ground state. Apparently there are no such states (see note added in proof at the end of the article).

I

1. Coefficient of Dielectronic Recombination

Let us consider the process of d.r. of an ion B_{Z+1} (in a state γ_0) via an intermediate auto-ionization level (γ, n, l) of the ion B_Z (see Fig. 1)



Here and throughout Z denotes the effective charge of the atomic residue, i.e., it exceeds the multiplicity of the ion by unity: $B_Z \equiv B^{(Z-1)+}$. We note that Z coincides with the spectroscopic symbol of the ion (for example, He II with $Z = 2$).

With practically no loss of generality we can assume that γ_0 is the ground state of the ion B_{Z+1} . The excited ion $B_Z(\gamma_0, n, l)$ produced as a result of the recombination can subsequently either go to the ground state (γ_0, n_0, l_0) emitting one or several quanta, or else again become ionized by an external particle or a photon. In this section we confine ourselves to consideration of the first possibility, which is realized at a sufficiently low plasma and radiation density. The possibility of repeated ionization will be considered in Ch. II.

To calculate the rate of the d.r. it is necessary to determine the population $N_{\gamma, n, l}$ of the auto-ionization state (γ, n, l) of the ion B_Z . To this end, we write

down the balance equation for the reaction (1)

$$N_{Z+1} N_e F(\Delta E) v \sigma_d(\gamma n l) \delta E = N_{\gamma, n, l} (W + A), \quad (2)$$

where $F(E)$ —electron energy distribution function, assumed to be Maxwellian, ΔE —energy of the level (γ, n, l) relative to the ionization limit B_Z (see Fig. 1), $\sigma_d(\gamma, n, l)$ —cross section of electron capture at the (γ, n, l) level, and W and A —probabilities of auto-ionization and radiative decays of the (γ, n, l) level. If we put $A = 0$, then (2) should satisfy the Saha distribution

$$\left(\frac{N_{Z+1} N_e}{N_{\gamma, n, l}} \right)_{\text{equil}} = S_{n, l} \equiv \frac{2g(\gamma_0)}{g(\gamma, n, l)} \left(\frac{m k T}{2\pi \hbar^2} \right)^{3/2} e^{\Delta E / k T}, \quad (3)$$

where $g(\gamma_0) = g(\gamma, n, l) = g(\gamma)(2l+1)$ —statistical weight of the corresponding states B_{Z+1} and B_Z . The argument of the exponential has a plus sign, since the (γ, n, l) level lies above the ionization limit. From (2) and (3) follows a relation between σ_d and W , namely:

$$W = S F v \sigma_d(\gamma, n, l) \delta E = \frac{g(\gamma_0)}{g(\gamma)} \frac{m^2 v^2}{\pi^2 \hbar^3} \frac{\sigma_d \delta E}{2l+1}. \quad (4)$$

For the rate of d.r.

$$\kappa_d = \frac{1}{N_{Z+1} N_e} \sum_{\gamma, n, l} N_{\gamma, n, l} A,$$

we get, using (2) and (4),

$$\kappa_d = \sum \frac{1}{S_{n, l}} \frac{W A}{W + A} = \sum_{\gamma, n, l} \left(\frac{2\pi \hbar^2}{m k T} \right)^{3/2} \frac{g(\gamma)}{g(\gamma_0)} \frac{(2l+1) A}{1 + (A/W)} e^{-\frac{\Delta E}{k T}}. \quad (5)$$

Capture of the electron occurs principally from the level with the higher value of n (see below). The electron at the (n, l) level has practically no influence on the internal electron in the state γ or γ_0 , so that the quantities pertaining to B_Z can be replaced by the

corresponding quantities for the ion BZ_{+1} :

$$\Delta E = E_{\gamma\gamma_0} - \frac{Z^2 Ry}{n^2} \cong E_{\gamma\gamma_0}, \quad A(\gamma, n, l) = A_\gamma = \sum_{\gamma' < \gamma} A_{\gamma\gamma'}, \quad (6)$$

where $E_{\gamma\gamma_0}$ —distance between the levels γ and γ_0 of the ion BZ , $Ry = 13.6$ eV—half of the atomic energy unit, and $1/A_\gamma$ —lifetime of the level γ . In most cases A_γ can be replaced with good accuracy by $A_{\gamma\gamma_0}$. Finally, the auto-ionization probability W can be expressed in terms of the partial cross section $\sigma_{\gamma_0\gamma}(l)$ for the excitation of the level γ by electron impact near threshold. Indeed, the inverse process—capture of an electron on the (n, l) orbit—corresponds to excitation of the ion BZ_{+1} by an electron with an energy somewhat below threshold.

As is well known, the partial cross section $\sigma_{\gamma_0\gamma}(l)$ for the excitation of the ion at threshold is finite (unlike the cross sections for the excitation of neutral atoms, which vanish at the threshold). Extrapolating $\sigma_{\gamma_0\gamma}(l)$ to the region below threshold, we obtain for the mean value of the capture cross section

$$\sigma_{\gamma_0\gamma} dE = \overline{\sigma_d} dE dn,$$

if dn corresponds to the energy interval dE . Consequently (assuming l to be specified)

$$\overline{\sigma_d}(\gamma, n, l) \delta E = \sigma_{\gamma_0\gamma}(l) \frac{dE}{dn} = \frac{2Z^2 Ry}{n^3} \sigma_{\gamma_0\gamma}(l). \quad (7)$$

Substituting (7) in (4) we obtain an expression for the auto-ionization probability

$$W = W_{\gamma, n, l} = \frac{Z^2 \Delta E g(\gamma_0)}{\pi \hbar n^3 g(\gamma)} \frac{\sigma_{\gamma_0\gamma}(l)}{\pi a_0^2 (2l+1)}. \quad (4')$$

In Appendix I we give a direct quantum-mechanical proof of relation (4') in first order of perturbation theory. A more detailed exposition of the properties of the Coulomb cross section at threshold and of the limiting transition, leading to formula (7), is contained in Appendix II, which was written by P. Paradoksov.*

Using (4'), (5), and (6) and assuming for the purpose of simplifying the future formulas that $A_\gamma = A_{\gamma\gamma_0}$, we can obtain the following expression for the d.r. rate:

$$\alpha_d = \sum_\gamma c(\gamma) \left(\frac{E_{\gamma\gamma_0}}{kT} \right)^{3/2} \exp\left(-\frac{E_{\gamma\gamma_0}}{kT}\right), \quad (8)$$

where

$$c(\gamma) = \frac{4\pi^{3/2} \hbar a_0 \alpha^3 Z f_{\gamma_0\gamma}}{m n_1} \sum_{n > n_1} \sum_{l=0}^{n-1} \frac{2l+1}{(1+(n/n_s))^3} \exp\left(\frac{Z^2 Ry}{n^2 kT}\right), \quad (9)$$

$$n_1 = \left(\frac{Z^2 Ry}{E_{\gamma\gamma_0}} \right)^{1/2}, \quad n_s \equiv n_s(l) = \frac{1}{\alpha} \left[\frac{n_1^2 \sigma_{\gamma_0\gamma}(l)}{\pi^2 a_0^2 (2l+1) f_{\gamma_0\gamma}} \right]^{1/3}. \quad (10)$$

Here $\alpha = e^2/\hbar c = 1/137$, $a_0 = \hbar^2/me^2 = 0.53 \times 10^{-8}$ cm, $f_{\gamma_0\gamma}$ —oscillator strength of the $\gamma_0\gamma$ -transition in the BZ_{+1} ion, and $(n_1 + 1)$ is approximately equal to the principal quantum number of the lowest auto-ionization level. The physical meaning of the number n_s will be discussed later.

The cross section $\sigma(l)$ is practically independent of n , but decreases very rapidly with increasing l (Fig. 2). It can therefore be assumed that the upper limit of the summation over l does not depend on n , and it is possible to interchange the order of the sum-

*The authors are grateful to P. Paradoksov for proposing to include the previously unpublished notes in the present review.

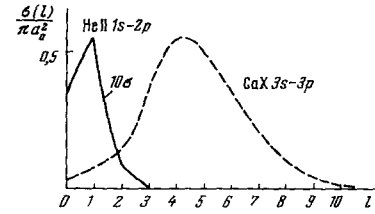


FIG. 2. Partial cross sections for excitation and resonant transition in He II and Ca X at threshold vs. l .

mation in (9). Replacing, further, the summation with respect to n by integration and omitting the factor $\exp(Z^2 Ry/n^2)$, which is close to unity, we can represent $c(\gamma)$ with good accuracy in the form

$$c(\gamma) = 6.55 \cdot 10^{-14} \frac{Z}{n_1} f_{\gamma_0\gamma} \sum_l \frac{(2l+1) n_s}{1+2.4 \left(\frac{n_1+1}{n_s} \right)^2} (\text{cm}^3 \text{sec}^{-1}). \quad (11)$$

$\sigma_{\gamma_0\gamma}(l)$ decrease in proportion to Z^{-4} as $Z \rightarrow \infty$, and $n_s \ll 1$ for all l . In this case we can carry out the summation with respect to n and l in (9):

$$c(\gamma) = 2.5 \cdot 10^8 \left(\frac{E_{\gamma_0\gamma}}{Ry} \right)^{1/2} \sigma_{\gamma_0\gamma} (\text{cm}^3 \text{sec}^{-1}), \quad Z \rightarrow \infty, \quad (11a)$$

where $\sum_l \sigma_{\gamma_0\gamma}(l) = \sigma_{\gamma_0\gamma}$ —total cross section of the $\gamma_0 \rightarrow \gamma$ transition and we neglect the difference between n_1 and $n_1 + 1$.

