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STRIATIONS

A. V. NEDOSPASOV

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THE formation of layers in a positive column is a widespread phenomenon known already to the pioneers of gas-discharge physics. This first manifestation of the instabilities which are so abundant in plasma has given rise to many contradictory opinions concerning its causes. The reviews[1-3] summarizing the extensive material accumulated during the century-old history of the study of striations gave no explanation of the basic phenomena. This explanation was obtained only in recent years.

The present article encompasses essentially research performed during the last 15 years. During this period, the ionization-diffusion nature of striations has been uniquely established.

The study of ionization instability is not only of general physical interest. This is apparently the basic instability of a low-temperature weakly ionized plasma, which can become manifest in MHD energy generators, plasma waveguides, gas lasers, and other technical devices.

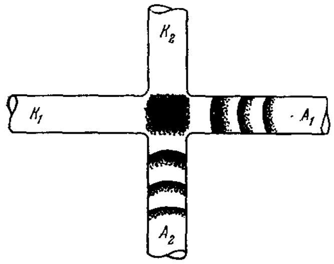


FIG. 1. Formation of striations behind the place of increased discharge density[4].

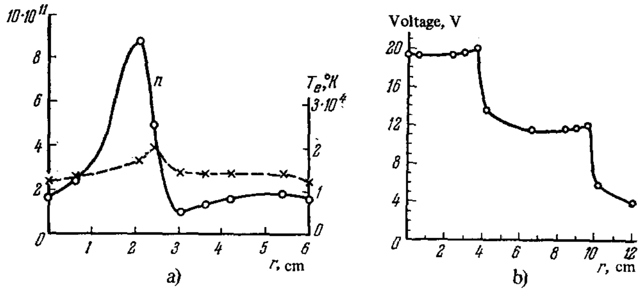


FIG. 2. a) Variation of electron density and temperature within the limits of the striation [7]. b) Variation of the potential on the tube axis in a layered positive column [7].

I. BASIC EXPERIMENTAL DATA ON STRIATIONS

1. Layered Positive Column

The layered discharge is the most widespread form of the positive column. Frequently a column which seems to be homogeneous turns out to be layered, but the presence of striations in it is masked by their rapid motion. In hydrogen, striations exist in a current-density range extending over almost eight orders of magnitude and in a pressure range of four orders of magnitude.[4] Striations in inert gases are observed in the pressure interval from several hundredths to several hundred mm Hg. Striations are either stationary (standing) or traveling. Stationary striations lie on the anode side of plasma homogeneity disturbances of any magnitude, for example on the cathode part of the discharge, on the side of a probe with large negative potential, etc.[4,5] Striations usually attenuate with increasing distance from their point of occurrence. The point of occurrence of stationary striations may be a region where the plasma density increases at the crossing of homogeneous positive columns (Fig. 1).

In pure inert gases, the striations move from the anode to the cathode (positive striations) with velocities v on the order of 10^3–10^5 cm/sec, and their length l reaches several column diameters. A typical picture of the variation of the plasma characteristics within the limits of clearly pronounced striations[6,7] is shown in Fig. 2. The electric potential increases jumpwise in the frontal part of the striation and varies little in the remaining part. Associated with the jump of the potential are an increase in the electron temperature and intense excitation and ionization in the front of the striation. The maximum of the concentration is shifted relative to the maximum of the temperature towards the anode.[4,6-8] In the tail of the striation, the electron energy is too small for ionization. In this part, the electrons are present because they had no time to recombine with the ions, and also because of the ambipolar diffusion in the axial direction. A stationary layered column is characterized by the fact that the balance of ionization and recombination takes place not at any instant of time (or not any transverse cross section), but only within the limits of the striation as a whole.

A numerical reduction of the experimental data on the concentration and temperature of the electrons in moving striations[7,9] shows that the concentration distribution satisfies the diffusion equation

$$\frac{\partial n}{\partial t} - D_a \Delta n = I(r, z, t);$$

here D_a is the coefficient of ambipolar diffusion and I the ionization per unit time. The ionization is concentrated in this case in a layer which is thin compared with the length of the striation (Fig. 3).

The radial propagation of density in a layer medium is the same as in a homogeneous one;[7,10] it is close to the Bessel function $J_0(\beta r/a)$. The eigenvalue of the boundary-value problem is $\beta = 2.4$ at $n(a) = 0$ and decreases with pressure when account is taken of more rigorous boundary conditions.[11-13]

Such a structure is characteristic of both moving and stationary striations of large amplitude. The relative changes of the concentration in the striations are always larger by several times than the relative changes of the electron temperature. This is connected with the strong dependence of the ionization coefficient on T_e in a weakly ionized plasma.

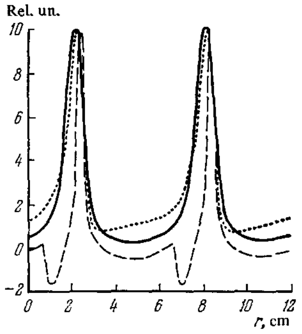


FIG. 3. Distribution of light intensity (solid line) and of ionization (dashed lines) in striations [7].

The moving striations produce oscillations with a fundamental frequency that lies in the interval from several hundred Hz to tens of kHz. In inert gases, these oscillations obey a similarity law, which was established experimentally by Pupp: the product of the frequency by the radius of the tube and the molecular weight of the gas $(fa\mu)$ is nearly the same function of the product of the gas pressure by the radius divided by the ionization potential (p_0a/u_i) for all gases (with the exception of helium)[14] (Fig. 4). The dependence of the striation velocity on the pressure is shown in Fig. 5.

The motion of the striations is frequently irregular, and a transition from regular motion to random and vice-versa may occur, for example, when the current is changed.[16] Disturbance to regular motion of striations in argon can be observed when the tube is cooled with liquid nitrogen. Such a characteristic of the striations as the length depends little on the discharge current and on the ballast resistance[15-17], although the parameters of the external electric circuit are of importance for the regular character of the striations.

Figure 6 shows photographs of striations, taken by A. A. Zaĭtsev with the aid of a rotating mirror. The rotation axis was parallel to the tube axis. Figure 6a corresponds to regular motion in the positive direction.

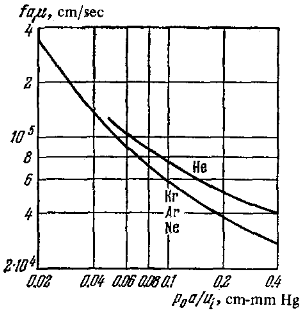


FIG. 4. Similarity law for striations [14].

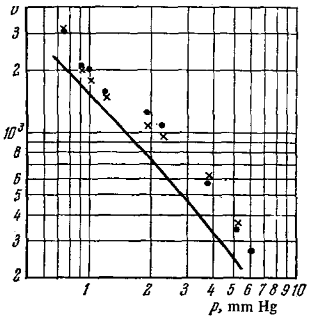


FIG. 5. Dependence of striation velocity in argon on the pressure (a = 1.15 cm) [15].

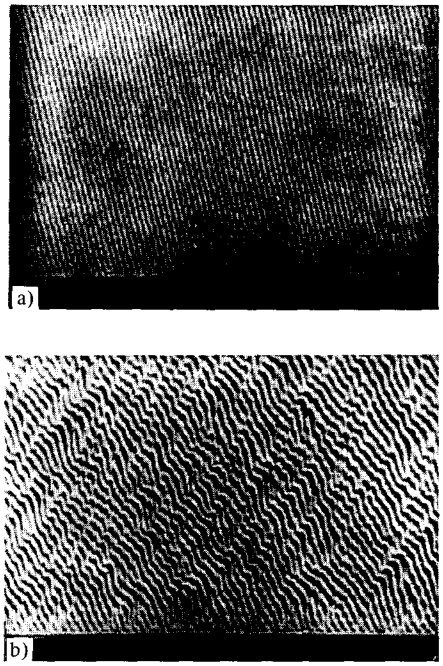


FIG. 6. Regular (a) and irregular (b) striations. Neon, a = 0.3 cm, p = 5 mm Hg. Time—from left to right, anode on top.

