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IONIZATION WAVES (STRIATIONS) IN A DISCHARGE PLASMA

L. PEKAREK

Physics Institute, Czechoslovak Academy of Sciences, Prague

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1. INTRODUCTION

THE stratification of the positive column of a discharge in a gas is the most readily observed type of waves and instabilities in a plasma. Nonetheless, this phenomenon has not been described even in the most complete review articles dealing with plasma wave phenomena and instabilities, whereas plasma waves and instabilities observed in the main only during the last 10–15 years in connection with investigations of hot plasma have been so thoroughly investigated that they are already described in textbooks. This is all the more strange, since striations, both standing and, more frequently, moving, are encountered in a discharge plasma so frequently, that it is precisely the stratified positive column which is the typical form of a discharge plasma of most gases or their mixtures.

One of the reasons for the little attention paid to striations lies in the physical nature of this phenomenon. The waves causing the stratification phenomenon belong to a special and rather peculiar class of wave phenomena. Their mechanism cannot be reduced to any of the presently known ones. The stratification mechanism does not contain a parameter similar to the momentum (as is the case, for example, with acoustic and other waves), and the principal role in the propagation is played by the change in the ionization rate in the plasma. Waves causing the stratification can therefore be called ionization waves. We shall adhere to this nomenclature in the present article.

The exceptional nature of ionization waves is reflected in the character of the wave equation describing them, which contains only the first derivative with respect to time. The wave solution is due to the integral term, which expresses the change in the rate of ionization in the plasma. Attempts to interpret striations as acoustic waves, electroacoustic waves (ion sound), longitudinal electric waves connected with Langmuir oscillations of the electron or ion gas, etc. turned out to be unproductive in light of new experimental results with artificially excited striations, although satisfactory agreement was obtained at first glance with data obtained by measurements of spontaneously existing striations.

In addition, ionization waves differ from all other types of waves primarily in the abundance of dispersion relations. This could be observed only by artificial excitation of ionization waves, whereas an investigation of spontaneously existing striations, in which the majority of workers were engaged, gave not even the least hint of the complexity of their phenomenological picture (in particular, concerning the direction of the group velocity, which is usually opposite that of the phase velocity) and did not make it possible to determine the source of the striations and the method of their self-excitation. For example, many workers considered striations moving from the anode to the cathode to be the results of localized self-exciting oscillations in the anode region of the discharge, and the decisive role of the plasma in the occurrence and the description of the properties of the striations long remained unclear.

Thus, the phenomenological complexity of the ionization waves, and at the same time the rather unique physical mechanism of their occurrence and propagation, have greatly hindered their explanation, and for a long time it was difficult to establish a common point of view even among those who directly investigated the stratification phenomenon. The situation changed only in recent years, and at present the ionization mechanism of striations is universally accepted. Recently, practical interest also increased in ionization waves and the associated longitudinal instability of the discharge plasma, since they produce harmful effects in gas lasers.

In the present article we present (following a historical summary) a review of the phenomenological properties of ionization waves in different gases, and show their main laws. Principal attention is paid to experimental work on artificially produced ionization waves of small amplitude. From among the investigations of spontaneously-existing ionization waves, we cite only certain most important results. Nor do we discuss experimental research on those wave phenomena in the plasma, in which the ionization interpretation of the waves is not obvious (as for example [16, 18, 19, 81, 116–118, 163]) and which the authors attribute to different mechanisms. One cannot exclude the possibility that some of these wave phenomena will nevertheless turn out to be connected with ionization mechanisms, but at the present time there are not enough data for such a statement.

In the technical part, principal attention is paid to an illustrative physical explanation of the main mechanism of propagation of ionization waves and of their instabilities.

Through the courtesy of the authors who furnished preprints of their papers we were able to include, besides papers published in 1966, also data obtained in 1966 and contained in still unpublished works. This has greatly supplemented, in particular, the review of the phenomenological properties of ionization waves.

2. HISTORY OF THE INVESTIGATIONS

Spontaneously-existing moving ionization waves (moving striations) were described by Wullner (1874),[2] followed by Spottiswoode (1876).[3–7] Stationary ionization waves (standing striations) were described in 1843 by Abria,[1] although this phenomenon was probably already known to Faraday.

By now there are approximately 400 papers devoted to the stratification of a discharge plasma. A characteristic of papers published prior to 1950 is that they deal with waves—moving or standing—which exist in the discharge spontaneously, without attempts to produce them by external artificial means. In this respect, the earliest papers[3-10, 24, 25, 27, 28, 32, 33] present the most abundant and systematic experimental material, although the employed experimental devices (stroboscope, rotating mirror) were very simple. However, the results obtained in the self-excitation modes are difficult to interpret, as a rule: the wave amplitude is usually large, the waveform differs greatly from sinusoidal, the luminescence accompanying the waves frequently falls between the regions of the peaks of the radiated light, and the charged-particle concentrations differ by a factor of several times at the minimum and maximum of the wave (see, e.g., [3, 14, 23]).

The picture is further aggravated by observation of several types of waves that exist simultaneously[11, 12, 15, 17] by observation of a double structure of the stationary waves,[9, 10, 35] by the ambiguity of the measurement results (which in the case of large-amplitude waves depend on the history of the occurrence of the discharge, on the magnitude of the discharge current, and on the pressure[45]), and by the irregularity of the waves and the oscillations associated with them.[38]

In spite of these difficulties, certain authors succeeded in obtaining repeated, simple, and at the same time sufficiently general results. Pupp[20, 24, 25, 27, 28, 32, 33] investigated systematically the properties of ionization waves in inert gases at currents on the order of several amperes, and established the limit of their spontaneous existence (now called the Pupp limit) (see also Fig. 2). He also found a number of simple empirical relations between the velocity and wavelength in inert gases, on the one hand, and the discharge conditions on the other (current, pressure, column diameter, type of inert gas). Working close to the limit of spontaneous existence of the waves, Pupp obtained consistently repeated results, and the waveform was close to sinusoidal. He also established that ionization waves are not generated by oscillations of the anode potential drop: after these oscillations were suppressed by means of an auxiliary discharge at the anode, the waves continued to exist and to have the same parameters as before. The independence of the ionization waves of the oscillations in the anode region was also demonstrated by Van Gorcum:[34] The positive column was homogeneous immediately after the ignition of the discharge, and the layered state set in with a delay on the anode side. The velocity, frequency, and length of ionization waves as functions of the discharge conditions, as well as the interaction between the ionization waves and the anode oscillations, were investigated in [4, 14, 15, 18, 19, 21, 22, 31] (see also [48, 67, 68]). Probe measurements were made of the temperature and density of the electrons at different locations in the striation.[8-10, 24, 33]

Concluding the stage of research of spontaneously existing waves were the investigations of Donahue and Dieke.[38, 46] They used for the investigations a modern oscilloscopic technique with photomultipliers, and observed rapidly moving pulsed disturbances (inappropriately called negative striations, although their wavelength is usually larger than the length of the tube), propagating against the direction of motion of the ordinary striations—in the direction of the anode. On the basis of abundant experimental material they concluded that the stratification of the column is a typical and most common phenomenon for a discharge plasma.

A new stage of research with artificial formation of waves began with the investigations of B. N. Klyarfel'd and A. A. Zaĭtsev. Klyarfel'd[42] dealt essentially with stationary ionization waves. In some mixtures, for example in a mixture of neon with hydrogen, Klyarfel'd was able to obtain weakly damped (towards the anode) stationary waves, which could be destroyed (on the anode side) by applying a negative voltage to the probe or by an external magnetic field. In the same manner he succeeded in a number of cases to set the stationary waves into motion, or else to stop moving waves. In the homogeneous sections of the column, Klyarfel'd produced artificial waves by means of a crossing discharge, a transverse magnetic field, or by narrowing the column with the aid of a circular aperture. He observed in hydrogen also ionization waves moving towards the anode. Klyarfel'd concluded from these experiments that there is no fundamental difference between stationary and moving striations—they are periodic repetitions of a local perturbation in the plasma in the direction of the electron drift, and he regarded them as a phenomenon with large amplitude. Zaĭtsev[40, 43, 52] investigated moving ionization waves and induced them externally either with the aid of an auxiliary internal electrode, or by modulating the discharge current with an alternating voltage having a frequency close to the frequency of the waves. He was also able, at a sufficient perturbation amplitude, to synchronize spontaneously existing waves with an external disturbance. In the regime without self-excitation, he artificially produced ionization waves of different frequency, and measured their velocity with the aid of a rotating mirror or a stroboscope, thereby performing the first measurements of the dispersion of ionization waves.

A pulsed perturbation having an amplitude known to be small (in order to obtain a linear phenomenon) was used by the present author.[56, 57, 60] The method of pulsed small-amplitude perturbation made it possible to produce a transient ionization wave (stratification wave; see Fig. 4 below), which contains complete information concerning the dispersion. It turned out that the group velocity of the ionization waves is directed from the cathode to the anode, whereas the phase velocity (the velocity of the striations themselves) is directed towards the cathode. Different types of such inverse ionization waves were obtained: for example, in neon two are observed simultaneously,[60, 75] and in some cases even three types of ionization waves,[83] differing from each other only quantitatively. Besides the inverse ionization waves, which are typical of inert gases, the small-pulse perturbation method revealed in hydrogen direct waves, in which the group and phase velocities are directed towards the anode.[74] It was established for all these waves that they can be amplified in the group-velocity direction, and that self-excitation sets in if the amplification is large enough: the perturbation is transported at the group velocity with an ever increasing amplitude towards the anode, from which it goes back to the cathode region, but not through the plasma (the transport of the perturbation does not take place at the phase velocity), but through the external circuits by means of current oscillations.[60] The described feedback scheme was confirmed by many experiments,[53, 72, 95, 100, 104, 129] and was proved quite illustratively by means of artificial feedback using a high-frequency barrier that kept the waves from reaching the anode region.[125] Only absolutely unstable ionization waves (see Sec. 3F) or stationary striations can occur spontaneously without feedback.

Investigations of ionization waves by the pulse excitation methods were carried out also in [71, 77, 112, 144] and elsewhere.

Direct measurements of the dispersion of ionization waves by means of a perturbation having a time periodicity, following Zaĭtsev, were performed by Wojaczek,[71, 88] Ruthscher,[105, 132] Pfau,[94, 139] and a number of other investigators.[95, 138, 147] Measurements of the dispersion in hydrogen and nitrogen[94] also demonstrated quite clearly the so frequently emphasized[4, 5, 42, 82] fundamental similarity between moving and stationary striations: if the dispersion curve is extrapolated, it reaches the k axis (or the λ axis) precisely at the point corresponding to the length of the stationary striations (see Fig. 12 below). Consequently, both phenomena belong to the same dispersion curve.

Ionization waves were obtained, both at pressures in the diffusion regime and at pressures on the order of 10^{-3} mm Hg, by Barrett[154, 159, 179] in mercury vapor and striations in neon were investigated at somewhat higher pressure (1.8×10^{-2} mm Hg). Pfau and Rutscher[158] observed ionization waves in inert gases at a pressure on the order of 10^2 mm Hg simultaneously with thermal contraction of the column.

Thus, ionization waves were obtained so far in a pressure region from 10^{-3} to 10^2 mm Hg and at currents from fractions of a milliampere to about ten amperes in different gases, gas mixtures with metal vapors, and in mercury vapor.

Abundant experimental material has been accumulated by now on artificially produced ionization waves, and is described for the most part in Sec. 3.

