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INTERACTION OF A MICROSYSTEM WITH A MEASURING INSTRUMENT

D. I. BLOKHINTSEV

Joint Institute for Nuclear Research

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1. INTRODUCTION

THE physical significance of quantum mechanics cannot be understood without a deep analysis of the measurement problem. This important circumstance already became clear in the very earliest time of development of quantum mechanics. The classical papers of N. Bohr[1], W. Heisenberg[2], and J. Von Neumann[3] laid the groundwork for the understanding of the interaction between the instrument and the measured object.

These papers, however, far from covered the entire problem. Later, different points of view were advanced, and the problem itself became the subject of various and frequently heated discussions. It suffices to recall the famous discussion following the publication of the well-known paper by A. Einstein, N. Rosen, and B. Podolsky[4], or the later disputes caused by the paper of D. Bohm[5]. The commemorative character of this issue makes it appropriate to recall that this journal has extensively and thoroughly discussed problems of quantum mechanics and allowed representatives of different points of view to advance their opinions.

"Different" ... Frequently we prefer to say "contradictory," "exclusive," etc. Now, however, when much has already been thought and much written, many of these "alternative" points of view are more reasonably regarded only as different aspects of the same scientific problem.

A story is told of a certain rabbi, who had a reputation for great cleverness. Somehow, two Jews, holding in their opinion, opposite views, appealed to him to resolve who is right.

After hearing them out, the rabbi said: "You Isaac, are right, and you Abraham, are also right." When the unsatisfied Jews complained to the rabbi's wife and asked her to use her influence and resolve the dispute, she told the rabbi: "I do not understand how you, being so clever, could admit that both were right, although they hold opposite views?" After thinking for awhile, the rabbi answered: "You, too, are also right." Thus, the problem consists apparently not so much of contrasting different points of view as in a successive development and deepening of the understanding of the problem.

It is not my purpose to describe in this review the history of the development of quantum mechanics. I wish to show that much progress has been made recently in the understanding of the measurement problem in quantum mechanics, and the purpose of the present article is to popularize this progress.

2. INFLUENCE OF MICROSYSTEM ON THE MEASURING INSTRUMENT

The very idea underlying this progress is not new. Many, *many years* ago the present author discussed with Professor A. A. Vlasov the advisability and the possibility of including in a quantum-mechanical analysis not only the measured object but also the measuring instrument, in order to be able to describe the entire measurement process by methods of mathematical physics as an objective physical process.

At that time, however, we could find no example of such a description clear enough to serve as a starting point for a new point of view. Furthermore, a highly influential concept of that time was that the instrument must not be included in the system described by quantum-mechanical methods.

It was assumed that this would call for the use of a new macroscopic instrument, which would again be described classically, and which would be necessary for the study of the situation in the complicated "microsystem plus instrument" system. Thus, the problem of the interaction of the microsystem and the instrument would be merely shifted elsewhere. We shall show below that in this lies the error of this widespread view of the relation between the instrument and the microsystem. The other side of the story was that the natural tendency was to emphasize the fact that the measurement influences the state of the measured macroscopic object, and the trivial but most important circumstance, that the micro-object is bound to influence the state of the measuring instrument, was left obscure. Otherwise, obviously, the instrument certainly does not perform its function. This influence of the macroscopic object on the measuring instrument can be investigated only if we combine the macroscopic object and the measuring apparatus into a single system and decide to consider it by methods of quantum mechanics. Before we proceed to discuss methods of such a unification, we recall the usual description of the measurement process in quantum mechanics.

From the purely formal point of view, measurement is described in quantum mechanics as a process of "reduction" of a wave function. Namely, if prior to the measurement the state of the microsystem was described by a wave function Ψ, which in the general case is a superposition of states Φ_L having definite dynamic variables:

$$\Psi = \sum_L C_L \Phi_L \tag{1}$$

(here C_L—amplitudes of the partial states), then the wave function Ψ "contracts" after the measurement of the dynamic variables L to one of the terms of the superposition (1), for example to Φ_L:

$$\Psi \longrightarrow \Phi_L. \tag{2}$$

This process is not described by any equation, and simply represents the results of the measurement: from a state of Ψ with an indefinite (in the general case) value of the dynamic variable L (so that the mean square value in this state is $\overline{\Delta L^2} \neq 0$) there arises after the measurement a state Φ_L with a definite value of this variable (in this state $\overline{\Delta L^2} = 0$). In the earlier stages of the development of quantum mechanics, the "reduction of the wave packet" was considered as a natural consequence of the interference of the measuring instrument with the state of the object.

However, in the already mentioned discussion between Einstein et al., on the one hand, and N. Bohr on the other, it became clear that the state of the microscopic object can change also in the case if the instrument does not interfere explicitly with its state. An explanation of the resultant paradox was presented from different points of view by N. Bohr[1] and by L. I. Mandel'shtam[6] (see also[7]).

What matters to us is that this discussion has paved the way to the interpretation of the wave function as the "log book" of the observer—a mathematical symbol containing the complete information on the possible results of any particular experiment and on the relative probability of these results.

From this point of view, the reduction of the wave packet is simply the mathematical notation for the measurement information obtained by the observer.

In this formulation of quantum-mechanical measurements, Einstein's paradox automatically disappears.

However, another problem is raised, noticed long ago by E. Schrödinger, and furthermore in a form that can jar the nerves of many of the readers. Namely, Schrödinger presents an example of an atomic system having two quantum states ψ_1 and ψ_2. In the general case its state is described by the wave packet

$$\psi = c_1\psi_1 + c_2\psi_2. \quad (3)$$

The first of these states causes operation of a Geiger counter; the second leaves the counter alone. Operation of the Geiger counter causes, by means of an auxiliary force, to break an ampoule with prussic acid in a chamber containing a cat[8].

