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ON THE PHOTON MASS

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1. INTRODUCTION

THIS article is devoted to the experimentally known data concerning the mass of the photon. This question has not been discussed recently in the physics literature, and it is usually implied that the photon mass is exactly equal to zero.

The arguments frequently advanced in favor of the idea that the photon mass is strictly equal to zero are as follows:

1. The existence of electromagnetic action as a distance implies that the photon mass is very small compared with the masses of other particles, and there should be no small parameters in the theory.

2. The theory (relativity theory, quantum electrodynamics) requires that the photon mass be equal to zero.

It is easy to see, however, that both arguments are wrong. To verify that the first argument is incorrect, it is sufficient to recall that the ratio of the constants of the gravitational and weak interaction is approximately 10^{-34}. Thus, small parameters are encountered in physics. We note that if the photon mass m_γ were smaller than the electron mass by 34 orders of magnitude, then its Compton wave length $\lambda_\gamma = \hbar/m_\gamma c$ would equal* approximately 10^{23} cm $\approx 10^5$ light years.

As regards the assertions that the vanishing of the photon mass follows from the theory, we can state the following.

If the photon mass were not equal to zero, then no harm would come to the special theory of relativity; the velocity that enters in the Lorentz transformation would simply be not the velocity of light, but the limiting velocity c to which velocities of all the bodies tend when their energy becomes much larger than their mass.

Within the framework of quantum electrodynamics, the vanishing of the photon mass is the consequence of the so-called gauge invariance of the second kind. However, the absence of gauge invariance of the second kind does not lead to any difficulties, unlike the gauge invariance of the first kind, violation of which denotes nonconservation of the charge.

Actually, gauge invariance of the second kind is not the cause but the mathematical expression of the vanishing of the photon mass. Quantum electrodynamics with a nonzero photon mass has no theoretical flaws: the charge in it is conserved and is renormalizable. In some respects it is even simpler than ordinary electrodynamics, since its quantization does not require an indefinite metric.

Thus, the question of the photon mass is not theoretical but experimental. This circumstance was appar-

ently first formulated distinctly by deBroglie[1,2], who obtained the first estimates of the upper limit of the photon mass. More rigorous estimates were obtained in 1943 by Schrödinger[3,4]. These two authors apparently exhausted[1-7] all the presently known methods of determining the upper limit of the photon mass. The question of limits for m_γ was discussed also by Ginzburg[8]. In the present brief review we shall refine somewhat the estimates on the basis of the latest experimental data, and compare the estimates obtained by different methods. We shall see how the photon mass will affect its free motion in vacuum (Sec. 2), the interaction between charges and currents realized as the result of exchange of virtual photons (Sec. 3), and finally, the properties of blackbody radiation (Sec. 4). The main results of this analysis are listed in a table at the end of the article.

We shall not discuss here the possible cosmological manifestation of a nonzero photon mass.*

2. PHOTON MASS AND VELOCITY OF LIGHT AND OF RADIO WAVES

As already mentioned, the presence of a photon mass would bring about a situation wherein the velocity of the photon in vacuum would no longer be a universal constant, and would depend on the photon energy, just as in the case of other particles with nonzero mass. For the group velocity of the electromagnetic waves we would have

$$\frac{v}{c} = \frac{k}{\sqrt{k^2 + m_\gamma^2 c^2}} = \frac{1}{\sqrt{1 + \left(\frac{\lambda}{\lambda_\gamma}\right)^2}}.$$

As a result the velocity of, say, blue light would be larger than that of red light. It was precisely this circumstance which was used by de Broglie to estimate the upper limit of the photon mass. He noted that the dispersion of the speed of light in vacuum would lead to the occurrence of color phenomena in the case of eclipses of double stars: the blue light would arrive earlier than

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*We shall use frequently also the system of units with $\hbar = c = 1$, and then $\lambda_\gamma = 1/m_\gamma$.

*De Broglie[6] gave an estimate $m_\gamma < 10^{-66}$ g, meaning $\lambda_\gamma > 10^{10}$ light years ($\lambda = \lambda/2\pi$). This estimate was obtained from the assumption that the masses of the photon and of the graviton, if they differ from zero, should be commensurate. The graviton mass in Einstein's equations for the gravitation field can be connected with the so-called cosmological constant. From a comparison of the Friedmann theory of the expanding universe with experiment it is seen that the Compton wavelength of the graviton cannot greatly exceed the radius of the visible part of the universe ($\sim 10^{10}$ light years). It is obvious, however, that such an estimate cannot be regarded as an experimental lower limit for the Compton wavelength of the photon.

