SOVIET PHYSICS USPEKHI VOLUME 11, NUMBER 1 JULY-AUGUST 1968

539.12.01

THE NONRELATIVISTIC QUARK MODEL

E. M. LEVIN and L. L. FRANKFURT

A. F. Ioffe Physico-technical Institute, Academy of Sciences of the U.S.S.R.

Usp. Fiz. Nauk 92, 243—288 (February, 1968)

CONTENTS

Introduction
I. The Nonrelativistic Quark Model and the Classification of Hadrons 109
II. Mass Formulas in the Quark Model 110
III. Electromagnetic Properties of Hadrons 112
IV. Weak Interactions of Hadrons 115
V. Scattering of High-energy Particles (Description of the Model) 116
VI. Main Consequences of the Quark Model for the Scattering of High-energy Hadrons 117
VII. Relations between Total Cross Sections 118
VIII. Relations between Differential Cross Sections for Inelastic Reactions 120
IX. The Density Matrix 122
X. Collisions of High-energy Hadrons (Relations between the Nonrelativistic Quark Model and other Theories) 123
XI. Conclusion. 124
What Should be Measured and Why 124
Appendix I. Hadron Wave Functions in the Quark Model 125
Appendix II. 126
Literature Cited 127

INTRODUCTION

THE discovery of a large and still growing number of new particles and the varied properties of these particles have brought into being the group-theory approach to the study of strongly interacting particles. The symmetry SU(3) is particularly important.[1,2] With it it has been possible to construct a classification of mesons and baryons. A brilliant confirmation of the theory was the discovery of the singlets and octets of pseudoscalar, vector, and tensor mesons, and the octet $J^P(1/2^+)$ and the decuplet $J^P(3/2^+)$ of baryons.[3] On the other hand, the large differences of the masses of particles belonging to the same multiplet indicates that the SU(3) symmetry is strongly broken, and this obliges us to look for a dynamical source of the symmetries. It is helpful to turn for analogies to nuclear physics, in which the group-theory approach has long been used with success.

In nuclear physics the source of the symmetries is quite apparent: all nuclei are constructed from protons and neutrons, which can be regarded as identical if we neglect electromagnetic interactions (isotopic symmetry arises in this way); if we further neglect the spin-spin and spin-orbit interactions of nucleons, we get the higher symmetry SU(4),[4] which plays the same role in nuclei as SU(6)[5,6] in elementary particles.

We can try to develop the analogy further, constructing a "quasinuclear" model of the strongly interacting particles (which have come to be called hadrons) with the assumption that all hadrons are composed of a few fundamental components. The first attempt was made by Fermi and Yang in 1949.[7] They pointed out that the π meson may be a bound state of a nucleon and an antinucleon. After the discovery of "strange" particles, Sakata increased the number of fundamental particles to three, adjoining to the proton and neutron the lambda hyperon and the corresponding antibaryon.[8] The model played a useful part in the classification of mesons and the construction of a theory of the weak interaction.[9] It turned out later, however, that the Sakata model leads to an incorrect classification of the baryons.[10] It was replaced by the octet version of SU(3),[1,2] which gave a basis for the successful classification of the mesons and baryons and even for predictions of the masses of new particles.[1] The discovery of the Ω^- was a triumph of the group-theory approach. On the other hand, purely theoretically it seems strange that the lower triplet representations of SU(3) are not realized in nature, the situation being quite different, for example, from that for the isotopic symmetry and the rotation group. Moreover, one does not understand why in SU(3) the nonets are singled out for the mesons and the octets and decuplets for the baryons.

Owing to this Gell-Mann and Zweig in 1964 raised the question of the possibility that particles exist which realize the triplet representation of SU(3) and possess unusual properties—fractional charge and baryon number equal to 1/3.[11,12] If these particles exist in the free state, then at least one of them must be stable. Let us denote by Q_p, Q_n the isodoublet with strangeness zero, and by Q_λ the isosinglet with strangeness -1. Gell-Mann called them quarks, and Zweig, aces. The antiquarks are denoted by $\overline{Q}_p$, $\overline{Q}_n$, $\overline{Q}_\lambda$.

If one interprets the observed elementary particles as compound systems made up of quarks and antiquarks, it is possible to construct the most economical classification of the hadrons, and form a sort of "Mendeleev table" for the elementary particles. In fact, the simplest baryon structure is Q-Q-Q, and therefore the baryons must group themselves into singlets, octets, and decuplets, since $3 \times 3 \times 3 = 1 + 8 + 8 + 10$. The simplets structure of a meson is Q-$\overline{Q}$, and consequently,

Table I. Quantum Numbers of Quarks and Antiquarks

Quarks \ Quantum numbers	Q	T	T_3	S	B	Y
Q_p	$\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\frac{1}{3}$	$+\frac{1}{3}$
Q_n	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$	$+\frac{1}{3}$
Q_λ	$-\frac{1}{3}$	0	0	-1	$\frac{1}{3}$	$-\frac{2}{3}$
$\overline{Q}_p$	$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$\overline{Q}_n$	$+\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$\overline{Q}_\lambda$	$+\frac{1}{3}$	0	0	$+1$	$-\frac{1}{3}$	$+\frac{2}{3}$

as in the Sakata model,[8,9] the mesons must come together in nonets—singlets and octets—since $3 \times 3^* = 1 + 8$. [An intuitive explanation of these resolutions of products of representations of SU(3) into irreducible representations is given in the review article[13].] The prediction that the mesons are grouped in nonets is in brilliant agreement with experiment; the existence of nonets of pseudoscalar and of vector mesons is firmly established, and that of a nonet of tensor mesons is very probable. In the case of the baryons the octet $1/2^+$ and the decuplet $3/2^+$ are completely filled; only some of the particles are observed in the other multiplets. At present there is no definitely known hadron which does not fit into the systematics constructed on the assumption that hadrons consist of a minimal number of quarks; baryons of three quarks, and mesons of a quark and an antiquark.

Interest in the quark model increased later in connection with the appearance of the group SU(6) and its generalizations.[5] These symmetries, in spite of successes in the explanation of the static properties of hadrons,[5,6] are evidently incorrect, since they contradict Lorentz invariance and the conservation of probability.[14] And it has turned out that the main results of these symmetries can be derived in a more traditional quark model,[12] in which symmetries of the type of SU(6) are only approximate (in exact analogy with the Wigner theory of the nucleus[4]).

There are a great many papers on quark models, but unfortunately lack of space allows us to deal with only a few of them. In this review we shall confine ourselves to the exposition of the simplest nonrelativistic quark model. This model is based on the assumption that a

Table II. Multiplets of Mesons in the Quark Model (according to data of[3])

L	G	J^P	$S=0$	L	G	J^P	$S=1$
0	$(-1)^T$	0^-	π (140), K (495) η (549), X_0 (958)	0	$(-1)^{T+1}$	1^-	ρ (780), K^* (890) ω (780), φ (1019)
1	$(-1)^{T+1}$	1^+	B (1208), K (1360) η (1400)	1	$(-1)^T$	2^+	A_2 (1300), K (1420) f (1250), f' (1500)
						1^+	A_1 (1080), K (1320) D (1285), E (1420)
						0^+	π_V (1003), K (1100) ω (1050), φ (1300)

strong interaction between the original quarks conserves their individuality, changing only the parameters of the quarks, and that the effective quasiparticles inside a hadron (we call them quarks also) interact weakly with each other and move slowly relative to each other.[15-17] The nonrelativistic nature of the motion of the quarks inside hadrons allows the application of the usual methods of atomic physics, and this makes it easy to construct a more complete systematics of hadrons of the type of L-S coupling, taking into account the spins and orbital angular momentum of the quarks. For example, for mesons the quark model (like any composite model based on a triplet of fermions) predicts both the ordinary parity P and also the G parity in agreement with experiment. Moreover, for the lowest states with orbital angular momentum zero it has been possible to explain some properties of hadrons. For the excited states of hadrons, because the experimental situation is unclear, we can assert only that the quark-model classification of the hadrons does not contradict the experimental results.

Using the methods of atomic physics, with the quark model we can not only rather easily explain the mass splitting of the hadrons within a multiplet,[1] but also connect the mass differences of particles belonging to different multiplets of SU(3)[12,15,18] (for example, the octet $1/2^+$ and the decuplet $3/2^+$), in agreement with experiment. There has been an equally successful explanation of the electromagnetic mass differences of hadrons.[55] We point out that in symmetries higher than SU(3) there are actually no predictions, since there are too many parameters in the mass formulas.

The quark model has achieved great success in the explanation of the electromagnetic properties of hadrons. In particular, the theory predicts the magnetic moments of baryons (see Table VI) in good agreement with experiment. [For example, $\mu_p/\mu_n = -3/2$ (theory),[5,6,15] and the experimental value of this ratio is -1.46]. The magnetic moment $\mu_{\Delta p}$ of the radiative decay differs from the theoretical value by a factor 1.28. The predictions derived for the form-factors are also not in contradiction with experiment (see Tables VII, VIII).[19,20] All of these results do not follow from SU(3).

If we assume that the radius of a quark is less than that of a hadron, we get from the quark model the equation $G_E^p(t) = G_M^p(t)/\mu_p$,[15] where $G_E^p(t)$ and $G_M^p(t)$ are the Sachs electric and magnetic form-factors of the nucleon. This result is in good agreement with experiment over a wide range of momentum transfers.[21]

In semileptonic weak interactions the quark model explains the well known selection rules: $\Delta T = 1$ in strangeness-conserving decays and $\Delta T = 1/2$, $\Delta S = \Delta Q = 1$ in decays which do not conserve strangeness. To get a description of semileptonic decays in the quark model which agrees with experiment[22] it suffices to fix the parameters of the β decay of quarks and the Cabibbo angle[23] (see Table X). As compared with SU(3), for the semileptonic processes now observed the quark model predicts a definite F/D ratio in the decays of the octet $1/2^+$ and gives a connection between the decays of the decuplet $3/2^+$ and the octet $1/2^+$ in agreement with experiment.[22] We note that the selection rules are not peculiar to the quark model; they hold in any model in which the hadrons are composed of three fermions and in which the weak interaction can be described as the decay of one fermion.[9] A nontrivial prediction follows for the axial form-factor of the nucleon: $F_A(t)/G_A = G_E^p(t)$, if, as before, we assume that the size of quarks is much smaller than that of nucleons. This last relation is also not in contradiction with experiment.[24]

The quark model allows us to explain some features of the scattering of high-energy hadrons, if we assume that the scattering of two particles reduces to a single scattering of the quarks of which the particles are composed. For this it is in any case necessary that the quarks be nonrelativistic inside the hadron and that they be much smaller than the hadron. On these assumptions one gets, for example, "selection rules" for the scattering of high-energy hadrons (see Chap. VI): $|\Delta Q| \leq 1$, $|\Delta T| \leq 1$, $|\Delta S| \leq 1$.[25,26] The mixing angles (cf. Chap. II) for the nonets 0^-, 1^-, 2^+, as determined from experiments on the scattering of hadrons at high energies,[28] are not in contradiction with the values calculated from the masses of known particles.[18,27,29] The quark model gives a less satisfactory description of the reactions $P(0^-) + N(1/2^+) \to P(0^-) + N_\delta(3/2^+)$. Here $P(0^-)$ denotes a meson of the octet 0^-. In this case the predictions for the spin correlations are rather good (cf. Chap. IX), but those for the differential cross sections are poorer. For the reaction $\pi^+ + p \to \eta + \Delta^{++}$ the cross section is isotropic at $p_\pi = 8$ GeV/c,[30] instead of having a minimum at $0°$. The inequality of the differential cross sections for the reactions $\pi^- + p \to \pi^0 + n$ and $\pi^+ + p \to \pi^0 + \Delta^{++}$ is violated by about a factor 1.5, with energy release $Q = 3$ GeV.

If we assume that the scattering amplitude of quarks depends only weakly on the momentum transfer, the main shape of the differential cross section is a universal function and is the same as the electromagnetic form-factor of the proton[31] (see Table XV).

A useful feature of the quark model is that in it it is easy to take into account breaking of SU(3); we have only to suppose that the parameters of Q_λ are different from those of Q_p and Q_n.[32,60,61] This is very important, since SU(3) symmetry is strongly broken in the interaction of hadrons.[13]

Interesting results are obtained on the assumption that the original parameters of the quarks are renormalized identically in mesons and in baryons. By means of the hypothesis of an equal gain of mass in mesons and in baryons Zweig explained the larger part of the mass difference of particles forming a single SU(3) multiplet, and derived formulas for the mass difference of mesons and baryons which turned out to be in good agreement with experiment.[12] The breaking of SU(3) in the spin-spin interaction of quarks is also of a universal character.[18] If the mass renormalization of the quarks is the same in mesons and in baryons, the ratio of the central masses of baryons and mesons must be 1.5, and experiment gives the value 1.8 (cf. Chap. II). Equality of the magnetic moments of quarks in mesons and in baryons allows us to express the width of the radiative decay $\omega \to \pi^0 + \gamma$ in terms of the magnetic moment of the proton. The theoretical value of this width is 1.2 MeV,[34] and the experimental value is $\Gamma = 1.15 \pm 0.25$ MeV.[3] If we assume that the amplitude for scattering of quarks by quarks (or antiquarks) does not depend much on whether the quark belongs to a meson or a nucleon, then in the Pomeranchuk limit $\sigma_{pp}/\sigma_{\pi p} = 3/2$.[25] This last result is valid only if baryons consist of three quarks. Let us assume that at the largest energies now achieved with accelerators σ_{pp} and $\sigma_{\pi p}$ are not very different from the Pomeranchuk limit; then experiment gives $2\sigma_{pp}/(\sigma_{\pi^+p} + \sigma_{\pi^-p}) = 1.58 \pm 0.05$.[35] All of the relations which are derived from the hypothesis that the quark parameters are the same in mesons and in baryons are not understandable outside of the quark model, and it is undoubtedly interesting that they agree with experiment.

One of the serious shortcomings of the approach in which quarks are regarded as actually existing particles is, of course, the fact that they have so far not been observed in the free state. Various possibilities for observing quarks have been discussed in detail in the review[36]. We shall present new results. Experiments with accelerators give a lower limit of 5 GeV on the mass of the quark.[37] If, indeed, we suppose that the cross section for quark production is depressed because there are many competing channels, the experimental lower limit is much lower: $m_Q \geq 3$ GeV.[38] If we believe the arguments based on the statistical model, it is also hard to observe quark production in cosmic rays.[39] Searches for quarks in cosmic rays give as an upper limit on the flux of quarks $I_Q \leq 10^{-9}$ quark cm^{-2} sec^{-1} sr^{-1}.[40] Probably it is easier to look for so-called relic or residual quarks, preserved as a result of the evolution of the Universe. The value predicted from the model of a "hot" universe for the most probable ratio of the number of residual quarks to the number of nucleons is 10^{-10}—10^{-13}.[41] Experimentally the upper limit on the number of quarks in graphite is 10^{-16}.[42,43] This sharp disagreement with the theory may be due to an unfortunate choice of material.[27]

The main shortcoming of the quark model is that it contains logical difficulties. For example, from the point of view of S-matrix theory, if the binding energy of the quarks is large it is natural to expect that other virtual states (π mesons, nucleon pairs, etc.) should play an important part, and including these could radically change all of the results; on the other hand, in our opinion the absence of quarks in the free state is not an argument against the model, since possibly quarks are quasiparticles of the type of phonons and excitons in solids. It is curious that M. Gell-Mann, one of the "discoverers" of quarks, regards them as quasiparticles.[44] Most, and perhaps all, of the results presented in this review can be explained by the presence of quasiparticles with an effective mass of the order of 300 MeV. This value of the effective mass of a quark is determined from the magnetic moment of the proton.[17]

The main difficulty in interpreting quarks as quasiparticles is of course the fact that the mechanism by which they arise is unknown. But our not understanding the causes of the existence of such unusual quasiparticles is indeed not surprising, since their appearance may be caused only by collective effects of virtual states. In the present inadequately developed state of S-matrix theory it is not excluded that the existence of quarks as quasiparticles may not lead to any contradictions. Moreover, the presence of quasiparticle-quarks does not contradict the widely held view that the strongly interacting particles are equally nonelementary. In our opinion, the impossibility of proving the existence of quasiparticle-quarks will not discredit the scheme as long as the model does not lose its heuristic value and its ability to explain experiments.

The purpose of this review is to acquaint the reader with the naive nonrelativistic quark model. The only requirement for an understanding of a large part of this paper is familiarity with the fundamentals of quantum mechanics. Remarks less easily accessible are those devoted to comparison of the predictions of the quark model with the symmetries. The writers have tried to give a brief explanation of the groups SU(3), SU(6), and $SU(6)_W$, but this is clearly insufficient for the understanding of some chapters (for example, Chap. X), and a more detailed knowledge of group theory is needed, for example, in the discussions of[1,13]. For the convenience of readers we give in Appendix II a table which lists the predictions of the quark model and the hypotheses which are necessary for them.

I. THE NONRELATIVISTIC QUARK MODEL AND THE CLASSIFICATION OF HADRONS

The basis of the model is the hypothesis that all hadrons consist of a few primeval particles, which have a fractional charge Q and a baryon number B equal to 1/3.[11,12] Following Gell-Mann, we shall call these particles quarks.[11] The quarks Q_p, Q_n form an isotopic doublet with strangeness S = 0, and the quark Q_λ is an isosinglet with S = −1. Assuming that the Gell-Mann-Nishijima formula

$$Q=T_3+\frac{S+B}{2},$$

holds for quarks, we can find the charges of the quarks. (The sum S + B is usually called the hypercharge Y.) Table I lists the quantum numbers of the quarks and antiquarks.

An additional hypothesis needed for the validity of most of the applications is that the strong interaction between quarks, which is responsible for the occurrence of bound states (i.e., mesons and baryons), preserves the individuality of the quarks, and only changes their parameters, and that inside a hadron the quarks interact weakly with each other and are nonrelativistic there. In the framework of quantum mechanics the possibility of such a picture can be explained with the example of a heavy quark which is at the bottom of a broad potential well.[15-17] In this case the ratio of the momentum of the quark in the well to its mass is a small quantity. (The condition for the quark to be nonrelativistic in the potential well is $MR \gg 1$, where M is the mass of the quark and R is the radius of the well.) In what follows, unless otherwise specified, the term "quark" is used only to refer to a quark which is inside a hadron.

If furthermore the forces between quarks are functions of the distance only, when other interactions are neglected (for example, spin-spin and spin-orbit interactions) we can put the wave function of a hadron in the form of the product of a coordinate part and a unitary-spin part (the analog of the charge-spin wave function in the Wigner model), which allows us to construct a classification of hadrons of the type of L-S coupling.[45] This sort of simple picture facilitates the intuitive interpretation of the broken symmetries SU(3), SU(6), and so on. In fact, if we assume that the forces between quarks do not depend on the charge and the strangeness, there is symmetry with respect to permutations of the quarks—SU(3) symmetry. If, on the other hand, the interaction potential does not depend on the spin, SU(6) symmetry holds. The dimension of the symmetry group is obviously determined by the dimension of the fundamental multiplet from which the particles are constructed and all of whose components have identical properties: in SU(3), the triplet Q_p, Q_n, Q_λ, and in SU(6), the sextet $Q_{p\uparrow}$, $Q_{p\downarrow}$, $Q_{n\uparrow}$, $Q_{n\downarrow}$, $Q_{\lambda\uparrow}$, $Q_{\lambda\downarrow}$ (the arrows denote the spin projection). If, however, the quark Q_λ interacts differently from the quarks Q_p and Q_n, then SU(3) is broken. The spin-spin and spin-orbit interactions in turn lead to breaking of SU(6). We can try to take the breaking of SU(3) and the spin-spin and spin-orbit interactions into account with perturbation theory. The smallness of the parameter in the perturbation-theory expansion assures the consistency of the scheme and (in the framework of the quark model) explains how symmetries of the type of SU(6) arise.