Many authors undertook to explain the mechanism of stratification of the positive column and to present its theory. Thomson himself[13, 26] obtained a (spatially) periodic solution of simplified equations for a positive column, and identified the solutions with stationary striations. Druyvesteyn[29, 30] succeeded in obtaining a solution of more accurate equations in the form of moving waves. However, the experimental material available before 1954 did not yield enough reference points for the theory, and sometimes even led to errors (e.g., the statement that a large pressure amplitude is necessary, that the amplitude of the waves in the column is constant, etc.). From the point of view of modern knowledge of ionization waves, the closest to reality from among the earlier theories is that of Druyvesteyn, who obtained in essence the correct dispersion, provided his work is supplemented by suitable calculation. The theoretical papers of the Forties and of the early Fifties[36, 37, 39, 41, 44, 49-51, 54, 58, 61-66] came much closer to the solution of the problem of the physical mechanism of stratification, in spite of the fact that in many of these papers no account was taken of some process of importance to the propagation of ionization waves (most frequently the dependence of the ionization coefficient on the electron temperature, as in [62, 63], or pair collisions in general,[39] etc.).

Based directly on the recent experimental data obtained by the method of artificial pulsed-wave excitation are the papers,[69, 85] proposing a chain of microphysical processes occurring in a plasma and causing stratification via ionization, with allowance for the relaxation time of the space charge. In particular, the existence of two types of waves (slow and fast) is explained as being due to parallel action of two ionization mechanisms (direct and stepwise) with different rates of equilibrium establishment. A further development of this theory is contained in [103, 106, 114, 127, 142], where the physical mechanism of the ionization waves is considered first by using a simplified plasma model[103, 106] and then with account taken of other processes of importance for the occurrence of these waves.[114, 127, 142].

Also based on new data concerning the dispersion properties of striations are the theoretical papers [78, 87, 89-91, 93, 101, 102, 107, 109, 115, 141, 147, 153, 155, 171, 178]. In particular, Rother,[89, 90, 91] Wojaczek,[93, 102, 155, 178] and Nedospasov and Ponomarenko[141] start out with very complete systems of equations and aim to obtain dispersion relations that are known from experiments with artificial excitation of the waves. However, only Wojaczek got as far as a numerical solution that turned out to reflect correctly the properties of the ionization waves in argon at large currents near the Pupp limit.[178]

The equation for ionization waves[127, 142] was used also by Barrett[159, 179] and Righetti et al.[161] to explain measurements made on the backward waves in the region of low pressures, and by Garscadden et al.[157, 160, 174] to explain the forward waves observed in a contracted column by Pfau and Rutscher.[158]

3. EXPERIMENTAL RESULTS

A. Types of Waves

By now, a large variety of phenomenological types of ionization waves has been found: backward waves with phase velocity v directed opposite to the group velocity u (i.e., $v/u < 0$), forward waves ($v/u > 0$) with phase velocity larger than the group velocity ($v/u > 1$) or smaller than the group velocity ($v/u < 1$) (in absolute magnitude) and also a number of intermediate types of waves. To facilitate the description of the experimental results, we first present a classification of the different types of ionization waves, and introduce notation for them in accordance with the main phenomenological properties. As is well known, these properties are described by the dispersion equation

$$F(\Omega, K) = 0, \tag{3.1}$$

which connects the complex frequency

$$\Omega = \omega - i\varphi \tag{3.2}$$

with the complex wave number

$$K = k + i\gamma. \tag{3.3}$$

Equivalent information is yielded also by the real function S(z, t) of the spatial coordinate z and the time t, describing the transient wave process (the magnitude of the perturbation of the medium at the point z at the instant of time t) that develops after a local perturbation of the medium in the form of a Dirac pulse:

$$S(z, t=0) = \delta(z). \tag{3.4}$$

The transient wave S(z, t) is connected by means of the Fourier integral (see [109, 115])

$$S(z, t) = \mathrm{Re}\,\frac{1}{2\pi}\int_{-\infty}^{\infty} Ce^{i(kz-\omega t)+\varphi t}\,dk \tag{3.5}$$

with the dispersion equation (3.1), if the latter is written in the form of two equations for the real frequency ω and the time increment as a function of the real wave number k at $\gamma = 1$, i.e.,

$$\omega = \omega(k), \tag{3.6}$$

$$\varphi = \varphi(k). \tag{3.7}$$

The plot of the time increment $\varphi(k)$ of the ionization waves has—in all the cases known so far—a simple form with a maximum for a definite "optimal" wave number k (see Fig. 23 below), and if this maximum is sharp, then only sections with a wave number close to k_1 appear in the transient wave S(z, t), just as in the direct measurement of dispersion. In such a case, the dispersion and the increment are described with sufficient accuracy by the relations (see [109])

$$\omega(k) = \omega(k_1) + \left(\frac{\partial\omega}{\partial k}\right)_{k_1}(k-k_1) + \frac{1}{2}\left(\frac{\partial^2\omega}{\partial k^2}\right)_{k_1}(k-k_1)^2 = \omega_1 + \omega_1'(k-k_1) + \frac{1}{2}\omega_1''(k-k_1^2), \tag{3.8}$$

$$\varphi(k) = \varphi(k_1) + \frac{1}{2}\left(\frac{\partial^2\varphi}{\partial k^2}\right)_{k_1}(k-k_1)^2 = \varphi_1 + \frac{1}{2}\varphi_1''(k-k_1)^2, \tag{3.9}$$

where $(\partial\varphi/\partial k)_{k_1} = 0$, thus determining k_1, and $(\partial^2\varphi/\partial k^2)_{k_1} < 0$. The integral (3.5) is calculated in this case accurately and gives the transient wave in the form[109, 115]

$$S(z, t) = \frac{C}{\sqrt{4\pi|B|t}}\exp\left\{-\frac{(z-\omega_1't)^2}{4\bar{b}_1 t} + \varphi_1 t\right\}\cos\left[k_1 z - \omega_1 t + \frac{(z-\omega_1't)^2}{4\bar{g}_1 t} - \frac{\psi}{2}\right], \tag{3.10}$$

where

$$\bar{b}_1 = -\frac{\omega_1''^2 + \varphi_1''^2}{2\varphi_1''}, \quad \bar{g}_1 = \frac{\omega_1''^2 + \varphi_1''^2}{2\omega_1''},$$

$$|B| = \frac{1}{2}\sqrt{\omega_1''^2 + \varphi_1''^2}, \quad \psi = \mathrm{arctg}\left(-\frac{\omega_1}{\varphi_1''}\right).$$

The group velocity of the maximum of the wave packet u_1 and the phase velocity at the center of the packet v_1 are connected with the dispersion by the relations

$$u_1 = \omega_1', \tag{3.11}$$

$$v_1 = \frac{\omega_1}{k_1}. \tag{3.12}$$

A practical method of calculating the dispersion from the parameter of the transient wave is given in [180].

Table I contains eight out of the sixteen possible phenomenological types of waves with group velocity different from zero ($u_1 \neq 0$), and without an inflection point in the measured section of the dispersion curve. The remaining eight possible types are mirror images of the first eight types with respect to the t axis (or k respectively).

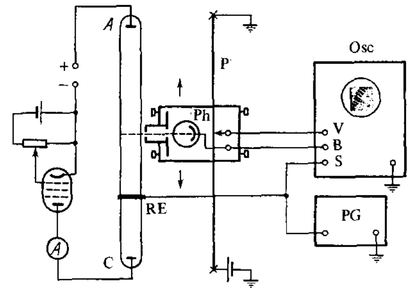


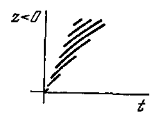
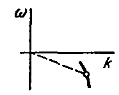
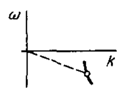
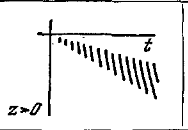
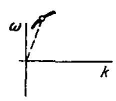
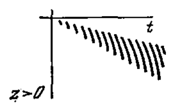
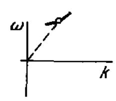
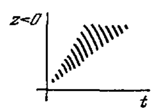
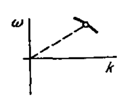
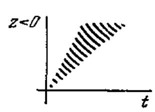
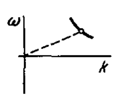
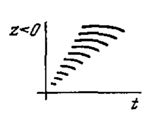
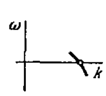
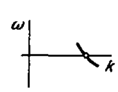
FIG. 1. Scheme for excitation of transition ionization waves (after [56, 60, 73]) and obtaining their space-time sweeps (after [164, 177]). A – anode; C – cathode of discharge tube; RE – external ring electrode; PG – pulse generator; Ph – photomultiplier; P – wire-wound voltage divider; V – input of vertical amplifier of oscilloscope Osc; B – input of beam-brightness modulation; S – input of beam sweep synchronization.

B. Artificial Excitation and Measurement of the Parameters of Ionization Waves

Figure 1 shows one of the variants of a scheme for artificially exciting transient ionization waves in a plasma.[56, 60, 73] The method of obtaining their oscillographic space-time sweep, developed by Stirand, Krejci, and Laska,[164, 177] is also clear from the figure. The plasma is perturbed locally by a brief pulse (or jump) of voltage applied to an external ring electrode RE. (A similar perturbation can be produced by a short-duration high-frequency field of a resonator,[79, 80] by a short-duration change of the current,[60] by an auxiliary internal electrode,[88, 111] or by a magnetic field of an external coil.[117, 126, 170, 179]) The wave propagates from the place where the ring is placed, and the change of plasma glow, which accompanies the wave, is received with a photomultiplier Ph. The signal modulates in this case (see [164, 177] the intensity of the beam of the oscilloscope, the time sweep being triggered by a pulse that serves simultaneously as the perturbation. A wire-wound voltage divider P supplies the vertical beam-deflection voltage, and the oscilloscope beam moves in a vertical direction in synchronism with the photomultiplier that moves along the tube. The successively produced individual horizontal strokes form on the oscilloscope screen the entire space–time sweep of the wave. The picture is equivalent to that obtained in a rapidly rotating mirror, but, of course, the electric method of obtaining the picture has better time resolution and higher sensitivity. A similar scheme can be used to investigate the self-excited waves, if the oscilloscope is synchronized by a signal from another stationary photomultiplier or by the oscillations of the discharge voltage. If the plasma is perturbed not with pulses having a low repetition frequency but with a signal having a frequency close to that of the striations, then it is possible to induce artificial waves that are periodic in time, and to measure, by varying the perturbation frequency, the corresponding wavelength $\lambda = 2\pi/k$ and the spatial gain γ by means of conventional oscillograms.[71, 105]

In some cases, photoelectronic converters were also used.[137, 140, 151, 172] In addition to measuring the oscillations of the light intensity, measurements were also made of the oscillations of the electron density and of

Table I. Types of Ionization Waves

Type designation	Transition wave	Dispersion	Plasma
F_D^-			Hydrogen [74,94,108], mixture of hydrogen with inert gas [120], deuterium [185]
F_A^-			Hydrogen, deuterium [185]
F_A^+			Nitrogen with traces of hydrogen [186], argon at thermal contraction [158]
F_D^+			
B_D^-			Neon [158], argon [184], mixture of neon with hydrogen (Fig. 11a, 13, and also [120,132,168])
B_A^-			Inert gases [110,111] (Fig. 5), nitrogen [94,139,161], oxygen [139,179], mercury vapor [143,179]
Z_D^-			Mixture of neon with hydrogen [120,168] (Fig. 11b), hydrogen [185]
Z_A^-			Hydrogen [185]

All the forward waves (v/u > 0) are designated F, the backward waves (v/u < 0) are marked B. Waves whose phase velocity passes through zero ($v_1 = 0$) are designated by the letter Z.

