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SECOND SOUND IN SOLIDS

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THe aim of the present paper is to discuss experimental observations of an interesting phenomenon, second sound in solid helium.[1,2]

It is well known that the heat motion in solids at low temperatures reduces to the presence in the solid of phonons - sound quanta. The energy of a quantum is connected with the sound frequency by the relation

$$\varepsilon = \hbar\omega.$$

Moreover, each phonon is characterized by a quasimomentum - a vector with similar properties as the momentum of a particle and equal to

$$\mathbf{k} = \hbar\varkappa,$$

where κ is the wave vector of the sound wave. At low temperatures there are only phonons with small ω. Then

$$\varepsilon = ck, \tag{1}$$

where c is the sound velocity.* In this respect phonons are analogous to photons, for which the connection between energy and momentum is given by the same formula (1) with c the light velocity. In equilibrium the phonons are distributed over quasi-momenta according to a Bose-Einstein distribution function

$$n(\mathbf{k}) = [e^{\varepsilon(\mathbf{k})/T} - 1]^{-1} \tag{2}$$

(T is the temperature in energy units).

Phonons can interact with one another - scatter and decay. In such collisions the energy and also the total quasi-momentum - with a restriction which we shall discuss below (see Eq. (10)) - are conserved. In that sense the system of phonons is analogous to a normal gas of particles. In particular, it turns out that a peculiar sound can propagate in such a gas. This "second order" sound propagating in the gas of normal sound quanta is called second sound. Since the phonon gas by its own character is a carrier of thermal motion, the quantity that oscillates in a second sound wave is the temperature, and not the density as in normal sound. One can say that second sound is an undamped thermal wave.

The existence of second sound was predicted theoretically for superfluid liquid helium by L. D. Landau.[3] V. P. Peshkov[4] observed second sound experimentally in liquid helium and he mentioned that such a phenomenon could also exist in solids.

We shall not give here Landau's general theory (it is given in the review[5]) but restrict ourselves to inexact but simple considerations which are convenient for our case of phonons in a solid.

First of all, we note that the phonon gas has an energy E and pressure p which, as in the case of isotropic radiation, i.e., a photon gas, are connected by the relation

$$E = 3p \tag{3}$$

(E is the energy per unit volume).

The phonon gas may move as a whole with respect to the crystal lattice. If the velocity of this motion is $\mathbf{v}$ the distribution function of such a gas is obtained from (2) by replacing $n(\epsilon)$ by $n(\epsilon - \mathbf{k}\cdot\mathbf{v})$. If we use such a distribution function to calculate the total quasi-momentum $\mathbf{K}$ of the gas, we find easily for small velocities $\mathbf{v}$

$$\mathbf{K} = \int \mathbf{k} n(\varepsilon - \mathbf{kv}) \frac{d^3k}{(2\pi\hbar)^3} \approx -\int \mathbf{k}(\mathbf{kv}) \frac{dn}{d\varepsilon} \frac{d^3k}{(2\pi\hbar)^3} = \frac{4E}{3c^2}\mathbf{v}.$$

The coefficient of proportionality between $\mathbf{K}$ and $\mathbf{v}$,

$$\rho_n = \frac{4E}{3c^2} \tag{4}$$

has the meaning of the "effective mass" density of the phonons just as the mass density of a normal gas is the coefficient of proportionality between its momentum and velocity.

We now introduce a set of equations describing the propagation of second sound. As a first equation we take the equation for the velocity of motion of the phonons which expresses in final reckoning the quasi-momentum conservation law. It must have the form of the usual hydrodynamic equation:

$$\rho_n \frac{\partial \mathbf{v}}{\partial t} = -\nabla p \tag{5}$$

(we assume the velocity v to be small).

As the second equation we choose, in accordance with the physical meaning of second sound as an undamped temperature oscillation, the equation expressing conservation of energy. It has the form

$$\frac{\partial E}{\partial t} + (E + p)\,\mathrm{div}\,\mathbf{v} = 0. \tag{6}$$

The second term in the brackets describes as in normal hydrodynamics the change in energy connected with the work done by the pressure forces of the phonon gas.