Let us proceed to the classification of hadrons. We begin with the mesons. It is natural to assume, in the spirit of Fermi and Yang, that the mesons are bound states of a quark and an antiquark. This at once fixes the parity $P=(-1)^{L+1}$ and the G parity $G=(-1)^{L+S+T}$ of a meson.[46] We first study the lowest state with the orbital angular momentum L = 0. It is obvious that, when the spin-spin interaction is neglected, the spin singlet and triplet are degenerate as to their masses; consequently, the quark model predicts the existence of nine pseudoscalar and nine vector mesons, in agreement with experiment.[3] That the mesons are grouped in nonets follows simply from the fact that the system "quark + antiquark" has precisely nine charge states. The concrete form of the unitary-spin part of the wave function is given in Appendix I. It is obvious that for bosons with L ≠ 0 the unitary-spin wave function is the same as for bosons with L = 0. Since L and S add vectorially, there is a degeneracy with respect to J. Table II lists the mesons with L = 0 and L = 1.

An important feature of this scheme is that in each nonet there are two particles having identical values T = S = 0. This is important in dealing with the mass splittings. Only the nonets 0^-, 1^-, 2^+, and, possibly, 1^+ are filled[3]; in other multiplets only some of the particles are at present observed, and the masses of the others have been estimated by means of the hypothesis of the increasing of the Q_λ-quark's mass (the methods in question are explained in Chap. II). The absence of these resonances is perhaps explained by the fact that resonances with large values of L are more weakly produced, and that furthermore for given L the states with large values of J have larger statistical weights.[45] A further indication in favor of the classification of mesons developed above after the type of L-S coupling is that the mixing parameters (see Chap. II) are nearly equal for the nonets 1^- and 2^+. A possible objection to this particular way of filling in the table is that the values of J^P are unreliable for all the mesons except the nonets 0^-, 1^-, and, probably, 2^+. The table we have written out includes only some of the observed resonances (but all of them reliable!); we shall not discuss the others in this review, since their quantum numbers J^P are not known. In principle resonances are possible in a system of two quarks and two antiquarks.[47] So far, however, not a single meson has been discovered with $|Q| > 1$, $|T| > 1$, $|S| > 1$, although there are some, not too reliable, indications of the existence of resonances with T = 3/2 and S = 2.[3]

The quark model also allows us to construct a systematics of the baryon resonances. It is simplest to construct baryons with three quarks. Let us first study the lowest state with L = 0. In this case there exist three types of unitary spin wave-function factors, which have definite symmetries with respect to permutations of the quarks: $\Psi_{[\alpha\beta\gamma]}$ (20)—completely antisymmetric, $\Psi_{\alpha\beta,\gamma}$ (70), of mixed symmetry corresponding to the Young scheme ⊟⊓,[48], and $\Psi_{\{\alpha\beta\gamma\}}$ (56)—completely symmetric. It can be shown that $\Psi_{\{\alpha\beta\gamma\}}$ includes 8 particles with spin 1/2 and 10 with spin 3/2. The quark structure of this function is given in Appendix I. If the quarks are fermions, then according to the Pauli principle the coordinate part of the wave function corresponding to $\Psi_{\{\alpha\beta\gamma\}}$ (56) must be antisymmetric with respect to permutations of the quarks, which is unusual for the wave function of a lowest state. The simplest way to avoid the difficulties associated with the choice of $\Psi_{\{\alpha\beta\gamma\}}$ (56) for the description of the ground state of the baryon is to ascribe to the quark a new quantum number, with respect to which the wave function will be antisymmetric.[15,16] This hypothesis is equivalent to increasing the number of quarks to nine, and consequently predicts particles which have not been observed experimentally. It has been suggested that the Fermi statistics be abandoned for the quarks.[49] (We note that all of the results given in this review are independent of the concrete form of the coordinate part of the wave function, even of its symmetry.) One can try to solve these problems by the introduction of many-particle forces.[50]

Still another attempt has been discussed in the literature; its authors reject the "nuclear" model of the baryon in favor of an atomic model: the quarks move in an attractive central potential around a nucleus, which is an SU(3) singlet, and the two-particle forces between the quarks are repulsive.[51] Then the special position of $\Psi_{\{\alpha\beta\gamma\}}$ has a simple analog in the form of Hund's rule, which explains the filling of the electron shells of atoms.[48] If we go further, we can replace the nucleus with a system of a quark and an antiquark, and we arrive at a model in which the baryons are bound states in a system of four quarks and one antiquark, after the fashion of the methane molecule CH_4.[18] This last scheme lets us escape both from the difficulty with the special position of $\Psi_{\{\alpha\beta\gamma\}}$ (56) and from the introduction of a new particle, the nucleus, which is necessary in the atomic model. Among disadvantages of the last model we must mention that it predicts not yet observed baryons with values of Q and Y larger than unity, although there are some indications of the existence of a baryon with Q = Y = 2 in the elastic scattering of K^+ by protons.[3] Hereafter in this review we shall postulate that the wave function of the hadron is $\Psi_{[\alpha\beta\gamma]}$ (56), since the most interesting applications of the quark model are associated with this wave function.

There are too many possibilities for the higher resonances in the quark model: $\Psi_{[\alpha\beta\gamma]}$ (20), $\Psi_{\alpha\beta,\gamma}$ (70), or a classification of the type of Russell-Saunders coupling.[45]

The possibility that the spectra of baryon resonances are of the pure rotational type has been discussed in[52]. The author was even able to predict the parity of the first excited states with L = 1, since the state of lowest energy will be that corresponding to rotation around an axis perpendicular to the plane in which the three particles lie. The unitary-spin wave function of such an excited state will also be $\Psi_{\{\alpha\beta\gamma\}}$ (56), and the systematics of the baryons with L = 1 is the same as that obtained on the basis of the idea of L-S coupling.[45] Unfortunately, an actual choice between the various schemes for classifying the baryon resonances is at present impossible, owing to the inadequacy of the experimental data, and therefore we shall confine ourselves in this review to the study of the multiplet with L = 0.

Accordingly, in the quark model (as in the Sakata model[8,9]) the mesons must be grouped into nonets, and in the case of nonrelativistic motion of the quarks it is natural to expect the appearance of nonets of mesons with high spins. For the baryons the model predicts the existence of octets and decuplets. We note that the existence of singlets is also not forbidden, since $\Psi_{[\alpha\beta\gamma]}$ (20) and $\Psi_{\alpha\beta,\gamma}$ (70) include the following representations of SU(3): 1(3/2) + 8(1/2) and 1(1/2) + 8(1/2) + 8(3/2) + 10(1/2) (the spin of the particles is shown in parentheses). In the explanation of the spectrum of baryons the nonrelativistic quark model encounters definite difficulties, since the coordinate part of the wave function of the lowest state must be antisymmetric with respect to permutations of the quarks. There is at present no convincing explanation of this in the framework of the nonrelativistic quark model.

It is interesting to compare the classification of hadrons which arises in the quark model with the assignments of the various symmetries. As compared with SU(3) the simplest type of quark model predicts (in agreement with experiment) the minimal number of hadrons: nonets for mesons, octets and decuplets for baryons. In the symmetries SU(6), $SU(6)_W$, and U(6, 6) it follows automatically from the existence of particles with large spin that there must exist particles with large isotopic spin, since in these symmetries the spin and the isotopic spin come in on the same footing. This is the main contradiction between SU(6), $SU(6)_W$, and U(6, 6) and experiment. [By the way, it is shown in[53], in the framework of $SU(6)_W$, that these resonances are hard to observe, since the usual decay channels are suppressed.] In the quark model the spin is different from the isotopic spin owing to the presence of an orbital angular momentum, and consequently there is no necessity for multiply charged resonances to exist.

II. MASS FORMULAS IN THE QUARK MODEL

In the preceding chapter a systematics of hadrons has been constructed on the assumption that the forces between quarks depend only on the distance and that the quarks Q_p, Q_n, Q_λ have identical properties. Actually there are also spin-spin and spin-orbit interactions between quarks; moreover, it can be seen from the analysis of experiments that the mass of the quark Q_λ must be different from that of Q_p and Q_n, and that Q_λ must interact differently. All of these interactions can be included only with perturbation theory.

Let us proceed to the study of the mass differences of particles which have L = 0 (the other quantum numbers are different for the different particles). We at first neglect the electromagnetic interaction, which gives mass corrections of the order of 10 MeV, whereas the actual mass differences are of the order of hundreds of MeV. We begin with the baryons, with quantum numbers $J^P = 1/2^+, 3/2^+$. It is easy to write the Hamiltonian responsible for the appearance of the mass differences[18]:

$$H_1 = a_B + c_B \sum |S_i| + d_B \sum_{n>m} (\mu_n \mu_m), \tag{2.1}$$

where $\mu_i = (1 - \alpha_B S_i)\sigma_i$, S_i is the strangeness operator of the quark, and σ_i is the Pauli spin matrix.

This form of the Hamiltonian corresponds to the assumption that the quark Q_λ is heavier than Q_p and Q_n by the amount c_B,[12] and that by definition it interacts more weakly. a_B and $c_B \sum S_i$ describe the contribution to the mass which does not depend on the spin, and $d_B(\mu_1\mu_2)$ describes the spin-spin interaction, with allowance for the weaker interaction of the quark Q_λ. Using the wave functions given in Appendix I, one can derive the mass formulas.[54] We shall proceed differently; from known particle masses we shall calculate the parameters of the Hamiltonian, and then the masses of the other particles. There is the best agreement with experiment for a_B = 1083 MeV, c_B = 180 MeV, d_B = 270 MeV, α_B = 0.42.[18] Table III shows the comparison of the predictions of this model with experiment.[18]

By means of four parameters we have correlated the masses of eight particles. There is good enough agreement with experiment. We can estimate the parameter of the perturbation-theory expansion by taking it equal to the ratio of the errors of the mass formula to the mass differences in the multiplet. It turns out of the order of 1/10, which assures the consistency of the scheme.

It is obvious that the same methods as for the baryons can be used for the mass differences of the mesons, with the important exception that in the derivation of the mass formulas for mesons we must also allow for the annihilation interaction, i.e., for the possibility of virtual transitions of the type $p\bar{p} \to \lambda\bar{\lambda}$, $n\bar{n}$, etc.[18] Let us write down the Hamiltonian of this sort of interaction:

Table III. Comparison of Baryon Masses in the Quark Model with Experiment (according to[18])

Particle	m_{theor}, MeV	m_{exp}, MeV	Particle	m_{theor}, MeV	m_{exp}, MeV
N	928	939	Δ	1238	1238
Σ	1195	1193	Σ_δ	1375	1385
Λ	1108	1115	Ξ_δ	1520	1530
Ξ	1340	1317	Ω_δ	1675	1675

$$H_2 = \sum f_s \begin{vmatrix} 1 & 1 & 1-\beta_s \\ 1 & 1 & 1-\beta_s \\ 1-\beta_s & 1-\beta_s & (1-\beta_s)^2 \end{vmatrix}. \tag{2.2}$$

The summation is taken over the spin operators. The fact that the interaction of the quark Q_λ is weaker is taken into account in H_1 and H_2. It is easy to see how H_2 acts on a state with T = S = 0:

$$H_2 |\pi_0\rangle = H_2 \left| \frac{p\bar{p} - n\bar{n}}{\sqrt{2}} \right\rangle = 0.$$

It has already been pointed out in Chap. I that the nonet contains two isotopic singlets with T = Y = 0. The presence of two completely degenerate states requires that a change of structure of the wave function be taken into account,[48] and this explains why the mass formulas that follow from SU(3) are only poorly satisfied for mesons (cf., e.g., the review[13]).

The actual particles will no longer belong to a definite representation of SU(3). It is convenient to describe this fact by introducing the concept of a mixing parameter[39]:

$$R_1 = -\omega_1 \sin\theta + \omega_8 \cos\theta, \qquad R_2 = \omega_1 \cos\theta + \omega_8 \sin\theta.$$

Here the R_j are the wave functions of the actual particles, and ω_1, ω_8 are the initial particles belonging to definite SU(3) multiplets (ω_1 to the singlet and ω_8 to the octet). For the nonet 1^- the mixing in the quark model is of an intuitive form: one of the particles, $R_1 = \varphi$, consists only of strange quarks, and the other, $R_2 = \omega$, only of nonstrange quarks, $\omega = (Q_p\bar{Q}_p + Q_n\bar{Q}_n)/2^{1/2}$.[17] In the language of the annihilation matrix H_2 this means that the matrix elements of H_2 that correspond to transitions $Q_\lambda\bar{Q}_\lambda \to Q_p\bar{Q}_p$, $Q_n\bar{Q}_n$ in the triplet state are strongly suppressed. It follows from this that φ must decay mainly into $K\bar{K}$, and ω into 3π, which is in good agreement with experiment.[17] It is easy to see that this sort of structure of φ, ω corresponds to

$$\mathrm{tg}\,\theta_V = \frac{1}{\sqrt{2}}.$$

For the nonet 0^- there is no such intuitive interpretation of the mixing effect. The quantity $\tan\theta_P$ can be calculated from the mass formulas, which in the nonrelativistic quark model are most naturally written in terms of the masses, not their squares, for mesons as well as for baryons. For the nonet 0^- we have $\mathrm{tg}\,\theta_P = -1/4$.[18]

The complete Hamiltonian responsible for the appearance of mass differences among mesons is of the form

$$H = a_M + c_M \sum |S_i| + d_M (\mu_1\mu_2) + H_2. \tag{2.3}$$

The best agreement with experiment is achieved for[27]

$$a_M = 598 \text{ MeV}, \quad c_M = 180 \text{ MeV}, \quad d_M = 620 \text{ MeV},$$
$$f_0 = 580 \text{ MeV}, \quad \alpha = 0.42, \quad \beta = 0.75.$$

Comparing these values with the analogous ones determined from the known baryon masses, we get $c_M = c_B \equiv c$, $\alpha_M = \alpha_B \equiv \alpha$, which is evidence that the increase of mass of the quark Q_λ as compared with the quarks Q_p, Q_n is the same in mesons and in baryons,[17] and that the breaking of SU(3) in the spin-spin interaction is of universal character.[18] Equality of the other

Table IV. Masses of Mesons in the Quark Model (according to [18])

Particle	m_{theor}, MeV	m_{exp}, MeV
π	133	137
K	490	494
ρ	753	780
K^*	868	890
φ	1010	1020
ω	753	780

parameters in the Hamiltonians (2.1) and (2.3) is hard to expect, owing to the different structures of mesons and baryons. The universality of the breaking of SU(3) ($c_M = c_B$, $\alpha_M = \alpha_B$) is characteristic for the quark model only, and lies outside the framework of the symmetries. This is the first example of a general phenomenon: the interaction between quarks is so constructed that it renormalizes the parameters of the quarks in the same way in mesons and in baryons. In this sense it is curious that $a_B/a_M = 1.8$ is not very different from the value 3/2,[33] which would be expected if the quark mass is renormalized in the same way in mesons as in baryons.

The theoretical predictions are compared with experiment in Table IV. It is curious that the main part of the mass difference within a single SU(3) multiplet is due to the increased mass of the quark Q_λ.

If we assume that the quark structure of φ is known from independent arguments (for example, from decays), that the increase of mass of the quark Q_λ is the same in mesons and baryons, and that the weakening of the interaction of the quark Q_λ in the spin-orbit interaction is universal (i.e., that α is the same for mesons and baryons), then by means of six parameters—a_B, a_M, c, d_B, d_M, α—we have correlated the masses of 14 particles—(π, K), (ρ, K*, φ, ω), (N, Σ, Λ, Ξ), (Δ, Σ_δ, Ξ_δ, Ω_δ^-), and the agreement between the relations so obtained and experiment is not bad.

In an analogous way, in the quark model we can derive relations between the electromagnetic mass differences of hadrons, taking into account the Coulomb interaction of quarks, the increased mass of the quark Q_n as compared with Q_p, and the interaction between the magnetic moments of quarks.[55] All of the predictions, and in particular the relations connecting the electromagnetic mass differences of particles belonging to the octet and the decuplet, which are peculiar to the quark model, are not in contradiction with experiment, but owing to the large experimental errors no more definite conclusions can be drawn. We note that for the octet $1/2^+$ the relations obtained in the quark model are the same as in SU(3) (see the review[65]). For purposes of completeness we give Table V, in which the theoretical predictions are compared with experiment.[55] The experimental data are taken from[3].

So far we have studied the relations between the masses of particles having L = 0. If it is reasonable to classify mesons after the type of L-S coupling [and a fact favoring this is that the change of boson mass with change of L is of the order of the mass difference of particles in the same SU(3) multiplet; see Table II], then we can correlate the masses of bosons that have different J with the same L and S.[45] The simplest way to remove the degeneracy in J is to include a spin-orbit interaction of the form α_L (L·S). Assuming that α_L depends only on L, we can calculate the masses of the nonet 0^+ in terms of the known masses of mesons with $J^P = 1^+, 2^+$ (see Table II):

$$m(0^+) = \frac{3m(1^+) - m(2^+)}{2}.$$

Substituting $m_{A_2}(2^+) = 1.306 \pm 0.008$ GeV, $m_{A_1}(1^+) = 1.079 \pm 0.008$ GeV, we calculate the mass of a particle with $T = 1$, $J^P = 0^+$: $m(0^+) = 0.965 \pm 0.016$ GeV. This agrees with the masses of the known particles π_V (1003) and δ (965).[3] Unfortunately, the J^P values are not known for these mesons.

As we have seen in this chapter, in the quark model the problem of accounting for the mass difference of hadrons is solved rather simply, in exact analogy with atomic and nuclear physics and in agreement with experiment. At the same time, in the symmetries higher than SU(3)—SU(6), $SU(6)_W$, and so on—there is actually no uniform method for deriving mass formulas, since such theories contain too many parameters.

The success of the application of perturbation theory to the calculation of mass differences in the quark model, in spite of the fact that these differences are large and the perturbation-theory expansion parameter is small, perhaps indicates that actually the mass differences are small, in some sense of the word, for example in comparison with the large mass of the free quark.

III. ELECTROMAGNETIC PROPERTIES OF HADRONS

In the study of the electromagnetic properties of hadrons it is assumed that the Hamiltonian for the interaction of the electromagnetic field with the particle can be written as the sum of the Hamiltonians for the interaction of the electromagnetic field with the quarks of which the hadron is composed, i.e.,

Table V. Electromagnetic Mass Differences of Baryons in the Quark Model

Electromagnetic mass difference	m_{theor}, MeV	m_{exp}, MeV	Electromagnetic mass difference	m_{theor}, MeV	m_{exp}, MeV
$n - p$	1,3 *)	1,3	$\Delta^- - \Delta^{++}$	3,9	$7,9 \pm 6,8$
$\Sigma^- - \Sigma^0$	4,8 *)	4,8	$\Delta^0 - \Delta^{++}$	0,89	$0,45 \pm 0,85$
$\Sigma^- - \Sigma^+$	7,9 *)	7,9	$\Sigma_\delta^0 - \Sigma_\delta^+$	1,3	
Σ^0/Λ	1		$\Sigma_\delta^- - \Sigma_\delta^+$	4,31	$5,8 \pm 3,9$
$\Xi^- - \Xi^0$	6,6	6,5	$\Xi_\delta^- - \Xi_\delta^0$	3,01	$4,9 \pm 3,0$
$\Delta^- - \Delta^{++}$	−0,41				

*The mass differences $n - p$, $\Sigma^- - \Sigma^0$, $\Sigma^- - \Sigma^+$ are taken from experiment, and the others are predicted. The experimental data are taken from [3].

$$H^{em} = \sum_j H_j^{em}. \tag{3.1}$$

Here the summation is over the quarks that belong to the particle.