The indices of the corresponding letters denote the following: upper right: "+" – positive group velocity (i.e., by definition, directed from the anode to the cathode), "–" – negative group velocity; lower right: A–phase velocity in the transition wave accelerates in time in the direction of the group velocity, D–phase velocity decelerates (e.g., the symbol F_D^- denotes a forward wave in the direction of the anode with a phase velocity that decelerates in the direction of the group velocity. The symbol B_A^- denotes a backward wave towards the anode, having a phase velocity that increases in the direction of the anode. Since the wave is backward, the increase of the phase velocity in the direction of the group velocity corresponds here to a decrease of the absolute magnitude of the phase velocity in the course of time.

Forward waves F differ qualitatively if their group velocity is larger than or smaller than the phase velocity (in absolute magnitude). This difference is connected with transition from the normal (v/u > 1) to the anomalous dispersion (v/u < 1). Inasmuch as so far all the observed ionization waves F^- have v/u > 1, and the F^+ waves have v/u < 1, we do not designate this difference by any additional index.

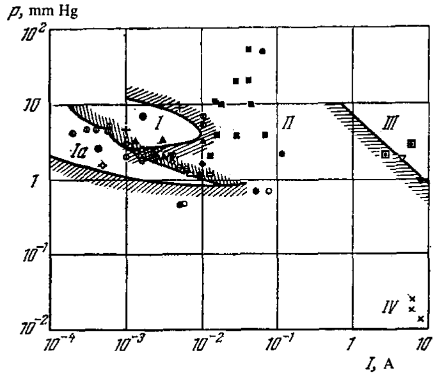


FIG. 2. Regions of dispersion measurements of ionization waves in neon at different pressures and currents (see Table II). Regions without spontaneously exciting waves: I – according to [84]; Ia – according to Michel [95]; III – according to Pupp [24]. II – region of self-exciting waves; IV – region of Barrett's measurements [179]. (For regions without spontaneously existing waves in different gases see also [128,131].)

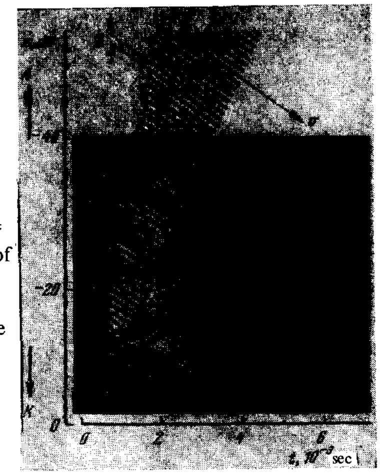


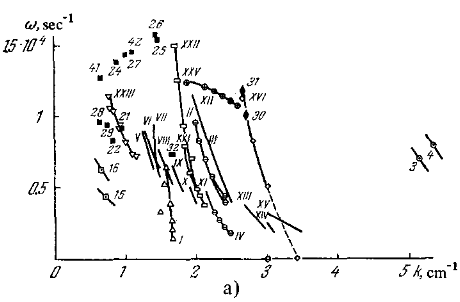
FIG. 4. Transition ionization wave (stratification wave) of type B^- in a positive-column plasma in neon. Tube diameter 1 cm, pressure 2 mm Hg, current 2.6 mA (wave velocity u = −200 m/sec, phase velocity of striations 29 m/sec). The perturbation was realized in this case by a jump of the discharge current. The photograph was obtained with the aid of a rotating mirror [56,57,60].

the electron temperature in the wave by means of high-frequency[123, 130, 135, 136, 148, 150] and probe methods.[59, 70, 152, 181]

C. Neon and Other Inert Gases

From the point of view of the properties of ionization waves, neon is the most investigated inert gas. Figure 2 shows, in the "current-pressure" plane, the regions in which the ionization waves were measured, the light circles denoting results of measurements made with artificially excited waves. The measured sections (or the points in the case of self-excited waves) of the dispersion curves are shown in Figs. 3a and b (see also Table II). All the observed ionization waves are backward ones, of the type B^{-1}, and in most cases B_A^-, i.e., the phase velocity increases towards the anode. Samples of the space-time sweeps of these waves are shown in Figs. 4 and 5. Both figures show the gradual stratification of the column following the perturbing pulse. (The positive column was homogeneous prior to application of the pulse.) At the point of perturbation, the first layer is produced instantaneously, and the succeeding layers are produced with a delay in the anode direction, although each of them individually moves in the opposite direction, i.e., towards the cathode. The transient process—the stratification wave—develops in such a way that the initial local aperiodic perturbation is gradually reproduced with alternating sign (i.e., the light region is followed by dark region, the dark one by a light one, etc.) with increasingly shorter distance to the anode, forming a layered structure, and in each layer the amplitude first increases, reaches a maximum, then decreases and the layer vanishes. Whereas the curvature of the phase trajectories cannot be determined reliably for the wave of Fig. 4, the wave of Fig. 5 shows clearly the bending of the phase trajectories (type B_A^-).

In addition to the indicated slow (or p_-) type of ionization wave, two fast types of ionization waves of type B^-, denoted r and s, were also observed in neon at low currents. By choosing carefully the current and the pressure, it is possible to find the place on the (I, p) diagram (Fig. 2) where all three waves (p, r, and s) are excited simultaneously with approximately the



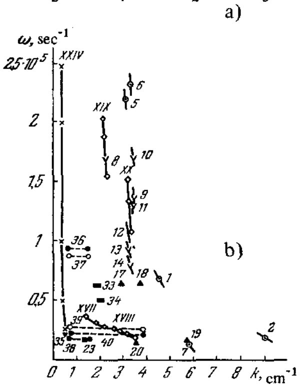


FIG. 3. Examples of results of measurements of dispersion in neon (the points without curves correspond to measurements in the self-excited regime). The symbols for the experimental points coincide with those of Fig. 2 (see Table II). Individual measurements and references are listed in Table II.

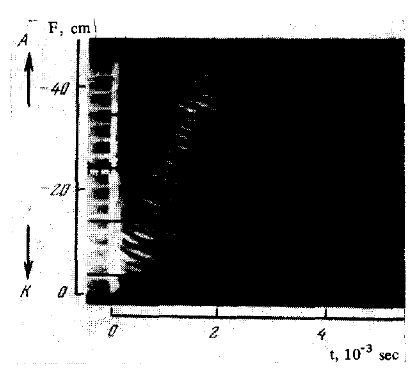


FIG. 5. Transition ionization wave of type B_A^- in neon, taken by an oscillographic space-time sweep method. Tube diameter 1.6 cm, pressure 1.92 mm Hg, current 5.5 mA. Velocities at center of packet: u = −290 m/sec, v = 39 m/sec (on the left side of the photograph – distance markers). The ending of the phase trajectories is clearly seen. (Photograph by K. Masek, from the files of the Physics Institute of the Czechoslovak Academy of Sciences.)

Table II. Dispersion measurements of ionization waves in the positive column in neon (pertains to Figs. 2 and 3)

Designation of experimental points	Number of curve or point	Pressure, mm Hg	Current, mA	Tube diameter, cm	Reference
△	I	2.6	1.5	3.5	43
⊖	II	1.4	6.1	2.2	139
	III	1.4	5	2	105
	IV	1.8	1.5	2.5	132
○	V	7	10		95
	VI	5.5	10		
	VII	3	1.0		
	VIII	5—7	0.6		
	IX	4—7	0.4		
	X	4—7	0.3	—	
	XI	4—6	0.2		
	XII	1.4—3	4		
	XIII	2	1.5		
	XIV	2	1		
	XV	5	0.55		
◇	XVI	1.7	4	2	121
	XVII *)	3.23	10	1.8	130
	XVIII	3.23	10	1.8	
	XIX *)	1.7	0.35	2	122
	XX	1.7	0.35	2	
▭	XXI	1.15	7	2.6	182
	XXII	1.15	12	2.6	
▽	XXIII	1.9	4200	3.1	111
×	XXIV **)	(18—26)·10^{-3}	6000	5	154, 179
⊕	XXV ***)	50	60	8	158
⊙	1 *)	7	1.7	0.6	75
	2				
	3	3	1.1	1.14	76
	4	3	1.1		
	5	3	0.35		
	6				
	7	2	2.6	1	60
V	8	2	2.6	1.1	83
	9				
	10		2.05	1.1	84
	11	1.9	3		
	12		4		
	13		5		
	14		6		
⊡	15	2.1	2630		119
	16	1.9	5750	4.55	
▲	17	3.2	10	1	53
	18		10		
	19		3		
	20		1.3		

*)One measurement, two waves.
**)One curve was obtained with the aid of experimental points from four different measurements. Magnetic field applied.
***)Contracted column, mercury-vapor pressure on the order of 10^{-4} mm Hg.

same amplification. This case is shown on the oscillograms of Fig. 6 (plotted in the usual manner). Both oscillograms show packets of the same three waves p, r, and s, but the time sweep of the lower curve is faster. Figure 7 shows the p, r, and s waves in a space-time sweep (on two photographs with different sweep velocities) together with the calculated sections of the dispersion curves. The parameters of these waves (of type B^-) are given in Table III. The p, r, and s waves should apparently be described by three individual dispersion curves (this follows also from the existence of two[146] or even three[182] types of stationary waves in neon).

Novak[99] found in the case of the three types of waves, p, r, s in neon (Fig. 8) that the product of the optimal wavelength $k_1 = 2\pi/\lambda_1$ and the electric field E_0 in the column is a constant quantity, so that the potential difference V_λ per wavelength in the low-current region is characteristic of a given type of wave:

$$\lambda_1 E_0 = V_\lambda. \qquad (3.13)$$

Table IV gives the values of the potential per wavelength for the ionization waves in neon, helium, and argon. The law (3.13) holds not only in a wide range of pressures, currents, and column diameters, but also when the column is placed in a strong magnetic field or is exposed to powerful light of constant spectral composition:[189] a change in the field E_0 by even a factor of 2 results in this case in an inversely proportional change of the optimal wavelength. The law of constant

FIG. 6. Oscillograms of packets of p, r, and s waves in neon. Top and bottom—the same phenomenon; on the lower photograph the sweep rate is larger. Tube diameter 1.1 cm, pressure 2.0 mm Hg, current 3.6 mA (according to [83]).

Table II. (Cont'd)

Designation of experimental points	Number of curve or point	Pressure, mm Hg	Current, mA	Tube diameter, cm	Reference
■	21	9.8	45	2.5	97
	22	10.3	18		
	23	3.9	16		
	24	3.9	30		
	25	3.9	65		
	26	2.1	12		
	27	21.9	40		
	28	21.9	40		
	29	53.2	40		
◆	30	1.6	10	1,9	133
	31	1.6	10	1,9	
▬	32	1.2	10	3	134
+	33	5	1	3	144
	34	9	5		
▼	35	1	750	2,5	137
●	36	0.48	5,7	0.7—1.5	147
○	37	0.48	6	1 —1.8	
●	38	2.2	110	0.9—1.3	
○	39	0.74	76	0,6—1.6	
●	40	0.74	50	0.6—1.6	
■	41	11	15	2.5	86
	42	21.9	27,5	2.5	98

I–XXV–dispersion curves measurement by the method of sinusoidal excitation.
1–16–dispersion curves obtained by the method of stratification wave;
17–42–experimental points determined from self-excited waves.