Figure 6b shows an example of irregular motion, wherein the length and the velocity of the striations changes, and a splitting of one striation into two is also observed.

Experiment has shown that striations are observed in definite intervals of variation of the discharge conditions. In many cases, pure inert gases exhibit[18-21] upper and lower limits of the current I, gas pressure p, and tube radius a in which the striations exist. In the space of these parameters, the region of existence of striations is bounded by a closed surface.[108, 110] The upper limit was first investigated by Pupp,[18] who showed that the maximum current I_{max} at which striations are still observed is inversely proportional to the pressure. It also follows from his data that a constant pressure I_{max} decreases with increasing molecular weight of the gas. These data are refined in [22]. It is shown, in particular, that the determination of the boundary depends on the sensitivity of the apparatus and on the measurement method. Using the dependence of the existence boundary on the magnitude of the tube radius, the authors of [22] have shown that striations can exist in part of a gas-discharge tube bounded by narrower sections which do not contain any regular oscillations. This proves that the striations are generated in the column itself, and are not produced by external disturbances.

In the general case the depth of modulation along a layered discharge is not constant. For moving striations it usually increases in the direction from the cathode to the anode.[21-25] Near existence boundary, the intensification of the striations takes place practically in the entire length of the column. With increasing distance from the boundary, the amplitude increases and reaches saturation at smaller distances, down to dimensions on the order of the length of one layer (Fig. 7).

Striations are observed in inert and molecular gases and are missing from discharges in alkali-metal va-

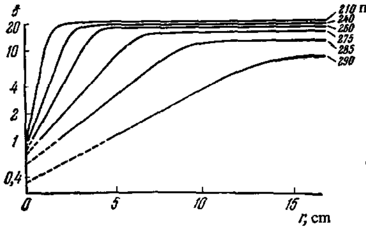


FIG. 7. Sharpness of striations along the tube [21]. Arton, p = 5 mm Hg, a = 0.25 cm. The numbers indicate the values of the current (in milliamperes). The upper limiting current of striation existence is 295 mA.

pors.[4, 24] Klyarfel'd reports that there are no striations in Cd and Hg vapors, and also in inert gases with small admixture of hydrogen.[4] In later investigations, however, striations were observed in mercury vapor in narrow pressure and current intervals.[25] In all gases, the striations vanish when the pressure is reduced, and when several electron mean free paths are spanned by the length of the striation.[4]

The dimensions of the striations and the regions of their existence can vary within very wide limits in the presence of different admixtures to the main gas. Thus, the addition of water vapor to hydrogen[4] decreases the length to 0.05 of the diameter, and addition of mercury vapor to helium decreases the maximum current I_{max} by three orders of magnitude.[16] Hydrogen impurity has a strong influence on the striation parameters in inert gases.[26, 27]

Besides broad positive striations, narrow "negative' striations moving from the cathode to the anode were also observed.[4]

2. Artificial Striations

An important step forward in the experimental study of the layered positive column was the discovery of artificial excitation of striations by A. A. Zaĭtsev. He has shown that external oscillations superimposed on a homogeneous discharge can produce and maintain moving striations.[28, 29] Different methods can be used to excite artificial layers: modulation of the current by an external generator, periodic disturbance of a section of the column with the aid of an auxiliary discharge or a probe, etc.

It is easiest to excite artificial striations near the limits of existence of the layered discharge, and the parameters of the artificial striations (dimensions, sign, and magnitude of the velocity) are close to those of spontaneously arising layers. Figure 8 shows a plot of the exponent d of spatial amplification of the oscillations, in the form $A \exp(-dx) \exp\{i(kx - wt)\}$, against the frequency of the external signal. (The positive direction of Ox is from the anode to the cathode.[30]) In these experiments, striations with natural frequency of 800 Hz existed at currents smaller than 1.9 A. At such a frequency, the artificial striations have the largest amplification factor (d_0). The value of d_0 increases when the current approaches the limiting value 1.9 A, and becomes positive at I = 2.1 A (Fig. 9). Artificial striations go over continuously into ordinary ones. The observed difference between the limiting current 1.9 A for self-excitation of striations and the current 2.1 A is discussed in Sec. 7 below. Figure 9 also shows the variation of d_0 in helium near the lower limiting current (according to Zaĭtsev's data). Spontaneous striations arose under these conditions at I = 370 mA.

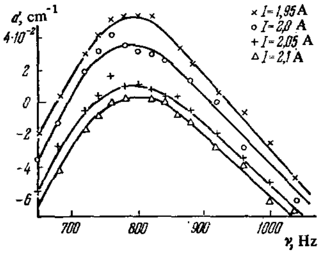


FIG. 8. Dependence of the spatial wave amplification exponent on the exciting frequency [30].

Unlike the large-amplitude oscillations which occur near the existence limit, the form of the artificial and spontaneous striations near the limit of stratification of the homogeneous column is close to sinusoidal.[28, 30] Changes in the discharge parameters, leading to stratification, can either cause a smooth increase of the oscillation amplitude from zero ("soft" mode), or a jumpwise change to a certain final value ("hard" mode). In the latter case, hysteresis phenomena are observed.[28]

It was shown in [28] that, within certain limits, the striation oscillations can assume the same frequency as the stimulating external generator. The external signal changes in this case the length and velocity of the striations. Identical changes occur also in the frequency and the velocity of artificial striations.[23, 30]

Artificial moving layers can also be obtained by applying to the plasma a single perturbing pulse. This method, first described by L. Pekarek,[31, 32] is particularly convenient for the study of the oscillatory properties of the positive column. The oscillations produced under the influence of the pulse attenuate, and the attenuation time near the stratification region can exceed the period of the oscillations T by several orders of magnitude. When the regime is changed in the direction of larger stability, the attenuation time decreases to a value on the order of T. Using a rotating mirror, Pekarek observed an essential singularity in the artificial excitation of striations. The excitation takes place in such a way, that the oscillatory regime propagates from the point of excitation towards the anode, although the resultant waves move towards the cathode. This phenomenon has been called a stratification wave.

The stratification wave was subsequently investigated by many authors[33-37] with the aid of a variety of procedures. Figure 10 shows typical oscillograms of light modulation (obtained with the aid of a photomultiplier) from the anode side of the disturbance location.[33] The oscillograms show that the delay of the oscillations increases with increasing distance. Since the striations attenuate in these experiments after 5–10 oscillations,

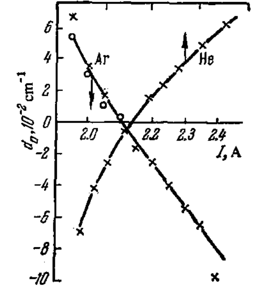


FIG. 9. Dependence of d_0 on the current strength near the upper limit (Ar) and lower limit (He) of striation existence.

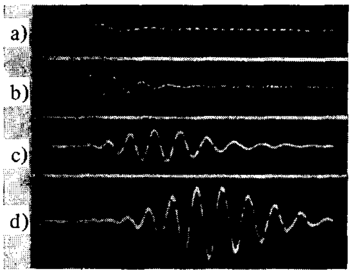


FIG. 10. Oscillograms of transient process at different distances d from the photomultiplier and the perturbation source. [33]. The time increases from left to right. The time-marker duration is 10^{-4} sec. The values of d are: a) 3 cm, b) 9.8 cm, c) 19.2 cm, d) 28 cm.

the zone of the nonstationary layers, moving towards the anode, exists only in a certain section of the column. The velocity of the stratification wave front, while differing in sign, is comparable with the velocity of the positive striations. Certain data on the velocities of the stratification wave and of the striations are given in [33]. For the stratification wave velocity (v_{str}) in helium, the following formula was obtained: $v_{str}(\text{cm/sec}) = 2.8 \times 10^{r}(ap)^{-1.6}$; p is in mm Hg and a is in cm.