2. Discussion of Formulas

We now stop to discuss the properties of the obtained formulas in greater detail.

1) The population of the auto-ionization levels (γ, n, l) is determined, in accordance with (2) and (4), by the ratio $A/W = (n, n_s)^3$. Therefore all the levels are subdivided in natural fashion into two groups. For the levels with $n < n_s$ we have $W \gg A$ and the radiative decay does not influence the population. These levels are in thermodynamic equilibrium with the continuous spectrum:

$$N_{\gamma, n, l} = N_{z+1} N_e S_{n, l}^{-1}, \quad n < n_s. \quad (12)$$

For the levels with $n > n_s$ we have $W \ll A$, and each capture of the electron is accompanied by recombination. The population at these levels is

$$N_{\gamma, n, l} = N_{z+1} N_e \frac{F_{\nu} \sigma_d(\gamma, n, l) \delta E}{A} = N_{z+1} N_e \frac{Z^2 Ry F_{\nu} \sigma_{\gamma_0\gamma}(l)}{2An^3}, \quad (13)$$

i.e., it decreases rapidly with increasing n .

It must be emphasized that the n_s is proportional to $[\sigma_{\gamma_0\gamma}(l)]^{1/3}$ and depends strongly on l . By way of illustration, Fig. 2 shows the $\sigma(l)$ dependence for two typical cases: the transition $1s - 2p$ in He II (large $E_{\gamma\gamma_0}$, $n_1 \sim 1$) and the transition $3s - 3p$ in Ca X (small $E_{\gamma\gamma_0}$, $n_1 \gg 1$). At sufficiently large l we have $n_s < n_1$, i.e., there are no levels that are in thermodynamic equilibrium with the continuous spectrum.

2) At small values of l , corresponding to the maximum values of $\sigma(l)$, we can put $\sigma(l) \sim \pi a_0^2 f_{\gamma_0\gamma} (Ry/E_{\gamma\gamma_0})^p$. For estimates it is usually possible to put $p = 2$, i.e.,

$$n_s \cong \frac{n_1^2}{\alpha Z} \gg n_1. \quad (14)$$

Actually the main contribution to $c(\gamma)$ is made by those l for which the inequality (14) is satisfied. It is seen from (9) that in this case the principal role is played by the auto-ionization levels with $n < n_S$, which are in thermodynamic equilibrium with the continuous spectrum. Inasmuch as $n_S \gg 1$ and all the levels with $n < n_S$ make the same contribution, we can henceforth put that only levels with $n \gg 1$ are significant.

3) In Secs. 1–2 we referred essentially to an external electron captured from the free state by a highly-excited level (n, l) of the B_Z ion. Let us now consider $\gamma_0 - \gamma$ transitions of an internal electron—the optical electron of the B_{Z+1} ion. In the sum over γ in (8) it is sufficient, as a rule, to take into account only one or two of the lowest excited configurations, which are optically coupled with the ground state (resonant states)*.

It is easy to show that if the values of $E_{\gamma\gamma_0}$ do not differ very strongly for the individual terms of the given electronic configuration, then the splitting into terms can be disregarded completely and the electronic configuration can be considered as one state. For this purpose it is sufficient to show that $c(\gamma)$ is additive over the terms. The quantity n_S in (10) is a function of the ratio $\sigma_{\gamma_0\gamma}/f_{\gamma_0\gamma}$ only, which does not depend on the term at equal values of $E_{\gamma\gamma_0}$. Consequently the entire dependence is concentrated in the factor $f_{\gamma_0\gamma}$ in (9). But the oscillator strength, as is well known, is additive in the upper levels, thus proving the additivity of $c(\gamma)$. As regards the splitting of the ground state γ_0 , it is possible to use the following general relation: let Γ_0 and Γ be the indices of the terms of the electronic configurations γ_0 and γ , then (see, for example^[13])

$$f_{\Gamma_0, \gamma} = \sum_{\Gamma} f_{\Gamma_0 \Gamma} = f_{\gamma_0 \gamma}$$

and consequently, in (8)–(9) we have

$$c_{\Gamma_0}(\gamma) = c_{\gamma_0}(\gamma). \quad (15)$$

It is obvious that all the foregoing pertains to an equal degree not only to the terms but also to the components of the fine structure, etc. Therefore the indices γ_0 and γ below pertain throughout to the electronic configurations of the ion B_{Z+1} .

The magnitude of the d.r. coefficient depends significantly on the ratio $E_{\gamma\gamma_0}/kT$, owing to the presence of the exponential factor in (8). If the $\gamma_0 - \gamma$ transition takes place with a change of the principal quantum number of the optical electron, then $E_{\gamma\gamma_0} \sim |E_{\gamma_0}| > kT$. Then the d.r. rate is small. For ions of this kind (an example is the $\text{He II} \rightarrow \text{He I}$ recombination), the d.r. plays a role only at such high temperatures that the concentration of this ion in the plasma is insignificant.

In those cases when the optical electron in the resonant state γ has the same principal quantum number as in γ_0 , we have $E_{\gamma\gamma_0} \ll |E_{\gamma_0}|$ and the exponen-

tial factor in (8) plays no role. At the same time, for such transitions the cross section is very large (see formula (14)). Therefore κ_d turns out to be very large. We note that in ions with $Z \geq 2$ an important role can be played by transitions from the filled subshell, for example $2s^2 2p^q \rightarrow 2s 2p^q$.

3. Comparison with Triple Recombination and Photorecombination

We now compare the efficiencies of different types of recombination.

a) Photorecombination $B_{Z+1} + e \rightarrow B_Z + \hbar\omega$. The frequency of this process in a plasma $N_e \langle v\sigma_\nu \rangle$ is of the order of $N_e \alpha^3 (a_0 \hbar/m)$, i.e., it is proportional to the first power of the density. Since this process is connected with emission of a quantum, the expression for its rate includes a small factor $\alpha^3 \cong 10^{-6}$. The photorecombination occurs in the ground and in the lower states of the ion B_Z , or more accurately in those states for which the energy $|E_n| > kT$.

b) Three-particle (nonradiative) recombination $B_{Z+1} + 2e \rightarrow B_Z + e$. The frequency of this process in a plasma $N_e^2 \langle v\sigma_\nu \rangle$ is proportional to the square of the electron density, and is much smaller than the photorecombination frequency at sufficiently low density. Estimates show that when $N_e < 10^{13} \text{ cm}^{-3}$ one can neglect as a rule the triple recombination in calculations of the ionization equilibrium. To be sure, it sometimes must be taken into account in estimates of the populations of very highly excited levels.

c) The frequency of the d.r. process (1) is $N_e \kappa_d$. Using (8), (9), and (14) we find that, in order of magnitude,

$$N_e \kappa_d \sim N_e \alpha^2 \frac{a_0 \hbar}{m} e^{-E_{\gamma\gamma_0}/kT}.$$

Just as in photorecombination, the frequency of the d.r. is proportional to the first power of the density. However, $N_e \kappa_d$ contains in lieu of the factor α^3 only α^2 , i.e., it can be smaller by two orders of magnitude than the photorecombination rate. Of course, it is necessary for this purpose that the exponential factor be of the order of unity, i.e., that $E_{\gamma\gamma_0} \ll kT$. Unlike photorecombination, the d.r. occurs essentially at high levels with $n \sim n_S$.

Thus, in a low-density plasma the total recombination frequency is $N_e [\langle v\sigma_\nu \rangle + \kappa_d]$. The dependence of κ_d on the temperature is given by formula (8). We can indicate for $\langle v\sigma_\nu \rangle$ only the asymptotic behavior at $kT \ll E_i$ and $kT \gg E_t$ (E_i —ionization energy):

$$\left. \begin{aligned} kT \ll E_i, \quad \langle v\sigma_\nu \rangle \sim (E_i/kT)^{1/2}, \quad \kappa_d \sim e^{-E_{\gamma\gamma_0}/kT}, \\ kT \gg E_i, \quad \langle v\sigma_\nu \rangle \sim (E_i/kT)^{3/2}, \quad \kappa_d \sim (E_{\gamma\gamma_0}/kT)^{3/2}. \end{aligned} \right\} \quad (16)$$

4. Influence of Dielectronic Recombination on the Level Populations and Plasma Radiation

1) As a result of the d.r., a shift of the ionization equilibrium takes place in the plasma. The additional recombination process obviously leads to a decrease of the degree of ionization, i.e., all the ionization curves shift towards larger temperatures. Therefore, failure to take d.r. into account has led in some papers to underestimates of the temperature obtained as a result of a spectroscopic measurements.

*A direct two-electron radiative transition to the ground state is possible from unstable excitations of the levels of a negative ion. However, the probability of such a transition is small (see note added in proof at the end of the article). We note, in addition, that in view of the absence of a Coulomb field the number of purely unstable levels of the negative ion is small.

In addition to the shift of the ionization equilibrium, the d.r. can, generally speaking, exert also a direct influence on the populations of the excited levels and, consequently, on the intensity of the radiation. Let us first consider in this connection the radiation in the $\gamma \rightarrow \gamma_0$ resonance line of the ion BZ_{+1} . The line intensity due to the direct excitation of the $\gamma_0 - \gamma$ transition by electron impact is $N_e \hbar \omega \langle v \sigma_{\gamma_0 \gamma} \rangle$, where

$$\langle v \sigma_{\gamma_0 \gamma} \rangle = \int_{E_{\gamma\gamma_0}}^{\infty} F(E) v \sigma_{\gamma_0 \gamma} dE \sim F(E_{\gamma\gamma_0}) v \sigma_{\gamma_0 \gamma} kT. \quad (17)$$

Each act of d.r. is accompanied by emission of the same quantum, albeit with a slight frequency shift due to the presence of the electron at the high level (n, l) .

