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THE PHYSICS OF MAGNETIC MATERIALS*

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1. If one tries to answer the question "What is magnetism?", great difficulties arise. The reason is that magnetic properties are inherent in literally everything around us—beginning with the elementary particles, which possess magnetic moments, and ending with infinite cosmic space, which is filled by an electromagnetic field one of whose components is the magnetic field. Therefore magnetism, like electricity and gravitation, is present everywhere. It is possible to separate out a narrower range of phenomena in which magnetic properties are paramount; this forms the content of the physics of magnetic phenomena. The universality of the magnetic properties of particles and of the field enables us to use these properties as a delicate source of information about the internal structure of microparticles and of macrobodies. It is sufficient to recall the great role of magnetism in the creation of a theory of the atom (the Zeeman effect), the quantum theory of solids (the Landau diamagnetism and Pauli paramagnetism of the conduction electrons in metals), and so on. And in the present development of physics and of other natural sciences, "magnetic information" is playing an outstanding role. Use of electron and nuclear magnetic resonance, of magnetic neutron diffraction, of the magnetic aspect of the Mössbauer effect, and so on has entered into the practice of almost every natural science.

The universality of magnetism has also opened up great possibilities for its application in technology: 1) by use of the magnetic properties of a substance for the creation of magnetic materials; 2) by use of magnetic properties for obtaining information about other—not easily accessible for direct study—physicochemical properties of materials; this lies at the basis of methods of magnetic structural analysis, magnetic flaw detection, and magnetic measurements.

If we address the question "What is magnetism?" to someone who is acquainted with physics only through a school course, there will immediately arise in his memory pictures of iron filings at the poles of magnets, the right- and left-hand rules, and such similar bits of knowledge, retained in memory from his youthful years.

These scanty bits of knowledge give a very superficial idea of magnetism. In order to understand more clearly the enormous significance that magnetism has acquired in the life of man, let us imagine for a moment that matter has lost its magnetic properties. This would produce catastrophic consequences. Over the whole world, all power would be completely paralyzed, because all electric generators and motors would go out of commission, since their action is based on use of the magnetic properties of substances. Telephones and radio sets would be silent; television sets would not operate. Electric trains, diesel locomotives, trolley cars, and motor vehicles would come to a stop. From this fortunately unreal picture, everyone, even one unacquainted with physics, will understand that magnetism is one of the most important properties of matter.

2. Magnetism is that particular form of material interaction that occurs between moving electrically charged particles. The material carrier of this interaction is the magnetic field, determined by the field-intensity vector **H** or the magnetic-induction vector **B**. In order to describe the action of a magnetic field on a substance composed of particles, it is necessary to determine their fundamental magnetic characteristic. In the case of electric forces, such a characteristic is the elementary electric charge $e = 4.80298 \times 10^{-10}$ cgs esu of the electron ($-e$) or proton ($+e$). In the case of magnetic forces, this characteristic is connected with the motion of the charge: with its velocity v or current i. A molecule, atom, or elementary particle can be likened to an elementary closed current circuit. It is known that the magnetic characteristic of such a circuit is the magnetic moment **M**, equal to the product of the current strength i and the area S of the circuit: $\mathbf{M} = iS\mathbf{n}_0$ ($\mathbf{n}_0$ = unit vector normal to the surface of the circuit). In the atomic world there are two characteristic magnetic moments that play the role of atoms of magnetism: the Bohr magneton $\mu_B = 0.92732 \times 10^{-20}$ cgs emu for electrons, and the nuclear magneton $\mu_N = 0.50505 \times 10^{-23}$ cgs emu for nucleons (the proton and the neutron). Nuclear magnetism is a thousand times weaker than electronic. This is connected with the large mass of nucleons ($m_p = 1.67252 \times 10^{-24}$ g) as compared with the electronic mass ($m_e = 0.91091 \times 10^{-27}$ g): $m_p/m_e = 1836 \gg 1$, since in the expression for the magnetons in terms of atomic constants the particle mass

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enters in the denominator: $\mu_B = e\hbar/2m_ec$, $\mu_N = e\hbar/2m_pc$, where $h = 2\pi\hbar$ (= 6.5256 × 10^{-27} erg sec) is Planck's constant and c is the velocity of light (= 2.997925 × 10^{10} cm/sec). The atomic magnetic moments μ_B and μ_N are very small in comparison with the magnetic moments of macrocurrents. For example, a circular current of radius 1 cm and strength 1 A has a moment of 1 cgs emu; that is, 10^{20} times larger than the Bohr magneton.

Magnetic interactions, as a rule, are weaker than electric. With the aid of the well-known Coulomb laws for electric and magnetic forces, it is possible to compare the electric and magnetic interaction energies, E_{el} and E_{mag}, of two particles with charges e and Bohr magnetons μ_B, at a distance of atomic order $a_0 \sim 10^{-8}$ cm. This comparison gives

$$E_{mag} \sim \frac{\mu_B^2}{a_0^3} \sim \frac{10^{-40}}{10^{-24}} \sim 10^{-16} \text{ erg};$$

$$E_{el} = \frac{e^2}{a_0} \sim \frac{10^{-20}}{10^{-8}} \sim 10^{-12} \text{ erg};$$

$$\frac{E_{mag}}{E_{el}} \sim 10^{-4} \ll 1.$$

From this it is evident that magnetic forces are ten thousand times weaker than electric. But in atoms of the heavy elements, where the resultant magnetic moments are large and may reach values above 10 μ_B, the energy E_{mag} increases a hundredfold, and magnetic forces begin to play an essential role. We remark also that in order to obtain a large effect, a very large energy is not always necessary. For example, compare the small energy expended by the finger in pressing the trigger with the enormous (in comparison with it) energy of the bullet as it leaves the gun. Just so, in many cases, weak magnetic interactions can play the role of pressing a "trigger" and can produce a large effect. This is observed in diffusion phenomena in crystals, in chemical reactions, and in biological processes. The directive role of "weak" magnetic forces in their influence on matter is beginning to play an important role in science and technology.

For later purposes it is important to know the energy of electronic and nuclear magnetons with respect to an external field H – E_H^{el} and E_H^{nuc} – and to compare it with the mean atomic thermal energy $E_{therm} \sim kT$ at temperature T (k = 1.37 × 10^{-16} erg/deg, Boltzmann's constant). This gives

$$E_H^{el} = \mu_B H \sim 10^{-20} \cdot H \text{ erg}; \quad E_H^{nuc} = \mu_N \cdot H \sim 10^{-23} \cdot H \text{ erg};$$

$$E_H^{el} \sim E_{el} \sim 10^{-12} \text{ erg} \quad \text{at} \quad H \sim 10^8 \text{ Oe};$$

$$E_H^{el} \sim E_{mag} \sim 10^{-16} \text{ erg} \quad \text{at} \quad H \sim 10^4 \text{ Oe};$$

$$E_{therm} \sim kT \sim 10^{-16} T \text{ erg};$$

$$E_H^{el} \sim E_{therm} \quad \text{at} \quad \begin{cases} T \sim 1^\circ \text{K} & \text{and } H \sim 10^4 \text{ Oe}; \\ T \sim 100^\circ \text{K} & \text{and } H \sim 10^6 \text{ Oe}. \end{cases}$$

The estimates given enable us to introduce equivalent magnetic fields, which divide external fields into "weak" and "strong." We see, for example, that from the point of view of electric atomic interaction, strong fields are fields of order 10^8 Oe and above, whereas from the point of view of magnetic interaction they are fields $\gtrsim 10^4$ Oe. On the other hand, from the point of view of thermal motion at low temperatures, T ~ 1°K, strong fields are fields above 10^4 Oe, whereas at room temperature, T ~ 300°K, they must be greater than 10^6 Oe. We see that the lower the temperature, the more clearly should magnetic interactions manifest themselves. This makes obvious the importance of magnetic investigations in the low-temperature region.