The effect exerted on v_{str} by a longitudinal magnetic field was investigated in [38].

II. NATURE OF STRIATIONS

3. Attempts of Theoretically Explaining the Striations

Different theoretical schemes were proposed to explain the mechanism of occurrence of striations.

A. A. Vlasov regarded striations as periodic structures within the framework of the well known kinetic equation of a collisionless plasma. The collisions with the neutral atoms, the ionization, and the recombination were assumed to be insignificant.[39,40]

The fact that the Vlasov equation does not hold for a layered discharge was noted in [4,41,42] As was already indicated, striations exist only when their length exceeds several electron mean free paths. Therefore collisions with the neutral gas cannot be neglected. The condition for the occurrence of periodic solutions in [39] is the presence of an electron drift velocity of the same order as their thermal velocity, a condition not satisfied in a column with striations.

Related to [39, 40] are a number of investigations[43-48] in which the striations are connected with Langmuir oscillations of the electrons and the ions. Among these papers, significant results are contained in a paper by Gordeev, who obtained the condition for the excitation of ion-acoustic waves by a current flowing through the plasma. The existence of such an excitation in a positive column was later confirmed experimentally.[19-49] However, striations are customarily observed under conditions when the ion sound cannot be excited as a result of frequent collisions between the ions and neutral atoms. Evidence that striations and ion sound have different natures is provided by the difference in the frequency spectra and wave dispersion, by the (frequently simultaneous) existence of these oscillations independently of each other, etc.[19, 36, 50-53, 100]

Periodic instabilities of the positive column were considered in [55-61] on the basis of the hydrodynamic equations for electrons and ions, and on the basis of the Poisson equation, while in [57] they were considered on the basis of the equation for the electron-gas energy. The ionization was assumed to be proportional to the electron density n_e. A common shortcoming of these investigations is that diffusion to the walls was neglected in the equation for small oscillations, while the term with the ionization was retained. This is incorrect not only from the quantitative point of view (the frequency of the striations is of the same order as the diffusion lifetime) but, more significantly, it leads to an investigation of a discharge state which is not in equilibrium with respect to ionization. The following dispersion equation was obtained in [57, 59] for quasineutral waves of the form $\exp\{i(\omega t - kx)\}$, which are identified with the striations:

$$i\omega = z - D_a k^2; \tag{3.1}$$

here z is the frequency of ionization by one electron. The expression given in [61] for the imaginary part of the frequency is

$$\operatorname{Im}\omega = -\frac{b_e E_0 z k}{z + \frac{D_e}{\lambda_D^2}} - iz; \tag{3.2}$$

here b_e and D_e are the mobility and the diffusion coefficient of the electrons, E_0 the longitudinal electric field, and λ_D the Debye screening radius. A characteristic feature of these expressions is the aperiodic growth of the perturbations at small values of k, regardless of the discharge conditions. This strange result is not related to the stationary positive column and is the consequence of the error indicated above. On the other hand, if the diffusion to the walls is correctly taken into account, then the positive column is stable[62] against small perturbations, under the assumption that z does not depend on n_e.

Several theoretical papers were published by L. Pekarek and his co-workers.[63-67] The occurrence of striations was attributed in these papers to deviations from quasineutrality of the plasma. In the first papers, the physical nature of the striations was related with the following assumptions: 1) λ_D and D_e are equal to infinity, and the ion mobility is equal to zero; 2) an important role is played only by fluctuations of the electron temperature and of the ion density, connected with the dependence of the ionization coefficient on T_e. Later on the authors considered the more general case and their viewpoint came closer to that developed in the present review.

As indicated above, the dimensions of the striations vary little with the electron density, and are usually larger by several orders of magnitude than the Debye radius λ_D. The need for regarding the striations as quasineutral waves was already noted by Druyvesteyn.[68] The treatment of striations as waves with a characteristic scale λ_D is therefore inconsistent. The fundamental equations in [64-66] have not been written out with sufficient accuracy, since the equation for the electron energy does not take into account the thermal conductivity. Although the ionization character of the striations was correctly assumed in [55, 61, 63-67], the results of these papers did not explain the main experimental facts on the layered positive column.

Chapnik[62, 69] attempted to interpret striations as waves of electron temperature T_e, assuming their concentration to be constant. The variation of T_e was determined in this case not from the energy equation, but from the continuity and motion equations. Such theoretical premises contradict the experimental data and exclude from consideration certain essential phenomena; they were therefore unsuccessful.

In many of the foregoing theoretical papers, the interpretation of striations as ionization-diffusion oscillations is ignored. But it is precisely this point of view, which has gradually developed in gas-discharge physics[70–73, 4], which is correct.

4. Large-amplitude Striations

The explanation of a number of regularities of striations was first presented in the framework of the semiphenomenological theory of relaxation oscillations of ionization-diffusion type.[74]

The intensified ionization in the front of the striation leads to a local increase of n, causing the longitudinal ambipolar diffusion to assume an important role. On the anode side, the electric diffusion field is opposite to the applied field, and the longitudinal field

$$E = \frac{j}{e_0 b_e n} - \frac{D_e}{b_e}\frac{\nabla n}{n} \tag{4.1}$$

is smaller than in a homogeneous column. The decrease of the Joule heating leads to a cooling of the electron gas and, consequently, to predominance of recombination over ionization in a considerable portion of the striation.

Let us consider, from the point of view of inert gases, the case when the ionization can be neglected everywhere except in a narrow layer in the front of the striation, and recombination takes place only on the tube walls. In a reference frame moving together with the striations at a velocity v, the distribution n is given by the equation

$$D_a n''_\xi + v n'_\xi - \frac{n}{\tau} = 0, \tag{4.2}$$

where

$$\xi = z - vt, \quad \tau = \frac{a^2}{\beta^2 D_a}.$$

Its solution is

$$n \sim n_0 \exp\left[-\frac{|\xi|}{2D_a}(v_1 - v)\right], \quad v_1 = \sqrt{v^2 + \frac{4D_a}{\tau}}. \tag{4.3}$$

The exponential decrease of the concentration in the axial direction, as seen from (4.1), has a certain limit. When this limit is reached, a potential jump takes place,[74,75] due to transport of current to the anode. This jump generates a new region of strong ionization, which is the start of the next striation. The notion that each striation is due to processes occurring in the cathode-facing tail of the preceding one was confirmed by a number of experiments described in [4], and by observations of the occurrence of striations upon ignition of a discharge.[76, 77]

A dispersion relation for the striations was derived in [74] on the basis of (4.1) and (4.3)

$$l \approx \frac{(v_1 + v)a^2}{2\beta^2 D_a} \ln\left[1 + \frac{D_e\left(1 - \frac{v}{v_1}\right) I}{D_a\left(1 + \frac{v}{v_1}\right) i}\right], \tag{4.4}$$

where I is the total recombination current to the walls within the confines of one striation and i the discharge current.

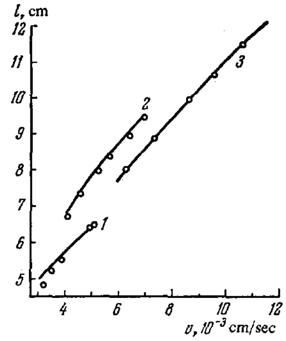


FIG. 11. Plot of l(v) in argon [15]. Pressure: 1 – 6, 2 – 4, 3 – 2 mm Hg.