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the red light. If the minimum delay time of the red light compared with the blue light, which can be noticed, is denoted by δt, and the time required for the light to travel from the sun to the earth is denoted by t, then we readily obtain

$$\frac{\delta t}{t} \approx \frac{\lambda_1^2 - \lambda_2^2}{2\lambda_\gamma^2} \approx \frac{1}{2}\left(\frac{\lambda_1}{\lambda_\gamma}\right)^2 ,$$

where λ_1 is the wavelength of the red light and λ_2 that of the blue light. Assuming, as did de Broglie, that $\lambda_1 = 10^{-5}$ cm, $\delta t = 10^{-3}$ sec, and $t = 10^{10}$ sec (on the order of 10^2 light years), we get

$$\lambda_\gamma \gg \lambda_1 \sqrt{\frac{t}{\delta t}} \sim 10 \text{ cm,}$$

corresponding to*

$$m_\gamma \lesssim \frac{10^{-37}\text{g·cm}}{10 \text{ cm}} = 10^{-38} \text{ g.}$$

Color phenomena lasting several minutes were observed in eclipses of variable stars (the Tikhov-Nordman effect). However, as noted by P. N. Lebedev[9], this effect is due to the fact that different sections of the stellar atmospheres have different spectral characteristics. Therefore the limit of λ_γ should be even lowered, and it is therefore too low to be of interest.

A somewhat better lower limit for λ_γ, but still too low, can be obtained by using data on radio-wave propagation. It follows from measurements made by a group headed by L. I. Mandel'shtam[10] that the phase velocity of propagation of radio waves with $\lambda = 300$ m coincides with the velocity of visible light accurate to 5×10^{-4}.

From this we get

$$\lambda_\gamma \gg 1.5 \text{ km.}$$

A further increase of the limit for λ_γ by this method encounters the difficulty that the velocity change, connected with the influence of the earth's surface already becomes significant at the attained accuracy, as was already noted in[10].

3. PHOTON MASS AND STATIC FIELDS

The lowest limit for the photon mass follows from data on the static magnetic field (the earth's field). As will be seen below, data on the static electric field give a much worse limitation. In order to understand how these limitations arise, let us consider in greater detail the properties of electrodynamics with $m_\gamma \neq 0$.

In such a theory, the expression for the electromagnetic field has the usual form, and consequently the photon-emission vertex is also usual. The only deviation from the usual electrodynamics is that the photon propagator $D_{\mu\nu}$ does not have the form

$$D_{\mu\nu} = \frac{\delta_{\mu\nu}}{k^2} ,$$

but becomes equal to the propagator of the neutral vector meson:

$$D_{\mu\nu} = \frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{m_\gamma^2}}{k^2 - m_\gamma^2} .$$

By virtue of the conservation of the current, the vertices Γ_μ are transverse:

$$k_\mu \Gamma_\mu = 0$$

(we assume here and throughout that the current with which the photon interacts is conserved, since nonconservation of the current, and in particular nonconservation of the charge, would of necessity lead to the appearance of a photon mass), and therefore in the simplest cases, which we shall consider, the second term $k_\mu k_\nu/m_\gamma^2$ in the numerator of the propagator makes no contribution and we can write

$$D_{\mu\nu} = \frac{\delta_{\mu\nu}}{k^2 - m_\gamma^2} .$$

It is well-known that the static potential corresponding to the propagator $\delta_{\mu\nu}/k^2$ is the Coulomb potential

$$V(r) = \frac{Q}{r} .$$

The potential corresponding to the propagator $\delta_{\mu\nu}/(k^2 - m^2)$ is the Yukawa potential

Thus, the presence of a photon mass should lead to an exponential decrease of the interaction at distances exceeding the photon Compton wavelength $\lambda_\gamma = 1/m_\gamma$.

