SOVIET PHYSICS USPEKHI VOLUME 11, NUMBER 1 JULY-AUGUST 1968

539.126.33

INTERACTION OF COSMIC MUONS OF HIGH ENERGY

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Usp. Fiz. Nauk 94, 91–125 (January, 1968)

CONTENTS

INTRODUCTION

RESEARCH on cosmic muons has become much more intense in recent years. This is due not only to the intriguing difference between the masses of the muon and electron while their interactions are seemingly identical (this is one of the most interesting problem of elementary-particle physics (see, e.g.,[1])); the study of the properties of cosmic muons can cast light on many characteristics of interactions at very high energies. We can indicate here the following important problems which are solved or have been solved (to a considerable degree) by investigating high-energy cosmic muons:

1. Searches for hypothetical anomalous (i.e., non-electromagnetic and non-weak) muon interactions that increase rapidly with increasing muon energy.

2. Observation of hypothetical particles with intermediate interaction (the interaction constant lies between the constants of the strong and electromagnetic interactions).

3. Clarification of the conditions of K-meson generation at very high energies.

4. Determination of the energy of the fastest pion produced in multiple processes occurring at energies $\gtrsim 10^{12}$ eV.

5. Determination of the cross section of photonuclear processes at energies unattainable with modern accelerators.

Problems 1 and 3–5 are considered in detail in Chap. IV, and therefore we shall not discuss here the methods of solving these problems.

The searches for particles with intermediate interaction, which were initiated in[2], are based on the simple idea—the investigation of the hard component of cosmic radiation at depths at which the nuclear-active components of the cosmic rays is no longer present, but the muon absorption is still insignificant. At such depths, equal to 10–100 m of water equivalent (m.w.e.), the relative interaction of such hypothetical particles would be the most effective.

We note further that for physicists engaged in the study of the properties of elementary particles, the problem of their sources has to a certain degree, secondary significance. Only the characteristics of the fluxes of the elementary particles (energy, intensity, etc.) are important. Our emphasis in the title of the article on the "interaction of cosmic muons" may seem unwarranted. The decisive justification for this is the fact that the subject of this article are muons with energies $\gtrsim 10^{11}$ eV, which have been produced so far only in interactions of cosmic rays. This is precisely why the study of high-energy muons remains confined to them.

At the same time, in order for the exposition to be complete, it is useful to discuss briefly the present status of that part of the problem which is touched upon in the article and is also solved with accelerators. We have in mind here, perhaps, the most "thrilling" question of the anomalous interaction, which regardless of its form should be manifest to some degree or another at small and large muon energies.* We can point to four approaches to the investigation of the anomalous interaction of muons by means of accelerators:

a) Measurement of the magnetic moment of the muon. This method is the best (as we shall show), since the theoretical value of the magnetic moment does not depend on the strong interactions.

The magnetic moment $\bar{\mu}$ of the muon, accurate to quantities $\sim \alpha^2$, is[3]

$$\bar{\mu} = \bar{\mu}_0 \left[1 + \frac{\alpha}{2\pi} + 0.75 \left(\frac{\alpha}{\pi} \right)^2 \right] = 1.0011654\bar{\mu}_0; \tag{I.1}$$

$\bar{\mu}_0$ is the Dirac magnetic moment of the muon. The most accurate experimental data were obtained[4] by measuring the factor (g − 2)†:

$$\bar{\mu} = (1.001162 \pm 5 \cdot 10^{-6}) \bar{\mu}_0. \tag{I.2}$$

The possible anomalies lie in the error incurred when measuring the magnetic moment; the contribution of the electromagnetic interactions to the magnetic moment is $\delta\bar{\mu} \lesssim 5 \times 10^{-6} \bar{\mu}_0$. This contribution can be treated from the following points of view (see, e.g.,[5]): Phenomenologically, the possible anomaly can be interpreted as violation of quantum electrodynamics at sufficiently large momentum transfers Λ or small distances $l \sim 1/\Lambda$.‡ The existence of the quantities Λ and l can also be interpreted as manifestations of the structure of the muon.

It can be shown that

$$\Lambda \sim \mu \sqrt{\frac{\alpha}{3\pi} \frac{\bar{\mu}_0}{|\delta\bar{\mu}|}}, \tag{I.3}$$

where μ is the muon mass. Such an anomaly can also be interpreted as a manifestation of the concrete additional interaction of the muon. In particular, if it is assumed that it results from exchange of a hypothetical vector meson with mass m_χ [6,7], then the following inequality holds:

$$\frac{f^2}{3\pi} \left(\frac{\mu}{m_\chi} \right)^2 \leqslant \frac{|\delta\bar{\mu}|}{\bar{\mu}_0}; \tag{I.4}$$

f is the constant of the coupling with the hypothetical field. For comparison with the weak interaction, it is convenient to introduce the dimensional constant

$$F = \frac{f^2}{m_\chi^2} \sim \frac{f^2}{\Lambda^2}.$$

It follows from (I.1)—(I.4) that $\Lambda \sim$ 2—3 GeV and $l \sim (2-3) \times 10^{-15}$ cm. The constant $F \lesssim 10^{-4}/\mu^2$, i.e., the upper limit of the constant F, obtained on the basis of (I.4), is larger by approximately three orders of magnitude than the constants of the weak interaction $g \sim 2 \times 10^{-7}/\mu^2$.**

b) Another method of analyzing muon interactions under conditions when there is no strong interaction is to study μ-e scattering. A difficulty of this method is the unpleasant decreasing dependence of the δ-electron production cross section σ_δ on the square of the 4-momentum transfer[8]:

$$\sigma_\delta \infty (q^2)^{-1/2}. \tag{I.5}$$

In addition, it is necessary to know with sufficient accuracy the spectrum of the incident muons, which is not a simple task (particularly in cosmic rays; see Chap. III).

Therefore, in spite of the fact that the possibility of investigating μ-e scattering has been pointed out long ago (see, e.g.,[8]), so far only one investigation of μ-e scattering has been performed with an accelerator, at an energy $E_\mu \sim 8$ GeV[9]. It turned out that, up to distances on the order of 7×10^{-14} cm, there is good agreement between experiment and calculations on the basis of quantum electrodynamics.

c) The production of muon pairs from nucleons, from the point of view of the question of the muon structure in which we are interested, has that shortcoming that nucleons, which certainly have a structure, take part in the reaction. This circumstance is particularly pronounced in the most interesting region—at high momenta transferred to the nucleon pairs. To decrease the influence of the nucleus, one usually selects cases with small momentum transfer to the nucleus, which naturally decreases the statistical accuracy of the experiments. Recent investigations of the production of muon pairs yielded $l \lesssim 2 \times 10^{-14}$ cm.[10]

d) In the study of elastic scattering of muons by nucleons, there are no ways of avoiding the influence of nucleon form factors. Therefore, strictly speaking, such experiments are intended for the investigation of nuclear structure rather than muon structure. It is possible, however, by comparing the curves of scattering of muons and electrons by nucleons, to attempt to determine the possible difference of the structures of the electron and the muon.* It must be emphasized that experiments on muon scattering are complicated, owing to the strong dependence of the scattering cross section on the muon momenta and owing to the need for determining quite accurately small scattering angles (in the presence of a strong background).

The most interesting experiments on the scattering of muons by protons were recently performed by the group of L. M. Lederman[11]. It can be concluded from these experiments that

$$\left[\left(\frac{1}{\Lambda} \right)^2 - \left(\frac{1}{\Lambda_e} \right)^2 \right]^{-1} \geqslant 2\,(\text{GeV})^2. \tag{I.6}$$

Using the previously obtained estimates of Λ, we can get $\Lambda_e \gtrsim 1$ GeV and $l_e = 3 \times 10^{-14}$ cm, where Λ_e and l_e are the momentum and the distance at which the electron structure plays no role. Thus, accelerator studies of processes in which muons take part at relatively large momentum transfers have led to the important (but to a certain degree negative) result that there are no additional interactions or "structure" at not very high muon energies (~ 3—5 GeV) or momenta Λ (~ 2—3 GeV).

In conclusion, a few words on the plan of the article.

*For a detailed exposition of the situation up to 1962 - 1963 at low energies see [1].

†Somewhat more accurate data were obtained in a recent paper by the same group [4] (F. Farley et al., Nuovo Cimento 45, 281 (1966)): $\bar{\mu} = (1.001165 \pm 3 \times 10^{-6})\bar{\mu}_0$ (Note added in proof).

‡Here and throughout $\hbar = c = 1$.

**A more accurate limiting value of the constant F is obtained on the basis of neutrino experiments (see Chap. IV, Sec. 1).

*Assuming (owing to the absence of deviations in the value of the magnetic moment) that the muon is a pointlike particle.

In Chapter I we analyze the accuracy of the calculations of the fast-muon interaction cross section. Such an analysis is necessary for quantitative comparisons with experiment.

In Chapter II we consider the passage of muons through large thicknesses of matter. Particular attention is paid to the calculation of the fluctuations of the mean free paths, which is necessary for an analysis of experiments at large depths.

In Chapter III we present the main results of the experimentally investigated cosmic muons. It is useful to note that we have chosen here characteristics that have been established quantitatively with relative reliability.

In Chapter IV we analyze the physical results obtained in experiments with cosmic muons. This chapter is of greatest interest to physicists who are not concerned with special problems.

We note, finally, that much attention is paid in the article to quantitative characteristics of the interaction of high-energy muons. The need for such an approach is dictated by the following causes: a) the increasing accuracy of the experimental research on cosmic muons, b) the need for rigorously justifying the conclusions drawn in the article, c) the author's desire to refute the frequently held opinion that it is impossible to obtain quantitative estimates of the characteristics of the interaction of elementary particles by cosmic-ray investigations.

We emphasize in conclusion that the questions touched upon in the article do not exhaust the entire problem of cosmic muons, and reflect more readily a subjective point of view on the importance of various trends in research.

I. FAST-MUON INTERACTION CROSS SECTION

1. Bremsstrahlung

The cross section for bremsstrahlung of relativistic muons on a stationary nucleus was calculated in the first Born approximation on the basis of the Bethe-Heitler method in a number of investigations[2-4]. In the most convenient form, which combines the entire range of emitted photons, the cross section is given in article[14]. However, in the approximation employed in[14] use was made of an obsolete value of the nuclear size. Inclusion of modern data concerning the radii of nuclei[15] leads to the following expression for the cross section:

$$\sigma_r(E_{0\mu}, v)\,dv = \alpha\left(2Zr_0\frac{m}{\mu}\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\frac{dv}{v}\ln\left(\frac{\frac{3}{2}k\frac{\mu}{m}Z^{-2/3}}{\frac{k\sqrt{e}}{2}\frac{\mu^2}{mE_{0\mu}}\frac{v}{1-v}Z^{-1/3}+1}\right), \tag{1.1}$$

where $v = E_\gamma/E_{0\mu}$, $E_{0\mu}$ and E_γ are the initial energy of the muon and the photon energy, and μ and m are the masses of the muon and the electron; $r_0 = e^2/m$, Z is the charge of the target, and $k \sim 190$ (constant). If

$$\gamma \sim 100\frac{\mu^2}{mE_{0\mu}}\frac{v}{1-v}Z^{-1/3} \gg 1$$

(absence of screening), then

$$\sigma_r(E_{0\mu}, v)\,dv = \alpha\left(2Zr_0\frac{m}{\mu}\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\frac{\ln\left(\frac{3}{\sqrt{e}}\frac{100}{\gamma}\frac{\mu}{m}Z^{-2/3}\right)}{v}\,dv. \tag{1.2}$$

If $\gamma \ll 1$ (total screening), then

$$\sigma_r(E_{0\mu}, v) = \alpha\left(2Zr_0\frac{m}{\mu}\right)^2\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)\ln\left(\frac{3}{2}\frac{\mu}{m}KZ^{-2/3}\right). \tag{1.3}$$

This raises the question of the accuracy of formulas (1.1)–(1.3) (see[16]). Expression (1.1) approximates the cross section calculated under definite physical assumptions, with accuracy up to 2%. The following were neglected in the calculation:

1) photon production in the field of the electrons of the atomic shell;
2) radiative corrections;
3) influence of the medium;
4) recoil.