According to (4.4), the length of the striations increases with their velocity. The increase of l is connected with the slower decrease of n, by virtue of which the minimum value of the concentration is attained at a large distance from the front of the striation. The law governing the concentration of the electrons in the moving striation depends on the ratio of the motion velocity to the velocity of the diffusion drift of the charges from the wall. The smaller the relative role played by the diffusion in the radial direction, the smaller the decrease of n along the discharge axis and the larger the length of the striation.

Relation (4.4) was verified experimentally[15] with the aid of the capture phenomenon described in [28]. When a voltage from an extraneous source is applied to an inductance in the tube supply circuit, it is possible to capture the striation oscillations by means of an external force in a certain frequency interval, close to the natural frequency of the striations. In this case the external signal varies simultaneously the velocity and the length of the striations. The l(v) dependence obtained in this manner is shown by circles in Fig. 11. The same figure shows the theoretical curves in accordance with (4.4), using the values of I measured in these experiments.

The length of the stationary striations is obtained from (4.4) as the particular case when v = 0:

$$l = \frac{a}{\beta} \ln\left(1 + \frac{D_e I}{D_a i}\right). \tag{4.5}$$

Formula (4.5) is valid also for the cathode region of a low-voltage arc,[75] in agreement with the notion that the phenomena in this region and in striations are similar. At a pressure of several mm Hg, the dimensions of the stationary striations as given by (4.5) turn out to be of the order of the tube diameter. On the other hand, the length of the moving striation can reach 4–5 diameters.

For strong striations, regarded as relaxation oscillations of the ionization-diffusion type, the characteristic scales of length, velocity, and oscillation frequency are respectively the quantities

$$\frac{a}{\beta}, \quad \frac{\beta D_a}{a} \quad \text{and} \quad \frac{1}{\tau} = \frac{\beta^2 D_a}{a^2}.$$

From this follows, in particular, the following similarity law: in a positive column $T_e \sim u_u$ and $D_a \sim u_i/p\mu$. We then get from $f \sim 1/\tau$ that $fa\mu \sim (pa/u_i)^{-1}$. The dashed curve in Fig. 12 shows the calculated values of $fa\mu$ on the basis of (4.4) and the experimental data con-

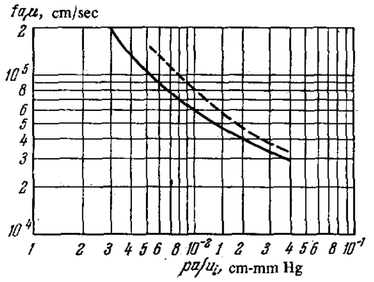


FIG. 12. Plot of $fa\mu$ vs. pa/u_i for argon [74].

cerning v and the ratio 1/i. They are in good agreement with the experimental curve.

If a longitudinal magnetic field is imposed on the discharge, such as to cause diffusion to the walls in accordance with the classical law, this reduces to a replacement of β by $\beta_{\parallel} = \beta_{\parallel}\sqrt{D_{\perp}/D_{\parallel}}$, where $D_{\perp}$ and $D_{\parallel}$ are the coefficients of ambipolar diffusion perpendicular to and parallel to the magnetic field.[78] Since the electron temperature varies sufficiently weakly with the magnetic field, the length of the striations is directly proportional to $\sqrt{D_{\parallel}/D_{\perp}}$, and the velocity is inversely proportional

$$l \approx l_0\sqrt{\frac{D_{\parallel}}{D_{\perp}}}, \quad v \simeq v_0\sqrt{\frac{D_{\perp}}{D_{\parallel}}}. \tag{4.6}$$

This was demonstrated experimentally by Zaitsev and Vasil'eva[38] for both moving and stationary striations.

The foregoing relaxation picture of the moving striations is not stable at all values of v.[80] It is possible to calculate the change in the length of striations following superposition of small velocity vibrations on their translational motion. In the case of small deviations of the start of the n-th striation from the unperturbed position

$$\xi_n \sim e^{i(\varkappa n-\omega t)}$$

and for small values of κ and $\omega \ll v_1 D_a$, the following dispersion equation holds true:

$$-i\omega\left(\frac{2D_a}{v_1^2}\frac{v_1+v}{v_1-v}-\frac{l}{v_1}\right)=i\varkappa-\frac{\varkappa^2}{2}. \tag{4.7}$$

The motion is stable if the expression in the brackets is larger than zero, which is always satisfied for sufficiently fast striations. The stability limit calculated from (4.7) is shown for the corresponding discharge conditions by the line in Fig. 5. It is close to the really observed motion. Neither the exact value of the velocity nor the magnitude of the jump can be obtained without a detailed analysis of the structure of the front of the striation, which is analogous to the cathode-drop zone in a low-voltage arc. No such analysis of the ionization region in striations of large amplitude has been performed so far.

5. Instability of Positive Column

As follows from the preceding section, the main regularities of the developed striations can be explained within the framework of ionization-diffusion oscillations of the relaxation type. This explanation, however, is not valid near the stratification boundaries and does not indicate in which cases the column becomes stratified and in which cases it remains homogeneous. Investigations near the existence boundaries, and especially data on striation excitations, show that the striations are a manifestation of vibrational instability of the discharge plasma.

It was shown in [81-84] that under certain conditions the increment of small oscillations of a homogeneous column becomes positive. An investigation of the stability was carried out within the framework of the following system of equations:

$$\frac{\partial n_m}{\partial t}-D\,\Delta n_m=\widetilde{P}, \tag{5.1}$$

$$\frac{\partial n}{\partial t}+\operatorname{div}(n\mathbf{v}_e)=n\widetilde{z}=\frac{\partial n}{\partial t}+\operatorname{div}(n\mathbf{v}_p), \tag{5.2}$$

$$\frac{\partial\left(\frac{3}{2}nT_e\right)}{\partial t}+\operatorname{div}\left(\frac{3}{2}T_e n\mathbf{v}^*\right)=-n\mathbf{v}_e\mathbf{E}-n\widetilde{H}, \tag{5.3}$$

$$\mathbf{v}_p=b_p\mathbf{E}, \tag{5.4}$$

$$\mathbf{v}_e=-b_e\left[\mathbf{E}+T_e\frac{\nabla n}{n}+\left(\delta-\frac{3}{2}\right)\nabla T_e\right], \tag{5.5}$$

$$\mathbf{v}_e^*=-\frac{2}{3}\delta b_e\left[\mathbf{E}+T_e\frac{\nabla n}{n}+\left(\delta^*-\frac{3}{2}\right)\nabla T_e\right], \tag{5.6}$$

$$n=n_m=0 \text{ when } r=a, \quad n'=n_m'=0 \text{ when } r=0; \tag{5.7}$$

here n_m and D are the concentration and the diffusion coefficient of the metastable atoms, n is the concentration of the electrons and ions, v_e and v_p are their mean velocities, $\widetilde{P}$ and $n\widetilde{z}$ are respectively the numbers of metastable atoms and pairs of electrons and ions arising in a unit volume per unit time, T_e is the temperature of the electrons in eV, and $\widetilde{H}$ is the energy lost by the electron in collisions per unit time. The numbers δ and δ^* are determined by the dependence of the electron mean free path on the velocity and by the form of their velocity distribution function.

It is assumed that the plasma is quasineutral in the presence of striations.

These diffusion equations, without account taken of the volume recombination and of the change of density of the neutral gas, are applicable to a rather limited region of the discharge parameters. However, since the positive column in inert gases is usually layered in this region, the parameters are suitable for the investigation of that instability which leads to the formation of the striations.