Expressing ρ_n and p in terms of E through (3) and (4) and eliminating v we get a wave equation for E

$$\frac{\partial^2 E}{\partial t^2} = \frac{c^2}{3}\Delta E, \tag{7}$$

from which it is clear that the second sound velocity is equal to

$$c_2 = \frac{c}{\sqrt{3}}. \tag{8}$$

In an anisotropic solid some averaged velocity $\bar{c}$ must occur in this formula.

*There are in a solid three acoustic branches with different velocities. We shall, however, forget about this, as it has no importance as far as principles are concerned.

Let us now consider the conditions under which second sound may be observed in a solid.[6]

It is, first of all, clear that the crystal must be of sufficiently good quality. The mean free path of thermal phonons connected with scattering by crystal defects or by impurities must be large compared with the second sound wavelength:

$$\lambda_2 \sim \frac{c_2}{\omega_2} \ll l_{\text{imp}} \qquad (9)$$

In actual fact it turns out that the most favorable object in this respect is a crystal of solid ^{4}He. This is connected with the fact that there are already hardly any impurities in liquid He: at low temperatures the solubility of all substances is very small. The only possible impurities are atoms of the other helium isotope, ^{3}He, and they can relatively easily be separated. On the other hand, defects such as vacancies which are formed when helium crystallizes relatively quickly go to the surface of the crystal. This is connected with the large amplitude of the zero-point vibrations of the atoms in solid and liquid helium. It is just the large magnitude of these oscillations which leads to the fact that under normal pressure helium remains liquid down to the absolute zero and solidifies only under pressures larger than 25 atm.

The second necessary requirement is of more importance as a matter of principle and is connected with the difference between the conservation laws for phonon quasi-momentum and for ordinary momentum. It is well known that in any process where the phonons interact with one another the conservation law has the form

$$\sum \mathbf{k}_i = \sum \mathbf{k}_f + 2\pi\hbar \mathbf{b} m \qquad (m = 0, 1, 2, \ldots); \qquad (10)$$

here $\Sigma \mathbf{k}_i$ is the sum of the phonon quasi-momentum before the interaction, $\Sigma \mathbf{k}_f$ the sum of the quasi-momenta after the interaction, and b a so-called reciprocal lattice vector which is characteristic for a given crystal.

If m = 0 (such interaction processes are called "normal") quasi-momentum is conserved. In those processes, however, in which $m \neq 0$ (such processes are called "Umklapp processes") the total quasi-momentum of the phonon gas is not conserved. This leads to a violation of Eq. (5): in it a "friction force" of the phonons on the crystalline lattice appears and this leads to damping of second sound. For the propagation of second sound the following condition is thus also necessary*

$$\lambda_2 \ll l_{\text{Umk}} \qquad (11)$$

where l_{Umk} is the phonon mean free path referring to Umklapp processes. Fortunately l_{Umk} increases steeply at low temperatures. In fact, combining (10) with the energy conservation law we understand easily that when $m \neq 0$ at least one of the initial phonons must have a quasi-momentum $\sim\hbar b$. Bearing in mind that

$$b \sim \frac{1}{a}, \quad \frac{\hbar c}{a} \sim \Theta,$$

where a is the interatomic distance, Θ the Debye temperature, we see that the energy of the phonon $\sim\Theta$. The number of such phonons is for $T \ll \Theta$ proportional to

$$e^{-\Theta/T}.$$

This means that

$$l_{\text{Umk}} \sim e^{\Theta/T}, \qquad (12)$$

i.e., it increases exponentially with decreasing temperature.

To derive the third condition we note that in Eqs. (5) and (6) we assumed that the phonon gas was in thermodynamic equilibrium. This equilibrium is secured by the "normal processes" of interactions of phonons with one another. In order that equilibrium can be established during a period of the sound vibrations it is thus necessary that

$$\lambda_2 \gg l_n, \qquad (13)$$

where l_n is the phonon mean free path with respect to "normal" interaction processes. Conditions (11) and (13) are compatible only when

$$l_{\text{nep}} \gg l_n. \qquad (14)$$

At low temperatures "three-phonon processes" occur the most often. These are the decay of one phonon into two others or the inverse process. A theoretical estimate shows that in that case[8]

$$l_n \sim T^{-5}.$$

This means that at low temperatures condition (14) must in all cases be satisfied. We note also that (13) means that l_n must be small compared with the characteristic crystal dimensions R. The first conditions when

$$l_{\text{Umk}} \gg R \gg l_n \qquad (15)$$

where realized in Mezhov-Delgin's experiments.[9] The results of[9] are thus an important stage along the path to observing second sound in solids.