In addition, diagrams in which virtual hadrons take part were taken into account approximately, by introducing a finite dimension of the nucleus.

5) The calculations were performed in the first Born approximation. The greatest correction to the emission cross section is due to the interaction of the muons with the electrons. This process was considered in[16-18]. The cross section σ_{er} for the interaction of the muon with the electrons can be approximately represented in the form of a sum of two quantities[17]*:

$$\sigma_{er} = \sigma_{1er} + \sigma_{2er} = 4\alpha r_0^2\left(\frac{m}{\mu}\right)^2\frac{\left(\frac{4}{3}-\frac{4}{3}v+v^2\right)}{v}\ln\frac{2E_{0\mu}(1-v)}{\mu v} + \frac{8}{3}\alpha r_0^2\frac{m}{E_\gamma}\frac{\ln\frac{m}{\mu v}}{v}, \tag{1.4}$$

$\sigma_{2er} \sim (\mu^2/mE_\gamma)\sigma_{1er}$; therefore at energies $E_{0\mu} \ll \mu^2/m$ the principal role is played by σ_{2er}; if $E_{0\mu} \gg \mu^2/m$, then the main contribution is made by the term σ_{1er}, which can be readily shown to coincide, accurate to an insignificant logarithmic factor (which reflects the difference in the radii of the nucleus and of the electron), with the cross section for the emission of a muon on a stationary charge Z = 1[16] (formula (1.2)). This can be readily understood, since the effective momentum transfer decreases with increasing $E_{0\mu}$, and therefore the influence of the electron recoil is negligibly small at sufficiently high energies. Then at energies $E_{0\mu} \gg \mu^2/m$ the total cross section σ_{rt} can be represented in the form

$$\sigma_{rt} = \alpha\left(2r_e\frac{m}{\mu}\right)^2 Z\left(\frac{4}{3}-\frac{4}{3}v+v^2\right) \times \frac{1}{v}\left\{Z\ln\left(\frac{\frac{3}{2}k\frac{\mu}{m}Z^{-2/3}}{\frac{k\sqrt{e}}{2}\frac{\mu^2}{mE_{0\mu}}\frac{v}{1-v}Z^{-1/3}+1}\right)+\ln\frac{2E_{0\mu}(1-v)}{\mu v}\right\}. \tag{1.5}$$

Formula (1.5) does not take into account the interference of the radiation from the electrons of the atomic shell and their mutual screening.

These factors were taken into account for the radiation from the electrons in the case of total screening[19-20].

When the relations obtained in the cited papers are applied to muon radiation, it is necessary to set the second term in the curly brackets equal to $1.2\ln[(3/2)k\mu Z^{-2/3}/m]$.

The most obscure point in the estimate of the accuracy of the presented formulas is the estimate of the radiative corrections. The contribution made to the cross section by the radiative corrections[21], for the case when the interaction of the atoms of the medium can be neglected, is

*An accurate expression for the cross section is contained in [8], but approximation (1.4) suffices for what follows.

$$-\Delta\sigma_r \sim 0.7\,\frac{\alpha}{\pi}\ln\frac{E_\gamma}{\Delta E}\,\sigma_r, \tag{1.6}$$

where ΔE is a certain minimum energy characteristic of the measuring instrument.

Unfortunately, we are interested in the cross section of the process in matter (i.e., when there is no measuring instrument and the density of the matter is sufficiently high). Then, recognizing that ΔE is under the logarithm sign (and therefore its exact value is not very important), we can attempt to estimate it by using dimensionality considerations[16]. The simplest expression having the dimension of energy and containing the atomic constants and the constants of matter is of the form

$$\Delta E \sim \frac{NZ^2}{A} e^2 \rho R_F^2 \sim 10 \text{ eV} \tag{1.7}$$

N is Avogadro's number, e is the electron charge, ρ the density, and R_F the radius of the atom after Thomas-Fermi. Putting $E_{0\mu} = 10^{12}$ eV and $E_\gamma = \bar{E}_\gamma \sim E_{0\mu}/\ln(E_{0\mu}/\mu) \sim 10^{11}$ eV, we get $\Delta\sigma_r \sim 0.05\sigma_r$.

The influence of the medium on the bremsstrahlung cross section can be readily estimated[16] on the basis of the simplified expression for the material factor, obtained by I. I. Gurevich[22]. It turns out that in the region $E_{0\mu} \sim 10^{12}$ eV the medium decreases the cross section of the bremsstrahlung by 10^{-3}–10^{-4}%.

The effective recoil momentum is

$$q_{\text{eff}} \sim \frac{\mu^2 E_\gamma}{E_{0\mu}(E_{0\mu}-E_\gamma)} \sim \frac{\mu^2 E_\gamma}{E_{0\mu}^2}. \tag{1.8}$$

Putting $E_\gamma = \bar{E}_\gamma$ and assuming that the recoil effect is insignificant when $q_{\text{eff}} \ll M$ (M—nucleon mass), we obtain the condition $\mu^2/10E_{0\mu} \ll M$ or $E_{0\mu} \gg 1$. Of course, such a weak condition is not sufficiently well founded, if for no other reason than that (1.8) has been obtained in the ultrarelativistic approximation. But for relativistic energies ($E_{0\mu} \gg \mu$) and not too large values of q, the influence of the recoil should be small. To estimate the deviation from the first Born approximation we recall that it is due to expansion in terms of two parameters: $\alpha Z/v$ and $\alpha Z/v'$ (v and v′ are the muon velocities before and after collision). In the relativistic case, the correction[23] connected with the finite nature of the first parameter, is on the order of $0.1(Z/82)^2\sigma_r$. The influence of the second parameter comes into play if $1 - v' \sim 1$. Then the correction is equal to[24]

$$\frac{\mu E_\gamma Z^{1/3}}{E_\mu(E_\mu - E_\gamma)}\sigma_r.$$

For medium and light elements ($Z \sim 10$–30), in which we shall be subsequently interested, both corrections are very small (~0.1–1% of σ_r).

Thus, taking all these remarks into account, the total cross section σ_{rt} can be represented at sufficiently high energies ($E_{0\mu} \gtrsim 10^{11}$ eV) in the form

$$\sigma_{rt} = 0.95\alpha\left(2r_0\frac{m}{\mu}\right)^2 Z(Z+\xi)\left(\frac{4}{3}+v^2-\frac{4}{3}v\right)\frac{1}{v} \times \ln\left(\frac{\frac{3}{2}k\frac{\mu}{m}Z^{-2/3}}{\frac{k\sqrt{e}}{2}\frac{\mu^2}{mE_{0\mu}}\frac{v}{1-v}Z^{-1/3}+1}\right). \tag{1.9}$$

The greatest uncertainty is due to the magnitude of the radiative corrections. Assuming that this uncertainty hardly exceeds the value of $|\Delta\sigma_r|$ itself, we can say that the accuracy of (1.9) is of the order of ~5%.

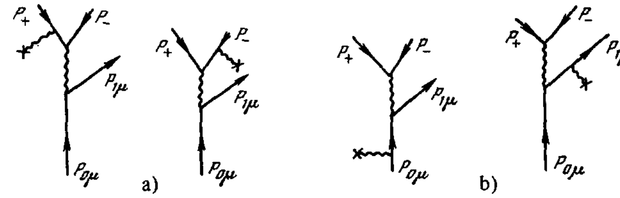

FIG. 1

2. Direct Pair Production

Direct pair production by muons is described by the diagrams shown in Fig. 1. An exact calculation of the cross section is a rather complicated problem, owing to the need for integrating the differential cross section with respect to the angles. A calculation with logarithmic accuracy is presented in[25], but certain inconsistencies crept into the integration with respect to the angles. A more consistent calculation in the logarithmic approximation was performed by F. F. Ternovskiĭ[26]. The most accurate calculation was made recently by S. R. Kel'ner[27]*. We present the results of the calculations of the cross section for direct pair production accurate to quantities $(m/\epsilon_p)^2$ and (μ/E_0)[27]:

$$\sigma_{p_3} = \sigma_{p_3}^a + \sigma_{p_3}^b, \tag{1.10}$$

$$\sigma_{p_3}^{a,b} = \frac{2}{\pi}(Zr_e\alpha)^2\frac{\Phi_{a,b}}{\varepsilon_p}\frac{E_{0\mu}}{E_{1\mu}} \tag{1.11}$$

($E_{1\mu}$ is the muon energy after collision).

$$\Phi_a^n = 2\left[\ln\frac{\varepsilon_p(1-v_p^2)}{2m\sqrt{1+x}}-\frac{1}{2}\right]\left[a_1\ln\left(1+\frac{1}{x}\right)-b_1-\frac{c_1}{1+x}\right]-a_1 f\left(\frac{1}{1+x}\right) + b_1 x\ln\left(1+\frac{1}{x}\right)+\frac{c_1}{1+x},$$

$$a_1 = \frac{1}{3}(2+v_p)^2\left(1+\frac{\varepsilon_p^2}{2E_{0\mu}E_{1\mu}}\right)+\frac{x}{3}(3+v_p^2),$$

$$b_1 = \frac{1}{3}(2+v_p^2), \qquad c_1 = \frac{1}{3}(x+v_p^2)+\frac{1}{6}\frac{\varepsilon_p^2}{E_{0\mu}E_{1\mu}},$$

$$v_p = \frac{\varepsilon_+ - \varepsilon_-}{\varepsilon_p}, \qquad \varepsilon_p = \varepsilon_+ + \varepsilon_-, \qquad x = \left(\frac{\mu}{m}\right)^2\frac{\varepsilon_p^2}{E_{0\mu}E_{1\mu}}\frac{(1-v_p^2)}{4}. \tag{1.12}$$

$$f(z) = -\int_0^z \frac{\ln|1-x|}{x}dx \text{—Spence function}^\dagger.$$

The condition for absence of screening is

$$\frac{\varepsilon_p}{\varepsilon_+\varepsilon_-}\left(m^2+\mu^2\frac{\varepsilon_+\varepsilon_-}{E_{0\mu}E_{1\mu}}\right) \gg Z^{1/3}m\alpha.$$

$$\Phi_a^s = 2\ln\left(kZ^{-1/3}\sqrt{1+x}\right)\left[a_1\ln\left(1+\frac{1}{x}\right)-b_1-\frac{c_1}{1+x}\right] + a_1 f\left(\frac{1}{1+x}\right)-d_1\ln\left(1+\frac{1}{x}\right)-\frac{2}{3}\frac{c_1}{1+x}+\frac{1}{18}(1-v_p^2);$$

$$d_1 = b_1 x + \frac{1}{18}(1-v_p^2)\left(\frac{E_{0\mu}}{E_{1\mu}}+\frac{E_{1\mu}}{E_{0\mu}}\right)+\frac{1}{18}x(3-v_p^2). \tag{1.13}$$

The condition of total screening is

$$\frac{\varepsilon_p}{\varepsilon_+\varepsilon_-}\left(m^2+\mu^2\frac{\varepsilon_+\varepsilon_-}{E_{0\mu}E_{1\mu}}\right) \ll Z^{1/3}m\alpha.$$

$$\Phi_b^{(n)} = \left\{\left[\ln\left(\frac{E_{0\mu}E_{1\mu}(1-v_p^2)}{(1+x)m^2}\right)-1\right]\left[a_2\ln(1+x)+\frac{b_2}{4}(1-v_p^2)+c_2\frac{x}{1+x}\right] - a_2 f\left(\frac{x}{1+x}\right)-b_2\frac{1-v_p^2}{4x}\ln(1+x)-c_2\frac{x}{1+x}\right\}\left(\frac{m}{\mu}\right)^2, \tag{1.14}$$

*When this article was written, formulas (1.10) and (1.11) were still unpublished. The author is grateful to S. R. Kel'ner for communicating his results prior to publication.