Thus, an observer looking in his "log book" in order to predict the result of a future experiment, finds among the possible results the "fact" that interference can be produced between the states of a live and dead cat! Indeed, it follows from (3) that

$$|\psi_M|^2 = |c_1\psi_1|^2 + |c_2\psi_2|^2 + 2\mathrm{Re}\, c_1^* c_2 \psi_1 \psi_2; \quad (4)$$

the last term indicating this strange possibility.

After observing the actual event (ψ_1 or ψ_2), criminal medicine demonstrates either the death of the unfortunate cat or its good health, and the wave function "contracts" in the court record to ψ_1 or to ψ_2!

It is easy to see that this terrible example can be made even more exciting, if we replace the cat by the observer himself together with the medical officers. Then in case ψ_1 there will no longer be anyone to "contract" the wave function.

Let us turn, however, to more realistic examples. Let us imagine that we are dealing with the decay of a radioactive atom. Let the state ψ_1 be the state of the atom before decay, and the state ψ_2 of the atom after decay. Theory predicts that $c_1 \cong \exp(-\lambda t)$, where t is the time and $T = 1/\lambda$ is the half-life; $|c_2|^2 = 1 - |c_1|$, so that the coefficient c_2 increases with time, and the coefficient c_1 decreases. Let us imagine that we are dealing with the distant past, when no possible observer could transmit to us information on the actual fate of the radioactive atom. Let this be the time of the ichthyosauri! If the period t separating us from that time greatly exceeds the half-life T of the atom, then we can state with a high degree of probability that the atom has decayed. However, the instant when the atom has actually decayed is not at all immaterial to the environment of the atom.

It is appropriate to recall a story by science fiction writer R. Bradbury, who describes how travelers in prehistoric time carelessly crushed a butterfly, and this small event influenced the outcome of the presidential elections in the USA in the year 2000!

The decay of the atom could cause some chain of events, the course of which could greatly depend on the instant of time when this decay took place. Yet an observer who is our contemporary still did not have the opportunity to "contract" the wave function into a function ψ containing the superposition of two possibilities: the atom has decayed—ψ_2, or the other possibility ψ_1—it is still in the initial state.

If the contemporary observer still takes the trouble to measure the state of the atom, he is most likely to find that the atom has decayed and is in the state ψ_2. However, if $t \gg T$, our contemporary is seriously late in his conclusions, since any other observer could arrive at the same conclusion much sooner. Thus, we wish to express in the language of quantum mechanic the statement that "the atom has decayed" independently of the observer. Actually, this event leads to different consequences, depending on the instant of decay of the atom, and cannot therefore be connected with changes occurring in the information received by the observer.

The observer does not take part in the events referred to, and therefore should be excluded from consideration.

If we imagine a succession of observers, one of whom is our contemporary and the rest precede one another, then one of this succession of observers will be distinguished by the fact that he was the first to note the decay of the atom. This instant should have an objective value and should find its reflection in the formalism of quantum mechanics without involving the observer. The presently described paradoxes, which are inherent in the understanding of the wave function as a collection of information, as a "log book" of an observer, become clear if one subjects to analysis not only the action of the measuring instrument on the macroscopic system, but also the action of the microsystem on the instrument.

The idea that this aspect of the problem is significant was the basis of a monograph by the author[9] and was developed independently, in somewhat different form in a paper by the Italian physicists A. Daneri, A. Loinger, and G. Prosperi[10]*.

3.

We start with a simple example, which illustrates the possibility of mathematically describing the evolution of a combined system consisting of a macrosystem (M) and a measuring instrument (I).

* See also [11].

We note two important circumstances:

a) Each measuring instrument consists of two functionally different parts: an analyzer (A) and a detector (D): I = A + D. The first part of the instrument ensures separation of the partial states Φ_L, contained in the super position (1), into individual channels (L); in other words, it effects in practice the spectral decomposition of the complicated initial state Ψ of the microsystem into partial states Φ_L.

The second part, the detector, produces a macroscopic signal, which indicates in which channel the microparticle is actually situated. The first function of the measuring instrument as an analyzer of the quantum ensemble has been discussed in sufficient detail in the literature (see, for example,[7]). To the contrary, the second function, the detection D, was either left in the shadow, or was considered quite superficially. Yet it is precisely the investigation of the operation of the detector which eliminates the paradoxes that are inherent in the information aspect of the measurement process.

b) The detector must be a macroscopically unstable system. Indeed, otherwise the microsystem will not have enough momentum-energy resources to produce a macroscopic phenomenon—the operation of the detector.

This circumstance, which is obvious to the experimental physicists, has not been subjected to a sufficient theoretical analysis in the discussion of the measurement problem.

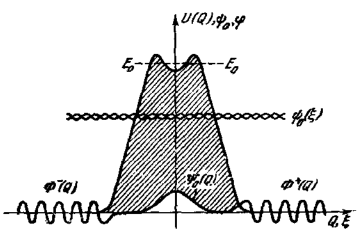
We now turn to simplest examples.

A. Determination of the Momentum of a Microparticle from its Interaction with a Macroscopic Body

We consider now a simple but somewhat formal example of the determination of the momentum k of a microparticle μ from its interaction with a macroscopic body M.*

It is obvious from the very outset that this body should be in an unstable (or in an almost unstable) equilibrium, otherwise the microparticle will not be able to "budge" it from its position.

We propose as such a body a small sphere with mass M; let the coordinate of its center of gravity be Q, and let the potential energy U(Q) be of the form shown in the figure. We consequently propose that the sphere is at the maximum of the potential energy U. Its relative stability is due to a small relative minimum of the energy U(Q). It is sufficient to impart to this sphere a slight energy $\Delta E > U_0 - E_0$, to make the sphere roll down the slope. Thus, the potential energy U(Q) has the form of a high volcano with a shallow crater (see the figure). The sphere serves as the detector determining the direction of the momentum of the microparticle (it can nudge this sphere to the right or to the left).