3. We shall discuss in more detail the problem of strong fields*. In nature, by means of the Mössbauer effect, of nuclear magnetic resonance, or of measurement of the nuclear contribution to the heat capacity of solids, magnetic fields up to values 10^6 Oe are detected in atomic nuclei in certain ferromagnetic and antiferromagnetic crystals [3]. Near magnetic stars the field can reach 3 × 10^4 Oe. Artificially, by means of the usual copper solenoids, through which pass strong constant currents, stationary magnetic fields of up to 2.5 × 10^5 Oe are produced; and by means of solenoids of superconductors, fields above 10^5 Oe [1,2]. Pulsed fields, obtained by discharge of electric condensers, reach 10^6 Oe and higher [1,2]. By means of explosion, pulsed fields above 10^7 Oe have been obtained [4]. In this last method, use is made of the law of constancy of magnetic flux Φ, equal to the product of the field intensity and the cross-sectional area of the flux: Φ = HS. In the explosion, the cross-section of the flux can be appreciably diminished, and then the field increases by the same factor. Before the explosion, there is produced in a coil, by discharge of condensers, a strong initial pulsed field ~ 5 × 10^5 Oe, which is then multiplied by the explosion.

Extra-strong magnetic fields, in whose production and application P. L. Kapitza [5] is the pioneer, are not an exotic matter; to obtain them is a practically necessary problem. They are necessary for localization of a thermonuclear plasma in magnetic traps, for magnetohydrodynamic generators, for study of hard magnetic materials. Recently altogether new possibilities have opened up for use of extra strong fields the technology: a) for obtaining technical details of complex profiles by plastic deformation of a metal in a pulsed magnetic field (magnetic stamping) [6], b) for stimulation of phase transitions in steels as a new method of obtaining high-stability materials [7]. In prospect, it can be expected that after treatment in pulsed magnetic fields up to 10^6 to 10^7 Oe, technical materials will be obtained with record-breaking magnetic, mechanical, and other physical properties. In principle it is possible to use extra-high magnetic fields for shielding in outer space against penetrating

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*See, for example, the reviews[1,2].

γ radiation[8]. It is well known that even the magnetic field of the earth, which is very weak (~ 0.5 to 1.0 Oe) but is distributed over a large space, and the atmosphere make possible the normal existence of everything living on our planet, by shielding against harmful cosmic radiation. The principle of using strong fields for shielding against radiation is connected with very delicate nonlinear quantum-electrodynamic effects, which occur when an electromagnetic field of very high frequency interacts with an electron-positron field in the presence of strong constant magnetic fields. It turns out that in the neighborhood of certain critical magnetic fields of the very high value $H_c = m^2e^3/ch \sim 10^{13}$ Oe, γ quanta of high energy ($\sim 10^{12}$ eV) can with high probability vanish by transformation into an electron-positron pair. Each particle of the pair can produce a second pair of a γ quantum. As a result there is obtained a shower of particles, in which the energy of the primary γ quantum is rapidly degraded (Fig. 1). Although the shielding problem here is very difficult, nevertheless, if it were possible to surround a space ship with a layer of magnetic field of order 10^8 Oe, 1 mm thick, this layer would be a safe shield against the most penetrating γ radiation, against which it is useless to apply any ordinary methods of shielding. This layer, in some sense, imitates on a very small scale (in a layer of order 1 mm) the effect of the enormous thickness of the earth's atmosphere. Therefore such magnetic shields can also be used as stimulators of shower production. This is an example of a future magnetic "material" without matter, from only a magnetic field in vacuum.

Still another example[8] can be cited of the nonlinear effect of interaction of electromagnetic and electron-positron fields in the presence of an external magnetic field; this is the effect of creation and subsequent absorption of a virtual electron-positron pair by a photon (Fig. 2). Here no high fields or large quantum energies are required. The fact is that the virtual pair exists only for a short time of order 10^{-22} sec. Therefore it follows from the uncertainty relation $\Delta E \cdot \Delta t \sim \hbar$ of quantum mechanics that an energy $\Delta E \sim \hbar/\Delta t \sim 10^{-27}/10^{-22} \sim 10^{-5}$ erg $\sim 10^7$ eV is sufficient for production of a pair. For this reason, even light quanta of low frequency can create virtual pairs. The practical significance of this effect consists in the fact that during their short lifetime, the pairs succeed in interacting with the field, and this leads to a diminution of the energy of the quantum that reappears after annihilation of the pair. In the great cosmic spaces, this can lead to a "magnetic red shift" (together with the predicted Einsteinian red shift in gravitational fields).

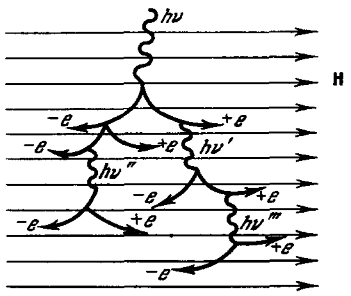

FIG. 1. Degradation of energy of a primary gamma quantum hν in a strong magnetic field **H** by formation of showers of electron-position pairs (−e, +e) and scattered quanta of lower frequency (hν', hν'', ... < hν).

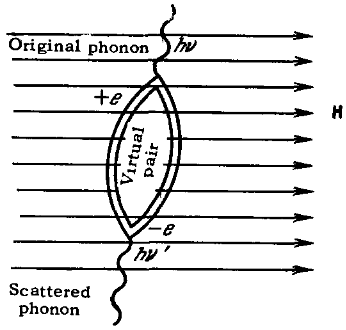

FIG. 2. Nonlinear interaction of photons hν with a magnetic field **H** by generation and absorption of virtual electron-positron pairs (−e, +e).

Very interesting, for the obtaining of strong magnetic fields, is the use of superconductors with large values of the critical magnetic fields ($H_{c.sc} \sim 10^5$ Oe) that remove the superconductivity. From such materials, wires for the winding of solenoids are prepared; and in these, at temperatures below the critical, magnetic fields $\gtrsim 10^5$ Oe are obtained[1,2].

Superconductors can also be used to obtain very weak fields, by means of superconducting magnetic-flux pumps, which evacuate the magnetic field from a region. The principle of this operation rests on use of the Meissner effect—the ejection of magnetic flux from a superconductor[9]. If a multiply connected superconducting specimen is cooled below its critical temperature Θ_{sc}, the magnetic flux will be "frozen" inside the hole in the specimen. Consider a specimen in the form of a double "doughnut" with two openings a and b, joined by a narrow gap c (Fig. 3). If such a specimen is placed in an external field

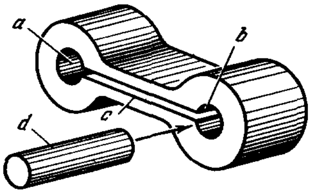

FIG. 3. Schematic diagram of a superconducting pump for obtaining strong ($\gtrsim 10^4$ Oe) or very weak ($\lesssim 10^{-4}$ Oe) fields.

parallel to the axes of the openings a and b, and if it is then cooled below Θ_{cs}, the magnetic flux will be stuck in the openings a and b. If a superconducting cylinder d is now inserted into the opening b, the magnetic flux from the opening b will be "squeezed out" into the remaining free space (through the gap c into the opening a), where the amount of the magnetic flux increases accordingly. If we assume that the total frozen-in flux remains constant in this process, then the field intensity H_a in the opening a upon insertion of the cylinder into the opening b will be connected with the initial field H_0 by the relation

$$H_a = H_0 \frac{S_a + S_b + S_c}{S_a + (S_b - S_d) + S_c},$$

where S_a, S_b, S_d, and S_c are the cross-sectional areas of the openings a and b, of the cylinder d, and of the gap c. Thus this pump, by operating repeatedly in one direction, can from a weak field produce a strong one. Fields up to 23 000 Oe have been successfully "pumped." By compelling the pump to operate in the reverse direction, it is possible to obtain weak fields down to 0.0002 Oe. Finally, superconductors can be used as ideal magnetic shields.