†The main properties of the Spence function are:

$$f(x)+f(1-x) = \frac{\pi^2}{6} - \ln x \cdot \ln|1-x|,$$

$$f(1) = -2f(-1) = \frac{2}{3}f(2) = 2f\left(\frac{1}{2}\right)+(\ln 2)^2 = \frac{\pi^2}{6}.$$

where

$$a_2=\left[\frac{E_{0\mu}}{E_{1\mu}}+\frac{E_{1\mu}}{E_{0\mu}}-\frac{2}{3}\right]\left[\frac{1+v_p^2}{4}-\frac{1-v_p^2}{4x}\right]-\frac{1}{3}\left(\frac{m}{\mu}\right)^2,$$

$$b_2=\frac{E_{0\mu}}{E_{1\mu}}+\frac{E_{1\mu}}{E_{0\mu}}-\frac{2}{3},\quad c_2=\frac{1-v_p^2}{3}-\frac{1}{12}\frac{\varepsilon_p^2}{E_{0\mu}E_{1\mu}}(1+v_p^2)+\frac{1}{3}\left(\frac{m}{\mu}\right)^2 \quad (1.15)$$

In this case the condition for the absence of screening is

$$\frac{E_{0\mu}E_{1\mu}m}{\mu^2\varepsilon_p}\ll\frac{1}{\alpha Z^{1/3}}\left(1+\frac{1}{x}\right).$$

$$\Phi_b^{(s)}=\Big\{2\ln\Big(\frac{\mu}{m}kZ^{-1/3}\sqrt{1+(1/x)}\Big)\Big[a_2\ln(1+x)+\frac{b_2(1-v_p^2)}{4}+c_2\frac{x}{1+x}\Big]$$
$$+a_2 f\left(\frac{x}{1+x}\right)+d_2\ln(1+x)+\frac{2}{3}c_2\frac{x}{1+x}+\frac{1}{18}(1-v_p^2)\Big\}\left(\frac{m}{\mu}\right)^2,$$
$$d_2=b_2\frac{(1-v_p^2)}{4}-\frac{2}{9}\left(\frac{1-v_p^2}{4x}-\frac{1+v_p^2}{4}\right)+\frac{1}{9}\left(\frac{m}{\mu}\right)^2. \quad (1.16)$$

The condition of total screening is

$$\frac{E_{0\mu}E_{1\mu}\cdot m}{\mu^2\varepsilon_p}\gg\frac{1}{Z^{1/3}\alpha}\left(1+\frac{1}{x}\right).$$

The ratios of $\sigma_{p_3}^a$ and $\sigma_{p_3}^b$ are

$$\frac{\sigma_{p_3}^b}{\sigma_{p_3}^a}\sim\left(\frac{\varepsilon_p}{E_{1\mu}}\right)^2\frac{1}{\ln\left(\frac{\varepsilon_p}{m}\right)},\quad m\ll\varepsilon_p\ll m\left(\frac{E_{1\mu}}{\mu}\right), \quad (1.17)$$

$$\frac{\sigma_{p_3}^b}{\sigma_{p_3}^a}\sim\left(\frac{\varepsilon_p}{E_{1\mu}}\right)^2\ln\left(\frac{\varepsilon_p}{m}\right),\quad m\left(\frac{E_{1\mu}}{\mu}\right)\ll\varepsilon_p\ll E_{1\mu}. \quad (1.18)$$

This raises further the question of the role of different corrections to the cross section (1.10). Naturally, owing to the complexity of the calculation of direct pair production, corrections similar to those estimated for bremsstrahlung have not yet been considered. However, for the main corrections (the influence of the orbital electrons and radiative corrections), their order of magnitude should coincide with the order of magnitude of the corrections for bremsstrahlung. Therefore the final expression for the cross section has the following form:

$$\sigma_p^{a,b}=2\frac{0{,}95}{\pi}Z(Z+\xi)(r_e\alpha)^2\frac{\Phi_{a,b}}{\varepsilon_p}\frac{E_{0\mu}}{E_{1\mu}}. \quad (1.19)$$

3. Electromagnetic-nuclear Reactions Induced by Muons

The formation of nuclear-active particles in muon-nucleon interactions is described by the diagram shown in Fig. 2. Unfortunately, the right side of this diagram, which contains the nucleon form factor, cannot be calculated by present-day methods. Therefore, the calculation of the cross section of such processes reduces to an estimate of the two diagrams of Fig. 3, where the "experimental" value of the cross section for pion production by real photons is used in the calculation of the diagram 3b. Such a crude approach to the solution of the problem makes, to a certain sense, the ever increasing accuracy of the calculations of the cross section of the "nuclear" muon interaction illusory. More readily, the calculations performed in recent years[28-30] confirmed the validity of the Weizsacker-Williams approximation, used back in the first paper by George[33]*, where the muon-nuclear cross section was calculated. Fortunately, however, two circumstances make such a crude approach justified. We are henceforth interested primarily in the energy lost by fast muons in matter. It turns out that: 1) this quantity depends little on the different assumptions on which the model for calculating

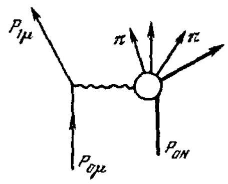

FIG. 2.

the muon-nuclear cross section is based, and 2) the loss to muon-nuclear processes in ground is itself less than 10% of the total loss (see Chapter II).

In general form, in the lab system, the differential cross section of the muon-nucleon process can be represented by[29]

$$\frac{d^2\sigma_{\mu N}}{dq^2\,d\varepsilon_\pi}=\frac{\alpha}{8\pi^2}\frac{1}{(E_{0\mu}^2-\mu^2)}\frac{1}{q^4}\left\{L^{(1)}(E_{0\mu}^2+E_{1\mu}^2)q^2-2\mu^2\varepsilon_\pi^2-\frac{q^4}{2}\right\}$$
$$+L^{(2)}(2\mu^2-q^2)q^2\}, \quad (1.20)$$

where $L^{(1)}$ and $L^{(2)}$ are certain arbitrary functions of the invariants characterizing the muon-nucleon interaction, ϵ_π is the summary energy of the nuclear-active particles, and q^2 is the square of the 4-momentum of the virtual photon (see Fig. 3).

The functions $L^{(1)}$ and $L^{(2)}$ take into account the structure of the nucleon, but, generally speaking, they need not coincide with the nucleon form factors obtained by studying the scattering of electrons by nucleons. The cross section of the process represented by the diagram of Fig. 3b is

$$\sigma_{\gamma N}=\frac{1}{4\pi\varepsilon_\pi}[L^{(1)}\varepsilon_\pi^2-L^{(2)}q^2]_{q^2\to 0}. \quad (1.21)$$

Assuming that $L^{(2)}\sim L^{(1)}$ and that the function $L^{(1)}$ has no singularities as $q^2\to 0$, we can rewrite (1.21) in the form

$$\sigma_{\gamma N}=\frac{L^{(1)}\varepsilon_\pi}{4\pi}\Big|_{q^2\to 0} \quad (1.22)$$

and neglect the term proportional to $L^{(2)}$ in (1.20). In this case it is easy to establish the connection between $\sigma_{\mu N}$ and $\sigma_{\gamma N}$. Let us establish the connection between formula (1.20) and the cross sections obtained by the Weizsacker-Williams[33] method and by the Kesslers[28], who used the pole approximation. To this end we integrate (1.20) (using $L^{(1)}$ as given by (1.22) and $L^{(2)}=0$) within the kinematic limits of the variation of q^2. In the ultrarelativistic case ($\epsilon_p\gg\mu$)

$$q_{min}^2\sim\frac{\mu^2\varepsilon_\pi^2}{E_{0\mu}E_{1\mu}},$$
$$q_{max}^2\sim 2M\varepsilon_\pi. \quad (1.23)$$

We put

$$L_1^{(1)}=\frac{4\pi\sigma_{\gamma N}}{\varepsilon_\pi}\left(\frac{\lambda^2}{q^2+\lambda^2}\right)^2,\quad \lambda=\mathrm{const}, \quad (1.24)$$

$$L_2^{(1)}=\frac{4\pi\sigma_{\gamma N}}{\varepsilon_\pi} \quad (1.25)$$

and integrate (1.20) with respect to q^2 within the limits from q_{min}^2 to q_{max}^2. In subsequent estimates, with $L=L_1^{(1)}$, we shall put $\lambda\sim 1$ GeV, corresponding approximately to the nucleon radius (specifically, $\lambda^2=$ $=0.365\ \mathrm{GeV}^2$) and $\epsilon_\pi\gg\lambda$, $\epsilon_\pi\ll E_{0\mu}$. Then

$$\frac{d\sigma_{\mu N}}{d\varepsilon_\pi}\sim\frac{2\alpha}{\pi\varepsilon_\pi}\sigma_{\gamma N}\ln\left(\frac{\lambda E_{0\mu}}{\mu\varepsilon_\pi}\right). \quad (1.26)$$

This expression (apart from an insignificant factor under

*The modern development of the Weizsacker-Williams method and its application to many problems (see, e.g., [31,32]).

the logarithm sign) coincides with the expression obtained by the Weizsacker-Williams method:

$$\frac{d\sigma_{\mu N}}{d\varepsilon_\pi} = \frac{2\alpha}{\pi\varepsilon_\pi}\sigma_{\gamma N}\ln\frac{E_{0\mu}}{\varepsilon_\pi}. \tag{1.27}$$

If we choose $L^{(1)} = L_2^{(1)}$, then

$$\frac{d\sigma_{\mu N}}{d\varepsilon_\pi} \sim \frac{\alpha}{\pi}\frac{\sigma_{\gamma N}}{\varepsilon_\pi}\ln\left(\frac{ME_{0\mu}^3}{\mu^2\varepsilon_\pi^2}\right). \tag{1.28}$$

If

$$\left(\frac{E_{0\mu}}{\mu}\right)^2 \gg \frac{E_{0\mu}M}{\varepsilon_\pi^2}$$

(this condition follows essentially from $\epsilon_\pi \gg \mu$), then

$$\frac{d\sigma_{\mu N}}{d\varepsilon_\pi} \sim \frac{2\alpha}{\pi}\frac{\sigma_{\gamma N}}{\varepsilon_\pi}\ln\frac{E_{0\mu}}{\mu}. \tag{1.29}$$

In the method indicated by a Japanese group[29], the uncertainty in the calculations reduces to the uncertainty in the choice of the function L. In[28,33], the results do not contain any arbitrary functions, but the possible source of error is essentially the same. For the analysis, we turn to Fig. 3. We have here two sources of errors: 1) neglect of the change of the state of the muon (the Weizsacker-Williams method) and 2) the replacement of $\sigma_{\gamma N}$ ($q^2 \neq 0$) by $\sigma_{\gamma N}$ ($q^2 = 0$). To estimate the error resulting from this replacement, we use the following reasoning. The cross section of the process shown in the diagram of Fig. 3b can depend, in particular, on the following invariants: q^2, qp_1, qp_2, ..., qp_n, where p_i is the 4-momentum of the i-th particle*. We assume further that the cross section $\sigma_{\gamma N}$ is a continuous function of only these invariants, and the true value of $\sigma_{\gamma N}$ corresponds to $q^2 = 0$. Then $\sigma_{\gamma N}$ ($q^2 \neq 0$) will practically not differ from $\sigma_{\gamma N}$ ($q^2 = 0$), if all the qp_i ($q^2 \neq 0$) differ little from qp_i ($q^2 = 0$).

More concretely,

$$qp_i(q^2 \neq 0) - qp_i(q^2 = 0) \ll qp_i.$$

This condition is equivalent to the condition

$$q^2 \ll m_i^2, \tag{1.30}$$

which leads in turn to the condition

$$\varepsilon_\pi \ll \frac{m_i}{\mu}E_{0\mu}. \tag{1.31}$$

Since $m_\pi \sim \mu$, this is practically equivalent to

$$\varepsilon_\pi \ll E_{0\mu}. \tag{1.32}$$

However, this condition is precisely the condition for the applicability of the Weizsacker-Williams method. Thus, an exact calculation with allowance for the diagram of Fig. 3a does not improve in practice the results of calculations with the diagram of Fig. 2 by the Weizsacker-Williams method†.

