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NONLINEAR EFFECTS IN A PLASMA

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INTRODUCTION

a) Scope

THE interest in problems involving the theory of nonlinear interaction of electromagnetic waves in a plasma and of nonlinear effects in a plasma dates back quite a while (see [1,2]). Only recently, however, has the theory of nonlinear-interaction effects been persistently pursued and developed to the extent of deriving clear-cut physical notions concerning the mechanisms of such an interaction. This development is connected with attempts at solving a number of problems in turbulent-plasma physics, strongly stimulated by the rapidly developing experimental research. We must limit from the very outset the scope of the nonlinear physical effects that will be considered in the present review. We confine ourselves to nonlinear effects arising in a plasma in the absence of collisions. This means that we consider only sufficiently rapid nonlinear interactions, occurring within times much shorter than the collision time τ_{coll}. A large number of problems involving nonlinear effects, in which collisions play a significant role, and in particular the dependence of the effective number of collisions on the electromagnetic fields, etc., is considered in the review by V. L. Ginzburg and A. V. Gurevich [3]. We confine ourselves henceforth to the interaction between waves having random phases. This means that the characteristic time during which the nonlinear interaction sets in is much longer than the characteristic time in which the wave phase changes. In the general case it is expedient to speak not of the characteristic phase-change time, but more accurately the characteristic field correlation time τ_c. Thus, we shall assume that the time of the nonlinear interaction of the waves satisfies the inequalities

$$\tau_c \ll \tau \ll \tau_{coll}. \tag{1}$$

Another essential limitation is the assumption of weak nonlinearity. To this end it is necessary, in general, that the wave energy in the plasma not exceed a certain value W_{max}, determined by the possibility of expanding the nonlinear interaction in terms of the amplitudes of the interacting waves. In the weak-nonlinearity approximation we must consider the interaction of linear oscillations and plasma waves, whose spectra are well known (see [4-6]). Particular interest attaches to weakly-damped waves, since such waves can be easily excited and the energy contained in them may be high. The intensity of strongly-damped waves is quite small and its order of magnitude is the same as of the thermal-fluctuation intensity. An essential assumption restricting the number of problems to be discussed further is that the energy of the interaction waves be much larger than the energy of the equilibrium fluctuations W_T. If W is the wave energy density per cm^3, then the required conditions are

$$W_T \ll W \ll W_{max}. \tag{2}$$

It must be emphasized that W_{max} is frequently quite high, and the weak-nonlinearity approximation describes the wave interaction in a wide range of values of the intensity W.

If $W \gg W_T$ then, as a rule, the interaction between particles via the waves, and also the nonlinear interaction between the waves themselves, prevails over pair collisions. It is precisely under these conditions that the collective processes in the plasma begin to play the dominating role. Usually the plasma is called turbulent when $W \gg W_T$, and weakly turbulent when $W \ll W_{max}$. We confine ourselves here to an analysis of the results obtained on wave interaction in a weakly turbulent plasma. It is appropriate to dwell here briefly on the history of research in the theory of nonlinear effects. As already noted, nonlinear effects for transverse waves in a plasma were considered relatively long ago in connection with ratio wave propagation [1-3]. The first to raise the question of nonlinear interaction of Langmuir oscillations was Sturrock [21], and the problem of interaction between Langmuir and transverse waves was raised by Ginzburg and Zheleznyakov [65]. Further development of the theory was by Drummond and Pines [22] and Shapiro [23]*. A specially important role was played by investigations of the quasilinear approximation [22-24,44,92-95], although these questions do not pertain directly to the problem of nonlinear interaction. In the quasilinear approach, account is taken only of the reaction of the generated oscillations on the object of generation (say, the beam particles).

*Closely related to nonlinear-interaction problems is research on the scattering of electromagnetic waves by a plasma, carried out by I. A. Akhiezer, A. I. Akhiezer, and A. G. Sitenko [82] (see [67]), by Rosenbluth and Rostocker [8], and by Salpeter [80].

These effects are similar to those well known as quantum-generator saturation effects [25]. The first to attempt an investigation of nonlinear effects proper in a plasma, within the framework of the kinetic approach, were Kadomtsev and Petviashvili [26] (see [16], Karpman [27], Galeev, Karpman, and Sagdeev [28], Camac [29] (see [30]), Rudakov, Vedenov, et al. [41,99], Akhiezer, Daneliya, and Tsintsadze [31,85], and others [32], and by Gaĭlitis and Tsytovich [18-20]. Many problems in the theory of nonlinear effects was considered by Silin [71], who used the correlation-function method developed by Bogolyubov [96]. Although the latter method is strictly speaking more consistant and permits allowance to be made for pair-collision effects, the greater complexity of the mathematical formalism does not permit its extensive use for various nonlinear interactions, with the exception of the case considered in [71,33], that of particles interacting in accord with Coulomb's law. In the investigation of more complicated interactions [34] it becomes necessary to use the usual formalism of perturbation theory, similar to that in [26]. It must perhaps be noted that the perturbation theory formalism [26,28] is quite cumbersome. Its complexity has led to numerous errors in the calculations presented in the original papers, and in the determination of the limits of applicability of the results. Sometimes no such limits are indicated at all. A large number of problems involving nonlinear interaction was investigated by using the kinetic equation for plasmons, with account taken of induced scattering processes (Gaĭlitis and Tsytovich [18,20], Kovrizhnykh and Tsytovich [19], Tsytovich [18a,15], Kovrizhnykh [73], and Liperovskiĭ and Tsytovich [74] (see [97,98])). With this method it is possible not only to present a simple interpretation in the language of probability theory, using diagram techniques, and to reveal certain inaccuracies, but also to investigate in simple manner problems which appear complicated when perturbation theory is used, such as the interaction between longitudinal and transverse waves [19], interaction of waves in a magnetoactive anisotropic plasma (Tsytovich and Shvartsburg [35]), relativistic effects [18], and others [35,34,20]. It has become clear recently, for example, that nonlinear effects play a very important role in the analysis of problems involved in the origin of cosmic rays (Ginzburg [38], Tsytovich [14,37,39]), when relativistic effects can no longer be disregarded.

Before we proceed to a detailed exposition of the material, we must make a few remarks concerning the variety of fields of application and procedures for calculating nonlinear effects.

b) Nonlinear Effects and Plasma Physics

We must point out first that the theory of nonlinear effects can be extensively used in the analysis of plasma instabilities (for a review of plasma instabilities see [7]). Frequently the development of the instability leads to the occurrence of intense random turbulent oscillations. The nonlinear effects determine the amplitudes of the oscillations and consequently the physical processes arising in the turbulent states of the plasma. The extent to which an analysis of the nonlinear effects is essential can be seen from the following simple example. Let us consider the well known plasma instability that sets in when beams interact with a plasma. The generation of Langmuir oscillations by the particle beam is under certain conditions perfectly analogous to the generation of waves in systems with so-called negative temperature. In this case the mechanism producing the radiation may be Cerenkov excitation of the Langmuir wave by the beam particle, and the "inverse population" is connected with the presence of the particle beam. As a rule, the generated oscillations have here phase velocities on the order of the beam velocity. If the nonlinear effects can divert the oscillations from that region of the phase velocity, in which they can be generated by the beam, with sufficient speed, then cascade-like development of the stimulated emission processes ceases and the instability is stabilized [36]. Thus, this effect makes the relativistic beams stable [39], a factor having important consequences for the astrophysics of cosmic rays.

This example illustrates the general premise that only the investigation of nonlinear effects can determine the practical significance of various plasma instabilities. The nonlinear effects can also determine the efficiency with which plasma is heated by different methods. Thus, if the heating is accompanied by intense plasma oscillations, then the nonlinear conversion of these oscillations into transverse waves leaving the plasma can serve as an additional source of plasma cooling [31]. Closely related to the problem of plasma confinement is the question of plasma diffusion. In the presence of intense oscillations in the plasma, the plasma diffusion is connected not with pair collisions of the particles, but principally with "collisions" between particles and oscillations. Such a diffusion is frequently called anomalous [16]. The nonlinear effects that limit the oscillation amplitudes determine by the same token the so-called anomalous diffusion. As a rule, the anomalous diffusion is determined by the long-wave oscillations. The nonlinear wave conversion that leads to the Langmuir oscillations from other types of waves is therefore of interest for plasma confinement problems.

The nonlinear effects also play an important role when the plasma is acted on by external fields. For example, if the plasma is in a sufficiently strong constant electric field $E > E_D$, where $E_D \sim e^2\omega_{0e}^2/v_{Te}^2$ is the so-called Dreicer field, then the pair collisions are unable to dissipate the energy acquired by the electron on the mean free path, and so-called runaway electrons are produced [88]. The appearance of the runaway electrons results in plasma instability, and oscillations are generated; then the principal role is taken not by the pair collisions, but the "collisions" between the runaway electrons and the plasma oscillations, which lead to anomalous plasma resistance, much higher than the usual resistance due to pair collisions [62, 89, 59, 101].

If a weak electromagnetic wave is incident on a plasma in which oscillations are excited, then this wave becomes scattered by the oscillations and can yield valuable information on the oscillation spectra and intensities and on other plasma parameters, particularly its temperature, density, etc.[15, 19, 66]. Recently there has been extensive development of experimental research in which nonlinear effects are used for plasma diagnostics [75, 76, 78, 83, 84] This method will apparently be most important in the investigation of cosmic-plasma turbulence [37]. Definite progress has by now been made also in experimental investigations of turbulent plasma [62]. Notice should be taken here first of investigation of two-stream plasma instabilities, particularly the observed high efficiency of interaction between the plasma and the beams (Faĭnberg et al.[8], Nezlin [9], Alekseff and Neidigh [10], Zavoĭskiĭ [11, 62]), the investigation of anomalous turbulent diffusion [12, 16], and the nonlinear coupling between high-frequency and low-frequency plasma oscillations [75]. Many problems in the turbulence of laboratory plasma were considered in the review [16], and the investigation of astrophysical problems involving turbulent plasma and cosmic-ray acceleration is the subject of the reviews [13, 86, 87]. Finally, great interest has been evinced in problems of interaction between intense electromagnetic waves and a plasma, particularly high-frequency fields, lasers [98, 90], and shock waves [94, 100].

c) Problems Involved in Procedures for Calculating Nonlinear Effects

Nonlinear effects give rise to coupling between different plasma waves, different oscillation modes, whose number in the plasma are quite large, and therefore the original articles deal as a rule with only some particular types of interaction. At the same time, it is necessary to cover more or less fully all types of interaction in order to be able to obtain a general picture that permits evaluation of the relative roles of the different interactions and the possibility of their different applications. The difficulties in the proper interpretation of the new important results obtained recently are connected in part with the very complicated mathematical formalism used in the original research. The purpose of this review is to describe the principal results obtained in this field in a simple form, which admits of a lucid physical interpretation. Our exposition is based on the coupling between the spontaneous and induced processes, which is familiar to a large circle of readers at least as a result of the vigorous development of the physics of quantum generators and amplifiers. In fact, nonlinear effects in a plasma can be related to a number of induced processes similar to those known in nonlinear optics [17, 91]. Thus, stimulated Raman scattering, which is well known in nonlinear optics, is produced also in a plasma. In a plasma such processes are usually called "decays." It is advantageous to illustrate here, by means of a simple example, how nonlinear decay interactions, which are obtained in simplest fashion from the coupling between spontaneous and induced transitions, can be investigated by Bloembergen's method [91], which is widely used in nonlinear optics. This example will also disclose clearly the limits of applicability of the customarily employed approach. The main quantity usually involved in Bloembergen's method [91] is the nonlinear polarization $\mathbf{P}$ or the nonlinear current $\mathbf{j} = \partial \mathbf{P}/\partial t$ which is connected with it. As shown in [19], in the case of high frequency waves such a nonlinear current produces in the plasma an interaction between the transverse and longitudinal plasma waves (but not between the transverse waves themselves). The dynamics of this interaction is determined by Maxwell's equations with account taken of the indicated nonlinear current ($\mathbf{j}^{(1)}$ and $\mathbf{j}^{(2)}$ are respectively the linear and nonlinear currents of the plasma) ($c = 1$):

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{\partial^2\mathbf{E}}{\partial t^2}=-4\pi\frac{\partial}{\partial t}(\mathbf{j}^{(1)}+\mathbf{j}^{(2)}),$$

$$\mathbf{j}^{(2)}=\int \mathbf{j}^{(2)}_{\mathbf{k},\omega}e^{-i\omega t+i\mathbf{kr}}\,d\lambda,$$

$$\mathbf{j}^{(2)}_{\mathbf{k},\omega}=-\frac{\omega_{0e}^2 e}{8\pi m_e}\left[\frac{\mathbf{k}}{\omega}(\mathbf{E}_{\mathbf{k}_1,\omega_1}\ \mathbf{E}_{\mathbf{k}_2,\omega_2})+2\mathbf{E}_{\mathbf{k}_1,\omega_1}\left(\frac{\mathbf{k}_2}{\omega_2}\mathbf{E}_{\mathbf{k}_2,\omega_2}\right)\right],$$

$$d\lambda=\frac{d\omega_1\,d\omega_2}{\omega_1\omega_2}\,d\mathbf{k}_1\,d\mathbf{k}_2\delta(\omega-\omega_1-\omega_2)\,\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2). \tag{3*}$$

It is simplest to find a solution of (3) by the Bogolyubov-van der Pol method [102], namely, it is known that in the linear approximation the solution of (3) corresponds for an isotropic plasma to independent propagation of longitudinal and transverse waves:

$$\mathbf{E}=\int \mathbf{E}^l_{\mathbf{k}}\,d\mathbf{k}\,e^{i\mathbf{kr}-i\omega^l(\mathbf{k})t}+\int \mathbf{E}^t_{\mathbf{k}}\,d\mathbf{k}\,e^{i\mathbf{kr}-i\omega^t(\mathbf{k})t}. \tag{4}$$

Taking the weak nonlinearity described by $\mathbf{j}^{(2)}$ into account, we can seek a solution of (3) in the form (4) assuming the amplitudes of the longitudinal waves $\mathbf{E}^l_{\mathbf{k}}$ and of the transverse waves $\mathbf{E}^t_{\mathbf{k}}$ to be slowly varying functions of the coordinates of the time (compared with $1/k$ and $1/\omega$). Then substitution of (4) in (3) in the simplest case, when the amplitudes depend only on the time, leads to the equations

$$\frac{\partial E^l_{\mathbf{k}}}{\partial t}=-\frac{\epsilon\omega_{0e}k}{4m_e}\int\frac{d\mathbf{k}_1\,d\mathbf{k}_2}{\omega^t(\mathbf{k}_1)\,\omega^t(\mathbf{k}_2)}E^t_{\mathbf{k}_1}E^{t*}_{\mathbf{k}_2}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)(\mathbf{e}_{\mathbf{k}_1}\mathbf{e}^*_{\mathbf{k}_2})\,e^{i\Delta\omega t}, \tag{5}$$

$$\frac{\partial E^t_{\mathbf{k}_1}}{\partial t}=\frac{\epsilon\omega_{0e}^2}{2\omega^t(\mathbf{k}_1)}\int\frac{(\omega^t(\mathbf{k}_2)+\omega^l(\mathbf{k}))\,k\,d\mathbf{k}\,d\mathbf{k}_1}{m_e\,\omega^t(\mathbf{k}_2)\,(\omega^l(\mathbf{k}))^2} \tag{6}$$

$$\times\ E^l_{\mathbf{k}}E^t_{\mathbf{k}_2}(\mathbf{e}^*_{\mathbf{k}_1}\mathbf{e}_{\mathbf{k}_2})\,e^{-i\Delta\omega t}\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2),$$

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*rot ≡ curl.

$$\frac{\partial E^t_{\mathbf{k}_2}}{\partial t} = -\frac{e\omega^2_{0e}}{2\omega^t(\mathbf{k}_2)} \int \frac{(\omega^t(\mathbf{k}_1) - \omega^l(\mathbf{k}))\,k}{m_e\,\omega^t(\mathbf{k}_1)\,(\omega^l(\mathbf{k}))^2}\,d\mathbf{k}\,d\mathbf{k}_2 E^{l*}_{\mathbf{k}} E^t_{\mathbf{k}_1} \times (\mathbf{e}_{\mathbf{k}_1}\mathbf{e}^*_{\mathbf{k}_2})\,e^{i\Delta\omega t}\delta(\mathbf{k} - \mathbf{k}_1 + \mathbf{k}_2), \tag{7}$$

where $\mathbf{e}_{\mathbf{k}_1}$ and $\mathbf{e}_{\mathbf{k}_2}$ are unit vectors of the polarization of the transverse waves, and $\Delta\omega = \omega^l(\mathbf{k}) - \omega^t(\mathbf{k}_1) + \omega^t(\mathbf{k}_2)$ is the frequency detuning, which characterizes the so-called phase synchronism of the interacting waves. If the interacting-wave phases are not random, then intense interaction can be expected only if the sign of the nonlinear interaction does not change appreciably during the characteristic interaction time τ, i.e., $\Delta\omega\tau \ll 1$. If furthermore the interacting waves are almost monochromatic (i.e., $E_{\mathbf{k}} \cong E\delta(\mathbf{k} - \mathbf{k}_0)$), then we easily obtain from the system (5)—(7) Bloembergen's algebraic equations (see (see [103,104]), which describe the mutual pumping of energy from the transverse to the longitudinal waves and back, the characteristic time of such pumping, as seen directly from (5) and (6), being proportional to the first power of the effective wave amplitude. Other results are obtained for random waves, when $\langle E_{\mathbf{k}}E^*_{\mathbf{k}'}\rangle = |E_{\mathbf{k}}|^2\delta(\mathbf{k} - \mathbf{k}')$. We note that in order for the wave to be regarded as random it is necessary that the width $\delta\omega$ of its spectrum satisfy the condition $\delta\omega\tau \gg 1$, and consequently $\Delta\omega\tau \gg 1$. Thus, the interaction between the random wave corresponds to a limiting case which is just the opposite of the phase-synchronism condition. The fact that $\exp(i\Delta\omega t)$ reverses sign many times during the interaction time has no particular significance in this case, since the random amplitude $E_{\mathbf{k}}$ varies even more rapidly. We can now obtain from the system (5)—(7) an equation for the mean-square values $|E_{\mathbf{k}}|^2$, multiplying, for example, each of the equations by $E^*_{\mathbf{k}}$. Account must be taken of the fact that the mean values of the cubes of the fields $\langle E_{\mathbf{k}}E_{\mathbf{k}_1}E_{\mathbf{k}_2}\rangle$ do not vanish and can be expressed with the aid of nonlinear equations in terms of $\langle E_{\mathbf{k}}E_{\mathbf{k}_1}E_{\mathbf{k}_2}E_{\mathbf{k}_3}\rangle$, and the latter can be expressed in first approximation in terms of $\langle E_{\mathbf{k}}E_{\mathbf{k}_1}\rangle\langle E_{\mathbf{k}_2}E_{\mathbf{k}_3}\rangle$, i.e., $|E_{\mathbf{k}}|^2|E_{\mathbf{k}'}|^2$. Let us write out, for example, the equation obtained for $|E^l_{\mathbf{k}}|^2$:

$$\frac{\partial N^l_{\mathbf{k}}}{\partial t} = \int w_{\mathbf{k}}(\mathbf{k}_1, \mathbf{k}_2)\,d\mathbf{k}_1\,d\mathbf{k}_2\,\{N^t_{\mathbf{k}_1}N^t_{\mathbf{k}_2} - N^t_{\mathbf{k}_2}N^l_{\mathbf{k}} + N^t_{\mathbf{k}_1}N^l_{\mathbf{k}}\},$$

$$N^l_{\mathbf{k}} = \frac{2}{\omega_{0e}}\pi^2|E^l_{\mathbf{k}}|^2; \qquad N^t_{\mathbf{k}} = 2\frac{\pi^2}{\omega t}|E^t_{\mathbf{k}}|^2;$$

$$w_{\mathbf{k}}(\mathbf{k}_1\mathbf{k}_2) = \frac{e^2\omega_{0e}k^2}{16\pi m_e^2\omega_1\omega_2}\left(1 + \frac{(\mathbf{k}_1\mathbf{k}_2)^2}{\mathbf{k}_1^2\mathbf{k}_2^2}\right)\delta(\omega - \omega_1 + \omega_2)\,\delta(\mathbf{k}_1 - \mathbf{k}_1 - \mathbf{k}_2).$$

$w_{\mathbf{k}}(\mathbf{k}_1\mathbf{k}_2)$ corresponds to the probability obtained in [19] for the stimulated Raman scattering of transverse waves by the generated longitudinal waves. We shall also see that the simple probability considerations and the coupling between the spontaneous and induced processes leads directly to equations of type (8). To calculate the probabilities, on the other hand, there is not need at all to derive the complete equation (8), and it is sufficient to use its limiting expression (for example as $N^l_{\mathbf{k}} \to 0$), and all the calculations become exceedingly simple. It must also be noted that inasmuch as a statistical description is used for the interaction of the random waves, the nonlinear mutual pumping (8), unlike the Bloembergen approximation [91,103], may be irreversible. Finally, the characteristic time intervals of nonlinear pumping (8) turn out to be already inversely proportional to the square of the wave amplitudes. Simple considerations connected with the probabilities were used in [18]. This method makes it possible to treat simply and in intuitive physical form the complicated problems of nonlinear interaction. We shall follow the procedure of [18], which was subsequently developed further in [19,20,15,36,73].

It should be noted that the approach used below is perfectly analogous to that extensively used in solid-state theory [40], and therefore allows us to consider from a unified point of view effects of nonlinear interaction of waves in the so-called solid-state plasma [41], interest in which has greatly increased recently in connection with new experimental advances. We note that the diagram method in the theory of turbulence of liquids was used first in [42] for a system of weakly interacting particles, such as a plasma, by Gaĭlitis and Tsytovich [18a], and for a strongly turbulent plasma by Mikhaĭlovskiĭ [43]. We shall use below diagram representations for the simplest processes, so as to clarify the elementary processes on which the nonlinear interaction is based.