The increase of the intensity is obviously equal to $N_e \hbar \omega \kappa_d(\gamma)$. For comparison with (17), we express $\kappa_d(\gamma)$ with the aid of (4), (5), and (7) in the form

$$\begin{aligned} \kappa_d(\gamma) &= \sum_{n, l} F(E) v \sigma_d(\gamma, n, l) \delta E [1 + (W/A)]^{-1} \\ &= \int_{E_{\gamma\gamma_0 - Z^2 Ry/n_s^2}}^{E_{\gamma\gamma_0}} F(E) v \sigma_{\gamma_0 \gamma} dE \\ &+ \int_{E_{\gamma\gamma_0 - Z^2 Ry/n_s^2}}^{E_{\gamma\gamma_0 - Z^2 Ry/n_s^2}} F(E) v \sum_l \sigma_{\gamma_0 \gamma}(l) [1 + (W/A)]^{-1} dE. \quad (18) \end{aligned}$$

We make use here of the fact that at energies corresponding to $n > n_s$ we have $W \ll A$. It is easy to see now that $\kappa_d(\gamma) \ll \langle v \sigma_{\gamma_0 \gamma} \rangle$. Indeed, the first term in (18) is small as a result of the small region of integration: $Z^2 Ry/n_s^2 \ll kT$. In the second term, the region of integration is $\sim kT$,* and the factor $[1 + (W/A)]^{-1} \ll 1$, since $n < n_s$. Thus, an increase of the intensity of the $\gamma \rightarrow \gamma_0$ resonance line, and consequently the radiation loss due to the d.r., is as a rule insignificant.

Of course, as already noted in the beginning of the section, the shift of the ionization equilibrium can greatly influence the intensity of the resonance lines of the ions.

In estimates of the thermal balance of the intergalactic medium, an important role is played by He, the number of atoms of which amounts to approximately 10% of the H atoms. The estimates presented above show that the d.r. does not influence directly the intensity of the He II resonance line. The large value $E_{\gamma\gamma_0} = 40$ eV denotes that the d.r. can affect the ratio He I/He II only at very high temperature $T \gtrsim 10^5$ °K. At such temperatures, the number of He II ions is determined by the equilibrium with the hydrogenlike ion He III, for which d.r. is impossible (see Fig. 4 below). Thus, the d.r. has likewise no effect on the ionization equilibrium of He II, and consequently does not influence the radiation of He II. The possibility of investigating the spectra of quasars in the far ultraviolet depends on the optical thickness of the intergalactic gas in the lines H I and He I. When $T \gtrsim 10^5$ °K a certain increase of the optical thickness in the He I resonance line ($\lambda 584 \text{ \AA}$) is possible. This is apparently

just the temperature of the intergalactic gas. However, the absence of absorption in L_{α} H I (in the spectra of remote quasars) points to a small thickness also in the He I line, since according to Fig. 4 the number of H I atoms is smaller than the number of He I atoms at $T \sim 10^5$ °K.

2) The situation is entirely different with the population of the high levels. As already noted above, the auto-ionization levels (γ, n, l) with $n < n_s$ are in thermodynamic equilibrium with the continuous spectrum. The corresponding stationary levels (γ_0, n, l) are populated at a rate $A_{\gamma\gamma_0}$, and decay as a result of a radiative transition to lower states much more slowly (at least by a factor n^3). Therefore the population of the levels (γ_0, n, l) relative to the continuous spectrum will be much higher than in the case of thermodynamic equilibrium. With increasing n , the population $N_{\gamma_0 n}$ will increase up to $n = n_s$.

To obtain a quantitative estimate, let us assume that during the lifetime of the BZ ion in the n -th (stationary) excited state there occurs a total mixing with respect to the quantum numbers l . At the same time, we shall disregard the collisions that lead to the ionization of the ion BZ or to transitions with change of n .* In addition, we neglect the underpopulation of the level n as a result of radiative transitions from higher levels.

Using formula (5), we can write the balance equation for the level $\gamma_0 n$ in the form

$$N_{\gamma_0 n} A_n = N_{Z+1} N_e \sum_l S_{n,l}^{-1} \frac{A}{1 + (A/W)}, \quad (19)$$

where A_n is the total probability of the radiative decay, which equals in the Kramers approximation

$$\begin{aligned} A_n &= A_0 \frac{3L}{n^3}, \quad A_0 = \frac{8\alpha^3 h Z^4}{3 \sqrt{3} \pi a_0^3 m} = 0.80 \cdot 10^{10} Z^4 \text{ sec}^{-1} \\ L &= \ln \frac{n}{(1.4n_0 - 1)^{2/3}}. \quad (20) \end{aligned}$$

Here n_0 —effective principal quantum number of the ground state of the ion BZ. For the probability of the resonant transition in the ion BZ_{+1} we have

$$A = A_0 \frac{e_{\gamma_0}}{e_{\gamma}} \left(\frac{E_{\gamma\gamma_0}}{Z^2 Ry} \right)^2 f_{\gamma_0 \gamma}. \quad (21)$$

From this we get the ratio of the population of the level $\gamma_0 n$ to the equilibrium (after Saha)

$$b(n) \equiv \frac{N_{\gamma_0 n}}{(N_{\gamma_0 n})_S} = \left(\frac{E_{\gamma\gamma_0}}{Z^2 Ry} \right)^2 f_{\gamma_0 \gamma} \frac{n^3}{3L} \sum_l \frac{2l - 1}{1 + (n/n_s)^3} e^{-E_{\gamma\gamma_0}/kT}. \quad (22)$$

We see that the factor $b(n)$ increases with n in proportion to n^3 up to $n \cong n_s$ (the presence of the sum over l complicates the variation somewhat, but does not change it qualitatively). When $n > \max n_s$ (we have in mind the maximum with respect to l), $b(n)$ remains almost constant. The corresponding maximum value of $b(n)$ with allowance for formula (n) can be written in the form

$$b_{\max} = \frac{137^3}{3\pi L} \frac{E_{\gamma\gamma_0} \sigma_{\gamma_0 \gamma}}{Z^2 Ry \pi a_0^3} e^{-E_{\gamma\gamma_0}/kT}. \quad (23)$$

If the exponential factor is not too small, as is the

*This relation is confirmed by a consideration of a number of typical but particular cases. It is difficult to prove it rigorously in the general case.

*Such an assumption is quite artificial, since the cross sections of the transitions $n, l \rightarrow n, l'$ at $n, l \rightarrow (n+1), l'$ do not differ very strongly. Therefore the estimates presented above should be regarded only as illustrations.

case in general, then $b(n)$ can reach very large values. Actually, however, at a certain sufficiently large n , collisions become significant and possibly also photoionization (see the following chapter II), and these restore the thermodynamic equilibrium (the Saha distribution; $b = 1$). Consequently, at a certain value of n , which depends on the electron density or on the radiation density, a decrease of $b(n)$ from the maximum value to unity begins.

5. Certain Results of Numerical Calculations

The rate of the d.r. can be calculated by means of formula (8) if one knows the coefficients $c(\gamma)$. As already noted above, in most cases it is sufficient to confine oneself to one term in the sum over γ , namely recombination through the resonance level of the ion BZ_{+1} . The transitions encountered in practical cases can be broken up in this case into two types:

$$\begin{aligned} \text{a) } & n_0 l_0^q \rightarrow n_0 l_0^{q-1}, nl; \\ \text{b) } & n_0 l_0^N, n_0 l_0^q \rightarrow n_0 l_0^{N-1}, n_0 l_0^{q+1}, \end{aligned}$$

with $l = l_0 + 1$, $N = 2(2l_0 + 1)$, i.e., $n_0 l_0^N$ -filled shell; $c(\gamma)$ for these two types of transitions are conveniently written in the form

$$\begin{aligned} \text{a) } & c(\gamma) = 10^{-10} \frac{q}{2l_0 + 1} \bar{c}; \\ \text{b) } & c(\gamma) = 10^{-10} \left(2 - \frac{q}{2l_0 + 1} \right) \bar{c}. \end{aligned} \quad (24)$$

Then the coefficients \bar{c} depend little on q . Figure 3 shows the dependence of \bar{c} on Z for the isoelectronic sequences from He to Ne.

The transition energies $E_{\gamma\gamma_0}$ in the ion BZ_{+1} can be written in the form

$$E_{\gamma\gamma_0} = RY [(Z+1)^2 a_0 + (Z+1) a_1 + a_2]. \quad (25)$$

We recall that $Z = 1$ corresponds to the neutral atom obtained in d.r. of a single ion. In Table I we list the corresponding $\gamma_0 - \gamma$ transitions and the values of the parameters a_0 , a_1 , and a_2 for the isoelectronic sequences He - Ne. In Table II are given the parameters

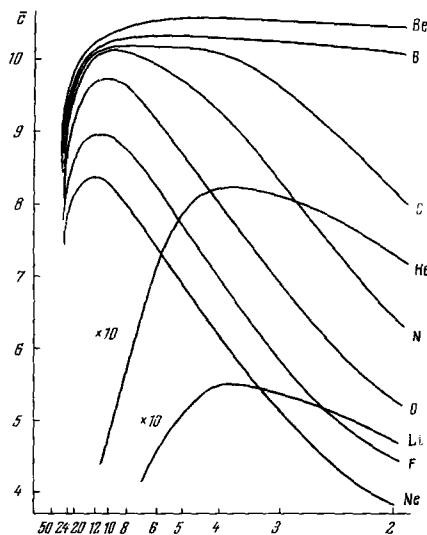


FIG. 3. Dependence of the d.r. coefficient \bar{c} on Z for a series of isoelectronic sequences (the abscissas represent $Z + 1$).