Potential energy U(Q) of heavy sphere M. E_0 - energy of its initial state. The figure shows also the wave functions $\Psi_0(Q)$, $\Phi^{\pm}(Q)$, and $\psi_0(\xi)$. The regions marked by the symbol $\Phi^{\pm}(Q)$ are the regions of quantum motion of the sphere; the symbol $\Psi_0(Q)$ denotes the region of classical macroscopic motion of the sphere.

*This example was first published in [12] (see also [9]).

In view of the fact that we assign to the sphere, for the sake of maximum simplicity, only one degree of freedom Q, it will be more convenient to describe the entire problem not by means of a density matrix but by means of wave functions.

We assume that at the initial instant of time t = 0 the micro-particle μ is described by the wave function

$$\psi_0(\xi) = A^+ e^{ik\xi} + A^- e^{-ik\xi}, \tag{5}$$

where ξ is the coordinate of the microparticle and k its momentum. Thus, it is assumed that there is a pure state, but with uncertain momentum $\pm k$. The task of our instrument is to determine the sign of the momentum (the direction of motion of the particle).

We denote the wave function of the macroscopic instrument (the sphere M) at t = 0 by

$$\Psi_0(Q) = \frac{1}{\sqrt{\pi}} e^{-Q^2/2a^2}, \tag{6}$$

where $a = \sqrt{\hbar/M\omega_0}$ and ω_0 is the frequency of oscillations of the sphere inside the crater. Thus, when t = 0 the wave function of the entire system "microparticle μ plus sphere M" will be

$$\Phi(Q, \xi, 0) = \Phi_0(Q, \xi) = \Psi_0(Q)\psi_0(\xi). \tag{7}$$

The Hamiltonian describing this system will obviously be

$$\mathscr{H}(Q, \xi) = -\frac{\hbar}{2M}\frac{\partial^2}{\partial Q^2} + U(Q) - \frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial\xi^2} + W(Q, \xi), \tag{8}$$

where $W(Q, \xi)$ is the energy of interaction between the sphere and the microparticle. The microparticle μ is regarded as free, and the sphere M has a potential energy U(Q). For simplicity we assume that $W(Q, \xi)$ is of the form

$$W(Q, \xi) = g\delta(Q - \xi) \tag{9}$$

and the wave function $\Phi(Q, \xi, t)$ for any instant of time t satisfies the equation

$$i\hbar\frac{\partial\Phi}{\partial t} = \mathscr{H}(Q, \xi)\Phi. \tag{10}$$

We seek this function in the form

$$\Phi(Q, \xi, t) = \Phi_0(Q, \xi) + \Phi^+(Q, \xi, t) + \Phi^-(Q, \xi, t). \tag{11}$$

Assuming that the coupling constant g is small, we find the functions Φ^+ and Φ^- in first approximation of perturbation theory. In this approximation

$$\Phi^+(Q, \xi, t) = \int U^+_{p'k'}(t)\,\Psi_{p'}(Q)\,e^{ik'\xi}e^{i(\omega_{p'}+\omega_{k'})t}\,dp'\,dk'$$

$$= e^{i(\omega_0+\omega_k)t}\int U^+_{p'k'}(t)\,\Psi_{p'}(Q)\,e^{ik'\xi}e^{-i\Omega t}\,dp'\,dk', \tag{12}$$

$$\Omega = \omega_0 + \omega_k - \omega_{p'} - \omega_{k'} = \frac{z}{t}; \tag{13}$$

here $\omega_0 = E_0/\hbar$—energy of the sphere in the initial state, $\omega_{p'} = E_{p'}/\hbar$, $E_{p'}$—energy of sphere in the final state, p′—momentum of sphere after going over into the excited state, and $\hbar\omega_k = \epsilon_k$ and $\hbar\omega'_k = \epsilon'_k$—energies of the particle before and after the interaction with the sphere. The function $\Phi(Q, \xi, t)$ has a similar form.

Further, integration of (10) after substitution of the function (11) with allowance of (12) yields

$$U^{+}_{p'k'}(t)=\frac{1}{\hbar}\frac{e^{i\Omega t}-1}{\Omega}U^{+}_{p'k'0k}, \qquad (14)$$

where the matrix element $U^{+}_{p'k'0k}$ is given by

$$U^{+}_{p'k'0k}=gA^{+}\int \Psi^{*}_{p'}(Q)\,e^{-ik'\xi}\delta(Q-\xi)\,\Psi_0(Q)\,e^{ik\xi}\,dQ\,d\xi. \qquad (15)$$

The function of the sphere in the excited state $\psi_{p'}(Q)$ can be written in the quasiclassical approximation in the form

$$\Psi_{p'}(Q)\sim N_{p'}e^{\frac{i}{\hbar}S_{p'}(Q)}, \qquad (16)$$

where $N_{p'}$ is a normalizing factor and $S_{p'}(Q)$ is the action function, approximately given by $S_{p'}(Q)\cong p'Q$. For this reason, the integral in (15) is equal to the Fourier transform $\Psi_0(\alpha)$ of $\Psi_0(Q)$ at $\alpha = p' + k' - k$. Therefore

$$\Phi^{+}(Q,\xi,t)=e^{i(\omega_0+\omega_k)t}\frac{gA^{+}}{\hbar}$$
$$\times\int N^{*}_{p'}\tilde{\Psi}_0(p'+k'-k)\frac{1-e^{-i\Omega t}}{\Omega}\Psi_{p'}(Q)\,e^{ik'\xi}\,dp'\,dk'. \qquad (17)$$