The review contains two chapters. The first is devoted to the general theory of wave interaction in spatially dispersive media, and the second to nonlinear interaction of waves in an isotropic plasma.

I. GENERAL THEORY OF WAVE INTERACTION IN SPATIALLY DISPERSIVE MEDIA

1. Concept of Number of Waves. Connection between Spontaneous and Induced Processes

As already noted, we shall consider the interaction of linear waves. In an arbitrary homogeneous anisotropic medium, Maxwell's equation, as is well known [4-6], are satisfied by solutions corresponding to plane waves $\sim \exp[-i\omega t + i\mathbf{k}\cdot\mathbf{r}]$ only for a definite relation between ω and $\mathbf{k}$, corresponding to different modes σ (normal waves) of the oscillations of the medium. The total electromagnetic field is a superposition of fields of different normal modes

$$\mathbf{E} = \sum_\sigma \mathbf{E}_{\mathbf{k}\sigma}\,d\mathbf{k}e^{-i\omega_\sigma(\mathbf{k}) + i\mathbf{k}\mathbf{r}}, \quad \omega_\sigma(-\mathbf{k}) = -\omega_\sigma(\mathbf{k}).$$

Here $\mathbf{E}_{\mathbf{k}\sigma}$ is the amplitude of wave σ, which depends on $\mathbf{k}$. It is convenient to introduce the unit vectors of polarization [4,5]

$$\mathbf{E}_{\mathbf{k}\sigma} = E_{\mathbf{k}\sigma}\mathbf{a}_{\mathbf{k}\sigma}, \quad \mathbf{a}_{\mathbf{k}\sigma}\mathbf{a}^*_{\mathbf{k}\sigma} = 1,$$

whose different components are connected by relations determined from Maxwell's equations

$$(k^2\delta_{ij} - k_ik_j - \omega_\sigma^2\varepsilon_{ij}(\omega_\sigma, \mathbf{k}))\, a_{ik\sigma} = 0, \tag{1.1}$$

where $\epsilon_{ij}(\omega, \mathbf{k})$ is the dielectric tensor.

A very convenient concept is that of the dielectric constant for a specified normal wave σ, defined by

$$\varepsilon^\sigma(\omega, \mathbf{k}) = \varepsilon_{ij}(\omega, \mathbf{k})\, a^*_{\mathbf{k}\sigma i} a_{\mathbf{k}\sigma j} + \frac{1}{\omega^2}(\mathbf{k}\mathbf{a}^*_{\mathbf{k}\sigma})(\mathbf{k}\mathbf{a}_{\mathbf{k}\sigma}). \tag{1.2}$$

Multiplying (1.1) by $\mathbf{a}^*_{\mathbf{k}\sigma i}$, we easily obtain

$$k^2 = \omega_\sigma^2(\mathbf{k})\,\varepsilon^\sigma(\omega_\sigma(\mathbf{k}), \mathbf{k}),$$

i.e., $\omega_\sigma(\mathbf{k})$ satisfies the dispersion equation

$$k^2 = \omega^2\varepsilon^\sigma(\omega, \mathbf{k}),$$

Let us consider first the transparency region, neglecting the anti-hermitian part of ϵ_{ij}, and also neglecting the imaginary part of the frequency ω_σ. In this case $|\mathbf{E}_{\mathbf{k}\sigma}|^2$ characterizes the intensity of the wave σ. From the quantum point of view, this intensity is the sum of the energy of the individual quanta (elementary excitations). The energy per cm^3 of the field of the quanta σ is equal to the energy of each individual quantum $\hbar\omega_\sigma$, multiplied by the number of quanta $N^\sigma_{\mathbf{k}} d\mathbf{k}/(2\pi)^3$, where $N^\sigma_{\mathbf{k}}$ is the density of the number of quanta per unit phase volume ($V = 1\ cm^3$),

$$W = \sum_\sigma \int \hbar\omega_\sigma(\mathbf{k})\, N^\sigma_{\mathbf{k}}\, d\mathbf{k}\, \frac{1}{(2\pi)^3}. \tag{1.3}$$

On the other hand, the energy of the electromagnetic field is given by the classical expression

$$W = \frac{1}{4\pi}\int_{-\infty}^{t} \mathbf{E}\frac{\partial \mathbf{D}}{\partial t}\, dt + \frac{B^2}{8\pi}, \qquad D_i(\omega, \mathbf{k}) = \varepsilon_{ij}E_j(\omega, \mathbf{k}),$$

$$B(\omega, \mathbf{k}) = \left[\frac{\mathbf{k}\mathbf{E}(\omega, \mathbf{k})}{\omega}\right]. \tag{1.4*}$$

By comparing (1.3) with (1.4) we can obtain the connection between $N^\sigma_{\mathbf{k}}$ and $|\mathbf{E}_{\mathbf{k}\sigma}|^2$. We note that waves with random phases are characterized by mean-square combinations of the fields $\mathbf{E}_{\mathbf{k}\sigma i}$ [45–47, 55]

$$\langle E_{\mathbf{k}\sigma}E^*_{\mathbf{k}'\sigma}\rangle = |\mathbf{E}_{\mathbf{k}\sigma}|^2\,\delta(\mathbf{k} - \mathbf{k}'). \tag{1.5}$$

The last relation enables us to find the mean value of the electromagnetic-field energy density by using relations (1.3) and (1.4):

$$W = \frac{1}{8\pi}\sum_\sigma \int d\mathbf{k}\,\frac{1}{\omega_\sigma(\mathbf{k})}\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma\right)_{\omega=\omega_\sigma(\mathbf{k})} |\mathbf{E}_{\mathbf{k}\sigma}|^2. \tag{1.6}$$

From a comparison with (1.3) we have

$$|\mathbf{E}_{\mathbf{k}\sigma}|^2 = N^\sigma_{\mathbf{k}}\frac{\hbar\omega_\sigma^2}{\pi^2}\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma\right)^{-1}_{\omega=\omega_\sigma} \tag{1.7}$$

For example, for longitudinal waves we get by virtue of $\epsilon^l = 0$

$$|\mathbf{E}^l_{\mathbf{k}}|^2 = \frac{\hbar}{\pi^2}N^l_{\mathbf{k}}\left(\frac{\partial\varepsilon^l}{\partial\omega}\right)^{-1}. \tag{1.8}$$

By introducing the number of quanta $N^l_{\mathbf{k}}$ we can obtain in simple fashion the connection between the spontaneous and induced processes.

Let us consider a simplest two-level system with $E_2 > E_1$. We denote by u_{21}, w_{12}, and w_{21} the respective per second probabilities of the spontaneous transition from E_2 to E_1, the induced transition from E_1 to E_2, and the induced transition from E_2 to E_1 [48]. From the detailed-balancing principle, which states that the direct and inverse transitions are equal in number, we have $N_2(u_{12} + w_{12}) = N_1w_{21}$, where N_2 and N_1 are the numbers of particles at the levels E_2 and E_1. Under equilibrium conditions we have $N_1 2$ $N_1/N_2 = \exp(\hbar\omega_{21}/T)$, where $\hbar\omega_{21} = E_2 - E_1$. On the other hand, the probability of the induced transition is proportional to $|E^\sigma_{\mathbf{k}}|^2$, or, in other words, $w_{12} = \widetilde{w}_{12}N^\sigma_{\mathbf{k}}$, and $\widetilde{w}_{21} = \widetilde{w}_{21}N^\sigma_{\mathbf{k}}$. It follows from this that

$$N^\sigma_{\mathbf{k}} = \frac{u_{12}}{\widetilde{w}_{21}\exp\left(\frac{1}{T}\hbar\omega_{21}\right) - \widetilde{w}_{12}} = \frac{1}{\exp\left(\frac{1}{T}\hbar\omega_{21}\right) - 1}. \tag{1.9}$$

The second equality in (1.9) is written out because $N^\sigma_{\mathbf{k}}$ should satisfy the Bose-Einstein distribution under equilibrium conditions [47]. Since (1.9) should hold true for all temperatures T, we have $\widetilde{w}_{12} = \widetilde{w}_{21}$, and $u_{12} = \widetilde{w}_{21}$. Consequently the total emission probability (both spontaneous and induced) is equal to

$$u_{12}(1 + N^\sigma_{\mathbf{k}}), \tag{1.10}$$

and the absorption probability is

$$u_{12}N^\sigma_{\mathbf{k}}, \tag{1.11}$$

i.e., to obtain the emission probability it is necessary to multiply the spontaneous-emission probability by $N^\sigma_{\mathbf{k}} + 1$, and to obtain the absorption probability, by $N^\sigma_{\mathbf{k}}$. This statement is well known in quantum electrodynamics [49] and in solid-state theory [40], and follows from general considerations of the theory of radiation in the presence of media (Ginzburg [50], Watson and Jauch [51], Ryazanov [52], Tsytovich [53]).

We should perhaps make here one more remark concerning the allowance for absorption or buildup of oscillations. The point is that in an unstable medium with $\gamma = \text{Im}\,\omega_\sigma > 0$ the question of spontaneous emission is much more complicated [54]. However, by virtue of the rapid growth of the oscillation intensity, the induced processes become decisive. Whereas in the absence of damping or buildup the energy conservation law gives

$$w_{21}^{\gamma=0} = w_{21}^0\,\delta(\omega_\sigma - \omega_{21})\,N_{\mathbf{k}\sigma}, \tag{1.12}$$

we get on the other hand

$$w_{21}^{\gamma\neq 0} = w_{21}^0\,N_{\mathbf{k}\sigma}\frac{|\gamma_{\mathbf{k}}|}{\pi[(\omega_{21} - \omega_\sigma)^2 + \gamma_{\mathbf{k}}^2]}. \tag{1.13}$$

When $\omega_{21} - \omega_\sigma$ are of the same order as $\gamma_{\mathbf{k}}$, (1.13) and (1.12) give in fact the same result. It is important that (1.13) can be used far from resonance. Then

*$[\mathbf{k}\mathbf{E}] \equiv \mathbf{k} \times \mathbf{E}$.

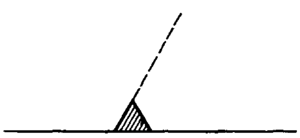


FIG. 1

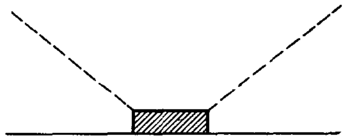


FIG. 2

the probability of the induced process turns out to be $\gamma_{\mathbf{k}}/\omega_\sigma$ times smaller. The interaction between nonresonant particles and waves, as shown in [16], can play an important role.

2. Emission, Scattering, and Decays

When the intensity of the oscillations (quanta) is large, the induced processes begin to dominate ($N_{\mathbf{k}}^\sigma \gg 1$). Under these conditions, the factor $N_{\mathbf{k}}^\sigma$ corresponds in the probability to both the absorbed and the emitted quantum. In the case of weak turbulence, as already noted, we are dealing with the first terms of the expansion of the interaction in terms of $N_{\mathbf{k}}^\sigma$ or $|E_{\mathbf{k}\sigma}|^2$. By virtue of the foregoing, the expansion in powers of $|E_{\mathbf{k}\sigma}|^2$ corresponds to expansion in terms of the number of the quanta participating in the process. The simplest process corresponds to participation of one quantum. It is shown graphically in Fig. 1. The next-order process is one in which two quanta participate. This is either emission and absorption of two quanta, or else emission of one quantum and absorption of another, which corresponds to scattering (Fig. 2). Finally, any wave moving in a medium produces polarization, and the waves can become scattered by this polarization, as shown schematically in Fig. 3. We must stipulate here that in any case it is impossible to divide the processes into the types shown in Figs. 2 and 3. The reason is that a particle participating in the process of Fig. 2 can be any one of the particles of the medium, including the particle that produces the polarization current that causes the nonlinear interaction of Fig. 3. For brevity we shall call the processes of Fig. 1 emission, those of Fig. 2 scattering, and those of Fig. 3 decay. The cross-hatched elements in Figs. 1—3 can be quite complicated in solids. In an ordinary laboratory gas plasma, owing to the weakness of interaction between the individual particles, these elements can be obtained in the first orders of perturbation theory. Let us write down the energy and momentum conservation laws for these processes. For Fig. 1, if the particle (or quasiparticle) is characterized by an energy spectrum $\epsilon_{\mathbf{p}}$, then

$$\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}+\hbar\mathbf{k}} + \hbar\omega_{\mathbf{k}}. \tag{2.1}$$

For small $\hbar\mathbf{k}$ we get the Cerenkov condition

$$\omega_{\mathbf{k}} = \mathbf{kv}, \tag{2.2}$$

where $\mathbf{v} = \partial\epsilon_{\mathbf{p}}/\partial\mathbf{p}$. For Fig. 2:

$$\varepsilon_{\mathbf{p}} = \varepsilon_{\mathbf{p}-\hbar\mathbf{k}+\hbar\mathbf{k}'} + \hbar\omega_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'} \quad \text{or} \quad (\mathbf{k}-\mathbf{k}')\,\mathbf{v} = \omega_{\mathbf{k}} - \omega_{\mathbf{k}'}. \tag{2.3}$$

Finally, for Fig. 3:

$$\omega_{\mathbf{k}}^\sigma = \omega_{\mathbf{k}-\mathbf{k}''}^{\sigma'} + \omega_{\mathbf{k}''}^{\sigma''}. \tag{2.4}$$

If $k'' \ll k$ and $\sigma = \sigma'$, then (2.4) yields $\mathbf{k}\cdot\mathbf{v}_{gr}^\sigma = \omega_\sigma$, where $\mathbf{v}_{gr}^\sigma = d\omega^\sigma/d\mathbf{k}$. There is a striking analogy between the decay and Cerenkov processes. In the decay process the role of the particle is assumed, roughly speaking, by one of the waves.

3. Kinetic Equation for Waves

1. It is easiest to obtain the kinetic equations describing the change in the number of waves $N_{\mathbf{k}}^\sigma$ from simple balance considerations. We introduce the scattering probability $w_{\alpha\sigma}^{\sigma'}(\mathbf{p}, \mathbf{k}, \mathbf{k}')$, which describes the transformation of a wave σ with momentum $\mathbf{k}$ into a wave σ' with momentum $\mathbf{k}'$ by scattering from a particle α with momentum $\mathbf{p}$, as well as the wave decay probability $u_{\sigma\sigma'}^{\sigma''}(\mathbf{k}, \mathbf{k}', \mathbf{k}'')$ describing the transformation of a wave σ with momentum $\mathbf{k}$ into a wave σ' with momentum $\mathbf{k}'$ and a wave σ'' with momentum $\mathbf{k}''$. We write first the expression for the change in the number of quanta $N_{\mathbf{k}}^\sigma$ per unit time as a result of scattering. The probability of adsorption of $\mathbf{k}$ and emission of $\mathbf{k}'$, taking induced processes into account, is equal to $w_\alpha^{\sigma\sigma'} N_{\mathbf{k}}^\sigma (N_{\mathbf{k}'}^{\sigma'} + 1)$. We take account here of the fact, demonstrated above, that each emitted wave gives rise to a factor $N_{\mathbf{k}} + 1$ and each absorbed wave to $N_{\mathbf{k}}$. To find $\partial N_{\mathbf{k}}^\sigma/\partial t$ we take account of the fact that the scattering can occur from particles having different momenta, and we multiply the probability by the particle distribution function $f_{\mathbf{p}}^\alpha$ and integrate with respect to $\mathbf{p}$:

$$-\int w_{\alpha\sigma}^{\sigma'} N_{\mathbf{k}}^\sigma (N_{\mathbf{k}'}^{\sigma'} + 1)\, f_{\mathbf{p}}^\alpha \frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3}. \tag{3.1}$$

In the process (3.1), the particle goes over from the state with momentum $\mathbf{p}$ into a state with momentum $\mathbf{p} + \hbar\mathbf{k} - \hbar\mathbf{k}'$. The inverse process is described, by virtue of the detailed-balancing principle, by the same probability

$$\int w_{\alpha\sigma}^{\sigma'} (N_{\mathbf{k}}^\sigma + 1)\, N_{\mathbf{k}'}^{\sigma'} f_{\mathbf{p}+\hbar\mathbf{k}-\hbar\mathbf{k}'}^\alpha \frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3}. \tag{3.2}$$

From (3.1) and (3.2) we obtain the total scattering-induced change in the number of quanta

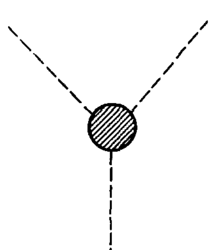


FIG. 3

$$\frac{\partial N_{\mathbf{k}}^{\sigma}}{\partial t}=N_{\mathbf{k}}^{\sigma}\sum_{\alpha\sigma'}\int w_{\alpha\sigma}^{\sigma'}N_{\mathbf{k}'}^{\sigma'}(f_{\mathbf{p}+\hbar\mathbf{k}-\hbar\mathbf{k}'}^{\alpha}-f_{\mathbf{p}}^{\alpha})\frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{k}'}{(2\pi)^3}$$

$$+\sum_{\alpha\sigma'}\int w_{\alpha\sigma}^{\sigma'}N_{\mathbf{k}'}^{\sigma'}f_{\mathbf{p}+\hbar\mathbf{k}-\hbar\mathbf{k}'}^{\alpha}\frac{d\mathbf{p}\,d\mathbf{k}'}{(2\pi)^6}-N_{\mathbf{k}}^{\sigma}\int w_{\alpha\sigma}^{\sigma'}f_{\mathbf{p}}^{\alpha}\frac{d\mathbf{p}\,d\mathbf{k}'}{(2\pi)^6}. \quad (3.3)$$

The first term of (3.3) describes induced scattering, which is significant in the case of intense turbulence, when the $N_{\mathbf{k}}^{\sigma}$ are large. By virtue of $\hbar|\mathbf{k}-\mathbf{k}'| \ll p$ we have for the induced scattering

$$\frac{\partial N_{\mathbf{k}}^{\sigma}}{\partial t}=N_{\mathbf{k}}^{\sigma}\sum_{\alpha\sigma'}\int w_{\alpha\sigma}^{\sigma'}N_{\mathbf{k}'}^{\sigma'}\hbar(\mathbf{k}-\mathbf{k}')\frac{\partial f_{\mathbf{p}}^{\alpha}}{\partial\mathbf{p}}\frac{d\mathbf{p}\,d\mathbf{k}'}{(2\pi)^6}. \quad (3.4)$$

Equally useful is an expression for the scattering produced when the $N_{\mathbf{k}}^{\sigma}$ are small and corresponding to the limit when only spontaneous scattering is significant

$$\frac{\partial N_{\mathbf{k}}^{\sigma}}{\partial t}=\sum_{\alpha\sigma'}w_{\alpha\sigma}^{\sigma'}N_{\mathbf{k}'}^{\sigma'}f_{\mathbf{p}}^{\alpha}\frac{d\mathbf{p}\,d\mathbf{k}'}{(2\pi)^6}. \quad (3.5)$$

The nonlinear interaction connected with the decay processes is also described by an expression which is quadratic in the number of quanta. The decrease in the number of quanta $N_{\mathbf{k}}^{\sigma}$ due to the absorption of waves σ and emission of waves σ' and σ'' is

$$\int u_{\sigma\sigma'}^{\sigma''}N_{\mathbf{k}}^{\sigma}(N_{\mathbf{k}'}^{\sigma'}+1)(N_{\mathbf{k}''}^{\sigma''}+1)\frac{d\mathbf{k}'\,d\mathbf{k}''}{(2\pi)^6}, \quad (3.6)$$

while the increase of $N_{\mathbf{k}}^{\sigma}$ due to the inverse process is

$$\int u_{\sigma\sigma'}^{\sigma''}(N_{\mathbf{k}}^{\sigma}+1)N_{\mathbf{k}'}^{\sigma'}N_{\mathbf{k}''}^{\sigma''}\frac{d\mathbf{k}'\,d\mathbf{k}''}{(2\pi)^6}. \quad (3.7)$$

We find therefore that the terms $(N_{\mathbf{k}}^{\sigma})^3$ drop out of the overall expression. Neglecting interactions that are linear in $N_{\mathbf{k}}^{\sigma}$, we obtain an equation describing the nonlinear effects (see (8)),

$$\frac{\partial N_{\mathbf{k}}^{\sigma}}{\partial t}=\int u_{\sigma}^{\sigma'\sigma''}(N_{\mathbf{k}'}^{\sigma'}N_{\mathbf{k}''}^{\sigma''}-N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}'}^{\sigma'}-N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}''}^{\sigma''})\frac{d\mathbf{k}'\,d\mathbf{k}''}{(2\pi)^6}. \quad (3.8)$$

We note that spontaneous decay corresponds to $N_{\mathbf{k}}^{\sigma}\to 0$. We note also that the equation for $N_{\mathbf{k}'}^{\sigma'}$ is obtained in the same manner as (3.8), but it is necessary to integrate (sum) over $\mathbf{k}'$ and $\mathbf{k}''$ and, in addition, it is necessary to reverse the signs in the expressions similar to (3.6) and (3.7):

$$\frac{\partial N_{\mathbf{k}'}^{\sigma'}}{\partial t}=\int u_{\sigma}^{\sigma'\sigma''}(N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}''}^{\sigma''}+N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}'}^{\sigma'}-N_{\mathbf{k}'}^{\sigma'}N_{\mathbf{k}''}^{\sigma''})\frac{d\mathbf{k}'\,d\mathbf{k}''}{(2\pi)^6}. \quad (3.9)$$