Table I. Types of transitions and parameters

Isoelectronic sequence	$\gamma - \gamma_0$ transition	a_0	a_1	a_2
He	$1s - 2p$	0.75	0	0
Li	$1s^2 - 1s2p$	0.75	0.78	—
Be	$2s - 2p$	0	0.135	-0.039
B	$2s^2 - 2s2p$	0	0.176	0.027
C	$2s^2 2p - 2s^2 p^2$	0	0.219	0.117
N	$2s^2 2p^2 - 2s^2 p^3$	0	0.256	0.258
O	$2s^2 2p^3 - 2s^2 p^4$	0	0.295	0.420
F	$2s^2 2p^4 - 2s^2 p^5$	0	0.333	0.609
Ne	$2s^2 2p^5 - 2s^2 p^6$	0	0.376	0.816

of the transitions in different ions of $Fe(Z)$.

The presented values of \bar{c} were obtained with the aid of formulas (9) and (10) by numerically calculating the cross sections of the excitation of the $\gamma_0 - \gamma$ transition at the threshold in the Born-Coulomb approximation, i.e., in first order of perturbation theory, when the outer electron is described by a Coulomb wave function with charge $Z + 1$. The energies E_{γ_0} and E_{γ} were determined as the eigenvalues of the single-electron Schrödinger equation without allowance for the volume. Comparison with the experimental data available for ions with small Z shows that the error in the calculation of the energies is of the order of the splitting of the state γ into terms.

To illustrate the influence of the d.r. on the ionization equilibrium, Figs. 4-6 show the ionization curves for H, He, O, and Ca. These curves correspond to the so-called "corona approximation" (small N_e), when the ionization by electron impact is in equilibrium with the photorecombination and with d.r. All the necessary cross sections were obtained numerically for each ion (for details see^[6])

II. INFLUENCE OF COMPETING PROCESSES

1. Introduction

As already noted, the d.r. occurs essentially on highly-excited levels. These levels are subject to the influence of the surrounding (electrons and radiation) much more strongly than the lower state, since the ratio of the interaction cross sections to the probability of the radiative decay increases rapidly with increasing n . Therefore in the case of d.r. the competing processes (collisions with electrons and absorption of radiation quanta) play a much more important role than in the case of photorecombination, which occurs essentially at lower levels. A correct quantitative calculation of the influence of the competing processes entails considerable difficulties and has not yet been performed. We therefore confine ourselves here only to a qualitative discussion and to some very simple estimates which are more readily of illustrative character.

With increasing electron density, the influence of the competing processes increases rapidly, and under the conditions of a laboratory plasma the role of the d.r. is in most cases apparently insignificant. We confine ourselves below to a discussion of only astrophysical applications ($N_e \lesssim 10^8 \text{ cm}^{-3}$).

Table II. Characteristics of d.r. in ions Fe(Z) (the quantities \bar{c} , \bar{c}' , and b_{\max} are defined in the text

Z	$\gamma_0-\gamma$ transition	$\frac{E_{\gamma_0\gamma_0}}{(Z+1)^2 Ry}$	$I_{\gamma_0\gamma}$	\bar{c}	\bar{c}'	$b_{\max} \cdot 10^{-3}$
1	4s-4p	0.090	1.0	1.37	0.20	1.1
2	3p ⁶ 3d ⁶ -3p ⁵ 3d ⁷	0.400	1.7	0.89	0.65	35
3	3p ⁶ 3d ⁵ -3p ⁵ 3d ⁶	0.23	2.0	1.57	1.25	23
4	3p ⁶ 3d ⁴ -3p ⁵ 3d ⁵	0.146	2.4	2.3	1.9	17
5	3p ⁶ 3d ³ -3p ⁵ 3d ⁴	0.102	2.8	3.2	2.7	13
6	3p ⁶ 3d ² -3p ⁵ 3d ³	0.076	3.04	4.0	3.6	10.7
7	3p ⁶ 3d-3p ⁵ 3d ²	0.059	3.4	4.8	4.4	9
8	3s ² 3p ⁶ -3d	0.047	3.4	5.3	4.8	7.4
9	3s ² 3p ⁵ -3d	0.038	2.75	5.8	5.3	4.6
	3s ² 3p ⁵ -3s3p ⁶	0.025	0.17	3.1	2.7	0.6
10	3s ² 3p ⁴ -3d	0.031	1.86	6.3	5.8	2.93
	3s ² 3p ⁴ -3s3p ⁵	0.021	0.326	3.5	3.0	0.93
11	3s ² 3p ³ -3d	0.026	1.3	6.8	6.3	1.8
	3s ² 3p ³ -3s3p ⁴	0.0173	0.47	3.8	3.3	1.1
12	3s ² 3p ² -3d	0.021	0.73	7.0	6.6	1
	3s ² 3p ² -3s3p ³	0.0148	0.6	4.1	3.7	1.26
13	3s ² 3p-3d	0.0179	0.33	7.4	7.0	0.4
	3s ² 3p-3s3p ²	0.0127	0.716	4.3	3.9	1.3
14	3s ² -3s3p	0.0111	0.82	4.5	4.1	1.32
15	3s-3p	0.0097	0.39	4.8	4.3	0.57
16	2p ⁶ -2p ⁵ 3d	0.19	3.0	0.55	0.55	0.048
17	2s ² 2p ⁵ -2s2p ⁶	0.023	0.05	0.76	0.75	0.0196
18	2s ² 2p ⁴ -2s2p ⁵	0.0191	0.086	0.79	0.78	0.034
19	2s ² 2p ³ -2s2p ⁴	0.0156	0.12	0.82	0.81	0.046
20	2s ² 2p ² -2s2p ³	0.0126	0.128	0.84	0.83	0.0546
21	2s ² 2p-2s2p ²	0.010	0.13	0.86	0.84	0.0616
22	2s ² -2s2p	0.0077	0.13	0.85	0.82	0.066
23	2s-2p	0.0056	0.05	0.82	0.79	0.030
24	1s ² -1s2p	0.78	0.8	0.0088	0.0088	0.00058
25	1s-2p	0.75	0.42	0.009	0.0090	0.00027

2. Influence of Secondary Ionization from an Excited Level

In the calculation of the d.r. coefficient in Ch. I, it was assumed that the excited ion $B_Z(\gamma, n, l)$ produced as a result of the recombination goes over subsequently to the ground state (γ_0, n_0, l_0) . However, at not too low an electron density N_e , or in the presence of high-density radiation in the plasma, the ion $B_Z(\gamma_0, n, l)$ can be again ionized prior to radiative decay to the ground state. This leads to a decrease of the coefficient of the d.r. [12]

For an estimate of the indicated effect, we shall use

a simplified model, which is already used above in Sec. 4 of Ch. I. Namely, we shall assume that total mixing of the states (γ_0, n, l) takes place with respect to the quantum numbers l but we shall disregard the population of the level (γ_0, n) as a result of transitions from the levels with other values of n . Then the contribution to the d.r. at the n -th level will be decreased by a factor

$$\frac{A_n}{A_n + A_p + N_e \langle v\sigma_i \rangle}$$

where A_n and A_p —probabilities of spontaneous decay and photoionization of the level n , and $N_e \langle v\sigma_i \rangle$ —probability of ionization by electron impact.

For a Planck radiation field with temperature T , we have in the Kramers approximation

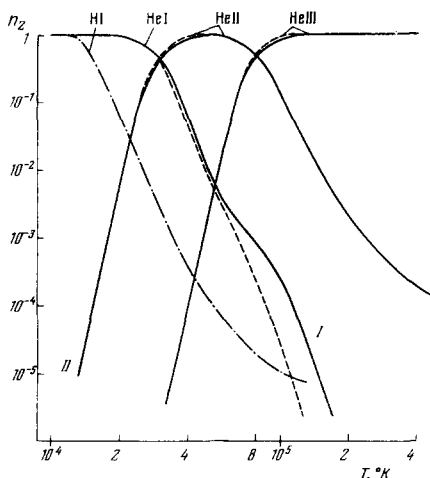


FIG. 4. Ionization curves $n_Z(T)$ for H and He. $n_Z = n_Z / \sum N_Z$ — relative concentration of the ion. The solid lines were obtained with allowance for the d.r., dashed lines — without allowance for the d.r.

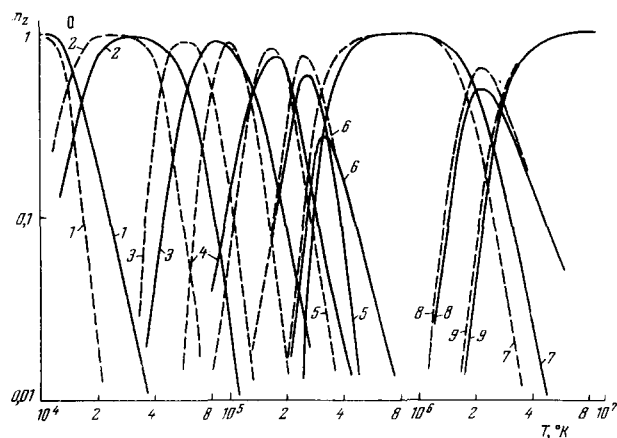


FIG. 5. Ionization curves for O. The symbols are the same as in Fig. 4.

