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Physics of our Time

MACROSCOPIC QUANTUM PHENOMENA

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1. VORTEX LINES IN SUPERFLUID HELIUM

SINCE the 1920's, when quantum mechanics was created, both theoretical and experimental physics have been divided into two more or less independent parts—classical and quantum physics. It is customary to consider that quantum mechanics, as the more general form, is required for the description of "microscopic," that is, atomic and nuclear, phenomena, and that ordinary classical mechanics is completely adequate for the investigation of "macroscopic" phenomena, especially for describing the motions of large "macroscopic" quantities of matter. This point of view is, of course extremely arbitrary. Even the existence of stable macroscopic bodies is a manifestation of quantum mechanical laws. Nothing in classical mechanics could prevent electrons from "falling into" atomic nuclei; only quantum mechanics can explain why this does not actually occur. Nevertheless, the foregoing distinction is true in a narrower sense. The Planck constant $\hbar$ that characterizes quantum processes does not appear in the equations of motion of solids, gases, and liquids. This constant can therefore be measured only in atomic experiments.

The behavior of some objects cannot be understood even on a macroscopic scale, however, purely on the basis of classical mechanics. This applies especially to liquid helium, or, more accurately, to the two isotopes liquid He^3 and liquid He^4. These liquids are absolutely unfreezable, since they remain liquids down to the absolute zero temperature. This is a specific quantum effect. According to classical mechanics, at absolute zero all atomic motions cease and all bodies must be solids.

It is therefore not surprising that the laws of motion of these liquids are inexplicable from a classical point of view. Specifically, at a sufficiently low temperature liquid He^4 becomes a superfluid; this means that it can then flow through fine capillaries as an ideal liquid, manifesting absolutely no viscosity. We find the properties of rotating superfluid helium to be especially interesting. In this case its quantum properties are revealed most directly, and the equations of motion of even large quantities of the liquid contain Planck's constant explicitly. Therefore even mechanical experiments with rotating superfluid helium enable us to measure this quantum constant in principle.

To understand the rotational properties of a superfluid let us consider a rotating cylinder filled with such a liquid. If the liquid in the cylinder also rotates about the axis of the cylinder, then the atoms of the liquid perform a rotational motion. This motion is quantized in accordance with the laws of quantum mechanics. To elucidate the consequences of this fact we shall use Bohr's semiclassical quantum rules, according to which an atomic orbit of circumference $2\pi r$ (where r is the distance from the axis of rotation) should be fitted by an integral number of de Broglie wavelengths $2\pi\hbar/Mv$ (where v is the velocity and M is the mass of the atom). In other words, the rotation of a liquid should satisfy the relations [1,2]

$$v = \frac{\hbar}{M}\frac{1}{r}n, \qquad (1)$$

where n is an integer. Equation (1) determines the distribution of velocity in a rotating superfluid, and this distribution differs from that of velocity in an ordinary liquid. The latter rotates as a unified whole with the velocity distribution $v = \Omega r$, where Ω is the angular velocity. The velocity increases without limit as the axis is approached. [Of course, Eq. (1) is meaningful only at distances much larger than atomic distances.] A line in a liquid that is rotating about the line in accordance with Eq. (1) is called a vortex line. In our case the only vortex line is the axis of the cylinder. If, however, a liquid performs other motions besides its rotation about the axis, this vortex line can become bent, thus complicating the distribution of velocities. Near the line the distribution will continue to be subject to (1), where r will mean the distance from the vortex line. Vortex lines in superfluid helium are the only macroscopic quantum objects of their kind. Indeed, on the one hand, Planck's constant appears explicitly in the basic Eq. (1), which determines all properties of the vortex line. And on the other hand a vortex line can be very long; in the very simple example that we have selected the line is as long as the cylinder.

A vortex line possesses energy that equals the kinetic energy of the liquid's motion about the line. It is evident that in a real liquid only vortex lines with $n = 1$ will be formed. We shall henceforth consider only such "unit" lines. The calculation of their kinetic energy yields

$$E = \int \frac{\rho v^2}{2} d^3r = \frac{\rho}{2} \int\limits_{-l/2}^{l/2} dz \int\limits_{0}^{R} \frac{\hbar^2}{M^2}\frac{1}{r} 2\pi r\, dr = \frac{\hbar^2}{M^2}\pi\rho l \ln\frac{R}{a}, \qquad (2)$$

where ρ is the density of the liquid, l is the length of the line, and R is the radius of the cylinder. The quantity a is a certain distance, of atomic order of magnitude, at which Eq. (1) becomes meaningless; a must be introduced as the lower limit of integration over r in (2). The magnitude of E is only slightly dependent on the exact value of a.