We shall concern ourselves first with the electromagnetic properties of the octet $1/2^+$ and the decuplet $3/2^+$. The allowed transitions for the vertex $p \to p + \gamma$ are E0, the charge form-factor of the nucleon, and M1, the magnetic form-factor; for the radiative decay $\Delta(3/2^+) \to p + \gamma$ only the M1 transition is allowed (E2 is impossible for two reasons: first, because the orbital angular momenta of Δ and p are zero, and second, because the spin functions of Δ and p are orthogonal[56]). From the meson photoproduction reaction $\gamma + p \to \Delta^+ \to \pi^0 + p$ one gets for the ratio A(E2)/A(M1) the value 0.02 ± 0.01 or less,[57] which does not contradict the model.

With the approximation that SU(3) symmetry holds, and using the wave functions given in Appendix I, we get for the electric (Sachs) form-factor of nucleons the result[6]

$$G_E(t) = QF(t) \tag{3.2}$$

for all momentum transfers allowed in the model (precisely the same prediction is obviously obtained for the charge form-factors of the meson). This result is evident without detailed calculation, since the operator for the E0 transition is proportional to the charges of the quarks and does not contain the spin operators. Therefore the quark model predicts that the charge form-factors of neutral particles are equal to zero. From experiments on the scattering of electrons by neutrons it is known that the electric form-factor of the neutron is approximately equal to zero,[58] which agrees with the quark model.

Let us now consider the M1 transition. For it

$$H(M1) = \sum_i \left(\frac{e_i}{rm_{\text{qu}}} \mathbf{L}_i + \mu_i\right) [\mathbf{k}\varepsilon], \tag{3.3*}$$

where $\mathbf{k}$ and ϵ are the momentum and polarization vector of the γ-ray quantum, and L_i and μ_i are the orbital angular momentum and magnetic moment of the i-th quark. The first term in (3.3) does not contribute to the M1 transition because the total orbital angular momentum of the hadron is zero. The contribution of the second term is easily calculated. It is given by

$$\langle a \,|\, H(M1) \,|\, b\rangle = \int \psi_0^+ (r_1, r_2, r_3)\, e^{ikr_1} \psi_b (r_1, r_2, r_3)\, d\tau \left\langle \chi_a(1, 2, 3) \middle| \sum \mu_i \middle| \psi_b (1, 2, 3) \right\rangle [\mathbf{k}, \varepsilon], \tag{3.4}$$

where a and b are the initial and final particles and χ is the unitary-spin part of the wave function. If for the nonstrange quarks we assume $\mu_i = \mu e_i \sigma$ where e_i is the charge of the quark [we note that when SU(3) holds this follows at once from the fact that the electromagnetic current in the quark model transforms according to the octet representation of SU(3)], then after some simple calculations (cf., e.g.,[59]) we get the ratio of the magnetic moments of the proton and neutron

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2}, \tag{3.5}$$

*$[\mathbf{k}\epsilon] \equiv \mathbf{k} \times \epsilon$.

Table VI. Magnetic Moments of Particles in the Quark Model (according to[3])

Particle	Magnetic Moment: theory	Magnetic Moment: experiment
p	2.79	2.79
n	−1.86	−1.91
Λ	−0.93	-0.73 ± 0.16
Σ^+	2.79	2.3 ± 0.6
Σ^0	0.93	
Σ^-	−0.93	
Ξ^0	−1.86	
Ξ^-	−0.93	
Ω^-	−2.79	

and for the magnetic transition in the decay $\Delta \to p + \gamma$ we have

$$\mu_{\Delta p} = \frac{2\sqrt{2}}{3} \mu_p. \tag{3.6}$$

The first relation agrees well with the experimental values $\mu_p = 2.79$, $\mu_n = -1.91$, or $\mu_p/\mu_n = -1.46$; experimentally[57] the second ratio differs by a factor 1.28 ± 0.02 from the prediction of the quark model.

The values of the magnetic moments of the baryons when there is SU(3) symmetry are shown in Table VI. As is well known, these relations were first derived[6] in SU(6) symmetry with the additional assumption that the electromagnetic current transforms according to the regular representation of that group.

In the quark model relations of the type (3.5), (3.6) are found also for momentum transfers which are not zero but are sufficiently small that the motion of the quarks can be regarded as nonrelativistic as before and the structure of the hadron is not changed. These predictions are in agreement with experiment. For example, the relation

$$\frac{G_M^p(k^2)}{G_M^n(k^2)} = -\frac{3}{2} \tag{3.7}$$

holds well over a wide range of k^2 (Table VII). The correspondence between the form-factor for the magnetic transition $\Delta^+ \to p + \gamma$ and the magnetic (Sachs) form-factor of the proton, which according to the model are connected by the relation

$$G_{\Delta p} = \frac{2\sqrt{2}}{3} G_M^p(k^2), \tag{3.8}$$

can be tested by studying the process of electroproduction of Δ_{33} (1238)[19] (Fig. 1). Taking $G_{\Delta p}(k^2)$ from the

Table VII. Comparison with Experiment of the Predictions of the Quark Model for the Electromagnetic Form-factors ($G_E^p = G_M^p/\mu_p = G_M^n/\mu_n$) (according to[21])

q^2, (GeV/c)^2	G_E^p	G_M^p/μ_p	G_M^n/μ_n
0.389	0.424 ± 0.017	0.409 ± 0.007	0.445 ± 0.021
0.623	0.281 ± 0.024	0.286 ± 0.006	0.310 ± 0.023
0.857	0.183 ± 0.026	0.228 ± 0.005	0.218 ± 0.028
1.17	$0.17^{+0.02}_{-0.03}$	0.155 ± 0.04	0.177 ± 0.014
1.75	$0.114^{+0.022}_{-0.029}$	0.0895 ± 0.005	0.120 ± 0.021
2.92	$0.00 + 0.004$	0.049 ± 0.006	$\leqslant 0.052$
3.89	$0.00 + 0.02$	0.0325 ± 0.0033	$\leqslant 0.036$

Table VIII. Comparison with Experiment of the Predictions of the Quark Model for the Form-factor of the Magnetic Transition $\Delta \rightarrow p$ $[G_{\Delta p}(k^2)]/(2^{3/2}/3\,\mu_p) = G_M^p(k^2)/\mu_p$ (according to[20])

k^2, F^{-2}	2	5	8	12
$\frac{G_{\Delta p}(k^2)}{2\sqrt{2}/3\mu_p}$	1.04±0.15	0.76±0.10	0.60±0.05	0.43±0.03
$\frac{G_M^p(k^2)}{\mu_p}$	0.81	0.61	0.48	0.36
k^2, F^{-2}	16	30	45	100
$\frac{G_{\Delta p}(k^2)}{2\sqrt{2}/3\mu_p}$	0.29±0.03	0.21±0.03	0.10±0.02	0.03±0.01
$\frac{G_M^p(k^2)}{\mu_p}$	0.28	0.16	0.09	0.03

experimental data and comparing it with $G_M^p(k^2)$, we see that the quark model has made a fairly good prediction of the quantity $G_{\Delta p}(k^2)$ and its behavior as the momentum transfer ranges from 2 F^{-2} to 100 F^{-2} (Table VIII). As is well known, (3.7) and (3.8) were first derived by the use of $SU(6)_W$ symmetry and the additional assumption that the electromagnetic current transforms according to the regular representation of that group [(3.7) in[20], and (3.8) in[19]].

If it is further assumed that the form-factors of the quarks themselves depend only weakly on the momentum transfer (i.e., that the radius of the quark is much smaller than the radius of the hadron), then in the quark model[15] the quantities $G_E^p(k^2)$ and $G_M^p(k^2)$ will be equal [cf. (3.4)], where $G_E^p(k^2)$ and $G_M^p(k^2)$ are the Sachs electric and magnetic form-factors of the proton.[70] This result is in good agreement with experiment,[21] much better agreement than should be expected from such a rough model (see Table VII).

An important feature of the quark model is that in it the breaking of SU(3) associated with the difference between the masses of strange and nonstrange quarks and with the weaker interaction of the strange quark (cf. Chap. II), which leads to a difference between the magnetic moments of the quarks Q_n and Q_λ, is uniquely taken into account. Relations between the electromagnetic parameters of hadrons which take into account the breaking of SU(3) are given in[60,61].

Let us now pass on to the radiative decays of vector mesons ($V \rightarrow P + \gamma$).[34] For these only the M1 transition is allowed. In the study of radiative decays of vector mesons in the quark model we encounter a new situation: owing to the large mass difference between vector and pseudoscalar mesons the pseudoscalar meson is relativistic, and therefore it is necessary to separate out kinematic factors. (We recall that we are regarding the quarks inside a hadron as nonrelativistic, and here we are separating out the motion of the pseudoscalar meson as a whole.) The quark model gives the correct quantity μ_{if} in the relativistic expression for the matrix element of the transition

$$A_{if} = \frac{\sqrt{4\pi}}{\sqrt{2q_0\,2p_0\,2k_0}}(2\mu_{if})\,\varepsilon_{\alpha\beta\gamma\delta}e_\alpha^V e_\beta q_\gamma k_\delta. \tag{3.9}$$

Here q, p, and k are the momenta of the vector meson, the pseudoscalar meson, and the photon, and $\mathbf{e}^V$ and $\mathbf{e}$ are the polarization vectors of the vector meson and the photon. The coefficients in (3.3) are chosen so that in the nonrelativistic approximation μ_{if} would have the meaning of the magnetic moment of the transition. For the decay probability we get the formula

$$\Gamma_{i\rightarrow f} = \frac{4}{3}\mu_{if}^2 k^3;$$

here k is the momentum of the particles which are formed. Taking the experimental value for $\Gamma_{\omega\rightarrow\pi^0+\gamma}$, we calculate the probabilities of the other radiative decays (see Table VI). In the calculation it was assumed that φ consists only of strange quarks, and ω of nonstrange quarks (in accordance with the results of Chap. II), and η is a pure octet state.

It can be seen from Table VI that all of the decays are strongly suppressed in relation to $\omega \rightarrow \pi^0 + \gamma$ (by an order of magnitude); i.e., for the decays of ρ, K^+, and φ the quark model actually gives the same results as the hypothesis of the conservation of A-parity.[62] In this sense it is curious that $\varphi \rightarrow \pi^0 + \gamma$ is forbidden and that $\varphi \rightarrow \eta + \gamma$ has a comparatively large value.

All of the results obtained in this chapter are based essentially only on a knowledge of the unitary-spin part of the hadron wave function and on the additivity of the electromagnetic interactions of the quarks making up a particle. If in addition we assume that the magnetic moments of the quarks are renormalized in the same way inside mesons and baryons, we can connect the decay width $\Gamma_{\omega\rightarrow\pi^0+\gamma}$ with the magnetic moment of the proton and get good agreement with experiment.[3] It is not hard to calculate $\mu_{\omega\rightarrow\pi^0+\gamma}$, using the unitary-spin wave functions of Appendix I

$$\omega^0 = \frac{1}{2}(p_\uparrow\bar{p}_\downarrow + n_\uparrow\bar{n}_\downarrow + p_\downarrow\bar{p}_\uparrow + n_\downarrow\bar{n}_\uparrow),\quad \pi^0 = \frac{1}{2}(p_\uparrow\bar{p}_\downarrow - n_\uparrow\bar{n}_\downarrow - p_\downarrow\bar{p}_\uparrow + n_\downarrow\bar{n}_\uparrow),$$

$$\mu_{\omega\rightarrow\pi^0} = \mu_p\left\langle \omega^0 \middle| \sum e_i\sigma_i \middle| \pi^0 \right\rangle =$$
$$= \frac{\mu_p}{4}\left\langle p_\uparrow\bar{p}_\downarrow + n_\uparrow\bar{n}_\downarrow + p_\downarrow\bar{p}_\uparrow + n_\downarrow\bar{n}_\uparrow \middle| \frac{4}{3}(p_\uparrow\bar{p}_\downarrow + p_\downarrow\bar{p}_\uparrow) + \right.$$
$$\left. + \frac{2}{3}(n_\uparrow\bar{n}_\downarrow + n_\downarrow\bar{n}_\uparrow)\right\rangle = \frac{\mu_p}{4}\left\{\frac{8}{3} + \frac{4}{3}\right\} = \mu_p,$$

that is,

$$\mu_{\omega\rightarrow\pi^0} = \mu_p. \tag{3.10}$$

Since μ_p = 2.79 e/2m$_p$, we can calculate $\Gamma_{\omega\rightarrow\pi^0+\gamma}$. It is found that $\Gamma_{\omega\rightarrow\pi^0+\gamma}$ is about 1.2 MeV, which agrees well with the experimental value[3] $\Gamma_{\omega\rightarrow\pi^0+\gamma}$ = (1.15 ± 0.25) MeV. This last prediction is probably one of the most serious successes of the quark

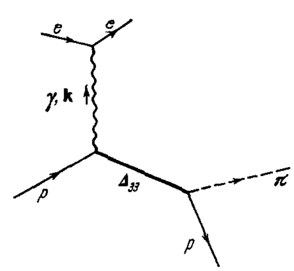

FIG. 1. Process of electroproduction of the isobar Δ_{33}.

Table IX. Predictions of the Quark Model for Radiative Decays of Vector Mesons

Reaction	Γ, MeV	$\frac{\Gamma}{\Gamma_{tot}}$ %	Reaction	Γ, MeV	$\frac{\Gamma}{\Gamma_{tot}}$, %
$\omega \to \pi^0\gamma$	1.15±0.25*)	10	$K^{*\pm} \to K^{\pm}\gamma$	0.067	0.145
$\omega \to \eta\gamma$	$0.61 \cdot 10^{-2}$	0.05	$K^{*0} \to K^0\gamma$	0.264	0.5
$\rho^{\pm} \to \pi^{\pm,0}\gamma$	0.115	0.07	$\varphi \to \eta\gamma$	0.278	7
$\rho^0 \to \eta\gamma$	0.043	0.031	$\varphi \to \pi^0\gamma$	0	0

*The experimental value is taken from [3].

model.[34] It is very important, since it emphasizes the common nature of mesons and baryons.

Let us list the main features of the application of the quark model to the description of the electromagnetic properties of hadrons.

1. All of the predictions of the model (Table IX), with the exception of $G_E^p(k^2) = G_M^p(k^2)/\mu_p$ and $\mu_{\omega \to \pi^0} = \mu_p$, also follow from the group $SU(6)_W$. We recall that $SU(6)_W$ assumes that the interaction of the particles is invariant with respect to simultaneous transformation of the group SU(3) and rotation of the spin around the direction of the momentum.[5] Therefore the only processes that are invariant with respect to $SU(6)_W$ are those characterized by a single momentum, for example two-particle decays (including also electromagnetic vertices) or amplitudes for scattering of two particles by an angle of zero or 180°.

2. It explains the fact that the (Sachs) electric and magnetic form-factors of the proton are equal, $G_E^p(k^2) = G_M^p(k^2)/\mu_p$.

3. The quark model allows us to get relations between the electromagnetic properties of mesons and baryons ($\mu_{\omega \to \pi_0} = \mu_p$).

The last two points are especially interesting, since at present they cannot be derived from other models.

In conclusion we can assert that the quark model gives a description of the electromagnetic properties of hadrons in good agreement with experiment.

IV. WEAK INTERACTIONS OF HADRONS

In this chapter we shall show that the nonrelativistic quark model gives a natural explanation of some peculiarities of semileptonic weak interactions, i.e., processes of the type of the β decay of the neutron. We shall not deal with purely leptonic and nonleptonic processes in this review, since the study of these processes requires new hypotheses which do not follow directly from the nonrelativistic quark model.

The application of the quark model to weak interactions is based on the fundamental hypothesis that the radius of the weak interaction is much smaller than the average distance between quarks in a hadron. If this is so, to first order in the weak interaction the amplitude for scattering of a lepton by a hadron is equal to the sum of the amplitudes for its scattering by the quarks[22] (Fig. 2). It is further assumed that the Hamiltonian of the weak interaction of a quark with leptons is of the form

$$H = G/\sqrt{2}\{(J_\mu)_\beta^\alpha l_\mu + \text{c.c.}\}, \tag{4.1}$$

where α and β are quark (antiquark) indices, $l_\mu = \bar{u}\gamma_\mu(1 - \gamma_5)u$ is the leptonic current, and $(J_\mu)_\beta^\alpha$ is

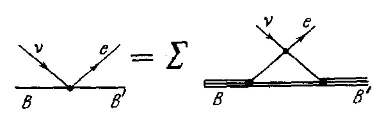

FIG. 2. Inelastic scattering of a neutrino by a baryon in the quark model.

the hadronic current, which in the Cabibbo scheme is given by[23]

$$\{(J_\mu)_\beta^\alpha\} = \begin{pmatrix} 0 & J_\mu^{\pi^-}\cos\theta & J_\mu^{K^-}\sin\theta \\ J_\mu^{\pi^+}\cos\theta & 0 & 0 \\ J_\mu^{K^+}\sin\theta & 0 & 0 \end{pmatrix}, \tag{4.2}$$

where the indices $\pi^\pm$ and $K^\pm$ are convenient for denoting the quantum numbers carried by the current $(J_\mu)_\beta^\alpha$, and θ is the Cabibbo angle. According to Cabibbo[23] the appearance of θ in (4.2) follows from the fact that the weak interaction singles out a definite direction in SU(3) space, in analogy with the singling out of charged particles in electrodynamics, and of strange hadrons in the breaking of SU(3). Then the strangeness-conserving hadronic current comes in with a factor cos θ, and the strangeness-changing current with the factor sin θ. The presence of sin θ in the quark model can be explained by the weakening of the interaction of the quark Q_λ in the strong interaction (cf. Chaps. II and III).[64] There are no visible reasons for the appearance of cos θ, and its existence is not firmly established experimentally, since the contribution of cos θ to the probability of β decay is of the order of the radiation corrections, which we cannot take into account rigorously at present (for more details, cf., e.g.,[65]). In Eq. (4.1) J_μ is the quark current, given by

$$(J_\mu)_\beta^\alpha = \bar{u}_\alpha(p_1)\{f_1\gamma_\mu + f_2\sigma_{\mu\lambda}(p_1 - p_2)_\lambda + g_1\gamma_\mu\gamma_5 + g_2\gamma_5(p_1 - p_2)_\mu\}u_\beta(p_2). \tag{4.3}$$

Here f_1, f_2, g_1, g_2 are unknown functions of $t = (p_1 - p_2)^2$, and for t = 0 we have $f_1(0) = 1$.