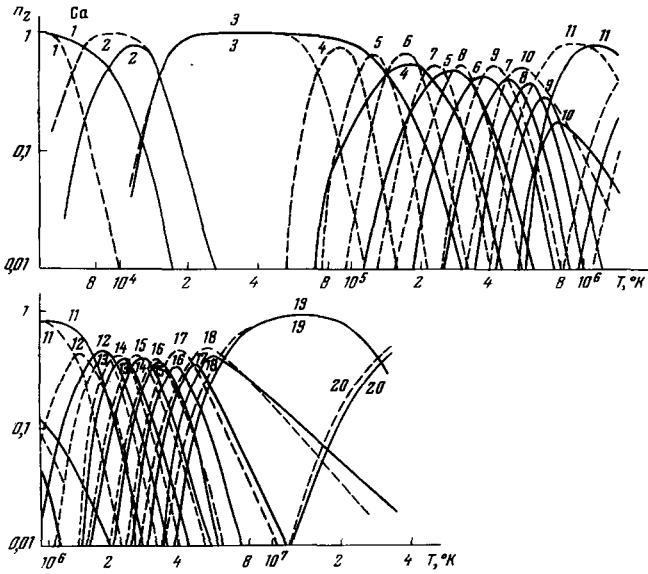


FIG. 6. Ionization curves for Ca. The symbols are the same as in Fig. 4.

$$A_v = \int_{\omega_n}^{\infty} \sigma_{iv}(\omega) J(\omega) \frac{d\omega}{h\omega} = \frac{A_0}{n^5} \int_{x_n}^{\infty} \frac{dx}{x(e^x - 1)}, \quad x_n = \frac{h\omega_n}{kT_v} = \frac{Z^2 R_y}{n^2 k T_v}, \quad (26)$$

where $\sigma_{iv}(\omega)$ —cross section for ionization from the level n , and $J(\omega)$ —Planck function of the radiation flux. The coefficient A_0 and the probability A_n of the spontaneous decay are given by formula (20). The rate of impact ionization $\langle v\sigma_i \rangle$ can be represented, in the case of a Maxwellian electron-velocity distribution with temperature T , by means of the semiempirical formula

$$\langle v\sigma_i \rangle = 10^{-7} \frac{n^3}{Z^3} \sqrt{\frac{\beta_n}{\beta_n + 1}} \frac{e^{-\beta_n}}{1 + \beta_n} \quad (\text{cm}^3 \text{ sec}^{-1}),$$

$$\beta_n = \frac{h\omega_n}{kT} = \frac{Z^2 R_y}{n^2 k T}. \quad (27)$$

Thus, allowance for the secondary ionization leads to replacement of $c(\gamma)$ in (8) and (9) by

$$c'(\gamma) = \frac{4\pi^{3/2} \alpha^3 h a_0 Z}{m n_1} f_{v\sigma} \sum_{n > n_1} \sum_l \frac{2l+1}{\left[1 + \left(\frac{n}{n_s}\right)^3\right] [1 + q_v + q_e]}, \quad (28)$$

$$q_v = \frac{1}{3L} \int_{x_n}^{\infty} \frac{dx}{x(e^x - 1)}, \quad q_e = \frac{N_e \langle v\sigma_i \rangle n^5}{3A_0 L}. \quad (29)$$

It is easy to see that $q_v \approx e^{-x_n}/3Lx_n$. When $x_n < 1$ we have

$$q_v = \frac{n^2}{n_s^2}, \quad n_s = Z \left(\frac{3LR_y}{kT_v} \right)^{1/2} \quad (n > n_v). \quad (30)$$

Analogously, q_e is exponentially small when $\beta_n > 1$. However, in all the cases of interest $\beta_n \ll 1$, i.e.,

$$q_e = \frac{n^2}{n_e^2}, \quad n_e = Z \left[\frac{2.4 \cdot 10^{17} L}{Z N_e} \left(\frac{R_y}{kT} \right)^{1/2} \right]^{1/7}. \quad (31)$$

Without taking into account the secondary processes, the d.r. occurs at levels with $n \lesssim n_s$. The photoionization and ionization by electron impact cause levels with $n > \min \{n_v, n_e\}$ to take practically no part in the d.r.

Let us consider by way of an example the case of the solar corona. We shall assume the radiation of the

photosphere to be Planck radiation with $T_\nu \approx 6000^\circ\text{K}$, electron density $N_e \approx 10^9$, and electron temperature $T \approx 10^6$ K. Then $n_\nu = 18Z$ and $n_e = 25Z^{6/7}$. For ions with $Z \sim 10$ we have $n_i \approx n_e$. For helium-like ions $n_s \sim 40 Z^{-4/3}$, i.e., impact ionization and photoionization do not play an important role when $Z \geq 2$. Ions of this kind produce radiation in the x-ray region ($\lambda \sim 1-30 \text{ \AA}$). The radiation in the lines of the visible and ultraviolet regions of the spectrum is connected with ions having a ground-state electronic configuration of the type $2s^2 2p^q$ or $3s^2 3p^q$. For such ions, the rate of the d.r. in the absence of photoionization is very large, since $n_s \approx 10^2 n_i Z^{-1} \sim 2 \times 10^3 Z^{-1}$. When $Z = 10$ we get $n_s \sim 200$, i.e., on the order of n .

Thus, photoionization by Planck radiation can lead to a noticeable weakening of the d.r., precisely in those cases when the role of this process is particularly large. The role of photoionization is apparently much more important under conditions prevailing in quasar envelopes (see the end of Sec. 2 of Ch. III below and [21]).

Of course, the foregoing estimates are quite crude and, as shown by comparison with the result of numerical calculations, somewhat overestimate the role of the photoionization. By way of illustration, Table II lists also the parameters \bar{c}' calculated for a number of Fe ions in accordance with (28)–(31) at $T_\nu = 6000^\circ\text{K}$, $N_e = 10^9$, and $T = 10^6$ K.

3. Populations of Excited Levels

In Sec. 4 of Ch. I we have shown that the d.r. leads to a very large over-population of the high levels. Without allowance for the secondary processes, the factor $b(n)$, which equals the ratio of $N_{\gamma n}$ to its equilibrium value (according to Saha), increases in proportion to n^3 when $n < n_s$, and remains practically constant when $n > n_s$. Impact ionization and photoionization cause the quantity $b(n)$ to start decreasing again when $n > n_\nu$ or n_e . To take these effects into account, it is sufficient to divide (22) or (23) by $[1 + (n/n_\nu)^2 + (n/n_e)^7]$.

It should be borne in mind, however, that the formula obtained in this manner is valid only at not too large values of n , namely so long as $b(n) \gg 1$. At still larger values of n , it is necessary to include in the balance equation the inverse processes—photorecombination (including induced) and triple recombination. For $n > \max n_s$, the factor $b(n)$ with allowance for all the indicated processes, can be written, using the known relations between the recombination and ionization rates, in the form

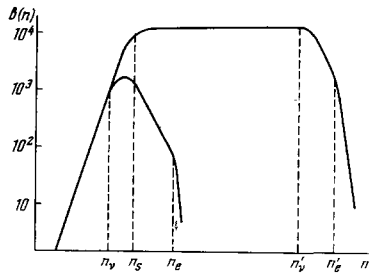
$$b(n) = \frac{b_{\max} + \Gamma q_v + q_e}{1 + q_v + q_e} \quad (n > \max n_s). \quad (32)$$

Here b_{\max} is determined by formula (23), $q_v = (n/n_\nu)^2$, and $q_e = (n/n_e)^7$ are given by formulas (29)–(31), and the coefficient Γ takes into account the possible difference between T_ν and T :

$$\Gamma = \frac{\int_0^\infty \sigma_v (1 - e^{-h\omega/kT_\nu})^{-1} e^{-E/kT} dE}{\int_0^\infty \sigma_v (1 - e^{-h\omega/kT_\nu})^{-1} e^{-E/kT_\nu} dE},$$

$$h\omega = E + \frac{Z^2 R_y}{n^2}. \quad (33)$$

Using the Kramers approximation for the photorecom-

FIG. 7. Approximate plot of the function $b(n)$.

bination cross section σ_ν , we get

$$\Gamma = \int_{x_n}^{\infty} \frac{e^{x(1-\tau)} dx}{x(e^x-1)} \bigg/ \int_{x_n}^{\infty} \frac{dx}{x(e^x-1)}, \quad \tau = \frac{T_\nu}{T},$$

$$x_n = \frac{Z^2 Ry}{n^2 k T_\nu}. \quad (34)$$

It is easy to see that in the case of greatest interest $Z^2 Ry/n^2 \ll kT_\nu \ll kT$ we have $\Gamma \approx 1$.

In the case $n < n_s$ it is necessary to substitute in (32) in lieu of b_{\max} the quantity defined by (22).

Thus, the factor $b(n)$ first increases with increasing n in proportion to n^3 . If $n_s < n_\nu, n_e$, then in the region $n_s < n < \min(n_\nu, n_e)$ the value of $b(n)$ remains constant at b_{\max} . When $n \gg n_\nu, n_e$ we get $b(n) \rightarrow 1$, corresponding to thermodynamic equilibrium with a continuous spectrum (Fig. 7). Table II lists also the values of b_{\max} for different Fe ions.

For a more consistent estimate of the populations it is necessary to include into consideration, besides ionization, the transitions between the discrete levels upon collision with electrons or upon absorption and stimulated emission of quanta. The electron in this case diffuses, as it were, over the levels of the atom (see, for example, [14, 15]). Detailed calculations of this effect have not yet been made. It can be expected that allowance for this effect can be to a noticeable smoothing of the population function over the levels, i.e., to a decrease of $b(n)$.