2. Let us stop to discuss the conservation laws that hold for nonlinear wave interactions. There are conservation laws that result from the conservation laws governing the elementary acts (2.3) and (2.4). Introducing the energy of the quanta of species σ:

$$W^{\sigma}=\int(2\pi)^{-3}\hbar\omega_{\sigma}(\mathbf{k})\,d\mathbf{k}N_{\mathbf{k}}^{\sigma} \quad (3.10)$$

multiplying (3.8) and (3.9) by ω_{σ} and $\omega_{\sigma'}$, and then integrating with respect to the corresponding $\mathbf{k}$, we can readily verify by virtue of (2.4) that

$$\frac{d}{dt}(W^{\sigma}+W^{\sigma'}+W^{\sigma''})=0. \quad (3.11)$$

This conservation law shows that for decay processes the energy can only go over from certain modes to others. In exactly the same way we can verify the validity of the momentum conservation law

$$\frac{d}{dt}(\mathbf{P}^{\sigma}+\mathbf{P}^{\sigma'}+\mathbf{P}^{\sigma''})=0,\qquad \mathbf{P}^{\sigma}=\int\frac{\hbar\mathbf{k}\,d\mathbf{k}}{(2\pi)^3}N_{\mathbf{k}}^{\sigma}. \quad (3.12)$$

There are also, however, other conservation laws. Thus, the total numbers of the quanta of species σ and σ' are conserved:

$$\frac{d}{dt}(N^{\sigma}+N^{\sigma'})=0,\qquad N^{\sigma}=\int N_{\mathbf{k}}^{\sigma}\frac{d\mathbf{k}}{(2\pi)^3}. \quad (3.13)$$

The validity of (3.13) can be readily verified directly from (3.8) and (3.9). In exactly the same way, $d(N^{\sigma}+N^{\sigma''})/dt=0$. In induced-scattering processes, the total number of quanta is conserved if the scattered quanta are of the same species as the scattering ones, i.e., $\sigma=\sigma'$. Indeed, integrating (3.4) with respect to $\mathbf{k}$ and taking into account the asymmetry of the right-hand side with respect to substitution of $\mathbf{k}'$ for $\mathbf{k}$, we have

$$\frac{d}{dt}N^{\sigma}=0. \quad (3.14)$$

It is also easy to see that the sum of the energies of the particles and of the waves is conserved in induced-scattering processes; this can be readily verified by using besides (3.4) the equation for the change in the distribution function of the scattering particles

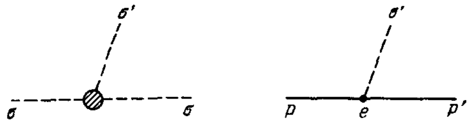
$$\frac{\partial f_{\mathbf{p}}^{\alpha}}{\partial t}=\frac{\partial}{\partial p_i}D_{ij}\frac{\partial f_{\mathbf{p}}^{\alpha}}{\partial p_j},\ D_{ij}=\int w_{\alpha\sigma}^{\sigma'}(k_i-k_i')(k_j-k_j')\hbar^2N_{\mathbf{k}}^{\sigma}N_{\mathbf{k}'}^{\sigma'}\frac{d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^6}. \quad (3.15)$$

Equation (3.15) is obtained in similar fashion from balance considerations. In this case the energy conservation law takes the form

$$\frac{d}{dt}\Big(W^{\sigma}+W^{\sigma'}+\sum_{\alpha}\int\varepsilon_{\mathbf{p}}f_{\mathbf{p}}^{\alpha}\frac{d\mathbf{p}}{(2\pi)^3}\Big)=0. \quad (3.16)$$

Let us examine the consequences of the foregoing conservation laws. First, in induced scattering of Langmuir waves into Langmuir waves, the total energy of the waves cannot change appreciably. Indeed, the energy of the Langmuir waves depends weakly on k, and the change in the total energy is small by virtue of the conservation of N^{l}. Second, in the decay of waves of high frequency ω_{σ} into waves of low frequency $\omega_{\sigma''}$, which is possible only when accompanied by production of another high-frequency wave $\omega_{\sigma'}$, $\omega_{\sigma}\gg\omega_{\sigma'}$, the total number of high-frequency quanta is conserved (see (3.13)). This also leads to the conclusion that the total energy of the Langmuir waves cannot change noticeably when Langmuir waves decay into low-frequency ones.

3. We note here the analogy between decay processes in high-frequency waves and Cerenkov effects for charged particles [19]. Indeed, even the diagram of the decay of a wave σ with transformation into a wave of the same species σ and emission of a wave σ' (Fig. 4a) is similar to the Cerenkov diagram (Fig. 4b). From simple balance considerations we can



a) b)

FIG. 4

write down an expression for the change in the number of quanta, connected with the induced Cerenkov effect. If w_α^σ is the Cerenkov-radiation probability, then

$$\frac{\partial N_{\mathbf{k}'}^{\sigma'}}{\partial t}=N_{\mathbf{k}'}^{\sigma'}\int w_\alpha^{\sigma'}\hbar\mathbf{k}'\frac{\partial f_{\mathbf{p}}^\alpha}{\partial\mathbf{p}}\frac{d\mathbf{p}}{(2\pi)^3}. \qquad (3.17)$$

It is assumed in (3.17) that the number of particles $f_{\mathbf{p}}^\alpha$ corresponds to absence of degeneracy. Therefore formulas similar to (3.17) can be obtained for Fig. 4b only for small N^σ (more accurately, $N^\sigma \ll N^{\sigma'}$). Then

$$\frac{\partial N_{\mathbf{k}'}^{\sigma'}}{\partial t}=N_{\mathbf{k}'}^{\sigma'}\int u_\sigma^{\sigma'\sigma}(N_{\mathbf{k}}^\sigma-N_{\mathbf{k}+(\mathbf{k}''-\mathbf{k})}^\sigma)\frac{d\mathbf{k}\,d\mathbf{k}''}{(2\pi)^6}. \qquad (3.18)$$

Recognizing that $\mathbf{k}-\mathbf{k}''=\mathbf{k}'$ by virtue of the momentum conservation law and assuming that $k' \ll k$ (small recoil), we obtain a formula which is perfectly analogous to the Cerenkov formula (3.17):

$$\frac{\partial N_{\mathbf{k}'}^{\sigma'}}{\partial t}=N_{\mathbf{k}'}^{\sigma'}\int u_\sigma^{\sigma'}\left(\mathbf{k}'\frac{\partial N_{\mathbf{k}}^\sigma}{\partial\mathbf{k}}\right)\frac{d\mathbf{k}}{(2\pi)^3},\qquad u_\sigma^{\sigma'}=\int u_\sigma^{\sigma'\sigma}\frac{d\mathbf{k}''}{(2\pi)^3}. \qquad (3.19)$$

We note that neglect of the terms $(N^\sigma)^2$, which enabled us to obtain (3.18) for waves of high frequencies $\omega_\sigma \ll \omega_{\sigma'}$, has frequently little effect on the accuracy [15]. The analogy with the Cerenkov effect can be naturally extended also to radiating waves σ. The equation for the change in the particle distribution function due to the Cerenkov effect is

$$\frac{\partial f_{\mathbf{p}}^\alpha}{\partial t}=\frac{\partial}{\partial p_i}D_{ij}\frac{\partial f_{\mathbf{p}}^\alpha}{\partial p_j}+\frac{\partial}{\partial p_j}(A_j f_{\mathbf{p}}^\alpha), \qquad (3.20)$$

$$D_{ij}=\int w_\alpha^{\sigma'}N_{\mathbf{k}'}^{\sigma'}k'_ik'_j\frac{d\mathbf{k}'}{(2\pi)^3},\qquad A_j=\int w_\alpha^{\sigma'}(\mathbf{k}')k'_j\,d\mathbf{k}'. \qquad (3.21)$$

The first term of (3.20) describes the diffusion of the particles in the field of the oscillation and the induced effects of Cerenkov absorption and emission of waves. The second term corresponds to spontaneous processes. (Equation (3.20), together with (3.17), describes the so-called quasilinear approximation [24].)

We shall show that similar equations hold in wave decay with small $N^\sigma \ll N^{\sigma'}$ and small recoil. We note that in this case Eq. (3.8) will not suffice, since it does not describe all the processes but only those shown in Fig. 5a (the direction of the arrows indicates emission or absorption), but not in Fig. 5b. If $\sigma'' \neq \sigma$ these processes correspond to other possible decays, whose probabilities are not expressed directly in terms of $u_\sigma^{\sigma'\sigma''}$. On the other hand, if $\sigma''=\sigma$, these probabilities can be expressed in terms of $u_\sigma^{\sigma'}$, namely the first diagram of Fig. 5b differs from the first diagram of Fig. 5a in that the initial state of the plasmon σ is replaced by the final state, which for Fig. 5a is $\mathbf{k}-\mathbf{k}'$, i.e., the probability of the process of the first diagram of Fig. 5b is $u_\sigma^{\sigma'}(\mathbf{k}+\mathbf{k}',\mathbf{k}')$. Therefore

$$\frac{\partial N_{\mathbf{k}}^\sigma}{\partial t}=-\int\{u_\sigma^{\sigma'}(\mathbf{k},\mathbf{k}')[(N_{\mathbf{k}'}^\sigma+1)N_{\mathbf{k}}^\sigma-N_{\mathbf{k}-\mathbf{k}'}^\sigma N_{\mathbf{k}'}^{\sigma'}]$$
$$+u_\sigma^{\sigma'}(\mathbf{k}+\mathbf{k}',\mathbf{k}')[N_{\mathbf{k}}^\sigma N_{\mathbf{k}'}^{\sigma'}-(N_{\mathbf{k}}^\sigma+1)N_{\mathbf{k}+\mathbf{k}'}^\sigma]\}\frac{d\mathbf{k}'}{(2\pi)^3}. \qquad (3.22)$$

In the case of small recoil we obtain equations analogous to (3.20):

$$\frac{\partial N_{\mathbf{k}}^\sigma}{\partial t}=\frac{\partial}{\partial k_i}D_{ij}^{\sigma\sigma'}\frac{\partial N_{\mathbf{k}}^\sigma}{\partial k_j}+\frac{\partial}{\partial k_j}(A_j^{\sigma\sigma'}N_{\mathbf{k}}^\sigma), \qquad (3.23)$$

$$D_{ij}^{\sigma\sigma'}=\int u_\sigma^{\sigma'}(\mathbf{k},\mathbf{k}')k'_ik'_jN_{\mathbf{k}'}^{\sigma'}\frac{d\mathbf{k}'}{(2\pi)^3},\quad A_j^{\sigma\sigma'}=\int u_\sigma^{\sigma'}(\mathbf{k},\mathbf{k}')k'_j\frac{d\mathbf{k}'}{(2\pi)^3}. \qquad (3.24)$$

We note that decay processes correspond to induced Raman scattering of waves by waves.

We note that processes similar to induced scattering of waves by particles correspond to four-plasmon decays. In this case processes are possible in which two waves are transformed into two waves and one wave into three waves, as well as all the inverse processes. By way of an example we write down the equation for the scattering of one quantum by another. Let $u_{\sigma\sigma'}^{\sigma\sigma'}(\mathbf{k},\mathbf{k}',\mathbf{k}_1,\mathbf{k}_1')$ be the probability of absorption of $\mathbf{k}$, $\mathbf{k}'$ and emission of $\mathbf{k}_1$, $\mathbf{k}_1'$. If the number of quanta $N^{\sigma'} \ll N^\sigma$, we readily obtain for the case of weak recoil an equation similar to (3.4):

$$\frac{\partial N_{\mathbf{k}}^\sigma}{\partial t}=N_{\mathbf{k}}^\sigma\int u_{\sigma\sigma'}(\mathbf{k},\mathbf{k}',\mathbf{k}_1)N_{\mathbf{k}_1}^\sigma(\mathbf{k}-\mathbf{k}_1)\frac{\partial N_{\mathbf{k}'}^{\sigma'}}{\partial\mathbf{k}'}\frac{d\mathbf{k}'\,d\mathbf{k}_1}{(2\pi)^6},$$
$$u_{\sigma\sigma'}=\int u_{\sigma\sigma'}^{\sigma\sigma'}\frac{d\mathbf{k}_1'}{(2\pi)^3}. \qquad (3.25)$$

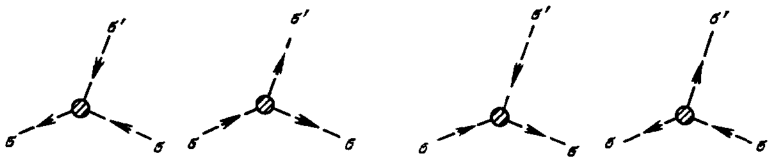
Let us examine now the possible qualitative consequences of the nonlinear effects. By virtue of the analogy between the decays into a single wave and the Cerenkov effect, it is clear that the decay processes can initiate instabilities that lead to wave generation. Scattering by plasma particles and by waves (i.e., the four-plasmon processes considered above) do not lead to a change in the total number of quanta. We shall show that in the case of scattering by equilibrium particles and waves, a nonlinear redistribution in the spectrum takes place, towards the lower-frequency oscillations. Assuming that $f_{\mathbf{p}}^\alpha$ in (3.4) depends only on the absolute value of the energy, we get

$$\frac{\partial f_{\mathbf{p}}^\alpha}{\partial\mathbf{p}}=\mathbf{v}\frac{\partial f_{\mathbf{p}}^\alpha}{\partial\varepsilon},$$

and recognizing that $(\mathbf{k}-\mathbf{k}')\mathbf{v}=\omega_{\mathbf{k}}^\sigma-\omega_{\mathbf{k}'}^{\sigma'}$ by virtue of the conservation laws, we obtain

$$\frac{\partial N_{\mathbf{k}}^\sigma}{\partial t}=N_{\mathbf{k}}^\sigma\sum_{\alpha\sigma'}\int w_{\alpha\sigma}^{\sigma'}N_{\mathbf{k}'}^{\sigma'}(\omega_{\mathbf{k}}^\sigma-\omega_{\mathbf{k}'}^{\sigma'})\frac{\partial f_{\mathbf{p}}^\alpha}{\partial\varepsilon}\frac{d\mathbf{p}\,d\mathbf{k}}{(2\pi)^6}. \qquad (3.26)$$

If $\partial f/\partial\epsilon < 0$, as is the case for equilibrium Maxwellian distribution, then, according to (3.26), the waves having lower frequencies build up. This is true both for like waves $\sigma=\sigma'$ and for energy transfer from one type of wave into another. If f is isotropic and $\partial f/\partial\epsilon > 0$ in a definite energy region, then it must be



a) b)

FIG. 5

borne in mind that there always exists also a region $\partial f/\partial \epsilon < 0$, by virtue of $f \to 0$ and $\epsilon \to \infty$. Thus, the question of the direction of the energy transfer is determined by which of these two regions plays the principal role in the scattering. The situation here differs somewhat, in spite of some similarity, from the situation that obtains in the case of the linear approximation, when damping takes place for any isotropic f [56,57].

Transfer towards higher frequencies occurs in the presence of directed particle beams (this result was first obtained for Langmuir waves in [33]). Indeed, assuming that $f = f(E)$, where $E = (p - p_0)^2/2m$, we obtain Eq. (3.26) in which $(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'})(\partial f/\partial \epsilon)$ is replaced by $(\omega_{\mathbf{k}} - \omega_{\mathbf{k}'} - (\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}_0)\,(\partial f/\partial E)$, and consequently the energy transfer is towards larger $\mathbf{k} \cdot \mathbf{v}_0$ when $(\mathbf{k} - \mathbf{k}') \cdot \mathbf{v}_0 > \omega_{\mathbf{k}} - \omega_{\mathbf{k}'}$.

A perfectly similar situation arises in the scattering, described by the decay interaction (3.25), of oscillations σ' by other oscillations σ. For example, we can speak of the scattering of Langmuir waves by high-frequency transverse waves. The presence of beams of high-frequency waves can lead to energy transfer towards higher frequencies and to linear absorption of Langmuir waves.

4. Nonlinear Plasma Current and Probability of Decay Processes

1. The equations written out above constitute expansions in the number of quanta, $N_{\mathbf{k}}^{\sigma}$, or in $|E_{\mathbf{k}\sigma}|^2$, the squared field amplitudes. Therefore the probabilities of different processes can be obtained if one knows the expansion of the nonlinear plasma current in the field amplitudes. If $E_k\,(k = \{\mathbf{k}, \omega\})$ is the Fourier component of the electric field, then in the linear approximation $j_{ki}^{(1)} = \sigma_{ij}(k) E_{kj}$. In the E^2 approximation

$$j_{ki}^{(2)} = \int S_{ijl}(k, k_1, k_2) E_{k_1 j} E_{k_2 l}\, d\lambda^{(2)}, \tag{4.1}$$

$$d\lambda^{(2)} = dk_1\, dk_2\, \delta(k - k_1 - k_2),\quad dk = d\mathbf{k}\, d\omega,\quad \delta(\mathbf{k}) = \delta(\omega)\delta(\mathbf{k}), \tag{4.2}$$

and in the E^3 approximation

$$j_{ki}^{(3)} = \int \Sigma_{ijlm}(k, k_1, k_2, k_3)\, E_{k_1 j} E_{k_2 l} E_{k_3 m}\, d\lambda^{(3)}, \tag{4.3}$$

$$d\lambda^{(3)} = dk_1\, dk_2\, dk_3\, \delta(k - k_1 - k_2 - k_3). \tag{4.4}$$

The presence of δ functions in (4.1) and (4.3) does not express any conservation laws, but is a consequence of the Fourier expansion. From the causality principle and the reality requirement we obtain (just as for the linear current [6,58]) a number of relations for the components σ_{ij}, S_{ijl}, and Σ_{ijlm} [28,35].

We note that the foregoing relations hold for weak stationary and homogeneous media. The concrete values of the components σ_{ij}, S_{ijl}, and Σ_{ijlm} can be obtained by using various equations describing the medium. It should be noted perhaps that although we are interested in the waves in the transparency region, it is nevertheless quite important in a number of cases to make allowance in the equations for the nonlinear current for absorption and, in particular, for collisions in the equations describing the plasma. This is connected with the fact that the nonlinear current may describe the interactions in those cases when one of the waves is virtual and consequently, it may turn out to be in the absorption region. Indeed, the nonlinear current (4.1) describes the vertex shown in Fig. 3, while (4.3) describes the vertex in Fig. 6. Each of these vertices can be included in the scattering effects (Fig. 7). This gives rise to a virtual line, whose frequencies and wave numbers lie most frequently outside the transparency region. In the case when the virtual line falls in the transparency region, the process represented by the diagrams of Fig. 7 breaks up into two independent processes, namely, a quantum is emitted first, and then decays (for a proof see [60]). Scattering corresponding to Fig. 7 is best

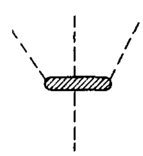
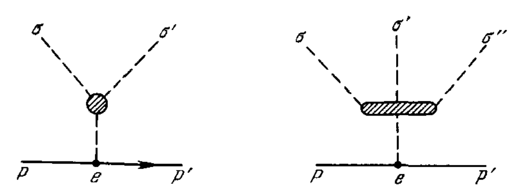


FIG. 6



a) b)

FIG. 7

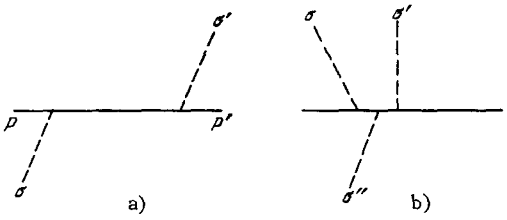


FIG 8

called nonlinear scattering, to distinguish it from the ordinary Compton scattering shown in Fig. 8.

2. By way of illustration we present the calculation of the nonlinear current for two models: a) Isotropic electron plasma described by the hydrodynamic equations ($c = 1$)

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=\frac{e}{m_e}(\mathbf{E}+[\mathbf{vH}]),$$

$$\frac{\partial n}{\partial t}+(\nabla n\mathbf{v})=0,\quad \frac{\partial \mathbf{H}}{\partial t}=-[\nabla\mathbf{E}]. \tag{4.5}$$

This system of equations can be readily rewritten in terms of Fourier components ($k = \{\mathbf{k}, \omega\}$):

$$\mathbf{v}_k=-\frac{e}{m_e i\omega}\mathbf{E}_k+\frac{1}{\omega}\int\{(\mathbf{v}_{k_2}\mathbf{k}_1)(\mathbf{v}_{k_1}-\mathbf{v}_{k_1}^{(1)})+\mathbf{k}_1(\mathbf{v}_{k_2}\mathbf{v}_{k_1}^{(1)})\}\,d\lambda^{(2)},$$

$$n_k=\frac{\mathbf{k}}{\omega}\int n_{k_2}\mathbf{v}_{k_1}\,d\lambda^{(2)},\quad \mathbf{v}_k^{(1)}=-\frac{e}{m_e i\omega}\mathbf{E}_k. \tag{4.6}$$

The sought quantity $\mathbf{j}_k$ takes the form

$$\mathbf{j}_k=e\int n_{k_1}\mathbf{v}_{k_2}\,d\lambda^{(2)}. \tag{4.7}$$

The unperturbed plasma density is n_0; $n_k^{(0)} = n_0\delta(k)$. Elementary expansion in terms of E_k leads to

$$S_{ijl}=-\frac{e\omega_{0e}^2}{8\pi m_e\omega_1\omega_2}\left(\delta_{ij}\frac{k_{2l}}{\omega_2}+\delta_{il}\frac{k_{1j}}{\omega_1}+\delta_{jl}\frac{k_i}{\omega}\right), \tag{4.8}$$

$$\Sigma_{ijls}=\frac{e^2\omega_{0e}^2}{8\pi m_e^2 i\omega_1\omega_2\omega_3}\left(\frac{k_i}{\omega}\cdot\frac{k_{2j}+k_{3j}}{\omega_2+\omega_3}\delta_{ls}\right.$$
$$+\frac{k_{2i}+k_{3i}}{\omega_2+\omega_3}\frac{k_{1j}}{\omega_1}\delta_{ls}+\frac{(\mathbf{k}_2+\mathbf{k}_3)^2}{\omega_2+\omega_3}\delta_{ij}\delta_{ls}$$
$$\left.+2\delta_{ij}\frac{k_{2l}}{\omega_2}\frac{k_{2s}+k_{3s}}{\omega_2+\omega_3}\right). \tag{4.9}$$

b) Isotropic electron plasma described by a kinetic self-consistent equation. Expanding f in terms E_k,

$$f=\sum_{i=0}^{\infty}f_iE^i,$$

we obtain the equations for the Fourier components

$$i(\omega-\mathbf{kv})f_{1k}=e\mathbf{F}_k\frac{\partial f_0}{\partial\mathbf{p}},$$

$$i(\omega-\mathbf{kv})f_{2k}=e\int\mathbf{F}_{k_1}\frac{\partial f_{1k_2}}{\partial\mathbf{p}}\,\partial\lambda^{(2)},\qquad \mathbf{F}_k=\mathbf{E}_k\left(1-\frac{\mathbf{kv}}{\omega}\right)+\frac{\mathbf{k}}{\omega}(\mathbf{vE}_k). \tag{4.10}$$

This yields for the current

$$\mathbf{j}_k^{(2)}=\int e\mathbf{v}f_{2k}\frac{d\mathbf{p}}{(2\pi)^3}$$

the expression

$$S_{ijl}=-e^3\int\frac{v_i}{\omega-\mathbf{kv}}\left[\left(1-\frac{\mathbf{k}_1\mathbf{v}}{\omega_1}\right)\frac{\partial}{\partial p_j}\right.$$
$$\left.+\frac{v_j\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}}\right)}{\omega_1}\right]\frac{1}{\omega_2-\mathbf{k}_2\mathbf{v}}\left[\left(1-\frac{\mathbf{k}_2\mathbf{v}}{\omega_2}\right)\frac{\partial}{\partial p_l}\right.$$
$$\left.+\frac{v_l}{\omega_2}\left(\mathbf{k}_2\frac{\partial}{\partial\mathbf{p}}\right)\right]f_0\frac{d\mathbf{p}}{(2\pi)^3}. \tag{4.11}$$

The hydrodynamic approximation (4.8) can be easily obtained from (4.11) by expanding in the parameter $\mathbf{k}\cdot\mathbf{v}/\omega$.