Attainment of strong magnetic fields is of great theoretical and practical interest. It is at present difficult even to predict what interesting and practically useful range of phenomena we shall open up if we attain fields of order 10^9 Oe and higher. To obtain and contain such fields is not an easy problem (if only because of the colossal pressures that arise in such fields; they amount to hundreds of millions of atmospheres). At present this problem is one of the most urgent in contemporary science.

4. Turning to the physics of magnetic materials, we must briefly review the magnetism of atoms, where there are three sources of magnetism: the orbital motion of electrons, their intrinsic "rotation" (spin), and the very weak magnetism of the atomic nucleus. As regards the action of an external magnetic field, it shows two basic effects upon the elementary atomic magnets: 1) an inductive effect on the electronic and nuclear currents, which according to Lenz's law produces an additional magnetic moment opposite in direction to the inducing field (this effect is called diamagnetism), and 2) an orienting effect of the field upon the intrinsic magnetic moments of the atoms, which exist in them even in the absence of an external field. This effect is called paramagnetism*.

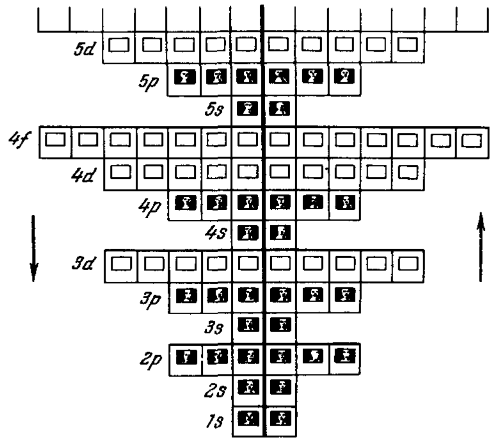

FIG. 4. Pictorial diagram of the filling up of electronic shells of atoms. ↑, ↓ · directions of the spin projections.

To explain the law of formation of the magnetic moments of atoms, we recall the order of construction of their electronic shells. According to the laws of quantum mechanics, there acts in a shell of the atoms a rigid population rule—the Pauli principle. An electronic shell can be likened to a many-storied building. In each story there is a definite number of places—"rooms"; in each of these only one electron can be accommodated. Furthermore, each story is divided into two halves—one for electrons with "right" spins, the other for those with "left" (Fig. 4). Each block of stories corresponds to one value of the principal quantum number (n = 1, 2, 3, ...), each story within a block to a definite form of orbit (determined by orbital quantum numbers $l = 0, 1, 2, \ldots, n-1$, denoted respectively by the letters s, p, d, f...). Each story is denoted by two symbols: nl (1s, 2s, 2p, 3s, ...). Each pair of rooms, symmetrically placed with respect to the middle line of the building, corresponds to one of the $(2l + 1)$ possible orientations of the orbit in space (spatial quantization). As, in the Mendeleev table, we go step-by-step from the first element, hydrogen, to the second, helium, the third, lithium, the fourth, beryllium, etc., we shall in sequence populate the "skyscraper" (see Fig. 4). In this process, in each story those rooms will first be populated that have some one direction of spin. This rule had already been established in 1927, by Hund, on the basis of an analysis of spectroscopic data. Hund's rule leads to the result that a large total magnetic moment can occur in atoms. In the rule for filling up an electronic shell of atoms, there is an important "infringement" of the successive population of stories. In each block with given principal quantum number n, only the first two stories are filled in sequence: ns and np. The population of stories with indices nd, nf, etc. always proceeds with a delay. It turns out, for example, to be energetically more favorable* to populate the highest 4s-story be-

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*Diamagnetism is a universal effect, therefore it is present in all atoms and substances. It is, however, a weak (second-order) effect, and therefore in those cases in which the atomic particles possess nonvanishing orbital and spin (or resultant) magnetic moments, the diamagnetism is practically always masked by the stronger paramagnetic effect.

*This is because the energy of the electrons increases more rapidly with increase of the orbital quantum numbers l than with increase of the principal quantum number n, that is, for example, the energy of a 4s state is lower than the energy of a 3d state.

fore the lowest 3d. Populating of the 3d story begins not with the element potassium, as would be supposed according to populating in sequence, but with the element scandium. These delayed populatings of the nd and nf stories lead to the appearance in the periodic table of a class of *transition* elements, with internal layers of an electronic shell of their atoms in a process of being added on to. In the periodic table there are altogether five groups of transition elements: the iron group (3d), the palladium group (4d), the platinum group (5d) (eight elements), the rare earths (RE) (4f) (14 elements), and the actinide group (5f and 6d) (four elements); to them are also still to be added the transuranium elements. Thus of the 92 stable chemical elements of the periodic table, 42 are transition elements; that is, almost half of the table.

Because of the presence of an incomplete internal nd or nf shell, all atoms of transition elements are strongly magnetic by virtue of Hund's rule. When the atoms combine into molecules or crystals, the external stories of their shells are greatly reorganized, the internal ones only slightly. Therefore the possibility arises of retaining, even in the crystalline state, the magnetism of atoms of the transition elements. Moreover, under certain favorable conditions the atomic magnetic moments not only are retained in the crystals, but also are arranged in an ordered manner with respect to the sites of the crystal lattice. Such an atomic *magnetic order* exists in crystals only below certain critical temperatures T_c, called the Curie point Θ_{fm} and the Néel point Θ_N.

5. Three types of magnetic order are distinguished: 1) parallel, or *ferromagnetic* (Fig. 5a); 2) antiparallel, or *antiferromagnetic*, with complete compensation of magnetic moments (Fig. 5b); 3) antiparallel, *ferrimagnetic*, with a nonvanishing resultant difference moment (Fig. 5c). There have recently been discovered in a number of substances, for example in the rare-earth metals, more complicated atomic magnetic structures of noncollinear type. The most rigorous theory of these structures was developed by I. E. Dzyaloshinskiĭ[10]. Magnetic diffraction of neutrons on ferro- and antiferromagnetic crystals gives us a graphic demonstration of the presence in them of atomic magnetic order of one or the other type.

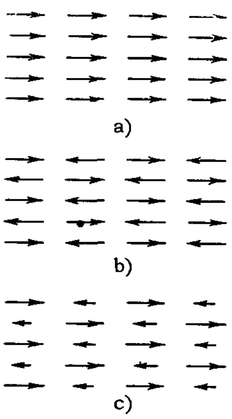


FIG. 5. Types of collinear atomic magnetic order. a) ferro-, b) antiferro-, and c) ferrimagnetic.

In typical ferromagnets—for example, in iron, cobalt, and nickel—the Curie point is rather high, of order 1000° K. Then from the equality $E_{int} \sim k\Theta_{fm}$ (E_{int} = the energy responsible for the magnetic order) it follows that in order of magnitude $E_{int} \sim 10^{-16} \times 10^3 = 10^{-13}$ erg. But in these metals the magnetic energy does not exceed a value of order 10^{-16} erg, and therefore it could guarantee magnetic order only with a very low Curie point, of order 1°K, not the 1000°K given by experiment. The only escape from this difficulty is to suppose that the interaction that produces the atomic magnetic order is not magnetic in nature, but *electric*. This paradox of classical physics was resolved in quantum mechanics in 1928 by J. Frenkel[11] and W. Heisenberg[12], who explained the nature of ferromagnetism by use of a deduction of quantum mechanics to the effect that by virtue of the Pauli principle, a part of the electric interaction of electrons (the so-called *exchange* energy) depends on the value of the resultant spin moment of the system of electrons. When it is the parallel orientation of spins that corresponds to a minimum of this energy, we have ferromagnetism (this occurs also in isolated atoms—Hund's rule); when it is the antiparallel, we have antiferromagnetism (predicted theoretically, independently, by L. D. Landau[13] and L. Néel[14]).