The question arises of the feasibility of using formulas (1.26)—(1.29), since they contain the unknown parameter $\sigma_{\gamma N}$ (the use of the direct data obtained with accelerators is not justified in view of the fact that the

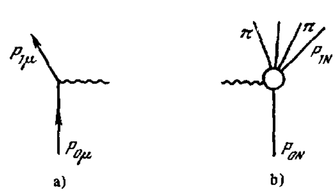

FIG. 3.

cross section $\sigma_{\gamma N}$ has been measured only at energies ~ GeV). In order to exclude this uncertainty, it is advantageous to employ the following procedure[16,34]: integrate the cross sections (1.26)–(1.29) over the spectrum of the cosmic muons (see Chapter III concerning this spectrum), and then compare the resultant expressions with the experimental data[34,35]. If we put $\sigma_{\gamma N} = h \times 10^{-28}$ cm²/nucleon, then the quantity h takes on the values listed in Table I. It must be stated that the value h ~ 1 agrees with the value of h obtained with an accelerator at much lower energies (~ GeV).

II. PASSAGE OF MUONS THROUGH MATTER

1. Average Muon Losses

a) Ionization loss. For $E_{0\mu} \gg \mu^2/m$, the ionization loss in standard ground (Z = 11, A = 22) can be expressed (following Sternheimer[36]) in the form

$$-\frac{dE}{dx}(u) = \left[1.9 + 0.08\ln\frac{E_{0\mu}}{\mu}\right][\mathrm{MeV \cdot g^{-1}cm^2}]. \tag{2.1}$$

Frequently, more accurate values are given for the coefficients of formula (2.1), but this is hardly justified, owing to the strong dependence of these coefficients on the details of the structure of the electron shell in the atoms, which is very difficult to take into account accurately.

b) Loss to bremsstrahlung. Using (1.9) after integrating with respect to dv, we can easily obtain

$$-\frac{dE}{dx}(r) = 0{,}95\alpha\left(2r_0\frac{m}{\mu}\right)^2 Z(Z+\xi)\left[\ln C - \frac{17}{18} + I\right]\frac{E_N N}{A}[\mathrm{g^{-1}cm^2}]$$

$$I = \frac{1}{1-B}\left\{[1 - B(1-\ln B)]\left[\frac{4}{3} + \frac{4}{3}\frac{B}{1-B} + \frac{B^2}{(1-B)^2}\right]\right.$$

$$\left. + \left[\frac{B^2}{2}\left(\frac{1}{2} - \ln B\right) - \frac{1}{4}\right]\left[\frac{4}{3} + \frac{2B}{1-B}\right] + \left[\frac{1}{9} - \frac{B^3}{3}\left(\frac{1}{3} - \ln B\right)\right]\frac{1}{(1-B)^2}\right\}$$

$$C = \frac{3}{\sqrt{e}}\frac{E_{0\mu}}{\mu}Z^{-1/3}, \quad B = \frac{2}{k\sqrt{e}}\frac{E_{0\mu}m}{\mu^2}Z^{1/3}. \tag{2.2}$$

Formula (2.2) was obtained by integrating with respect to v from 0 to 1. Actually, it is necessary to integrate up to a certain $v_{max} < 1$, determined by the applicability of formula (1.10), which is obtained in the relativistic approximation. In order of magnitude, $v_{max} \sim 1 - t\mu/E_\mu$, t ~ 1. However, the error due to setting v_{max} equal to unity is small and its order of magnitude is

$$\frac{\mu}{E_\mu}\ln\left(\frac{E_\mu}{t\mu}\right).$$

Putting t = 10, we find that even when $E_\mu \sim 10^{12}$ eV the correction is smaller than 1%. If $E_\mu/\mu \to \infty$, then in this asymptotic limit

Table I

Model	Weizsacker-Williams	Kesslers	$L^{(1)}=L_1^{(1)}$	$L^{(1)}=L_2^{(1)}$
h	1.4	0.18	0.8	0.38

*We disregard the trivial invariants $p_i^2 = m_i^2$.

†We note that condition (1.31) has a rather general character. Thus, for direct pair production this condition is of the form $\epsilon_p \ll mE_{0\mu}/\mu$. Inasmuch as the cross section has a maximum in the region $\epsilon_p \lesssim mE_{0\mu}/\mu$, it is not at all surprising that the Weizsacker-Williams method gives good results for this process.

$$\frac{dE_\mu}{dx}(r) = 0.95\alpha\left(2r_0\frac{m}{\mu}\right)^2 Z(Z+\xi)$$
$$\times\left[\ln\left(\frac{3}{2}\frac{k\mu}{m}Z^{-2/3}\right) - \frac{17}{18}\right]\frac{E_\mu N}{A}\ [\mathrm{g^{-1}cm^2}]. \quad (2.3)$$

It should be noted that this asymptotic limit is not reached even at rather high energies (~ 10^{12} eV).*

We present the numerical values of $-dE/dx(r)$ for standard ground (Table II). Approximately we have $dE/dx(r) \sim Z^2/A$.

c) Loss to pair production. In accordance with[26] (taking into account, in addition, pair production on electrons and radiative corrections), the loss to pair production is given by[16]

$$-\frac{dE}{dx}(p) = 0.95\frac{2}{3\pi}\frac{(Z\alpha r_e)^2}{A}NL\int_0^E \Phi(R')\,d\varepsilon_p\ [\mathrm{g^{-1}cm^2}] \quad (2.4)$$

where

$$\Phi(R') = \left[\frac{64}{15}(R')^2 + \frac{38}{15} + \frac{19}{15(R')^2} + \frac{1}{1+(R')^2}\right] \times \frac{\sqrt{1+(R')^2}}{R}$$
$$\ln\frac{\sqrt{1+(R')^2}+R'}{\sqrt{1+(R')^2}-R'} - 4\left[\frac{7}{3}+\frac{32}{15}(R')^2\right]\ln(2R') - \frac{32}{15} - \frac{38}{15}(R')^2$$

and

$$R' = \frac{\varepsilon_p \mu}{2E_{0\mu} m}.$$

This formula is valid in the case of total screening.

Integration of the more accurate formula (1.19) yields[27]

$$-\frac{dE}{dx}(p) = 0.95\frac{19\pi}{9}(\alpha r_e)^2 Z(Z+\xi)$$
$$\times\frac{m}{\mu}\left(\ln\frac{E_\mu}{4\mu} - \frac{11}{6}\right)\frac{N}{A}E\ [\mathrm{g^{-1}cm^2}] \quad (2.5)$$

in the absence of screening, and

$$-\frac{dE}{dx}(p) = 0.95\frac{19\pi}{9}(\alpha r_e)^2 Z(Z+\xi)\frac{m}{\mu}\left(\ln 2kZ^{-1/3} - \frac{2}{57}\right)\frac{N}{A}E\ [\mathrm{g^{-1}cm^2}] \quad (2.6)$$

in the case of total screening.

We present the numerical values of the loss to pair production, obtained in accordance with formulas (2.4) and (2.6) (Table III). In the last column of Table III we give the values of the losses calculated in accordance with the theory of Murota et al.[25,15] Unfortunately, there is still no formula analogous to (1.9) for the pair-production cross section in the entire range of energies†. Only expressions for the limiting cases (from the point of view of screening) exist. It is therefore of interest to compare roughly the energy intervals for the region of total screening in the bremsstrahlung and pair-production processes.

For the estimate we use the following procedure. We find the average values $\bar{v}_p$ and $\bar{\varepsilon}_p$, which we then substitute in the relations for the condition of total screening:

$$\bar{v}_p \sim \frac{\int_0^1 dv_p}{\int_{v_{p\,\min}}^1 \frac{dv_p}{v_p}} = -\frac{1}{\ln v_{p\,\min}}. \quad (2.7)$$

In order of magnitude $v_{p\,\min} \sim \mu/E_\mu$. Therefore

$$\bar{v}_p \sim \frac{1}{\ln\frac{E_\mu}{\mu}},\quad \bar{\varepsilon}_p \sim \frac{m}{\mu}E_\mu.$$

*This circumstance was noted earlier by the authors of [14].

†The important problem of calculating the losses for the intermediate cases was solved only very recently (see the paper of S. R. Kel'ner and Yu. D. Kotov at the International Conference on Cosmic Rays, Canada, 1967) (Note added in proof).

Table II

$\frac{E_\mu}{\mu}$	10^4	10^5	∞
$\left(-\frac{dE}{dx}(r)\Big/E\right)$ $\mathrm{g^{-1}cm^2}$	$1.7\cdot10^{-6}$	$1.8\cdot10^{-6}$	$1.82\cdot10^{-6}$

Table III

Formulas	(2,4)	(2,6)	Theory of Murota et al.
$\left[-\frac{dE}{dx}(p)\Big/E\right]\cdot$ $\mathrm{g^{-1}cm^2}$	$1.9\cdot10^{-6}$	$2.3\cdot10^{-6}$	$1.8\cdot10^{-6}$

The total screening condition for bremsstrahlung is

$$\frac{1}{\alpha}\frac{\mu^2}{m}\frac{\bar{v}}{E_\mu}Z^{-1/3} \ll 1,$$

or

$$\frac{E_\mu}{\mu} \gg \frac{\mu\bar{v}Z^{-1/3}}{\alpha m} \sim 10^3. \quad (2.8)$$

For direct pair production, the screening condition has the approximate form

$$\frac{E_\mu}{\mu} \gg \frac{Z^{-1/3}}{\alpha} \sim 10^2. \quad (2.9)$$

Condition (2.9) is weaker than (2.8); this indicates that total screening for the process of direct pair production occurs earlier than for bremsstrahlung.

d) Loss to muon-nuclear processes

$$-\frac{dE_\mu}{dx}(N) = N\int_0^{E_\mu}\varepsilon_\pi\,d\sigma_{\mu N}, \quad (2.10)$$

where $d\sigma_{\mu N}$ is determined by relations (1.26)–(1.29).

Substituting the cross sections for the photonuclear processes, listed in Table I, we can easily calculate the quantity $dE_\mu/dx(N)$[16,30]. A characteristic feature of the results obtained for different models is the surprising insensitivity of the losses to the chosen models. In[16] the value obtained for all models is

$$-\frac{dE_\mu}{dx}(N) \sim 4\cdot10^{-7}E_\mu\ [\mathrm{g^{-1}cm^2}]. \quad (2.11)$$

K. Kobayakava[30] obtained also for different models

$$-\frac{dE_\mu}{dx}(N) \sim 3\cdot10^{-7}E_\mu\ [\mathrm{g^{-1}cm^2}]. \quad (2.12)$$

The stability of the nuclear losses against changes in the models, on the one hand, and the relatively small contribution of these losses (~ 6–8%) to the total loss, on the other, give substantial grounds for assuming that the lack of rigorous expressions for the muon-nuclear processes lowers only insignificantly the accuray with which the total muon losses are determined*.

2. Average Muon Range

The dependence of the quantities a and b on the energy is, strictly speaking, logarithmic. Therefore the equation for the average range

$$-\frac{dE_\mu}{dx} = a + bE_\mu \quad (2.13)$$

must be solved numerically. However, it can be approximated in a certain interval by

*A graphic comparison of the losses can be found in the paper by A. D. Erlykin (Proc. Internat. Conf. Cosmic Rays, London, 1965, v. 2) (Note added in proof).

$$a = \text{const}, \quad b = \text{const} \tag{2.14}$$

which do not depend on E_μ. Then

$$E_\mu(x) = \frac{(a + bE_{0\mu})\,e^{-bx} - a}{b} \tag{2.15}$$

and the range is

$$R = \frac{1}{b}\ln\left(1 + \frac{b}{a}E_{0\mu}\right). \tag{2.16}$$

To estimate the error due to the substitution (2.14), we can use the following procedure. We calculate the range by means of formula (2.16), putting $a = \bar{a}$ and $b = \bar{b}$ ($\bar{a}$ and $\bar{b}$ are the mean values of these quantities in the given interval), and then compare the calculations with the values of R obtained numerically by Kobayakava[30]. Of course, for such comparisons it is necessary to choose the values of $\bar{a}$ and $\bar{b}$ obtained in[30]. We choose the most interesting interval $E_{0\mu} = 10^{12}-10^{13}$ eV. In this interval $\bar{a} = 2.6$ MeV-g^{-1}cm^2 and $\bar{b} = 3.5 \times 10^{-6}$ g^{-1}cm^2.

Table IV lists the results of the comparison.