Using the simplest scheme of an atom with a single metastable level, going over to one-dimensional equations, and linearizing these equations with respect to small perturbations of the form $\exp\{i(\omega t-kx)\}$, we can obtain the following dispersion equation:

$$\begin{vmatrix} (i\omega+z_m)\,n_m & -\frac{n_m}{\tau_m} & -\widetilde{P}_T T_e \\ \frac{n_m}{n_g}z_{mi} & i\omega+D_a k^2 & \delta_1 k^2 D_a-\widetilde{z}_T T_e \\ 0 & \frac{2j_0^2}{n_0 D_e}-ikj_0 & \delta_0 D_e n k^2+\frac{3}{2}ikj_0+nH_T' \end{vmatrix}=0. \tag{5.8}$$

Here

$$\left.\begin{aligned} &\widetilde{z}_T=\frac{\partial}{\partial T_e}\left\{z_i+\frac{n_m}{n_g}z_{mi}-\frac{1}{\tau}\right\}, \\ &\widetilde{P}_T=\frac{\partial}{\partial T_e}\left\{n_e z_m-n_e\frac{n_m}{n_g}z^*-\frac{n_m}{\tau_m}\right\}, \\ &\delta_1=\delta-\frac{3}{2}, \quad \delta_0=\delta(\delta^*-\delta), \end{aligned}\right\} \tag{5.9}$$

z_m, n_m/n_g, z_m^*, z_i, and $n_m z_{mi}/n_g$ are respectively the frequencies of the acts of excitation, de-excitation, direct and stepwise ionization per electron. The quantity τ_m characterizes the vanishing of the metastable atoms as a result of diffusion to the walls and collisions with the neutral atoms.

For striations we have $\omega \sim D_a k^2$, $E \sim T_e k$, and $D \ll D_a$, we have therefore neglected in (5.8) the derivatives with respect to time in the energy balance and the diffusion of the metastable particles along the tube axis. Since the ac component of the voltage at the terminals of the balanced resistance limiting the current in the tube with striations is usually much smaller than the dc component, the current density $j_0 = -nv_e$ does not depend on the time.

To determine the increment of the oscillations it is possible to assume, without an appreciable error, that

$$\frac{1}{n}\frac{\partial n}{\partial x} \gg \frac{1}{T_e}\frac{\partial T_e}{\partial x}.$$

Then when $\omega \ll z_m$ (i.e., neglecting the oscillations of the metastable atoms we get

$$\gamma = \operatorname{Re}(i\omega) = -\left\{k^2 D_a + \frac{2\tilde{z}_T T_e}{\frac{H'_T D_e}{b_e^2 E_2} + \delta_0\left(\frac{kT_e}{E_0}\right)^2}\right\} + \frac{1}{\tau_m}\frac{z_{mi}\frac{n_m}{n_g}}{z_m\frac{n_e}{n_m}} \cdot \quad (5.10)$$

The first term in this expression, γ_0, is always negative, and if the thermal conductivity is sufficiently large it has a maximum at $k = k_{max}$. Added to it is a positive quantity which, in principle, can make the increment positive near k_{max}. For this purpose it is necessary to have a noticeable stepwise ionization and processes in which the excited atoms are destroyed without collisions with the electrons ($1/\tau_m > 0$). However, if the latter processes predominate, the column is stable.

The intensification of ionization with increasing temperature, as follows from (5.10), is a stabilizing factor. The reason that the intensification of the ionization and the corresponding increase in the plasma conductivity lead to a decrease of the Joule heating of the electrons.

An important role in the development of the instability is played by the oscillations of the density of the metastable atoms.[83, 84] Even when $1/\tau_m \approx 0$ the column is unstable if

$$\frac{\gamma_0}{\omega_r} \leqslant \omega_r \tau \frac{\tilde{P}_T T_e}{z_m T_e} \frac{z_{mi}\frac{n_m}{n_g}}{z_m\frac{n_e}{n_m}}, \quad (5.11)$$

where ω_r is the real part of the frequency.

It follows from (5.10) and (5.11) that the larger $\tilde{z}_T$ and the smaller $\tilde{q}_T$, the more stable the plasma column. For a Maxwellian electron distribution we have

$$\tilde{z}_T \sim \frac{u_i}{T_e} \text{ and } \tilde{P}_T \sim \frac{u_m}{T_e}\left(1 - \frac{(u_i - u_m)}{T_e}\right).$$

where u_i and u_m are the ionization and excitation potentials of the atom. This apparently explains the increased stability on going from light inert gases to heavy ones, and the relative stability of a mercury positive column.

The appreciable role played by stepwise ionization in the stratification of a discharge was noted earlier in [85]. It was confirmed by many experimental facts. The disintegration of metastable states by external irradiation decreases the amplitude of the striations.[16] Addition of mercury impurity to an inert gas, as already noted, decreases the limiting currents by 2-3 orders of magnitude. In alkali-metal vapors, where there are no metastable particles, there are likewise no striations.[86] The existence of an upper and lower current instability limit follows from (5.10) and (5.11), since the ratio $(z_{mi}n_m/n_g)/(z_m n_e/n_m)$ tends to zero at small and large currents.

Thus, the main experimental facts are qualitatively explained within the framework of the theory of ionization instability of the discharge column. It is apparently difficult to obtain from it reliable quantitative data on the stratification limits. To determine the numerical values it is necessary to assume a model in which the atom has many energy levels and to take into account the deviation of the electron distribution function from Maxwellian.[87-89] It might be assumed that more complete data on the atom level populations and on the excitation and ionization frequencies will make it possible, within the framework of the same physical concepts, to obtain satisfactory quantitative agreement with experiment concerning the limits of existence of striations.

The deviation of the distribution from Maxwellian, with a deficit of fast electrons, can greatly intensify the ionization instability of the column. In the places where the plasma has a larger density, the number of fast electrons is increased as a result of the increase in the number of the electron-electron collisions. As a result, the number of ionization and excitation acts increases with increasing n; this, as shown above, is necessary for the development of instability. The stability of the positive column, with due allowance for the variation of the distribution function, was recently investigated by Gentle,[90] who obtained fair agreement with experiment.

In addition, such a deviation from Maxwellian distribution decreases the coefficient of the thermal conductivity that stabilizes the discharge.

6. Velocity of Ionization Waves

From (5.8) we get the phase velocity of the wave:[91]

$$\frac{\omega}{k} = \frac{j_0 D_a}{e_0 n_e D_e} \frac{\alpha_1 - \left(\frac{ka}{2,4}\right)^2}{\alpha_2 + \alpha_3 k^2}, \quad (6.1)$$

where $\alpha_{1,2,3}$ are numbers on the order of several units and depend on $\tilde{z}_T$ and H_T.

Usually the largest increment is possessed by waves whose length is several column diameters ($ka \lesssim 1$). For these we have from (6.1)

$$\frac{\omega}{k} \approx cb_i E_0, \quad (6.2)$$

where c is a coefficient on the order of unity. This is the known velocity of the positive striations. It turns out to be of the order of the velocity of the longitudinal drift of the ions and usually exceeds it by several times.

The motion is due to the fact that the region of maximum ionization does not coincide with the maximum electron concentration. The diffusion field $-T_e(\nabla n/n)$ is superimposed on the external electric field, as a result of which the region of the maximum Joule heat release, followed by the region with the maximum concentration, shift in broad striations towards the anode (Fig. 13).

For striations with $l \lesssim a$, the phase velocity reverses sign, in accord with (6.1), the narrow striations

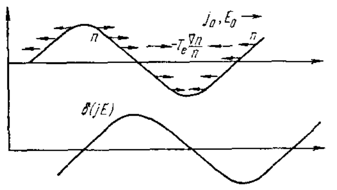


FIG. 13

moving towards the anode. The reversal of sign takes place when $\delta_1 k^2 D_a > \tilde{z}_T T_e$ in (5.8). Physically this is connected with the fact that in short waves the local increase of T_e leads to a decrease of n, since the additional ionization cannot compensate for the thermal expansion of the electron gas.