Substances with atomic magnetic order can be provisionally divided into four classes: 1) f-metals (RE) and alloys; 2) d-metals and alloys; 3) very dilute alloys of d- or f-metals in a nonmagnetic metallic matrix; 4) nonmetallic crystals.

In RE f-metals, an important role in the establishment of magnetic order is played by the conduction electrons (the previous valence electrons of the isolated atoms), which become collectivized in the crystal and form a Fermi liquid (according to Landau[15]) permeating the lattice of ionic cores. On the other hand, in f-metals the magnetically active incomplete 4f-layers have a very small radius (in comparison with the interatomic distances in the crystal). Neighboring 4f-layers do not overlap, and therefore they do not experience direct interaction. The active carrier of interaction between these 4f-layers is the Fermi liquid of conduction electrons, which brings about an *indirect* exchange between the magnetic moments of the 4f-layers localized on the lattice sites; this leads to atomic magnetic order when $T < \Theta_{fm}$. Figure 6 represents this graphically in the form of buoys, with arrows, located under the water (inside the Fermi liquid), firmly tied to anchors (4f-shells localized near nuclei). Regular motion of the

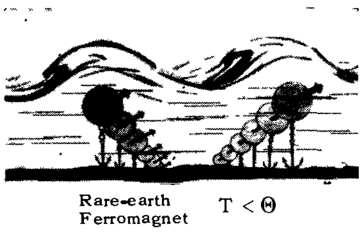


FIG. 6. Pictorial representation of ferromagnetic order in f-metals with indirect exchange interaction of f-shells via the conduction electrons (at temperatures T below the Curie point Θ).

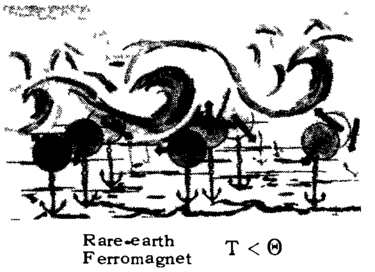


FIG. 7. Pictorial representation of ferromagnetic order in f-metals with indirect exchange interaction of f-shells via the conduction electrons (at temperatures T above the Curie point Θ).

water (parallel waves) aligns all the arrows on the buoys parallel to one another (ferromagnetic order of the atomic magnetic moments). When $T > \Theta_{fm}$, the heat motion destroys this order (as is graphically represented in Fig. 7). In many re cases, there is first a temperature range in which a ferromagnetic state exists (from 0°K to Θ_{fm}), then an antiferromagnetic state (from Θ_{fm} to Θ_N); the magnetic structures, as a rule, have a noncollinear character[10].

In the case of d-metals the situation is more complicated, because the incomplete d-layers do not have as small effective radii as do the f-layers, and therefore they overlap the neighboring atoms in the crystal appreciably. This leads to the result that the d-electrons as well as the valence electrons take part in the formation of a Fermi liquid. In Fig. 8, the d-electrons are shown in the form of an ordered series

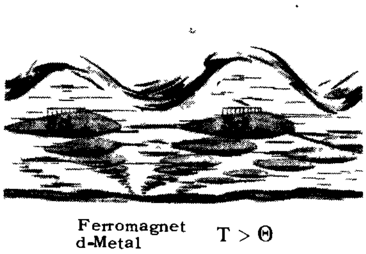


FIG. 8. Pictorial representation of ferromagnetic order in d-metals—mixture of two Fermi liquids of s- and d-electrons (at temperatures T below the Curie point Θ).

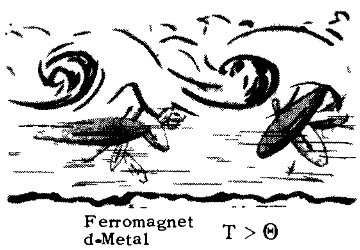


FIG. 9. Pictorial representation of ferromagnetic order in d-metals—mixture of two Fermi liquids of s- and d-electrons (at temperatures T above the Curie point Θ).

speak of a mixture of two Fermi liquids: a light one, of conduction electrons, and a heavy one, of former d-electrons of the incomplete layers). Above the Curie point, heat motion destroys the ordered arrangement of the submarines (Fig. 9). In the case of d-metals, the order arises from exchange interaction of electrons within the mixture of Fermi liquids.

In the case of nonmetals with atomic magnetic order, we always have to do not with pure elements, but with chemical compounds in which atoms of transition elements are separated by magnetically neutral ions of oxygen, fluorine, sulfur, etc. The localized external (valence) electronic layers of these ions are also the carriers of the exchange coupling between the magnetic ions. Thus in nonmetallic ferro- and antiferromagnets, as well as in f-metals, there is indirect exchange: not, however, through conduction electrons, but through bound electrons of a diamagnetic medium. This is shown graphically in Fig. 10. In this case the interaction as a rule is antiferromagnetic, and a resultant moment appears only when there is ferrimagnetism. Representatives of this class of ferromagnet are oxides of the transition metals and primarily the ferrites (for which ferrimagnetism is named).

From the short description that has been given of the various types of material with magnetic order—from which, alone, it is possible to prepare magnetic materials—it is evident that in the chemical make-up of these substances, at least one of the components must be a transition element. It is in fact the transi-

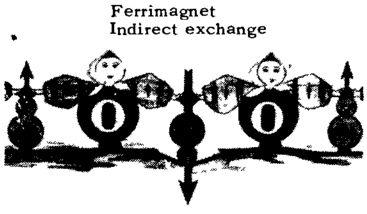


FIG. 10. Pictorial representation of antiferromagnetic order in a ferrimagnet (uncompensated antiferromagnetism). O, magnetically neutral anion of a nonmetal; Me, magnetically active cation of a d- or f-metal.

tion elements that provide the basic "building material" for magnetic materials.

6. Modern technical magnetic materials belong to the class of ferromagnetic metals and alloys and of semiconductor ferromagnets—ferrites. Because of the complexity of the practical conditions under which they are exploited in technology, they should possess a definite complex of magnetic and other physical properties—electrical, mechanical, thermal, etc. The most important primary magnetic characteristics of a ferro- or ferrimagnetic material are: the value of the resultant magnetic moment—the spontaneous magnetization I_s (moment of unit volume), or magnetic saturation—and the value of the Curie point or the Néel point. The largest saturation magnetizations are possessed by the RE's. (The maximum saturation, $I_s \sim 3000$ G, is observed in holmium.) The RE's, however, because of the weakness of the exchange coupling (small energy E_{int}), have very low Curie points (except gadolinium, with $\Theta_{\text{fm}} = 16°\text{C}$; the others are much lower, for example $\Theta_{\text{fm}} = -253°\text{C}$ for holmium); in the 3d-metals—Fe, Co, Ni—the saturation magnetization (the largest is $I_s \sim 1700$ G for iron) is less than in RE's, since the number of electrons in the 3d-layer and their resultant moment are less. The latter is lowered still more in a crystal because of the metallic coupling in which the 3d-electrons also take part. But the Curie and Néel points in 3d-metals are higher, for example $\Theta_{\text{fm}} = 1130°\text{C}$ in Co. To obtain ferromagnetic compounds with high I_s and Θ_{fm} is a very difficult problem. So far there has been success in raising the magnetization of iron in its alloy with cobalt (by 13%). Apparently the possibility of production of magnetic materials with the highest I_s and Θ_{fm} must be sought in systems of alloys and compounds of f- and d-metals. Intensified search in this direction is now on its way (see, for example,[16]).