3. The probabilities of the decay processes can be obtained directly from the nonlinear plasma current. To this end we use the correspondence principle. Namely, we consider the equations that describe the decay processes in the limit when the number of emitted quanta is small. From (3.8) with $N_k \to 0$ we can easily obtain the intensity of radiation from a unit volume of the plasma

$$Q_\sigma=\frac{\partial}{\partial t}\int\frac{\hbar\,\omega_\sigma N_{\mathbf{k}}^\sigma\,d\mathbf{k}}{(2\pi)^3}=\int\frac{\hbar\omega_\sigma u_\sigma^{\sigma'\sigma''}N_{\mathbf{k}'}^{\sigma'}N_{\mathbf{k}''}^{\sigma''}\,d\mathbf{k}\,d\mathbf{k}'\,d\mathbf{k}''}{(2\pi)^9}. \tag{4.12}$$

On the other hand, the intensity Q_σ can be determined as the emission energy of the nonlinear current excited in the plasma by the waves σ' and σ''. The emission energy of the current $j^{(2)}$ in the plasma is the work done by the forces of the field, produced by the current, on the current itself

$$Q_\sigma=-\int\mathbf{j}(\mathbf{r},t)\,\mathbf{E}(\mathbf{r},t)\,d\mathbf{r}. \tag{4.13}$$

Expressing E in terms of j by means of Maxwell's equation with allowance for the linear part of the plasma current

$$(k^2\delta_{ij}-k_ik_j-\omega^2\varepsilon_{ij})E_{kj}=4\pi i\omega j_{ki}, \tag{4.14}$$

we obtain an expression for Q^σ in terms of the plasma current

$$Q^\sigma\quad\frac{(2\pi)^6}{TV}\int_{\substack{T\to\infty\\ V\to\infty}}d\mathbf{k}|\omega^\sigma|\frac{|\,\mathbf{a}_{\sigma\mathbf{k}}^*\mathbf{j}_{\mathbf{k}}\,|^2}{\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma\right)_{\omega=\omega_\sigma}}. \tag{4.15}$$

Omitting the details of the calculations (see [18, 35] we point out that if we use the nonlinear current (4.1) and average over the phases, then we can obtain from (4.13) an expression such as (4.12). Comparison of (4.15), (4.12), and (1.7) enables us to find an expression for the probability $u_\sigma^{\sigma'\sigma''}$:

$$u_\sigma^{\sigma'\sigma''}(k,k',k'')=16(2\pi)^7\,\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')$$
$$\times\,\delta(\omega_{\mathbf{k}}^\sigma-\omega_{\mathbf{k}'}^{\sigma'}-\omega_{\mathbf{k}''}^{\sigma''})(\omega_{\mathbf{k}'}^{\sigma'})^2(\omega_{\mathbf{k}''}^{\sigma''})^2$$
$$\times\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^{\sigma'}\right)^{-1}_{\omega=\omega_{\mathbf{k}'}^{\sigma'}}\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^{\sigma''}\right)^{-1}_{\omega=\omega_{\mathbf{k}''}^{\sigma''}}$$
$$\times|S_{\sigma\sigma'\sigma''}(\omega_{\mathbf{k}}^\sigma\mathbf{k},\omega_{\mathbf{k}}^{\sigma'}\mathbf{k}',\omega_{\mathbf{k}''}^{\sigma''}\mathbf{k}'')|^2\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma\right)^{-1}_{\omega=\omega_{\mathbf{k}}^\sigma},$$

$$S_{\sigma\sigma\,\sigma''}=a_{\sigma i}^*a_{\sigma' j}a_{\sigma'' l}S_{ijl}. \tag{4.16}$$

We can obtain in similar fashion the probabilities of four-plasmon decay interactions [98]

$$u_\sigma^{\sigma'\sigma''\sigma'''}(k, k', k'', k''') = 48\,(2\pi)^5\,\delta\,(\omega_{\mathbf{k}}^{\sigma} - \omega_{\mathbf{k}'}^{\sigma'} - \omega_{\mathbf{k}''}^{\sigma''} - \omega_{\mathbf{k}'''}^{\sigma'''})$$
$$\times\ \delta\,(\mathbf{k} - \mathbf{k}' - \mathbf{k}'' - \mathbf{k}''')$$
$$\times\,|\Sigma_{\sigma\sigma\ \sigma''\sigma'''}|^2\,(\omega_{\mathbf{k}}^{\sigma'}\omega_{\mathbf{k}''}^{\sigma''}\omega_{\mathbf{k}'''}^{\sigma'''})^2 \prod_{i=0}^{3} \left(\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^{\sigma i}\right)^{-1}_{\omega=\omega_{\mathbf{k}_i}^{\sigma i}}. \tag{4.17}$$

By way of illustration let us find the decay probability for three longitudinal waves when $\mathbf{a_k} = \mathbf{k}/k$. From (4.11) it follows that

$$S_{lll} = -\frac{e^3\omega}{kk_1k_2}\int \frac{1}{\omega - \mathbf{kv}}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}}\right)\frac{1}{\omega_2 - \mathbf{k}_2\mathbf{v}}\left(\mathbf{k}_2\frac{\partial}{\partial\mathbf{p}}\right) f_0\,\frac{d\mathbf{p}}{(2\pi)^3}. \tag{4.18}$$

The expression for S_{lll} is particularly simple in the case of one-dimensional plasmons, when the directions of all three plasmons coincide. Symmetrizing S_{lll} with respect to 1 and 2, we obtain

$$S_{lll} = -\frac{e^3\omega}{2}\int \frac{m^2}{\varepsilon_p^2}\,\frac{1}{(\omega - kv)(\omega_1 - k_1v)(\omega_2 - k_2v)}\,\frac{\partial f}{\partial p}\,\frac{dp}{(2\pi)^3}. \tag{4.19}$$

For a plasma in which all particles are nonrelativistic, expression (4.19) together with (4.15) gives the result obtained in [62] and [30].

5. Nonlinear and Compton Scattering and Their Interference

Let us consider the nonlinear scattering represented by the diagram in Fig. 7a. To find the scattering probability we shall assume one of the N_{k}^{σ} to be small when the scattering is spontaneous and is described by (3.5), and the radiation intensity of the wave σ is equal to

$$Q^{\sigma} = \sum_{\alpha\sigma'}\int u_{\alpha\sigma}^{\sigma'}\omega_\sigma N_{\mathbf{k}'}^{\sigma'} f_{\mathbf{p}}^{\alpha}\,d\mathbf{p}\,d\mathbf{k}\,d\mathbf{k}'\,\frac{1}{(2\pi)^9}. \tag{5.1}$$

The radiation intensity is a linear function of the number of waves, and consequently the current generating it is a linear function of the wave field. It is obvious that to obtain such a current, which describes the *nonlinear scattering, it is necessary to place in one of the vertices the field produced by the charge.* We denote the field produced by the charge by E^Q. Since we are interested in a current which is linear both in the wave field and in the charge field, we get from (4.1)

$$(j_{ki}^{(2)})_{\mathrm{n.s}} = \sum_{\sigma'}\int S_{ijs}(k, k_1, k_2)\,(E_{jk_1}^{Q}E_{sk_2}^{\sigma'} + E_{jk_1}^{\sigma'}E_{sk_2}^{Q})\,d\lambda^{(2)}. \tag{5.2}$$

The role of the current $j_{\mathrm{n.s.}}^{(2)}$ can be visualized as follows: The field E_{k}^{Q} of the charge produces around the charge a polarization screening charge, and the field of the waves E_{k}^{σ} causes this screening charge to oscillate. The radiation produced as a result is indeed the one corresponding to the nonlinear scattering. Integrating over the δ-functions contained in $d\lambda^{(2)}$, we can readily represent (5.2) in the form

$$(j_{ki}^{(2)})_{\mathrm{n.s}} = \sum_{\sigma'}\int \Lambda_{ij}^{\mathrm{n.s}}(k, k_1)\,E_{jk_1}^{\sigma'}\,dk_1\delta\,(\omega - \omega_1 - (\mathbf{k} - \mathbf{k}_1)\,\mathbf{v}), \tag{5.3}$$

where

$$\Lambda_{ij}^{\mathrm{n.s}}(k, k_1)\,\delta\,(\omega - \omega_1 - (\mathbf{k} - \mathbf{k}_1)\,\mathbf{v})$$
$$= [S_{ilj}(k, k - k_1, k_1) + S_{ijl}(k, k_1, k - k_1)]\,E_{l,\,k-k_1}^{Q} \tag{5.4}$$

Relation (5.4) is quite important, since it makes it possible to calculate the nonlinear scattering from the known nonlinear current. We call attention to the fact that in (5.4) are contained the values of the components S from the difference frequencies $\omega - \omega'$ and difference wave vectors $\mathbf{k} - \mathbf{k}'$, which can be quite small for two mutually interacting waves of even very high frequencies, so that the collisions must be taken into account in a number of cases.

We note that the field E^Q which enters in (5.4) must be assumed to correspond to a charge moving in the plasma uniformly and in a straight line,* since allowance for the perturbation produced in the motion of the charge by the wave would lead in (5.3) to expressions of higher order in the wave amplitude. The field E^Q is therefore determined by Maxwell's equations (4.14), where $j = j^Q$ is the current produced by a uniformly and linearly moving charge

$$j^Q = \frac{e\mathbf{v}}{(2\pi)^3}\,\delta\,(\omega - \mathbf{kv}).$$

Denoting the inverse Maxwellian operator by Π_{ij}

$$\Pi_{is}\,(k^2\delta_{ij} - k_ik_j - \omega^2\varepsilon_{ij}) = \delta_{is}, \tag{5.5}$$

we get

$$E_{jk}^{Q} = \frac{4\pi i\omega e}{(2\pi)^3}\,\Pi_{js}v_s\delta\,(\omega - \mathbf{kv}). \tag{5.6}$$

In view of the fact that the charge motion can be regarded as uniform and linear when nonlinear scattering is considered, the effect of the nonlinear scattering can be interpreted as radiation produced when a uniformly moving charge passes through inhomogeneities produced by the waves in the plasma density and in its polarization. Such radiation is similar to transition radiation in periodic structures [63] and to radiation produced in media whose properties vary periodically in time and in space [64]. However, nonlinear scattering is not the only type of scattering, namely, besides moving uniformly the scattering charge executes small oscillation in the field of the wave and produces a current whose magnitude is proportional to the wave field E_{jk}^{σ}

$$(j_{ik})^{\mathrm{c.s}} = \sum_{\sigma'}\int \Lambda_{ij}^{\mathrm{c.s}}(k, k_1)\,E_{jk_1}^{\sigma'}\,dk_1\delta\,(\omega - \omega_1 - (\mathbf{k} - \mathbf{k}_1)\,\mathbf{v}). \tag{5.7}$$

This current produces radiation corresponding to the ordinary Compton scattering. It must be especially emphasized that in calculating the radiation intensity it is necessary first to add both expressions (5.7) and (5.3), and then square in accord with (4.14), i.e., it is necessary to take into account the interference of both types of scattering. In quantum language this means simply that it is necessary to sum the matrix elements and not the probabilities. The total scattering probability can be assumed to be equal to the sum of the probabilities of the two indicated effects [65]

*For the sake of simplicity we assume no external magnetic field. For similar calculations in external magnetic fields see[35].

only as an estimate.* It should be noted that frequently (see below) the matrix elements of two scattering processes have opposite signs and strongly cancel each other. Therefore, in many cases it is impossible to get any idea concerning the order of magnitude of the nonlinear interaction of the wave without allowance for interference. The reason why the amplitudes of two types of scattering have different signs can be intuitively understood from the fact that the charge responsible for the nonlinear scattering is screening and has a sign opposite that of the trial charge. To find the scattering probability it is necessary to follow a simple procedure in which the total current is substituted in (5.3), averaged over the phases, and the result compared with (5.1). As a result we have

$$w_{\alpha\sigma}^{\sigma'}(\mathbf{p},\ \mathbf{k},\ \mathbf{k}') = 4(2\pi)^9 \frac{\delta(\omega_{\mathbf{k}}^{\sigma}-\omega_{\mathbf{k}'}^{\sigma'}-(\mathbf{k}-\mathbf{k}')\mathbf{v}_\alpha)(\omega_{\mathbf{k}'}^{\sigma'})^2}{\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma(\omega,\mathbf{k})\right)_{\omega=\omega_{\mathbf{k}}^{\sigma}}\left(\frac{\partial}{\partial\omega}\omega^2\varepsilon^\sigma(\omega,\mathbf{k}')\right)_{\omega=\omega_{\mathbf{k}'}^{\sigma'}}} \times |\Lambda_{\sigma\sigma'}(k,\ k')|^2,$$

$$\Lambda_{\sigma\sigma'} = a_{i\mathbf{k}}^{\sigma*}(\Lambda_{ij}^{\mathrm{C},s}+\Lambda_{ij}^{n,s})\,a_{j\mathbf{k}'}^{\sigma'};\quad k=\{\mathbf{k},\ \omega_{\mathbf{k}}^{\sigma}\};\quad k'=\{\mathbf{k}',\ \omega_{\mathbf{k}'}^{\sigma'}\}. \qquad (5.8)$$

We note that nonlinear scattering for high frequencies frequently takes place on the screening electrons of the plasma. Therefore Compton scattering by ions, which is inversely proportional to the square of the ion mass, turns out to be negligibly small. In this case the interference disappears, and the scattering by the ions may exceed by many times the scattering by electrons. The foregoing physical interpretation of the high efficiency of scattering by ions was made in [18a]. The large role played by the ions in the scattering of transverse waves into transverse ones was made evident relatively long ago [3,82]. It should be noted here that the large role of ions in nonlinear effects is due also to the small value of the quantity df/dp which enters in (3.4).

II. NONLINEAR INTERACTION OF WAVES IN AN ISOTROPIC PLASMA

Introduction. Waves of isotropic plasma. There are numerous well known different types of isotropic-plasma oscillations [4-7]. We summarize here the principal data concerning the dispersion properties of such waves.

a) Transverse waves are described by a spectrum $\omega^t = \sqrt{k^2+\omega_{0e}^2}$ and have phase velocities v_{ph}^t exceeding the speed of light:

$$v_{\mathrm{ph}}^t = \frac{\sqrt{k^2+\omega_{0e}^2}}{k} > 1.$$

Their group velocity is smaller than the speed of light

$$v_{\mathrm{gr}}^t = \frac{d\omega}{dk} = \frac{k}{\sqrt{k^2+\omega_{0e}^2}};\qquad 0 < v_{\mathrm{gr}}^t < 1.$$

The dispersion properties of transverse waves are affected little by spatial dispersion [70]. The spectra of a relativistic plasma, with allowance for quantum effects, were determined in [105]. The wave damping is usually due to collisions. The damping decrement

$$\gamma = \frac{\mathrm{Im}\,\varepsilon^t\omega^2}{\mathrm{Re}\,\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^t}$$

is of the order of

$$\gamma \sim \nu_{\mathrm{coll}}\frac{\omega_{0e}^2}{(\omega^t)^2},$$

where ν_{coll} is the collision frequency.

b) Langmuir waves have a spectrum $\omega^l \simeq \omega_{0e} + (3/2)k^2v_{Te}^2/\omega_{0e}$. The phase velocities lie in the interval $v_{Te} < v_{\mathrm{ph}}^l < \infty$, and the group velocities in the interval $0 < v_{\mathrm{gr}}^l < v_{Te}$. Waves with $v_{\mathrm{ph}} < v_{Te}$ are strongly damped by Landau absorption; the order of magnitude of absorption due to collision is ν_{coll}. The spectra of the waves in a relativistic and quantum plasma are discussed in [105].

c) Ion-sound oscillations have a spectrum [72]

$$\omega_s = \frac{kv_s}{\sqrt{1+k^2v_{Te}^2/\omega_{0e}^2}} + \frac{3}{2}\frac{k^2v_{Ti}^2}{\omega_{0i}}$$

When $k \ll \omega_{0e}/v_{Te} \equiv \lambda_{De}$, the spectrum is sonic, $\omega^s = kv_s$, $v_s = \sqrt{m_e/m_i}\,v_{Te}$, whereas when $k \gg 1/\lambda_{De}$ the wave frequency is close to ω_{0i}. The phase velocities of the waves lie in the interval $v_{Ti} < v_{\mathrm{ph}}^s < v_s$, and the group velocities lie in the interval

$$\left(\frac{T_i}{T_e}\right)^{1/4} v_{Ti} < v_{\mathrm{gr}}^s < v_s.$$

The absorption due to the Landau damping by the electrons is of the order of

$$\gamma = \sqrt{\frac{\pi m_e}{8m_i}}\,\frac{kv_s}{(1+k^2\lambda_{De}^2)^{3/2}}.$$

The spectra of the possible waves are shown schematically in Fig. 9. Our task is to compare the different types of nonlinear interactions between the indicated three types of plasma waves.

6. Induced Scattering of Langmuir Waves by Plasma Electrons with Transformation into Langmuir Waves

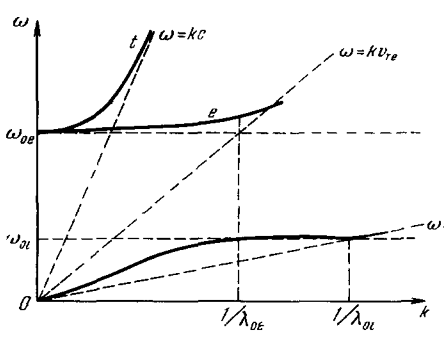


FIG. 9

*Such estimates were first carried out in [65] for effects involving the transformation of longitudinal waves into transverse ones. We shall explain later the condition under which one of the processes dominates and interference is insignificant, thus determining the region of applicability of the estimates of [65].

1. For the purpose of illustration we present in this section a consistent derivation of the nonlinear interaction of Langmuir waves in accordance with the general theory developed above. In the subsequent sections we present only the result of a similar calculation and pay principal attention to an analysis and comparison of the different nonlinear interactions.

We consider first Compton scattering. The oscillations of the charge in the wave field that determines the Compton scattering are described by the equation ($c = 1$)

$$\frac{d}{dt}\frac{v_i}{\sqrt{1-v^2}}=\frac{e}{m}\int F_{ik_1}e^{i\mathbf{k}_1\mathbf{r}-i\omega_1 t}\,dk_1,\quad \mathbf{F}_k=\mathbf{E}_k\left(1-\frac{\mathbf{kv}}{\omega}\right)+\frac{\mathbf{k}(\mathbf{vE}_k)}{\omega}. \tag{6.1}$$

Considering only terms of first order in the field intensity E_k, we get as a solution of (6.1)

$$r_i(t)=v_{0i}t-\frac{e\sqrt{1-v_0^2}}{m}(\delta_{ij}-v_{0i}v_{0j})\int F_{jk_1}\frac{e^{i(\mathbf{kv}_0-\omega)t}}{(\mathbf{kv}_0-\omega)^2}\,dk_1. \tag{6.2}$$

The part of the current which is linear in the field and which is due to the charge oscillations is

$$j_{ik}=\frac{ie^2}{(2\pi)^3 m}\sqrt{1-v_0^2}\int\frac{dk_1}{(\omega-\mathbf{kv}_0)^2}$$
$$\times[v_{0i}k_j(\delta_{js}-v_{0j}v_{0s})-(\mathbf{kv}_0-\omega)(\delta_{is}-v_{0i}v_{0s})]$$
$$\times F_{k_1 s}\delta(\omega-\omega_1-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}_0). \tag{6.3}$$

Comparing (6.3) with (5.7) we can readily obtain the value of Λ_{ij}, which determines the Compton scattering

$$\Lambda_{ij}^{\text{C.s}}(k,k_1)=\frac{ie^2}{(2\pi)^3 m}\frac{\sqrt{1-v^2}}{(\omega-\mathbf{kv})^2}[v_i k_l(\delta_{ls}-v_l v_s)$$
$$-(\mathbf{kv}-\omega)(\delta_{is}-v_i v_s)]\left[\delta_{sj}\left(1-\frac{\mathbf{k}_1\mathbf{v}}{\omega_1}\right)+\frac{k_{1s}v_j}{\omega_1}\right] \tag{6.4}$$

The Compton scattering of longitudinal waves is determined from

$$\Lambda_{ll}^{\text{C.s}}=\Lambda_{ij}^{\text{C.s}}\frac{k_i k_{1j}}{kk_1}=\frac{ie^2\sqrt{1-v^2}\,\omega\,[(\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v})]}{kk_1 m(\omega-\mathbf{kv})^2(2\pi)^3} \tag{6.5}$$

To find the nonlinear interaction we make use of (5.4), in which we substitute the expression (5.6) for the charge field and the nonlinear current (4.11).