A similar change should take place also in the well-known expressions for the magnetic field. Thus, for example, in place of the ordinary expression for the field of a magnetic dipole*

$$A = \frac{[mr]}{r^3}$$

we get the expression

$$A = \left(\frac{[mr]}{r^3}\right)(1 + m_\gamma r)e^{-m_\gamma r}.$$

using this expression to describe the earth's magnetic field, and making use of the fact that the earth's magnetic field extends to distances on the order of 10^4 km, Schrödinger concluded that $\lambda_\gamma > 10^4$ km[3,4]. Data on the earth's magnetic field, obtained up to the time of Schrödinger work from cosmic-ray studies and studies of the northern lights, have by now been greatly supplemented by satellite measurements (see, for example,[11]). These measurements show that the magnetic field of the earth has the form of a magnetic dipole up to distances on the order of $6R_+ \approx 30\,000$ km. This indeed gives the best presently available limit of m_γ. Further refinement of this limit by measuring the earth's magnetic field is impossible, since at large distances the earth's magnetic field becomes comparable with the magnetic field of the solar plasma flowing around the earth—the so-called solar wind.

As noted by Gintsburg[8], the limit of λ_γ can be refined further by measuring the magnetic field of Jupiter. Results of radio observations of Jupiter in the decimeter band (200—300 MHz) and in the dekameter band (5—43 MHz) point to the existence on Jupiter of radiation belts at distances on the order of $2R_{11}$, and conse-

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*The estimate $m < 10^{-44}$ g given in [2,5] is apparently due to a misprint.

*$[m\,r] \equiv m \times r$.

quently of a strong magnetic field (from 1 to 10 G in accordance with various estimates). The radius of Jupiter $R_{,}$ is larger than the radius of the earth by approximately one order of magnitude ($R_{,}$ = 11.2R_{+} = 71 400 km); therefore the measurement of the magnetic field of Jupiter at distances of several times $R_{,}$ would make it possible to raise the limit of λ_γ by approximately one order of magnitude, possibly up to a million kilometers.

From the point of view of the question considered by us, it is of interest to increase further the accuracy of interference radio observation of Jupiter, which give a high spatial resolution, and particularly to perform an accurate measurement of Jupiter's magnetic field with the aid of rockets.

Let us consider now the limitations imposed on the photon mass by experiments aimed at verifying the Coulomb law. From among the experiments of this type known to us, the most accurate was performed in 1936 by Plimpton and Lawton[15]. In this experiment they measured the potential difference between two concentric spheres, the outer one being charged to a certain potential V_1. At an outer-sphere radius $R_1 \approx 75$ cm, inner-sphere radius $R_2 \approx 60$ cm, and potential V_1 = 3000 V they obtained $V_1 - V_2 \lesssim 10^{-6}$ V. (In the case of $m_\gamma = 0$, as is well-known, we should get $V_1 - V_2 = 0$.) The deviations from the Coulomb law were sought by Plimpton and Lawton in the form

$$V(r) = \frac{Q}{\gamma^{1+\varepsilon}}.$$

In this case we should get

$$\frac{V_2 - V_1}{V_1} \sim \varepsilon,$$

and it follows from the experimental result that $\varepsilon \lesssim 10^{-9}$. However, this experiment is less sensitive to the type of modification of Coulomb's law which is of interest to us. Indeed, in the case when

$$V(r) = \frac{Qe^{-m_\gamma r}}{r}$$

the field intensity at the outer surface of the charged sphere is

$$E = \frac{Qm_\gamma^2}{3},$$

and we get for the potential difference in the Plimpton-Lawton experiment

$$\frac{V_1 - V_2}{V_1} \sim \frac{1}{3} m_\gamma^2 (R_1 - R_2) R_1 \lesssim 3 \cdot 10^{-10}.$$

It follows therefore that the lower limit for the Compton wavelength of the photon is

$$\lambda_\gamma \approx 3 \cdot 10^4 \sqrt{(R_1 - R_2) R_1} \sim 10 \text{ km}.$$

This limitation is weaker by three orders of magnitude than that obtained from data on the earth's magnetic field.