Let now $\mathcal{P}$ be the value of the sphere momentum which corresponds to the conservation of energy during the interaction. It follows from (15) that this will occur at $z/t = 0$ and

$$\omega_p=\frac{E_p}{\hbar}=\omega_0+\omega_k-\omega_{k'},\quad E_p=\frac{\mathcal{P}^2}{2M}+\text{const};$$

therefore

$$\frac{\mathcal{P}'^2}{2M}-\frac{\mathcal{P}^2}{2M}=\frac{z}{t}. \qquad (18)$$

From this we get that $\mathcal{P}' - \mathcal{P} = -z/vt$ and $d\mathcal{P}' = -dz/vt$, where $v = \mathcal{P}/M$ is the velocity of the sphere. Further, when $z \cong 0$ we have

$$\frac{k'^2}{2\mu}-\frac{k^2}{2\mu}=\omega_0-\frac{\mathcal{P}'^2}{2M}, \qquad (19)$$

or

$$(k'-k)(k'-k)=2\mu\omega_0-\frac{\mu}{M}\mathcal{P}'^2. \qquad (20)$$

if the function $\Psi_0(Q)$ is not too sharp (the amplitude is not very small, as is the case when the crater is not very deep!), then its Fourier transform $\tilde{\Psi}_0(p' + k' - k)$ will differ noticeably from zero only when

$$p'+k'-k\cong 0. \qquad (21)$$

From (20) and (21) it follows that as $M \to \infty$

$$k'=-k, \qquad (22)$$

$$\mathcal{P}=2k, \qquad (23)$$

as expected when a light particle collides with a heavy weakly-bound sphere, namely, an elastic reflection of the light microparticle took place, with a small energy transfer (vanishingly small when $M \to \infty$).

We now put $p' + k' - k = \mathcal{P} + k' - k - z/vt = q$, so that $vk' = dq$, $k' = q - \mathcal{P} + k + z/vt$. Introducing in the integral (17) the new integration variables q and z, we get from (17)

$$\Phi^{+}(Q,\xi,t)=\frac{gA^{+}}{\hbar}e^{i(\omega_0+\omega_k)t}\frac{|N_p|^2}{v}\psi_0(\xi)\,e^{i\mathcal{P}Q}e^{i(k-\mathcal{P})\xi}I^{+}\left(\frac{Q-\xi}{vt}\right), \qquad (24)$$

where

$$I^{+}\left(\frac{Q-\xi}{vt}\right)=\int_{-\infty}^{\infty}\frac{1-e^{-iz}}{z}e^{-\frac{iz}{vt}(Q-\xi)}\,dz. \qquad (25)$$

If we write down the well-known discontinuous integral

$$J(a)=\int_{-\infty}^{\infty}\frac{e^{iaz}}{z}\,dz=\begin{cases}\pi i, & a>0,\\ -\pi i, & a<0,\end{cases} \qquad (26)$$

then

$$I^{+}\left(\frac{Q-\xi}{vt}\right)=J\left(\frac{\xi-Q}{vt}\right)-J\left(\frac{vt+\xi-Q}{vt}\right). \qquad (27)$$

We note that $vt > 0$ for I^+, since $\mathcal{P} = 2k > 0$ in this case. It follows therefore from (27) that $I^+ = -2\pi i$ for $vt > Q - \xi$ and $Q - \xi > 0$; otherwise $I^+ = 0$. We recall that owing to the presence of the factor $\Psi_0(\xi)$ in Φ^+, only small values $|\xi| \lesssim a$ are important. Therefore the result denotes that $\Phi^+(Q, \xi, t)$ differs from zero when $t \to \infty$ only in the region $0 < Q < +\infty$, i.e., to the right of the vertex of the crater, corresponding to a positive momentum $\mathcal{P} = 2k$ acquired from the microparticle.

The function $\Phi^-(Q, \xi, t)$ is calculated in exactly the same manner. In this case $\mathcal{P} < 0$ and $v < 0$, and in place of the factor I^+ we obtain the factor

$$I^{-}\left(\frac{Q-\xi}{vt}\right)=J\left(\frac{Q-\xi}{vt}\right)-J\left(\frac{vt-Q+\xi}{vt}\right), \qquad (27)$$

which differs from zero only when $vt < Q - \xi < 0$. In this case the sphere rolls out of the crater to the left.

We now construct the density matrix for our case:

$$\begin{aligned}\rho(Q,\xi,Q',\xi')=\Psi^{*}(Q,\xi,t)\Psi(Q',\xi',t)=&\,\Phi_0^{*}(Q,\xi,t)\Phi_0(Q',\xi',t)\\&+\Phi_0^{*}(Q,\xi,t)\Phi^{+}(Q',\xi',t)-\Phi_0^{*}(Q,\xi,t)\Phi^{-}(Q',\xi',t)\\&+\Phi^{+*}(Q,\xi,t)\Phi_0(Q',\xi',t)+\Phi^{-*}(Q,\xi,t)\Phi_0(Q',\xi',t)\\&+\Phi^{+*}(Q,\xi,t)\Phi^{+}(Q',\xi',t)+\Phi^{+*}(Q,\xi,t)\Phi^{-}(Q',\xi',t)\\&+\Phi^{-*}(Q,\xi,t)\Phi^{+}(Q',\xi',t)+\Phi^{+*}(Q,\xi,t)\Phi^{+}(Q',\xi',t)\\&+\Phi^{-*}(Q,\xi,t)\Phi^{-}(Q',\xi',t).\end{aligned} \qquad (28)$$

When $t \to \infty$ and $|Q|, |Q'| > a$, all but the last two terms of this matrix vanish. Namely, the terms containing Φ_0 vanish when $Q, Q' \to \pm\infty$ like $\exp(-Q^2/a^2)$ or $(-Q'^2/a^2)$, and the interference terms, which contain products of the type $\Phi^{+*}\Phi^-$, vanish when $t \to \infty$, owing to the properties of the function $I^{\pm}(Q - \xi/vt)$. Therefore, when $t \to \infty$ and $|Q|, |Q'| \gg a$ we get

$$\rho(Q,\xi;Q',\xi',t)=\Phi^{+*}(Q,\xi,t)\Phi^{+}(Q',\xi',t)$$
$$+\Phi^{-*}(Q,\xi,t)\Phi^{-}(Q',\xi',t),\quad t\to\infty,\ |Q|,|Q'|\gg a. \qquad (29)$$

We see that the macroscopic instrument destroys the interference of the states of the microparticle $A^+e^{ik\xi}$ and $A^-e^{-ik\xi}$; further, when Q and $Q' \to +\infty$ we have

$$\rho(Q,\xi;Q',\xi',t)\to\Phi^{+*}(Q,\xi,t)\Phi^{+}(Q',\xi',t) \qquad (30)$$

and when $Q, Q' \to -\infty$ we have

$$\rho(Q,\xi;Q',\xi',t)\to\Phi^{-*}(Q,\xi,t)\Phi^{-}(Q',\xi',t). \qquad (30')$$

These two cases correspond to observation of the sphere either on the right of the crater (30) or on the left (30′).