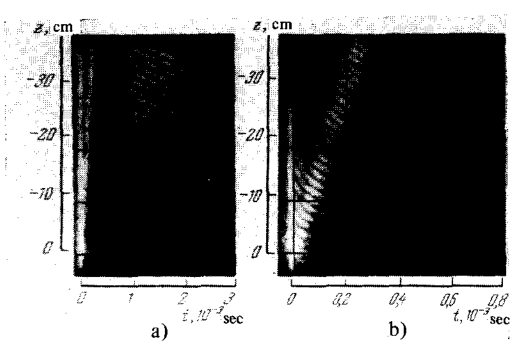
potential per wavelength does not hold for large currents (on the order of several amperes).

When the current and gas pressure are varied, the parameters of all three waves change without jumps, but in different manners for different types of waves: thus, for example, the phase velocity of the r and s waves and their amplification with increasing current decrease, while those of the p wave increase. In the middle of the region I (or Ia) on Fig. 2, all three waves are strongly attenuated.

Figure 9 shows a comparison of the velocity of the fast ionization waves in neon with the velocity of the atomic ions, calculated from the measured electric field and the assumed ion mobility. The velocities do not agree, although their order of magnitude and the character of their dependence on the current are the same (Table V).

Under the influence of light of constant spectrum composition, which lowers the concentration of the atoms in metastable states, the amplification of the slow wave decreases and that of the fast wave increases.[76] Cooling the gas to liquid-nitrogen temperature slows down the wave peak, and conversely, accelerates the r and s waves[183] (see Table V). This favors the assumption[69] that the p-wave is due to changes in the density of the atoms in metastable states.

In the case of large currents in excess of the Pupp limit, the picture of the ionization waves in neon is much simpler: a single wave of the type $B_{\bar{A}}$ is always observed, and the absolute magnitude of the group velocity of the wave is always approximately equal to the phase velocity, i.e., $u \approx -v$. The measured part of the dispersion curve is represented in this case in the (ω, k) plane by the equilateral hyperbola



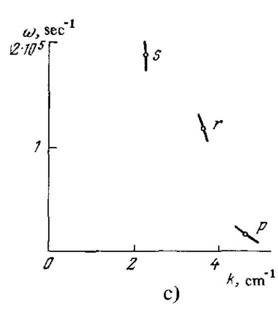


FIG. 7. Space-time sweep of p, r, and s waves in neon (a, b) and corresponding dispersion curves (c). Tube diameter 1.0 cm, neon pressure 2.0 mm Hg, current 3.4 mA. Photographs a) and b) differ only in the rate of the time sweep. (Photograph by K. Masek, from the files of the Physics Institute of the Czechoslovak Academy of Sciences.)

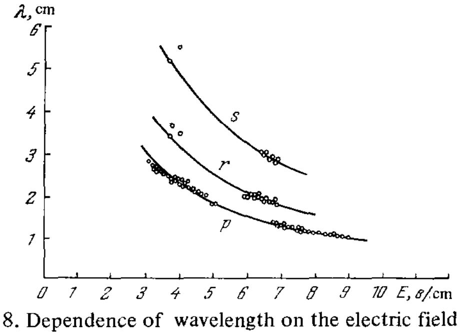


FIG. 8. Dependence of wavelength on the electric field in the positive column of neon for p, r, and s waves. Experimental points obtained at pressures from 1.0 mm Hg to 5.5 mm Hg, currents from 0.2 mA to 8.0 mA, and tube diameters from 1.1 cm to 2.1 cm (from [99].

$$\omega k = \text{const}. \quad (3.14)$$

Table VI lists numerical values of the wave parameters near the Pupp boundary in neon. The amplification curve has in this region a rather sharp maximum.

The same dispersion law (3.14) was observed also for ionization waves in neon at low pressures on the order of 10^{-2} mm Hg and at a current on the order of several amperes[154, 179] (see Figs. 2 and 3). In this case the $\varphi(k)$ curve is quite flat, and no gain is usually observed. To decrease the attenuation, a longitudinal magnetic field on the order of 15 Oe was applied to the plasma in these cases. To obtain a sufficiently distinct signal, a synchronous amplification technique was employed. Table VII shows certain numerical parameters of these waves as given by Barrett.[179]

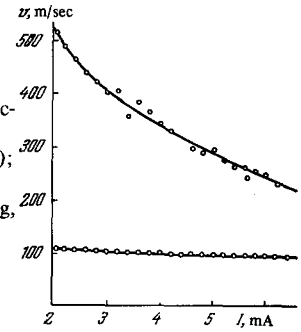


FIG. 9. Dependence of phase velocity v of fast ionization waves in neon on the discharge current (upper curve); lower curve—calculated velocity of atomic ions. Neon pressure 1.9 mm Hg, tube diameter 1.1 cm (from [84]).

In other inert gases in the region near the Pupp boundary, the position does not differ qualitatively from that observed for neon. Only waves of one type are observed here, with dispersion $\omega k \approx$ const. In the region of pressures near 1 mm Hg and of low currents 1–10 mA only one wave of the B_A^- type was observed so far in helium,[92, 173] three waves were observed in argon[184] (just as in neon), while krypton and xenon have in this region only self-excitated striations. The region of very low pressures was investigated by Barrett in argon. He found that at currents on the order of 5 A and

Table III. Parameters of three simultaneously excited waves of Type B^- in neon
Tube diameter 1 cm, current 3.4 mA, pressure 2 mm Hg (see Fig. 7)

Type of wave	u, m/sec	v, m/sec	λ, cm	k, cm^{-1}
p	−231	37.2	1.37	4.58
r	−1570	330	1.72	3.65
s	−4520	840	2.78	2.26

Table IV. Vales of potential per wavelength (Novak's constants) V_λ for Ne, Ar, and He (in volts)

Gas	Type of wave p	r	s	Reference	Gas	Type of wave p	r	s	Reference
Ne	9.2	12.67	19.48	99	He			30.05	99
Ar	6.7	9.5	12.0	184	He	~14.2			Unpublished measurements by V. Perina

Table V. Measurement of parameters: Wavelength λ, phase and group velocity v, u—for different types of ionization waves of type B^- in neon (I—current, E_0—electric field intensity, T_g—temperature of neutron gas, $p_0 = 298\ p/T_g$—reduced pressure).

A. Influence of irradiation (tube diameter 1.14 cm)

	I, mA		p_0, mm Hg		E_0, v/cm		λ, cm		v, m/sec		u, m/sec	
Type of wave	p	s	p	s	p	s	p	s	p	s	p	s
Non-irradiated gas	1.1	0.35	—	—	7.56	9.53	1.22	2.10	13.50	730	−81.40	−3150
Irradiated gas	1.1	0.35	—	—	7.66	9.67	1.17	1.99	14.80	736	−84.00	−2890

B. Influence of cooling (tube diameter 1.6 cm)

	I, mA		p_0, mm Hg		E_0, v/cm		λ, cm		v, m/sec		u, m/sec	
Type of wave	p	r	p	r	p	r	p	r	p	r	p	r
$T_g = 298°$ K	8.4	6.0	1.9	1.2	4.77	4.92	1.95	2.73	58.4	525	−273	−1350
$T_g = 77°$ K	8.4	6.0	1.9	1.2	5.34	5.17	1.76	2.52	41.1	620	−199	−1630

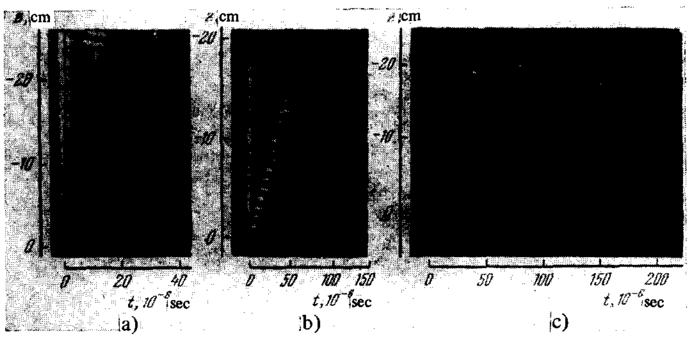


FIG. 10. Ionization waves in a mixture of neon and hydrogen. a) 71% Ne, 29% H_2, total pressure 1.55 mm Hg, current 20 mA. b) 60% Ne, 40% H_2, total pressure 1.6 mm Hg, current 13.4 mA. c) 55% Ne, 45% H_2, total pressure 2 mm Hg, current 6.0 mA. Tube diameter 1.7 cm. (Photographs from the files of the Physics Institute of the Czechoslovak Academy of Sciences.) (In the upper left of Fig. 10a—wave starting from the internal probe; the first wave in Fig. 10b is from the ring electrode, and the second from the cathode.)

Table VI. Numerical examples of ionization wave parameters near the Pupp boundary in neon

Type of wave	Tube diameter, cm	Current, mA	Pressure, mm Hg	u, m/sec	v, m/sec	λ, cm
B_A^-	3.1	4200	1.9	to −80 from −180	<150	to 5.4 from 8.3

pressures 10^{-2} mm Hg there exist simultaneously two ionization waves of type B_A^-.[154, 179]

D. Transition from the Backward Wave B^- to the Forward Wave F^-. Connection with Stationary Striations

In the current range from 0.1 mA to 1 mA and at pressures 1–3 mm Hg, where backward ionization waves of type B^- are observed in neon, forward waves of the type F_D^- with v/ < 1 were observed in hydrogen.[74] In a mixture of neon with hydrogen it is possible to observe both backward B^- and forward F^- waves.[120,132, 168] These waves are smoothly transformed into each other when the concentrations of the mixture components are varied. At the transition point there is observed a transition wave Z_D^-, the phase velocity of which reverses sign. The transition is shown for three ratios of the mixture components in Figs. 10a–c. The sections of the dispersion curves obtained from the parameters of the transition waves are shown in Fig. 11 by the heavy curves.

The production of the Z_D^- wave, which attenuates quite weakly towards the anode (as, for example, in Fig. 10b), is made difficult by the following circumstance: stationary striations, which begin from the cathode end of the column and attenuate quite weakly towards the anode, are produced in the column without any external perturbation. In order to operate with a homogeneous plasma, the waves were excited locally, at a point located 40–50 cm away from the cathode (i.e., more than 50 wavelengths). At such a distance, the stationary striations are already practically unnoticeable, and they have no influence whatever on the form of the Z^- wave (this was verified also in the following manner: the photomultiplier was fastened securely at one point, and the ring used for external excitation of the waves was moved). The spatial damping of the Z^- wave was identical, within the limits of error, with the damping of the stationary striations. On the other hand, the spatial period of the stationary striations (the corresponding wave number is designated by crosses in Fig. 11 for all three measurements) coincides, within the limits of measurement errors, with the wavelength of Z_D^- at the point where its phase velocity is equal to zero.

From this it is easy to conclude that the stationary periodic structure occurs when a dispersion curve $\omega(k)$ crosses the k axis, i.e., passes through the point with zero phase velocity.

When the dispersion curves for the B_D^- and F_D^- waves are extrapolated in Fig. 11, they cross the k axis at a point corresponding to the experimentally observed value of the wave number k for the stationary striations. This was already known on the basis of dispersion measurements made by the method of periodic (in time) perturbation[132] (Fig. 12).