This effect is particularly important in the region $n > n_s$. Indeed, the decrease of the function $b(n)$ as a result of photoionization begins at

$$n = n_\nu = Z \left(\frac{Ry L}{k T_\nu} \right)^{1/2}.$$

The transitions between the neighboring levels, upon absorption of Planck radiation, lead to diffusion of the atomic electron from the level $n < n_\nu$ to the level $n = n_\nu$. As a result, the decrease of $b(n)$ begins already at a level for which $\Delta E_n \approx kT_\nu$, i.e., with

$$n \approx \left(\frac{2Z^2 Ry}{k T_\nu} \right)^{1/3} \ll n_\nu.$$

In concluding this section, we shall discuss briefly the following question. If high-density radiation is present in the volume it is necessary, in general, to take into account the stimulated photorecombination. If $T_\nu \ll T$, then it occurs only at high levels with $n > n_\nu$. The total photorecombination probability increases for such n by a factor $kT_\nu/\hbar\omega_n \sim n^2$. In other words, the probability of photorecombination at a level $n > n_\nu$ is proportional to $(1/n^3)n^2 = 1/n$. However, when summing over n it is necessary to bear in mind that at sufficiently large n ($n > n_e$) the levels

are in equilibrium with the continuous spectrum as a result of the much stronger collision effect. A simple estimate shows that in all real conditions the increase of the total probability of photorecombination as a result of stimulated transitions is insignificant.

It is seen from the foregoing estimates of the population of the higher levels that stimulated recombination is insignificant in this case, too. Indeed, it affects only the expression for Γ , which is close to unity in the most interesting cases.

III. CERTAIN ASTROPHYSICAL APPLICATIONS

1. Solar Corona

The physical interpretation of the spectrum of the solar corona is a rather complicated problem, principally in view of the appreciable inhomogeneity of the corona. In particular, the temperature of the corona varies with depth, and within the limits of one layer there are active regions whose temperature is much higher than the average temperature of the "quiescent corona." Nonetheless, there are observational data in a broad spectral region, from the visible to the x-ray region, which make it possible to obtain a considerable amount of information in the chemical composition and on the temperatures of both the quiescent corona and its active regions. We shall stop to discuss certain questions for which the d.r. may turn out to be significant.

The line radiation in the visible region of the spectrum is connected with transitions between terms or fine-structure components of the ground-state electron configuration of ions of large multiplicity. Obviously, the ions should have an unfilled electron shell. In this case, the ionization equilibrium curve $N_Z(T)$ has a sharp maximum. The lines of the given ion are emitted from the regions of the corona with an electron temperature close to the temperature T_Z of the maximum of the corresponding ionization-equilibrium curve. On the other hand, it is possible to determine the temperature T_Z from the line width, which, under the conditions of the corona, is due entirely to the Doppler broadening mechanism.

Of course, both T_Z and T_D for lines of different ions are essentially different, since the lines can be emitted from different regions of the corona. A stranger fact is that for each line T_D turns out to be systematically larger than T_Z (see, for example, [16]). In all the calculations of T_Z performed prior to 1964, the d.r. was not taken into account. Burgess and Seaton^[4] were the first to show that allowance for the d.r. in the calculation of the curve of the ionization equilibrium leads to an increase of T_Z and accordingly to an appreciable decrease of the difference between T_D and T_Z . By way of illustration, Table III lists the temperatures for three lines.

If a consistent account is taken of the secondary effects, then the role of the d.r. can possibly decrease. A more accurate knowledge of the difference $T_D - T_Z$ is important for an estimate of the velocities of the macroscopic motions in the corona.

Lines in the x-ray region of the spectrum are due essentially to emission of H- and He-like ions (the

Table III

Ion	$\lambda \text{ \AA}$	$T_D \cdot 10^{-6}$	$T_Z \cdot 10^{-6}$	
			With out d.r.	With d.r.
Fe X	6374	1.9	0.6	1.25
Fe XIV	5303	2.5	1.2	2.7
Ca XV	5694	3.0	2.5	3.2

only exceptions are the Ca and Fe ions). For the former, the d.r. is impossible at all. For He-like ions it leads to an appreciable increase of the concentration in those cases when the ionization energy E_Z is smaller than kT , i.e., to the right of the maximum of the ionization-equilibrium curve. The shift of the maximum is in this case insignificant. It should be noted that with increasing Z the rate of photorecombination increases in proportion to Z , and the rate of ionization decreases like Z^{-3} . Therefore the value of E_Z/kT at the maximum of the ionization curve of the ion Z becomes smaller. As a result, the role of the d.r. increases with increasing Z for He-like ions.

Table IV lists the intensities of certain lines of the solar corona, calculated with and without allowance for the d.r., for three values of T . The data in the table correspond to a uniform model of the corona with an emission measure after Baumbach ($N_e^2 V = 3.2 \times 10^{49} \text{ cm}^{-3}$), and with the photospheric values of the abundance of the elements^[17]. Although the uniform model cannot claim a complete description of the x-ray spectrum, it does give qualitatively correct ideas of the temperature of those corona regions which are responsible for the emission of the indicated lines. Recent data indicate that the abundance of a number of elements in the corona differs greatly from the photospheric values. A comparison of the results listed in Table IV with experiment^[18] confirms this conclusion^[19,6,7], but the presently available data apparently are still insufficient for a reliable determination of the abundance of the elements in the corona.

The d.r. process leads also to rather interesting peculiarities in the dependence of the ratio of the line intensities on the temperature. Thus, the ratio of the

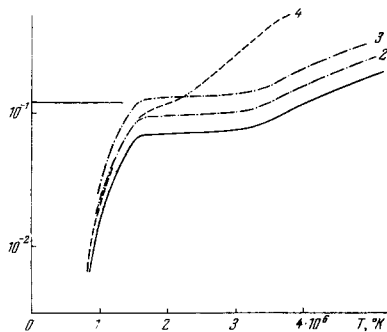


FIG. 8. Ratio of the intensities of the resonant lines of the ions N VII and O VII as a function of the temperature. 1, 4 — abundance in accordance with photospheric data^[17], 2 — abundance in accordance with data on ultraviolet radiation^[19], 3 — abundance in accordance with data on solar cosmic rays^[19]; 1 — 3 — calculations with allowance for d.r., 4 — without allowance for d.r.; horizontal line — experimental value^[18].

intensities of the resonant lines N VII and O VII has a “plateau” in the temperature region $(1.5-3) \times 10^6 \text{ K}$ (Fig. 8). Inasmuch as the temperature can hardly exceed $3 \times 10^6 \text{ K}$ by a large amount, this makes it possible to draw certain qualitative conclusions concerning the ratio of the abundances of O and N in the solar corona. Indeed, the curve presented in the same figure, corresponding to the abundances of O and N is obtained from data on the ultraviolet radiation of the corona and of solar cosmic rays, clearly favors the latter.

2. Emission and Absorption Lines in Transitions Between High Levels

D.r. leads to a considerable repopulation of the high levels $b(n) \gg 1$ at large values of n . This can strongly influence the intensities of the emission line connected with the transition between the high levels. The study of such lines (with $n \sim 100$) in the spectra of gas nebulas in the radial band plays an important role in the determination of the physical characteristics of the nebulas. The properties of levels with such large n for atoms of all elements are practically identical with those of hydrogen. This pertains also to the rate of population of the levels in radiative and triple recombination. d.r. is the only exception, for in the case of hydrogen it is impossible. The large values of $b(n)$ denote an anomalously large intensity of the radio lines of Mg, O, C, etc. compared with H (with allowance for the relative abundance of the elements. This is precisely the explanation proposed by Goldberg and Dupree^[20] for the results of the observations of the lines 109α (i.e., the transition $n = 109 \rightarrow 108$) in C and H).

In the case of ions of large multiplicity, the lines corresponding to the transition $n \rightarrow n - 1$ fall in the infrared invisible regions of the spectrum (at $Z \sim 10$ and $n \sim 10-15$). An important fact is that the intensity of such lines does not depend on n , whereas when they are excited by electron impact or in photorecombination, the intensity is proportional to n^{-3} . A rough estimate shows that when $n > n_S^{1/3} \sim 4-6$, the population due to the d.r. becomes more effective than population by electron impact.

When $n > n_p$ or n_e , i.e., on the decreasing branch of the $b(n)$ curve (see Fig. 7), it is possible in principle to observe the absorption lines^[11,20]. In this case the optical thickness is

$$\tau = \left(N_n - \frac{n^2}{(n+1)^2} N_{n+1} \right) kl = N_Z N_e \left(\frac{2\pi\hbar^2}{mkT} \right)^{3/2} n^2 \frac{\partial b(n)}{\partial n}, \quad (35)$$

where k —absorption coefficient

$$k = \frac{2\pi^2\hbar}{137m\Delta\omega} f_{n, n+1}, \quad f_{n, n+1} = \frac{n}{5} \quad (\text{large } n), \quad (36)$$

and $\Delta\omega$ —line width. Assuming Doppler broadening (since the collisions become significant only when $n \ll n_e$), we get ultimately

$$\tau \approx 10^{-27} N_Z N_e l \sqrt{A} \frac{n^6}{Z^2 T^2} \frac{\partial b(n)}{\partial n}. \quad (37)$$

When $n > n_S$, with allowance for secondary processes of photoionization and impact ionization, we get in accordance with (32)

$$b(n) = \frac{b_{\max}}{1 + q_n + q_e} \quad (38)$$

Table IV. Relative concentrations n_Z and x-ray fluxes J (erg/cm² sec) in the lines of the most abundant elements