Since finite energy is required for the production of a vortex line, this can only occur at some finite rotational velocity of the cylinder. It can be shown that this "critical" angular velocity of rotation is

$$\Omega_{cr} = \frac{\hbar}{MR^2} \ln R/a. \quad (3)$$

If the rotational velocity is smaller than Ω_{cr}, the liquid will remain at rest even in a rotating cylinder. When $\Omega \geq \Omega_{cr}$ a vortex line is generated in the center of the cylinder and the aforedescribed rotational pattern is found. If $\Omega \gg \Omega_{cr}$ many vortex lines are formed in the cylinder. As a result the motion of the liquid on the average resembles its rotation as a whole, although the velocity increases in accordance with (1) near each vortex line.

We note that when the rotating cylinder is a fine capillary the angular velocity Ω_{cr} is very appreciable. With R measured in centimeters we have

$$\Omega_{cr}\sec^{-1} \approx 1.4 \cdot 10^{-4} \frac{\ln(R/4\cdot 10^{-8})}{R^2}.$$

The time when a vortex line is formed in the cylinder can be observed. Therefore Ω_{cr} is the first example of a macroscopic mechanical quantity whose definition contains Planck's constant explicitly.

Experiments with vortex lines for the, in principle, possible determination of Planck's constant by mechanical measurements were performed in 1960 by by the English physicist Vinen.[3] The vibrations of a string placed along the axis of a cylinder filled with superfluid helium were observed. Each vibration can be represented as the sum of two oppositely polarized circular motions (clockwise and counter-clockwise) of a bent string. The sinusoidal shape of the string has a wavelength that in the simplest case equals the fundamental tone of a string twice as long. The rotational frequency can be obtained by equating the centrifugal force to the elastic force tending to return the string to equilibrium. This elastic force is

$$-\frac{k}{l^2} r,$$

where r is the deviation of the string from the axis, k is a characteristic constant of the string, and l is the length of the strength. The frequency ω_0 is then defined by

$$\varkappa\omega_0^2 r = \frac{k}{l^2}, \quad \omega_0 = \frac{1}{l}\sqrt{\frac{k}{\varkappa}} \quad (4)$$

(with κ representing the mass of the string per unit of its length). When the string is immersed in a liquid κ must include the mass of liquid that is entrained by the string. In a liquid at rest the clockwise and counterclockwise frequencies of the string are, of course, exactly equal; this is equivalent to a doubly degenerate vibration.

In the rotating cylinder, when $\Omega \geq \Omega_{cr}$ the liquid begins to rotate about the string according to (1). In other words, a vortex line is created with its axis along the string. The line vibrates together with the string. Since the moving string now coincides with a vortex line it will be acted on by an additional force having the same nature as the lift force on an aircraft wing, represented by the Joukowski formula

$$F = \Gamma\rho\,[\mathbf{n}\times\mathbf{u}],$$

where **n** is a unit vector having the orientation of the angular velocity, $\Gamma = 2\pi r v = 2\pi\hbar/M$ is the circulation round the string, and **u** is the velocity of motion of the string. The magnitude of the force is

$$\frac{2\pi\hbar}{M} u\rho \quad (5)$$

directed perpendicular to the string and to the direction of the latter's motion. One can easily comprehend that for the oscillations where the direction of string rotation coincides with that of the liquid, the force is directed toward the axis; for the other oscillation the force is directed away from the axis. In the first case the force must be added to the elastic force in the equation for the frequency. Substituting $u = \omega r$, we have

$$\varkappa\omega^2 r = \frac{k}{l^2} r + \frac{2\pi\hbar}{M}\rho\omega r. \quad (6)$$

We assume $\omega = \omega_0 + \Delta\omega/2$. If, as actually does occur, $\Delta\omega \ll \omega_0$, Eq. (6) yields approximately

$$\Delta\omega \approx \frac{2\pi\hbar}{M}\frac{\rho}{\varkappa}. \quad (7)$$

The frequency of the other independent oscillation decreases by the same amount; the frequency difference between the two oscillations is thus $\Delta\omega$.