The dependence of the magnitude and of the sign of the striation velocity on their dimensions was recently investigated in [92, 93]. A continuous variation of the velocity of the artificial layers with a reversal of the sign was observed as a function of k. The experiment was performed with a mixture 54% Ne and 46% He. In a tube of 1.7 cm diameter, the phase velocity reverses sign at k = 6 – 7. The gradual transition from broad positive striations to narrow negative striations is observed in a neon-hydrogen mixture with increasing hydrogen content.

In a high-frequency discharge, ionization instability and plasma stratification exist just as when E = const (t).[94–98] In particular, expressions (5.10) and (5.11) remain in force. The resultant striations are usually stationary, since the term $jT_e(\nabla n/n)$ has a zero average value. Superposition of an additional constant field sets the striations into motion, and in accordance with (6.1) their velocity increases in direct proportion to the dc current and decreases with increasing plasma density (Fig. 14).[96, 98]

As follows from (6.1), the phase velocity decreases with increasing k. This dispersion law, confirmed by numerous experiments, is brought about, as already noted in Sec. 4, by the finite recombination rate on the walls. Another essential factor is the electronic thermal conductivity. The larger this conductivity, the smaller the changes of T_e and the additional ionization due to the concentration perturbations.

Whenever it is possible to put $\delta_1 k^2 D_a \ll \tilde{z}_T T_e$ and $\delta_0 D_e k^2 \gg H'_T$ in Eq. (5.8), the dispersion formula (6.1) takes on the simple form

$$\omega k = \text{const.} \tag{6.3}$$

Such a dependence was observed in [36] at low gas pressures. More frequently $l(f)$ is linear (Fig. 15).

The group velocity of the waves is $d\omega/dk < 0$, i.e., it is directed towards the anode. This circumstance, first pointed out by A. A. Zaĭtsev, explains L. Pekarek's stratification wave. The wave packet moves towards the anode, becoming stronger or weaker, depending on whether the discharge is unstable or stable.

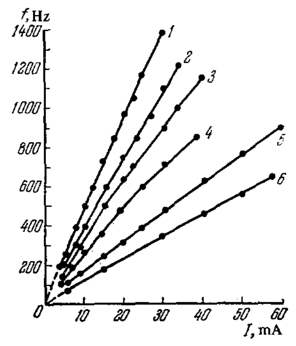


FIG. 14. Frequency of moving layers in a high-frequency discharge vs. the dc component of the current. Neon, p = 1.8 mm Hg, 2a = 3.1 cm; the current in milliamperes is: 1 – 35, 2 – 60, 3 – 76, 4 – 98, 5 – 117, 6 – 164.

7. Oscillatory Properties of Column

As already noted, the oscillations of the current in the external circuit are not necessary for the appearance of striations. However, such oscillations, as well as oscillations in the cathode or anode parts of the discharge, can greatly influence the steady-state wave spectrum. At low pressures, the ionization instability near the boundary becomes manifest in the appearance of noise with a continuous spectrum in a certain interval of frequencies.[101] The maximum of the amplitude corresponds to the frequency from which the gain factor of the waves is the largest, and the customary dispersion relations for striations are satisfied. With increasing amplitude, one wave becomes singled out and synchronizes the remaining oscillations, and a line spectrum is produced. An important role is played in this case by the feedback between the anode and the cathode ends of the column, which is usually effected via the external electric circuit.[103, 31] In such a column there is satisfied the phase-balance condition[48, 21]

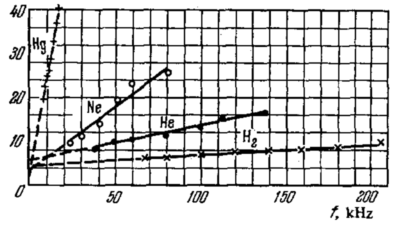
$$\frac{2\pi}{l} L + \varphi_0 = 2\pi n, \tag{7.1}$$

where $2\pi L/l$ is the phase shift of the column, φ_0 is the phase shift introduced by the rest of the circuit, and n is an arbitrary integer. In accordance with (7.1) it is possible, by smoothly varying the length of the column, to observe a jumpwise change in the frequency and the length of the striations at intervals equal to the average striation length.

Near the plasma stability limit, the gain factor of the waves is small, and the occurrence of self-maintaining oscillations depends on the feedback coefficient and on the tube length. For this reason, the stratification limit in short tubes does not coincide with the condition for the vanishing of the increment. The striations can vanish completely when the column length is decreased and still spans five wavelengths.[21]

Other oscillations can cause irregular striations which are frequently observed under different conditions.[104–106] It is seen from Fig. 6b that the irregular character of the striations is produced by perturbations that propagate, like the stratification wave, towards the anode. The source of these perturbations can be the natural oscillations in the cathode and anode parts of

FIG. 15. Dependence of length of striations on the frequency in different gases (a = 1.5 cm).



the discharge, and the anode oscillations can be transmitted through the external circuit. Such oscillations can change the frequency and consequently other characteristics of the striations, and can influence the limits of their occurrence, although the ionization intensification of the waves in the column does not by itself depend on these oscillations.

The fact that one nonlinear wave with definite l and f is frequently observed in a layered positive column, and not a turbulent regime with many degrees of freedom, has made it possible to investigate theoretically the properties of striations of small finite amplitude,[83, 107-109]. The conditions of "soft" excitation of oscillations were obtained in [83, 107], where it was shown that near the excitation limit the form of the striations should differ little from sinusoidal, and their amplitude is proportional to $\sqrt{\lambda - \lambda_{cr}}$, where λ_{cr} is the critical value of a parameter determining the state of the plasma. Such a parameter may be the current, the gas pressure, or the radius of the column.[110] The wave decrement near the stability limit is proportional to $\lambda - \lambda_{cr}$.[83, 17, 108] These results of the quasilinear theory are confirmed by experiment.[28, 30, 21] With increasing $\lambda - \lambda_{cr}$, the striations gradually assume the form of the relaxation oscillations considered in Sec. 4. In the front of the striations, where the electric field and the directional velocity of the electrons are larger, new types of oscillations may become excited.

Thus, ion-acoustic oscillations with frequency 0.2–1 MHz were observed[19] in an argon discharge at the starting points of the striations. Plasma oscillations with frequency 2–4 GHz, connected with the front of the striations, are described in [111].

III. MAGNETIC STRIATIONS

8. Theory

The superposition of a transverse magnetic field on a positive discharge column in a glass tube changes the characteristics of the striations, and in particular, decreases their length.[112] These changes of ordinary striations are connected apparently with the decrease of the longitudinal diffusion and thermal conductivity and with a redistribution of the plasma density over the section of the column, such as to increase the loss at the walls.

In addition, a unique mechanism of ionization instability exists in a plasma placed in crossed electric and magnetic fields. Certain proposals of magneto-gasdynamic conversion of thermal energy into electricity are based on the use of a low-temperature plasma in cross crossed E and H fields. It is therefore no accident that the new type of instability, called magnetic striations, was first investigated in plasma of inert gases containing as an additive easily ionized metal vapors (usually cesium or potassium). Kerrebrock[113] proposed that such a plasma could be used in mhd generators, and the electron temperature in such a plasma can exceed the gas temperature since the electrons are heated by the Joule heat of the working current of the generator. This makes it possible to maintain the required electric conductivity of the plasma and still lower the temperature of the working gas (T_g) below 2000° C.