Ion, transition	Abundance of element	$\lambda, \text{\AA}$	Without d.r.						With d.r.						Experiment
			10 ⁶ °K		2 · 10 ⁶ °K		4 · 10 ⁶ °K		10 ⁶ °K		2 · 10 ⁶ °K		4 · 10 ⁶ °K		
			n_Z	J	n_Z	J	n_Z	J	n_Z	J	n_Z	J	n_Z	J	
N VI	-4 15*)		0 95	-2 22	-1 39	-3 93	-3 30	-4 21	0 87	-2 22	0 13	-2 24	-3 17		
1s-2p		28.75	-2 22	-3 93	-3 30	-4 21	0 87	-2 22	0 13	-2 24	-3 17				
-3p		24.95	-3 26	-3 16	-5 42	-3 26	-3 26	-3 26	-3 53	-4 34					
N VII			0 15	-3 10	0 46	-2 44	-1 47	-2 15	0 12	-4 86	0 41	-2 40	-2 16	-3 40	
1s-2p		24.77	-5 90	-3 63	-3 29	-5 73	-5 73	-5 73	-3 57	-3 29					
-3p		20.97													
O VII	-3 91		0 99	-2 46	0 38	-1 37	-2 53	-2 22	0 99	-2 45	0 56	-1 55	-1 12	-2 32	
1s-2p		21.56	-3 37	-2 51	-3 39	-3 39	-3 39	-3 39	-3 37	-2 76	-2 76	-2 76	-2 21	-3 32	
-3p		18.57													
O VIII			-2 72	-5 77	0 54	-1 20	0 18	-1 34	-2 69	-5 73	0 38	-1 14	-1 33	-3 36	
1s-2p		18.87	-6 52	-2 25	-2 25	-2 60	-2 60	-2 60	-6 50	-2 18	-2 18	-2 18	-2 59	-4 59	
-3p		16.08													
Ne IX	-4 81		0 88	-5 78	0 94	-3 85	0 21	-2 21	0 88	-5 57	0 95	-3 85	-2 42		
1s-2p		13.69	-7 32	-4 13	-4 13	-4 13	-4 13	-4 50	-7 32	-4 13	-4 13	-4 13	-3 10		
-3p		11.49													
Ne X			-4 23	---	-1 41	-4 14	0 63	-2 32	-4 23	---	-1 35	-4 12	0 45	-2 23	
1s-2p		11.98	---	---	-5 12	-5 12	-5 12	-3 47	---	---	---	-5 11	-3 34		
-3p		10.25													
Na X	-5 20		0 67	-7 12	0 97	-5 82	0 71	-3 12	0 64	-7 11	0 96	-5 82	0 81	-3 14	
1s-2p		10.98	-9 34	-6 67	-6 67	-6 67	-6 67	-4 17	-7 98	-9 33	-6 69	-6 69	-4 20		
-3p		9.43													
Na XI			-6 10	---	-2 20	-8 48	0 28	-4 18	-7 98	---	-2 19	-8 46	0 16	-4 14	
1s-2p		9.99	---	---	-9 26	-9 26	-9 26	-5 19	---	---	-9 24	-9 24	-5 11		
-3p		8.38													
Mg XI	-4 25		0 28	-8 46	0 92	-4 27	0 91	-2 10	0 20	-8 34	0 92	-4 27	0 94	-2 10	
1s-2p		9.16	---	---	-5 18	-5 18	-5 18	-3 12	---	---	---	-5 17	-3 13		
-3p		7.83													
Mg XII	-		-9 75	---	-3 48	-8 17	-1 78	-4 38	-9 55	---	-3 48	-8 16	-1 46	-4 22	
1s-2p		8.44	---	---	---	---	---	-5 44	---	---	---	---	---	-5 25	
-3p		7.11													
Al XII	-5 16		-1 44	---	0 79	-6 35	0 96	-4 32	-1 20	---	0 78	-6 34	0 95	-4 32	
1s-2p		7.74	---	---	-6 35	-6 35	-6 35	-5 35	---	---	-6 34	-6 34	-7 17	-5 35	
-3p		6.62													
Al XIII	-		---	---	-4 13	---	-1 18	-6 23	---	---	-4 13	---	-1 17	-6 21	
1s-2p		7.09	---	---	---	---	---	-7 21	---	---	---	---	---	-7 20	
-3p		5.99													
Si XIII	-4 32		-2 32	---	0 54	-5 10	0 93	-3 29	-2 42	---	0 50	-6 98	0 92	-3 29	
1s-2p		6.65	---	---	-7 46	-7 46	-7 46	-4 13	---	---	-6 65	-7 14	-4 13	-4 13	
-3p		5.49													
Si XIV	-		---	---	-6 70	---	-2 38	-6 46	---	---	-6 65	---	-2 36	-6 44	
1s-2p		6.19	---	---	---	---	---	-7 44	---	---	---	---	---	-7 40	
-3p		5.24													
S XIV	-4 20		-4 14	-9 19	0 30	-4 68	0 21	-3 18	-7 98	---	-4 46	-4 40	0 20	-3 17	
2s-4p		23.38	---	---	-4 24	-4 24	-4 24	-4 74	---	---	-5 36	-5 36	-4 73		
-5p		20.98													
S XV	-		-7 18	---	-1 56	-8 39	0 78	-4 38	-9 12	---	-2 86	-9 60	0 76	-4 37	
1s-2p		5.2	---	---	---	---	---	-5 10	---	---	---	---	---	-5 10	
-3p		4.19													
S XVI	-		---	---	-9 32	---	-3 13	-8 18	---	---	---	---	-3 12	-8 17	
1s-2p		4.78	---	---	---	---	---	---	---	---	---	---	---	---	
-3p		3.99													
Ca XV	-5 14		-7 40	---	-1 96	-6 92	-1 66	-5 22	-9 40	---	-2 57	-7 56	0 29	-5 94	
2p-3s		26.1	---	---	-6 15	-6 15	-6 15	-6 54	---	---	-8 88	-8 88	-5 24	-5 24	
-4s		19.1	---	---	-4 21	-4 21	-4 21	-3 12	---	---	-5 30	-5 30	-3 54	-3 54	
-3d		24.4	---	---	-5 96	-5 96	-5 96	-4 36	---	---	-6 37	-6 37	-3 46	-3 46	
-4d		18.7													
Ca XVI	-		---	---	-2 66	-7 22	0 17	-5 22	---	---	-3 11	-9 30	0 26	-5 32	
2p-3s		23.9***)	---	---	-8 27	-8 27	-8 27	-6 44	---	---	---	---	---	-6 68	
-4s		17.5*)	---	---	-5 12	-5 12	-5 12	-3 12	---	---	-7 20	-7 20	-3 20	-3 20	
-3d		22.7*)	---	---	-6 18	-6 18	-6 18	-4 30	---	---	-9 30	-9 30	-4 45	-4 45	
-4d		17.2*)													
Ca XVII	-		---	---	-3 41	-7 20	0 42	-3 10	---	---	-5 15	---	0 49	-4 46	
2s-3p		20.1*)	---	---	-8 32	-8 32	-8 32	-4 28	---	---	---	---	---	-4 43	
-4p		15.1*)													
Ca XVIII	-		---	---	-5 47	-9 11	0 28	-4 34	---	---	-8 83	---	-1 72	-5 89	
2s-3p		19.07	---	---	-5 56	-5 56	-5 56	-5 56	---	---	---	---	---	-5 14	
-4p		14.28													
Ca XIX	-		---	---	-7 13	---	-1 56	---	---	---	---	---	-1 15	-9 31	
1s-2p		3.19	---	---	---	---	---	---	---	---	---	---	---	---	
Ca XX	-		---	---	---	---	-8 62	---	---	---	---	---	-8 16	---	
1s-2p		2.99	---	---	---	---	---	---	---	---	---	---	---	---	
Fe XVII	-5 37		---	---	0 42	-4 56	0 38	-2 14	---	---	-2 71	-5 27	0 34	-2 10	
2p-3d		15.26	---	---	-5 69	-5 69	-5 69	-3 31	---	---	---	-6 32	---	-3 20	
-4d		12.16													
Fe XVIII	-		---	---	-3 58	-8 82	0 30	-4 22	---	---	-4 16	---	-1 27	-5 70	
2p-3s		15.88	---	---	-9 75	-9 75	-9 75	-5 40	---	---	---	---	---	-5 12	
-4s		12.02	---	---	-6 17	-6 17	-6 17	-3 58	---	---	-8 28	-8 28	-4 44	-4 44	
-3d		14.33	---	---	-7 48	-7 48	-7 48	-3 41	---	---	-9 28	-9 28	-2 80	-2 80	
-4d		11.48													

*The order and mantissa of the number are given, for example, 4 95 denotes 0.95 × 10⁻⁴.
 **The intensity of this line is less than 10⁻¹⁰.
 ***Possible error ~ 0.5 Å.

(radiative recombination is insignificant under the conditions of interest to us). The value of b_{\max} can be estimated with the aid of (23) and the semiempirical formula for $\sigma_{\gamma_0\gamma}$:

$$\sigma_{\gamma_0\gamma} \approx 10 f_{\gamma_0\gamma} \left(\frac{Ry}{E_{\gamma\gamma_0}} \right)^2 \pi a_0^2, \quad (39)$$

whence

$$b_{\max} \approx 10^6 f_{\gamma_0\gamma} \frac{Ry}{Z^2 E_{\gamma\gamma_0}} e^{-E_{\gamma\gamma_0}/kT}. \quad (40)$$

Let us consider by way of an example the situation in the solar corona: $T = 10^6$ °K, $T = 6000$ °K, $N_e \sim 10^8 - 10^9$ cm $^{-3}$, and values of n for which $q_\nu > 1$ and $q_\nu > q_e$. Such values of n are the most interesting, since they lead to the maximum of $\partial b(n)/\partial n$. Using (38) and (40), and introducing the wavelength of the absorption line $\lambda = 0.45 \times 10^{-5} n^3/Z^2$ cm, we get

$$\tau \approx 10^{-13} N_e N_Z l \sqrt{A} \frac{\lambda f_{\gamma_0\gamma}}{T^2} \left(\frac{Ry}{E_{\gamma_0\gamma}} \right) e^{-E_{\gamma_0\gamma}/kT}. \quad (41)$$

When $\lambda < \lambda_\nu$, the value of τ is smaller than that given in (41). When $\lambda > \lambda_e \approx 0.1Z$, where λ_e corresponds to $q_e \approx q_\nu$, the optical thickness τ decreases with increasing λ in proportion to $1/\lambda$. In the estimate of τ it is necessary also to summarize the contribution of ions at different elements with given Z , since λ depends only on Z and n . According to our estimates^[11], the total value of τ is of the order of 10^{-2} when $Z = 10$ (the values of $N_Z N_e l$ were taken from^[7]). An optical thickness of the same order, $10^{-3} - 10^{-2}$, can be expected also for other ions.