We note that the inverse Maxwellian operator Π_{ij} which enters in (5.4),

$$\Pi_{ij}=-\frac{1}{k^2}\left\{\frac{k_i k_j}{\omega^2\varepsilon^l(\omega,\mathbf{k})}-\frac{k^2\delta_{ij}-k_i k_j}{k^2-\omega^2\varepsilon^t(\omega,\mathbf{k})}\right\} \tag{6.6}$$

describes in an isotropic plasma two types of nonlinear scattering, namely the first term (6.6), which contains the Green's function of the longitudinal field (ϵ^l and ϵ^t are the longitudinal and transverse dielectric constants) describes the process of scattering via a virtual longitudinal wave, while the second term, via a virtual transverse wave. We obtain

$$\Lambda_{ll}^{\text{n.s}}=\frac{ie^2\omega}{kk_1}4\pi e^2\frac{1}{(2\pi)^6}\sum_\alpha\int\frac{d\mathbf{p}'}{\omega-\mathbf{kv}'}$$
$$\times\left[\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}'}\right)\frac{1}{\omega-\omega_1-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}'}\left(\tilde{\boldsymbol{\alpha}}\frac{\partial}{\partial\mathbf{p}'}\right)\right.$$
$$\left.+\left(\tilde{\boldsymbol{\alpha}}\frac{\partial}{\partial\mathbf{p}'}\right)\frac{1}{\omega_1-\mathbf{k}_1\mathbf{v}'}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}'}\right)\right]$$
$$\times f_{\mathbf{p}}^\alpha\quad\tilde{\boldsymbol{\alpha}}=\left(1-\frac{(\mathbf{k}-\mathbf{k}_1)\mathbf{v}'}{\omega-\omega_1}\right)\boldsymbol{\alpha}+\frac{(\mathbf{k}-\mathbf{k}_1)(\boldsymbol{\alpha}\mathbf{v}')}{\omega-\omega_1}, \tag{6.7}$$

$$\boldsymbol{\alpha}=\frac{\mathbf{k}-\mathbf{k}_1}{(\mathbf{k}-\mathbf{k}_1)^2\varepsilon^l(\omega-\omega_1,\mathbf{k}-\mathbf{k}_1)}-\frac{(\omega-\omega_1)\left(\mathbf{v}-\frac{(\omega-\omega_1)(\mathbf{k}-\mathbf{k}_1)}{(\mathbf{k}-\mathbf{k}_1)^2}\right)}{(\mathbf{k}-\mathbf{k}_1)^2-(\omega-\omega_1)^2\varepsilon^t(\omega-\omega_1,\mathbf{k}-\mathbf{k}_1)} \tag{6.8}$$

The total scattering cross section is determined by the quantity $|\Lambda_{ll}^{\text{n.s.}}+\Lambda_{ll}^{\text{C.s.}}|^2$ and, consequently takes the form [18b,20]

$$w_{\alpha l}^l(\mathbf{p},\mathbf{k},\mathbf{k}_1)=\frac{32\pi^3 e^4|\Lambda_{ll}^\alpha|^2\delta(\omega_{\mathbf{k}}-\omega_{\mathbf{k}_1}-(\mathbf{k}-\mathbf{k}_1)\mathbf{v})}{\left.\frac{\partial\varepsilon^l(\omega,\mathbf{k}_1)}{\partial\omega}\right|_{\omega=\omega_{\mathbf{k}_1}}\left.\frac{\partial\varepsilon^l(\omega,\mathbf{k})}{\partial\omega}\right|_{\omega=\omega_{\mathbf{k}}}}, \tag{6.9}$$

where

$$\Lambda_{ll}^\alpha=\frac{((\mathbf{kk}_1)-(\mathbf{kv})(\mathbf{k}_1\mathbf{v}))\sqrt{1-v^2}}{m_\alpha(\omega_{\mathbf{k}}-\mathbf{kv})\,2kk_1}+\sum_{\alpha'}\frac{4\pi e^2}{kk_1}\int\frac{d\mathbf{p}'}{\omega_{\mathbf{k}}-\mathbf{kv}'}$$
$$\times\left[\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}'}\right)\frac{1}{\omega_{\mathbf{k}}-\omega_{\mathbf{k}_1}-(\mathbf{k}-\mathbf{k}_1)\mathbf{v}'}\left(\tilde{\boldsymbol{\alpha}}\frac{\partial}{\partial\mathbf{p}'}\right)\right.$$
$$\left.+\left(\tilde{\boldsymbol{\alpha}}\frac{\partial}{\partial\mathbf{p}'}\right)\frac{1}{\omega_{\mathbf{k}_1}-\mathbf{k}_1\mathbf{v}'}\left(\mathbf{k}_1\frac{\partial}{\partial\mathbf{p}'}\right)\right]f_{\mathbf{p}}^{\alpha'}\frac{1}{(2\pi)^3}. \tag{6.10}$$

In the right side of (6.10), account is taken of the fact that the nonlinear scattering described by the second term of (6.10) can be produced both by the electrons and by the ions of the plasma. In the case of the Langmuir waves, in which we are not interested, an important role is played only by the nonlinear scattering in which the nonlinear element is determined by the electrons. We write out here also the equation describing the nonlinear interaction of the Langmuir waves

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t}=N_{\mathbf{k}}^l\sum_\alpha\int N_{\mathbf{k}_1}^l\frac{d\mathbf{k}_1}{(2\pi)^3}w_{\alpha l}^l(\mathbf{p},\mathbf{k},\mathbf{k}_1)(\mathbf{k}-\mathbf{k}_1)\frac{\partial f_{\mathbf{p}}^\alpha}{\partial\mathbf{p}}\frac{d\mathbf{p}}{(2\pi)^3}. \tag{6.11}$$

2. We now proceed to an analysis of Eqs. (6.10)—(6.11). It must be noted first that the thermal particles of the plasma take part in the scattering of the Langmuir waves. Indeed, in order for the scattering to be possible it is necessary that the velocity of the scattering particle satisfy the law of energy conservation in each individual scattering act

$$\omega-\omega_1=\frac{3}{2}\frac{v_{Te}^2}{\omega_{0e}}(k^2-k_1^2)=(\mathbf{k}-\mathbf{k}_1)\mathbf{v}$$

or for the projection of the velocity v on $\mathbf{k}-\mathbf{k}_1$ we have

$$v_{\mathbf{k}-\mathbf{k}_1}\sim\frac{v_{Te}^2}{v_{\text{ph}}}<v_{Te}$$

The remaining two velocity projections can be arbitrary. The use of these inequalities together with $\mathbf{k}\mathbf{v}\ll\omega$ and $\mathbf{k}_1\mathbf{v}\ll\omega_1$ allows us to obtain the approximate form of the nonlinear interaction, which contains only the value of the dielectric constant as a function of the arguments $\omega-\omega_1$ and $\mathbf{k}-\mathbf{k}_1$ A very important factor here is the translation of the two types of scattering, brought about by the fact that the nonlinear scattering (by the screening charge) completely cancels out, in the first approximation in $\mathbf{k}\cdot\mathbf{v}/\omega$, the Compton scattering. Indeed, let us consider a plasma of nonrelativistic temperatures, when $v\ll 1$, and let us take into account only the scattering via a virtual longitudinal wave. We then have

$$\Lambda^{e}_{ll} \simeq \frac{(\mathbf{kk}_1)}{kk_1}\frac{1}{\omega^2 m_e}\left(1-\frac{2(\mathbf{kv})}{\omega}\right)-\frac{(\mathbf{kk}_1)}{kk_1}\times\frac{\varepsilon^l_e(\omega-\omega_1,\ \mathbf{k}-\mathbf{k}_1)-1}{\varepsilon^l(\omega-\omega_1,\ \mathbf{k}-\mathbf{k}_1)}\frac{1}{\omega^2 m_e} \tag{6.12}$$

$\epsilon^l = \epsilon^l_e + \epsilon^l_i - 1$, ϵ_e and ϵ_i are respectively the electronic and ionic parts of the plasma permittivity. In the first term of (6.12), which describes the Compton scattering, we took into account in first approximation the small correction $\sim \mathbf{k}\cdot\mathbf{v}/\omega$, connected with the Doppler change in the frequency as sensed by the scattering charge. This is necessary because the nonlinear scattering cancels the Compton scattering almost completely. The Doppler effect in the Compton scattering determines the scattering effect in an electron plasma ($m_i \to \infty$):

$$\varepsilon^l_e(\omega-\omega_1,\ \mathbf{k}-\mathbf{k}_1) \simeq \varepsilon^l(\omega-\omega_1,\ \mathbf{k}-\mathbf{k}_1) \gg 1,$$

$$\Lambda^l_{ll}\,|_{m_i\to\infty} = \frac{2(\mathbf{kk}_1)}{kk_1}\frac{(\mathbf{kv})}{\omega^2 m_e}. \tag{6.13}$$

As a result, substituting (6.13) in (6.11), we find that the nonlinear interaction in scattering by thermal plasma electrons ($m_i \to \infty$) is of the form

$$\frac{\partial N^l_{\mathbf{k}}}{\partial t} = \frac{3v_{Te}N^l_{\mathbf{k}}}{2m_e n\omega_{0e}}\int \frac{N^l_{\mathbf{k}_1}}{(2\pi)^{5/2}}\frac{(\mathbf{kk}_1)^2}{k^2k_1^2}\frac{[\mathbf{kk}_1]^2}{|\mathbf{k}-\mathbf{k}_1|^3}(k_1^2-k^2)\,d\mathbf{k}_1. \tag{6.14}$$

This form of the nonlinear interaction was first obtained in [33,16]. We must note the characteristic features of such an interaction: a) the vanishing of interaction of waves with mutually perpendicular and parallel directions; b) the transfer of energy in the direction of lower wave numbers (lower frequencies), which follows from the general relations. The latter signifies that if the wave distribution is such that they are for example, concentrated about $k = k_0$, then the nonlinear effects lead to the generation of waves with $k < k_0$ and to a decrease of the waves with $k \approx k_0$. It is clear here that if the "bare" waves with $k < k_0$ are present at all angles, then the waves that grow will be those having directions different from the initial spectrum, i.e., the energy-transfer process is accompanied by isotropization of the plasma-wave distribution. The characteristic times of the indicated energy-transfer and isotropization processes turn out to be different, depending on the characteristic wave number of the waves into which the energy is transferred. If k is of the order of k_0 and $|\mathbf{k} - \mathbf{k}_1|$ is of the order of k_0 and the angle between $\mathbf{k}$ and $\mathbf{k}_1$ is of the order of unity, then the characteristic time is of the order of [33,20]

$$\frac{1}{\tau} \sim \omega_{0e}\frac{W^l}{nm_e v_{Te}^2}\left(\frac{v_{Te}}{v^0_{\text{ph}}}\right)^3, \qquad v^0_{\text{ph}} \simeq \frac{\omega_{0e}}{k_0}, \tag{6.14$'$}$$

where W^l is the energy of the initial oscillations per cm^3. The characteristic energy-transfer time can reach relatively large values, much higher than the frequency of the Coulomb collisions of the particles. The characteristic time of energy transfer into oscillations whose wavelengths are much larger than the wavelengths of the initial oscillations

$$\frac{1}{\tau'} \sim \omega_{0e}\frac{W^l}{nm_e v_{Te}^2}\left(\frac{v_{Te}}{v^0_{\text{ph}}}\right)^3\left(\frac{v^0_{\text{ph}}}{v_{\text{ph}}}\right)^2; \qquad v_{\text{ph}} = \frac{\omega_{0e}}{k},$$

is larger than τ by a factor $(v_{\text{ph}}/v^0_{\text{ph}})^2 \gg 1$. This shows that the energy transfer should be realized in a "relay" fashion, or, roughly speaking, the mean value of v_{ph} increases linearly with time. This can be derived more rigorously by solving the integral equation (6.14) (see [73]). It must also be noted that the vanishing of the interaction when $\mathbf{k} \parallel \mathbf{k}_1$ and $\mathbf{k} \perp \mathbf{k}_1$ is the consequence of the approximate character of (6.14). The interaction of the $\mathbf{k} \parallel \mathbf{k}_1$ and $\mathbf{k} \perp \mathbf{k}_1$ waves is much weaker, since it is described by the terms of next higher order in the small parameter $(v_{Te}/v_{\text{ph}})^2$. Naturally, it is meaningful to take account of such effects only in the case of strictly parallel or strictly perpendicular wave vectors. We then have for an electronic plasma ($m_i \to \infty$) $\mathbf{k} \parallel \mathbf{k}_1$ [21,26,23,33]

$$\frac{\partial N^l_{\mathbf{k}}}{\partial t} = N^l_{\mathbf{k}}\int N^l_{\mathbf{k}_1}\,d\mathbf{k}_1\frac{27}{8(2\pi)^{5/2}}\frac{(k_1^2-k^2)}{|\mathbf{k}_1-\mathbf{k}|}\frac{v_{Te}}{n\omega_{0e}m_e}\frac{k^2k_1^2v_{Te}^2}{\omega_{0e}^2}. \tag{6.15}$$

and for $\mathbf{k} \perp \mathbf{k}_1$ the interaction differs from (6.15) by the factor $1/9$ [33]. The characteristic energy-transfer time is shorter than (6.14) by approximately v_{Te}^2/v_{ph}^2 times. The condition under which the vectors for which the interaction is described by (6.15) are parallel is of the form $\theta \ll \theta_0 = v_{Te}/v_{\text{ph}}$, where θ is the angle between k and k_1, or the difference between the angle and $\pi/2$. Since the interaction between one-dimensional spectra is relatively weak, a more probable process for one-dimensional spectra is isotropization with transfer of the waves into the angle region $\theta > \theta_0$.

It must be emphasized, however, that in the cases of practical interest allowance for the ionic part of the dielectric constant in the nonlinear scattering can be important and can limit noticeably the possibility of using (6.14). Indeed, recognizing that $\epsilon^l = e^l_e + \epsilon^l_i - 1$, we get from (6.12)

$$\Lambda^{ll} = \frac{\mathbf{kk}_1}{kk_1}\frac{1}{\omega^2 m_e}\left(\frac{2(\mathbf{kv})}{\omega} + \frac{\varepsilon^l_i(\omega-\omega_1,\ \mathbf{k}-\mathbf{k}_1)}{\varepsilon^l(\omega-\omega_1,\ \mathbf{k}-\mathbf{k}_1)}\right). \tag{6.16}$$

It must be emphasized that the foregoing corrections do not describe the scattering by the ions, and we are referring only to effects of scattering by a screening charge produced by the electrons and ions. The ions affect the screening only because the Compton and nonlinear scattering cancel each other in the first approximation. Let us estimate the conditions under which the influence of the ions on the screening of the scattering by electrons begins to dominate over the Doppler corrections to the Compton scattering. It should be noted that such an estimate is best made under conditions when the scattering by ions is impossible or, as will follow from the results of the next section, under conditions when scattering by ions is possible and the scattering by electrons which is considered here is as a rule a small effect. For scattering by the ions to be possible it is necessary that the velocity of the ion which participates in the scattering, $v_i = (\omega - \omega_i)/|\mathbf{k} - \mathbf{k}_1|$, be smaller than v_{Ti} (otherwise the number of scattering ions is exponentially small for a Maxwellian distribution). Thus, under the conditions $\omega - \omega_1 \gg |\mathbf{k} - \mathbf{k}_1| v_{Ti}$ and $\omega - \omega_1 \ll |\mathbf{k} - \mathbf{k}_1| v_{Te}$ we have

$$\varepsilon_i^l \simeq -\frac{\omega_{0i}^2}{(\omega-\omega_1)^2}, \quad \varepsilon_e^l \simeq \frac{\omega_{0e}^2}{(\mathbf{k}-\mathbf{k}_1)^2 v_{Te}^2}$$

For $k_1 \sim k$ and $|\mathbf{k} - \mathbf{k}_1| \sim k$, the order of magnitude of ϵ_i^l is $(m_e/m_i)(v_{ph}/v_{Te})^4$, and when

$$\frac{v_{ph}}{v_{Te}} \ll \left(\frac{m_i}{m_e}\right)^{1/2}$$

the order of magnitude of ϵ^l is v_{ph}^2/v_{Te}^2, i.e.,

$$\frac{\varepsilon_i^l}{\varepsilon^l} \sim \frac{m_e}{m_i}\frac{v_{ph}^2}{v_{Te}^2},$$

and the first term of (6.16) is of the order of v_{Te}/v_{ph}. Thus, the ions greatly influence the scattering even when

$$\frac{v_{ph}}{v_{Te}} \gtrsim \left(\frac{m_i}{m_e}\right)^{1/3}$$

Recognizing in addition that $v_{ph}/v_{Te} \gg 1$, we see that for a hydrogen plasma with $m_i/m_e \sim 2\times 10^3$ the results of (6.14) are applicable in the narrow region $1 \ll v_{ph}/v_{Te} \ll 10$. For a plasma of a heavy gas, such as a cesium plasma, the limits of applicability of the results of [33] are wider. If we neglect the Doppler corrections in (6.15), then under the conditions

$$\left(\frac{m_i}{m_e}\right)^{1/3} \ll \frac{v_{ph}}{v_{Te}} \ll \left(\frac{m_i}{m_e}\right)^{1/2}$$

the nonlinear interaction is described by the approximate formula [36]

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = N_{\mathbf{k}}^l \times \int N_{\mathbf{k}_1}^l d\mathbf{k}_1 \frac{2}{27}\frac{m_e^2}{m_i^2}\frac{\omega_{0e}^5}{v_{Te}^5}\frac{(\mathbf{k}-\mathbf{k}_1)^4(\mathbf{k}\mathbf{k}_1)^2}{|\mathbf{k}-\mathbf{k}_1|(k_1^2-k^2)^3 k^2k_1^2(2\pi)^{5/2} n m_e v_{Te}}, \tag{6.17}$$

and when $v_{ph}/v_{Te} \gg (m_i/m_e)^{1/2}$ we have

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = N_{\mathbf{k}}^l \int N_{\mathbf{k}_1}^l \frac{3\,\omega_{0e}(k_1^2-k^2)(\mathbf{k}\mathbf{k}_1)^2 d\mathbf{k}_1}{8(2\pi)^{5/2} n m_e v_{Te}|\mathbf{k}-\mathbf{k}_1| k_1^2k^2}. \tag{6.18}$$

The role of the ions in the screening of the scattering by the electrons was clarified in [36], where the nonlinear interaction (6.17) was obtained for a one-dimensional spectrum ($\mathbf{k} \parallel \mathbf{k}_1$), and in the non-onedimensional case in [73]. It should be noted that the interaction (6.18), unlike (6.14), depends little on the angles of the interacting waves when $\theta \lesssim 1$.

The order of magnitude of the characteristic time under conditions of (6.18) and when $k - k_1$ is of the order k is

$$\frac{1}{\tau} \sim \omega_{0e}\frac{W^l}{n m_e v_{Te}^2}\frac{v_{Te}}{v_{ph}},$$

which is much larger (by a factor $(v_{ph}/v_{Te})^2$) than (6.14). We call attention to the fact that polarization effects of the ions are especially important when one-dimensional spectra are considered. The general expression for the nonlinear interaction of one-dimensional spectra, an expression suitable also for waves whose phase velocity can be of the same order as or larger than the velocity of light, was obtained in [15]:

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = N_{\mathbf{k}}^l \int N_{\mathbf{k}_1}^l d\mathbf{k}_1 \frac{3\omega_{0e}}{8(2\pi)^{5/2}}\frac{k_1^2-k^2}{|k-k_1| n m_e v_{Te}}\left\{v_{Te}^4 - \left| v_{Te}^2 \frac{1-\varepsilon_i^l(\omega-\omega_1, k-k_1)}{\varepsilon^l(\omega-\omega_1, k-k_1)} + \frac{3kk_1}{(k-k_1)^2\varepsilon^l(\omega-\omega_1, k-k_1)}\right|^2\right\}. \tag{6.19}$$

We note that the result (6.15) follows from (6.19) for waves having nonrelativistic phase velocities only when

$$\frac{v_{ph}}{v_{Te}} \ll \left(\frac{m_i}{m_e}\right)^{1/4}.$$

For waves whose phase velocities are close to the velocity of light, the nonlinear interaction can be determined by a formula that differs from (6.15) by a numerical coefficient 17/9 (see [15]) only at very high plasma temperatures

$$v_{Te} \gg \left(\frac{m_e}{9m_i}\right)^{1/4}$$

If one of the latter inequalities is not satisfied, the nonlinear interaction of one-dimensional waves has the same order of magnitude as that of non-one-dimensional waves (6.18). We note that in (6.19) we took into account only the nonlinear scattering via a virtual longitudinal wave, just as in all the preceding formulas. Scattering via a virtual transverse wave in the one-dimensional case is strictly forbidden, and (6.19) describes the interaction of arbitrary one-dimensional spectra.