When $Q \to +\infty$ and $Q' \to -\infty$ or $Q \to -\infty$ and $Q' \to +\infty$ (this is a case of interference of the results of the observations from the right and from the left) we have $\rho(Q, \xi; Q', \xi', t) \to 0$. This is to be expected from a "good" instrument: its "pointer" should occupy one of the possible definite positions. In our example the "pointer" is the heavy macroscopic sphere. It is clearly seen how a quantum phenomenon—the scattering of a quantum by the sphere M—changes by itself, owing to the weak stability of the sphere located on the top of the potential mountain, into a macroscopic classical phenomenon—motion of a heavy sphere to the right or to the left of the crater. The macroscopic nature of the phenomenon is ensured by the sufficient height of the potential mountain, on the top of which the sphere was initially at rest.

B. Thermodynamically Unstable Detector

Let us consider schematically an example of a thermodynamically unstable microparticle detector[9]. The microparticle is an atom having one valence electron, so that the entire atom has a magnetic moment $M_B\sigma$ equal to the magnetic moment of this electron, where M_B is the Bohr magneton, and $\sigma(\sigma_x, \sigma_y, \sigma_z)$ is the Pauli spin matrix. The wave function Ψ of the atom can be written in the form

$$\Psi(Q, x) = \Psi_1(Q)\psi_1(x) + \Psi_2(Q)\psi_2(x), \tag{31}$$

where $\Psi_1(Q)$ and $\Psi_2(Q)$ are functions describing the motion of the atom as a whole; $\psi_1(x)$ and $\psi_2(x)$ are functions describing the internal states of the atom and corresponding to the two possible orientations of the magnetic moment of the atomic electron.

For concreteness we shall assume that the magnetic field is parallel to the Oz axis, so that the function ψ_1 corresponds to the orientation of the moment along the Oz axis, and the function ψ_2 corresponds to the opposite orientation. We assume that an external magnetic field H, which we assume to be inhomogeneous, has already been to separate spatially beams of atoms having different magnetic-moment orientations, so that

$$\Psi_1(Q)\Psi_2(Q) = 0. \tag{32}$$

Thus, we shall assume that the first task of the measuring instrument is to destroy the interference of the states $\psi_1(x)$ and $\psi_2(x)$ corresponding to different spin orientations of the valence electron, and has already been performed. In other words, once the beams have passed through the inhomogeneous magnetic fields, it remains for us only to "install" under each of the beams a separate detector, which registers the arrival of a particle belonging to the corresponding beam, i.e., which actually registers one of the two states of the particle ("state" in the sense of its spin orientation).

We use as such a detector a system consisting of a large number of oscillators $s = 1, 2, \ldots, N$, $N \to \infty$, which, to avoid the use of additional symbols, will be assumed to be two-dimensional, oscillating in the (x, y) plane. Further, we assume that oscillations of type "x" and oscillations of type "y" do not interact in practice with each other.

This allows us to assign different temperatures θ to the "x" oscillations and to the "y" oscillations. Namely, we assume that in the initial state of the detector ("initial" in the sense of the interaction with the microparticle μ) the "x" oscillations are connected with a Gibbs thermostat having a temperature θ; therefore the "x" oscillations themselves also have at t = 0 the same temperature; as to the "y" oscillations, we assume that at t = 0 they are at absolute zero temperature. Thus, the detector is in a thermodynamically unstable state: any action, even small, coupling the "x" and "y" oscillations, immediately leads to intense energy transfer from the "x" oscillations to the "y" oscillations.

The "heating" the "y" oscillations is just that macroscopic phenomenon which identifies the state of the given individual microparticle, in our example the atom.

Let us consider now mathematically the operation of such a detector. The Hamiltonian of the unperturbed system of our oscillators is written in the form

$$\mathcal{H}_0 = \sum_{s=1}^{N} \mathcal{H}_0(x_s y_s) - E_0, \tag{33}$$

$$\mathcal{H}_0(x_s y_s) = -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial x_s^2} + \frac{\partial^2}{\partial y_s^2}\right) + \frac{M\omega_0^2}{2}(x_s^2 + y_s^2); \tag{34}$$

here E_0 is the zero-point energy, $E_0 = \hbar\omega_0/2N$, M is the oscillator mass, and ω_0 is the oscillator natural frequency.

The interaction energy W of these oscillators with the beam microparticle incident on the detector is assumed to be in the form

$$W = \omega \sum_{s=1}^{N} \mathbf{M}_{sz}\sigma_{sz} = -i\hbar\omega \sum_{s=1}^{N} \sigma_z \frac{\partial}{\partial y_s}, \tag{35}$$

where M_s is the mechanical moment of the oscillator and σ_z is the spin matrix of the optical electron of the atom. Since we have proposed that the atoms in the beam are oriented along the z axis, we have written $M_z\sigma_z$ in lieu of $\mathbf{M}\cdot\sigma$, where

$$M_z = -i\hbar\left(x_s\frac{\partial}{\partial y_s} - y_s\frac{\partial}{\partial x_s}\right) = -i\hbar\frac{\partial}{\partial\varphi_s},$$

and in lieu of σ simply $\sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. Obviously, for one detector it is necessary to take $\sigma_z = +1$ and for the other -1. We note that

$$x_s = r_s\cos\varphi_s, \quad y_s = r_s\sin\varphi_s, \tag{36}$$

$$r_s = +\sqrt{x_s^2 + y_s^2}. \tag{36'}$$

The detector D will be described by a density matrix ρ, which we take in the "x, y" representation.