The last column of the table lists the values of the differences of the "exact" and approximate values of R in per cent. To estimate whether the errors that have crept in are of practical significance, let us calculate dR/da and dR/db. In the interval of $E_{0\mu}$ of interest to us we have

$$\ln\left(1 + \frac{b}{a}E_{0\mu}\right) \sim 1 \div 3.$$

Therefore, in order of magnitude

$$\frac{dR}{db} \sim -\frac{R}{b} \text{ and } \frac{dR}{da} \sim -\frac{R}{a}.$$

We have seen that b is determined accurate to ~10%. Therefore the inaccuracy due to the error of b exceeds the error due to replacement of the "exact" solution of Eq. (2.13) by the approximate solution (2.15) and (2.16).

3. Absorption Curve

Let $P(E_\mu, x)$ be the differential cross section of the muons at a depth x. The intensity $P(E_\mu, x)$ is connected with the spectrum at sea level $P(E_\mu, 0)$ by the relation

$$P(E_\mu, x) = P[\varphi(E_\mu, x); 0]\,\frac{d\varphi}{dE_\mu}, \tag{2.16'}$$

where the function φ is defined by $E_{0\mu} = \varphi(E_\mu, x)$. In the first approximation we can use (2.15) for the calculation of φ. Then, if

$$P(E_\mu, 0) = DE_\mu^{-(\gamma+1)}, \quad D = \text{const}, \quad \gamma = \text{const}, \tag{2.17}$$

we get

$$P(E_\mu, x) = D\,\frac{b^{\gamma+1}e^{bx}}{[(a + bE_\mu)\,e^{bx} - a]^{\gamma+1}}. \tag{2.18}$$

Table IV

$E_{0\mu}\cdot 10^{-12}$, eV	$R_1\cdot 10^{-5}$, g - cm^{-2} (according to (2.16))	$R_2\cdot 10^{-5}$, g cm^{-2} (numerical calculations of [30])	$\left(\frac{R_1-R_2}{R}\right)$, %
1	2,43	2.60	0.07
2	3.65	3.89	0.07
3	4.63	4,75	0.03
4	5.32	5.4	0.015
5	5.89	5.93	0.01
7	6.70	6.74	0.005
10	7.64	7.63	0

The integral spectrum of the muons at the depth is

$$T(E_\mu, x) = D\frac{b^\gamma}{\gamma}\,[e^{bx}(E_\mu b + a) - a]^{-\gamma}. \tag{2.19}$$

The absorption curve (i.e., the dependence of the total intensity of the muons on the amount of matter above the setup) is of the form

$$T(0, x) = \frac{D}{\gamma}\left(\frac{b}{a}\right)^\gamma (e^{bx} - 1)^{-\gamma}. \tag{2.20}$$

It is of interest to determine the dependence of the absorption curve on the errors in the determination of the parameters a and b. A change of the parameter a leads to a shift of the absolute value of T(0, x), whereas an error in the value of b causes also a change in the exponent γ.*

From (2.20) we easily get

$$\frac{dT(0, x)}{da} = -\frac{T(0, x)}{\gamma a}. \tag{2.21}$$

The dT(0, x)/db dependence is much more complicated: for example, dN/db = 0 if

$$\frac{xe^{bx}}{e^{bx} - 1} = \frac{\gamma}{b}$$

(if $\gamma \sim 2.5$ and $b = 4 \times 10^{-6}$ g^{-1}cm^2, corresponding to $x \sim 5 \times 10^5$ g/cm^2).

For the extreme values we have

1) $xb \ll 1$:

$$\frac{dT(0, x)}{db} \sim \frac{T(0, x)}{b}(\gamma - 1), \tag{2.22}$$

2) $xb \gg 1$:

$$\frac{dT(0, x)}{db} \sim T(0, x)\left(\frac{\gamma}{b} - x\right). \tag{2.23}$$

4. Muon Range Fluctuations

The combination of a decreasing muon spectrum and fluctuating muon energy losses leads to fluctuations of the muon ranges and to a certain characteristic variation of the absorption curve, as first noted in[37,13]. The physical reason for the change in the absorption curve lies in the fact that a given range R will be possessed by particles having energies that are smaller or larger than given by (2.16). However, owing to the decreasing spectrum, the number of particles with smaller energies is much larger than the number of particles with larger energies. This leads to a change of the absorption curve.

The calculation of the intensity with allowance for the fluctuations reduces to a solution of an equation similar to that used to describe cascade processes:

$$-\frac{\partial P(E_\mu, x)}{\partial x} + a\frac{\partial P(E_\mu, x)}{\partial E_\mu} = \int_0^1 \left[P(E_\mu, x) - \frac{1}{1-v}P\left(\frac{E_\mu}{1-v}, x\right)\right]\sigma_t(E_\mu, v)\,dv, \tag{2.24}$$

where $\sigma_t = \sigma_r + \sigma_p + \sigma_N$.

This equation was solved analytically for extremely small and extremely large depths[13]. In general form, the analytic expression was given recently by Nishimura[38] (see also[30]). In a simple approximate form, a solution of (2.24) was obtained in[39].

Equation (2.24) was solved also by the Monte Carlo method[40,41]. We emphasize that all the solutions of (2.24) were obtained for power-law spectrum (2.17) and at constant values of the functions a and b. For integer values of γ, the solution has a relatively simple form[30,13].

*If b is determined from the absorption curve.

The total intensity $T_f(0, x)$ at a depth x, with allowance for the fluctuations, is given by the expansion

$$T_f(0, x) = D\frac{1}{a^{\gamma}\gamma!}\sum_{s=\gamma}^{\infty}\frac{\partial A(s)}{\partial s}e^{-A(s)x}\prod_{i=1}^{\gamma-1}[A(s)-A(i)], \quad (2.25)$$

where

$$A(s) = A_r(s) + A_p(s) + A_N(s), \quad (2.26)$$

$$A_{r,p,N} = \int_0^1 [1-(1-v)^s]\,\sigma_{r,p,N}\,dv. \quad (2.27)$$

The ratio is

$$r = \frac{T(0, x)}{T_f(0, x)} = \left\{\frac{1}{(\gamma-1)!}\left(\frac{e^{bx}-1}{b}\right)^{\gamma}\sum_{s=\gamma}^{\infty}\frac{\partial A(s)}{\partial s}e^{-A(s)x}\prod_{i=1}^{\gamma-1}[A(s)-A(i)]\right\}^{-1}. \quad (2.28)$$

A comparatively simple expression for the integral spectrum $T(E_\mu, x)$ is obtained for arbitrary γ if $E_\mu > E_\mu$ [13]:

$$T(E_\mu, x) = De^{-6\cdot 10^5 xA(\gamma)}F(E_\mu), \quad (2.29)$$

$$F(E_\mu) = E_\mu^{-(\gamma+1)}\sum_{n=0}^{\infty}(-1)^n C_{\gamma+n}^{n}K_{\gamma,n}\left(\frac{\varepsilon_\mu}{E_\mu}\right)^n, \quad (2.30)$$

$$K_{\gamma,n} = \frac{nK_{\gamma,n-1}}{A(\gamma+n)-A(\gamma)}, \quad K_{\gamma,0} = 1.$$

$\epsilon_\mu \sim 1.5 \times 10^{12}$ eV has the meaning of the total ionization losses in one radiation length for the muons. We present the values of the coefficients $A_{r,p,N}$ and $A'_{r,p,N} = \partial A/\partial s$, calculated in accordance with the function $\sigma_{r,p,N}$ assumed in [30]* (Table V). We present further the values of r calculated by K. Kobayakava [30] for different values of γ and x (Table VI).

For r we can propose a rather simple approximation†

$$r \sim e^{-0.1b(\gamma-1.3)x}. \quad (2.31)$$

In the last column of Table VI are given the values of the coefficient r, calculated by formula (2.31). The absorption curve with allowance for the fluctuations was calculated in a relatively simple form by G. T. Zatsepin and E. D. Mikhal'chi [39].

We note that a comparison of the values of r calculated by different methods is presented in [30]. The difference does not exceed ~ 10% as a rule. At the same time, the accuracy with which the intensity was measured at large depths (~ (6–8) × 10^5 g/cm^2), the only ones at which fluctuations are really significant, is for the time being much worse. Therefore the errors in the calculation of the fluctuations can hardly play any noticeable role for a while. We note, in particular, that the approximations used for the cross sections σ_p and σ_N are insignificant (owing to the complexity of σ_p and the uncertainties connected with the calculation of σ_N). The point is that the main role (~ 80%) in the fluctuations of the muon ranges is played by bremsstrahlung, the cross section of which is well known and has a relatively simple form.

Table V

s	$A_r\cdot 10^6$	$A_p\cdot 10^6$	$A_N\cdot 10^6$	$A'_r\cdot 10^6$	$A'_p\cdot 10^6$	$A'_N\cdot 10^6$
1	1.74	2.06	0.28			
2	2.72	4.08	0.50	0.75	2.0	0.19
3	3.32	6.04	0.60	0.56	1.95	0.15
4	3.81	7.97	0.81	0.45	1.90	0.13
5	4.23	9.87	0.93	0.37	1.88	0.12
6	4.56	11.74	1.05	0.32	1.84	0.11
7	4.87	13.58	1.15	0.28	1.82	0.10
8	5.13	15.38	1.23	0.24	1.80	0.09
9	5.38	17.16	1.32	0.22	1.78	0.08
10	5 59	18.84	1.40	0.21	1.76	0.07

*The functions σ given in the present paper and in [30] differ slightly from each other. The fact that this difference is immaterial for estimates of the fluctuations will be discussed later.

†For the zeroth approximation of the function r see [71].

Table VI

$x\cdot 10^{-5}$, g·cm^{-2}	r				(2,31)
	γ=2	γ=2.5	γ=3	γ=3.5	γ=2.5
2	0.99	0.96	0.92	0.88	0.91
4	0.95	0.87	0.78	0.69	0.83
6	0.89	0.75	0.62	0.50	0.76
8	0.81	0.62	0.47	0.34	0.68
10	0.71	0.49	0.33	0.22	0.61

III. FUNDAMENTAL CHARACTERISTICS OF HIGH-ENERGY COSMIC MUONS

1. Energy Spectrum of Vertical Muons at Sea Level

The energy spectrum of muons at sea level is a very important characteristic in a large number of problems connected with fast cosmic muons. For muons with energy $\gtrsim 10^{12}$ eV, there are three main methods of measuring the energy spectrum. The only direct method is to measure the deflection of the muons in a magnetic field at small depths underground (~ 10 m.w.e.). The magnetic method has a major advantage in that all the quantities determining the muon momentum are measured directly. This method is based on the relation

$$P_\mu = 300H\rho \quad (3.1)$$

(H is the intensity of the magnetic field in Oe and ρ is the radius of curvature of the deflection of the muons in the magnetic field in cm; then P_μ is in electron volts). In spite of the seeming simplicity of this method, it encounters appreciable difficulties in the measurement of momenta at energies $\gtrsim 10^{12}$ eV. The reasons lie in the existence of limits on the measurement accuracy of the value of ρ and in the difficulties of producing a strong magnetic field in an appreciable region of space. On the other hand, if the region of action of the magnetic field is decreased, then the transmission of the setup decreases, causing an inevitable decrease of the statistical accuracy.

The most thorough measurements have been made with the aid of a magnet and neon tube by A. W. Wolfendale and co-workers for many years [40,42,43]. The latest papers give data for the differential and integral spectra. These results show that γ = 2.2 in the energy region $E_\mu \lesssim 3 \times 10^{11}$ eV and γ = 2.6 in the region 3×10^{11} eV $\le E_\mu \le 3 \times 10^{12}$ eV. The estimated error in the determination of γ is $\Delta\gamma$ = 0.2. Magnetic measurements of the muon spectrum were made also by Holmes et al. [44] and by Nash et al. [45] In [45] the muon spectrum was determined up to an energy 4×10^{11} eV; it agrees well with the data of [43].

The measurements of Holmes agree with [45] up to ~ 2×10^{11} eV, and then diverge strongly (γ ~ 1.8–1.9). The reason for the divergence lies apparently in the fact that it was possible to measure directly with the apparatus of[44] momenta up to $E_{\mu} \sim 10^{11}$ eV, and then corrections were introduced for the geometry of the setup, this being a rather ambiguous procedure in the case under consideration.