7. Nonlinear Scattering via Virtual Transverse Waves

Going over to the analysis of nonlinear scattering via a virtual transverse wave, it should be noted that the important role of such a nonlinear interaction was discussed for the first time for scattering of longitudinal waves into transverse ones in [18a]. The role of scattering via a virtual transverse wave for nonlinear interaction of Langmuir waves with one another was pointed out in [34]. However, the role of such an interaction was overestimated there because the effects connected with this scattering were compared with formula (6.14), which is not valid in the region of large phase velocities, at which scattering via a virtual transverse wave may come into play (as a rule, this is $v_{ph} \gg 1$). Under these conditions, scattering via a longitudinal wave is described by the interaction (6.18), which was not taken into account in [34]. The polarization effects of the ions can also affect the scattering via a virtual transverse wave [15]. We therefore present the results [15], which describes all three types of interaction (Compton scattering, scattering via a longitudinal wave, and scattering via a transverse wave), obtained in the approximation when no account is taken of weak interactions corresponding to one-dimensional spectra (of the type (6.19))

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t}=N_{\mathbf{k}}^l\int N_{\mathbf{k}_1}^l d\mathbf{k}_1\frac{3\omega_{0e}(k_1^2-k^2)}{8(2\pi)^{5/2}|\mathbf{k}-\mathbf{k}_1|nm_e v_{Te}}\left\{\frac{[\mathbf{k}\mathbf{k}_1]^2 v_{Te}^2}{k^2k_1^2\omega_{0e}^2|\mathbf{k}_-|^2}\left|2(\mathbf{k}\mathbf{k}_1)+\frac{\omega_-^2(\varepsilon_e^t(\omega_-,\mathbf{k}_-)-1)\,2(\mathbf{k}\mathbf{k}_1)-k_-^2\omega_{0e}^2}{k_-^2-\omega_-^2\varepsilon^t(\omega_-,\mathbf{k}_-)}\right|^2+\frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2k_1^2}\left|\frac{\varepsilon_i^l(\omega_-,\mathbf{k}_-)}{\varepsilon^l(\omega_-,\mathbf{k}_-)}\right|^2\right\},\quad \omega=\omega-\omega_1,\ \mathbf{k}_-=\mathbf{k}-\mathbf{k}_1 \tag{7.1}$$

The term containing the denominator $k_-^2 - \omega^2\epsilon^t(\omega_-, k_-)$ takes into account the contribution of the scattering via a virtual transverse wave. We can also see from the foregoing expression that scattering via a virtual transverse wave is possible only for waves whose wave vectors are not strictly parallel. Let us therefore ascertain when scattering via a transverse wave will be the decisive one, assuming that the angle between k and k_1 is of the order of unity. The sum of the Doppler correction to the Compton scattering and nonlinear scattering via a transverse wave is determined, in accordance with (7.1), by the square of the modulus of the quantity

$$\Lambda=\frac{\omega_{0e}^2k_-^2-2(\mathbf{k}_1\mathbf{k})(k_-^2-\omega_-^2\varepsilon_i^t(\omega_-,\mathbf{k}_-))}{k_-^2-\omega_-^2\varepsilon^t(\omega_-,\mathbf{k})} \tag{7.2}$$

A large value of Λ is possible only because of the small denominators (7.2). If k_1 and k are of the same order of magnitude, then the order of magnitude of $|k_-|$ when

$$\theta\sim 1\left(\cos\theta=\frac{(\mathbf{k}\mathbf{k}_1)}{kk_1}\right)$$

is equal to k, whereas $\omega \sim 3k\Delta k v_{Te}^2/\omega_{0e}$, with $\Delta k = |k_1| - |k|$ can, generally speaking, be a rather small quantity. Further, since scattering by ions is disregarded, we have $\omega_- \gg |k_-| v_{Ti}$ or

$$\Delta k\gg\frac{v_{Ti}}{v_{Te}}\frac{v_{\mathrm{ph}}}{v_{Te}}k,$$

$$\varepsilon^t(\omega_-,\mathbf{k}_-)\simeq-\frac{m_e}{9m_i}\frac{v_{\mathrm{ph}}^4}{v_{Te}^4}\frac{k^2}{(\Delta k)^2}+\frac{i}{3}\sqrt{\frac{\pi}{2}}\frac{v_{\mathrm{ph}}^3}{v_{Te}^3}\frac{k}{\Delta k},$$

with the first term of ϵ^t corresponding to the contribution of the ions, and the second to the contribution of the electrons. If

$$\frac{\Delta k}{k}\gg\frac{v_{\mathrm{ph}}}{v_{Te}}\frac{m_e}{9m_i} \tag{7.3}$$

then the main contribution to ϵ^t is made by the electrons. We can then obtain the approximate form of the nonlinear interaction, in the particular cases $k_-^2 \gg \omega^2\epsilon^t(\omega_-, k_-)$, which is equivalent under our conditions to $\Delta k/k \ll 1/v_{ph}v_{Te}$. If the conditions $\Delta k/k \gg 1/v_{ph}v_{Te}$ and (7.3) are satisfied, we obtain an interaction in the form indicated in [34] (the very important condition (7 3), as well as a number of conditions that follow, was not indicated in [34]),

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t}=N_{\mathbf{k}}^l\int\frac{2}{3}\frac{\omega_{0e}}{(2\pi)^{7/2}}\frac{[\mathbf{k}\mathbf{k}_1]^2}{(k_1^2-k^2)}\frac{|\mathbf{k}-\mathbf{k}_1|^3}{nm_e v_{Te}}\frac{N_{\mathbf{k}_1}^l d\mathbf{k}_1}{k^2k_1^2} \tag{7.4}$$

It was assumed in (7.4) that $v_{ph}^l \gg 1$ for only under these conditions can scattering via a longitudinal wave take place. Indeed, the characteristic time of the spectral energy transfer (7.4) is of the order of

$$\frac{1}{\tau}\simeq\frac{k}{\Delta k}\frac{W^l}{nm_e v_{Te}^2}\frac{v_{Te}}{v_{\mathrm{ph}}}\omega_{0e}$$

On the other hand, in scattering via a longitudinal wave with $\epsilon_i^l \ll \epsilon_e^l$, i.e.,

$$\frac{\Delta k}{k}\gg\frac{v_{\mathrm{ph}}}{v_{Te}}\sqrt{\frac{m_e}{9m_i}},$$

we have the estimate

$$\frac{1}{\tau}\sim\left(\frac{k}{\Delta k}\right)^3\omega_{0e}\left(\frac{m_e}{9m_i}\right)^2\left(\frac{v_{\mathrm{ph}}}{v_{Te}}\right)^3\frac{W^l}{nm_e v_{Te}^2}$$

From the comparison we obtain the conditions under which (7.4) predominates

$$\frac{\Delta k}{k}\gg\frac{v_{\mathrm{ph}}^2}{v_{Te}^2}\frac{m_e}{9m_i}$$

By virtue $\Delta k/k < 1$ we have

$$\frac{v_{\mathrm{ph}}}{v_{Te}}\ll\sqrt{\frac{9m_i}{m_e}}$$

and by virtue of

$$\frac{\Delta k}{k}\gg\frac{1}{v_{\mathrm{ph}}v_{Te}}$$

we obtain from $\Delta k/k < 1$ that $v_{ph} \gg 1/v_{Te}$, i.e.,

$$v_{Te}\gg\left(\frac{m_e}{9m_i}\right)^{1/4}$$

This condition shows that the interaction (7.4) can be decisive only in a plasma with very high electron temperatures (>100 keV for a hydrogen plasma), and furthermore $v_{Ti} \ll v_{Te}^3$. If the inequality

$$\frac{\Delta k}{k}\ll\frac{v_{\mathrm{ph}}}{v_{Te}}\sqrt{\frac{m_e}{9m_i}}$$

holds, then the interaction via the longitudinal wave is of the order of

$$\frac{1}{\tau}\sim\omega_{0e}\frac{\Delta k}{k}\frac{v_{Te}}{v_{\mathrm{ph}}}\frac{W^l}{nT_e}$$

and the condition under which the scattering via the transverse wave predominates takes the form [15] $\Delta k \ll k$. The condition on the electron temperature can again arise if we take account of the fact that by virtue $\Delta k < k$ and

$$\frac{\Delta k}{k}\gg\frac{v_{\mathrm{ph}}m_e}{9v_{Te}m_i}$$

we get $v_{ph} \ll v_{Te}(9m_i/m_e)$ and by virtue $v_{ph} \gg 1/v_{Te}$ we have $v_{Te} \ggg (m_e/9m_i)^{1/2}$. It must be borne in mind here that each of the foregoing inequalities should be satisfied at least with a margin of 2 or 3 orders of magnitude, something which is hardly possible for a hydrogen plasma at nonrelativistic temperatures.

Scattering via a virtual transverse wave under the conditions

$$\frac{v_{\Phi}m_e}{9v_{Te}m_i} \ll \frac{\Delta k}{k} \ll \frac{1}{v_{ph}v_{Te}}$$

corresponds to the following result [34], which is derivable from (7.1):

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = N_{\mathbf{k}}^l \int \frac{3\omega_{0e}^3 (k_1^2 - k^2) [\mathbf{k}\mathbf{k}_1]^2 v_{Te} N_{\mathbf{k}_1} d\mathbf{k}_1}{8 (2\pi)^{5/2} |\mathbf{k} - \mathbf{k}_1|^3 n m_e k^2 k_1^2} \qquad (7.5)$$

The characteristic energy-transfer time is $1/\tau \approx \omega_{0e}(\Delta k/k)(W^l/nT_e)v_{Te}^3 v_{ph}$. From a comparison with scattering via a longitudinal wave we obtain the condition under which scattering via a transverse wave predominates $v_{ph} \gg 1/v_{Te}$. On the other hand, from a comparison of (7.3) with

$$\frac{\Delta k}{k} \gg \frac{v_{ph}}{v_{Te}} \frac{m_e}{9m_i}$$

we get $v_{ph} \lll (9m_i/m_e)^{1/2}$, which together with $v_{ph} \gg 1/v_{Te}$ yields $v_{Te} \ggg (m_e/9m_i)^{1/2}$ (the inequality with a margin of 2—3 orders of magnitude). Comparison of (7.5) with scattering via a longitudinal wave at

$$\frac{\Delta k}{k} \gg \frac{v_{ph}}{v_{Te}} \left(\frac{m_e}{9m_i}\right)^{1/2}$$

gives the condition under which scattering via a transverse wave dominates

$$\frac{\Delta k}{k} \gg \frac{1}{v_{Te}} \left(\frac{v_{ph}m_e}{9v_{Te}m_i}\right)^{1/2}$$

From $\Delta k/k < 1$ and $v_{ph} \gg 1$ it follows that $v_{Te} \gg (m_e/9m_i)^{1/3}$. In addition, from the inequalities preceding (7.5) it follows that

$$1 \ll v_{\Phi} \ll \left(\frac{9m_i}{m_e}\right)^{1/4},$$

which is a stringent condition for a hydrogen plasma. The foregoing analysis shows that the particular forms of the interactions (7.4) and (7.5), obtained in [34], have narrow regions of applicability, at least for light gases. We now discuss the particular forms, which follow from (10.1), of the nonlinear interaction connected with scattering via a virtual transverse wave, in the case when the ions determine the polarization effects, i.e., when

$$\frac{\Delta k}{k} \ll \frac{v_{ph}}{v_{Te}} \frac{m_e}{9m_i}$$

If at the same time $v_{ph} \gg (m_i/m_e)^{1/2}$ and $\Delta k/k < v_{ph}/v_{Te}^2$, then the nonlinear interaction takes the form

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = \mathrm{v}_{\mathbf{k}}^l \int N_{\mathbf{k}_1}^l d\mathbf{k}_1 \frac{3(k_1^2 - k^2)[\mathbf{k}\mathbf{k}_1]^2 v_{Te} |\mathbf{k} - \mathbf{k}_1| m_i^2}{8(2\pi)^{5/2} n m_e k^2 k_1^2 \omega_{0e} m_e^2}. \qquad (7.6)$$

The characteristic time of the nonlinear interaction is of the order

$$\frac{1}{\tau} \sim \omega_{0e} \frac{W^l}{nT_e} \frac{v_{Te}^3}{v_{ph}^3} \frac{\Delta k}{k} \frac{m_i^2}{m_e^2}$$

An analysis shows that the interaction (7.6) cannot dominate over scattering via a longitudinal wave. On the other hand, if $1 \ll v_{ph} \ll (m_i/m_e)^{1/2}$, then the interaction takes the form (7.5), and upon comparison with scattering via a longitudinal wave we get $v_{ph} \gg 1/v_{Te}$, i.e., $v_{Te} \ggg (m_e/m_i)^{1/2}$. The foregoing analysis shows that allowance for the ions in the screening of the scattering via the transverse wave can relax somewhat the conditions required for the nonlinear interaction to be determined by scattering via a virtual transverse wave, especially for heavy gases with large m_i: the interaction can be described by (7.5) if the derived inequalities are satisfied.

8. Induced Scattering of Langmuir Waves by Ions

In order for the plasma ions to be able to scatter Langmuir waves, it is necessary to satisfy the inequality

$$\frac{3\Delta k}{k} < \frac{v_{Ti}}{v_{Te}} \frac{v_{ph}}{v_{Te}}. \qquad (8.1)$$

This inequality is the consequence of the conservation of energy during scattering. When $\Delta k \sim k$ it reduces to [36]

$$T_i > T_e 9 \frac{m_i}{m_e} \frac{v_{ph}^2}{v_{Te}^2}. \qquad (8.2)$$

For a nonisothermal plasma with strongly heated electrons $T_e \gg T_i$ we can always indicate small values of T_i such that conditions (8.1) and (8.2) are not satisfied, and the scattering by the ions does not play any role. However, even when $T_i \sim T_e$ the waves with large phase velocities (for (8.2), $v_{ph}/v_{Te} \sim (9m_i/m_e)^{1/2}$) and small $\Delta k/k$ (see (8.1)) will essentially interact in scattering by ions.

The point is that induced scattering by ions, if it is allowed by conservation laws, usually exceeds scattering by electrons. There are two reasons for this. The first was indicated in [18a], namely: The probability of scattering of high-frequency waves by the ions is quite large because the Compton scattering is small, owing to the small mass of ions, and cannot cancel out the nonlinear scattering determined by m_e. The second reason for the large role of nonlinear effects in scattering by ions is that when (8.1) and (8.2) are satisfied the number of ions capable of scattering the waves is larger by a factor of v_{Te}/v_{Ti} than the number of the scattering electrons.

In accord with the foregoing, the scattering by the ions is purely nonlinear and is determined by the second term of (6.12), which by virtue of (8.1), (8.2), $\omega_- \ll |\mathbf{k}_-| v_{Ti}$, and $v_{Ti} \ll v_{Te}$ takes the form

$$\Lambda_{ll} - \frac{\mathbf{k}\mathbf{k}_1}{kk_1} \left(1 + \frac{T_e}{T_i}\right)^{-1} \frac{1}{\omega^2 m_e}$$

The nonlinear interaction is described by the formula [33,34,36,28,15]

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = N_{\mathbf{k}}^l \frac{3}{8} \frac{\omega_{0e} T_e}{n m_e v_{Ti} T_i} \left(1 + \frac{T_e}{T_i}\right)^{-2} \int \frac{N_{\mathbf{k}_1}^l d\mathbf{k}_1}{(2\pi)^{5/2}} \frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2 k_1^2} \frac{k_1^2 - k^2}{|\mathbf{k} - \mathbf{k}_1|} \qquad (8.3)$$

The order of magnitude of the characteristic energy-transfer time is

$$\frac{1}{\tau} \sim \omega_{0e} \frac{\Delta k}{k} \frac{W^l}{n m_e v_{Te}^2} \frac{v_{Te}}{v_{Ti}} \frac{v_{Te}}{v_{\text{ph}}} \frac{T_e}{T_i} \left(1+\frac{T_e}{T_i}\right)^{-2}$$

In comparing the nonlinear interaction (8.3) with the nonlinear scattering by electrons, we must take into account the fact that the latter takes under conditions (8.1) and (8.2) the form [36]

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = N_{\mathbf{k}}^l \int N_{\mathbf{k}_1}^l d\mathbf{k}_1 \frac{3\omega_{0e}(k_1^2-k^2)\left(\frac{T_e}{T_i}\right)^2 (\mathbf{k}\mathbf{k}_1)^2}{\left(1+\frac{T_e}{T_i}\right)^2 8(2\pi)^{5/2} |\mathbf{k}-\mathbf{k}_1| k^2 k_1^2 n m_e v_{Te}} \tag{8.4}$$

The characteristic time of this interaction exceeds (8.3) when $T_e/T_i \ll m_i/m_e$. In this case the scattering by the ions dominates under these conditions over the nonlinear interaction connected with the Doppler corrections to the Compton scattering via the electrons (6.14), subject to satisfaction of the criterion

$$\frac{v_{\text{ph}}}{v_{Te}} \gg \left(\frac{m_e}{m_i}\right)^{1/4} \left(1+\frac{T_e}{T_i}\right) \left(\frac{T_e}{T_i}\right)^{-\frac{3}{4}}, \tag{8.5}$$

which is satisfied automatically when $T_e \gg T_i$ and $T_e \ll T_i(m_i/m_e)$ by virtue of $v_{\text{ph}} \gg v_{Te}$. In the case when $T_e \ll T_i$, the foregoing condition does not impose any limitations only when $T_i \ll T_e(m_i/m_e)^{1/3}$. On the other hand, at large values of T_i, the interaction of waves with small v_{ph} close to v_{Te} may not be determined by the scattering by ions. It must be borne in mind here that for waves with $\Delta k \sim k$ at $T_i \gg T_e$, (8.5) and (8.2) are incompatible, so that we can speak of small Δk when (8.1) is not violated.

So far we have been dealing with comparison of the spectral energy redistribution of the electron and ion energies at identical Δk. However, it is advantageous to carry out the comparison in a different manner, namely, by considering such $\Delta k_0 \lesssim k$ for which scattering by ions is impossible, assuming that (8.2) is violated. But then "relay" scattering by ions is still possible. In each step $\Delta k \ll \Delta k_0$ and satisfies the condition (8.1). Such a scattering should be compared with the degree to which a single energy transfer by electrons is allowed. It follows from (8.1) that even for small T_i there are such small Δk for which the nonlinear interaction is determined by the ions. However, the characteristic time of the spectral energy redistribution increases with decreasing Δk which are allowed by inequality (8.1):

$$\frac{1}{\tau} \sim \frac{\omega_{0e}}{3} \frac{W^l}{n T_e} \frac{T_e}{T_i} \left(1+\frac{T_e}{T_i}\right)^{-2}.$$

This time corresponds to energy transfer at $\Delta k \ll \Delta k_0$, the transfer when Δk is of the order of Δk_0 being larger by a factor $\Delta k_0/\Delta k$, i.e., its order of magnitude is

$$\frac{1}{\tau} \sim \omega_{0e} \frac{W^l}{n T_e} \frac{v_{Ti} v_{\text{ph}}}{v_{Te}^2} \frac{\Delta k_0}{k} \frac{T_e}{T_i} \left(1+\frac{T_e}{T_i}\right)^{-2}$$

When

$$\frac{v_{\text{ph}}}{v_{Te}} \ll \left(\frac{m_i}{m_e}\right)^{1/3}$$

this time should be compared with (6.14), thus providing a criterion for the predominance of scattering by ions

$$\frac{v_{\text{ph}}}{v_{Te}} \gg \left(1+\frac{T_e}{T_i}\right)^{1/2} \left(\frac{T_i}{T_e} \frac{m_i}{m_e}\right)^{1/8} \tag{8.6}$$

A similar criterion for

$$\frac{v_{\text{ph}}}{v_{Te}} \gg \left(\frac{m_i}{m_e}\right)^{1/2}$$

is

$$\frac{v_{\text{ph}}}{v_{Te}} \gg \left(1+\frac{T_e}{T_i}\right) \left(\frac{T_e m_i}{T_i m_e}\right)^{1/4} \tag{8.7}$$

Attention should also be called to the fact that in the case of one-dimensional spectra, if all the interacting waves have identical directions, the criterion for the possibility of scattering by ions is (8.2). Then the characteristic energy transfer time

$$\frac{1}{\tau} \simeq \omega_{0e} \frac{W^l}{n T_e} \frac{v_{Te}^2}{v_{Ti} v_{\text{ph}}} \frac{T_e}{T_i} \left(1+\frac{T_e}{T_i}\right)^{-2} \tag{8.8}$$

does not increase with decreasing Δk. On the other hand, the process analogous to "relay like" transfer at small Δk occurs in the one-dimensional case in such a way that during each stage there is a reversal in the direction of the waves [36].

9. Stimulated Scattering of Ion-sound Waves

The interaction of ion-sound waves greatly differs from the interaction of Langmuir waves. First, the phase velocities of the waves are bounded from above by the quantity $v_s = v_{Te}(m_e/m_i)^{1/2}$, and transfer to smaller ω brings the oscillation energy into the region where the absorption due to collisions becomes all the more significant. Therefore such an energy transfer can serve as a mechanism for absorbing the oscillations generated as a result of the plasma instabilities. Second, the appreciable dependence of the oscillation frequency on the wave number in the acoustic part of the branch causes the transfer process itself, in which the number of quanta is conserved, to be accompanied, unlike the case of Langmuir waves, by an appreciable decrease in the energy of the ion-sound oscillations. Third, the principal role in the scattering is played only by the plasma ions, and on the acoustic part of the branch the transfer can have only a "relay-like" character, in other words, it can be approximately described by differential equations (transfer to wave-number values that are close in modulus). The latter follows directly from the law of energy conservation during scattering

$$(k-k')v_s = (k-k')v_{Ti}\sqrt{\frac{T_e}{T_i}} < |\mathbf{k}-\mathbf{k}'| v_{Ti},$$

$$\text{i.e., } \frac{\Delta k}{|\mathbf{k}-\mathbf{k}'|} \ll \sqrt{\frac{T_i}{T_e}}.$$

In the case of small angles θ between the interacting waves, Δk should be especially small

$$\frac{\Delta k}{k} \ll \theta \sqrt{\frac{T_i}{T_e}}$$

At the same time, for short-wave ion-sound oscillations $k^2\lambda_{De}^2 \gg 1$, the frequencies of which are close to ω_{0i}, the interaction can have an integral character (transfer at Δk of the order of k). This follows from the form of the spectrum of such oscillations. Thus, when

$$1 \ll k^2\lambda_{De}^2 < \sqrt{\frac{T_e}{T_i}}, \quad \omega - \omega' \simeq \frac{\Delta k}{k^3\lambda_{De}^4}\omega_{0i} < |\mathbf{k}-\mathbf{k}'|\, v_{Ti}$$

and Δk is of the order of k, this is possible if

$$k^2\lambda_{De}^2 > \left(\frac{T_e}{T_i}\right)^{1/3},$$

and if

$$\left(\frac{T_e}{T_i}\right)^{1/2} \ll k^2\lambda_{De}^2 < \frac{T_e}{T_i}$$

the transfer can always be integral.