Let us still emphasize that, for a given wavelength λ_2, condition (11) limits the temperature from above, and condition (13) from below.

Let us now immediately turn to a description of the experiments.[1] In these experiments they did not study the propagation of sinusoidal vibrations but thermal pulses of length $\tau \sim 0.1$ to 5.0 μsec. The experimental setup is illustrated in Fig. 1. The space which is not hatched is filled with a single crystal of solid helium of dimensions 9×8 mm grown under a pressure of 54.2 atm. The authors studied 13 samples, of which four turned out to be sufficiently good to observe second sound. The number 1 in the figure indicates the position of the emitter of thermal pulses - a carbon resistance which is heated by pulses of an electrical current. The number 2 shows the position of the receiver (detector) of the vibrations - a carbon thermometer. The change in temperature of the detector was of the order of 10^{-2}. The copper bar 3 served to remove the heat when the crystal was grown.

In Fig. 2 we show typical experimental curves. Curves a) and c) show the time-dependence of the detector temperature and the curves b) and d) the time-dependence of the rate of change in temperature $d(\delta T)/dt$.

*Phenomena occurring when the number of Umklapp processes is small are discussed in detail in the survey [7].

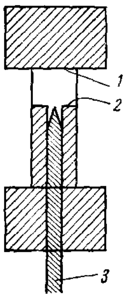

FIG. 1.

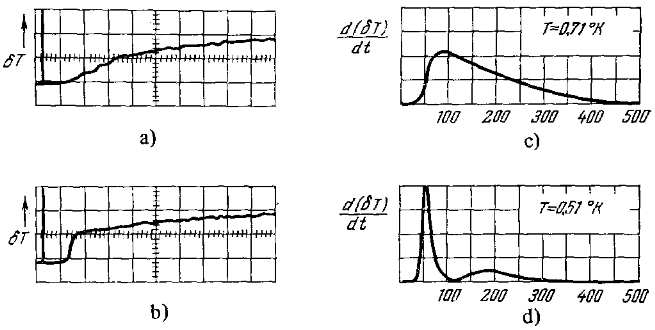

FIG. 2.

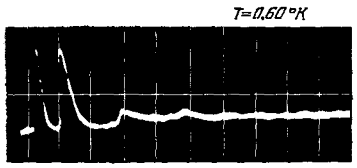

FIG. 3

Pressure, atm	33	54	100	130
c_2, m/sec	130	160	180	210

Curves a) and b) refer to $T = 0.71°K$. At this temperature, condition (11) is not satisfied. (For a thermal pulse this condition must be rewritten in the form $c_2/\tau \ll l_{Umk}$. The pulse in a point of the apparatus has a smearedout character.

When $T = 0.51°K$ (curves c) and d)), the conditions for the propagation of second sound are satisfied. In a point inside the apparatus there is a sharp pulse. Its speed of propagation $c_2 \sim 160$ m/sec corresponds to the estimate (8). Moreover, one sees clearly on the curve a second pulse of smaller intensity. This "echo" is the pulse reflected from the plane of the receiver and from the plane of the emitter, and again reaching the receiver. The existence of such an echo is a conclusive indication of the wave character of the process of the propagation of heat under the given circumstances.

Afterwards the authors repeated the experiments for different pressures. The distance between the emitter and the receiver was 0.77 cm, while the thickness of the crystal was approximately 2.5 cm.[2] This enabled them to reduce the scattering of the thermal pulses by the boundary of the sample. Under those conditions they succeeded in observing not one, but two reflected pulses (Fig. 3). The table of the dependence of the second sound velocity on pressure has the form

It is of interest to note that at temperatures below 0.5°K one can no longer satisfy condition (13). In that case heat begins to be transferred by the phonon current from the emitter to the receiver without collisions. The pulse broadens again and its velocity approaches the ordinary sound velocity.

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