A less direct method of measuring the energy spectrum is based on registration of the distribution of the showers produced by the muons. An advantage of the method is the very large transmission, and a shortcoming is the need for converting from the primary data to the muon spectrum. The distribution of the showers with respect to the number of particles at a depth x is

$$\Psi(n)\,dn = dn\int_0^\infty P(E_\mu, x)\,dE_\mu \int_0^\infty \sigma(E_\mu, E_s)\,dE_s \times \int_\Omega W(E_s, z, \theta, \varphi, n)\,dz\,d\theta\,d\varphi, \tag{3.2}$$

where $W(E_s, z, \theta, \varphi, n)$ is the probability of production of a shower with n particles by secondary particles having a total energy E_s, the shower moving at zenith and azimuthal angles θ and φ and passing through an area element dz of the setup. Ω is a certain region of integration, defined by the geometry of the setup. The integral (3.2) is simplified by two circumstances: 1) The main contribution to the cross section σ is made by bremsstrahlung. Thus, in the energy region 10^{11}–10^{12} eV, pair production and the δ-process contribute approximately several per cent (see, for example,[46]). The cross section is $\sigma_r \sim 1/E_\gamma$. 2) The cascade curves have a sharp maximum, and $n_{max} \sim E_\gamma$. Therefore in the simplest cases the $\Psi(n)$ curve is approximately similar to the $P(E_\mu, x)$ curve. However, for accurate calculations it is necessary to calculate the function W, usually a difficult matter. For the case when the measurements are made with apparatus in which the cascade curve is registered at one point*, there is a basic difficulty: the dependence on θ is obtained from calculations that contain several indeterminate factors. Therefore a step forward is the use of an ionization calorimeter[47], first employed for muon research at MIFI (Moscow Engineering Physics Institute)[48]. The shower produced by the muons is registered in this case at many points, making it possible to estimate directly the direction of muon motion (the angle θ) and to estimate the energy of the shower produced by the muons. Table VII lists the values of γ determined recently by the ionization method.

It must be emphasized that all the data obtained by the ionization method give results that agree well, $\gamma \sim 2.4$–2.6 (with the exception of the work of the NIIYaF (Nuclear Physics Research Institute) group[52], which incidentally, agrees with the data by others within the limits of two standard deviations).

The next method of determining the muon spectrum reduces to a calculation from the γ-quantum spectrum measured at high altitudes[54,55]. This method, developed in[54-56], is based on the calculation of the pion and kaon spectrum with subsequent calculation of the number of photons and muons. For direct pion generation, the usual decay schemes are assumed

$$\pi^\pm \to \mu^\pm + \nu_\mu,$$
$$\pi^0 \to 2\gamma,$$
$$\varphi_{\pi^0}(E_0, E_\pi) = \frac{1}{2}\varphi_{\pi^\pm}(E_0, E_\pi). \tag{3.3}$$

For kaon generation followed by their decay into γ quanta and muons, the following modes of decay are taken into consideration:

$K^\pm \to \mu^\pm + \nu_\mu$ (60% of all $K^\pm$-particle decays),
$K^0 \to 2\pi^0$ (30%),
$K^0 \to \pi^+ + \pi^-$ (70%),
$$\varphi_{K^0}(E_0, E_K) = \varphi_{K^\pm}(E_0, E_K). \tag{3.4}$$

φ_{π^0}, $\varphi_\pi^\pm$, φ_{K^0}, and $\varphi_{K^\pm}$ are respectively the differential energy spectra of the π^0, $\pi^\pm$, K^0, and $K^\pm$ mesons produced by the primary cosmic particle (which we henceforth assume to be a nucleon) of energy E_0. We shall use in what follows the following formula for the probability $w_{\pi,K}$ of the decay of a pion or kaon on the segment $x - x_1$ of the atmosphere (the path is reckoned from the top of the atmosphere)

$$w_{\pi,K}(x - x_1) = u_{\pi,K}\, e^{\frac{x_1}{L_{int}}}\left(\frac{x_1}{L_{int}}\right)^{u_{\pi K}} \int_{x_1/L_{int}}^{x/L_{int}} \frac{e^{-x}}{x^{u_{\pi,K}+1}}\,dx, \tag{3.5}$$

where $L_{int} \sim (85 \pm 5)$ g/cm^2 is the nucleon range relative to the interaction;

$$u_{\pi,K} = \frac{l m_{\pi,K}}{E_{\pi,K}\tau_{0,\pi,K}};$$

$m_{\pi,K}$ is the mass of the pion or the kaon; $l \sim 8000$ m; $\tau_{0,\pi,K}$ is the lifetime of the meson at rest. Then the total number $N_{\pi,K}dE$ of particles with energy lying between E and E + dE, decaying on the path from the top of the atmosphere to the level x, will be

$$N_{\pi,K}\,dE = dE\,u_{\pi,K}\int_E^\infty \Phi(E_0)\,\varphi_{\pi,K}(E_0, E)\,dE_0 \times \int_0^{x/l_{int}} dy\,\frac{e^{-y}}{y^{u_{\pi,K}+1}} \int_0^y e^{\left(1 - \frac{l_{int}}{L_{abs}}\right)} t^{u_{\pi,K}}\,dt. \tag{3.6}$$

L_{abs} is the nucleon absorption range, $L_{abs} \sim (120 \pm 5)$ g/cm^2; $\Phi(E_0)$ is the differential energy spectrum of the primary nucleons. The integral (3.6) has the following features: 1) The pion and kaon generation spectrum enters exclusively in the first integral, which can be readily estimated from (3.3) or (3.4) and the experimental data of[54], by assuming that the γ quanta measured at very large altitudes are produced directly in π^0-meson decay (and not as a result of cascade processes). 2) The integral (3.6) can be readily estimated for the important limiting case $u_{\pi,K} \ll 1$ and

Table VII

Method	γ	Energy interval, eV; Bursts
Method[49]	2.4±0.1	10^{11}–$6\cdot10^{12}$
Method[50]	2.5±0.3	10^{12}–$4\cdot10^{12}$
Method[51]	2.5±0.3	$3\cdot10^{11}$–$2\cdot10^{12}$
Method[52]	2.15±0.15	$4\cdot10^{11}$–$4\cdot10^{12}$
Calorimeter[53]*	(2.5±0 2)	$3\cdot10^{11}$–$3\cdot10^{12}$

*The measurements of [53] were made with the aid of a calorimeter located in Tbilisi.

*For example, in the customarily employed method of measuring the distribution of bursts in ionization chambers.

$x \gg L_{int}$:

$$N_{\pi,K}\,dE \sim dE\,\frac{u_{\pi,K}\int\limits_E^\infty \Phi(E_0)\,\varphi_{\pi,K}(E_0,E)\,dE_0}{1-\frac{L_{int}}{L_{abs}}}. \qquad (3.7)$$

From the spectra (3.6) and (3.7) it is easy to go over to the differential spectrum of the muons $P(E_\mu, 0)$ (neglecting the muon absorption in the atmosphere). To this end, it is necessary to take into account the kinematics of the decays (3.3) and (3.4). For example, for the scheme (3.3) we have $E_\mu \sim 0.8E_\mu$. From relation (3.7) we can draw the following conclusion. The decay scheme (at sufficiently large energies E_μ) affects only the absolute intensity of the muons, without affecting its form. The only parameter of the calculation is the ratio L_{int}/L_{abs}, which is known with sufficient accuracy (~ 10%). The value of the exponent γ determined in this manner is approximately 2.6 in the energy region $3 \times 10^{11}-10^{12}$ eV and $\gamma = 2.8$ in the region $10^{12}-6 \times 10^{12}$ eV.

Thus, all three methods give results that agree well in the muon energy region $3 \times 10^{11}-3 \times 10^{12}$ eV, namely $\gamma \sim 2.5-2.6$. It is quite difficult to estimate the total error of γ in the different methods, but in most papers a value on the order of 0.1—0.2 is cited. The value $\gamma = 2.5 \pm 0.2$ is at present quite likely.

The absolute value of the muon intensity can be estimated by using the data of[43]:

$$T(E_\mu, 0) = 10^{-8}\left(\frac{E_\mu}{3\cdot 10^{11}}\right)^{-2.5\pm 0} \text{cm}^2\text{sec}^{-1}\text{sr}^{-1}$$
$$(3\cdot 10^{11}\text{eV} < E_\mu < 3\cdot 10^{12}\text{ eV}) \qquad (3.8)$$

2. Energy Spectrum of Fast Muons Moving at Large Zenith Angles

The energy spectrum of muons with energy $\gtrsim 10^{11}$ eV moving at large zenith angles was studied in a number of investigations. The first tentative measurements[57] led to an exponent $\gamma_2 \sim 1.75-2.0$ at $E_\mu > 10^{11}$ eV and zenith angles $\theta > 75°$. A more detailed investigation of this problem was made with the MIFI calorimeter[48,58] and with the aid of a magnetic spectrometer[59]. In both investigations, the apparatus was situated such that the recorded mesons moved in directions close to horizontal. The calorimeter was used to investigate the energy spectrum in the region $2 \times 10^{11}-3 \times 10^{12}$ eV and $55° < \theta < 90°$. The results of both methods were in good agreement. For the muon flux averaged over the angular interval 55—90° it is possible to propose the following approximation[58]*:

$$T_r(E_\mu, 0) = 10^{-5}\left(\frac{E_\mu}{10^{11}}\right)^{-2.1\pm 0.15} \text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1} \qquad (3.9)$$

Figure 4 shows the energy spectra of the muons for large zenith angles (78.75°, 81.25°, 83.75°, 86.25°, and 88.75°)[59].

3. Muon Absorption Curve in Ground

The dependence of the muon intensity on the underground depth of the apparatus was investigated many times. The most detailed measurements of the absorption curve were made with the aid of fourfold coincidences by a Japanese-Indian group[61] in gold mines (India). The average density of the ground over the pit was 3.02 g/cm³, the average $\bar{Z}$ was 12.9, and the average $\bar{A}$ was 26.3. The error is estimated by the authors at 2%. We shall henceforth assume this value, although it must be noted that it is based on the assumption that the ground over the pit consists of hornblende. The authors state that this mineral occupies more than 90%, and more likely 98%, of the entire volume. At the same time, it follows from Table II (see[61]) that only ~ 60% of all samples contain this mineral. Figure 5, taken from[61], shows data on the intensities of the muons at different depths.

In spite of probable variations of the ground composition in different measurements, the results agree well with one another, particularly at $x \lesssim 2000$ m.w.e.

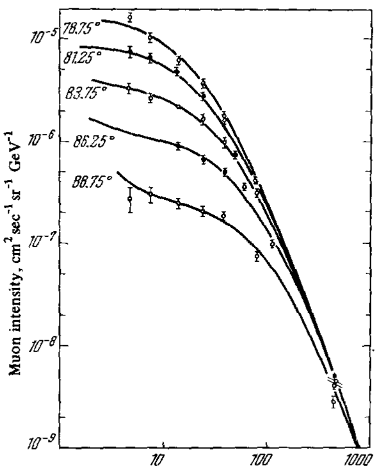

FIG. 4. Energy spectra of muons at large zenith angles. Solid curves – results of calculations under the assumption that all muons are produced in π-μ decay [59].

4. Angular Distribution of Muons at Large Depths

The angular distribution of the muons was measured by Randall and Hazen[62] at a depth 850 m.w.e., by

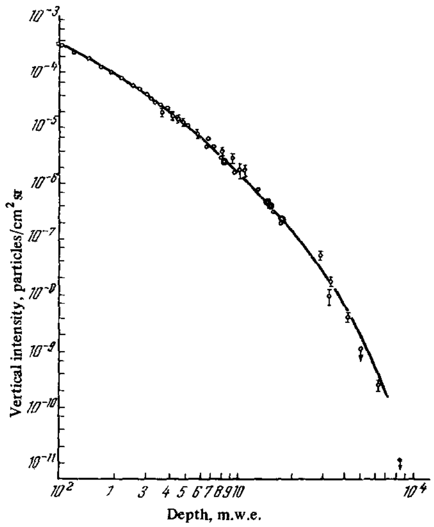

FIG. 5. Dependence of the muon intensity on the depth. Data by numerous authors, gathered by Miyake et al [61].

*We note that preliminary results on the measurements of the spectrum of the horizontal muons, obtained from the transition radiation, agree with other data [60].