The differential form of the nonlinear interaction for the acoustic part of the spectrum was first obtained in [26] and subsequently in [61, 74]. It follows from the foregoing that the energy transfer has an integral character with respect to the angles, as was most clearly demonstrated in [61]. At the same time, it must be specially emphasized that the differential form of the nonlinear interaction is approximate. This can be seen already from the fact that the resultant equations of the nonlinear interaction begin to depend on the boundary conditions in the wave-number space and can accordingly be multiply valued [61] Actually the differential character of the equations is possible only in a limited region of wave numbers, and outside this region the energy transfer is integral and, roughly speaking, determines the only boundary condition connected with the distribution of the oscillations in the region of the integral transfer of the oscillations.

Since the phase velocities of the ion-sound oscillations exceed the average thermal velocity of the ions, their scattering by the ions is quite similar to scattering of Langmuir waves by electrons. To obtain the scattering probability we can therefore simply use (6.9), taking in the nonlinear scattering account of only the ion contribution

$$\Lambda_{ss} = \frac{\mathbf{kk}_1}{kk_1}\frac{1}{\omega^2}\left(1 - \frac{2(\mathbf{kv})}{\omega}\right)\frac{1}{m_i}$$

$$-\frac{(\mathbf{kk}_1)}{kk_1}\frac{\varepsilon_i^l(\omega_-, \mathbf{k}_-)}{\varepsilon^l(\omega_-, \mathbf{k}_-)}\frac{1}{\omega^2}\frac{1}{m_i} \simeq \frac{(\mathbf{kk}_1)}{kk_1}\frac{2(\mathbf{kv})}{\omega^2}\frac{1}{m_i}$$

We have retained here only the Doppler corrections to the Compton scattering, the order of which is kv_{Ti}/ω, i.e., $v_{Ti}/v_s \sim (T_i/T_e)^{1/2}$ in the acoustic part of the spectrum and $v_{Ti}/v_{ph} \gg (T_i/T_e)^{1/2}$ in the region of the ion oscillations. When $v_{Ti} \ll v_{Te}$ the order of magnitude of the neglected corrections is $(T_i/T_e)^{1/2}$. In the acoustic part of the spectrum the probability is proportional to $\delta(\omega_-)(\omega_- \gg |k_-|v_{Ti})$, so that part of the Doppler effect, due to the motion of the ions along $\mathbf{k} - \mathbf{k}_1$, is insignificant*. At the same time, in the region of ion oscillations the parallel component of the Doppler effect is also insignificant by virtue of $\omega \ll |k_-|v_{Ti}$. Thus, the scattering probability is described by the approximate expression

$$w_{is}^s(\mathbf{p},\ \mathbf{k},\ \mathbf{k}_1) = \pi(2\pi)^3\frac{[\mathbf{kk}_1]^2(\mathbf{kk}_1)^2}{|\mathbf{k}_-|^2k^2k_1^2}\delta(\omega_- - \mathbf{k}_-\mathbf{v}). \tag{9.1}$$

The probability (9.1) enables us to obtain readily from (6.11) an expression for the nonlinear interaction for the entire region of existence of the ion-sound oscillations

$$\frac{\partial N_{\mathbf{k}}^s}{\partial t} = N_{\mathbf{k}}^s\int N_{\mathbf{k}_1}^s\frac{d\mathbf{k}_1}{4\pi^2}\frac{(\mathbf{kk}_1)^2}{k^2k_1^2}\frac{T_i}{m_i^2n}\frac{[\mathbf{kk}_1]^2}{\sqrt{2\pi}}\frac{\omega_-}{v_{Ti}^3|\mathbf{k}_-|^3}e^{-\frac{\omega_-^2}{2v_{Ti}^2k_-^2}} \tag{9.2}$$

In the acoustic part of the spectrum, using

$$-\frac{\omega}{\sqrt{2\pi}v_{Ti}^3k^3}e^{-\frac{\omega^2}{2v_{Ti}^2k^2}} \to \delta'(\omega)$$

we get from (9.2) the result [26] (see [68, 74])

$$\frac{\partial N_{\mathbf{k}}^s}{\partial t} = N_{\mathbf{k}}^s\int N_{\mathbf{k}_1}^s\frac{d\mathbf{k}_1}{4\pi^2}\frac{(\mathbf{kk}_1)^2}{k^2k_1^2}\frac{T_i}{m_in}[\mathbf{kk}_1]^2\delta'(\omega - \omega_1), \tag{9.3}$$

which is equivalent to the differential form

$$\frac{\partial N_{k\Omega}^s}{\partial t} = N_{k\Omega}^s k^2\frac{\partial}{\partial k}\int\frac{T_i\sin^2 2\theta}{T_e 16\pi^2 m_i n}N_{k\Omega_1}^s k^4\,d\Omega_1; \tag{9.4}$$

θ is the angle between $\mathbf{k}$ and $\mathbf{k}_1$, and $d\Omega_1$ is the solid angle of $\mathbf{k}_1$. When $\theta \ll 1$ the last equation goes over into Eq. (4) of [68]. It must be emphasized that in the region of small angles, where (9.4) tends to zero, and also in the region $\theta \to \pi/2$, the next terms of the expansion in the parameter T_i/T_e become significant. With this, (9.4) is valid only if $\theta \gg T_i/T_e$ [74]. The characteristic transfer time for (9.4) with $\theta \sim 1$ is of the order of

$$\frac{1}{\tau} \sim \omega_s\frac{W^s}{nT_e}\frac{k}{\Delta k}\frac{T_i}{T_e}.$$

We must emphasize the fact that the analytic properties of the approximate kernel of the integral equation (9.3) differ greatly from the properties of the more exact kernel (9.2), and that this must be taken into account when solving the equations (see [68]). In the spectral region corresponding to ion oscillations,

$$\exp\left\{-\frac{\omega_-^2}{2v_{Ti}^2k_-^2}\right\} \simeq 1,$$

and the interaction turns out to be similar to that characterized by the nonlinear interaction of the Langmuir waves

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$$^*\quad k_{\parallel}v_{\parallel} = \frac{(\mathbf{kk}_-)(\mathbf{vk}_-)}{k} = \omega_-\frac{\mathbf{kk}_-}{k_-^2} \to 0$$

by virtue of $\delta(\omega)$ More rigorously, the nonlinear interaction makes no contribution, owing to the fact that the integrand is odd in $v_{\parallel}$. The result is proportional to the next term in the expansion of $\delta(\omega_- - k_-v)$, namely, $\delta'(\omega_-)$. However, the contribution made to the coefficient preceding the δ-function by the parallel velocity component is $\omega_-^2\delta'(\omega_-)$, which is rigorously equal to zero.

$$\frac{\partial N_{\mathbf{k}}^{s}}{\partial t}=N_{\mathbf{k}}^{s}\int N_{\mathbf{k}_1}^{s}\,\frac{d\mathbf{k}_1}{(2\pi)^{5/2}}\,\frac{\omega_-[\mathbf{k}\mathbf{k}_1]^2}{v_{Ti}nm_i\,|\,\mathbf{k}_-\,|^3}\,\frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2k_1^2}. \qquad (9.5)$$

The corrections to the nonlinear scattering, due to the polarization effects of the electrons, are quite small*.

10. Nonlinear Interaction of Langmuir and Ion-sound Waves

a) Decay processes. Interest in these interactions is due to the fact that experimental research results are available for the interaction between high-frequency and low-frequency waves [75]. The principal role in such interactions is apparently played by decay interactions. It must be emphasized that in an isotropic plasma the decay processes of waves of the same type, such as sss, *lll*, and ttt, are forbidden by the laws of energy and momentum conservation in each individual decay act, which can be readily seen directly from the dispersion properties of these waves. It is also easy to understand that the processes of coalescence of two ion-sound waves into a Langmuir wave are also forbidden, because the sum of the energies of two ion-sound waves does not exceed $2\omega_{0i}$, and the frequency of the Langmuir waves is larger than ω_{0i}. This leaves only one decay process describing the interaction between Langmuir and ion-sound waves, corresponding to emission or absorption of an ion-sound wave by a Langmuir wave.

An analysis [74a] shows that the main contribution to the nonlinear current, which determines the foregoing decay, is made by the electrons. Recognizing that the average thermal velocity of the electrons is much larger than the phase velocity of the ion-sound waves, we obtain from (4.14) and (4.11) [74a, 73b]†

$$u_{\mathbf{p}}^{ls}=\frac{e^2\omega_s^3m_i\,(2\pi)^6}{16\pi m_e^3v_{Te}^4k_s^2}\,\frac{(\mathbf{k}_1^l\mathbf{k}_2^l)^2}{(k_1^lk_2^l)^2}\,\delta\,(\mathbf{k}_1^l-\mathbf{k}_2^l-\mathbf{k}_s)\,\delta\,(\omega_1^l-\omega_2^l-\omega_s). \qquad (10.1)$$

Here $\mathbf{k}_1^l$, $\mathbf{k}_2^l$, ω_1^l, and ω_2^l are the wave vectors and frequencies of the Langmuir waves, while $\mathbf{k}_s$ and ω_s are those of the ion-sound waves. In those cases when the momentum of the radiation of the s-wave is small compared with the momentum of the Langmur wave, the conservation laws during the decay are perfectly analogous to the Cerenkov conditions $\mathbf{k}_s\cdot\mathbf{v}_{gr}^l=\omega_s$. It must be noted, incidentally, that frequently k_s can become comparable with $\mathbf{k}^l$ in decays, and it is necessary to use the exact conservation laws. They lead to the conclusion that s-wave emission is not always possible. Indeed, denoting by θ_+ the angle between the initial *l*-wave and the radiated s-wave, we have

$$\cos\theta_+=\frac{|\,k_s\,|+2k_0}{2\,|\,k_1^l\,|}; \qquad k_0=\frac{\omega_{0e}}{3v_{Te}}\sqrt{\frac{m_e}{m_i}}.$$

From $-1<\cos\theta<1$ and $|\,k_s\,|>0$ it follows that $|\,k_1^l\,|>k_0$, i.e.,

$$v_{\text{ph}}^l<3v_{Te}\sqrt{\frac{m_i}{m_e}}. \qquad (10.2)$$

It must be noted that the maximum momentum which can be obtained by the radiated s-wave is $2\,|\,k_1^l\,|$ when $k_1^l\gg k_0$, and since $k_1\ll 1/\lambda_{De}$, it follows that $(k_s)_{max}\ll 1/\lambda_{De}$, i.e., a wave in the acoustic part of the spectrum is emitted in the decay. For one-dimensional decay, the equality $|\,k_s\,|+2k_0=2\,|\,k_1^l\,|$ is satisfied, and when $k_1^l\gg k_0$ we have $k_s\simeq 2k_1^l$. This shows that as a result of emission of the sound wave, the direction of the propagation of the Langmuir wave is reversed. When $k_1<3k_0$ ($k_1>k_0$), only a single one-dimensional decay is possible, when $k_1<5k_0$ a double decay is possible, etc. The total number of decays is therefore finite and amounts to approximately $\sqrt{m_i/m_e}$. This circumstance can be readily understood by recognizing that the energy of the Langmuir waves decreases in the decays, i.e., k decreases and the phase velocity of the waves increases. The end result is an increase in the phase velocities of the *l*-waves to $3v_{Te}\sqrt{m_i/m_e}$. It should be noted that if the Langmuir waves have narrow spectra, the emission of the sound waves may lead to formation of a Langmuir satellite spectrum that does not overlap the initial spectrum. This is possible if the width of the Langmuir wave spectrum is $\Delta\omega^l<\omega^s$. For a one-dimensional spectrum this reduces to

$$\Delta k^l<\frac{2k_0}{k_1^l}\,(k_1^l-k_0),$$

which yields $\Delta k^l<2k_0$ when $k\gg k_0$. It must be emphasized that the number of Langmuir waves is conserved both in decay and in coalescence. Therefore the decay processes lead only to an energy redistribution in the Langmuir-wave spectrum, i.e., to the same effects as the induced scattering considered above. Such an energy transfer does not occur in decay without limit, as in the case of the induced scattering, but is limited by the maximum phase velocity $3v_{Te}\sqrt{m_i/m_e}$. The direction of the energy transfer depends in the case considered here on the ratio of the intensities of the ion-sound waves to the Langmuir waves. This can be readily understood by recognizing that the absorption of the ion-sound oscillations lead to a decrease in the phase velocities, while emission leads to an increase. It is clear, for example, that

*Scattering by electrons must frequently be regarded as small. This can be readily understood by recognizing that the ion-sound oscillations interact with the electrons in the first linear approximation, and therefore the nonlinear effects are corrections. In addition, under conditions when the quasilinear effects are strong it is in general impossible to assume that the electron distribution is Maxwellian. By decreasing $\partial f_e/\partial p$, to which the nonlinear interaction is proportional, the quasilinear effects decrease the scattering by the electrons.

†See also [76, 93a], where the probability of the process differs from that in [74a, 73b] by the factor

$$\frac{k_s^2\lambda_{De}^2}{2}=\frac{1}{\omega\,\dfrac{\partial\varepsilon^s}{\partial\omega}}.$$

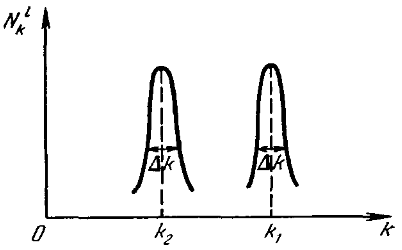

FIG. 10

when the ion-sound waves have high energy, the absorption will predominate and the transfer will be towards smaller values of v_{ph}^l. Let us illustrate this by means of a simple example. Assume, for example, that we have a one-dimensional narrow spectrum of Langmuir waves which has only one non-decaying satellite spectrum ($k_0 < k_1 < 3k_0$) (see above). The waves of the first packet can emit a sound wave and become transformed into the waves of the second packet. On the other hand, the waves of the second packet can only absorb sound waves (Fig. 10)* Let us assume that at the initial instant of time the entire energy of the Langmuir waves was concentrated in the waves of the first packet. Then emission of sound waves is induced, i.e., the waves subsequently emitted are initiated by the previously emitted waves. Such a system is analogous to a two-level system with negative temperature, which is considered in the theory of quantum generators. When a noticeable number of waves of the second packet appears, absorption is produced, and becomes equal to the emission when the intensities of the first and the second packets become comparable in order of magnitude†. This means that the number of waves of the first packet has decreased by an amount on the order of the initial number of waves; the number of emitted waves is of the same order. Since the energy of each of the s-quanta is smaller than the energy of the l-quanta by a factor of ω_s/ω_{0e}, the total energy W^s of the generated s-quanta amounts to

$$\frac{W^s}{W^l} \simeq \frac{\omega_s}{\omega_{0e}},$$

where W^l is the energy of the l-quanta. In the process under consideration, the number of quanta of the second packet increases, i.e., the phase velocities increase. This has been the result of the fact that at the initial instant of time W^s was small. What will occur if W^s is large? It is clear that if W^s is smaller than $W^l\omega_s/\omega_{0e}$, then the picture will change little. On the other hand, if in the initial state the number of quanta of the second packet exceeds the number of quanta of the first, then absorption of s-waves and transfer to the first packet takes place. For the transfer to be appreciable it is necessary that W^s exceed $W^l\omega_s/\omega_{0e}$. It is easy to estimate the characteristic time of the transfer process for both cases. In the first we can assume the intensity of the first packet to be specified, that of the second to be small, that of the sound waves to be small, and the characteristic time τ_1 can be assumed equal to the growth time of the waves of the second packet

$$\frac{\partial N^l_{\mathbf{k}_2}}{\partial t} = N^l_{\mathbf{k}_2} \int N^l_{\mathbf{k}_1} u_l^{ls} \frac{d\mathbf{k}^l_1\, d\mathbf{k}_s}{(2\pi)^6}\,.$$

In the second case, when intense sound waves are present, we can regard the intensity of the sound waves as specified, and the characteristic time τ_2 can be assumed equal to the decrement time of the intensity of the waves of the second packet*

$$\frac{\partial N^l_{\mathbf{k}_1}}{\partial t} = -N^l_{\mathbf{k}_1} \int N^s_{\mathbf{k}_s} u_l^{ls} \frac{d\mathbf{k}^l_2\, d\mathbf{k}_s}{(2\pi)^6}\,.$$

It is easy to compare the foregoing characteristic times†

$$\frac{\tau_2}{\tau_1} \simeq \frac{W^l\omega_s}{W^s\omega^l}\,.$$

If the energy of the sound waves exceeds $W^l\omega_s/\omega_{0e}$, then the second of the considered processes is faster by a factor $W^s\omega_{0e}/W^l\omega_s$. An estimate for τ_1 is given by

$$\frac{1}{\tau_1} \simeq \frac{k_0}{k_1 \cos\theta_+}\,\omega_{0e}\,\frac{W^l}{nT_e}\,\frac{k^l_1}{\Delta k^l_1}\,. \qquad (10.3)$$

It follows from (10.3) that when $k_1 \gg k_0$ the highest probability is possessed by emission of an s-plasmon almost perpendicularly to the momentum of the l-plasmon. By virtue of (10.2) it is advantageous to compare the time (10.3) with the characteristic time (6.14) of the energy transfer in scattering by electrons. This comparison shows that the time of appearance of one satellite can be smaller than the time of the induced scattering. However, it is also advantageous to know what kind of a redistribution with respect to phase velocities of the l-waves is brought about by the decays. The scattering (6.14) is effective for energy transfer at Δk^l of the order of k^l, whereas in the case of energy transfer given by (10.3), all that changes in first approximation for the one-dimensional case is the fact that the direction of k is reversed; when the process is repeated, an l-wave is emitted, travelling in the initial direction but having a momentum smaller by $2k_0$. Therefore for energy transfer at

*Sound waves can be absorbed also by the first packet, but as a rule these waves have other lengths and propagation directions.

†A more accurate calculation [74a] shows that this occurs when the intensity of the second packet is approximately three times larger than the intensity of the first packet.

*Calculation [74a] shows that the characteristic energy-transfer times differ from those estimated by a logarithmic factor which depends on the initial intensity.

†The last of the given order-of-magnitude estimates follows from the fact that the phase volumes of the s- and l-waves are approximately equal, $\Delta k^2 \simeq 2\Delta k^l$.

$\Delta k^l \sim k^l$ with $k^l \gg k_0$ it is necessary to have $\sim k^l/2k_0$ decays. Since the energy W^l is redistributed in this case among $k^l/2k_0$ packets, the characteristic transfer time is approximately $k^l/2k$ times larger than (10.3). Yet it follows from (10.3) that a more probable process is emission of an s-plasmon at a large angle, when the momentum transfer k_s, and consequently also the scattering angle of the l-plasmon, is small. The picture of the decays will therefore be in general non-uniform. With this, the s-plasmon can already be emitted and absorbed by waves of the same packet, and the resultant s-plasmon generation effects will be differential. Such a generation will be quite similar to Cerenkov generation,

$$\frac{1}{\tau}=\frac{1}{N^s_{\mathbf{k}}}\frac{\partial N^s_{\mathbf{k}}}{\partial t}\simeq\int u^s_l\mathbf{k}_s\frac{dN^l_{\mathbf{k}^l}}{d\mathbf{k}^l}\frac{d\mathbf{k}^l}{(2\pi)^3};$$

$$u^s_l=\int u^{ls}_l\frac{d\mathbf{k}^l_2}{(2\pi)^3}=\frac{e^2\omega_s^3 m_i(2\pi)^3}{16\pi m_e^3 v_{Te}^4 k_s^2}\,\delta(\omega_s-\mathbf{k}_s\mathbf{v}^l_{\text{gr}}). \qquad (10.4)$$

The approximation (10.4) is suitable only when $k_s \ll k_0$, $k^l_1 > k_0$, and $k_s < (k^l/k_0)\Delta k^l$. If the l-wave packet has an angle aperture $\Delta\theta \sim 1$, then

$$\frac{1}{\tau}\sim\omega_{0e}\frac{W^l}{nT_e}\frac{k_s k_0}{k^l_1\Delta k^l}, \qquad (10.5)$$

which is k_s/k_0 times smaller than (10.3). However, the transfer time decreases with increasing k_s; more probable is emission having the maximum k_s allowed by the given approximation, i.e., of the order of k_0. Thus, if we are considering energy transfer with Δk^l of the order of k^l, of a spectrum of width $\Delta k^l \sim k^l$, then (10.5) yields ($k_s \sim k_0/3$):

$$\frac{1}{\tau}\sim\omega_{0e}\frac{W^l}{nm_e v_{Te}^2}\left(\frac{v_{\text{ph}}}{v_{Te}}\right)^3\left(\frac{m_e}{3m_i}\right)^{3/2}, \qquad (10.6)$$

and for the maximum permissible (10.5) small Δk we have

$$\frac{1}{\tau}\approx\omega_{0e}\frac{W^l}{nT_e}\frac{v_{\text{ph}}}{v_{Te}}\left(\frac{m_e}{3m_i}\right)^{1/2}.$$

We note that the efficiency of energy transfer in this case, unlike in scattering by electrons, increases with increasing phase velocities of the waves. The transfer (10.6) dominates over scattering by electrons when

$$\frac{v_{\text{ph}}}{v_{Te}}>\left(\frac{3m_i}{m_e}\right)^{1/4}.$$

Thus, when $T_e \gg T_i$ and for specially narrow packets (small Δk), the transfer of the l-waves as a result of decays into s-waves can become decisive.