The hypothesis of conserved vector current,[63] which is equivalent to the assumption that the electromagnetic and "weak" vector currents are different components of the same isotopic vector, allows us to express f_1 and f_2 in terms of the electromagnetic and magnetic form-factors of the quark, and consequently, in terms of these quantities for the nucleon (cf., e.g.,[9]). Only g_1 and g_2 remain unknown. If we assume that the proper form-factor $g_1(t)$ of the quark depends only weakly on the momentum transfer t [for $g_2(t)$ this last hypothesis is probably less satisfactory, since the π-meson pole contributes to $g_2(t)$], the axial form-factor of the nucleon is equal to the electric form-factor:

$$\frac{F_A(t)}{G_A(t)} = G_E^p(t).$$

This last prediction is not in contradiction with experiment.[24]

It is not hard to see that for SU(3) symmetry the quark current has the same quantum numbers (Q, S, T) as the nonet of mesons consisting of a quark and an antiquark, and transforms under SU(3) like a singlet and an octet. Because there are no neutral components[65] [this fact is already reflected in (4.2)], only charged currents having the quantum numbers of $\pi^\pm$ and $K^\pm$ mesons take part in the weak interactions. Obviously, owing to the additivity of the quark currents, the total hadronic current, equal to the sum of the quark currents, has these same properties, i.e., the total

Table X. Semileptonic Decays in the Quark Model (according to [67])

Decay	Theory	Experiment
$n \to p$	1	1
$\Lambda \to p$	$0.95 \cdot 10^{-3}$	$(1 \pm 0.1) \cdot 10^{-3}$
$\Sigma^- \to n$	$1.2 \cdot 10^{-3}$	$(1.39 \pm 0.2) \cdot 10^{-3}$
$\Xi^- \to \Lambda$	$0.67 \cdot 10^{-3}$	$(2.4 \pm 1.4) \cdot 10^{-3}$
$\Sigma^- \to \Lambda$	$0.63 \cdot 10^{-3}$	$(0.75 \pm 0.28) \cdot 10^{-3}$
$\Omega^- \to \Xi^0$	$5 \cdot 10^{-3}$	

hadronic current transforms according to the octet representation of SU(3). This is usually postulated in treatments of the weak interactions outside the quark model (see the review[65]). From the octet character of the hadronic current there follow the selection rules:

1. In strangeness-conserving semileptonic processes (the current $J_{\pi^\pm}$) the selection "rule" $\Delta T = 1$ must hold.
2. In strangeness-changing semileptonic processes (the current $J_{K^\pm}$) the selection "rules" $\Delta T = Y$, $\Delta S = \Delta Q$ must hold.

These selection rules are valid in any model in which hadrons are constructed from three fermions and in which the weak interaction occurs as a decay of one fermion.[9,46] The predictions are in good agreement with experiment.[65]

If we suppose that G is known from the decay of the muon, $\sin\theta$ from leptonic decays, and G_A from β decay, then, neglecting the contribution of the pseudoscalar, we can describe all semileptonic decays of hyperons in a way in agreement with experiment (Table X). As compared with the phenomenological approach, in which the octet character of the SU(3) transformation of the hadronic current is assumed, for the semileptonic decays of the octet $1/2^+$ the quark model gives an additional condition, fixing the ratio F/D for the matrix element of the axial current. For the reaction $\gamma + p \to \mu + \Delta$ the theory predicts all of the parameters, since

$$\begin{aligned} J(\Delta_\uparrow^+ \to n_\uparrow)_V &= -\frac{2\sqrt{2}}{5} J(p_\uparrow \to n_\uparrow)_{\text{mag}}, \\ J(\Delta_\uparrow^+ \to n_\uparrow)_A &= \frac{-2\sqrt{2}}{5} J(p_\uparrow \to n_\uparrow)_A, \end{aligned} \tag{4.4}$$

where $J(\ )_{V,A}$ is the matrix element of the hadronic vector (axial) current. These relations have been compared in detail with experiment in[66]. They are not in contradiction with experiment.

All of these predictions, except $F_A(t)/G_A = C_E^p(t)$, can also be derived by means of the group approach in the framework of $SU(6)_W$,[67] but in $SU(6)_W$ one needs, besides a knowledge of the spin-spin part of the wave function, the additional assumption that the hadronic current transforms according to a representation[35] of that group. Therefore in some sense the quark model can be taken as a reason for believing that SU(6) symmetry is effective in semileptonic processes.

V. SCATTERING OF HIGH-ENERGY PARTICLES (DESCRIPTION OF THE MODEL)

As we have seen in the preceding chapters, the static properties of hadrons can be rather easily understood in the nonrelativistic quark model. The correctness of the relations between form-factors for nonzero momentum transfers is still stronger evidence in favor of the quark model, with quarks interacting weakly inside the hadrons. We may suppose that characteristic consequences of the quark structure of the particles will manifest themselves in the scattering of hadrons.

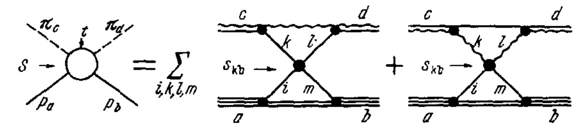

FIG 3. High-energy scattering of a meson by a baryon in the quark model. A straight line represents a quark, a wavy line an antiquark.

In the region of low and moderate energies it is hard to get results without specific assumptions which are beyond the scope of the naive quark model (but see[68]). On the other hand, we may hope that when the momentum of the incident particles is much larger than the momenta of the quarks inside a hadron, the main contribution to the amplitude will come from single scattering of a quark of one particle by a quark (or antiquark) of the other particle (Fig. 3).[25,26] We can explain the possibility of such an approximation with a simple example. Let us suppose that the quarks—heavy particles—are at the bottom of a broad potential well; then they will be nonrelativistic inside the well (owing to the large mass of the quarks and the width of the well). If the size of the quark and the range of the interaction of a quark of one hadron with a quark (or antiquark) of another is much smaller than the average distance between the quarks in a particle (the width of the well), then at small momentum transfers the main contribution is from single scattering of a quark of one particle by a quark (or antiquark) of the other, subject to the condition that the momentum of the colliding particles is much larger than that of a quark inside the well.[25] We recall that we are regarding the quarks as nonrelativistic only inside the particle, and the particles themselves are ultrarelativistic. In the framework of this picture the corrections to the sum of the quark amplitudes fall off with increasing energy, or are of the order of the ratio of the range of the interaction of the quarks to the width of the well. Here the physical meaning of the approximation is obviously different from that in the problem of deuteron-deuteron scattering,[69] since during the interaction the quark is inside the well, and because of its great depth edge effects are unimportant. The quark amplitudes in terms of which the scattering amplitude for high-energy particles is expressed are naturally very different from the amplitudes for scattering of free quarks (if such exist!).

We note that the hypotheses that have been stated suffice for the derivation of several predictions. For example, the selection rules and relations between the elements of the density matrix follow solely from the unitary-spin structure of the quark-quark scattering amplitude. For most applications, however, it is necessary to make the further assumption that at large energies, when the momentum of the incident particle is much larger than the momentum of a quark inside a hadron, we can neglect the dependence of the quark-quark scattering amplitude on the momenta of the quarks inside the particles. Then the scattering amplitude for high-energy hadrons is of the form

$$A_{ab}^{cd}(s,t)=F_{ab}(t)F_{cd}(t)\sum_{\substack{i\in a,\, m\in b,\\ k\in c,\, l\in d}} M_{im}^{kl}(s_{qu},t). \tag{5.1}$$

In this expression all of the notations correspond to Fig. 3; $s = (p_1 + p_2)^2$ is the square of the energy of the colliding particles in the center-of-mass system, $t = (p_1 - p_1')^2$ is the square of the momentum transfer (with sign reversed), and

$$F_{ab}(t)=\int \Psi_a^*(r_1r_2r_3)\,l^{i(\Delta r_1)}\Psi_b(r_1r_2r_3)\,d\tau, \quad \Delta_\mu=(p_a-p_b)_\mu.$$

$F_{ab}(t)$ characterizes the probability that the quark remains inside the hadron after the scattering. Since in the quark model the coordinate part of the wave function for a definite sort of particle (mesons or baryons) is the same if there is SU(3) symmetry, $F_{ab}(t)$ is a universal function: $F_{ab}(t) = F(t)$. If we also assume that the radius of a quark is much smaller than that of a hadron, then F(t) is equal to the electric (Sachs)[70] form-factor of the proton (see Chap. III), i.e., $F(t) = G_E^p(t)$ for small momentum transfers, as long as the structure of the quark remains unimportant.[31] $M_{im}^{k}(s_q, t)$ is the amplitude for scattering of a quark by a quark (or an antiquark). We write it in the two-component form, assuming that the effective potential is sufficiently smooth:

$$M=a+b(\sigma_1\sigma_2)+c(\sigma_1+\sigma_2,\ \mathbf{v})+d(\sigma_1\mathbf{k})(\sigma_2\mathbf{k}) + e(\sigma_1\mathbf{m})(\sigma_2\mathbf{m})+f(\sigma_1-\sigma_2,\ \mathbf{v}), \tag{5.2}$$

where

$$\mathbf{v}=\frac{[\mathbf{nn}']}{|[\mathbf{nn}']|},\quad \mathbf{m}=\frac{\mathbf{n}-\mathbf{n}_1}{|\mathbf{n}-\mathbf{n}'|},\quad \mathbf{k}=\frac{\mathbf{n}+\mathbf{n}'}{|\mathbf{n}+\mathbf{n}'|},$$

$\mathbf{n}$ is the unit vector along the momentum of the incident particle, and $\mathbf{n}'$ is that for the scattered particle; a, b, c, e, f are matrices in charge space. If we assume that the quarks are identical [which corresponds to the symmetry SU(3)], then f = 0, and each term in (5.2) can be written more conveniently in charge space, for example

$$a=a'I+a''(\lambda_1\lambda_2),$$

where I is the unit matrix and λ_j are the eight unitary-spin matrices.[1]

VI. MAIN CONSEQUENCES OF THE QUARK MODEL FOR THE SCATTERING OF HIGH-ENERGY HADRONS

Before proceeding to a detailed comparison of the predictions of the model with experiment, we give our attention to those results that can be derived without complicated calculations.

At high energies and small momentum transfers the model forbids (in good agreement with experiment) reactions in which the changes of the charge, the hypercharge, and the isotopic spin of the initial particles do not satisfy the "rules" $|\Delta Q| \le 1$, $\Delta T \le 1$, $\Delta Y \le 1$.[25] This is due to the fact that the system quark + antiquark does not contain states with $|Q| > 2$, $T > 1$, and $|Y| > 1$ (see Table I). Beginning at 6–8 GeV/c the cross sections for forbidden reactions such as $K^-p \to \pi^-\Sigma_\delta^-$ ($|\Delta Q| = |Q_K - Q_\pi| = 2$) and $K^-p \to \Sigma^-\pi^+$ ($|\Delta Q| = 2$) are an order of magnitude smaller than those for the corresponding allowed reactions [$K^-p \to \pi^-\Sigma_\delta^+$ and $K^-p \to \Sigma^+\pi^-$ ($|\Delta Q| = 0$)]. The differential cross sections have been measured for some reactions which are forbidden in the model. They show a distinct minimum at the angle 0°.[30]

The fact that the predictions of the model are well satisfied experimentally is to some extent a verification of the assumptions that have been made, and moreover enables us to judge the energy value above which it is reasonable to compare the model with experiment. The model clearly does not work if the momentum of the incident hadron is less than 4 to 6 GeV/c. (For example, in the reaction $K^-p \to K^+\Xi^-$ there is a forward peak for $p_K = 3.5$ GeV/c.[30])

The only exception is the annihilation processes, for which at P = 7 GeV/c the forbidden process $p\bar{p} \to \Xi\bar{\Xi}$ ($|\Delta s| = 2$) has an appreciable cross section, and it is isotropic.[96] This clearly means that the asymptotic behavior for annihilation processes begins at higher energies.

2. Reactions of the type

$$\pi+N\to M_\lambda+N\ (N_\delta), \tag{6.1}$$

where M_λ is a pair $(Q_\lambda\bar{Q}_\lambda)$ are forbidden in this model,[77] since in the range of applicability of the hypothesis that the amplitudes are "additive" the only allowed reactions are those in which there is a change of the properties of only one quark in a hadron, and this "rule" does not hold for the case (6.1).

The weakening of the interaction of the quark Q_λ leads to a change of the structure of boson wave functions, which it is convenient to describe by introducing a mixing parameter (see Chap. II). By this means we can express M_λ in terms of the physical particles and state the forbiddenness of (6.1) in a more convenient form:

$$\frac{\sigma(\pi^-p\to\varphi n)}{\sigma(\pi^-p\to\omega n)}=\frac{\sigma(\pi^-p\to\varphi\Delta^{++})}{\sigma(\pi^+p\to\omega\Delta^{++})}=\left(\frac{1-\sqrt{2}\,\mathrm{tg}\,\theta_V}{1+\sqrt{2}\,\mathrm{tg}\,\theta_V}\right)^2, \tag{6.2}$$

$$\frac{\sigma(\pi^-p\to X^0n)}{\sigma(\pi^-p\to\eta n)}=\frac{\sigma(\pi^+p\to X^0\Delta^{++})}{\sigma(\pi^+p\to\eta\Delta^{++})}=\left(\frac{1-\sqrt{2}\,\mathrm{tg}\,\theta_P}{1+\sqrt{2}\,\mathrm{tg}\,\theta_P}\right)^2. \tag{6.3}$$

Here φ and ω belong to the nonet 1^-, and X^0 and η to the nonet 0^-; θ_V and θ_P are the corresponding mixing angles.

The relations (6.2), (6.2) allow us to determine the mixing angle independent of the mass formulas, and even to find its sign! For example, the fact that the cross section for production of the φ meson is small in comparison with that for production of the ω meson,

$$\sigma(\pi^+p\to\varphi\Delta^{++})=10\ \mu\mathrm{b} \qquad \text{at } 3.65\ \mathrm{GeV/c}$$

while

$$\sigma(\pi^+p\to\omega\Delta^{++})=(650\pm100)\ \mu\mathrm{b} \qquad \text{at } 3.6\ \mathrm{GeV/c}$$

and

$$400\ \mu\mathrm{b} \qquad \text{at } p=4.0\ \mathrm{GeV/c}$$

indicates that φ consists mainly of strange quarks (this is in good agreement with the mass formulas and the absence of decay of φ into 3π).

The relation (6.3) is still more interesting; it allows us to test the idea of mixing in relation to the nonet 0^-. If the L-S coupling type of classification is correct for the higher boson resonances, we can try to apply the model to processes in which these mesons are produced. In particular, for the nonets 2^+ and 1^+ we get

$$\frac{\sigma(\pi^- p \to f' n)}{\sigma(\pi^- p \to f^0 n)} = \left(\frac{1 - \sqrt{2}\,\mathrm{tg}\,\theta_T}{1 + \sqrt{2}\,\mathrm{tg}\,\theta_T}\right)^2, \tag{6.4}$$

$$\frac{\sigma(\pi^- p \to En)}{\sigma(\pi^- p \to Dn)} = \left(\frac{1 - \sqrt{2}\,\mathrm{tg}\,\delta}{1 + \sqrt{2}\,\mathrm{tg}\,\delta}\right)^2, \tag{6.5}$$

where θ_T and δ are the mixing angles in the nonets 2^+ and 1^+. The relations (6.4) are of great interest, since they are an independent check of the quark structure of the nonets 2^+ and 1^+. It is stated in[28] that an analysis of the experiments on the scattering of particles made by means of Eqs. (6.2)–(6.4) gives values of θ_P, θ_V, and θ_T that are comparable with the mixing angles expected from the masses of the nonets, with $\theta_P < 0$, $\theta_V > 0$, as expected in the quark model.[18]

3. Another important prediction relates to reactions in which there is a pseudoscalar meson in the initial and final states. For example, $P + N \to P + N$ (N_δ), where P denotes the nonet of mesons with $J^P = 0^-$ and N_δ the decuplet $J^P = 3/2^+$. Because the spin of the meson P is zero we have $\langle P|\sigma_q|P\rangle = 0$, and consequently only part of the quark-quark (or quark-antiquark) scattering amplitude contributes to this reaction, namely the terms a and $c(\sigma_i v)$ [cf. (5.1), (5.2)], and these amplitudes must be averaged over the nucleon wave functions (the index i refers to a quark belonging to the nucleon). Because the spins of N and N_δ are different, for the reaction $P + N \to P + N_\delta$ only the term $c(\sigma v)$ remains, and the amplitude with spin reversal in reactions $P + N \to P + N$ is also expressible in terms of this quantity. This makes it possible to derive a number of interesting relations[72]:

$$\left.\begin{aligned}
&\frac{d\sigma^{fl}}{dt}(\pi^- p \to \pi^0 n) = \frac{25}{24}\frac{d\sigma}{dt}(\pi^+ p \to \pi^0 \Delta^{++}) = \frac{25}{8}\frac{d\sigma}{dt}(\pi^- p \to \pi^0 \Delta^0),\\
&\frac{d\sigma^{fl}}{dt}(\pi^- p \to \eta n) = \frac{25}{24}\frac{d\sigma}{dt}(\pi^- p \to \eta \Delta^0) = \frac{25}{8}\frac{d\sigma}{dt}(\pi^+ p \to \eta \Delta^{++}),\\
&\frac{d\sigma^{fl}}{dt}(K^- p \to \overline{K}^0 n) = \frac{25}{8}\frac{d\sigma}{dt}(K^- p \to \overline{K}^0 \Delta^0),\\
&\frac{d\sigma^{fl}}{dt}(K^+ p \to K^0 n) = \frac{25}{24}\frac{d\sigma}{dt}(K^+ p \to K^0 \Delta^{++}),\\
&\frac{d\sigma^{fl}}{dt}(K^- p \to \pi^- \Sigma^+) = \frac{1}{8}\frac{d\sigma}{dt}(K^- p \to \pi^- \Sigma^+_\delta).
\end{aligned}\right\} \tag{6.6}$$

Here $d\sigma^{fl}/dt$ is the differential cross section which comes from the amplitude with spin reversal. Two conclusions follow from the relations (6.6):

a) The cross sections for all processes of the type $P + N \to P + N_\delta$ must approach zero for small scattering angles ϑ like $\sin^2 \vartheta/2$, owing to the properties of the amplitudes with spin reversal for the reactions $P + N \to P + N$.

A minimum of this sort is observed in the reaction $\pi^+ p \to \pi^0 \Delta^{++}$[73] at $p_\pi \sim 6-8$ GeV/c.[30] In the reaction $\pi^+ p \to \eta \Delta^{++}$ at $p_\pi = 8$ GeV/c there is no minimum in the scattering at small angles,[30] but on the other hand there is no forward peak, the cross section being isotropic. Possibly this is evidence that the asymptotic behavior has not yet been reached for this reaction.

b) A comparison of (6.6) with experiment is possible only with the additional hypothesis that the main contribution to the differential cross section for $\pi^- p \to \pi^0 n$ is from the amplitude with change of helicity (there is evidence in favor of this hypothesis in the absence of a minimum at the angle 0° in this reaction[30]). A study of the differential cross sections for the reactions $\pi^- p \to \pi^0 n$ and $\pi^+ p \to \pi^0 \Delta^{++}$ shows that there is a qualitative correspondence between them, in the positions of the maxima and minima. In the Regge-pole model this sort of correspondence is due to the fact that these reactions occur only through the exchange of a reggeized ρ meson. Comparison of the top relation at the maximum for $t = -0.1$ $(\mathrm{GeV/c})^2$ shows that it agrees to 50-percent accuracy.

4. If we assume that the amplitude for scattering of a bound quark by a quark (or antiquark) does not depend much on whether the quark (or antiquark) belongs to a meson or to a baryon, then in the high-energy limit, where the Pomeranchuk theorem holds, there is a connection between the meson-baryon and baryon-baryon total cross sections. From a simple counting of the number of quark amplitudes one easily gets[25] $\sigma_{pp} = 9a$, $\sigma_{\pi p} = 6a$, $\sigma_{\pi\pi} = 4a$, where a is the total cross section for scattering of a quark by a quark (or antiquark), in the limit where the Pomeranchuk theorem holds. From this it follows that

$$\sigma_{pp} : \sigma_{\pi p} : \sigma_{\pi\pi} = 9 : 6 : 4.$$

If we assume that at the largest energies now attainable σ_{pp} and $\sigma_{\pi p}$ do not differ much from the Pomeranchuk limit, then this prediction is in fair agreement with experiment[35]

$$\frac{\sigma_{pp}}{\sigma_{\pi p}} = 1.58 \pm 0.05.$$

When the total cross sections are processed by means of the Regge-pole model, the ratio of the contributions of the vacuum pole to these cross sections is 1.8 ± 0.02.[79] We note that the connection of the baryon-baryon and meson-baryon cross sections is not understandable outside the framework of the quark model, and therefore the fact that the last relation agrees with experiment is a serious argument in favor of the quark structure of hadrons.