Taking "x" to mean the entire aggregate of x coordinates of the oscillators $(x_1, x_2, \ldots, x_s, \ldots, x_N)$, and by "y" analogously all the coordinates $(y_1, y_2, \ldots, y_s, \ldots, y_N)$, we can write the matrix ρ in the form $\rho = (x, y; x', y', t)$.

The matrix ρ satisfies the equation

$$\frac{\partial\rho}{\partial t} + [\mathcal{H}_0 + W, \rho] = 0. \tag{37}$$

It will be more convenient for us to use in place of the matrix ρ the matrix

$$\tilde{\rho} = e^{\frac{i\mathcal{H}_0 t}{\hbar}}\rho e^{-\frac{i\mathcal{H}_0 t}{\hbar}}. \tag{38}$$

we note that $[W, \mathcal{H}_0] = 0$, and therefore $\tilde{W} = W$, and, substituting in place of ρ its expression in terms of $\tilde{\rho}$, we obtain

$$\frac{\partial\tilde{\rho}}{\partial t} + [W, \tilde{\rho}] = 0. \tag{39}$$

When substituting the operator W from (35), it is necessary to adhere to the rules for the multiplication of matrices with continuous rows and columns. To employ this rule, W must be written in matrix form. For example, it is necessary to write in lieu of the operator

$$-i\hbar\frac{\partial}{\partial q}$$

$$\mathcal{P}_{q'q''} \equiv i\hbar\frac{\partial}{\partial q'}\delta(q'-q''). \quad (40)$$

The multiplication $\mathcal{P}\rho$ denotes

$$(\mathcal{P}\rho)_{q'q'''} = \int \mathcal{P}_{q'q''}\rho(q''', q'')\,dq''' = -i\hbar\frac{\partial}{\partial q'}\rho(q', q'') \quad (40')$$

etc. If we use these simple rules, then the substitution of W in (39) will lead, in expanded form, to the simple partial differential equation

$$\frac{\partial\widetilde{\rho}}{\partial t} + \omega\sum_{s=1}^{N}\left(\frac{\partial\widetilde{\rho}}{\partial\varphi_s} + \frac{\partial\widetilde{\rho}}{\partial\varphi'_s}\right) = 0. \quad (41)$$

This equation can be solved in elementary fashion. Its general integral is

$$\widetilde{\rho} = \widetilde{\rho}(\omega t + \varphi_1, \omega t + \varphi_2, \ldots, \omega t + \varphi_s, \ldots, \omega t + \varphi_N, r_1, r_2, \ldots, r_s; \omega t + \varphi'_1, \omega t + \varphi'_2, \ldots, \omega t + \varphi'_s, \ldots, \omega t + \varphi'_N, r'_1, r'_2, \ldots, r'_s, \ldots, r'_N), \quad (42)$$

in which $r_1, r_2, \ldots, r_N$ and $r'_1, r'_2, \ldots, r'_N$ enter as parameters.

We now turn to the initial data for this matrix. In order not to clutter up the formulas with factors, we introduce as the unit length the quantity $l = \sqrt{\hbar/2H\omega_0}$, and we use in place of the temperature θ the reciprocal quantity $\beta = \hbar\omega_0/\theta$. In these units, all our quantities become dimensionless. When $t = 0$ we have $\widetilde{\rho}(x, y; x', y', 0) = \rho_\theta(x, x')\rho_0(y, y')$. In accordance with the assumptions made concerning the absolute zero temperature of the "y" oscillations, we have

$$\rho_0(y, y') = C_0 e^{-\frac{1}{2}\sum_{s=1}^{N}(y_s^2 + y_s'^2)}, \quad (43)$$

where C_0 is a certain constant normalization factor and $\exp(-y_s^2/2)$ is a wave function describing the angular oscillation of the s-th oscillator along the Oy axis.

The situation is much more complicated in the case of calculation of the matrix $\rho_\theta(x, x')$, since the "x" oscillations are at a temperature θ. In this case the state is mixed and the weights of the individual states $\psi_n(x)$ with energy E_n will be $\exp(-E_n/\theta) = \exp(-\beta E_n)$; therefore the matrix $\rho_\theta(x, x')$, which describes the ensemble in equilibrium with the Gibbs thermostat at temperature θ, is written in the form

$$\rho_0(x, x')\,e^{\beta F(\beta)}\sum_n e^{-\beta E_n}\psi_n^*(x)\,\psi_n(x') = e^{\beta F(\beta)}Z_\theta(x, x'), \quad (44)$$

where

$$Z_\theta(x, x') = \sum_n e^{-\beta E_n}\psi_n^*(x)\,\psi_n(x'). \quad (45)$$

The sum extends here first over all the states n having the energy E_n, and then over all states with different energies E_n. Even in the case of oscillators, the direct calculation of such a sum is very difficult. We shall therefore use a round-about way, based on the fact that if $\mathcal{H}(x)$ is the Hamiltonian of the system under consideration and $\psi_n^*(x)$ is its eigenfunction, then

$$\mathcal{H}(x)\,\psi_n^*(x) = E_n\psi_n^*(x) \quad (46)$$

and therefore

$$f(\mathcal{H})\,\psi_n^*(x) = f(E_n)\,\psi_n^*(x).$$

Therefore (45) can be written in the form

$$Z_\theta(x, x') = \sum_n e^{-\beta\mathcal{H}(x)}\psi_n^*(x)\,\psi_n(x') \quad (45')$$

and, differentiating with respect to β, we find that the sum $Z_\theta(x, x')$ satisfies the differential equation

$$\frac{\partial Z_\theta}{\partial\beta} + \mathcal{H}Z_\theta = 0. \quad (47)$$

In place of $\mathcal{H}(x)$ we should substitute here the unperturbed Hamiltonian operator for the "x" oscillations, i.e.,

$$\mathcal{H}_0(x) = \sum_{s=1}^{N}\left(-\frac{1}{2}\frac{\partial^2}{\partial x_s^2} + \frac{1}{2}x_s^2\right) - \frac{1}{2}N, \quad (48)$$

which we take from (33) and (34), taking into account the new units in which the length x is measured.