Fig. 11 shows that the behavior of a mixture of neon with hydrogen can be described by a single dispersion curve: with increasing hydrogen pressure in the mixture, this curve shifts towards higher wave numbers k and simultaneously the maximum of the amplification (the minimum of the damping) shifts in the same direction, and at a rate faster than that of the dispersion curve itself. As a result, the point of the maximum gain (ω_1, k_1) on the dispersion curve slides to the right with increasing hydrogen content, downward along the dispersion curve, crosses the k axis, and enters the region of negative values of ω.

The stationary periodic structure, produced in the mixture in the anode direction, by any time-constant local perturbation of the column (which may be precisely the cathode end of the column), does not always attenuate. To obtain the waves shown in Fig. 10, the conditions (current, pressure, ratio of mixture components) are specially chosen such that attenuation is obtained at the point k_S where the dispersion curve crosses the k axis. By varying the current and the pressure it is easy

Table VII. Numerical examples of parameters of ionization waves in region IV of Fig. 2 (from [179])

Type of wave	Tube diameter, cm	Current, mA	Pressure, mm Hg	u, m/sec	v, m/sec	λ, cm
B_A^-	5	6000	0.018 and 0.026	to −1900 from −75 000	to 600 from 50 000	to 11.8 from 62.8

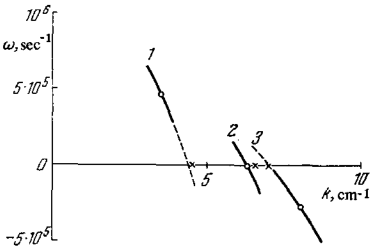


FIG. 11. Sections of dispersion curves obtained from the photographs of Fig. 10. The crosses denote the wave numbers measured on stationary striations near the cathode (from [162]).

to obtain amplification for k_s at a given mixture, i.e., $\varphi(k_s) > 0$. In this case the stationary structure fills the entire positive column and assumes over the entire length of the column the well known form of the stationary striations. The amplitude of this structure is limited by nonlinear phenomena.

E. Hydrogen

It follows in particular from recent investigations by Stirand et al.[185] that the picture of ionization waves in hydrogen is highly varied (compared with all the hitherto investigated gases). Figure 13 shows the regions of existence of ionization waves of different types at different currents and pressures. At a pressure exceeding 0.7 mm Hg and a tube diameter of 2.2 cm, only waves with negative phase velocity are observed in hydrogen (self-excited or artificially excited of type F^-), or else a stationary undamped periodic structure. However, by selecting the pressure and the current such as to eliminate the stationary undamped stratification and to make the artificially-produced ionization waves weakly growing or weakly damped, we obtain different types of waves and dispersions. Examples of the parameters of these waves are listed in Table VIII.

A very interesting phenomenon is observed in the cross-hatched region (0.25 mm Hg, 1.8 mA): whereas a type-B_A^- wave is observed at a smaller current and a type-F_D^- wave at a larger one, the transition between them does not occur via a wave similar to Z_D^- or Z_A^-, but via a unique wave which can be called a crossing wave (Fig. 14). The corresponding dispersion curve calculated from the wave parameters has an inflection point precisely on the k axis (Fig. 15). The group velocity of the wave is the same in the parts with positive and negative phase velocity, and the phase trajectories intersect, reaching zero velocity at the extreme right part of the space-time sweep (this part of the wave, however, is no longer resolved on the photograph). In spite of the complicated form, the wave $(F_D^- \times B_A^-)$ is a single wave, with a single dispersion curve.

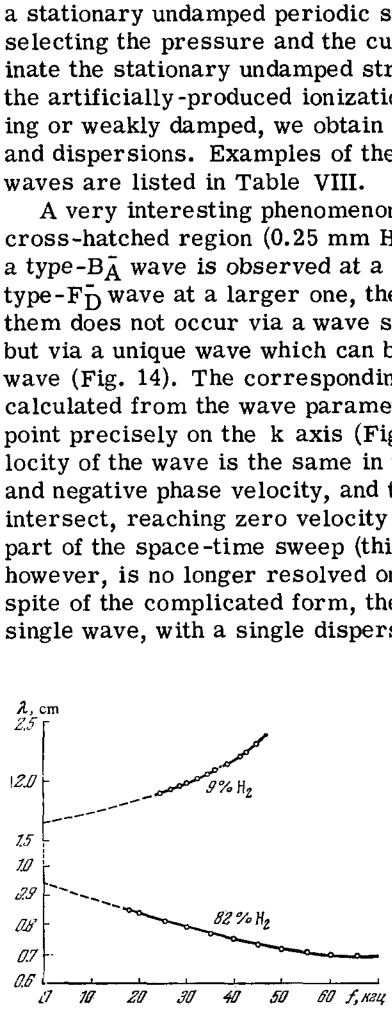


FIG. 12. Extrapolation of dispersion curves of a hydrogen-neon mixture. Pressure 1.8 mm Hg, current 5 mA and 1.75 mA, tube diameter 2.5 cm. The extrapolated curves reach the λ axis at points corresponding to the lengths of the stationary striations (from [132]).

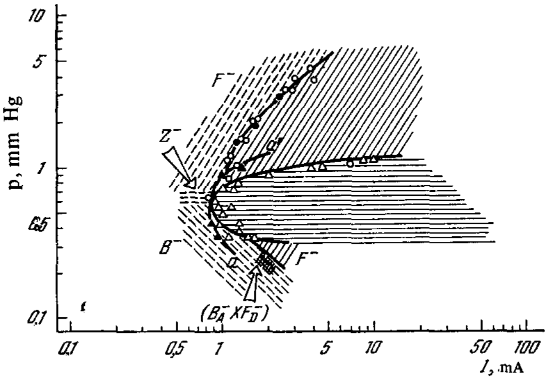


FIG. 13. Regions with different types of ionization waves in hydrogen. Curve aa′ corresponds to the places of constant spatial attenuation of the standing waves, $\gamma = -0.133$ cm^{-1}. Other curves – boundaries between regions of spontaneously existing standing waves (≡), moving waves (//), and region without self-excitation (dashed-line oblique hatching). The individual measurements are shown by points (e.g., see Table VIII). Tube diameter 2.2 cm (from [185]).

F. Oxygen

Unique ionization phenomena, not seen in other gases, are observed in oxygen. As is well known, the positive discharge column in oxygen can exist in two forms,[47, 55] low-gradient T-form and high-gradient H-form. The T-form is connected without exception with moving pulses of large amplitude.[96] In the H-form there are observed forward and backward waves of the type B^- and F^-.[176] Diagram 16, from Sabadil's paper,[176] shows the regions in the (I, p) plane with different types of waves. The measurements were made only by the method of periodic perturbation. The curvature of the dispersion curves can therefore not be determined with reliability.

In spite of the fact that the propagation of the pulse disturbances of the T-form belong undoubtedly to phenomena with large amplitude, it is characterized by simple dependences of the quantities on one another (e.g., the pulse velocity does not depend on the pulse frequency), and the nature of this propagation is apparently connected with ionization.[176]

G. Waves with Positive Group Velocity

In the region of medium pressures, near 10^2 mm Hg (see Fig. 2), the ionization waves appear simultaneously with the thermal narrowing of the positive column. It is difficult to produce here artificial excitation of these waves in such a way as to leave their amplitude small, and the phenomenon is made more complicated by simultaneous contraction of the column, which greatly changes all the plasma parameters. Nonetheless, Pfau

Table VIII. Examples of parameters of different types of ionization waves in hydrogen (see Fig. 14)

Type of wave	I, mA	p, mm Hg	E_0, v/cm	u, m/sec	v, m/sec	k, cm^{-1}	k_s, cm^{-1}
F^-	1.25	1.5	29.00	−2890	−293	7.66	6.28
Z^-	0.7	0.6	16.70	−3510	0	4.62	—
B^-	0.80	0.4	13,45	−6060	+484	3.43	4,33
$(B_A^- \times F_D^-)$	1.56	0.3	11.20	−7340	−149÷+1620	5.52	3,93

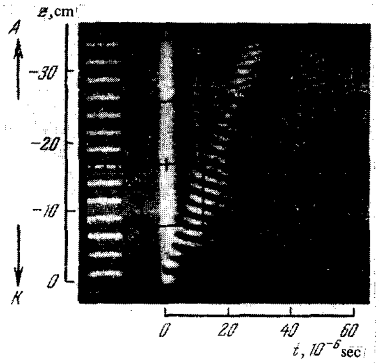


FIG. 14. Crossing wave ($F_D^- \times B_A^-$) in a hydrogen plasma. The lower part shows more clearly the backward part B_A^-, and in the upper part the intensifying part, the forward wave F_D^- begins to prevail. Tube diameter 1.7 cm, pressure 0.4 mm Hg, current 1.5 mA; group velocity at the center u = −9310 m/sec (from [188]).

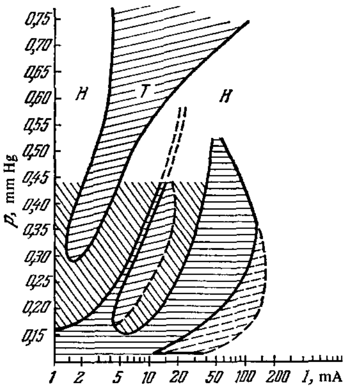


FIG. 16. Regions with different types of waves in oxygen. \\\ – waves of positive phase velocity; ≡ – standing striations; // – waves moving in negative direction (towards the anode). T – low-gradient form of column; H – high-gradient form (from [176]).

and Rutscher[158] succeeded, under these conditions, not only in investigating the process of propagation of the state of contraction and stratification of the column, but also measure, by the method of time-periodic perturbation, the dispersion of these waves. In neon, the group velocity of the waves (and the propagation velocity of the contracted state of the column) is negative, and the dispersion has the same character as in backward waves in inert gases at lower pressures. In argon, however, an anomaly is observed, in that the propagation of the contracted state of the column, and with it the group velocity of the waves accompanying the contraction, have a positive direction, i.e., from the anode to the cathode, corresponding to a wave of the F^+ type.

Recently Stirand et al.[186] were able to excite in a nitrogen plasma with traces of hydrogen a transition wave with positive group and phase velocities (type F_A^+), excited by a small-amplitude pulse and not connected with contraction (Fig. 17).

The form of the corresponding section of the dispersion curve for this case is shown in Table I. When the current and pressure are changed, the group velocity changes and can become negative: the wave F_A^+ goes over into a B_D^- wave. At the transition point itself, the group velocity should equal zero. The form of such a wave is shown in Table IX, together with the corresponding section of the dispersion curve.

H. General Dispersion Curve

All the heretofore observed types of artificially excited waves can be described—in spite of their surprising variety—by a single form of dispersion curve, which changes its position in the (ω, k) plane when the plasma parameters are changed, without changing its general form. The point (ω_1, k_1) with the smallest attenuation (the largest gain $\varphi(k)$ shifts smoothly along the dispersion curve when the parameters of the plasma are changed, causing the change in the type of waves.

Figure 18 shows such a hypothetical dispersion curve in three different positions relative to the k axis. The F_A^+ wave lies on the rising part of the curve and has a positive group and phase velocity. The remaining waves lie on the decreasing part of the curve, and their group velocity is negative. In view of the fact that the group of the temporal increment $\varphi(k)$ has usually a rather sharp maximum, only a small section of the entire dispersion curve can be measured without varying the plasma parameters.