VII. RELATIONS BETWEEN TOTAL CROSS SECTIONS

We shall now proceed to a systematic comparison of the predictions of the model with experiment. Everywhere in what follows we shall assume that the quarks Q_p and Q_n interact in the same way (i.e., there is isotopic invariance of the strong interaction), and the quark Q_λ interacts differently, i.e., in the quark model there is a unique type of breaking of SU(3) in the interaction.[32,60,61]

The simplest problem is the study of the elastic scattering amplitude at zero angle, which is connected with the total cross section by the optical theorem. Since the optical theorem involves the imaginary part of the forward scattering amplitude, summed over the spins, the total cross section does not depend on the spin interaction of the quarks. Therefore the calculation of the relations between total cross sections is extremely simple: it reduces to counting the numbers of different quark amplitudes. For example, for the scattering of a π^+ meson by a proton we have

$$\begin{aligned}
\mathrm{Sp}\,\mathrm{Im}\,A^{el}_{\pi^+ p}(t=0) &= \langle (Q_p\overline{Q}_n)(Q_pQ_pQ_n) \,|\, (Q_p\overline{Q}_n)(Q_pQ_pQ_n) \,|\rangle \\
&= 2\langle Q_pQ_p \,|\, Q_pQ_p\rangle + 2\langle Q_p\overline{Q}_n \,|\, Q_p\overline{Q}_n\rangle + \langle Q_n\overline{Q}_n \,|\, Q_n\overline{Q}_n\rangle + \langle Q_pQ_n \,|\, Q_pQ_n\rangle.
\end{aligned} \tag{7.1}$$

In a similar way we can express the cross sections for other processes in terms of the cross sections for scattering of quarks:

$$\left.\begin{aligned}
\sigma(\pi^+p) &= 2a\,(pp) + 2a\,(p\bar{n}) + a\,(pn) + a\,(n\bar{n}),\\
\sigma(\pi^-p) &= 2a\,(pn) + 2a\,(p\bar{p}) + a\,(nn) + a\,(n\bar{p}),\\
\sigma(K^+p) &= 2a\,(pp) + a\,(pn) + 2a\,(\bar{\lambda}p) + a\,(\bar{\lambda}n),\\
\sigma(K^-p) &= 2a\,(\bar{p}p) + a\,(\bar{p}n) + 2a\,(\lambda p) + a\,(\lambda n),\\
\sigma(K^0p) &= 2a\,(pn) + a\,(nn) + 2a\,(\bar{\lambda}p) + a\,(\bar{\lambda}n),\\
\sigma(\bar{K}^0p) &= 2a\,(p\bar{n}) + a\,(n\bar{n}) + 2a\,(\lambda p) + a\,(\lambda n).
\end{aligned}\right\} \quad (7.2)$$

If isotopic invariance $[a(pp) = a(nn),\ a(\lambda p) = a(\lambda n)]$ is assumed, there is obviously only one valid relation[62]

$$\sigma(K^-p) - \sigma(\bar{K}^0p) - \sigma(K^+p) + \sigma(K^0p) = \sigma(\pi^-p) - \sigma(\pi^+p). \quad (7.3)$$

Owing to the isotopic invariance of the strong interactions

$$\sigma(K^0p) = \sigma(K^+n), \qquad \sigma(\bar{K}^0p) = \sigma(K^-n),$$

and since $\sigma(K^+n)$ and $\sigma(K^-n)$ are measured experimentally we express (7.3) in terms of these cross sections:

$$[\sigma(K^-p) - \sigma(K^-n)] + [\sigma(K^+n) - \sigma(K^+p)] = \sigma(\pi^-p) - \sigma(\pi^+p). \quad (7.4)$$

This equation is in agreement with experiment within the limits of experimental error (Table XI; the experimental data are taken from[35]).

If SU(3) symmetry holds in the interaction of the quarks there are other predictions.[25,26] They are not given here, since from the point of view of all present-day models the experimental data on total cross sections indicate serious breaking of unitary symmetry. In the Regge-pole model this breaking manifests itself in the necessity of introducing mixing parameters for the vector and tensor reggeons.[95,96] In the quark model

$$\frac{a\,(\lambda p) + a\,(\bar{\lambda}p)}{a\,(pn) + a\,(p\bar{n})} = \frac{\sigma(K^-p) + \sigma(K^+n) - \sigma(\pi^-p)}{2\sigma(\pi^+p) - \sigma(\pi^-p) + 3\,[\sigma(K^+n) - \sigma(K^+p)]}. \quad (7.5)$$

In exact SU(3) the right member must be equal to unity. Experimentally, when the energy of the incident particle is 18 GeV/c the value of this ratio is 0.6 ± 0.1. The same relations are retained for the total scattering cross sections for vector mesons, and furthermore $\sigma_{tot}(\pi N) = \sigma_{tot}(\rho N)$, since the quark amplitudes that contribute to $\sigma_{tot}(\rho N)$ are the same as those for $\sigma_{tot}(\pi N)$.

For the baryon-baryon and baryon-antibaryon total cross sections one does not get a single relation whose experimental testing would be possible in the immediate future. All of the relations are written out in[25,26], and we may refer the reader to those papers.

If we assume that the amplitude for scattering of a quark by a quark (or an antiquark) does not depend on the nature of the hadron to which the quark belongs, we can get relations between the meson-baryon and baryon-baryon total cross sections which are characteristic of this model alone. Namely, not assuming SU(3) symmetry, we have[25,26]

$$\frac{1}{2}(\sigma(pp) + \sigma(p\bar{p})) = \frac{3}{4}[\sigma(\pi^+p) + \sigma(\pi^-p)] + \frac{1}{4}[\sigma(K^+p) + \sigma(K^-p) - \sigma(K^+n) - \sigma(K^-n)], \quad (7.6)$$

$$\sigma(pp) - \sigma(pn) = \sigma(K^+p) - \sigma(K^+n), \quad (7.7)$$

$$\sigma(p\bar{p}) - \sigma(\bar{p}n) = \sigma(K^-p) - \sigma(K^-n). \quad (7.8)$$

The derivation of these relations is simple if we note that

$$\left.\begin{aligned}
\sigma(pp) &= 4a\,(pp) + 4a\,(pn) + a\,(nn),\\
\sigma(\bar{p}p) &= 4a\,(\bar{p}p) + 2a\,(p\bar{n}) + 2a\,(\bar{p}n) + a\,(n\bar{n}).
\end{aligned}\right. \quad (7.9)$$

The relation (7.6) agrees with experiment to 20 percent (Table XII), and the left and right members of (7.7) and (7.8) are close to zero, their differences being of the order of the experimental errors.

In comparing (7.6)–(7.8) with experiment we must remember that baryons and mesons consist of different numbers of quarks and have different masses. Therefore, strictly speaking, one must compare the right and left members of (7.6)–(7.8) at different energies, so that the quark amplitudes may depend on the same invariants (s_q, see Fig. 3). For example, if we assume that hadrons consist of weakly bound quarks with effective mass of the order of 300 MeV, we must compare the right and left members of the relations (7.6)–(7.8) at

$$\frac{s_{pp}}{s_{\pi p}} = \frac{3}{2}.$$

Therefore it is most reasonable to make the comparisons for (7.6)–(7.8) at the highest possible energy, for which we can hope that the quark amplitudes do not depend much on energy. Unfortunately, at attainable energies $\sigma(p\bar{p})$ changes considerably with increasing energy, so that, strictly speaking, present energies are not sufficient for the testing of these relations. In the limit in which the Pomeranchuk theorem holds (7.6) takes the form

$$\frac{\sigma(pp)}{\sigma(\pi p)} = \frac{3}{2}. \quad (7.10)$$

This relation has already been discussed in the preceding chapter. We note that in this model the relation of[30] for the Pomeranchuk limits,

$$\sigma_{\pi p}^2 = \sigma_{\pi\pi}\sigma_{pp},$$

is satisfied, since

$$\frac{\sigma_{\pi p}}{\sigma_{\pi\pi}} = \frac{\sigma_{pp}}{\sigma_{\pi p}} = \frac{3}{2}.$$

From this it is easy to calculate the Pomeranchuk limit for $\sigma_{\pi\pi}$ ($\sigma_{\pi\pi} = 14.6$ mb).

Table XI. Comparison of the Experimental Results with the Relation

$\sigma(\pi^-p) - \sigma(\pi^+p) = [\sigma(K^-p) - \sigma(K^-n)] + [\sigma(K^+n) - \sigma(K^+p)]$

p, GeV/c	$\sigma(\pi^-p) - \sigma(\pi^+p)$, mb	$[\sigma(K^-p) - \sigma(K^-n)] + [\sigma(K^+n) - \sigma(K^+p)]$, mb
6	2.3±0.5	2.6±1.2
8	2.4±0.5	4.2±1.1
10	1.7±0.5	2.1±1.1
12	1.7±0.5	1.7±1.1
14	1.5±0.5	1.5±1.1
16	1.7±0.5	1.4±1.5
18	1.5±0.5	2.2±2.4

Table XII. Comparison with Experiment of the Relation

$\sigma_T(pp) + \sigma_T(p\bar{p}) = \frac{3}{2}[\sigma_T(\pi^+p) + \sigma_T(\pi^-p)] + \frac{1}{2}[\sigma_T(K^+p) + \sigma_T(K^-p) - \sigma_T(K^+n) - \sigma_T(K^-n)]$ (according to[35])

p, GeV/c	$\sigma_T(pp) + \sigma_T(\bar{p}p)$, mb	$3/2[\sigma_T(\pi^+p) + \sigma_T(\pi^-p)] + \frac{1}{2}[\sigma_T(K^+p) + \sigma_T(K^-p) - \sigma_T(K^+n) - \sigma_T(K^-n)]$, mb
6	99.9±1.7	82.9±1.4
8	96.4±1.4	82.5±1.4
12	91.1±1.5	77.9±1.4
14	89.9±1.5	75.2±1.4
16	87.9±1.4	73.4±1.5
18	89.0±4.2	72.8±1.6

Table XIII. Table of Values of Cross Sections of Bound Quarks

p	10 GeV/c	16 GeV/c	p	10 GeV/c	16 GeV/c
$a(pp)$	4.3	4.3	$a(\bar{p}n)$	3.3	3.15
$a(pn)$	4.4	4.3	$a(p\lambda)$	3	2.95
$a(p\bar{p})$	5.2	4.5	$a(\bar{p}\lambda)$	1.5	1.4

Using the expressions (7.2) and (7.9) and the known total cross sections,[35] we can calculate the cross sections for scattering of a bound quark by a quark (or an antiquark). A set of these quantities is given in Table XIII, for values 10 and 16 GeV/c of the momentum of the incident particle. It can be seen that a(pp) = a(pn)—i.e., the imaginary part of the charge-transfer amplitude is small (but this is not true of the real part, as was assumed in[81]). An important feature of the data of Table XIII is the large difference between $a(p\lambda)$ and $a(p\bar{\lambda})$, on one hand, and a(pn) and $a(p\bar{n})$, on the other, which is evidence of a serious breaking of SU(3) symmetry in the interaction of quarks. This strong breaking of SU(3) symmetry is not surprising; in the mass analyses of Chap. II we saw that SU(3) is seriously broken in spin-spin and annihilation interactions of quarks.[18] In this sense it is curious that the quark model enables us to derive without the use of SU(3) symmetry predictions which are usually derived by means of SU(3) [for example, (7.3)].

We note that for the real parts of the scattering amplitudes, averaged over the spins of the incident particles, we get the same relations as for the total cross sections.[76] Assuming that for high-energy scattering at zero angle the contribution of the amplitude with spin reversal is small, and that charge transfer can be neglected, we get the experimentally verified equation

$$\frac{\langle \mathrm{Re}A(pp)\rangle+\langle \mathrm{Re}\,A(p\bar{p})\rangle}{\langle \mathrm{Re}\,(\pi^+p)\rangle+\langle \mathrm{Re}\,(\pi^-p)\rangle}=\frac{3}{2}. \tag{7.11}$$

It is convenient to use (7.10) to write (7.11) in the form

$$\frac{\alpha_{pp}+\alpha_{\bar{p}p}\dfrac{\sigma(p\bar{p})}{\sigma(pp)}}{\alpha_{\pi^+p}+\alpha_{\pi^-p}}=1, \tag{7.12}$$

where α = Re A/Im A. Experimentally[93]:

$$\alpha_{pp}=-0.22\pm0{,}05,\quad \alpha_{p\bar{p}}=+0{,}07\pm0{,}032 \text{ or } +0.02\pm0.032,$$
$$\alpha_{\pi^+p}+\alpha_{\pi^-p}=-0.28\pm0.05.$$

Because of the difficulty which arises from the impossibility of unambiguously separating off the electromagnetic corrections to the high-energy scattering amplitude, two values are given for $\alpha_{p\bar{p}}$.[93] It can be seen that (7.12) is not in contradiction with experiment.

VIII. RELATIONS BETWEEN THE DIFFERENTIAL CROSS SECTIONS FOR INELASTIC REACTIONS

It is a much more complicated matter to study with the quark model the relations between the differential cross sections for inelastic reactions. This is primarily due to the fact that the spin-spin and spin-orbit interactions of the quarks become important. Furthermore, for scattering at nonzero angles it is necessary to take the form-factors of the particles into account [cf. Eq. (5.1)].

There exists, however, a whole class of relations which do not depend on any additional assumptions and follow simply from the fact that reactions in which values of ΔS, ΔQ, and ΔT larger than unity are transferred in the t channel are forbidden.

For example, the reaction $\pi^-p \to \pi^+\Delta^-$ is forbidden by these "rules" (the charge transfer is two units). From this there at once follows a connection between the isotopic amplitudes (there are two of them in this reaction), which gives the following relations between the cross sections[72]:

$$\begin{aligned}\sigma(\pi^+p\to\pi^+\Delta^+)&=\frac{2}{3}\sigma(\pi^+p\to\pi^0\Delta^{++})\\ &=\sigma(\pi^-p\to\pi^-\Delta^+)=2\sigma(\pi^-p\to\pi^0\Delta^0).\end{aligned} \tag{8.1}$$

All of the predictions similar to this one are written out in[82]. At p_π = 8 GeV/c we know only the cross sections[56]

$$\sigma(\pi^+p\to\pi^0\Delta^{++})=(0.12\pm0.02)\text{ mb}$$
$$\sigma(\pi^+p\to\pi^+\Delta^+)=\left(0.08\pm{}^{0.04}_{0.03}\right)\text{ mb}$$

These values agree with (8.1) within the limits of experimental error.

More interesting relations follow from the quark model for reactions in which vector mesons are produced[71]:

$$\begin{aligned}\sigma(K^-p\to\varphi\Lambda)&=\sigma(\pi^-p\to K^*\Lambda),\\ \sigma(K^-p\to\omega\Lambda)&=\sigma(K^-p\to\rho^0\Lambda).\end{aligned} \tag{8.2}$$

The known experimental data for these reactions are at not too high energies, and the errors are large. At p_K = 3 GeV/c, $\sigma(K^-p\to\varphi\Lambda)$ = (40 ± 8) μb, and at p_π = 2.7 GeV/c, $\sigma(\pi^-p\to K^{0*}\Lambda)$ = (53 ± 8) μb; at 4.1 GeV/c, $\sigma(K^-p\to\rho^0\Lambda)$ = (41 ± 17) μb, and $\sigma(K^-p\to\omega\Lambda)$ = (41 ± 15) μb; accordingly, (8.2) is not in contradiction with experiment.

Similar relations can be written for the nonet 0^- (and in general for any nonet of mesons consisting of a quark and an antiquark):

$$\left.\begin{aligned}&\sigma(K^-p\to\eta\Lambda)+\sigma(K^-p\to X^0\Lambda)=\sigma(\pi^-p\to K^0\Lambda)+\sigma(K^-p\to\pi^0\Lambda),\\ &\sigma(\pi^-p\to\pi^0n)+\sigma(\pi^-p\to\eta n)+\sigma(\pi^-p\to X^0n)\\ &\qquad=\sigma(K+n\to K^0p)+\sigma(K^-p\to\bar{K}^0n),\\ &\sigma(\pi^+p\to\pi^0\Delta^{++})+\sigma(\pi^+p\to\eta\Delta^{++})+\sigma(\pi^+p\to X^0\Delta^{++})\\ &\qquad=3\sigma(K^-p\to\bar{K}^0\Delta^0)+\sigma(K^+p\to K^0\Delta^{++}).\end{aligned}\right\} \tag{8.3}$$

All of these equations can not as yet be derived from hypotheses other than the quark model, and therefore it is very important to verify them. Unfortunately, because of our present ignorance of the cross sections for the production of X^0 these relations cannot be tested.

Besides the equations we have given, which in general connect the differential cross sections for scattering at any angle ϑ, further consequences for scattering at zero angle appear in the model.[82,83] This can be easily understood if we note that for zero scattering angle the terms responsible for the spin-orbit interaction drop out of Eq. (5.2):

$$\left.\begin{aligned}&\frac{d\sigma}{dt}(\pi^-p\to K^0\Sigma^0)=\frac{1}{3}\frac{d\sigma}{dt}(\pi^-p\to K^0\Lambda),\\ &\frac{d\sigma}{dt}(K^-p\to M^0\Sigma^0)=\frac{1}{3}\frac{d\sigma}{dt}(K^-p\to M^0\Lambda)\quad (M^0=X^0 \text{ or } \eta),\\ &\frac{d\sigma}{dt}(K^-p\to\pi^-\Sigma^0)=\frac{2}{3}\frac{d\sigma}{dt}(K^-n\to\pi^-\Lambda),\\ &\frac{d\sigma}{dt}(\pi^-p\to\rho^0n)=\frac{25}{24}\frac{d\sigma}{dt}(\pi^+p\to\rho^0\Delta^{++}),\\ &\frac{d\sigma}{dt}(\pi^-p\to\omega n)=\frac{25}{24}\frac{d\sigma}{dt}(\pi^+p\to\omega\Delta^{++}).\end{aligned}\right\} \tag{8.4}$$

If we make the further assumption that SU(3) symmetry holds in the strong interactions, there is a much larger number of relations between the differential cross sections.[25,82,83] Since at present these relations cannot be tested at sufficiently high energies, they are not given here. We note only that with SU(3) symmetry in the quark model the relations between the differential cross sections for zero-angle scattering of particles belonging to the same SU(3) multiplet agree exactly with the predictions of SU(6). In this sense it is a matter of interest to test the relations (8.4), since their existence when there is violation of SU(3) is natural only in the quark model.

The relations most characteristic of the quark model are those between baryon-baryon and meson-baryon cross sections. As in the case of the total cross sections, to derive them it is necessary to assume that the amplitude for the scattering of a quark by a quark (or antiquark) do not depend much on what sort of hadron the quark belongs to. We give examples of such relations, which are derived in[83] without the assumption that there are any symmetries higher than the isotopic symmetry:

$$\left.\begin{aligned}
&\sigma(\pi^-p\to K^0\Lambda)=\frac{9}{4}\sigma(\bar{p}p\to\bar{\Sigma}^0\Lambda)-\frac{1}{12}\sigma(p\bar{p}\to\bar{\Lambda}\Lambda),\\
&\sigma(\pi^-p\to K^{0*}\Lambda)=\frac{9}{4}\sigma(\bar{p}p\to\bar{\Sigma}_0^0\Lambda),\\
&\sigma(\pi^+p\to K^+\Sigma^+)+\frac{1}{9}\sigma(\pi^+p\to K^{*+}\Sigma^+)=\sigma(p\bar{p}\to\bar{\Sigma}^+\Sigma^+),\\
&\sigma(K^+p\to K^0\Delta^{++})+\frac{25}{9}\sigma(K^+p\to K^{0*}\Delta^{++})=\sigma(pp\to n\Delta^{++}),\\
&\sigma(K^+p\to K^{0*}\Delta^{++})=\frac{9}{8}\sigma(pp\to\Delta^0\Delta^{++}).
\end{aligned}\right\}\qquad(8.5)$$

In comparing the theory of inelastic processes with experiment it is necessary to include corrections which arise because the masses of the colliding particles are different. The fact that the hypothesis that the coordinate part of the wave function of a hadron depends only weakly on its mass is successful in explaining the electromagnetic properties of hadrons gives us the hope that here, as in the case of the decay $\omega\to\pi^0\gamma$, it is sufficient to take only the kinematics into account.