Numerous investigations of such a low-temperature plasma, performed in recent years,[114-124] have shown that under certain conditions ionization equilibrium with the electron temperature is maintained in it, and the energy lost by the electrons is due essentially to elastic collisions with the gas atoms. Sufficiently slow variations of the plasma parameters occur in a quasistationary manner.[125, 126]

The ionization instability, which leads to stratification of the plasma in crossed E and H fields, was theoretically considered by Velikhov and Dykhne[127, 129] and by Kerrebrock.[128] The state of the plasma is described by the following system of equations:

1) The generalized Ohm's law

$$\sigma\left[\mathbf{E}+T_e\frac{\nabla n}{n}+\left(\delta-\frac{3}{2}\right)\nabla T_e\right]=\mathbf{j}+[\mathbf{j}\Omega\tau]. \tag{8.1*}$$

2) The energy conservation law for electrons

$$J\frac{\partial n}{\partial t}+\frac{3}{2}\frac{\partial}{\partial t}(nT_e)-\delta\nabla T_e\frac{j}{e}=\frac{Ej}{e}-\varkappa\frac{nT_e}{\tau}. \tag{8.2}$$

3) The Saha equation

$$\frac{n^2}{n_g-n}=AT_e^{3/2}e^{-J/T_e}. \tag{8.3}$$

In (8.1)–(8.3), n and n_g are the concentrations of the plasma electrons and neutral atoms, T_e and J are the electron temperature and the ionization potential (in volts), κ is the average fraction of the energy lost by the electron by collision with heavy particles, τ is its free path time, σ is the conductivity, δ is a number determined by the dependence of the mean free path of the electrons on the velocity and by the form of their velocity distribution function, A is a known constant, and $\mathbf{\Omega} = e\mathbf{B}/mc$.

The term with the thermal conductivity was left out from (8.2), and it was assumed for simplicity that $T_e \gg T_g$.

The linearized system of equations (8.1)–(8.3) yields, under the conditions curl $\mathbf{E} = 0$ and div $\mathbf{j} = 0$, the following dispersion equation for small-amplitude waves of the form exp $i(\mathbf{k}\cdot\mathbf{z} - \omega t)$

$$i\omega=\frac{j_0^2\left\{-2\omega\tau\sin\varphi\cos\varphi-2\cos^2\varphi+2\sin^2\varphi\,\frac{\partial\ln\tau}{\partial\ln n}-\frac{\partial\ln T}{\partial\ln n}\right\}}{n_0\sigma J\left[1+\frac{3}{2v}\left(1+\frac{\partial\ln T_e}{\partial\ln n}\right)\right]}+\frac{i(\mathbf{k}\mathbf{j}_0)\left(v-\frac{3}{2}-xv\right)}{en_0\left(v+\frac{3}{2}\right)^2(1-x)+\frac{3}{2}(2-x)}; \tag{8.4}$$

here φ is the angle between the current j_0 and the front of the wave,

$$v=\frac{J}{T_e},\quad x=\frac{n}{n_g}\ \text{and}\ \frac{\partial\ln T}{\partial\ln n}=\frac{2-x}{\left(v+\frac{3}{2}\right)(1-x)}. \tag{8.5}$$

The oscillation increment has a maximum if the following inequality is satisfied

$$\operatorname{tg}2\varphi=-\frac{\omega\tau}{1+\frac{\partial\ln\tau}{\partial\ln n}}. \tag{8.6}$$

Its value is

$$\gamma_{\max}\simeq\frac{j_0^2}{\sigma nJ}\left\{\sqrt{(\omega\tau)^2+\left(1+\frac{\partial\ln\tau}{\partial\ln n}\right)^2}+\frac{\partial\ln\tau}{\partial\ln n}-1-\frac{\partial\ln T}{\partial\ln n}\right\}. \tag{8.7}$$

If the degree of ionization of the additive is weak, then

---

*$[\mathbf{j}\Omega\tau] \equiv \mathbf{j} \times \Omega\tau$.

$$\frac{\partial \ln T}{\partial \ln n} \approx \frac{2T}{J} \sim 10^{-1}$$

and $\gamma = \gamma_{\max}$ when $\omega \gtrsim 1$.

The physical mechanism of the ionization instability can be understood by considering the fluctuation of the Joule heating connected with the fluctuation of the plasma density. Let us assume that in the shaded region of Fig. 16 the concentration is larger than in the surrounding space by a small amount δn. We assume that the main current is directed along Oy, the magnetic field is perpendicular to the plane of the figure and directed away from us. In accordance with the generalized Ohm's law we have

$$\frac{E_{0x}}{E_{0y}} = \omega\tau. \tag{8.8}$$

The electron velocity in this plane is

$$\mathbf{v} = -\frac{b_e \mathbf{E}}{1+(\omega\tau)^2} + \frac{(\omega\tau)^2}{1+(\omega\tau)^2} \frac{c\,[\mathbf{EH}]}{H^2}, \tag{8.9}$$

where the first term corresponds to the usual mobility and the second to the drift in the crossed fields. For simplicity we neglect the fluctuations of τ. The normal component of the current is conserved; therefore electric charges appear on the boundary of the layer and change v and E inside the layer. The magnitudes of these changes are determined by the first term of (8.9), since the tangential field cannot change (curl $\mathbf{E} = 0$):

$$\delta v_\perp = -v_\perp \frac{\delta n}{n} = -\frac{b_e \delta E_\perp}{1+(\omega\tau)^2}\,; \tag{8.10}$$

here

$$v_\perp = -\frac{j_0 \cos\alpha}{e n_0}.$$

Owing to change of $\mathbf{E}_\perp$, the longitudinal velocity also changes:

$$\delta v_{\parallel} = \frac{(\omega\tau)^2}{1+(\omega\tau)^2} \frac{c\,\delta E_\perp}{H}. \tag{8.11}$$

The longitudinal component of the fluctuation current in the layer

$$\delta j_{\parallel} = j_{0\parallel} \frac{\delta n}{n} - e n_0 \delta v_{\parallel},$$

is equal to, according to (8.8) and (8.11),

$$\delta j_{\parallel} = j_0 (\sin\alpha + \omega\tau \cos\alpha) \frac{\delta n}{n}. \tag{8.12}$$

The additional power released in a unit volume of the layer is

$$\delta W = (\delta j_{\parallel} E_0) + j_{0\perp} \delta E_\perp = \frac{j_0^2}{\sigma} (2\omega\tau \sin\alpha \cos\alpha - 2\cos^2\alpha + 1) \frac{\delta n}{n}. \tag{8.13}$$

Thus, in a layer with $\delta n > 0$ the value of δW can exceed the increase in the energy lost to collision with the gas; the latter is equal to

$$\frac{j_0^2}{\sigma}\left(\frac{\delta n}{n} + \frac{\delta T}{T}\right).$$

This in turn increases T_e and the degree of ionization, i.e., it leads to a further deviation of the concentration from the equilibrium value.

The developed inhomogeneities move in analogy with the striations in a positive column.[91] As shown in Sec. 6, the motion is due to the release of additional heat, which is proportional to the gradients of n and T_e. In the case of a low degree of ionization and when the energy loss in the elastic collisions predominates, the velocity of the magnetic striations is

$$v \simeq \frac{T_e}{J} \sqrt{\frac{T_e}{M}} \tag{8.14}$$

(M—molecular weight of the main gas).

When the degree of ionization of the additive is increased, the velocity, in accordance with (8.4), reverses sign when

$$\frac{n_e}{n_g} = 1 - \frac{3}{2}\frac{T_e}{J}.$$

The backward motion when $d \ln T/d \ln n \gtrsim 1$ is connected with the transport of heat by the drifting electrons.

Formulas (8.4), (8.6), and (8.7) pertain to an unbounded plasma. In a plasma of finite dimensions, the form and the dimensions of the inhomogeneities depend on the boundary conditions for the perturbations. The fluctuation currents are then shortcircuited inside the volume by the external electric circuit (Fig. 17).

With increasing degree of ionization of the alkali additive and with decreasing temperature difference $T_e - T_g$, the critical magnetic field increases. According to (8.7), the plasma becomes stable when $x \to 1$, if $\partial \ln \tau/\partial \ln T < 0$. However, weakly-conducting columns (not necessarily of round cross section), elongated along the magnetic field,[130] can arise in such a plasma with a fully ionized additive.