Table VIII

x, m.w.e.	n
816 [64]	1.9±0.2
850 [62]	2.3±0.3
1812 [64]	3.0±0.2
1840 [63]	3.15±0.2

Bollinger[63] at a depth 1840 m.w.e., and by a British-Indian group[64] at depths 816, 1812, and 4100 m.w.e. The angular distribution at a depth x, denoted I(x, 0), is usually approximated by the function

$$I(x, \theta) = T(0, x)\cos^n\theta, \quad n = \text{const.} \tag{3.10}$$

Table VIII summarizes the values of the exponent n, obtained in the cited papers.

5. Positive Excess

Measurement of the ratio $P_+(E_\mu, 0)/P_-(E_\mu, 0)$ of the number of positive muons with energy E_μ to the number of negative muons with the same energy at sea level was performed many times in magnetic fields. angles $\theta < 75°$ and $\theta > 75°$. The most remarkable is the approximate independence of the value of this ration of the energy in a rather wide interval.

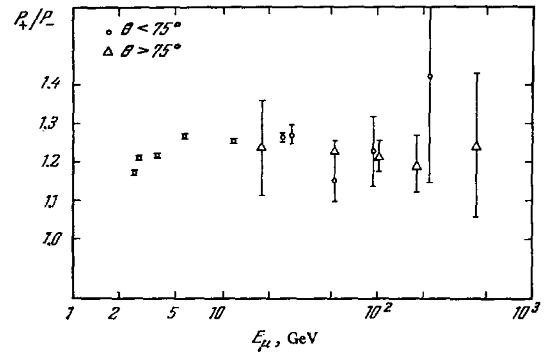

FIG. 6. Positive excess vs. energy [65].

IV. FUNDAMENTAL PHYSICAL RESULTS

1. Upper Limit of Anomalous Muon Interaction

By anomalous interaction we should mean here an interaction different from those described in Chapter I. Searches for such interactions offer, within the framework of modern field concepts, the only hope of understanding the puzzling difference between the muon and electron masses. If the additional interactions are characteristic also of the muonic neutrinos, then experiments with neutrinos contained in the cosmic rays are also highly significant[54].

Without stopping to describe the many attempts of constructing such interactions*, we shall touch upon two investigations[7,66]. I. Yu. Kobzarev and L. B. Okun'[7] considered the hypothetical existence of a vector boson interacting with the muon but not interacting with the electron. Such a vector boson decays within a nuclear time interval into a pair of muons. However, the vector interaction introduced in this manner changes slowly with energy, and the value of its constant is bounded from above by results obtained in measurements of the magnetic moment (see the Introduction), and also of the cross section for the interaction of muonic neutrinos, as was noted by the authors of[7] themselves. If we use the value of the constant F obtained on the basis of a determination of the magnetic moment, then the cross section σ_V of the anomalous interaction of the muonic neutrino is $\sim 10^{-31}$–10^{-32} cm^2. In[68] they investigated the elastic scattering of neutrinos by protons

$$\nu_\mu + p \rightarrow \nu_\mu + p. \tag{4.1}$$

It turned out that $\sigma_{\nu N} < 2 \times 10^{-37}$ cm^2. This experiment moves the upper limit of the value of F to $\sim 10^{-6}/\mu^2$, which coincides in order of magnitude with the weak-interaction constant, practically closing the model of the muon anomalous interaction that varies slowly with the energy. M. A. Markov[66] introduced a pseudovector interaction that increases rapidly with the energy. On the basis of experiments performed with accelerators, the constant for this model is $g^2 < 10^{-6}$–10^{-7}. It is interesting that such an interaction is a source of μ-pair production, which leads to additional energy loss by the muons.

*A review of this question is contained in [67].

Experimental searches for the anomalous interaction of the high-energy muons can now be performed in three ways.

a) Comparison of the energy spectrum of the muons at sea level with the absorption curve. It follows from (2.20) that measurement of the absorption curve makes it possible to establish a unique relation between the experimental values of the quantities b and γ. Inasmuch as the exponent γ is measured independently, it is possible to determine b from measurement of the absorption curve. Unfortunately, the spectrum has been measured directly up to an energy $\sim 3 \times 10^{12}$ eV (see (3.1)), corresponding to depths $\sim (3$–$4) \times 10^3$ m.w.e., whereas the experimental data on the absorption curve were obtained to a depth ~ 8000 m.w.e., corresponding to energies $E_\mu \sim 10^{13}$ eV. Therefore, strictly speaking, the reasoning in question pertains to energies $E_\mu \sim 10^{12}$ eV; it is possible, however, to estimate the upper limit of b, by extrapolating the muon sea-level spectrum to the region $E_\mu \sim 10^{13}$ eV. So far, all the measurements of the cosmic-ray spectra have shown that γ increases (albeit very weakly) with increasing energy. It is therefore natural to assume that such an extrapolation gives a lower limit of the values of γ. As follows from (4.2), a decrease of γ leads to an increase of b, and calculation using the extrapolated value of γ gives the upper limit of b. Such a procedure was developed by Miyake et al.[41], who obtained $b \le 4.8 \times 10^{-6}$ g^{-1}cm^2. The theoretical value of b, calculated from formulas (2.2), (2.6), and (2.11) gives a value 5.1. The uncertainty in this quantity is $\sim 5\%$ (owing to the incomplete screening (see Table II), the inaccuracy of the calculation of the radiative losses and σ_N), giving $\Delta b_{theor} \sim 0.3$. The inaccuracy in the experimental determination is connected with the errors in the determination of γ*. In order to find the connection between the error $\Delta\gamma$ and the inaccuracy with which Δb is calculated, it is necessary to equate to zero the total derivative of T(0, x), determined

*The small discrepancies in the estimate of the fluctuations are smaller than the experimental errors in the measurement of the intensity, which, say for depths of 6000 m.w.e., amount to ~25% of the average value.

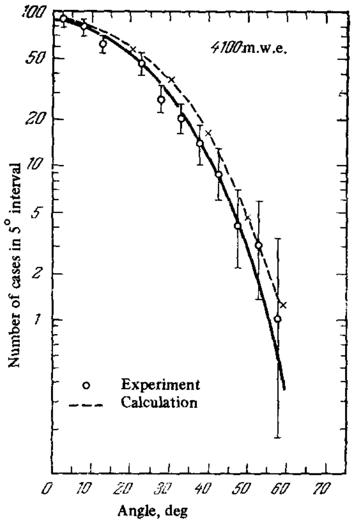

FIG. 7. Angular distribution of the muons at a depth of 4100 m.w.e. Solid line – experiment, dashed – calculation.

from (2.20). We obtain

$$db = -\frac{\ln\left[\frac{E_\mu b}{a(e^{bx}-1)}\right]}{\gamma\left(\frac{1}{b}-\frac{xe^{bx}}{e^{bx}-1}\right)}\,d\gamma, \tag{4.2}$$

where $E_\mu = 3\times10^{11}$ eV is the normalized constant in the muon spectrum (3.8). For values $\gamma = 2.5$, $x = 5\times10^5$ g/cm^2, and $\Delta\gamma = 0.2$ we get $\Delta b \sim 6\times10^{-7}$ g^{-1}cm^2. Thus, the most probable error in the determination of b is ~10%; this figure is the most probable limit of the anomalous interaction. An extreme case is obtained if the contribution made to the loss by the photonuclear processes is set equal to zero. Then the possible contribution due to the hypothetical interactions yields $\lesssim 0.2$b*.

b) Comparison of the muon energy spectra measured by the magnetic and ionization methods. In calculating the spectrum from the ionization bursts or with the aid of calorimeters, it is assumed that the ionization is due to bremsstrahlung. If an anomalous muon interaction exists, causing the appearance of photons or electrons with energy close to the muon energy, then the spectrum obtained from the ionization should lie higher than that measured by the magnetic method. Unfortunately, the measurement errors are large, reaching about 50% in the region $E_\mu \gtrsim 3\times10^{12}$ eV. At this accuracy, both spectra coincide. Therefore, even the omission of the photonuclear processes, which amount to ~15% of the bremsstrahlung, in the calculation of the spectrum cannot be detected experimentally.

c) Comparison of the theoretical and experimental angular distributions at large ground thicknesses. The calculation of the angular distribution of the muons at sea level was performed by many authors. Detailed calculations are given in[69]. The differential spectrum of the muons at different angles is expressed in the form

$$\mathcal{P}(E_\mu, 0, \theta) = \mathcal{P}(E_\mu, 0, 0)\,p(E_\mu, \theta), \tag{4.3}$$

where $p(E_\mu, \theta)$, generally speaking, is a complicated function of both parameters, but for large energies

*We emphasize that this conclusion is based on Eq. (2.20) with a constant value of b. In the region $E_\mu \sim 10^{13}$ eV, the value of b changes very little. It is very probable that the resultant error in the determination of b changes little.

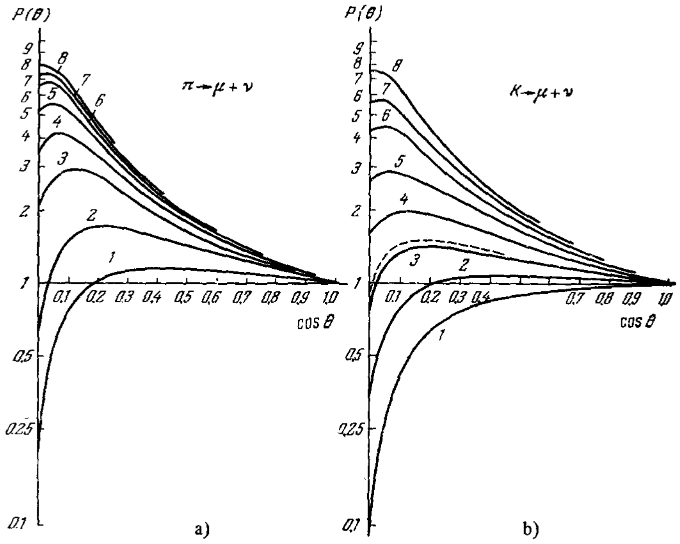

FIG. 8. Angular distribution of muons at sea level for energies E_μ equal to 10^{11} eV (1), 2×10^{11} eV (2), 5×10^{11} eV (3), 10^{12} eV (4), 2×10^{12} eV (5), 5×10^{12} eV (6), 10^{13} eV (7), and 10^{14} eV (8). All the curves are normalized to the vertical intensity. The dashed curve in Fig. 8b takes into account the K-π-μ process [69].

($E_\mu > 10^{12}$ eV), this function simplifies

$$p(E_\mu, \theta) \sim e^{2,3\frac{1-\cos\theta}{1+\cos\theta}}. \tag{4.4}$$

Starting from (4.3), (4.4), (2.31), and (2.16), we can obtain the angular distribution of the muons at large depths[70]:

$$\mathcal{P}(E_\mu, x, \theta) = \frac{T(E_\mu, 0, 0)}{\gamma}\exp\left\{-\gamma\left[\ln\frac{a}{b}+\ln\left\{\exp\left(\frac{xb}{\cos\theta}\right)-1\right\}\right] + 2,3\frac{1-\cos\theta}{1+\cos\theta}+\frac{0.1b(\gamma-1,3)x}{\cos\theta}\right\}. \tag{4.5}$$

Equation (4.5) was used to calculate the angular distributions of muons at a depth of 4100 m.w.e. Comparison of the theoretical distribution (formula (4.5) with $b = 5\times10^6$ g^{-1}cm^2) with the experimental one[64] revealed good agreement (Fig. 7). Comparison of the theoretical and experimental distributions obtained by other authors likewise showed no noticeable deviations (see[70–72]).