Probably it is most natural to compare the cross sections for the same energy release Q and the same sum of kinetic energies of the emerging particles, since this quantity takes the thresholds of the reactions into account correctly.[84] For the same reason it is necessary to separate off a phase-volume factor $F = p_f/sp_i$ (where p_i and p_f are the initial and final momenta of the hadrons in the center-of-mass system) from the cross sections in the standard way,[84] since the relations were derived for the squares of the matrix elements. In the quark model, besides these corrections which are customary with the symmetries, one further correction is necessary, which is due to the fact that the scattering amplitude contains the form-factors F(t) [cf. Eq. (5.1)], which are different for mesons and for baryons. It is hard to make these corrections exactly.

In the literature[83] there has been discussion of the consequences of the additional hypothesis that the dependence of the quark amplitudes on the momentum transfer t is weak (a polynomial), and that all of the strong (exponential) dependence on the momentum transfer is due to the form-factor F(t) of the particle. For the differential cross sections it is then predicted that

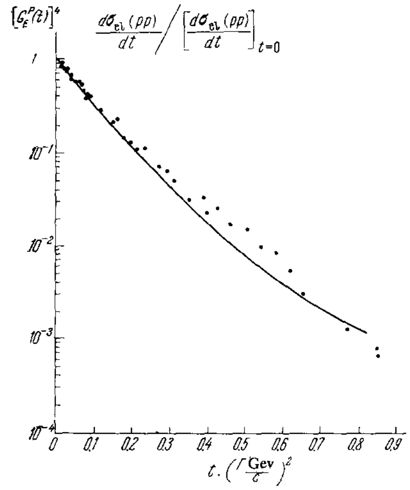

FIG. 4. Behavior of the quantities $\frac{d\sigma_{el}(pp)}{dt}\Big/\left[\frac{d\sigma_{el}(pp)}{dt}\right]_{t=0}$ and $[G_E^p(t)]^4$ as functions of the momentum transfer t. The solid line shows the behavior of $\frac{d\sigma_{el}(pp)}{dt}\Big/\left[\frac{d\sigma_{el}(pp)}{dt}\right]_{t=0}$ for p = 12.8 GeV/c; the points are values of $[G_E^p(t)]^4$.

they have the same main dependence on the momentum transfer t, and that this dependence is the same as for the electromagnetic form-factor of the hadron[31]; this is reminiscent of the droplet model of Yang and Wu.[85] This prediction is in agreement with the known differential cross section for pp scattering[31] (see Fig. 4, which is taken from[92]).

Observed deviations from a universal t-dependence of the differential cross sections in inelastic reactions are on this hypothesis explained by the presence of a kinematic dependence of the quark amplitudes on t. [For example, the maximum at t = 0.1 (GeV/c)^2 in the reaction $\pi^-p\to\pi^0n$ is explained by the large value of the amplitude with spin reversal, which is proportional to $(-t)^{1/2}$ at small scattering angles.]

Within the framework of these last assumptions we can take F(t) into account by dividing the cross section by e^{At}, using a value of A which is different for baryon-baryon and for meson-baryon scattering and is taken from experiment. After such a procedure we are dealing essentially only with the quark amplitudes. It is quite clear that the errors incurred here by neglecting the polynomial dependence of the quark amplitudes on $t^{1/2}$, and also the large experimental errors, can be rather impressive. Moreover, as before there is still a theoretical uncertainty in the quark model, as to the choice of energies for the comparison of the right and left members of the relations (8.5). We have already discussed this in some detail. Essentially we have only one rigorous relation between the meson-baryon and the baryon-baryon cross sections, namely the ratio of the total cross sections in the Pomeranchuk limit. In spite of this, the comparison with experiment made in[83] for Q from 0.4 to 0.8 GeV shows that all of the relations are not in contradiction with experiment, except the inequality $\sigma(\pi^-p\to K^0\Sigma^0) < (2/3)\sigma(p\bar{p}\to\Lambda\bar{\Lambda})$, which follows from (8.5). For it the left member is experimentally twice the right member (but the Q's are small, ~0.4–0.6 GeV).

Relations between the zero-angle scattering cross sections for meson-baryon and baryon-baryon reactions are of interest, especially because there has recently appeared a paper[81] in which it is asserted that relations of this type are seriously violated. First of all we note that the equations of[81] are valid only for scattering at angle 0° and with neglect of the spin-spin interaction [the amplitudes b and d in Eq. (5.2)]. Actually it is necessary to take the spin into account, as we shall see in what follows.

We write out some relations for scattering at 0° angle:

$$\left.\begin{aligned}
&\frac{d}{dt}\sigma(pn\to np)=\frac{d}{dt}\sigma(K^+n\to K^0p)+\frac{25}{9}\frac{d}{dt}\sigma(K^+n\to K^{0*}p),\\
&\frac{d}{dt}\sigma(p\bar{p}\to n\bar{n})=\frac{d}{dt}\sigma(K^-p\to K^0n)+\frac{25}{9}\frac{d}{dt}\sigma(K^-p\to \overline{K}^{*0}n),\\
&\frac{d}{dt}\sigma(p\bar{p}\to\Lambda\bar{\Lambda})=\frac{9}{4}\frac{d}{dt}\sigma(\pi^+p\to K^+\Sigma^+)+\frac{81}{4}\frac{d}{dt}\sigma(\pi^+p\to K^{*+}\Sigma^+),\\
&\frac{d}{dt}\sigma(pn\to np)+\frac{d}{dt}\sigma(p\bar{p}\to n\bar{n})=\\
&\qquad=\frac{d}{dt}\sigma(\pi^-p\to\pi^0n)+3\frac{d}{dt}\sigma(\pi^-p\to\eta n)+\\
&\qquad+\frac{25}{24}\frac{25}{9}\left[\frac{d}{dt}\sigma(\pi^+p\to\rho_0\Delta^{++})+\frac{d}{dt}\sigma(\pi^+p\to\omega\Delta^{++})\right].
\end{aligned}\right\} \tag{8.6}$$

In order to get the results of[81], one must neglect in (8.6) the cross sections for production of vector mesons. Only the last relation can be compared with experiment. At 8 GeV/c the right member is about three times the left member. It can be seen from the analysis of the experiment that the main contribution to the right member is from the term $(d/dt)\sigma(\pi^+p\to\rho^0\Delta^{++})$, which is different from zero only because of the presence of the spin-spin interaction [the amplitudes b and d in Eq. (5.2)]. The too large value of $(d/dt)(\pi^+p\to\rho^0\Delta^{++})$ is possibly due to the fact that this differential cross section is known only for large values $t=-0.05\ (\mathrm{GeV/c})^2$, where the spin-orbit interaction not included in (8.6) may be important. For example, the differential cross section for the reaction $\pi^+ + p\to\rho^+ + p$ falls by a factor three when the momentum transfer is decreased from $t=-0.05\ (\mathrm{GeV/c})^2$ to $t=-0.02\ (\mathrm{GeV/c})^2$. Therefore it is not clear at present whether or not (8.6) is in contradiction with experiment.

IX. THE DENSITY MATRIX

In this chapter we shall consider the properties of the density matrix of a particle with spin S in the quark model,

$$\hat{\rho}_{\mu\mu'}=\frac{\Sigma_{\lambda_a\lambda_c\lambda_d}A^{\lambda_a\mu}_{\lambda_c\lambda_d}(A^{\lambda_a\mu'}_{\lambda_c\lambda_d})^*}{\Sigma_{\lambda_a\lambda_b\lambda_c\lambda_d}A^{\lambda_a\lambda_b}_{\lambda_c\lambda_d}(A^{\lambda_a\lambda_b}_{\lambda_c\lambda_d})^*}, \tag{9.1}$$

where $A^{\lambda_a\lambda_b}_{\lambda_c\lambda_d}$ are helicity amplitudes.[74] Since $\hat{\rho}_{\mu\mu'}$ is expressed in terms of the quark amplitudes, by comparing it with experiment one can get more detailed information in regard to the nature of the interaction of quarks at high energies.[88] There is a different quantity which is more convenient for comparison with experiment

$$\rho_{mm'}(\Psi_b)=\Sigma_{\mu\mu'}d^s_{m\mu}(-\Psi_b)\,\hat{\rho}_{\mu\mu'}\,d^s_{m'\mu'}(-\Psi_b). \tag{9.2}$$

Here m and m′ are projections of the spin of particle b (see Fig. 3) in the direction of the motion of the initial particle a in the system in which b is at rest. Ψ_b is the angle between the momenta p_a and $+p_d$ in this system. For forward scattering the two matrices are equal, since in this case $\Psi_b=0$ and $d^S_{m\mu}(0)=\delta_{m\mu}$.[74,89] In the quark model it is simplest to calculate $\hat{\rho}_{\mu\mu'}$, because if we consider only single scattering of a quark of one particle by a quark of the other particle the quark amplitudes are additive.

The relations between elements of the density matrix are free from many kinematic uncertainties, in particular from the need to take the phase-space volume into account. Moreover, all of the predictions refer to a single reaction, so that there are no effects of mass differences and no question as to the s and t values at which the relations are to be compared.

The most interesting processes from the point of view of the model are those of scattering of pseudoscalar mesons by nucleons with the production of a particle belonging to the decuplet with $J^P=3/2^+$: $P+N\to P+N_\delta$. For these reactions the quark model predicts all of the elements of the density matrix at all angles (permissible in the model)[88]:

$$\rho_{\mu\mu'}=\frac{1}{8}\begin{pmatrix}3&0&\sqrt{3}&0\\0&1&0&\sqrt{3}\\\sqrt{3}&0&1&0\\0&\sqrt{3}&0&3\end{pmatrix}. \tag{9.3}$$

The comparison with experiment is shown in Table XIV, taken from[88] and[97]. It can be seen that in general (9.3) agrees fairly well with experiment, although the energies are clearly not high enough, especially for $K^+p\to K^0\Delta^{++}$. Since the cross section for a reaction of this type must approach zero for $\vartheta\to 0$ (cf. Chap. VI), the value of the density matrix at $\vartheta=0$ can be due to a large extent to corrections to the model. For greater definiteness one must make the comparison for scattering at large angles, where the main parts of the matrix elements are those calculated from the model.

An important consequence of the quark model for the reactions $P+N\to P(V)+N(N_\delta)$ is that the elements $\hat{\rho}_{\mu\mu'}$ are real.[88] As an example let us consider the density matrix for the reaction $\pi p\to\rho N_\delta$. First, any helicity change larger than unity is forbidden in the model, and therefore $A^{\lambda}_{3/2,-1/2}=0$. Second, the amplitudes responsible for a change of the helicity for only one pair of particles in the t channel are equal to zero [this can be seen from Eq. (5.2) and the fact that the spins of the particles are different]. Consequently,

$$A^1_{1/2,\,1/2}=A^{-1}_{1/2,\,1/2}=A^0_{1/2,\,-1/2}=A^0_{3/2,\,1/2}=0;$$

and furthermore

$$\sqrt{3}A^{+1}_{-1/2,\,1/2}=A^{-1}_{3/2,\,1/2},\qquad \sqrt{3}A^{-1}_{-1/2,\,1/2}=A^{+1}_{3/2,\,1/2},$$

since

$$\sqrt{3}\langle N_{+1/2}|\sigma_+|N_{\delta-1/2}\rangle=\langle N_{+1/2}|\sigma_-|N_{\rho+3/2}\rangle.$$

Then, if we write $A^0_{1/2,\,1/2}=F$, $A^{-1}_{-1/2,\,1/2}=B/2^{1/2}$, $A^{+1}_{-1/2,\,1/2}=C/2^{1/2}$, we get for the density matrices of the ρ and Δ produced in this reaction

$$\{\hat{\rho}_{\mu\mu'}\}=\frac{1}{N}\begin{pmatrix}2(|B|^2+|C|^2)&0&2\,\mathrm{Re}\,BC^*\\0&2|F|^2&0\\2\,\mathrm{Re}\,BC^*&0&2(|B|^2+|C|^2)\end{pmatrix}, \tag{9.4}$$

$$\{\hat{\rho}_{\mu\mu'}\} = \frac{1}{N}\begin{pmatrix} \frac{3}{2}(|B|^2+|C|^2) & 0 & \sqrt{3}\,\mathrm{Re}\,BC^* & 0 \\ 0 & |F|^2+\frac{1}{2}(|B|^2+|C|^2) & 0 & 0 \\ \sqrt{3}\,\mathrm{Re}\,BC^* & 0 & |F|^2+\frac{1}{2}(|B|^2+|C|^2) & 0 \\ 0 & 0 & 0 & \frac{3}{2}(|B|^2+|C|^2) \end{pmatrix} \quad (9.5)$$

The elements of the density matrix can be calculated in analogous ways for the other reactions.

The comparison with experiment can be made only at comparatively small energies. It shows that for all of the processes the forbidden matrix elements $\hat{\rho}_{\mu\mu'}$ are in fact small in comparison with some of the allowed elements. On the other hand, some elements which were predicted to be different from zero are as small as the forbidden elements, which indicates that some quark amplitudes are small at the energies attained so far. For example, in the reaction $\pi^+p \to \rho\Delta^{++}$ the element ρ_{00} is much larger than all the other elements of the density matrix. For p_π = 8 GeV/c and $\vartheta = 0$[30]: $\rho_{00} = 0.77 \pm 0.04$, $\rho_{1,-1} = -0.031 \pm 0.02$, $\rho_{10} = -0.124 \pm 0.03$. The experimental data do not correspond to an exactly zero angle, since $\rho_{1,-1}$ and ρ_{10} are different from zero. It can be seen from the comparison of (9.4), (9.5) with experiment that $F \gg B$ (i.e. the dominant amplitude for the scattering of a quark by a quark (or antiquark) is $a(\sigma_z^{(1)}, \sigma_z^{(2)})$. This is in good agreement with the large value of ρ_{00} in the reactions $\pi p \to \rho p$, $\pi p \to \omega\Delta$, $K^+p \to K\Delta^{++}$ when the momentum of the incident particle is of the order of 8 GeV/c.[88]

The study of the density matrix in the quark model probably indicates that spin effects are important in the scattering of high-energy hadrons at small angles in inelastic processes. To reach more reliable conclusions we need to measure $\hat{\rho}_{\mu\mu'}$ at smaller angles.

X. COLLISIONS OF HIGH-ENERGY HADRONS (RELATIONS BETWEEN THE NONRELATIVISTIC QUARK MODEL AND OTHER THEORIES)

We shall first discuss the connection of the quark model with higher symmetries of the type of the collinear group $SU(6)_W$ (see Chap. III). We shall not consider higher symmetries of the type of U(6, 6), since they are in contradiction with unitarity.[14] From the point of view of the quark model there are no reasons to expect that the interactions of high-energy particles will be $SU(6)_W$ invariant. We recall that the successes of $SU(6)_W$ symmetry in explaining the static properties of hadrons in the quark model are due precisely to the nonrelativistic character of the motions of quarks inside hadrons, and in the problem now considered quarks belonging to different particles are relativistic with respect to each other. This last remark is all the more interesting, because it is well known that a number of predictions of $SU(6)_W$ symmetry relating to the scattering of hadrons are in complete contradiction with experiment.[88,89] We give examples below which demonstrate the degree of violation of $SU(6)_W$ symmetry in

Table XIV. Comparison with Experiment of Values of the Spin Density Matrix in the Quark Model

Reaction	$\rho_{\mu\nu}$	Theory	Experiment	
			8 GeV/c:	4 GeV/c:
	ρ_{33}	0.375	0.22 ±0.06	0,40±0,06
$\pi N \to \pi\Delta$	$\rho_{3,-1}$	0.215	0.132±0.07	0.21±0.08
	ρ_{31}	0	0.066±0.0	−0.03±0.07
			3 GeV/c:	
	ρ_{33}	0.375	.28±0,06	
$K^+\rho \to K^0\Delta^{++}$	$\rho_{3,-1}$	0.215	0.21±0.05	
	ρ_{31}	0	0.04±0.05	
			5.5 GeV/c:	4,5 GeV/c:
	ρ_{33}	0.375	0.30±0.15	0.35±0.09
$K_p^- \to \Sigma_\delta^+\pi^-$	$\rho_{3,-1}$	0.215	0.25±0.06	0.16±0.11
	ρ_{31}	0	0.00±0.15	0.16±0.14

the interactions (at $\vartheta = 0$):

$$\frac{2}{3}\frac{d}{d\Omega}\sigma(K^+ + p \to K^{*+} + p) = \frac{d}{d\Omega}\sigma(K^+ + p \to K^{*0} + \Delta^{++}); \quad (10.1)$$

experimentally, at Q = 1.42 GeV, the left side is equal to (200 ± 70) μb/sr, and at Q = 1.12 GeV the right side is (1720 ± 200) μb/sr;

$$\frac{32}{9}\frac{d}{d\Omega}\sigma(K^- + p \to K^{*-} + p) = \frac{d}{d\Omega}\sigma(K^- + p \to \overline{K}^0 + n); \quad (10.2)$$

at Q = 0.95 GeV the left member of (10.2) is equal to (1408 ± 213) μb/sr, and at Q = 0.97 GeV the right member was found to be (142 ± 20) μb/sr. Of course there is a large uncertainty in the experimental determination of the differential cross section for scattering at zero angle, but this cannot completely alter the situation. Inclusion of breaking of SU(3) symmetry also does not save the situation, since the relations (10.1) and (10.2) follow from the subgroup $SU(6)_W - SU(4)_W$, which does not bear on the breaking of $SU(3)_W$. All of these relations are absent in the quark model. The absence of $SU(6)_W$ symmetry manifests itself formally in the fact that in the amplitude (5.1), (5.2) for the scattering of a quark by a quark (or an antiquark) the functions occurring with the spin matrices are not connected with each other. It is all the more interesting that the predictions of the quark model, given in the preceding chapters, are in much better agreement with experiment.

If we neglect the breaking of SU(3) symmetry, it is obvious that many results of the quark model which connect reactions of the scattering of particles belonging to the same SU(3) multiplet can be derived from the more usual hypothesis that certain nonets of reggeized mesons are dominant in the t channel.[25,75] This remark relates to the "rules" $|\Delta Q| \le 1$, $|\Delta V| \le 1$, $|\Delta T| \le 1$ (cf. Chap. VI), and also to the Johnson-Treiman relations for the total cross sections.[74] If, on the other hand, one takes SU(3) breaking into account it is already hard to relate the predictions of the quark model to other models. In this sense the most interesting results of the quark model are those relating to reactions in which neutral mesons are produced, Eqs. (6.2)–(6.5). The study of these reactions provides a test of values of the mixing parameters as determined from the mass formulas.[70] The relations (6.2)–(6.5) have been compared with experiment in[28], and it was found that they are not in contradiction with experiment.