The most important technical characteristics of a magnetic material are the magnetization curve and the hysteresis loop, that is the dependence of the magnetization on the external magnetic field. Figure 11 shows a typical magnetization curve and hysteresis loop of a soft magnetic material (an iron-nickel alloy). The form of the curve and loop determines the basic practical magnetic parameters and thereby the range of technical applicability of the material.

First of all, it is necessary to understand the physical nature of those processes in a ferromagnet that lead to the very occurrence of a magnetization curve. Here it is first necessary to answer the question: why, on cooling of a ferromagnetic specimen below the Curie point, does there not arise in it a uniform spontaneous moment throughout its whole volume; that is, why is there not parallelism of all the atomic moments throughout the whole volume of the body, as, it would seem, there should be in a ferromagnet? Experiment always shows that on cooling of a ferromagnetic body in a magnetic shield (with H = 0), it always remains superficially in a demagnetized state. As early as 1907, the French physicist P. Weiss[17] advanced the hypothesis of the existence of regions of spontaneous magnetization—domains. According to this hypothesis, a ferromagnet at H = 0 is divided into separate spontaneously magnetized regions—domains—in such a way that their resultant moment, for the whole specimen, is zero. Only in 1935 was the domain hypothesis given a theoretical basis, in the well-known work of L. D. Landau and E. M. Lifshitz[18]. From Fig. 12 it is evident that when the specimen is magnetized uniformly, there arises around it a strong self-field, possessing large positive energy. If the specimen is divided into domains (see Fig. 12 b and c), then the more there are of them, the weaker will be the self-field outside the specimen, and the less will be the volume occupied by it and its energy. It is possible to divide a specimen into domains in such a way that the magnetic flux is completely closed inside the specimen (see Fig. 12 d, e), and outside it the field vanishes; conse-

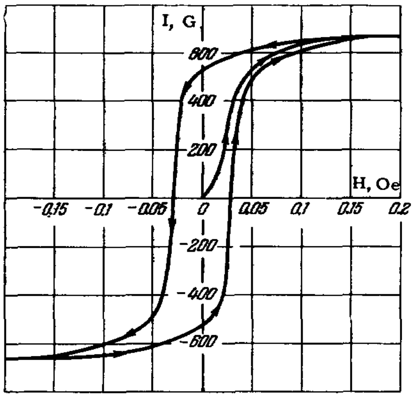


FIG. 11. Typical magnetization curve and hysteresis loop of a soft magnetic material (supermalloy).

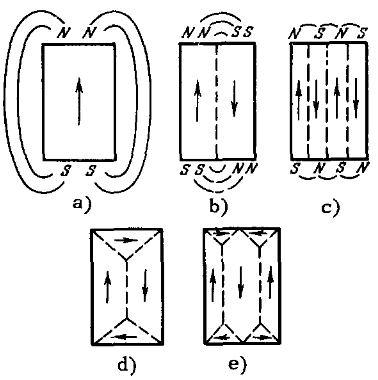


FIG. 12. Simplest types of division of a ferromagnetic specimen of rectangular shape into domains. a), specimen magnetized to saturation; b) and c), domain structure without closure domains; d) and e), domain structure with closure domains.

quently its energy also vanishes. Detailed consideration of the mechanism of domain formation shows that between the domains there arise boundary layers of finite thickness; their formation requires energy, but it is of course less than the volume energy of the vanished self-field of the specimen. Thus the formation of a domain structure is an effect of the self-demagnetization of ferromagnetic bodies.

The direction of the spontaneous magnetization in the domains and the arrangement of the boundaries between them are not random. They are connected with the orientations of the crystallographic axes in the crystallites of the ferromagnetic specimen. This relation is determined by the phenomenon of magnetic anisotropy.

The domain structure can be observed visually by various methods[19]. The simplest is the method of powder deposits. At places of emergence of the boundary layers on to the surface of the specimen, there is always, in small portions of the volume, a stray magnetic self-field. A magnetic powder is attracted to these places and portrays the emergence of the domain structure on to the surface of the crystal (Fig. 13) (Bitter-Akulov bands).

For observation of the domain structure, it is also possible to apply a magneto-optic method, by use of the Kerr or the Faraday effect: that is, rotation of the plane of polarization of light on reflection or transmission, respectively, of it through a magnetized medium. For example, a picture obtained by means of the Kerr effect on reflection of light from the surface of a ferromagnetic single crystal, possessing a plane-parallel domain structure, has the form of alternating dark and light regions, the magnetization directions in which are antiparallel. In contrast to the powder-deposit method, the magneto-optic methods show the whole surface of the domains (Fig. 14). In the case of thin ferromagnetic films, the electron-microscope method of studying domain structure is beginning to be applied successfully. It is evident from Fig. 15 that this method allows determination even of small perturbations of the uniformity of the magnetization inside the domains themselves.

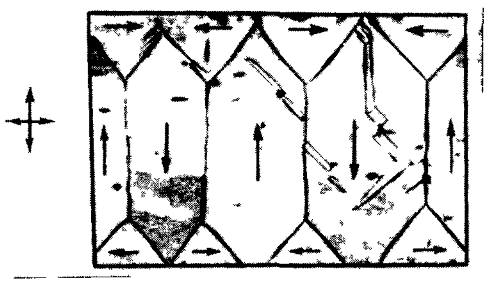


FIG. 13. Picture (obtained by the powder-deposition method) of the domain structure on the surface of a single crystal of an iron-silicon alloy.

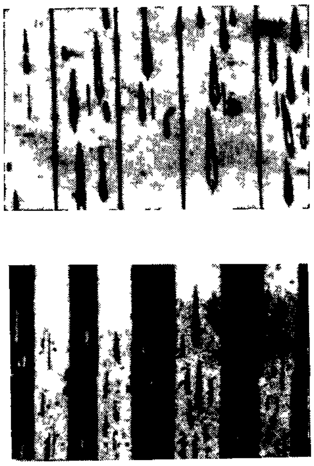


FIG. 14. Domain structure on the surface of a ferromagnetic crystal shown by the powder method (above) and the magneto-optic method (below).

Figure 16 shows a typical magnetization curve with an initial demagnetized state. Here also are shown photographs of the domain structures corresponding to various sections of the curve. Because the direction of the field in this case is parallel (or antiparallel) to the magnetization vector in the domains, the whole process of magnetization proceeds exclusively by displacement of the domain boundaries, until the domain structure is completely annihilated at saturation. If the angle between the field and the domain magnetization differs from 0° and 180°, then after disappearance of the domain structure there begins a process of rotation of the resultant moment into the direction of the external field.

The dimensions of the domains, their shapes, and the shape and energy of their boundaries are strongly dependent on structural inhomogeneities and on the shape and dimensions of the specimens and of the crystallites. This is the reason for the pronounced structure-sensitivity of the magnetic properties of materials. Near a defect the magnetization cannot be uniform, because this leads to the appearance of a stray magnetic field with large energy. Near a defect, therefore, there are formed local flux-closing domain structures (Fig. 17). Thanks to the closure regions that form on them, defects show a delaying effect on the motion of domain boundaries during the magnetization of a specimen in an external field Figure 18 shows how a structural defect ''glues'' a boundary to it during the passage of the boundary

FIG. 15. Picture of the domain structure of a thin ferromagnetic film (≈175 Å). Obtained by means of the electron microscope (× 1400).