2. Production of K Mesons at Very High Energies

The question of the fraction of K mesons produced in collisions of high-energy particles can be approached, on the basis of cosmic-muon data, from different points of view.

a) Angular distribution of muons at sea level. Owing to the differences in the lifetimes and masses of the muons and kaons, the quantities u_π and u_K (see (3.5) and (3.6)) differ appreciably. Therefore, in the energy region where the ranges with respect to decay and interaction of a given sort of particles become equalized ($E_\mu \sim 10^{11}-10^{12}$ eV), an appreciable role in the generation of the muons will be played by the distribution of the air density along the meson path. The density distribution, naturally, depends on the zenith angle, which leads to a dependence of the angular distributions of the muons on the mechanism of their generation (K-μ decay, π-μ decay, direct muon generation). Figure 8 shows the angular distributions of the muons for two generation mechanisms[69]. The solid lines in Fig. 8 represent the calculated data of[59] on the muon energy distributions obtained under the assumption that all result from π-μ decay. As seen from the figure, good agreement is observed between the experimental results and the calculations. Wolfendale and co-workers[59] estimate that the best agreement in the region $E_\mu \lesssim 5 \times 10^{11}$ eV is obtained if $n_K/n_\pi \lesssim 0.2$; in any case, $n_K/n_\pi < 0.4$ (n_K and n_π are the numbers of K and π mesons produced in the elementary act). The energy of the K and π mesons responsible for the generation of the cosmic muons is smaller by a factor of approximately 5–10 than the energy of the primary particles (see (4.3)). Therefore the quantities n_K/n_π presented above pertain to a primary-particle energy equal to $E_0 \lesssim (2-5) \times 10^{12}$ eV. The question of the muon generation mechanism was considered also on the basis of the angular distribution measured with the aid of a calorimeter[48,58]. It was found that the overwhelming part of the muons is generated in π-μ or K-μ decays.

b) Absolute measurement of the muon intensity. From (3.5) and (3.6) it follows that the absolute muon intensity is determined by the quantities u. Figure 9 shows the muon spectra obtained on the basis of measurements of the photon spectra at high altitudes (see (3.1)) and calculations under the assumption that the muon sources are the π-μ and K-μ decays[56] (see also the earlier paper[73]). The same figure shows the muon spectrum measured by A. Wolfendale and co-workers by a magnetic method. We see that the data agree well with the assumption that in practice all the muons are produced in π-μ decay. The fraction due to K-μ decay does not exceed 10–20%. It is appropriate to note here the following: formulas (3.5) and (3.6) were obtained under the assumption that the pion or kaon range is equal to the nucleon range. Actually, the cross sections for the interaction of the pions and kaons is apparently somewhat smaller than the nucleon interaction cross section[74]. This circumstance increases the ordinates of both calculated curves of Fig. 9, which increases still further the discrepancy between the measured spectrum of the muons and the calculated one, based on the K-μ decay hypothesis.

c) Measurement of the polarization of cosmic muons. Without going into details of the interesting question of polarization of cosmic muons (see the theory and literature in[1]), we note merely that measurements of its value have led to the conclusion that in the E_μ = 2–5 GeV region the ratio is $n_K/n_\pi \lesssim 0.2-0.3$[75,76]. Two remarks should be made in this connection. The ratio $n_K/n_\pi \lesssim 0.2$ agrees with direct measurements made at energies $\gtrsim 10^{12}$ eV[77]. The muons are due to the decay of particles that carry away approximately 15–20% of the primary energy (see (4.3)). Therefore all the conclusions of this section pertain precisely to these fastest secondary particles.

3. Fraction of Energy Carried Away by Fastest Pions

It follows from (3.6) that, owing to the rapidly drooping spectrum $\Phi(E_0)$, an effective contribution to muon spectrum is made by pions with the highest energies. As noted by N. L. Grigorov[78], the ratio of the number of pions to the number of primary nucleons in the same energy interval is determined by the quantity

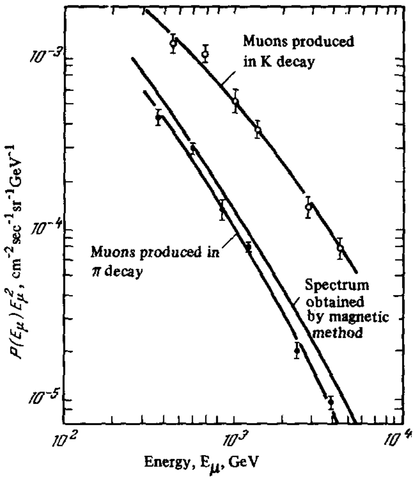

FIG. 9. Muon energy spectrum measured directly (magnetic method) and calculated under the assumption of K-μ decay (upper curve) or π-μ decay (lower curve) [56].

$n_e \alpha^{\gamma-1}$,*, where $n_e \sim 1$ is the effective number of pions with maximum energy, and α is the fraction of the energy carried away by them. Independent measurements of the spectra of the primary nucleons and muons make it possible therefore to determine $n_e\alpha^{\gamma-1}$, which turned out to equal ~ 0.07–0.1. Hence $\alpha \sim 0.15-0.20$ ($n_e = 1$). This conclusion was arrived at, on the basis of different models, by many authors (for example,[55,78-80]; see also the earlier papers[81,82]).

4. Role of Isobars in Multiple Processes

Peters[83] advanced the hypothesis that hyperons are produced in multiple processes. Hyperon decay leads to the appearance of fast pions, which ensures the required muon spectrum[79,80]; this hypothesis, however, leads to an incorrect sign of the charge excess[80,83]. Therefore an isobar mechanism, which can yield the correct values of the muon intensity and of the charge-excess sign, was proposed for the first time in[80].

Generally speaking, quite frequently the appearance of isobars in multiple processes followed by generation of fast pions would be a rather arbitrary hypothesis†, were it not for one important circumstance, namely the approximate dependence of the positive excess on the energy (see (3.5)).

Indeed, if the positive charge of the primary particle is uniformly distributed among the secondary particles, then the positive excess should decrease with increasing energy of the primary particle. The magnitude of this decrease can be estimated in the following manner. Assume that the number of charged particles is equal to $n_c^{(1)}$ at a certain energy $E_0^{(1)}$, and to $n_c^{(2)}$ at an energy $E_0^{(2)} > E_0^{(1)}$. In such a simple model we have

$$\frac{P_+}{P_-} \sim \frac{n_c+2}{n_c}.$$

Putting $n_c \sim E_0^{1/4}$, $n_c(10^{10}\text{ eV}) = 5$, and recognizing that the measurements of the positive excess were performed in the interval of E_0 from 10^{10} to 10^{12} eV, we find that

*More accurately, $n_e\alpha^{\gamma_0}$, where γ_0 is the exponent of the primary-particle spectrum; $\gamma_0 \sim \gamma - 1$ (if $E_\mu > 10^{11}$ eV).

†The possible role of isobars in interactions of high-energy particles was postulated earlier [84].

P_+/P_- should decrease by approximately 20%.*

The approximate constancy of the positive excess can be explained by assuming that the primary positive charge is not uniformly distributed among the secondary ones, but is predominantly transferred to the fastest pions [80]. The mechanism of such a phenomenon can be understood within the framework of modern concepts, using the simplest one-meson diagrams. But even in such a scheme it is necessary to make the following additional assumptions:

a) the incident particle is as a rule a proton;

b) the baryons carry away a greater part of its fraction in energy than the secondary pions.

In spite of the very strong assumptions, such an approach has attracted attention recently, owing to its uniqueness [85,86]. It must be emphasized, however, that within the framework of the isobar mechanism one must still expect, unconditionally, some change in the positive excess. We refer here to the consequences of the trivial assumption that when the energy E_0 changes the contribution of isobars of a given sort changes, causing a change in the isotopic relations in the nodes of the single-valued diagrams.

5. Cross Section of Photonuclear Processes at High Energies

As already mentioned (see (1.3)), the cross section of inelastic photonuclear processes, up to energies $\sim 10^{11}$ eV, remains approximately constant and equal to $\sim 10^{-28}$ cm^2/nucleon.

V. COSMIC MUONS AND NEUTRINOS

We shall not touch upon the entire major problem of cosmic neutrinos, and confine ourselves only to those questions that are common to research on both types of particles.

a) The muons are the main source of the neutrinos produced in the atmosphere and under ground. The energy spectra of the secondary neutrinos were calculated in a number of papers [87,88].

b) The muon absorption curve can be used to estimate the upper limit of the cross section of the neutrino interaction (B. M. Pontecorvo, A. E. Chudakov [89]). Indeed, at any measurement depth, the count intensity is the summary intensity of events produced by muons and neutrinos. Assuming the muon contribution to be equal to zero, we can obtain the upper limit of the neutrino interaction cross section. An analysis of the absorption curve obtained by the Japanese-Indian group [61] gave as first estimates for the limit of the neutrino-nucleon scattering cross section [89] $\sigma_{\nu N} < 10^{-34}$ cm^2†.

c) The cosmic muons constitute a background that hinders the investigation of the neutrino interaction. Since the muon interaction cross section exceeds the neutrino interaction cross section by more than ten orders of magnitude, and the fluxes of both types of particles are approximately the same, it is necessary to eliminate the muon background as much as possible in the study of cosmic neutrinos. To this end it is necessary to make use of two devices: 1) place the apparatus deep underground and 2) measure the neutrinos coming from the earth (the earth is practically transparent to the neutrinos [90]).

VI. CERTAIN UNEXPLAINED PHENOMENA CONNECTED WITH MUONS

We discuss here a few phenomena that lead to interesting conclusions. However, owing to the unique nature of the experimental premises and the inaccuracies of the calculations, these conclusions should be approached with appreciable caution.

Barret et al. [91] measured at a depth ~ 1600 m.w.e. the dependence of the frequency of the coincidences of discharges in two systems of counters on the distance between them. A kink was observed in this dependence at small distances ($\sim 1-2$ m) between the counter systems. The authors of [91] interpreted this kink as a manifestation of two different processes: the muons of extensive showers were responsible for the coincidences at large distances, whereas at small distances ($\leq 1-2$ m) the coincidences were due to local showers produced in the ground. However, a more detailed analysis of the latter assumption [92] has shown that such an explanation encounters considerable difficulties, if account is taken of only the presently known muon interactions. The greatest contribution (owing to the geometry of the setup) at small distances is made by direct production of muon pairs by muons in the electromagnetic field. But even this process leads to a smaller effect than observed.

For many years, the Moscow University group (see [2]) has been investigating extensive muon showers and correlated bursts in ionization chambers and the energy spectrum of the muons. To explain their experimental data, the authors advanced the hypothesis that a new heavy particle with intermediate interaction exists.*

It is interesting to note a case [34,35] in which the energy of the muon-nuclear shower is ~ 300 GeV, thus indicating a relatively large cross section for the muon-nuclear processes, with a large energy release.

VII. CONCLUDING REMARKS

An investigation of high-energy cosmic muons has led to a number of interesting physical consequences.

1. At energies $E_\mu \lesssim 10^{12}$ eV, the anomalous interaction of the muons contributes apparently not more than 10% of the total electromagnetic interaction (within the framework of the scheme of our calculations under the most extreme assumptions, this contribution can reach not more than 20%) (see Chapter IV, 1).

2. In interactions between nucleons of energy $\sim 10^{12}$ eV and light nuclei, a pion is produced and carries away approximately 15–20% of the energy of the primary particle (Chapter IV, 3).

3. This pion carries away a larger fraction of the primary charge than the remaining pions (Chapter IV, 4).

4. At nucleon energies $\lesssim 10^{12}$ eV, $n_K/n_\pi < 0.2$, but at any rate < 0.4 (Chapter IV, 2).

5. Initially, up to energies $E_\mu \sim 10^{11}$ eV, the cross section of the photonuclear processes remains approximately constant (Chapter I, 3 and Chapter IV, 5).

Graphical results of a similar estimate are found in [85].

†The limit of the cross section $\sigma_{\nu N}$ was brought down to 10^{-37} cm^2 as a result of accelerator experiments [68].

*The latest measurements of the NIIYaF group (S. N. Vernov et al.[97]) did not confirm the existence of such particles (Note added in proof).

The results make it possible to point to certain prospects in the study of cosmic muons.

From our point of view, the following experiments are of importance:

1) Study of the energy spectrum up to high energies with high accuracy and at different angles. These measurements must be carried out with the aid of different procedures.

2) Refinement of the dependence of the positive excess on the energy.

3) Study of the nuclear interaction of high-energy muons. In particular, it is necessary to use a calorimeter for this purpose (see[48]).

4) Refinement of the absorption curve in ground and study of the absorption curve in water. A certain interest in this question has been noted in recent years[93-96].

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Translated by J. G. Adashko