4. PHOTON MASS AND BLACKBODY RADIATION

A photon with nonzero mass, unlike a photon with zero mass, has not two but three polarization states. It may therefore turn out that if the photon has a mass

then an additional factor 3/2 appears in Planck's formulas for the density of blackbody radiation, and thus the Boltzmann black-body constant should differ by a factor 1.5 from the observed one. It is easy to see, however, that this will not take place. The reason why the statistical description of black-body radiation remains practically unchanged[7] lies in the fact that the probabilities of transitions in which "longitudinal" photons take part contain a small factor m_γ^2/ω^2, where ω is the photon frequency.*

Let us consider, following[7] a photon gas situated in a cavity with linear dimensions L. For the time of transformation of the transverse photons in the cavity into longitudinal ones we get

$$t \sim \frac{L}{c} \left(\frac{\omega}{m_\gamma}\right)^2 = \frac{L}{c} \left(\frac{\lambda_\gamma}{\lambda}\right)^2.$$

Assuming that ω is in the optical region ($\lambda \sim 10^{-5}$ cm) and putting L = 10 cm and $\lambda_\gamma = 10^9$ cm, we get $t \sim 10^{18}$ sec $\sim 10^{11}$ years. Thus, no real effect arises. We note that at such small values of w_z the walls of practically any cavity will be transparent to the "longitudinal" photons.

CONCLUSION

Thus, the Compton wavelength of the photon is certainly larger than 30 000 km. It is possible to increase this limit by measuring the magnetic field of Jupiter at distance from the planet where this field is small. At present we see no other experiment capable of yielding a comparable accuracy.

We can raise the question whether it is necessary to strive to lower the limit of the photon mass if the theory with a zero photon mass seems more attractive from the esthetic point of view than the theory with nonzero mass, and if there are no theoretical grounds whatever for introducing the photon mass. After all, we do not see how the solution of any of the numerous problems of

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*In this note we present a simple derivation of this statement. Let the photon-emission amplitude A be given by

$$A = M_\alpha \epsilon_\alpha,$$

where ϵ_α is the 4-vector-potential of the photon normalized to a single photon, and M_α is a 4-vector that depends on the remaining 4-vectors of the problem (4-momenta of the particles and their spins). Let the photon momentum be directed along the z axis. In the system in which the photon is at rest, the probabilities of emission of transverse photons (w_x and w_y) and of a "longitudinal" photon (w_z) are respectively

$$w_x = |M_x^c|^2, \qquad w_y = |M_y^c|^2, \qquad w_z = |M_z^c|^2,$$

Where the index c denotes the rest system.

The same probabilities, expressed in terms of the components of the 4-vector M in the laboratory system, are respectively

$$w_x = |M_x|^2, \qquad w_y = |M_y|^2, \qquad w_z = |M_z|^2 - |M_0|^2.$$

We now use the transversality of the 4-vector M:

$$k_0 M_0 - k_z M_z = 0,$$

and obtain

$$w_z = |M_z|^2 \frac{k_0^2 - k_z^2}{k_0^2} = |M_z|^2 \frac{m_\gamma^2}{\omega^2}.$$

since our currents are the same as in ordinary electrodynamics, M_z is finite when $m_\gamma^2 \to 0$, and we can assume that M_x, M_y, and M_z are of the same order of magnitude. It follows therefore that

$$w_x \sim w_y, \qquad w_z \sim \frac{m_\gamma^2}{\omega^2} w_x.$$

Upper limit of the photon mass m_γ (lower limits of the
Compton wavelength of the photon λ_γ), which follow
from different experimental data

Physical phenomenon	λ_γ	$m\gamma/m_e$*	Literature**
Dispersion of the velocity of light from double stars	0.1 km	10^{-9}	de Broglie [2]
Velocity of radio waves	1 km	10^{-15}	Mandel'shtam [9,10]
Coulomb's law (measurement of the field inside a charged sphere)	10 km	10^{-16}	Plimpton-Lawton [15]
Extent of the earth's magnetic field	30 000 km	10^{-20}	Schrodinger [3,4]
Extent of the magnetic field of Jupiter***	10^6 km	10^{-21}	Gintsburg [8]

*m_e — electron mass.

**The numerical values of m_γ/m_e and χ_γ listed in the table differ in a number of cases from those presented in the cited papers.

***The measurements were not made.

modern theory of elementary particles can be facilitated by the presence of a photon mass.

However, esthetic arguments are frequently in error, and it is necessary to understand clearly that all we know at present is that $\lambda_\gamma > 30\,000$ km. We cannot guarantee that attempts to increase this limit will not lead to unexpected results.

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