We note that Eq. (7) contains, in addition to Planck's constant, only certain parameters, characterizing the liquid and the string, which can be measured in advance. The experiment consisted in measuring the frequency difference $\Delta\omega$.

The experimental arrangement is shown in Fig. 1. The string W was a beryllium bronze wire 5 cm long and 2.54×10^{-3} cm in diameter. This wire was in-

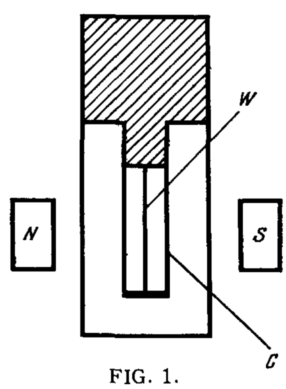

FIG. 1.

serted into a helium-filled cylinder C having a 4-mm inside diameter. The apparatus was rotatable at rates from 0.1 to 2 rpm. The natural vibrational frequency of the wire was about $\omega_0 \approx 500$/sec. The experiment was performed at the temperature 1.3°K. The apparatus was mounted between the poles of a magnet, so that the wire was acted on by a field of 3×10^3 Gauss.

When an electric current flowed through the wire it was deflected by the magnetic field. The initial current pulse induced vibrations of the string. Vibrations of the string in the magnetic field induced a potential difference between its ends; this p.d. was proportional to the velocity of the string's motion and could therefore be used to study the vibrations. In the absence of a vortex line and with equality of the two circular frequencies the vibrational amplitude is damped monotonically following the initial pulse. If the frequencies differ beats are produced, and the monotonic damping is modulated with the frequency $\Delta\omega$. Under our experimental conditions the frequency difference was about 0.45/sec. Figure 2 is a typical recording of the string vibrations. We observe that the initial decrease of amplitude is followed by an increase; the occurrence of beats is thus manifest.

The experiments were not actually performed for the purpose of determining $\hbar$, which is known with extremely greater accuracy from ordinary "atomic" experiments, but rather to test the theory of vortex lines. The results agreed well with the theoretical predictions, and thus proved the theoretical possibility of determining Planck's constant in a purely mechanical manner. The electromagnetic devices comprised only a subsidiary portion of the apparatus.

The foregoing discussion omitted mention of certain secondary effects that interfered with the experiments. First, the vortex line did not always coincide with the string over its entire length. In such instances the beat frequency did not agree with Eq. (7). It also appeared that some beats existed in a nonrotating cylinder when no vortex line was present. These beats are accounted for by the hypothesis that the wire was not completely symmetrical and uniform from a mechanical point of view. For example, some segments of the wire could have been twisted. Such "parasitic" beats had to be taken into account in treating the experimental data.

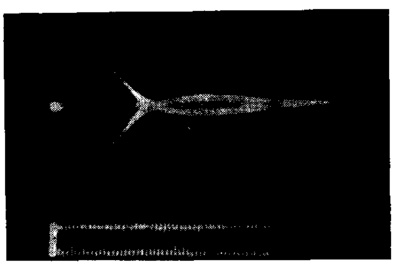

FIG. 2.

Vinen's experiments were performed with straight vortex lines parallel to the rotational axis of the cylinder. As already mentioned, the lines may also be curved or even closed to form vortex rings. The formation of such rings in a rotating cylinder is energetically unfavored; they appear only in the flow of helium along capillaries or in the motion of particles in the helium. Rayfield and Reif[4] have studied vortex rings formed around an ion moving in helium. In these experiments the helium was irradiated by a radioactive source and the resultant ions were accelerated by an electric field. Below 0.6°K practically the entire energy obtained by the ion from the field is expended in the formation of a vortex ring moving with the ion. The energy of the ring is given by (2) with l replaced by the ring circumference $2\pi b$ (where b is the radius of the ring) and with R replaced by b. This formula shows that when we know the energy, i.e. the potential difference through which the ion has passed, we can calculate the ring radius b. On the other hand, the radius determines the velocity of motion of the ring:

$$v = \frac{\hbar}{2Mb} \ln b/a. \qquad (8)$$

Since the velocity of a ring associated with an ion can also be measured directly, all quantities in this relation are known. The equation can therefore also be used for the experimental determination of $\hbar$. Even when the uncertainty of the denominator a of the logarithm is taken into account, $\hbar$ can be determined to within 30%; this is a fairly good result for such an unusual method. The maximum ring radius b in these experiments was $\sim 10^{-4}$ cm. The rings were thus completely macroscopic, having radii that were many times larger than atomic separations in liquid helium, which are of the order 4×10^{-8} cm. [This last quantity is taken as the value of a in (2) and (3)].