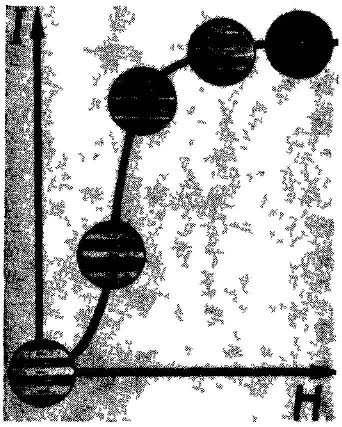


FIG. 16. Change of the domain structure of a ferromagnet in a magnetic field, in various sections of the magnetization curve.

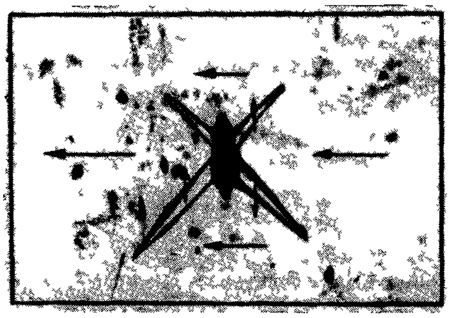


FIG. 17. Dagger-shaped secondary closure domains near a structural defect in a ferromagnetic crystal. The arrows show the direction of the magnetization.

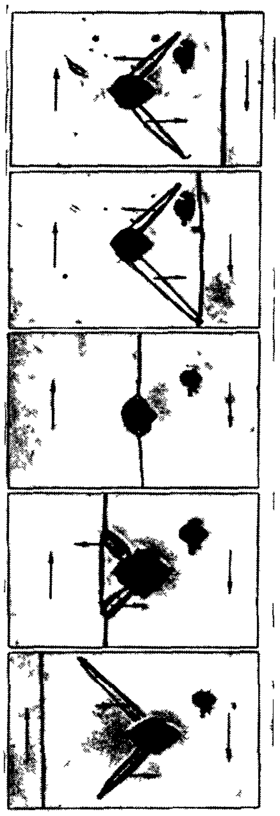


FIG. 18. Delay of the motion of a domain boundary in a ferromagnetic crystal by a structural defect. The arrows show the direction of the magnetization.

across a defect region of the crystal. Consequently, to facilitate the process of domain-boundary displacement it is necessary to have a material that is very pure and homogeneous with respect to structure and composition

It is very important to know the mechanism of magnetization reversal in ferromagnets, since in practical applications of magnetic materials they are magnetized alternately in the direct and in the reverse directions This takes place, for example, in all alternating-current equipment—in transformers, generators, and motors—and also in microwave technology. If after attainment of saturation the strength of the field is decreased, the magnetization as a rule will not decrease along the initial curve but will lag; in a complete cycle of change of the field (from saturation in one direction to saturation in the other, and return to the initial state), it describes a magnetic hysteresis loop (see Fig. 11). The magnetization on the loop at $H = 0$ is called the remanent magnetization I_r (the remanent induction is $B_r = 4\pi I_r$). The value of the field at $I = 0$ or $B = 0$ is called the coercive force, ${}_IH_c$ and ${}_BH_c$ respectively. In connection with the phenomenon of magnetic hysteresis we arrive at the concept of nuclei of magnetization reversal. Figure 19 shows the mechanism of appearance and growth of nuclei of magnetization reversal on the edge of a cobalt crystal. It is seen from the figure that upon diminution of the field from a state of saturation ($H = 2000$ Oe), there first appears a "paling" of fine, dagger-shaped secondary magnetic-flux-closing domains, with reverse magnetization. Then at fields ~ 500 Oe, some of these regions begin to grow, becoming transformed into nuclei of magnetization reversal. At a field ~ 100 Oe, we already observe a domain structure.

The magnetic properties of ferromagnetic specimens are greatly influenced by change of their geometric dimensions, especially when the dimensions of the specimen become comparable with the dimensions of a domain; that is, when the magnetic structure changes from multidomain to single-domain. In the case of pure iron, for example, solely as a result of a change of the dimensions of the specimen, from large dimensions to a powder with a diameter of a hundred angstroms, the coercive force increases from hundredths of an oersted to 1000 Oe, i.e., by five orders of magnitude.

7. In the classification of magnetic materials it is necessary to start from two basic requirements that are presented to them from the point of view of technology[20] It is necessary to have materials by means of which the largest possible magnetic flux can be produced with the smallest possible external magnetic

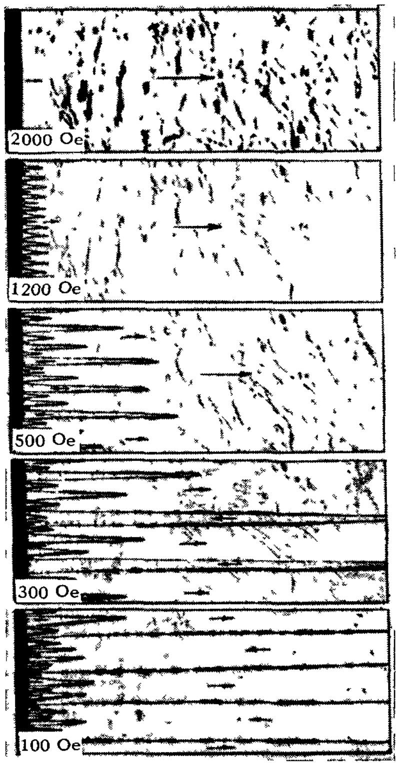


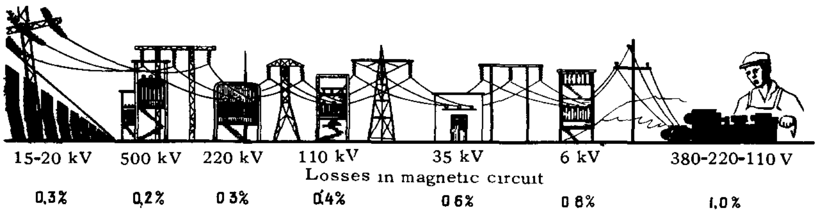
FIG. 19. Appearance and growth of nuclei of reversed magnetization at an edge of a single crystal of cobalt. The numbers give the size of the magnetic field intensity in oersteds, the arrows show the directions of the magnetization in the specimen.

field and with the smallest possible energy losses (this is possible if the magnetization in the material "follows easily" all the changes of external field), and also materials that by themselves, without the participation of external magnetizing sources, act as sources of a strong and stable (with respect to external fields, temperature, etc.) magnetic flux. Such materials are initially magnetized by means of external sources of field and are thereafter used in a state of remanent magnetization, i.e., as permanent magnets.

The first requirement is satisfied by soft magnetic materials, the second by hard or high-coercivity materials.

A soft magnetic material should have a magnetization curve with a large permeability (describing the rapidity with which the curve rises), attainable in very weak fields, and should have a very narrow hysteresis loop with a negligibly small coercive force. It is also desirable that these materials possess a high saturation and a high Curie point, and that they be poor conductors of electric current; this insures smallness of the losses due to Foucault currents in magnetization reversal. A material that can serve as an extreme example of "softness" is supermalloy, an iron-nickel alloy with molybdenum added, with initial permeability $\sim 10^5$ G/Oe, maximum permeability $\sim 10^6$ G/Oe, and $H_c \sim 0.002$ Oe.