The variables in (47) separate, by virtue of the additivity of the Hamiltonian (48), and we can solve (48) in explicit form for one variable x; in this case we have

$$\frac{\partial Z_\theta(x, x')}{\partial\beta} - \frac{1}{2}\frac{\partial^2 Z_\theta(x, x')}{\partial x^2} + \left(\frac{1}{2}x^2 - \frac{1}{2}\right)Z(x, x') = 0. \quad (49)$$

We shall seek the solution of this equation in the form

$$Z_\theta(x, x') = \exp\{a + bx^2 + cxx' + bx'^2\} \quad (50)$$

and with boundary condition

$$Z_\theta(x, x') \sim \frac{1}{\sqrt{\beta}}e^{-\frac{1}{2\beta}(x-x')^2}, \quad \theta\to\infty, \ \beta\to 0, \quad (51)$$

corresponding to evaporation of the oscillators as $\theta\to\infty$. Equation (51) has the form of a sum for ideal-gas particles.

Substitution of (40′) in (49) leads to the equations

$$\frac{\partial a}{\partial\beta} = b - \frac{1}{2}, \quad \frac{\partial b}{\partial\beta} = 2b^2 - \frac{1}{2}, \quad (52)$$

$$\frac{\partial c}{\partial\beta} = 2bc, \qquad \frac{\partial b}{\partial\beta} = \frac{1}{2}c^2. \quad (52')$$

This system is compatible and has a solution

$$b = -\frac{1}{2}\frac{e^{2\beta}+1}{e^{2\beta}-1} \to -\frac{1}{2\beta}, \quad (53)$$

$$a = \int\left(b - \frac{1}{2}\right)d\beta = -\frac{\beta}{2} + \int b\,d\beta \to -\frac{1}{2}\lg\beta, \quad (53')$$

$$c^2 = (4b^2 - 1), \qquad c = -\sqrt{4b^2-1}; \quad (53'')$$

we introduce also

$$\gamma = \frac{c}{2b} = -\sqrt{1 - \frac{1}{4b^2}} \to -1. \quad (53''')$$

From these data for a, b, and c we see that the initial condition (51) is satisfied.

On the basis of (50), (53) and (53′), (53″), and (53‴) we can write the matrix $\rho_\theta(x, x')$ in explicit form:

$$\rho_\theta(x, x') = C_0 e^{b\sum_{s=1}^{N}(x_s^2 - 2\gamma x_s x'_s + x_s'^2)}; \quad (54)$$

here C_θ is a certain normalizing factor, namely $C_\theta = \exp[\beta F(\beta)]$, where $F(\beta)$ is the free energy of the oscillator. Taking now (43) into account, we obtain the total density matrix $\tilde{\rho}$ describing the state of the detector at t = 0:

$$\tilde{\rho}(x, y; x', y', \theta) = C_0 C_\theta e^{b \sum_{s=1}^{N} (x_s^2 - 2\gamma x_s x_s' + x_s'^2) - \frac{1}{2} \sum_{s=1}^{N} (y_s^2 + y_s'^2)}. \tag{55}$$

In order to find now the matrix $\tilde{\rho}$ at the instant of time t, we must take (36) and (36′) into account and replace everywhere in (55), in accordance with (42), the angles φ_s by $\varphi_s + \omega t$ and the angles φ_s' by $\varphi_s' + \omega t$. As a result we get

$$\tilde{\rho}(x, y; x', y', t) = C_0 C_\theta e^{\Delta + A\cos 2\omega t + B \sin 2\omega t}, \tag{56}$$

where

$$\Delta = \frac{b}{2} \sum_{s=1}^{N} [r_s^2 + r_s'^2 - 2\gamma r_s r_s' \cos(\varphi_s - \varphi_s')] - \frac{1}{2} \sum_{s=1}^{N} (r_s^2 + r_s'^2), \tag{57}$$

$$A = \frac{b}{2} \sum_{s=1}^{N} [r_s^2 \cos 2\varphi_s + r_s'^2 \cos^2 \varphi_s' - 2\gamma r_s r_s' \cos(\varphi_s + \varphi_s')] - \frac{1}{2} \sum_{s=1}^{N} (r_s^2 \cos 2\varphi_s + r_s'^2 \cos 2\varphi_s'), \tag{58}$$

$$B = -\frac{b}{2} \sum_{s=1}^{N} [r_s^2 \sin 2\varphi_s + r_s'^2 \sin 2\varphi_s' - 2\gamma r_s r_s' \sin(\varphi_s + \varphi_s')] + \frac{1}{2} \sum_{s=1}^{N} (r_s^2 \sin 2\varphi_s + r_s'^2 \sin 2\varphi_s'). \tag{59}$$

This somewhat cumbersome result must be averaged over the period $\omega/2$, if we assume that the frequency characterizing the coupling of the atom with the detector is sufficiently large.