The absorption lines should be manifest also in the quasar spectra^[21]. Here, however, the conditions differ greatly from the conditions in the corona. The continuous spectrum differs from the Planck spectrum and has a very large effective temperature. The electron density in the quasar shell is apparently much smaller than in the solar corona. As a result, the decreasing branch of $b(n)$ is connected exclusively with photoionization. Estimates presented in^[21] show that it is possible to expect $\tau \sim 10^{-3} - 10^{-1}$ (the emission measure $N_e^2 l$ is not very well known) for ions with $Z = 1-4$.

APPENDIX

I.

In first-order perturbation theory in the interelectron interaction $V = \sum_j e^2 |\mathbf{r} - \mathbf{r}_j|^{-1}$, the probability of

the auto-ionization $(\gamma, n, l) \rightarrow (\gamma_0, E, l')$ (E, l' —energy and angular momentum of the electron in the continuous spectrum) is determined by the formula^[22]

$$W = \frac{2\pi}{\hbar g} |\langle \Psi_{\gamma_0} \Phi_{E, l'} | V | \Psi_\gamma \Phi_{n, l} \rangle|^2, \quad (I.1)$$

$$g = g_\gamma (2l + 1).$$

If n is sufficiently large, the dependence of (I.1) on n can be determined in explicit form. It is necessary to take into account here that $\varphi_{n, l}$ is the wave function of the electron in the Coulomb field of the atomic remainder $-Ze^2/r$, i.e., its radial part equals

$$\varphi_{n, l}(r) = \left(\frac{Z}{a_0} \right)^{1/2} \left[\frac{(n+l)!}{(n-l-1)!} \right]^{1/2} \frac{1}{n(2l+1)!} \left(\frac{2Zr}{a_0 n} \right)^{l+1} \times e^{-Zr/a_0 n} F \left(-n+l+1, 2l+1; \frac{2Zr}{a_0 n} \right) \rightarrow_{n \rightarrow \infty} \left(\frac{Z}{a_0 n} \right)^{3/2} \sqrt{\frac{2Zr}{a_0}} J_{2l+1} \left(\sqrt{\frac{8Zr}{a_0}} \right) = \frac{Z\hbar}{a_0 \sqrt{mn}^{3/2}} \varphi_{0, l}(r), \quad (I.2)$$

where $\varphi_{0, l}(r)$ is the radial Coulomb wave function of the continuous spectrum with zero energy.

It follows from (I.1) and (I.2) that

$$W = \frac{2\pi Z^2 \hbar}{m a_0 n^3 g} |\langle \Psi_{\gamma_0} \Phi_{E, l'} | V | \Psi_\gamma \Phi_{0, l} \rangle|^2. \quad (I.3)$$

The matrix element in (I.3) coincides exactly with the matrix element which determines the partial cross section for the excitation of the $\gamma_0 - \gamma$ transition in the Born-Coulomb approximation at an incident-electron threshold energy:

$$\sigma_{\gamma_0\gamma}(l) = \frac{4\pi^2 a_0^2}{g \gamma_0} \frac{Ry_l}{E_{\gamma\gamma_0}} |\langle \Psi_{\gamma_0} \Phi_{E, l'} | V | \Psi_\gamma \Phi_{0, l} \rangle|^2. \quad (I.4)$$

Comparing (I.3) with (I.4) we get (4').

II. QUASICLASSICAL PROPERTIES OF HIGHLY EXCITED LEVELS IN A COULOMB FIELD AND DIELECTRONIC RECOMBINATION*

High excited levels with $n \gg 1$ of an electron in a Coulomb field describe states in which the electron is almost always far from the nucleus: in this sense, the states are similar to the states of electrons in the continuous spectrum at low positive energy. It is therefore natural that suitably-averaged quantities pertaining to the discrete spectrum near its condensation point (as $n \rightarrow \infty, E \rightarrow 0$) do not differ from the corresponding values for the continuous spectrum near the threshold, i.e., when $0 < E \ll Z^2 Ry$.

As is well known, processes in which slow electrons in a Coulomb field are obtained in the final state differ in the fact that the corresponding cross sections tend to a finite nonzero value σ_0 at threshold when $E \rightarrow 0$.[†] The vanishing of the phase volume ($dN/dE \sim p \sim \sqrt{E}$) as $E \rightarrow 0$ is compensated by the infinite growth of the matrix elements due to the deformation of the wave function by the Coulomb field: $|\psi(0)|^2/|\psi(\infty)|^2 \sim 1/\sqrt{E}$. We express quantities pertaining to the discrete spectrum when $-Z^2 Ry \ll E < 0$, i.e., when $n \rightarrow \infty$, in terms of σ_0 :

$$\frac{1}{E_{n_2} - E_{n_1}} \sum_{n=n_1}^{n_2} \sum_l \sum_m \int \sigma_{n, l, m}(E) dE = \sigma_0. \quad (II.1)$$

In the discrete spectrum, the cross sections are δ -functions of the energy (without allowance for the

*See footnote on p. 413.

[†]It is well known that without the Coulomb interaction the cross section of the photoeffect at similar processes in which low particles are produced tends to zero in proportion to the particle momentum $p \sim \sqrt{E}$. Accordingly, the cross section for the capture of slow particles is proportional to $1/p \sim 1/v$ in the absence of a Coulomb field. To the contrary, in the Coulomb field the cross section for capture of slow change particles is proportional to $1/p^2$, i.e., it is inversely proportional to the energy.

level widths) or else very narrow resonance curves.

It follows from formula (II.1) that the integral of the cross section of the energy for each individual sublevel with fixed l and m is on the average proportional to $1/n^3$. If we take the interval $\Delta n = n_2 - n_1 \ll n$ and recognize that the difference $E_{n_2} - E_{n_1} = Z^2 Ry [(n_2^2 - n_1^2)/n^2 n_2^2]$ we get

$$\sum_{l,m} \int \sigma_{n,l,m} dE = \frac{2Z^2 Ry \sigma_0}{n^3}. \quad (\text{A.3})$$

We can qualitatively assume that the process of obtaining the slow electron (for example when an electron in the ground state absorbs the quantum) always occurs near the nucleus. Then, if $\hbar\omega = I + E$ (I —ionization energy, E —small but positive), then the slow electrons will off to infinity.

$$\text{If} \quad \hbar\omega = I - \xi,$$

the slow electron also goes far away from the nucleus, but still remains bounded. As a result of quantization of the bound states, ξ can no longer be arbitrary, but $\xi = \xi_n = Ry/n^2$.

However, for the probability of the process when white light "continuous spectrum" is used for the irradiation, the sign of E is not important, and the total probability of the process is the same when $E > 0$ (ionization), and when $E = -\xi < 0$ the total probability of the process is determined by the events that take place near the nucleus, and does not depend on the subsequent fate of the electron or on whether the electron moves far away or all the way to infinity.

Formulas of the type (II.1) and (II.2) pertain to several types of processes:

1) Absorption of light by an atom in the ground state. We assume here that

$$\hbar\omega = I + E$$

and compare the photoeffect $\hbar\omega > I$, $E > 0$ and the excitation of the high levels at

$$\hbar\omega_n = I - (Z^2 Ry/n^2).$$

2) Emission of light upon interaction of a fast electron with an ion. The electron energy is A ; it can radiate

$$\hbar\omega = A - E, \quad \hbar\omega < A, \quad E > 0$$

—bremsstrahlung or

$$\hbar\omega_n = A - Z^2 Ry/n^2$$

upon recombination at a high level n .

3) The same formula pertains also to nonradiative interaction of an electron with a positive ion, connected with the internal excitation of the ion. Let ΔE be the energy of excitation of the ion. Then inelastic scattering of the electron takes place at an electron energy $A = \Delta E + E$, $E > 0$, with the slow electron (energy E) going off to infinity. At an electron energy $A_n = \Delta E - Z^2 Ry/n^2$, an atom is produced in a doubly-excited state with positive energy. In this state, an internal electron is excited, and furthermore, a bound

incoming electron is on a remote orbit. Thus, for example, the process $\text{He}^+(1s) + e \rightarrow \text{He}(2p, n, l)$ is possible.

It is precisely this process—d.r.—which is considered in detail in the fully orthodox article to which these remarks are appended.

Note added in proof (to p. 412). According to (9) we have $c(\gamma) \sim f_{\gamma_0\gamma}$. Therefore we do not consider at all optically forbidden $\gamma_0 - \gamma$ transitions from which $f_{\gamma_0\gamma}$ is small, although $\sigma_{\gamma_0\gamma}$ can be appreciable, together with the corresponding capture cross sections (for example, exchange excitation of levels with $s \neq s_0$).

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