From an analysis of the density matrix for the reactions $\pi + p \to \rho + p$, $\pi^+ + p \to \pi^0 + \Delta^{++}$, it can be seen that spin effects must be taken into account (the quantity ρ_{00} is large). It was the neglect of the quark spins that led the authors of[81] to the conclusion that the quark model does not agree with experiment. We note without proof that if we assume that the main contribution to the amplitude for scattering of a quark by a quark (or an antiquark) is given by reggeized mesons,[77] agreement with experiment requires the introduction of reggeons with $P_r = -1$, which are not usually used in the analysis of experiments [$P_r = (-1)^j\eta$, where j is the spin and η is the intrinsic parity of the reggeon].

An important feature of the quark model is the possibility of connecting the cross sections for scattering of mesons by nucleons with nucleon-nucleon cross sections (Chaps. VI, VII), which cannot be done with other models. These predictions are not in contradiction with experiment, and the most rigorous of them, $\sigma_{pp}/\sigma_{\pi p} = 3/2$, agrees with experiment. This last relation is especially interesting, since it brings out the very simple quark structure of baryons, showing that a nucleon consists of three quarks.

We have so far in effect assumed that the forces between quarks are mainly pair forces. If, on the other hand, we start from the hypothesis that the stability of the structure of a baryon is a consequence of the dominance of three-particle forces, then in the scattering of hadrons we have as before pair forces acting between quarks and antiquarks, but the forces between quarks are of three-particle type, i.e., the main contribution to the scattering is from diagrams of the type of Fig. 5. In this latter case we can get a relation between the nucleon-nucleon and meson-nucleon cross sections[90,91]

$$\sigma_{\text{tot}}(\pi^+p) + \sigma_{\text{tot}}(\pi^-p) = \frac{1}{3}[\sigma_{\text{tot}}(\bar{p}p) + \sigma_{\text{tot}}(\bar{p}n)] + \frac{1}{6}[\sigma_{\text{tot}}(pp) + \sigma_{\text{tot}}(pn)]. \tag{10.3}$$

If we assume that the dependence of the cross sections on the energy of the incident particle is weak, the two sides of (10.3) can be compared at the same energy, and then (10.3) is satisfied to 5–10 percent accuracy. Inclusion of the kinematics makes the agreement worse, about 20 percent, so that the situation is the same as for (8.5).

We must count among the shortcomings of this approximation its unnaturalness at large energies, for which the cross sections are close to their Pomeranchuk limits. With this sort of model it is hard to understand the similarity of the forward-scattering peaks for the reactions[93]

$$p + p \to p + p, \quad \bar{p} + p \to \bar{p} + p.$$

Accordingly, we have shown that a considerable part of the predictions of the quark model does not follow

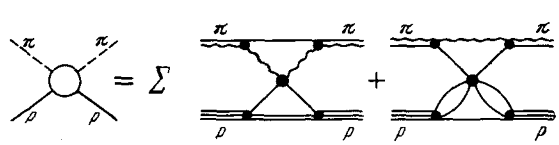

FIG. 5. Scattering of a meson by a baryon in the quark model, if three-particle forces between quarks are dominant. A straight line represents a quark, a wavy line an antiquark.

from the theories known at present, and if the agreement with experiment persists as the experimental data are made more precise—and, most important of all, as the energies of the colliding particles are increased—this will be a serious argument in favor of the quark structure of hadrons.

XI. CONCLUSION

With the appearance of the quark model an unusual situation has arisen in elementary-particle physics. On one hand, the quark model, which is essentially based only on the intuitive picture that mesons consist of two, and baryons of three, independent particles, explains the experiments quite satisfactorily. As can be seen from this review, the agreement with experiment is considerably better than could have been expected. The search for free quarks, however, has been without result so far. On the other hand, a consistent application of the nonrelativistic quark model encounters deep difficulties: it is not understood why heavy free quarks retain their individuality inside a hadron and play the main role in its structure; we do not understand how to explain the antisymmetry of the coordinate part of the wave function of the ground state of a baryon; it is not clear whether the hypotheses which have to be made to obtain the results can be reconciled with each other (for example, the point nature of a quark with the presence of a "meson coating" on it, and the nonrelativistic properties with the large binding energy). If we regard quarks as "quasiparticles" which do not exist outside the hadron, some of these difficulties automatically drop out. At present, however, we do not have even a crude physical idea about the formation of quasiparticles with a fractional charge. In spite of these difficulties, the writers would like to point out that the assumption of quarks as point nonrelativistic quasiparticles is clearly the most economical expression of the hypotheses that must be made to achieve results which agree with experiment. It may be hoped that the situation will become clearer in the next few years through the rapid accumulation of experimental data. If the predictions of the quark model are confirmed as the experiments are improved, the quark model will possibly be the phenomenological basis of the future theory of the strong interactions.

WHAT SHOULD BE MEASURED AND WHY

1. The most important task is experiments on the observation of quarks, using accelerators at higher and higher energies. These will make it possible to set a more accurate lower limit on the mass of the quark. Equally interesting, though less unambiguous, are searches for so-called "residual" quarks, since a raising (sic!) of the upper limit for the number of quarks in the space surrounding us may throw light both on the nature of quarks and on the consistency of cosmological models. The same purpose can be served by a raising of the upper limit for the quark flux in cosmic rays.

2. The observation of new particles and of their spins, parities, and masses would make possible a much more accurate systematics of hadrons, and would perhaps establish the nature of the forces between quarks.

It seems to us that the development, and perhaps also the demolition, of the model will be due to the discovery of new particles. It is especially important to look for multiply charged resonances with large Q, T, and Y.

3. More accurate measurements of the magnetic moments of baryons will allow a test of an important property of the quark model—the additivity of the quark amplitudes. These same remarks also apply to semileptonic weak interactions and radiative decays of vector mesons.

4. A decrease of the error in the measurement of the axial form-factor of the nucleon will allow a test of the fundamental hypothesis that the size of the quarks is much smaller than that of a nucleon. In this case at small momentum transfers $F_A(t)/G_A = C^p_E(t)$; here $G^p_E(t)$ is the Sachs electric form-factor of the proton and $F_A(t)$ is the axial form-factor.

5. Measurement of the differential cross sections for inelastic processes at higher and higher energies would allow independent test of the quark model. The most interesting processes are those of production of the $3/2^+$ decuplet in the scattering of a pseudoscalar meson by a nucleon: $P(0^-) + N \to P(0^-) + N_\delta(3/2^+)$. Here the model completely predicts the density matrix, even for scattering by nonzero angles, and at the angle $0°$ there should be a minimum of the differential cross section.

6. A knowledge of the cross section for production of X^0 at large energies in reactions of the type $\pi^- + p \to X^0 + n$ would allow an independent test of the idea of the grouping of the 0^- mesons into a nonet. An analogous remark is also valid in the case of production of f_0 in the reaction $\pi^- + p \to f_0 + n$, which will make it possible to determine whether the model can be applied to the production of baryon resonances with $L \neq 0$.

7. Measurement of the spin-flip amplitude for reactions of the type of $\pi^- + p \to \pi^0 + n$, $\pi^- + p \to \eta + n$, and so on, will permit a test of the important prediction of the quark model that connects these amplitudes with the scattering cross sections for processes of the type $P(0^-) + N \to P(0^-) + N_\delta(3/2^+)$.

8. Measurement of total cross sections at ever higher energies will permit a test of a fundamental prediction of the quark model: the ratio $\sigma_{pp}/\sigma_{\pi p} = 3/2$ in the Pomeranchuk limit. This relation is important because it fixes the three-quark structure of baryons.

Measurements of differential cross sections for inelastic reactions at angle $0°$ will permit tests of relations connecting the cross sections for scattering of mesons by nucleons and of nucleons by nucleons, and consequently will give us more accurate knowledge about the structure of hadrons.

APPENDIX I

HADRON WAVE FUNCTIONS IN THE QUARK MODEL

We here display the unitary-spin parts of hadron wave functions in the quark model for the lowest multiplets. Their structure is completely determined by giving the quantum numbers: the spin J, its projection J_z along the axis of quantization, the isotopic spin T, its projection T_3, the hypercharge Y, and in addition to these, for baryons, the symmetry of $\Psi_{\alpha\beta\gamma}$ (α, β, γ are quark indices) with respect to permutations of the quarks. For convenience in writing the formulas we have adopted the notations:

1) $Q_{p\uparrow} = 1^+,\quad Q_{p\downarrow} = 1^-,\quad Q_{n\uparrow} = 2^+,\quad Q_{n\downarrow} = 2^-,\quad Q_{\lambda\uparrow} = 3^+,\quad Q_{\lambda\downarrow} = 3^-,\quad \bar{Q}_{p\uparrow} = \bar{1}^+.$

2) $$\Phi(ij,k) = Q_iQ_jQ_k + Q_jQ_iQ_k + Q_jQ_kQ_i + Q_iQ_kQ_j + Q_kQ_jQ_i + Q_kQ_iQ_j,$$
$$\Phi(iik) = Q_iQ_iQ_k + Q_iQ_kQ_i + Q_kQ_iQ_i,\quad \Phi(iii) = Q_iQ_iQ_i,$$
$$|\Phi(ijk)|^2 = 6,\quad |\Phi(iik)|^2 = 3,\quad |\Phi(iii)|^2 = 1.$$

The introduction of the function $\Phi(ij, k)$ considerably simplifies the calculation. This is easy to understand, since any one-particle operator in the unitary-spin space reduces to a permutation of the quarks. For example, the effect of the operator for changing the spin of the particle, $J_+ = \sum_i \sigma_i^+$ (where the index i corresponds to a quark), is trivial: $1^-, 2^-, 3^- \to 1^+, 2^+, 3^+$. Therefore we give here only the hadron wave functions with the maximum value of the projection of the spin along the axis of quantization.

The wave functions of η and X^0 in the nonet 0^- are written on the assumption that φ is a pure singlet with respect to SU(3). Since, however, η, X^0 (ω, φ) have the

1. Baryons (the octet $1/2^+$)

Particle	Mass, MeV	Y	T	T_3	J_z	Wave function
p	938.2	1	1/2	+1/2	+1/2	$\frac{1}{3\sqrt{2}}(2\Phi(1^+, 1^+, 2^-) - \Phi(1^+, 2^+, 1^-))$
n	939.5	1	1/2	−1/2	+1/2	$\frac{(-1)}{3\sqrt{2}}(2\Phi(2^+, 2^+, 1^-) - \Phi(1^+, 2^+, 2^-))$
Σ^+	1189.5	0	1	+1	+1/2	$\frac{1}{3\sqrt{2}}(2\Phi(1^+, 1^+, 3^-) - \Phi(1^+, 3^+, 1^-))$
Σ^0	1192.6	0	1	0	+1/2	$\frac{1}{6}(2\Phi(1^+, 2^+, 3^-) - \Phi(1^+, 3^+, 2^-) - \Phi(2^+3^+1^-))$
Σ^-	1197.5	0	1	−1	+1/2	$\frac{1}{3\sqrt{2}}(2\Phi(2^+2^+3^-) - \Phi(2^+3^+2^-))$
Λ	1115.6	0	0	0	+1/2	$\frac{1}{2\sqrt{3}}(\Phi(1^+, 3^+, 2^-) - \Phi(2^+, 3^+, 1^-))$
Ξ^0	1314.7	−1	1/2	+1/2	+1/2	$\frac{(-1)}{3\sqrt{2}}(2\Phi(3^+, 3^+, 1^-) - \Phi(1^+, 3^+, 3^-))$
Ξ^-	1321.2	−1	1/2	−1/2	+1/2	$\frac{(-1)}{3\sqrt{2}}(2\Phi(3^+, 3^+, 2^-) - \Phi(2^+, 3^+, 3^-))$

2. Baryons (the decuplet $3/2^+$)

Particle	Mass, MeV	Y	T	T_3	J_z	Wave function
Δ^{++}	1236.0	1	3/2	+3/2	+3/2	$\Phi(1^+, 1^+, 1^+)$
Δ^+	1236.0	1	3/2	+1/2	+3/2	$\frac{1}{\sqrt{3}}\Phi(1^+, 1^+, 2^+)$
Δ^0	1236.0	1	3/2	−1/2	+3/2	$\frac{1}{\sqrt{3}}\Phi(1^+, 2^+, 2^+)$
Δ^-	1236	1	3/2	−3/2	+3/2	$\Phi(2^+, 2^+, 2^+)$
Σ_δ^+	1385	0	1	+1	+3/2	$\frac{1}{\sqrt{3}}\Phi(1^+, 1^+, 3^+)$
Σ_δ^0	1385	0	1	0	+3/2	$\frac{1}{\sqrt{6}}\Phi(1^+, 2^+, 3^+)$
Σ_δ^-	1385	0	1	−1	+3/2	$\frac{1}{\sqrt{3}}\Phi(2^+, 2^+, 3^+)$
Ξ_δ^0	1530	−1	1/2	+1/2	+3/2	$\frac{1}{\sqrt{3}}\Phi(1^+, 3^+, 3^+)$
Ξ_δ^-	1530	−1	+1/2	−1/2	+3/2	$\frac{1}{\sqrt{3}}\Phi(2^+, 3^+, 3^+)$
Ω^-	1674	−2	0	0	+3/2	$\Phi(3^+, 3^+, 3^+)$

3. Mesons (the nonet 0^-)

Particle	Mass, MeV	Y	T	T_3	J_z	Wave function
π^+	139.6	0	1	+1	0	$\frac{1}{\sqrt{2}}(1^-\bar{2}^+ - 1^+\bar{2}^-)$
π^0	135	0	1	0	0	$\frac{1}{2}(1^+\bar{1}^- + \bar{2}^+ - 2^+\bar{2}^- - 1^-\bar{1}^+)$
π^-	139.6	0	1	−1	0	$\frac{1}{\sqrt{2}}(2^+\bar{1}^- - 2^-\bar{1}^+)$
K^+	494	1	1/2	+1/2	0	$\frac{1}{\sqrt{2}}(1^+\bar{3}^- - 1^-\bar{3}^+)$
K^0	498	1	1/2	−1/2	0	$\frac{1}{\sqrt{2}}(2^-\bar{3}^+ - 2^+\bar{3}^-)$
K^-	494	−1	1/2	−1/2	0	$\frac{1}{\sqrt{2}}(3^-\bar{1}^+ - 3^+\bar{1}^-)$
$\bar{K}^0$	498	−1	1/2	+1/2	0	$\frac{1}{\sqrt{2}}(3^+\bar{2}^- - 3^-\bar{2}^+)$
η	550	0	0	0	0	$\frac{1}{2\sqrt{3}}((1^+\bar{1}^-) + (2^+\bar{2}^-) - 2(3^+\bar{3}^-) - (1^-\bar{1}^+) - (2^-\bar{2}^+) + 2(3^-\bar{3}^+))$
X^0	960	0	0	0	0	$\frac{1}{\sqrt{6}}(1^+\bar{1}^- + 2^+\bar{2}^- + 3^+\bar{3}^- - 1^-\bar{1}^+ - 2^-\bar{2}^+ - 3^-\bar{3}^+)$

4. Mesons (the nonet 1^-)

Particle	Mass, MeV	Y	T	T_3	J_z	Wave function
ρ^+	760	0	1	+1	+1	$-(1^+\bar{2}^+)$
ρ^0	760	0	1	0	+1	$\frac{1}{\sqrt{2}}(1^+\bar{1}^+ - 2^+\bar{2}^+)$
ρ^-	760	0	1	−1	+1	$2^+\bar{1}^+$
K^{*+}	890	1	1/2	+1/2	+1	$1^+\bar{3}^+$
K^{*0}	890	1	1/2	−1/2	+1	$-2^+\bar{3}^+$
K^{*-}	890	−1	1/2	−1/2	+1	$-3^+\bar{1}^+$
$\bar{K}^{*0}$	890	−1	1/2	+1/2	+1	$3^+\bar{2}^+$
ω	783	0	0	0	+1	$\frac{1}{\sqrt{2}}(1^+\bar{1}^+ + 2^+\bar{2}^+)$
φ	1020	0	0	0	+1	$3^+\bar{3}^+$

Appendix II

Prediction	SU (3)	SU (6), SU (6)$_W$	The quark model		Comparison with experiment
				basic assumptions	
Classification of hadrons					
Existence of nonets of mesons	Yes, but it is not understood why other representations are not realized	Yes	Yes	The structure with the smallest number of quarks is realized	The existence of nonets $0^-, 1^-, 2^+$ is firmly established, and the existence of 0^+ and 1^+ messons is probable
Existence of singlets, octets, and decuplets of baryons	The same	Yes	Yes	The same	The existence of the octet ½$^+$ and the decuplet $3/2^+$ is firmly established
Classification of mesons after the type of L–S coupling. Prediction of parity and G parity	No	No	Yes	It is probably necessary that the quarks be nonrelativistic	Agreement (see Table II)

Appendix II(Continued)

Prediction	SU (3)	SU (6), SU (6)$_W$	The quark model		Comparison with experiment
				basic assumptions	
Classification of Hadrons					
Existence of octet $1/2^+$ and decuplet $3/2^+$ with nearly equal masses	Yes, but it is not understood why the masses are nearly equal	Yes	Yes	Quarks nonrelativistic, unitary-spin part of wave function symmetric	Existence of octet $1/2^+$ and decuplet $3/2^+$ firmly established
Mass formulas					
Connection between masses of particles belonging to the same representation of SU(3)	Yes, if we assume that the breaking of SU(3) transforms like the eighth component of an octet	Yes, if we assume a definite law of transformation of the mass operator under SU(6). The minimal law 35 leads to disagreement with experiment	Yes	The quarks are nonrelativistic	Agrees (see Table III)
Connection between masses of particles belonging to different representations of (SU(3)	No	The same	Yes	The same	Agrees (see Tables III and IV)
Relation between electromagnetic mass differences in octet and decuplet	No	The same	Yes	The same	Not in contradiction (see Table V)
Connection between differences of baryons and of mesons	No	No	Yes	Mass increase of strange quark is same in mesons and baryons	Agrees (see Chap. II)
Electromagnetic properties					
$\frac{\mu_p}{\mu_n} = -\frac{3}{2}$	No	Yes, if the electromagnetic current transforms according to the regular representation of SU(6)$_W$	Yes	Quarks nonrelativistic, unitaryspin wave function of baryon symmetric	Agrees, $\frac{\mu_p}{\mu_n} = -1.46$
$\mu_{\Delta p} = \frac{2\sqrt{2}}{3}\mu_p$	No	The same	Yes	The same	Differs by factor 1.28
$G_n(t) = 0$	No	The same	Yes	The same	Agrees within limits of error
$\frac{G_M^p(t)}{G_M^n(t)} = -\frac{3}{2}$	No	The same	Yes	The same	Agrees (see Table VII)
$\frac{G_{\Delta p}(t)}{\frac{2\sqrt{2}}{3}\mu_p} = G_E^p(t)$	No	The same	Yes	The same	Agrees (see Table VIII)
$G_E^p(t) = \frac{G_M^p(t)}{\mu_p}$	No	No	Yes	Also assumed that $r_q \ll r_{had}$	Agrees up to 3 (GeV/c)^2 (see Table XII)
Relations between widths of radiative decays of vector mesons	Except for connection between decays of φ and ω. The electromagnetic current then transforms with the octet representation of SU(3)	Yes, if we assume that the electromagnetic current transforms according to the representation 35	Yes	Quarks nonrelativistic	Not in contradiction (see Table IX)
$\mu_{\omega\to\pi_0} = \mu_p$	No	No	Yes	The magnetic moments of the quarks are the same in mesons and in baryons	Predicted, $\Gamma(\omega \to \pi_0 + \gamma) = 1.2$ MeV; experimentally, $\Gamma_{exp} = (1.15 \pm 0.25)$ MeV

Appendix II(Continued)