Therefore the observed results will be determined by the matrix

$$\tilde{\rho}(x, y; x', y', t) = C_0 C_\theta e^{\Delta} \frac{1}{\pi} \int_{-\pi/2}^{+\pi/2} e^{[A\cos 2\omega t + B \sin 2\omega t]}\, dt. \tag{60}$$

The last integral reduces to a Bessel function:

$$\frac{1}{\pi} \int_{-\pi/2}^{+\pi/2} e^{[A\cos 2\omega t + B \sin 2\omega t]}\, dt = I_0(R), \tag{61}$$

where $R = (A^2 + B^2)^{1/2}$. Therefore the time-averaged matrix $\rho(x, y; x', y', t)$ is equal to

$$\tilde{\rho}(x, y; x', y', t) = C_0 C_\theta e^{A} I_0(R); \tag{62}$$

$$I_0(R) = 1 + \frac{1}{4} R^2 + \ldots, \quad |R| \ll 1, \tag{63}$$

$$I_0(R) \simeq \frac{e^R}{\sqrt{2\pi R}} + \ldots, \quad |R| \gg 1, \tag{63'}$$

so that at small values of R we have

$$\overline{\tilde{\rho}(x, y; x', y', t)} = C_0 C_\theta e^{\Delta} \tag{64}$$

and at large R

$$\overline{\tilde{\rho}(x, y; x', y', t)} = C_0 C_\theta \frac{e^{\Delta + R}}{\sqrt{2\pi R}}. \tag{64'}$$

If we recall that

$$b = -\frac{1}{2\beta} = -\frac{\theta}{2},$$

then the appearance of the factor e^{Δ} with Δ from (58) indicates that the energy has been distributed among the "x" oscillations and "y", and the temperature has dropped from θ to $\theta/2$.

At large values of R, the result also offers evidence of a redistribution of the energy among the "y" and "x" oscillations, but is not as clear as in the case of small R.

We see thus that a microparticle penetrating into a thermodynamically unstable detector has produced there a complete redistribution of the energy, i.e., a macroscopic phenomenon. It is seen from the foregoing example that the macroscopic measuring instrument must be an unstable system (more accurately, almost unstable).

By virtue of this instability, the initial quantum phenomenon changes automatically into a macroscopic phenomenon, with the aid of which the microsystems announce their appearance in the various channels.

Bearing in mind the tremendous scale of such a macroscopic phenomenon as compared with the microscopic initial phenomenon inducing it, we can regard the former as an explosion.

Thus, the microparticle announces its states by means of an explosion in the microworld.

CONCLUSION

From the point of view developed here, the reduction of the wave packet (2) reflects an objective process, consisting in the fact that the micromanifestation gives rise to a macrophenomenon.

This transformation of a quantum phenomenon into a macroscopic phenomenon can be traced mathematically.

When the measurement is regarded from this point of view, the paradoxes connected with the seemingly direct influence of the change of the observer's information on the course of real events drop out automatically, and the entire physical picture of the phenomena investigated by quantum mechanics can now be summarized as follows:

Quantum mechanics studies microsystems μ in a definite macroscopic setup $\mathfrak{M}$, or symbolically—it studies the sum $\mu + \mathfrak{M}$. The macroscopic setup can be resolved into two parts:

$$\mathfrak{M} = M + \mathrm{I}.$$

The first part dictates the conditions of motion of the microsystem μ, in other words, it determines the state of the microsystem. The second part I of the macrosetup is macroscopically unstable, and the microparticle is capable of producing macroscopic phenomena in it*. This part can be used by the observer as a measuring instrument, provided the presence of I, if possible, does not influence that part of the macrosetup M which organizes the initial state of the microparticle. The repetition (or in other words, the reproduction) of identical aggregates $\mathfrak{M} + \mu$ forms a quantum ensemble. This ensemble can be characterized by a wave function Ψ_μ (or in the general case by a density matrix ρ_M).

The repetition of the aggregates $\mathfrak{M}$ (or in other words, their reproduction) also forms a quantum ensemble, which can be described by a density matrix $\rho_{\mathfrak{M}}$. At the initial instant of time this density matrix can be written in the form of a product of two density matrices:

$$\rho_{\mathfrak{M}}^0 = \rho_M^0 \rho_{II}^0,$$

*It is clear that $\mathfrak{M}$ does not always contain I.

where ρ_I^0 is the density matrix describing the macroscopically unstable part of the macrosetup I at the initial instant of time.

The ensemble described by the density matrix ρ_M has that distinguishing feature that a macroscopic phenomenon initiated by the microsystem develops in it in the course of time.

The development of this phenomenon is indeed the physical mechanism causing the reduction of the wave function (2).

It is clear that a different organization of the macroscopically unstable part of the macrosetup I will lead to different types of reduction and will correspond, in the customary sense, to different measuring devices.

[1]N. Bohr, Usp. Fiz. Nauk **16** (4), 446 (1936); Experiment and Theory in Physics (Physics in My Generation, Pergamon, 1956, p. 205).

[2]W. Heisenberg, Physical Principles of Quantum Mechanics, Dover, 1930.

[3]J. von Neumann, Mathematical Foundations of Quantum Mechanics, Princeton, 1955.

[4]A. Einstein et al., Usp. Fiz. Nauk **16** (4), 440 (1936).

[5]D. Bohm, in: Voprosy prichinnosti v kvantovoĭ teorii (Problems of Causality in Quantum Theory) [Translations], IL, 1955.

[6]L. I. Mandel'shtam, Lectures on Quantum Mechanics, Collected Works, v. 5, AN SSSR, 1950.

[7]D. I. Blokhintsev, Osnovy kvantovoĭ mekhaniki (Principles of Quantum Mechanics), 4th ed., Vysshaya shkola, 1963.

[8]E. Schrödinger, Reports of Solvay Congress, 1928.

[9]D. I. Blokhintsev, Printsipal'nye voprosy kvantovoĭ mekhaniki (Fundamental Problems in Quantum Mechanics), Nauka, 1966.

[10]A. Daneri, A. Loinger and G. Prosperi, Nucl. Phys. **33**, 297 (1962).

[11]H. S. Green, Nuovo Cimento 9, 88 (1958).

[12]D. I. Blokhintsev, Voprosy filosofii (Problems of Philosophy), No. 9, 108 (1963).

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