The point of intersection of the k axis and the dispersion curve gives the wave number of the standing periodic structure that arises on the group-velocity side, i.e., towards the anode, following any constant local perturbation in the plasma. If $\varphi(k_s) > 0$, i.e., if amplification takes place in this point, a stationary undamped periodic structure (standing striations) is produced in the column, with an amplitude that is determined by the nonlinear phenomena and is constant over the entire length of the column. This stationary structure must be distinguished, of course, from the standing waves which are produced, for example, when two opposing harmonic waves of equal frequency and amplitude interfere: with the exception of the nodal points, all the points in a standing wave oscillate periodically in time. Standing striations, to the contrary, are stationary waves, i.e., a structure that is periodic in space without temporal oscillations.

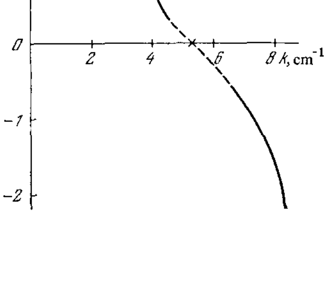


FIG. 15. Dispersion curve of crossing wave $F_D^- \times B_A^-$) obtained by calculating the sweep on Fig. 14. The cross on the k axis corresponds to the wave number k_s measured in standing weakly-damped striations near the cathode (from [188]).

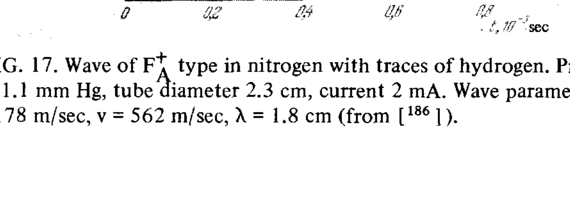
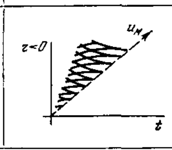
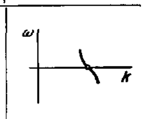
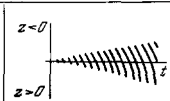
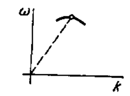


FIG. 17. Wave of F_A^+ type in nitrogen with traces of hydrogen. Pressure 1.1 mm Hg, tube diameter 2.3 cm, current 2 mA. Wave parameters: u = 178 m/sec, v = 562 m/sec, λ = 1.8 cm (from [186]).

Table IX. Types of ionization waves

Type designation	Transition wave	Dispersion
$(B_A^- \times F_D^-)$		
$(F_A^+ O B_D^-)$		

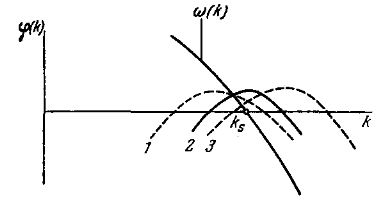


FIG. 19. Possible positions of time-increment curve $\varphi(k)$ for the case undamped standing striations.

Figure 19 shows three possible positions of the curve of the temporal increment $\varphi(k)$ for the case when undamped standing striations are produced. In curves 1 and 3 the maximum does not coincide with the position of the wave number k_S of the stationary structure, and the stationary striations have a tendency to move (in curve 3—towards the anode, in 1—towards the cathode).

The instability (i.e., the growth of the perturbations with time) which is obtained when $\varphi(k_1) > 0$ is usually of the convective type: at the point $z = 0$ of the initial pulsed perturbation, the amplitude of the perturbation decreases and an equilibrium state is restored if the perturbation does not repeat. Nonetheless, in some cases the instability of the ionization waves is absolute. This is clearly seen in the case $u = 0$, when the maximum of the perturbation does not propagate. In such a case it is sufficient to have $\varphi(k) > 0$ for the point where $u = 0$ to make the instability absolute: the ionization waves are then produced spontaneously and without feedback through the external circuit, and it is impossible to make the positive column homogeneous, thus making it impossible to excite artificially waves of small amplitude. Apparently this was the case observed by Pfau and Rutscher simultaneously with the contraction of the positive column in argon after a sudden current jump.[158] Far from the self-excitation boundaries, when the value of the increment $\varphi(k_1)$ is large, the instability of the ionization waves becomes apparently absolute also in other cases. The simultaneous existence of several types of waves (e.g., p, r, s in neon) cannot be explained by means of a single dispersion curve: in this case, each of the forms of the waves must have its own individual dispersion curve.

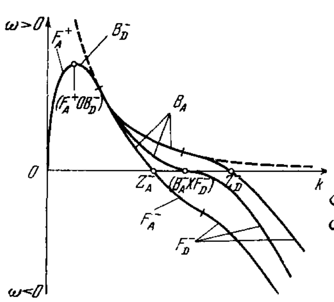


FIG. 18. General dispersion curve (schematic). Dashed curve — ωk = const.

4. PHYSICAL MECHANISM

A. Theoretical Approach

The wide range of ionization-wave velocities (from 10 m/sec to 10^3 m/sec and more) and the strong dependence of their velocity on the discharge current and on the gas pressure, completely exclude any identification of these waves with ordinary sound waves in a neutral gas. The velocity of the ion sound (electroacoustic waves) depends on the discharge conditions, primarily on the electron temperature, but at gas pressures on the order of 1 mm Hg the collisions between the ions and the neutral-gas atoms are so frequent ($> 10^5\ \mathrm{sec}^{-1}$) that the resultant wave damping practically destroys the wave propagation. The same holds for longitudinal waves of the Langmuir type and Langmuir oscillations: their frequency in the electron gas ranges from 10^6 to $10^{10}\ \mathrm{sec}^{-1}$ (under conditions when striations are observed), and the damping due to collisions is of the order of $10^9\ \mathrm{sec}^{-1}$. The ion gas has a Langmuir frequency 10^3–$10^5\ \mathrm{sec}^{-1}$, which is of the same order as the frequency of the ionization waves, but the damping is here of the order of $10^5\ \mathrm{sec}^{-1}$, owing to the high frequency of collision between the ions and the neutral atoms. In addition, none of the indicated wave types has an instability or a dispersion comparable with the dispersion found in ionization waves. Single-velocity electron beams are not retained in the plasma of the diffusion regime, and it is impossible for wave instability (or compensation for their damping) to occur when energy is transferred from the beam to the wave.

Nor can the phenomena be attributed to hydromagnetic types of waves, since the ionization waves exist without a magnetic field. The physical mechanism of the io-

FIG. 20. Initial state of perturbation (schematic). n_+ — perturbation of ion density, n_-, ρ, e, ϑ — corresponding perturbations of the electron density, space charge, electric field intensity, and electron temperature, respectively.

$$n_+ = n_0 e^{-0{,}361\left(\frac{z}{\Lambda}\right)^2},$$

$$n_- \approx n_+ + \frac{1}{4\pi q_0}\frac{E_0}{N_0}\frac{\partial n_+}{\partial z} - \frac{1}{4\pi q_0}\frac{kT_-}{q_0 N_0}\frac{\partial^2 n_+}{\partial z^2},$$

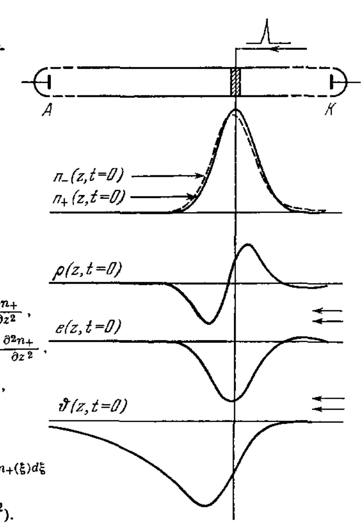
$$\rho = q_0(n_+ - n_-) \approx -\frac{1}{4\pi}\frac{E_0}{N_0}\frac{\partial n_+}{\partial z} - \frac{1}{4\pi}\frac{kT_-}{q_0 N_0}\frac{\partial^2 n_+}{\partial z^2},$$

$$e = 4\pi\int \rho\, dz \approx -\frac{E_0}{N_0}n_+ - \frac{kT_-}{q_0 N_0}\frac{\partial n_+}{\partial z},$$

$$\frac{\partial\vartheta}{\partial z} - a_1\vartheta = -b_1 e,$$

$$\vartheta = \frac{kT_-}{q_0 N_0} b_1\left[n_+ - \left(a_1 + \frac{q_0 E_0}{kT_-}\right)\right]\int_z^\infty e^{a_1(z-\xi)} n_+(\xi)\, d\xi$$

$\left(b_1 \approx \frac{3}{2} q_0,\ \text{see.}\ ^{106,142}\right)$ (from [142]).



nization waves must therefore be sought in other processes.

There is, of course, no doubt that all the foregoing phenomena can be obtained by solving the Boltzmann kinetic equation for the discharge-plasma conditions. But the difficulties are masked by the fact that in a plasma with a low degree of ionization the collision term is given by a very complicated expression, and the cross sections for many types of collisions are unknown. As a result, all the theoretical treatments developed to date were based essentially on the hydrodynamic equations.

Assuming the neutral gas of the plasma to be weakly ionized, it is customary in the solution to neglect variations of its density, temperature, or pressure and their influence on the ionization phenomena. This, of course, makes it necessary to forego an explanation of the interaction between the sound in the neutral gas and the ionization, which was observed, for example, in [166]. The continuity equation for the neutral gas and the equation for its momentum thus do not enter at all in the initial system of equations, and the presence of the gas affects the values of the diffusion, mobility, and other coefficients for the charged particles.

The initial system then consists of the following: a) the continuity equations for: 1) ions, 2) electrons, 3) atoms in excited states and, if necessary, 4) other types of charged or neutral particles; b) the Poisson equation, which is sometimes replaced by the quasineutrality requirement; c) the energy-balance equation, usually for the electrons only. The equations are linearized by introducing small deviations of the variables from the equilibrium state, are reduced to motions with respect to one variable—the spatial coordinate along the discharge axis—and a solution in the form of waves $\exp\{i(\Omega t - Kz)\}$ is introduced in the obtained system of linear differential equations. The calculations, which are almost always approximate, yield the dispersion equation $F(\Omega, K) = 0$. In spite of the large number of time derivatives, only one is taken into account in the approximate calculation in final analysis, so that Ω enters in the resultant final dispersion equation only in the first degree: this precludes before hand the obtaining of two separate dispersion curves and the explanation of several types of simultaneously existing waves. In addition to using the dispersion method for solving this system, another method, particularly convenient for the investigation of the transition wave, is used: the system of equations is reduced to a single integro-differential equation of the type

$$\frac{\partial n}{\partial t} = \int_{-\infty}^{\infty} K(z-\xi)\, n(\xi, t)\, d\xi, \qquad (4.1)$$

where K is a real function containing all the parameters of the system of equations (see [114, 124]). From the form of the kernel K it is possible to determine rapidly the character of the solution (in particular, whether the response of the medium to a pulsed perturbation is oscillatory or aperiodic[114,145]). Without entering into details, we present in what follows an illustrative physical interpretation of the main mechanism of the ionization waves.