2. QUANTIZATION OF A MAGNETIC FLUX IN SUPERCONDUCTORS

Another interesting class of macroscopic quantum effects is associated with the properties of superconductors. Superconductivity has much in common with superfluidity. We can attribute the absence of resistance in the superconducting state to the formation of a superfluid liquid of electrons in a superconducting metal; this liquid flows through the metal without friction. Let us consider a superconducting ring or hollow cylinder. When a current is generated in such a ring it will flow for an infinitely long time because resistance is absent. However, it is found that the current cannot be of arbitrary magnitude, because a current round a ring or cylinder represents the circular motion of electrons, which is quantized like the rotational motion of helium atoms about a vortex line. The quantization rules are different, a current flowing through a conductor generates a magnetic field that alters the properties of the system.

We shall not here derive the quantization rules for this case; the derivation has already been published in the review article[5]. The current-induced magnetic flux is quantized within the cylinder cavity:

$$\Phi = H \cdot \pi R^2 = \Phi_0 n = \frac{\pi \hbar c}{e} n. \quad (9)$$

Here Φ is the magnetic flux, H is the magnetic field inside the cylinder, R is the inside radius of the cylinder, e is the charge of an electron, and c is the velocity of light. One quantum Φ_0 of the magnetic flux has the numerical value 2.06×10^{-7} gauss-cm^2.

Although Φ_0 has a very small value, the corresponding magnetic field could be measured experimentally. Deaver and Fairbank[6] used tin tubes about 1 cm long with 1.5×10^{-3} cm inside diameter as superconducting cylinders. Corresponding to a magnetic flux of one quantum Φ_0 the magnetic field inside the tube would be about 0.1 gauss, which is quite appreciable. (We recall that the earth's magnetic field is 0.5 gauss.) Small coils (having 10 000 turns each) positioned near the ends of the tube measured the magnetic field generated by the tube. During the measuring period the tube vibrated with a 1-mm amplitude at 1000 cps along its axis; the emf generated in the measuring coils was registered. The instrument had been calibrated to give the field in the tube directly from this emf. It was found that within the experimental error limits the magnetic flux inside the tube actually varied only discontinuously in accordance with Eq. (9). The experimenters estimated the measurements to be 20% accurate. It can be affirmed that Planck's constant may be measured with similar accuracy by means of this completely macroscopic, although not mechanical, procedure.

Doll and Näbauer[7] experimented with a lead tube of 10^{-3}-cm diameter and 0.6-mm length. In this case a magnetic field of 0.25 gauss corresponded to a single quantum of flux Φ_0. In these experiments the magnetic moment of the tube was measured, or, speaking more accurately, the perpendicular force acting on the tube in the external magnetic field. The tube, which had been formed by the deposition of lead on a quartz fiber, was weighed using a torsion balance (Fig. 3). An alternating magnetic field H_x of known magnitude was applied in the perpendicular direction. This field produced an alternating moment of force that set the system into vibration. The field frequency was automatically set equal to the natural vibrational frequency of the system. Knowing the logarithmic decrement, which had been measured in advance by observing the damping of free vibrations, and having measured the vibrational amplitude in the field, the moment of force was determined. The magnetic moment of the tube and the flux passing through it were then calculated. The results confirmed Eq. (9).

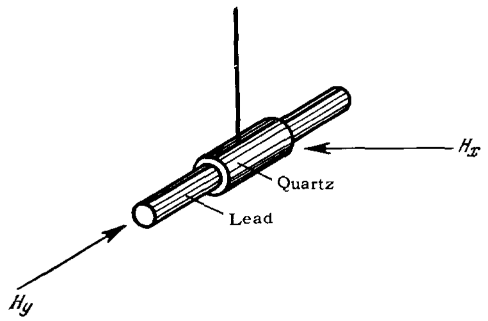

FIG. 3.

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Translated by I. Emin