The paramount importance of soft magnetic materials in the economy of a country is evident, for example, from the role played in it by sheet iron for transformers and dynamos. The quantity and quality of these materials determine the country's resources for energy equipment and consequently its economic power as a whole. The annual production of these materials in the whole world amounts to many millions of tons. Hence follows the enormous importance of the problem of producing very-high-quality magnetic materials for electrotechnology. To emphasize the public importance of the problem of improving the properties of these magnetic materials, it is sufficient to recall that in multistage transmission of electrical energy from power plants to the consumer, much energy is lost in generators, motors, and transformers through magnetization reversal and Foucault currents. One must also keep in mind that this transmission involves many stages of conversion. It is seen from Fig. 20 that in the year 1965 two hydroelectric power plants on the Volga were operating in the Soviet Union to compensate the losses in transmission lines, generators, and motors. In monetary terms this means losses in the billions of rubles. Improving the quality



Losses in whole line (depending on load) **2.5%–3.5%**

Total losses, with allowance for losses in windings and electric circuits, 6%

Total quantity of electrical energy produced

1965, 509 billion kWh
1966, 560 billion kWh } Losses } 30-40 billion kWh, or ≈2 hydroelectric power plants on the Volga
1980, 3000 billion kWh

FIG. 20. Losses in transmission of electrical energy in magnetic circuits of electrical machinery and of transformers.

of electrotechnical material not only results in economy through lowering of the losses, but also permits appreciable decrease in the size of the machines and the transformers, and this entails great saving of nonferrous metals and other scarce materials. Thus the problem of improving the quality of sheet iron for transformers and dynamos, especially in connection with the plans for swift growth of our power engineering, will acquire increasingly great urgency. It is no accident that very intensive work on this problem is going on in hundreds of laboratories in the world.

Besides electrotechnical magnetic sheet materials, the industry of all countries each year consumes an increasingly larger quantity of other high-quality soft magnetic materials, principally iron-nickel alloys of the permalloy type. Especially to be mentioned are the most important soft magnetic materials of the semiconductor class—the ferrites, which because of their high electrical resistivity are indispensable materials for microwave technology. Ferrites also play a major role in the construction of computers: magnetic memory devices are made from them. Recently much attention has been paid to the development of new magnetic materials—thin ferromagnetic films (of thickness ~ 1000 Å), principally of permalloy, which offer great prospects for future improvement of the operation of computing devices, because their magnetization-reversal speed is two to three orders of magnitude larger than that of ferrite toroids. This makes possible as great an increase of the speed of action of electronic mathematical machines (provided similar speeds can be attained in the other, nonmagnetic units of the machine).

A hard magnetic material, in order to fulfill its purpose as a stable source of strong magnetic field, should possess as wide a hysteresis loop as possible—as large as possible coercive force and remanent induction. The principal application of these materials is as permanent magnets. They are widely applied in very diverse fields of technology, and especially in instrument making: in electrical measuring apparatus, loudspeakers, sound recording, telephones, electric generators, magnetic lenses in electron microscopes, cathode-ray oscilloscopes, magnetic compasses, etc. The basic magnetic properties that determine the quality of a permanent magnet are described by the descending part of the hysteresis loop (Fig. 21), which describes the change of induction in a demagnetizing field from the state of remanent induction to zero induction ($B = 0$) at a reversed field equal to the coercive force ${}_BH_C$. Magnets usually serve to produce a field in an air gap between poles. It must be remembered that the field produced by a permanent magnet inside the magnet itself is opposite to its magnetization and therefore demagnetizes it, lowering the remanent induction to a value determined by the intersection of the descending loop with the shearing line (see Fig. 21). The larger the gap, the stronger is this demagnetization effect and, consequently, the smaller is the induction in the magnet. Therefore the choice of a magnet must be strictly compatible with its shape. It is seen from the curves in Fig. 21 that in order to retain a higher induction in a magnet, it is necessary to have the highest possible coercive forces ${}_BH_C$. The figure also gives the values of the maximum demagnetization energy $(BH)_{max}$ in G-Oe and the equivalent volumes for operation of the material at the point of maximum energy. Also presented here is a comparison of the dimensions of magnets in the form of disks (the least advantageous shape in the demagnetization sense) made of Co-Pt alloy and of cobalt steel. Because of the high coercive force of Co-Pt, a disk of this alloy with diameter 2 mm and thickness 0.05 mm produces the same magnetic effect as does a disk of cobalt steel with D = 16 mm and d = 0.4 mm, though the volume of the latter is some 500 times larger. Hence it is clear that the quality of high-coercivity materials can in a number of cases limit the development of technology, when it is necessary to product miniature technological devices: for example, in use of magnets in radio-electronic devices, in a watch mechanism, etc. The physics of high-coercivity (h-c) materials faces great and difficult problems in the production of new materials with demagnetization energy above 10^7 G-Oe, with coercive force greater than 5000 Oe, whose composition would not include scarce materials like platinum.

Besides soft and hard magnetic materials, still to be mentioned are magnetostrictive materials, from which ultrasonic radiators and receivers are prepared (in these, use is made of the phenomen of magnetostriction, i.e., the change of dimensions of a ferromagnet upon magnetization of it).

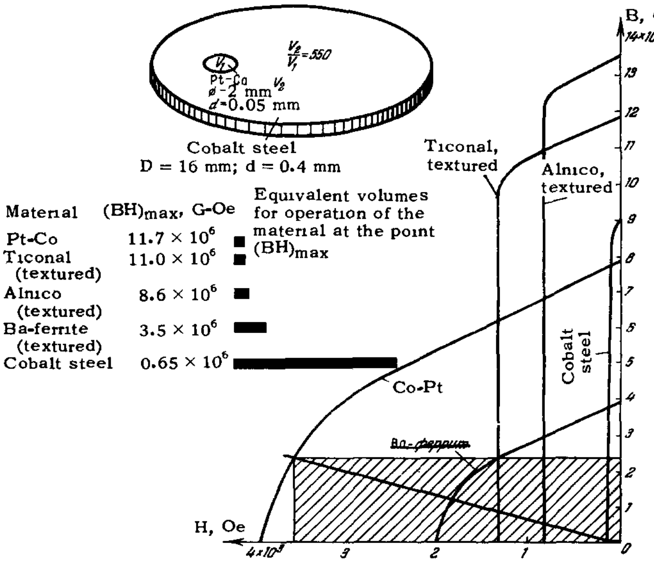


FIG. 21. Basic magnetic characteristics of hard (high-coercivity) magnetic materials.

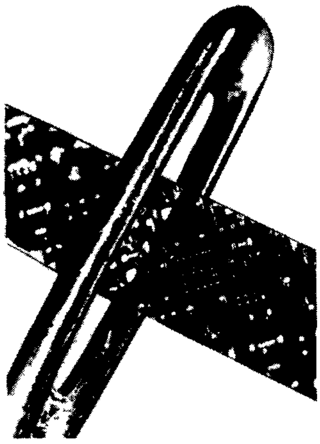


FIG. 22. Graphic illustration of the miniaturization of radio-electronic systems by use of magnetic materials. An element of the system can be threaded through the eye of an ordinary darning needle.

In modern technology, use is made of the most diverse magnetic materials, with various physical properties. This is entirely natural, because the problems that are solved in technology with the aid of magnetic materials are also very diverse. By way of illustration of this diversity, it can first of all be pointed out that the range of coercive forces in the magnetic materials already in use in technology extends from 10^{-3} Oe to 10^4 Oe, and the range of maximum permeabilities from 1 to 5×10^6 G/Oe. A second clear example is provided by a comparison of the miniature magnetic units of a computing device (Fig. 22) and of the gigantic magnetic circuit of an elementary-particle accelerator, still far from the limiting size (Fig. 23).