Prediction	$SU(3)$	$SU(6)$, $SU(6)_W$	The quark model		Comparison with experiment
				basic assumptions	
Semileptonic weak interactions					
Selection rules: if $\lvert \Delta S = 0$, then $\Delta T = 1$; if $\lvert \Delta S \rvert = 1$, then $\Delta S = \Delta Q$, $\Delta T = 1/2$	Yes, if the current transforms according to the representation 35 (sic)	Yes, if the current transforms according to the representation 35	Yes	Hadrons consist of three fermions and the decay is due to the decay of one quark	Good agreement[46]
$\frac{F}{D} = \frac{2}{3}$	No	The same	Yes	The same; quarks are nonrelativistic	Not in contradiction (Table X)
$J(\Delta^+_\uparrow \to n_\uparrow)_V = -\frac{2\sqrt{2}}{5} \times J(p_\uparrow \to n_\uparrow)_M$,	No	The same	Yes	The same; quarks are nonrelativistic	Not in contradiction[66]
$J(\Delta^+_\uparrow \to n_\uparrow)_A = -\frac{2\sqrt{2}}{5} \times J_A(p_\uparrow \to n_\uparrow)$	No				
$\frac{F_A(t)}{C_A} = G_E^p(t)$	No	No	Yes	The same; $r_q \ll r_{had}$	Not in contradiction[24]
Scattering of high-energy particles					
Selection rules in t channel of reaction: $\Delta Q \rvert \leqslant 1$, $\lvert \Delta T \rvert \leqslant 1$, $\lvert \Delta V \rvert \leqslant 1$	Yes, if we assume main contribution is from exchange of nonet of reggeized mesons	No	Yes	Quarks nonrelativistic. Hypothesis that single scattering of a quark of one hadron by a quark of the other dominates	Good agreement (see Chap. VI)
$\frac{\sigma(\pi^- p \to R_1 n)}{\sigma(\pi^- p \to R_2 n)} = \left(\frac{1-\sqrt{2}\,\mathrm{tg}\,\vartheta}{1+\sqrt{2}\,\mathrm{tg}\,\vartheta}\right)^2$, where R_1 and R_2 are particles with T = Y = 0, and ϑ is the mixing angle	No	No	Yes	The same	Agrees with the mixing parameter determined from the mass formulas
$\frac{d}{dt}\sigma(\pi^- p_\uparrow \to \pi^0 n_\downarrow) = \frac{25}{8} \times \frac{d\sigma}{dt}(\pi^- p \to \pi^0 \Delta^0)$	No	No	Yes	The same	Disagrees by factor 1.5
Minimum at angle 0° for reactions of the type $\pi p \to \pi\Delta$	No	Yes	Yes	The same	Does not agree with the behavior of the differential cross section for $\pi^- p \to \eta\Delta$ (see Chap VIII)
Spin density matrix for reaction $\pi p \to \pi\Delta$	No	No	Yes	The same	Not in contradiction (see Table XIV)
$\sigma_t(K^+p) - \sigma_t(K^-p) = \sigma_t(\pi^+p) - \sigma_t(\pi^-p) + \sigma_t(K^+n) - \sigma_t(K^-n)$	Yes, if we assume that the reaction goes by the exchange of nonets of vector and tensor mesons	Yes, but together with the following	Yes	The same	See Table XIV
$\frac{1}{2}[\sigma_t(K^+p) - \sigma_t(K^-p)] = \sigma_t(\pi^+p) - \sigma_t(\pi^-p)$	Yes, if there is pure F coupling of the vector mesons with the baryons	Yes, but together with the foregoing	Yes	The same, with conservation of SU(3) in the interaction of quarks	Agrees within the limits of error
$\sigma_t(K^+p) + \sigma_t(K^-p) = \frac{1}{2}[\sigma_t(\pi^+p) + \sigma_t(\pi^-p) + \sigma_t(K^+n) + \sigma_t(K^-n)]$,	Yes, if there is pure F coupling of the vector mesons with the baryons	No	Yes	The same, with conservation of SU(3) in the interaction of quarks	Agreement to 10-12 percent

Appendix II (Continued)

Prediction	$SU(3)$	$SU(6)$, $SU(6)_W$	The quark model		Comparison with experiment
				basic assumption	
Scattering of high-energy particles					
$\frac{\sigma_t(pp)}{\sigma_t(\pi p)} = \frac{3}{2}$ for the Pomeranchuk limits	No	No	Yes	The same, and the scattering amplitudes are equal for quarks in a meson and in a baryon	Experimentally ratio is 1.58 ± 0.05
$\sigma_t(pp) + \sigma_t(\bar{p}p) = \frac{3}{2}[\sigma_t(\pi^+p) + \sigma_t(\pi^-p)] + \frac{1}{2}[\sigma_t(K^+p) + \sigma_t(K^-p)] - \frac{1}{2}[\sigma_t(K^+n) + \sigma_t(K^-n)]$	No	No	Yes	The same	Is satisfied to 20 percent accuracy (see Table XII)
$\frac{d}{dt}\sigma(p\bar{p} \to n\bar{n}) = \frac{d}{dt} \times \sigma(K^-p \to \bar{K}^0 n) + \frac{25}{9}\frac{d}{dt} \times \sigma(K^-p \to \bar{K}^0 n)$ and other equations (see Chap. VIII)	No	No	Yes	The same	Violated. For comments see Chap. VIII

same quantum numbers, when SU(3) is broken these particles undergo reconstruction. In particular, for the nonet 1^- one of the particles, φ, consists only of strange quarks, and the other, ω, of nonstrange quarks; then, obviously,

$$\varphi = Q_\lambda \bar{Q}_\lambda = \frac{\sqrt{3}\,\omega_1 - \sqrt{6}\,\omega_8}{3} = \frac{1}{\sqrt{3}}\,\omega_1 - \sqrt{\frac{2}{3}}\,\omega_8,$$

$$\omega = -\frac{Q_p\bar{Q}_p + Q_n\bar{Q}_n}{\sqrt{2}} = \sqrt{\frac{2}{3}}\,\omega_1 + \frac{1}{\sqrt{3}}\,\omega_8,$$

where

$$\omega_1 = \frac{Q_p\bar{Q}_p + Q_n\bar{Q}_n + Q_\lambda\bar{Q}_\lambda}{\sqrt{3}}, \qquad \omega_8 = \frac{Q_p\bar{Q}_p + Q_n\bar{Q}_n - 2Q_\lambda\bar{Q}_\lambda}{\sqrt{6}}.$$

It can be seen from this that the mixing parameter for the nonet 1^- (see Chapter II) is given by $\mathrm{tg}\,Q_V = 1/2^{1/2}$.

---

[1] M. Gell-Mann, Report CTSL-20, 1961.

[2] Y. Ne'eman, Nuclear Phys. **26**, 222 (1961).

[3] A. H. Rosenfeld et al., Revs. Mod. Phys. **39**, 1 (1967).

[4] E. Wigner, Phys. Rev. **51**, 106 (1937).

[5] F. Gürsey and L. Radicatti, Phys. Rev. Letters **13**, 173 (1964). M. Gell-Mann, Phys. Rev. Letters **14**, 77 (1965). H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 80 (1965). A. Pais, Phys. Rev. Letters **13**, 175 (1964). F. Gürsey, Physics Letters **14**, 330 (1964). F. Gürsey, A. Pais, and L. A. Radicatti, Phys. Rev. Letters **13**, 299 (1964). M. A. B. Beg and A. Pais, Phys. Rev. Letters **13**, 517 (1964). A. Salam, R. Delbourge, et al., Proc. Roy. Soc. **A284**, 146 (1965).

[6] M. A. B. Beg, A. Pais, and B. W. Lee, Phys. Rev. Lett. **14**, 508 (1964).

[7] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).

[8] S. Sakata, Progr. Theor. Phys. **16**, 686 (1956).
[9] L. B. Okun', Zh. Eksp. Teor. Fiz. **34**, 489 (1958) [Sov. Phys.-JETP **7**, 322 (1958)]. M. A. Markov, Giperony i K-mezony (Hyperons and K Mesons), Moscow, Fizmatgiz, 1958.
[10] Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **40**, 319 (1961) [Sov. Phys.-JETP **13**, 216 (1961)].
[11] M. Gell-Mann, Physics Lett. **8**, 214 (1964).
[12] G. Zweig, CERN, Preprint 8419/TH 412 (1964).
[13] V. M. Shekhter, Itogi SU(3) [Consequences of SU(3)], in collection Voprosy fiziki élementarnykh chastits (Topics in Elementary-particle Physics), Erevan, 1966, page 436.
[14] R. Blankenbecler et al., Phys. Rev. Letters **14**, 518 (1965). V. V. Geshkenbeĭn, B. L. Ioffe, M. S. Marinov, and V. N. Roginskiĭ, ZhETF Pis. Red. **1**, No. 6 (1965) [JETP Lett. **1**, 164 (1965)]. H. J. Lipkin, Physics Letters **14**, 336 (1965). O. G. Bokov, Yad. Fiz. **4**, 391 (1966) [Sov. J. Nucl. Phys. **4**, 280 (1967)].
[15] N. N. Bogolyubov, V. V. Struminskiĭ, and A. N. Tavkhelidze, Preprint OIYaI (Joint Inst. Nucl. Res.) D-1968, 1965.
[16] Y. Nambu, Coral Cables Conference, 1965; G. Morpurgo, Physics **2**, 95 (1965).
[17] B. Sakita, Phys. Rev. **137**, B1756 (1964).
[18] Ya. B. Zel'dovich and A. D. Sakharov, Yad. Fiz. **4**, 395 (1966) [Sov. J. Nucl. Phys. **4**, 283 (1967)].
[19] K. J. Barnes et al., Phys. Rev. Letters **14**, 82 (1965).
[20] V. V. Geshkenbeĭn, Yad. Fiz. **3**, 549 (1966) [Sov. J. Nucl. Phys. **3**, 398 (1966)] (This paper gives reference to the author's other papers).
[21] J. K. Walker and R. Wilson, Phys. Rev. **141**, 1267, 1286 (1966).
[22] C. H. Albrait, L. S. Lin and M. Tycia, Phys. Rev. Letters **16**, 921 (1966).
[23] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1965).
[24] G. Bernardini, Twelfth International Conference on High-energy Physics, Dubna, 1964, Vol. 2, page 37.
[25] E. M. Levin and L. L. Frankfurt, ZhETF Pis. Red. **3**, 125 (1965) [JETP Lett. **3**, 79 (1965)].
[26] H. J. Lipkin and F. Scheck, Phys. Rev. Letters **16**, 71 (1966).
[27] R. H. Dalitz, Symmetries and the Strong Interactions, Berkeley, XIII Int. Conf. on High-energy Physics, 1966.
[28] G. Benson et al., Papers submitted to the XIII Int. Conf. on High-energy Physics, 1966. K. Lai and T. Schuman, reference in[27].
[29] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).
[30] P. R. O. Morrison, Invited papers presented at the Conf. on High-energy Two Body Reactions, Stony Brook, April 1965.
[31] L. Van Hove, CERN, Preprint TH 676 (1966).
[32] H. J. Lipkin, Phys. Rev. Letters **16**, 1015 (1966).
[33] P. G. O. Freund, Phys. Rev. Letters **15**, 929 (1965).
[34] W. E. Thirring, Phys. Lett. **16**, 335 (1965). L. D. Soloviev, Phys. Lett. **16**, 345 (1965). S. Badier and C. Bouchiat, Phys. Lett. **15**, 98 (1965). Ya. I. Azimov, V. V. Anisovich, A. A. Ansel'm, G. S. Danilov, and I. T. Dyatlov, ZhETF Pis. Red. **1**, No. 2 (1965) [JETP Lett. **1**, 72 (1965)].
[35] W. Galbraith et al., Phys. Rev. **138**, B913 (1965).
[36] Ya. B. Zel'dovich, L. B. Okun', and S. V. Pikel'ner, Usp. Fiz. Nauk. **87**, 113 (1965) [Sov. Phys.-Usp. **8**, 702 (1966)].
[37] D. E. Dorfan et al., Phys. Rev. Letters **14**, 999 (1965).
[38] V. M. Maksimenko, I. N. Sisakyan, E. L. Feĭnberg, and D. S. Chernavskiĭ, ZhETF Pis. Red. **3**, 340 (1966) [JETP Lett. **3**, 219 (1966)].
[39] E. L. Feĭnberg, ZhETF Pis. Red. **3**, 497 (1966) [JETP Lett. **3**, 324 (1966)].
[40] Arizona, 1966 (Bowen et al.); Brookhaven, 1966 (Adair et al.); Argonne, 1966 (Lamb et al.); CERN, 1966, cited in[27].
[41] Ya. B. Zel'dovich, Usp. Fiz. Nauk **89**, 647 (1966) [Sov. Phys.-Usp. **9**, 602 (1967)].
[42] V. B. Braginskiĭ, Ya. B. Zel'dovich, V. K. Martynov, and V. V. Migulin, Zh. Eksp. Teor. Fiz. **52**, 29 (1967), [Sov. Phys.-JETP **25**, 17 (1967)].
[43] G. Callinaro, G. Morpurgo, Phys. Lett. **23**, 609 (1966).
[44] M. Gell-Mann, Preliminary Talk at XIII Int. Conf. on High-energy Physics, Berkeley, 1966.
[45] E. Borchi and R. Gatto, Phys. Lett. **14**, 352 (1965). Ya. I. Azimov, V. V. Anisovich, A. A. Ansel'm, G. S. Danilov, and I. T. Datlov, ZhETF Pis. Red. **2**, 109 (1965) [JETP Lett. **2**, 68 (1965)].
[46] L. B. Okun', Slaboe vzaimodeĭstvie élementarnykh chastits (The Weak Interaction of Elementary Particles), Moscow, Nauka, 1961.
[47] V. V. Anisovich, ZhETF Pis. Red. **2**, 554 (1965), [JETP Lett. **2**, 344 (1965)]. M. Resnikoff and R. N. Silber, Phys. Rev. **149**, 1245 (1966).
[48] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Moscow, Nauka, 1964 [Addison-Wesley, 1958].
[49] V. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
[50] T. K. Kuo, L. A. Radicatti, Phys. Rev. **B139**, 746 (1964); L. I. Schiff, Phys. Rev. Letters **17**, 612 (1966); O. W. Greenberg and P. Zwanziger, Phys. Rev. **150**, 1177 (1966); G. Morpurgo, Phys. Lett. **20**, 684 (1966).
[51] Z. Maki, Progr. Theor. Phys. **31**, 331 (1964). Y. Hara, Phys. Rev. **134**, B701 (1965).
[52] M. P. Leontovskiĭ, ZhETF Pis. Red. **5**, 448 (1967) [JETP Lett. **5**, 362 (1967)].
[53] H. J. Lipkin et al., Phys. Rev. **146**, 1052 (1966).
[54] References to the original articles are given in[18].
[55] A. D. Dolgov et al., Physics Lett. **15**, 1 (1965).
[56] C. Becchi and G. Morpurgo, Physics Lett. **17**, 352 (1965).
[57] R. H. Dalitz and D. A. Sutherland, Phys. Rev. **146**, 1180 (1966).
[58] N. F. Ramsey, Twelfth Int. Conf. on High-energy Physics, Dubna, 1964, Vol. 1, page 950.
[59] Ya. B. Zeldovich, Usp. Fiz. Nauk **86**, 303 (1965) [Sov. Fiz.-Usp. **8**, 489 (1965)].
[60] E. M. Levin and L. L. Frankfurt, Yad. Fiz. **3**, 1135 (1965) [Sov. J. Nucl. Phys. **3**, 825 (1965)].
[61] S. B. Gerasimov, Zh. Eksp. Teor. Fiz. **50**, 1559 (1966), [Sov. Phys.-JETP **23**, 1040 (1966)].
[62] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).
[63] S. S. Gershteĭn and Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. **29**, 698 (1955), [Sov. Phys.-JETP **2**, 576 (1956)]. R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[64] Ya. B. Zel'dovich, ZhETF Pis. Red. **4**, 381 (1966) [JETP Lett. **4**, 257 (1966)].
[65] L. B. Okun', Unitarnaya simmetriya i slabye

vzaimodeĭstviya (Unitary Symmetry and the Weak Interactions), Moscow, 1965.

[66] N. J. Papastamation and P. G. Sutherland, Phys. Lett. **14**, 246 (1965). C. H. Albright and L. S. Liu, Phys. Rev. Letters **14**, 324, 532 (1965); Phys. Rev. **B140**, 1611 (1965).

[67] L. J. Muzinich, Phys. Lett. **14**, 252 (1965).

[68] R. W. Royen and V. F. Weisskopf, Preprint No. AT (30.1), page 209. Here one can find references to papers in which the quark model has been applied to strong interactions.

[69] I. M. Shushkevich, Dissertation (Physicotechnical Inst., Acad. Sci. of the U.S.S.R.), 1952.

[70] K. G. Sachs, Proc. of the Int. Conf. on High-energy Physics, 1962; F. Ernest et al., Phys. Rev. **119**, 1105 (1960).

[71] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters **17**, 412 (1966).

[72] C. Itzykson and M. Jacob, Saclay Preprint, 1966.

[73] M. Jacob and G. C. Wick, Ann. Physics **26**, 322 (1964).

[74] K. Johnson and S. B. Treiman, Phys. Rev. Letters **14**, 189 (1965).

[75] R. F. Sawyer, Phys. Rev. Letters **14**, 471 (1965).

[76] J. J. J. Kokkede and L. Van Hove, Nuovo Cimento **42**, 711 (1966).

[77] N. Cabibbo, L. Korwitz, and Y. Neeman, Phys. Lett. **22**, 336 (1966).

[78] R. Davis et al., cited in [27].

[79] K. A. Ter-Martirosyan, Preprint ITÉF (Inst. Theor. Exp. Phys.) No. 417, 1966.

[80] V. N. Gribov and I. Pomeranchuk, Phys. Rev. Letters **8**, 343 (1962). V. N. Gribov, V. L. Ioffe, I. Ya. Pomeranchuk, and A. P. Rudik, Zh. Eksp. Teor. Fiz. **42**, 1419 (1962) [Sov. Phys.-JETP **15**, 984 (1962)].

[81] V. Barger and L. Durand, III Phys. Rev. **156**, 1525 (1967).

[82] P. Locher and H. Römer, Phys. Rev. Letters **23**, 496 (1966).

[83] H. J. Lipkin, P. Scheck, and H. Stern, Phys. Rev. **152**, 1375 (1966).

[84] S. Meshuov et al., Phys. Rev. Letters **13**, 212 (1964).

[85] T. T. Wu and C. W. Yang, Phys. Rev. **137**, B708 (1965).

[86] J. L. Friar and J. S. Trefil, CERN, Preprint TH 723 (1966).

[87] J. D. Jackson, Nuovo Cimento **34**, 644 (1964).

[88] J. D. Jackson, Phys. Rev. Letters **15**, 990 (1965).

[89] J. C. Carter et al., Phys. Rev. Letters **15**, 373, 768(E) (1965).

[90] V. V. Anisovich, ZhETF Pis. Red. **2**, 439 (1965) [JETP Lett. **2**, 272 (1965)].

[91] R. J. Oakes, CERN, Preprint TH 748 (1967).

[92] J. J. J. Kokkede, CERN, Preprint TH 745 (1967).

[93] S. L. Lindenbaum, Report on XIII Int. Conf. on High-energy Phys., Berkeley, 1966.

[94] C. Y. Chun et al., Phys. Rev. **152**, 1171 (1967).

[95] V. Barger and M. Olsson, Phys. Rev. **146**, 1080 (1966).

[96] ZhETF Pis. Red. **5**, 407 (1967) [JETP Lett. **5**, 332 (1967)].

[97] Y. E. Mott, Dissertation (Illinois, 1967).

Translated by W. H. Furry