The finite amplitude of the layers in an unbounded plasma was taken into account by Vedenov and Velikhov.[130] They have shown that for fluctuations whose amplitude $\delta n > n/\omega\tau$, layers perpendicular to the direction of the average current are most likely to develop. The dimensions of the arising inhomogeneities remain arbitrary, within the framework of Eqs. (8.1)–(8.3). In fact, the energy transport by radiation and by ambipolar diffusion limits the development of short waves. Long waves are limited by the dimensions of the plasma and depend on the conditions under which the plasma is maintained.

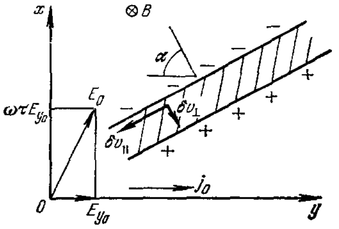


FIG. 16

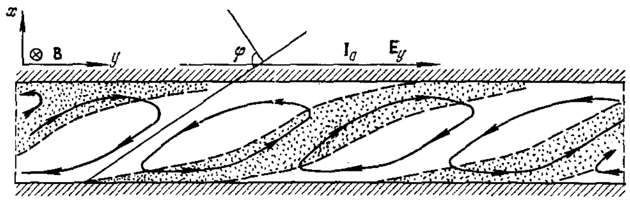


FIG. 17

9. Experimental Investigation of Magnetic Striations

The first experiments aimed at testing the theory of [127] were performed with a pulsed discharge in a glass tube of rectangular form, placed between poles of an electromagnet.[131] The tube contained several pairs of electrons connected to independent discharge circuits. The experimental setup is shown in Fig. 18. The tube contained argon at a pressure of approximately 100 mm Hg and mercury vapor at a pressure of approximately 10^{-2} mm Hg. A system of electrostatic probes was used to determine the averaged electric fields along the current (E_y) and the Hall current (E_x), to determine the relative changes of the plasma density, and to observe the oscillations. The electron temperature was measured by optical methods. The time necessary for the current to drift under the influence of the force $(1/c)\,\mathbf{j}\times\mathbf{B}$, determined by the mobility of the ions in E_x, was much longer than the measurement time.

It was observed that oscillations with frequency on the order of 10^4 Hz are excited in a plasma when $\omega\tau > 3$. The ratio $\bar{E}_x/\bar{E}_y$ ceases in this case to depend on the magnetic field intensity (Fig. 19), and the conductivity decreases in inverse proportion to $\omega\tau$. Similar results were obtained in [138].

The nature of these phenomena was established by Shipuk and his co-workers.[129, 132, 133] Their experimental setup was similar to that shown in Fig. 18. To observe the structure of the inhomogeneities of the plasma they used photomultipliers and also multistage electron-optical converters, with which photographs were taken at 10–14 μsec exposures. The discharge was produced at different pressures of the inert gas (argon or helium) with cesium vapor added, the degree of ionization of the additive being variable.

When the magnetic field intensity is increased, the plasma with weak degree of ionization loses homogeneity. Striations inclined at 25–35° to the current are produced in the plasma in a plane perpendicular to B (Fig. 20). The critical value for the occurrence of the instability is $(\omega\tau)_{cr} \sim 1-2$. With increasing $\omega\tau$, the striations lose their regular form as a result of development of inhomogeneities of smaller dimensions, and the picture becomes that of turbulence. When the direction of the current or of the magnetic field is reversed, the inclination of the striations changes in such a way that $k_x/k_y < 0$ always, where k_y and k_x are the components of the wave vector along the current $\mathbf{j}_0$ and along $\mathbf{j}_0 \times \mathbf{B}$. These data agree well with the ionization-instability theory developed in the preceding section.

In the case of small $\omega\tau$, the dimensions of the magnetic striations are determined by the configuration of the discharge and are equal in order of magnitude to the width of the plasma column. The measured velocity of the regular striations is close to the theoretical one determined by formula (8.14) (Fig. 21). The density and plasma-potential oscillations connected with this motion lie, in the case of small excess over critical, in a narrow interval of frequencies on the order of several kHz. With increasing $\omega\tau$ and with development of small-scale inhomogeneities, the spectrum of the oscillations broadens towards larger frequencies. It was also shown in [132,133] that the stability increases with

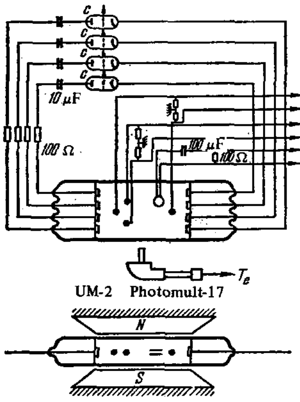


FIG. 18. Setup for experiments with a discharge in a transverse magnetic field.

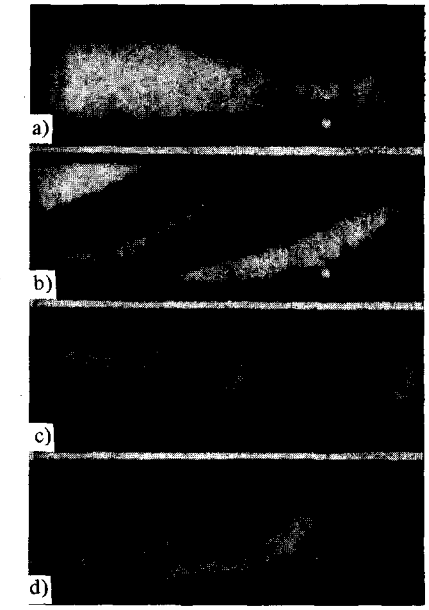


FIG. 20. Photographs of the discharge. Values of $\omega\tau$: a) 0; b) 2; c) 3; d) 8.

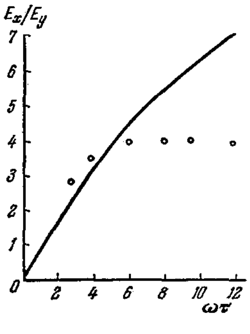


FIG. 19. Plot of E_x/E_y vs. $\omega\tau$ [131]. Solid line–calculation for a homogeneous plasma.

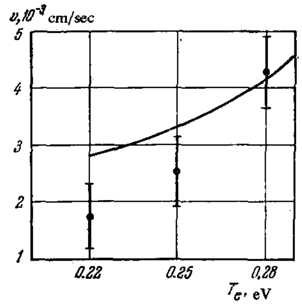


FIG. 21. Velocity of magnetic striations in an argon-cesium plasma.

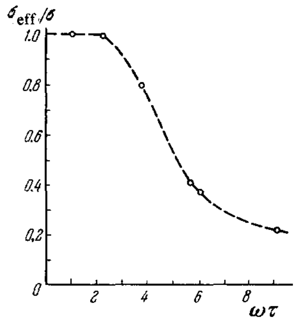


FIG. 22. Dependence of the ratio of the effective conductivity to the average conductivity on $\omega\tau$ [132].

increasing degree of ionization of the additive.

The occurrence of magnetic striations and closed Hall currents influences the effective Hall parameter (the ratio of the average field E_x/E_y) in such a way, that this ratio ceases to depend on **B**. A theoretical justification for this law is given in [127, 134].

The effective conductivity of the plasma in a direction perpendicular to **B** is decreased in this case (Fig. 22).

The ionization character of the described phenomena confirms the experiments of Brederlow, Feneberg, and Hodson,[135] who investigated plasma in an argon–potassium mixture at 2000°K. At low current densities, when the Joule heating of the electrons is insignificant, the plasma conductivity retained its laminar character and the effective Hall parameter increased linearly with B, up to a value on the order of 10. With increasing current density and electron temperature, oscillations set in with frequency 7 kHz and the E_x/E_y ratio saturated at the values 2–3.

Many new data confirming the theory of occurrence of magnetic striations were obtained also in [137, 139, 140].

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