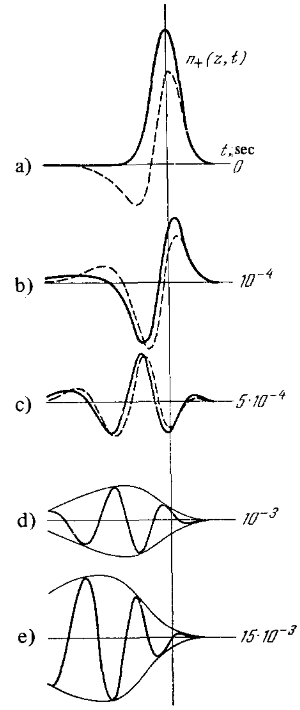


FIG. 21. Time development of perturbation. a) Initial perturbation; b – e) states after 10^{-4}, 5×10^{-4}, 10^{-3}, and 1.5×10^{-3} sec, respectively. The dashed curves on diagrams a – c) correspond to a time longer by 5×10^{-5} sec than for the solid curves. Anode to the left, cathode to the right. Calculated for the following values of the parameters: $D_a = 200$ cm² sec⁻¹, $\alpha = 1500$ sec⁻¹, $A = 8500$ sec⁻¹ cm⁻¹, and $a_1 = 0.3$ cm⁻¹.

B. Development of Wave

The physical mechanism of ionization waves is best understood by considering the initial stage of the stratification wave, for it is precisely the development of the oscillatory process following a local perturbation that is perfectly aperiodic in time and in space which distinguishes the stratification phenomenon in a plasma.

Let us imagine that a small local perturbation of an initially homogeneous plasma produced in the vicinity of the point $z = 0$ an excess of positive ions $n_+ = N_+ - N_0 > 0$ and electrons $n_- = N_- - N_0 > 0$ (Fig. 20). Such a local increase in the density of the charged particles cannot stay in the given place for a long time. If the perturbation were not to cause a change in the ionization rate per electron, then there would be no oscillatory or wave phenomena, for the produced concentration peak would in this case gradually spread out along the tube axis in accordance with the equation of ambipolar diffusion

$$\frac{\partial n_+}{\partial t} = D_a \frac{\partial^2 n_+}{\partial z^2}. \qquad (4.2)$$

Here $D_a \approx kT_- \mu_+ / q_0$ is the coefficient of ambipolar diffusion due to the electric field of the space charge (see Fig. 20).

However, the additional electric field due to the space charge influences also the average energy of the electrons: the electrons, arriving in the perturbation region (in the direction of the arrows in Fig. 20) lose their energy by collision as before, but as a result of the decreased electric field in this region their energy balance is upset and their temperature decreases. The electrons emerging from the region of the perturbation, in the direction of the drift in the electric field, have a lower temperature (see lower part of Fig. 20), and only after covering a certain distance, usually about 1 cm or

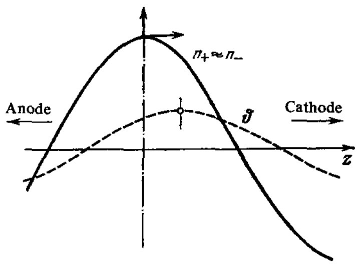


FIG. 22. Mechanism of phase velocity of ionization wave. The maximum of the electron temperature is shifted towards the cathode, and the ionization shifts the maximum of the ions to the right. The shift towards the cathode can also exceed $\pi/2$.

more under discharge plasma conditions, do they restore the equilibrium value of the temperature corresponding to the homogeneous column. Thus, the localized ion and electron density perturbation at the point $z = 0$ changes in final analysis the temperature of the electrons in a region that lies already beyond the limits of the initial perturbation, in the anode direction, and in such a way that an increase of the concentration causes a decrease of the temperature.

Since the ionization rate per electron in a discharge plasma depends strongly on the electron temperature, the production of ions and electrons in the region where the ionization rate is reduced slows down, and consequently a region with decreased carrier density is produced. Thus, the local increase of the density of the charged particles generates gradually, by lowering the electron energy and lowering the ionization, a region with decreased charged-particle concentration closer to the anode. In turn, beyond this region, a new region, closer to the anode but with increased concentration, is produced in the same manner. The processes repeat closer and closer to the anode, leading to a gradual occurrence of regions with alternating sign of the deviation from the equilibrium state.

The foregoing can be expressed mathematically in rather simple fashion, by adding to ambipolar-diffusion equation (4.2) a term expressing the change in the ionization rate as a function of the electron temperature:

$$\frac{\partial n_+}{\partial t} = D_a \frac{\partial^2 n}{\partial z^2} + Z'_\vartheta N_0 \vartheta, \qquad (4.3)$$

where $\vartheta = k(T_- - T_0)$ is the deviation of the electron temperature from the equilibrium value kT_0, and $Z_\vartheta = (\partial Z/\partial\vartheta)_{T_0}$ is the slope of the plot of the ionization coefficient Z against the electron temperature (it is not important here whether the ionization is produced directly or stepwise), and N_0 is the unperturbed density of the ions and electrons. Taking for ϑ the simplified relation given in the caption of Fig. 20, we note that Eq. (4.3) assumes the form

$$\frac{\partial n_+}{\partial t} = D_a \frac{\partial^2 n}{\partial z^2} + \alpha n_+ - A \int_z^\infty e^{a_1(z-\xi)} n_+(\xi)\, d\xi \qquad (4.4)$$

for the single variable n_+ (here $\alpha = Z'_\vartheta b_1 kT_-/q_0$ and $A = Z'_\vartheta b_1(kT_- a_1/q_0 + E_0)$). A numerical solution of (4.4) for an initial ion-density distribution in the form of a Gaussian curve and for particular numerical values of the coefficients is shown in Fig. 21. The solution has indeed the character of the backward stratification wave typical of inert gases: the group velocity is directed towards the anode, a packet with optimal wavelength is produced in the center, the phase (e.g., the maxima of the densities) moves towards the cathode,

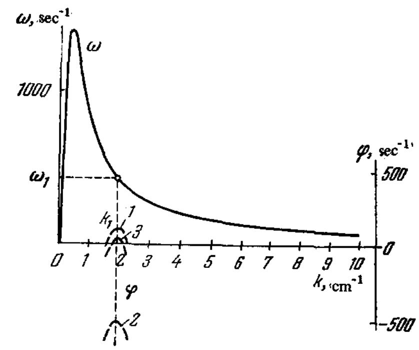


FIG. 23. Dispersion curve $\omega(k)$ (thick line) for $D_a = 200\ cm^2 sec^{-1}$, $A = 8500\ cm^{-1} sec^{-1}$, $a_1 = 0.3\ cm^{-1}$, and $\varphi(k)$ curve (dashed) for three different values of the parameter. 1 – Case corresponding to equations in the caption of Fig. 20; 2 – with allowance for the thermal conductivity of the electrons at a Maxwellian distribution according to Eq. (4.5) [165]; 3 – with allowance for $dZ/dN_- > 0$.

the amplitude of the wave increases in the propagation direction.

It is easy to see that the mechanism of phase displacement is of the ionization type: the maxima of the ionization in the waves do not coincide with the maxima of the electron and ion densities, and are shifted towards the cathode. As a result of this shift (Fig. 22), more new ions are produced on the cathode side of the ion maximum than on the anode side, and consequently the maximum shifts towards the cathode.

Figure 22 also explains readily the mechanism of the wave instability for the given case: at the point of the maximum ion density the temperature of the electrons, and consequently also the production of the ions, is larger than in the equilibrium state. The ionization tends in this case to increase the density of the ions also at the location of the maximum, whereas the ambipolar diffusion, to the contrary, tends to equalize the density deviations. If the effect of increased ionization at the location of the ion maximum prevails over the lowering of the amplitude as a result of ambipolar diffusion, the amplitude of the pertubation increases and the wave is amplified. This can occur only in a certain wavelength interval. At short wavelengths, the influence of the ambipolar diffusion prevails over the influence of the ionization, and at large wavelengths the phase shift of the electron temperature always exceeds $\pi/2$ rela-

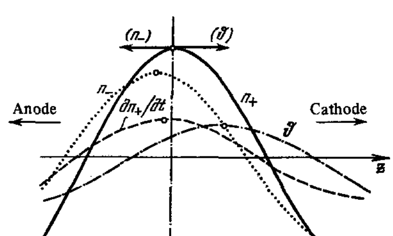


FIG. 24.Scheme of possible mechanism of negative phase velocity of ionization wave F^-. Whereas the maximum of the deviation of the electron temperature ϑ is shifted towards the cathode and tends (by means of ionization) to shift the maximum of the ion density to the right, the shift of the electron density displaces (also via ionization) the maximum of the ions towards the cathode. The result may be positive, negative, or zero phase velocity.

tive to the electron and ion densities, which leads to damping rather than amplification: the ionization acts in this case as a stabilizing factor.

In spite of its simplicity and clarity, this mechanism of ionization instability is ineffective at a current density such that the equation for ϑ in the caption of Fig. 20 does not express with sufficient accuracy the energy balance of the electron gas (primarily as a result of neglecting the electron thermal conductivity). This situation is encountered also at currents near the Pupp limit. In the case of a strong thermal conductivity of the electron gas, say Maxwellian, the phase shift between the ion density and the electron temperature is always larger than $\pi/2$, and the instability of the ionization waves results from an additional phenomenon, namely the dependence of the ionization coefficient on the electron-gas density (see below).

C. Allowance for Additional Processes

The processes represented by the equations in the caption of Fig. 20 are expressed in simplified form, and are accompanied in a discharge plasma by additional phenomena. Let us dwell briefly on some of them, which can exert a strong influence on the parameter of the ionization waves.

First, when the electron temperature changes, a change takes place not only in the ionization rate, but also in the electron density, since the pressure of the electron gas and the values of the transport coefficients change. This leads to a broadening of the region of the space-charge field, and is equivalent to a certain degree to an increase of the Debye length for the electrons.[165, 187] Sharp gradients of the electron temperature become smoothed out by the electron thermal conductivity, thus lowering the tendency to wave instability. The use of the energy-balance equation for electrons with a Maxwellian or near-Maxwellian velocity distribution leads to the following expression for the deviation of the electron temperature (see [165])

$$\vartheta(z) = -C_1 \int_z^{\infty} e^{a_1(z-\xi)} n_+(\xi)\, d\xi + C_2 \int_{-\infty}^{z} e^{-b(z-\xi)} n_+(\xi)\, d\xi. \quad (4.5)$$

Here again a_1 is the reciprocal of the relaxation length of the electron temperature in the electron drift direction, b is the reciprocal of the relaxation length of the electron temperature in the direction opposite to the electron drift and due to the thermal conductivity of the electron gas, and C_1 and C_2 are amplitudes expressed with the aid of the electron-gas transport coefficients.

Since usually $b \gg a_1$, we have

$$C_2 \int_{-\infty}^{z} e^{-b(z-\xi)} n_+(\xi)\, d\xi \approx \frac{C_2}{b}, \quad (4.6)$$

and Eq. (4.5) differs from the equation (for ϑ) in the caption of Fig. 20 only in the values of the amplitudes C_1 and C_2. Although the variation of the temperature corresponding to the Gaussian deviation of the ion density (see Fig. 20 and [165]) is not altered appreciably, nevertheless, in view of the fact that the amplitude C_2 (which is frequently also negative) is small, the instability of the ionization waves arises when (4.5) is employed usually only when an additional instability mechanism acts in the plasma. In inert gases at sufficiently large currents, a mechanism is always present as a result of stepwise ionization. The average ionization frequency per electron increases in this case not only with increasing average electron energy, but also with increasing electron density (at constant electron temperature), i.e., $\partial Z/\partial N > 0$—the ionization rate increases more rapidly than the first power of the electron density.[178] As a result of this dependence, there appears in (4.4) an additional term proportional to the concentration n, which is equivalent to an increase of the coefficient α. It can lead to ionization instability of the waves also in the case of appreciable thermal conductivity of the electron gas, when $C_2 < 0$. The curve of the temporal increment (see Fig. 23 below) shifts upward as a result, but the dispersion and position of the optimal wave number k_k do not change.