We shall digress on some questions about the use of magnetic materials in the field of magnetic measurement technology[22]. For example, for measurement of constant magnetic fields use is made of a high-sensitivity ferroprobe apparatus with permalloy cores. It is universal practice to use ferroprobe magnetometers, which measure very weak fields in very small volumes. For example, magnetometers with ferroprobes of length 50 to 70 mm permit measurement of a field of order 10^{-6} Oe (0.1 gamma). These magnetometers are used for measurement of the earth's magnetic field and its variations, for geophysical exploration (with airplanes or on the ground), for determination of the stray magnetic field around electric machinery and transformers, and for detection of ferromagnetic inclusions in objects made of nonmagnetic or weakly magnetic materials (for example, in disks of turbojet engines). Magnetometers with ferroprobes of length 5 to 1.5 mm permit measurement of constant fields of 0.05 Oe and below. They are applied in magnetic-ferroprobe flaw-detection for detecting disruptions of continuity in steel objects. Figure 24 shows the core of a ferroprobe of this sort in comparison with a match-head. In the last few years, ferroprobe magnetometers have found application also in medicine, in the surgical removal of ferromagnetic foreign objects from the body of a human being (fragments of sewing or hypodermic needles, bullets, mine and shell splinters, etc.). The chief advantage of this method lies in the fact that ferroprobes make it possible to discover and make precise the location of the object in the course of the operation. Their application has led to the saving of many human lives.

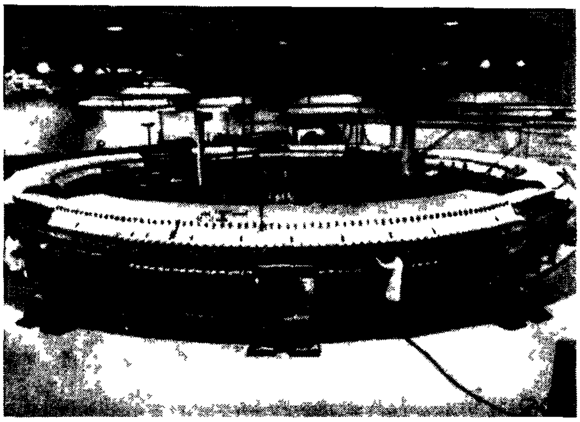


FIG. 23. General view of the magnetic cosmotron at Brookhaven (USA).

Ferroprobe magnetic measurements are only a negligible part of all the inexhaustible possibilities of magnetic measurements, which can be placed at the service of science, technology, and man's way of life.

In view of the fact that the bulk of technical objects are ferromagnetic (iron, steel), a broad possibility is opened for magnetic control of their structure (magnetic structure analysis) and, consequently, of their mechanical stability properties; also for exposure of defects of the disruption-of-continuity type (magnetic flaw detection). The principal advantage of magnetic methods of control lies in the fact that they make it possible to test industrial parts without destroying them. This guarantees the possibility of continuous control of all products at various stages of their production technology and at the time of its technical exploitation. These methods are open to automatization, and therefore their application is especially important at the present stage of broad development of automatization of industrial processes.

Imagine, for example, modern automatized tube-rolling production, in which a rolling mill discharges many meters of tubes in one second. After this they drop on to a section of the control where a whole army

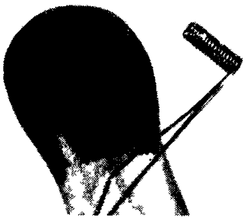


FIG. 24. Graphic comparison of a miniature ferroprobe (located inside the coil) with the head of a match.

of several hundred inspectors, with magnifying glasses, visually check the quality of this important product. Such control is an anachronism: it invalidates the whole advantage of automatic production by introducing a large number of subjective errors; it has no quantitative criterion for correct rejection of faulty pieces. Finally, under mass-production conditions this type of control requires an enormous staff of workers on an unproductive operation. Application of automatic magnetic methods of control in this manufacture and in other branches of mechanical and metallurgical industry makes it possible to put the control of the quality of the product on an incomparably higher level, and thereby to increase the reliability and durability of the machines and apparatus being manufactured. Very important also is the application of magnetic methods of control to transportation, in the control of tracks being laid in a railroad (magnetic flaw-detection cars), control of the wheel inclinations of railroad cars, etc.

To illustrate how broad a range of magnetic information about material structure can be obtained by magnetic measurements, it is worth while to cite the following two examples: 1) use of the magnetic properties of nucleons and of other heavy, strongly interacting particles (hadrons) for verification of the most general contemporary group-theoretical approach in the theory of strong interactions of elementary particles (the quark hypothesis) [23] or in the determination, by magnetic scattering of fast electrons, of the magnetic form factors of nucleons, measurement of which reveals the internal geometry of elementary particles [24]; 2) use of a magnetostructural method of investigation in the study of the details of decomposition processes in solid solutions, by determination of the properties of the superparamagnetism of very fine (tens of Å) precipitates of a ferromagnetic phase; then their spontaneous moment behaves like a giant molecule of a paramagnetic gas [25]. These two examples show, sufficiently convincingly, the extremely broad range of "magnetic" scientific information about the structure of matter. In the first example we have to do with elementary particles; in the second, with macroscopic properties of a solid.

Even within the "classical" magnetic-materials field, we learn each day that determination of the magnetization curve—that "macro-passport" of magnetic materials—is only the very first step in their study. Technological practice in their use and theoretical and experimental physical investigations of these materials require much deeper penetration inside their atomic structure. It is now quite insufficient, for example, to study only the domain structure. It is necessary to know in detail the inhomogeneities of the distribution of atomic magnetic moments within the volume of an individual domain. It is therefore no accident that physicists are throwing into the study of the mechanism of magnetization processes, and of the details of domain and atomic structure, the whole arsenal of powerful modern experimental resources of electron and nuclear physics—such as ferromagnetic resonance, nuclear magnetic resonance, the Mössbauer effect, neutron diffraction, x-ray spectroscopy, electron microscopy, study of the complex of physicochemical properties at low and very low temperatures and high pressures, etc. It is important to us to know not only the over-all macrocharacteristics of a magnet, for example its magnetization, Curie point, etc., but also the distribution and orientation of the individual atomic magnetic moments on individual sites of the crystalline lattice, and even the spatial distribution of the density of this moment, etc. Only such depth and detail in microscopic physical investigations is capable of opening up before us new perspectives for the practical application of magnetism in all conceivable aspects, or will put into the hands of the physicist and engineer reliable means for the conscious and skillful control of the processes that go on in magnetic materials. Production of the most modern magnetic materials is one of the most important conditions for technical progress as a whole. This problem has and will have very great importance in our country in connection with the great program for building a Communist society.

The physics of magnetic phenomena as a whole and the physics of magnetic materials in particular are receiving great attention over the whole world. Much attention is being devoted to this important branch of science in the Soviet Union, too. It must be remembered that the present state of the theory of ferromagnetism gives us so far only a general qualitative indication of a direction in which to seek a solution of developing new magnetic materials. Therefore for specific solution of this most important practical problem, the path followed everywhere will of necessity be the manufacture of large experimental batches of varied alloys and compounds, with the transitional elements as a basis, and the choice from among them of the best according to their technological magnetic characteristics. For successful and most rapid completion of this laborious program, it is necessary to have a large quantity of extremely pure materials, in single-crystal form, with a powerful modern basis of metallurgy. Specimens of the experimental alloys must be subjected to careful and throrough study with the aid of the whole arsenal of modern means of physical experiment mentioned above. These investigations will therefore require complicated and costly equipment and the expenditure of large resources. It is nevertheless necessary to make these expenditures, because the discovery and introduction of new magnetic materials, new magnetic methods of measurement and control, and new magnetic control apparatus will bring incomparably greater technico-economic advantages and will strengthen still further the economic power of the State.

Magnetism has been open to man for several thousand years. Yet in that time it has not aged but has grown continuously more youthful and has become ever more inexhaustible in the diversity of its manifestations—so complicated is the dialectic of the development of science. The physics of magnetic phenomena—a branch of science old in origin, but perpetually young in its development—always opens up a wide field of activity for young talents, that they by their inspired work may discover and place at the service of man new and ever more unexpected manifestations of the universal magnetic interaction of natural matter.

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