By taking into account the stepwise ionization, the deviation of the distribution of the electron energy from Maxwellian, and the influence of the change of electron temperature on the transport coefficients, Wojaczek[178] obtained full quantitative agreement with experiment for the parameters of the backward ionization wave in argon at large currents. In spite of the complexity of the processes taken into account, the resultant equation coincides with (4.4) but, of course, the coefficients α, A, and a_1 take into account all the processes indicated above. This case of complete quantitative agreement between the theory of ionization waves and experiment was obtained after very laborious calculations and preliminary auxiliary calculations, which turned out to be essential, in spite of the fact that the inert-gas plasma with a large discharge current (on the order of several amperes) is simpler than all others and has been most thoroughly investigated.

At low currents (on the order of several milliamperes or less) in inert gases, the direct ionization begins to prevail over stepwise ionization.[169] The atoms in the metastable states begin to play a new role: they constitute essentially an admixture of another gas with a very low ionization potential, and the amount of this gas depends on the state of the plasma and vice versa. At low currents their lifetime becomes larger than the lifetime of the positive ions, and the gas of the metastable atoms can become manifest separately. This gives rise to a slow ionization wave.[74] The instability of the fast wave connected with the direct ionization[84] should be caused in this case by the phase-shift mechanism (see Fig. 22), whereas the slow wave that develops in parallel is connected with the rate of change of the density of the metastable, i.e., neutral atoms, and the phase shifts in it can differ strongly from the shifts on Fig. 22.

So far there is no consistent theory of ionization waves at low temperatures in inert gases, and all the more in molecular gases and mixtures. The described mechanism, as well as Eq. (4.4) itself, which contains only the first derivative with respect to time, reveal the unusual character of the resultant wave: it is connected with a relaxation process (with respect to time). The wave solution and its oscillatory character are due to the integral long-range term in (4.4), which expresses the influence of the local perturbation of the ion density on the ionization rate. Unconnected with the wave is the motion of matter forward and backwards during the propagation (as, e.g., in the case of gas molecules in a sound wave) or transversely (as, e.g., for atoms in a transverse elastic wave in a solid). The oscillations of the ion and electron density in the ionization wave are not accompanied by a change of their momentum and are not connected with the energy stored in inertial processes.

D. Dispersion

Substituting in (4.4) a solution in the form of a wave that is periodic in space, $\exp\{i(\omega t - kz) + \varphi t\}$, we obtain equations for the dispersion $\omega(k)$ and for the temporal increment $\varphi(k)$ in the form

$$\omega = A\frac{k}{k^2+a_1^2}, \tag{4.7}$$

$$\varphi = -D_a k^2 + \alpha - \frac{Aa_1}{k^2+a_1^2}. \tag{4.8}$$

Figure 23 shows a plot of $\omega(k)$ for the same values of the parameters of (4.4) as in Fig. 20, and a plot of $\varphi(k)$ for three different values of the coefficient α. Since k_1 lies, at the employed numerical values, in the region of a descending dispersion curve $\omega(k)$, the dispersion corresponds to a backward transition wave of the type B_A^-.

If $a_1 \ll k_1$, i.e., at relatively large relaxation lengths of the electron temperature, the dispersion equation (4.7) assumes the form (see [159])

$$\omega \approx \frac{A}{k} \quad \text{and} \quad \omega k \approx A. \tag{4.7a}$$

The corresponding backward wave has a velocity $u \approx -v = -A/k^2$. Such a case is observed, for example, in inert gases at large currents.

1. Forward waves of type F^+ and waves with zero group velocity. When $a_1 \gg k_1$, the dispersion equation (4.7) goes over into

$$\omega \approx \frac{A}{a_1^2} k\left(1 - \frac{k^2}{a_1^2}\right). \tag{4.7b}$$

A forward transition wave of type F_A^+ with velocities $v > u > 0$ is obtained:

$$v \approx \frac{A}{a_1^2}\left(1 - \frac{k^2}{a_1^2}\right) \tag{4.9}$$

and

$$u \approx \frac{A}{a_1^2}\left(1 - 3\frac{k^2}{a_1^2}\right). \tag{4.10}$$

Consequently, the forward waves of type F^+ (see Fig. 18) can be explained by means of the same mechanism as the B_A^- waves, within the framework of the dispersion curve (4.7) (see Fig. 23). It is sufficient to have the wave number k_1, for which the maximum of the temporal gain $\varphi(k)$ is obtained, correspond to the rising part of the dispersion curve (4.7); this can be satisfied for the wave on Fig. 17 when

$$D_a = 7.636\cdot 10^3 \text{ cm/sec} \quad \alpha = 3.65\cdot 10^5 \text{ sec}^{-1}$$
$$a_1 = 4.846 \text{ cm}^{-1} \quad A = 2.004\cdot 10^6 \text{ cm}^{-1}\text{sec}^{-1}$$

Garscadden et al.[157, 160] obtained good agreement between (4.7) and measurements of the forward ionization wave in a contracted column. If the increment $\varphi(k)$ is maximal precisely at the place where $\omega(k)$ has a maximum, then a type $(F_A^+ O B_A^-)$ wave is obtained with zero group velocity at the center of the packet.

2. Forward waves of type F^- and waves with zero phase velocity. Forward waves of the F^- type with group and phase velocities directed towards the anode cannot be obtained from (4.7), since the dispersion curve does not cross the k axis at all. In certain theories (see, e.g., [114, 175]) the phase velocity turns out to be negative for short waves (large k), but the maximum of the increment $\varphi(k)$ lies in these cases in the region $\omega > 0$.

One cannot exclude the possibility that the mechanism of negative direction of the phase velocity is connected with a certain process which is unique to hydrogen and oxygen, where these waves are observed. But it can also be explained within the framework of the aforementioned processes as being due to the fact that the maximum of the ionization in a wave of type F^- is shifted with respect to the maximum of the density towards the anode. In principle, two causes of such a shift are possible: a) the maximum of the electron temperature itself is shifted towards the anode; b) the maximum of the electron density is shifted towards the anode relative to the maximum of the ions, and the effect of ionization due to changes in the electron density, which always tends to shift the phase of the wave towards the anode, is stronger than the effect of ionization due to the change of the electron temperature (Fig. 24).

Explanation b) is more probable, since the same shift between the temperature and the density was observed in hydrogen for standing striations as for backward ionization waves in inert gases.[156, 167] An appreciable shift of the maximum of the electron density relative to the maximum of the ions can be observed either when the wavelength is comparable with the Debye length,[113] or in the case when the changes of the electron temperature have a strong influence on the electron density, thus increasing the polarization and being equivalent to a certain degree to an increase of the Debye length (see [187]).

A wave of type Z^- is a transition between B^- and F^- waves: the maximum of polarization in the Z^- wave should also coincide with the maximum of the density precisely for a wave number k for which the increment is maximal in this case.

E. Other Phenomena

In the preceding sections we touched upon only the mechanism causing the main properties of the ionization waves. In experimental investigations of more specific phenomena, for example the influence of the magnetic field on the parameters of ionization waves, the influence of irradiation,[76, 189] the influence of active impurities[92, 132] on the properties of the waves, etc., it is customary to present also qualitative explanations of the observed phenomena.

Quantitative explanations of all these relations (as well as the law of constant potential difference per wave length) can be expected only after a theory is developed for ionization waves at weak currents, a development which is still in the initial stage.

5. CONCLUSION

The nature of ionization waves or striations in a discharge plasma is due to a change of quite general processes. These are: a) the occurrence of space charge at the place where the plasma homogeneity is locally disturbed (e.g., by an excess of ions and electrons); the charge is the result of polarization of the positive ions and electrons, which have different velocity of free diffusion and different mobility in the electric field; b) a change of the average electron energy as a result of violation of their energy balance due to a local change of the electric field by the space charge; c) a change of the ionization rate together with a change of the average electron energy; d) a gradual change (lowering) of the ion density together with a change in the ionization rate.

The chain of these processes leads to propagation of the perturbation, with alternation of the positive and negative deviations of the density and of the remaining plasma parameters from the equilibrium state.

In order for the ionization waves to be unstable (amplified) it is necessary to have: e) a dependence of the ionization coefficient Z on the electron-gas density.

In spite of the fact that main properties of the ionization waves can be explained as due to the foregoing general processes in the plasma, an exact quantitative explanation of the waves of all the observed types, their numerical parameters, and dependences on the discharge conditions still call in the overwhelming number of cases for additional theoretical research.

It is possible that in more complicated plasma, such as in a molecular-gas plasma, in a plasma with electronegative gas, in mixtures of different gases and vapors, etc., the behavior of the ionization waves depends significantly on the specific processes, such as formation of negative ions, which change the electric field of the space charge, on the excitation of vibrational and rotational states of the molecules, which changes strongly the form of the electron-energy distribution curve and their average kinetic energy, on electron capture processes, etc.

There is no full assurance that the one-dimensional approximation is sufficiently accurate for a complete quantitative agreement between theory and experiment.

The most difficult problem is to obtain from the theory the correct plot of $\varphi(k)$ of the temporal increment of the wave as a function of the discharge conditions. It is known from the experiments that the damping or intensification, and all the more the position of the maximum, are usually quite sensitive to even small changes of the state of the plasma. Intensification (or damping) is obtained in ionization waves as a result of many competing frequently strong processes, which cancel each other to values close to zero. It is therefore not surprising that in the theory it is easier to obtain the correct form of the dispersion curve than that of the temporal increment. However, it is the value of the temporal increment $\varphi(k)$ at the point of the maximum which determines predominantly whether ionization waves will appear in the discharge spontaneously or not. Therefore an exact explanation (or even prediction in the case of different gases and their mixtures) of the regions of current, pressure, and tube diameter at which ionization waves are unstable (i.e., $\varphi(k_1) > 1$) is impossible at present. This situation is also due to the fact that so far there are no exact data not only concerning such parameters as the relaxation length of the electron temperature, the lifetime of the excited levels under plasma conditions, etc., but even the cross sections of different types of collisions and for the formation of unstable particles in a discharge plasma are unknown. A great difficulty is caused also by the strong sensitivity of ionization processes to the distribution of the electron energy, which in turn depends strongly on the type of gas and on the energy levels of its atoms or molecules.

On the other hand, the connection between the parameters of the ionization waves precisely with these measurable phenomena in a plasma makes it possible, in principle, to investigate these phenomena by means of theoretical relations at definite parameters of the ionization waves. At the present time, however, this possibility is limited only by the small number of particular cases.

In order to describe in conclusion the status of the problem of ionization waves, we can use the following comparison: The present situation is similar to the initial stage in the understanding of chemical reactions, when certain simplest processes have been made clear, the principle of chemical joining of elements is understood, but the details and variety of these reactions are not yet revealed. The comparison, incidentally, goes deeper than may appear at first glance: the creation and vanishing of particles of different types in the plasma and their mutual transformation are phenomena which are very close to phenomena of chemical kinetics not only with respect to the form of the equations describing them, but also with respect to their very nature. Unlike ordinary chemical reactions, for which a nonoscillatory development and aperiodic damping are characteristic, ionization reactions in a plasma can develop in an oscillatory manner, when the rate of creation and vanishing of the particles of a definite type alternately increases and decreases. This behavior is connected with the influence of the electric field on the rate of the ionization reactions and imparts a wave character to the ionization reactions in a plasma.

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