It is clear further that in the case of a large level of turbulence of the ion-sound waves the absorption of the latter leads to a decrease in the phase velocities of the Langmuir waves. Absorption of sound waves by Langmuir waves, in analogy with emission, is not always allowed by the conservation laws. Thus, the Langmuir waves cannot absorb ion oscillations $\omega_s \simeq \omega_{0i}$, since the momentum of the latter greatly exceeds $1/\lambda_{De}$ and the Langmuir wave resulting from the absorption *would* have $v_{ph} \ll v_{Te}$, which is impossible. Absorption of sound waves is possible only if $2(k_0 - k^l_1) < k_s < (k_0 + k^l_1)$, if $k^l_1 < k_0$. When $k^l_1 > k_0$, the region where only absorption is permissible is $2(k^l_1 - k_0) < k_s < 2(k_0 + k^l_1)$. When $k_s < 2(k^l_1 - k_0)$, both absorption and emission are possible. An analysis [37] has shown that in the case of isotropic acoustic turbulence, in a wide range of spectral distribution of the oscillations, induced emission of sound waves can predominate when $k_s < 2(k_1 - k_0)$, in other words, transfer of l-waves takes place in the direction of larger v^l_{ph}. When $k_1 \gg k_0$, the region where only absorption is possible is quite narrow, and in order for absorption to predominate it is necessary that the greater part of the sound-oscillation energy be concentrated in this region. The transfer processes occurring in the nonlinear interaction will rapidly take the s-oscillations out of this region. With increasing k^l_1, the relative width of the indicated region increases, but the absolute width remains equal to $2k_0$ at $k^l_1 < k_0$. When $k^l_1 > k_0$, the absolute width of the region decreases with decreasing k^l_1, and consequently the energy ΔW^s, which determines the intensity of the l-wave transfer, also decreases in this region.

b) Scattering processes. We now proceed to consider the scattering processes in which the l-wave is transformed into an s-wave. An example of such a process is absorption of an l-wave by a plasma particle, with emission of an s-wave, or emission of two waves (l or s) and, naturally, the inverse processes. These processes, unlike all those considered above, lead to a change in the number of the l-waves, and consequently it is expedient to regard them as responsible for the effects of "true" nonlinear absorption.

The energy and momentum conservation laws readily allow us to establish that the interaction between the l-waves and the sound waves ($\omega_s = \mathbf{k}_s\cdot\mathbf{v}_s$) is exponentially small in a plasma in which there are no particle beams. Indeed, it follows from

$$\omega^l\pm\omega^s=(\mathbf{k}^l\pm\mathbf{k}^s)\mathbf{v}$$

that the projection of the velocity on $\mathbf{k}^l \pm \mathbf{k}^s$ is

$$v_{\parallel}\simeq\frac{\omega_{0e}}{|\mathbf{k}^l\pm\mathbf{k}^s|}\gg v_{Te}$$

with

$$k^l\ll\frac{1}{\lambda_{De}},\qquad k^s\ll\frac{1}{\lambda_{De}},$$

i.e., the particles which can participate in this scattering should have a velocity much higher than the mean thermal velocity of the electrons. In the region of ion oscillations, $k^s \gg k^l$ and

$$\frac{\omega_{0e}}{k_s}\gg\frac{\omega_{0i}}{k_s}\gg v_{Ti},$$

i.e., the ions cannot participate in the scattering. By virtue of $\omega_{0e}/k_s \ll v_{Te}$, the scattering is determined by the electrons when $\omega_s \simeq \omega_{0i}$.

Thus, whereas decay interactions are allowed only for the acoustic part of the ion-sound oscillations, scattering by electrons is possible only for the region of ion oscillations, and both processes supplement each other in this respect. The probability of scattering, w_-, or of emission of two waves, w_+,[73b] leads to the following expression for the nonlinear absorption of waves in a Maxwellian plasma:

$$\frac{\partial N_{\mathbf{k}}^l}{\partial t} = -N_{\mathbf{k}}^l \int \frac{d\mathbf{k}_s}{2} \frac{N_{\mathbf{k}_s}^s \omega_{0i}\omega_{0e}}{(2\pi)^{5/2} nm_e v_{Te}^2} \left(\frac{\mathbf{k}\mathbf{k}_s}{kk_s}\right)^2 \frac{\omega_{0e}}{|k_s| v_{Te}}. \quad (10.7)$$

The characteristic decrement can be estimated from the formula

$$\gamma \sim \frac{1}{\tau} \sim \pi\omega_{0e} \frac{W^s}{nT_e} \frac{1}{k_s\lambda_{De}},$$

where W^S is the energy of the ionic oscillations. The maximum decrement corresponds to k_s of the order of $1/\lambda_{De}$, and the minimum one is smaller by a factor $\sqrt{T_i/T_e}$.

Attention should be called to the fact that the sign of (10.7) can reverse if the plasma contains electron beams of low velocity $v \ll v_{Te}$. The latter is essential for many astrophysical applications. The change in the number of ion-sound waves in this process turns out to be:

$$\frac{\partial N_{\mathbf{k}_s}^s}{\partial t} = N_{\mathbf{k}_s}^s \int N_{\mathbf{k}}^l \frac{d\mathbf{k}}{(2\pi)^{5/2}} \frac{\omega_{0i}\omega_{0e}}{nm_e v_{Te}^2} \left(\frac{\mathbf{k}\mathbf{k}_s}{kk_s}\right)^3 \frac{k\omega_{0e}}{k_s^2 v_{Te}}. \quad (10.8)$$

Attention should be called to the dependence of the sign of the effect (10.8) (generation or absorption) on the mutual orientation of k and k_s (whether k and k_s, for example, are parallel or antiparallel). Such a possibility, with the sign of (10.7) unchanged, is connected with the fact that in this process part of the energy and of the momentum of the waves is taken up by the plasma particles.

11. Nonlinear Interaction of Transverse Waves

The nonlinear interaction of transverse waves of lower order in terms of wave energy is described by scattering by the plasma electrons and ions. Although scattering of transverse waves in a plasma has been discussed many times and in detail[79-82], the effects of stimulated scattering have many specific features, which we shall now discuss. Although stimulated scattering is determined by the same cross sections, nevertheless the resultant effects are, on the one hand, proportional not to the particle distribution function, as for spontaneous scattering, but to the derivative of the momentum distribution function; on the other hand, unlike spontaneous scattering, stimulated scattering occurs only in those reactions in which there is intense transverse radiation to initiate the stimulated transitions. By virtue of $v_{ph}^t > 1$, the corrections connected with the Doppler effect in scattering are in this case of the order of $(kv/\omega) < v_{Te} \ll 1$, i.e., not larger than the relativistic corrections to the thermal motion, and can be disregarded. Thus, the Compton scattering is described by (see (6.4))

$$\Lambda_{tt}^{C.s} = \frac{ie^2 (\mathbf{e}_{\mathbf{k}}^t \mathbf{e}_{\mathbf{k}_1}^t)}{m_\alpha \omega (2\pi)^3},$$

where $\mathbf{e}_{\mathbf{k}}^t$ are the unit vectors of the polarization of the two interacting transverse waves. Nonlinear scattering with

$$\omega_- \ll |\mathbf{k}_-| v_{T\alpha} \; (\omega_- = \omega^t - \omega_1^t, \; \mathbf{k}_- = \mathbf{k}^t - \mathbf{k}_1^t)$$

is determined for scattering via a longitudinal wave by

$$\Lambda_{tt}^{C.s} = -\frac{ie^2}{m_\alpha \omega} \frac{\varepsilon_e^l(\omega_-, \mathbf{k}_-) - 1}{\varepsilon^l(\omega_-, \mathbf{k}_-)} \frac{(\mathbf{e}_{\mathbf{k}}^t \mathbf{e}_{\mathbf{k}_1}^t)}{(2\pi)^3},$$

which differs from the corresponding expression for the longitudinal waves in the directions of the vectors $\mathbf{e}_{\mathbf{k}}^t$. Averaging over the polarizations yields

$$\overline{(\mathbf{e}_{\mathbf{k}}^t \mathbf{e}_{\mathbf{k}_1}^t)^2} = \frac{1}{2}\left(1 + \frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2 k_1^2}\right).$$

It is easy to obtain an expression for the probability of scattering by electrons[73b]; this leads to the following expression for the nonlinear interaction:

$$\frac{\partial N_{\mathbf{k}}^t}{\partial t} = N_{\mathbf{k}}^t \int N_{\mathbf{k}_1}^t \frac{\omega_- \omega_{0e}^4 \left(1 + \frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2 k_1^2}\right)}{8(2\pi)^{5/2} |\mathbf{k}_-| v_{Te} \omega^t \omega_1^t nT_e} \left|\frac{\varepsilon_i^l(\omega_-, \mathbf{k}_-)}{\varepsilon^l(\omega_-, \mathbf{k}_-)}\right|^2 d\mathbf{k}_1. \quad (11.1)$$

The maximum energy transfer of the transverse waves is characterized by a time[15]

$$\frac{1}{\tau} \sim \frac{W^t}{nm_e v_{Te}^2} \left(\frac{\omega_{0e}}{\omega^t}\right)^3 \omega_{0e} \frac{\Delta\omega}{|\mathbf{k}_-| v_{Te}}.$$

In the case when $\omega_- \ll |\mathbf{k}_-| v_{Ti}$, the waves interaction is determined only by the nonlinear scattering by ions

$$\frac{\partial N_{\mathbf{k}}^t}{\partial t} = N_{\mathbf{k}}^t \int N_{\mathbf{k}_1}^t \frac{\omega_- \left(1 + \frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2 k_1^2}\right) \omega_{0e}^4}{8(2\pi)^{5/2} |\mathbf{k}_-| v_{Ti} \omega^t \omega_1^t nm_i v_{Ti}^2} \frac{1}{\left(1 + \frac{T_e}{T_i}\right)^2}. \quad (11.2)$$

The characteristic time of the process (11.2) is smaller under these conditions than (11.1) when $T_e < T_i m_i/m_e$. It should be noted that the condition $\omega_- \ll |\mathbf{k}_-| v_{T\alpha}$ can be satisfied for large frequencies $\omega \gg \omega_{0e}$, where this condition indicates that the interaction in question describes energy transfer with small $\Delta\omega^t$, $\Delta\omega^t/\omega^t \ll \Delta\theta v_{T\alpha}$. When $\Delta\theta \sim 1$, the time of transfer with $\Delta\omega^t$ of the order of ω^t is larger than (11.1) or (11.2) by approximately $1/v_{T\alpha}$ times. When the inverse inequalities $\omega_- \gg |\mathbf{k}_-| v_{T\alpha}$ are satisfied it is necessary to use the hydrodynamic approximation (4.8) for the nonlinear current described in the nonlinear scattering; the first and last terms of (4.8) make no contribution, owing to $(\mathbf{e}_{\mathbf{k}}^t \cdot \mathbf{k}) = 0$, while the second term is directed along $\mathbf{k} - \mathbf{k}_1$ and consequently only scattering via a longitudinal wave is possible in this approximation, scattering via a transverse wave being impossible. This points immediately to a criterion that establishes when the nonlinear scattering becomes a negligibly small effect. In this case

$$\frac{\varepsilon_e^l(\omega_-, \mathbf{k}_-) - 1}{\varepsilon^l(\omega_-, \mathbf{k}_-)} \simeq \frac{-\frac{\omega_{0e}^2}{\omega_-^2}}{1 - \frac{\omega_{0e}^2}{\omega^2}} \ll 1$$

when $\omega_- = \omega^t - \omega_1^t \gg \omega_{0e}$. Naturally, at least one of the interacting waves should be of high frequency, $\omega \gg \omega_{0e}$. Therefore when $\omega_- \gg \omega_{0e}$ the scattering can be regarded as Compton scattering. Under these conditions the energy transfer becomes differential and is determined by the value of $\Lambda_{tt}^{C.s.}$ written out above*

$$\frac{\partial N_{\mathbf{k}}^t}{\partial t}=\frac{\omega_{0e}^4 N_{\mathbf{k}}^t}{8(2\pi)^2 nm_e}\int N_{\mathbf{k}_1}^t\left(1+\frac{(\mathbf{k}\mathbf{k}_1)^2}{k^2k_1^2}\right)\times\frac{(\mathbf{k}-\mathbf{k}_1)^2}{\omega\omega_1}\delta'(\omega-\omega_1)\,d\mathbf{k}_1. \tag{11.3}$$

The characteristic energy-transfer time is estimated at

$$\frac{1}{\tau}\sim\frac{W^t}{nm_ec^2}\omega_{0e}\frac{\omega_{0e}^3}{\omega_t^3}\frac{\omega^t}{\Delta\omega^t}.$$

The differential character of the energy transfer (11.3), just as in the case of the sound oscillations, serves as an approximate expression for the energy transfer under conditions when the physically infinitesimally small $\Delta\omega$ satisfies the requirement $\Delta\omega \gg \omega_{0e}$ †. From a comparison of (11.3) and (11.1) we readily see that when $\Delta\theta \sim 1$ the "relay type" transfer by an amount $\Delta\omega$ of the order of ω, in which each stage is by a small amount $\Delta\omega$ satisfying $\omega_- \ll |k_-| v_T$, it is more effective by a factor $1/v_{Te}$ than "relay type" transfer in which each $\Delta\omega$ satisfies the inequality $\omega_- \gg |k_-| v_{Te}$ and $\Delta\omega \gg \omega_{0e}$.

We now consider scattering by electrons via a virtual transverse wave. Such a process can be significant when $\omega_- \ll k_- v_{T\alpha}$ for long-wave transverse waves, whose frequencies differ little from ω_{0e} [34]. From (5.4) we can obtain in this case

$$\frac{\partial N_{\mathbf{k}}^t}{\partial t}=\frac{N_{\mathbf{k}}^t}{8(2\pi)^{5/2}}\int\frac{N_{\mathbf{k}_1}^t\omega_-(k_-^2+(\mathbf{k}\mathbf{k}_1))[\mathbf{k}\mathbf{k}_1]^2\,d\mathbf{k}_1\omega_{0e}^4}{nm_ev_{Te}|\mathbf{k}_-|k^2k_1^2|k_-^2-\omega_-^2\varepsilon^t(\omega_-,\mathbf{k}_-)|^2}. \tag{11.4}$$

If we neglect the contribution of the ions in $\epsilon^t(\omega_-, k_-)$, then (11.4) goes over into the result obtained in [34]. The characteristic energy-transfer time in scattering through an angle θ of the order of unity can be estimated from (11.4) in two limiting cases. If $k_-^2 \gg |\omega^2\epsilon^t(\omega_-, \mathbf{k}_-)|$, i.e., for k_- of the order of k, if $\omega_- \simeq k\Delta k/\omega_{0e}$, and the inequalities

$$\frac{\Delta k}{k}\ll\frac{kv_{Te}}{\omega_{0e}};\qquad\omega_{0i}\ll k\ll\omega_{0e}$$

are satisfied, we get the estimate

$$\frac{1}{\tau}\simeq\frac{W^t}{nm_ec^2}\omega_{0e}\left(\frac{\Delta k}{k}\frac{\omega_{0e}}{kv_{Te}}\right).$$

On the other hand, if $\Delta k \ll v_{Te}\omega_{0i}$*, then the characteristic time of scattering via a longitudinal wave is of the order of

$$\frac{1}{\tau}\simeq\frac{W^t}{nm_ec^2}\omega_{0e}\left(\frac{\Delta k}{k}\frac{\omega_0}{kv_{Te}}\right)\frac{k^2}{\omega_{0e}^2v_{Te}^2}.$$

Consequently the scattering via a transverse wave can predominate only when $k \ll \omega_{0e} v_{Te}$, which by virtue of the inequality $\omega_{0i} \ll k \ll \omega_{0e}$ is possible only when $v_{Te} \ggg (m_e/m_i)^{1/2}$. Thus, scattering via a transverse wave is possible only under conditions of very high plasma temperatures †. If $k_-^2 \ll |\omega_-^2\epsilon^t|$, but $|\epsilon_e^t| \gg |\epsilon_i^t|$, i.e., $\Delta k \gg k^2v_{Te}/\omega_{0e}$ and $\Delta k \gg (m_e/m_i)\omega_{0e}v_{Te}$, then the characteristic time of transfer via a transverse wave is

$$\frac{1}{\tau}\simeq\frac{W^t}{nm_ec^2}\omega_{0e}\left(\frac{k^2v_{Te}}{\Delta k\omega_{0e}}\right).$$

In this case the scattering via a transverse wave predominates only when $\Delta k \ll kv_{Te}^2$, which is compatible with the previously written inequalities when

$$\frac{m_e}{m_i}\omega_{0e}v_{Te}\ll k\ll\omega_{0e}v_{Te}.$$

Finally, when $k_-^2 \ll |\omega^2\epsilon^t|$ and $|\epsilon_e^t| \ll |\epsilon_i^t|$, i.e.,

$$\frac{k^2v_{Te}}{\omega_{0e}}\ll\Delta k\ll\frac{m_e}{m_i}\omega_{0e}v_{Te}\text{ or }k\ll\omega_{0i},$$

the characteristic energy-transfer time is estimated at

$$\frac{1}{\tau}\simeq\frac{W^t}{nm_ec^2}\omega_{0e}\frac{k^2}{\omega_{0i}^2}\frac{\Delta km_i}{m_ev_{Te}\omega_{0e}},$$

and is smaller than that for scattering via a longitudinal wave only when

$$k\gg\omega_{0i}\frac{1}{v_{Te}}\left(\frac{m_e}{m_i}\right)^{1/2},$$

which is compatible with the inequalities written out above only if $v_{Te} \ggg (m_e/m_i)^{1/2}$.

12. Nonlinear Interaction of Transverse and Langmuir Waves

The interaction of transverse and Langmuir waves in a plasma plays a special role. The point is that, as a rule, the Langmuir waves are most effectively excited in a plasma under certain conditions. This occurs, for example, in important applications in which a beam passes through a plasma, and in other methods of plasma turbulization. At the same time, longitudinal oscillations cannot exist outside the plasma. The nonlinear interactions convert the Langmuir oscillations into transverse ones, which

*Actually, the first approximation, proportional to $\delta(\omega - \omega_1)$, makes no contribution in the nonlinear equations, and only the term corresponding to the expansion of the δ-function in terms of $(\mathbf{k} - \mathbf{k}_1)\cdot \mathbf{v}/(\omega - \omega_1)$, namely $(\overline{\mathbf{k}} - \overline{\mathbf{k}}_1)\cdot \mathbf{v}\delta'(\omega - \omega_1)$, is important. This raises the question of the role of the Doppler corrections, which have the same order of magnitude in the probability as the terms that are included. Calculation with the aid of (6.4) shows that for an isotropic particle distribution they are insignificant (they are proportional to $(\omega - \omega')\delta(\omega - \omega') \to 0$).

†We note in this connection that scattering by ions, for example, cannot be represented analytically in differential form even approximately.

*The region $\Delta k \gg v_{Te}\omega_0$ but $\Delta k \ll k^2v_{Te}/\omega_{0e}$ is quite narrow, since the inequalities $\omega_{0e}^2 \gg k^2 \gg \omega_{0i}\omega_{0e}$ are also required.

†The conclusion in [3-4] that scattering via a transverse wave always dominates if $\Delta k/k \ll kv_{Te}/\omega_{0e}$ is inaccurate, since no attention was paid in [34] to the role of the ions in the screened scattering via a longitudinal wave (see (11.1)) and no account was taken of the ions in the screening of the scattering via the transverse wave.

are usually easy to observe. The question of conversion of longitudinal waves into transverse ones becomes particularly acute in astrophysical problems, where the main information is obtained in the form of radiation. The question of conversion of longitudinal waves into transverse ones by spontaneous scattering was first considered by Ginzburg and Zheleznyakov in [65]. We analyze here in detail the conversion due to induced scattering, with allowance for the disclosed effects of compensation of Compton and nonlinear scattering, the role of the ions in the screening of the scattering by electrons, scattering by ions, and scattering via a virtual transverse wave.

a) Interaction effects in stimulated scattering by plasma particles. It is easy to see that in scattering by plasma particles the Langmuir waves can become transformed into transverse ones, whose frequencies are close to the electric plasma frequency. This follows directly from the conservation of energy during scattering

$$\omega^t - \omega^l = (\mathbf{k}^t - \mathbf{k}^l)\,\mathbf{v} = \frac{(k^t)^2}{2\omega_{0e}} - \frac{3\,(k^l v_{Te})^2}{2\omega_{0e}}.$$

Energy is transferred always in the direction of lower frequencies. In the presence of intense Langmuir oscillations and of weak transverse oscillations, the principal role is played by transfer into transverse waves, where by virtue of $\omega^t < \omega^l$ we get $k^t < \omega_{0e}\sqrt{3}(v_{Te}/v_{ph})$. On the other hand, in the case of intense transverse waves with frequencies $\omega \sim \omega_{0e}$, plasma waves can become excited with phase velocities satisfying the inequality

$$\frac{v_{ph}}{v_{Te}} > \left(\frac{2\,(\omega^t - \omega_{0e})}{\omega_{0e}}\right)^{1/2}.$$

A general expression for the scattering probability was obtained in [20], and a general formula for the nonlinear interaction in [15]. Neglecting scattering via a virtual transverse wave, we have

$$\begin{aligned}\frac{\partial N^t_{\mathbf{k}_2}}{\partial t} = &-N^t_{\mathbf{k}_2}\int d\mathbf{k}_1 N^l_{\mathbf{k}_1}\frac{\omega_{0e}^2\omega_-}{4m_e n v_{Te}^3 (2\pi)^{5/2}|\mathbf{k}_-|}\\ &\times\left\{\frac{[\mathbf{k}_1\mathbf{k}_2]^2}{k_1^2k_2^2}\left[\left|\frac{\varepsilon_i^l(\omega_-,\mathbf{k}_-)}{\varepsilon^l(\omega_-,\mathbf{k}_-)}\right|^2\right.\right.\\ &+\frac{v_{Te}^2k_1^2k_2^2}{\omega_{0e}^2|\mathbf{k}_-|^2}+\frac{2v_{Te}^2(\mathbf{k}_2\mathbf{k}_-)(\mathbf{k}_1\mathbf{k}_2)}{\omega_{0e}^2|\mathbf{k}_-|^2}-\left.\frac{2v_{Te}^2(\mathbf{k}_1\mathbf{k}_2)^2}{\omega_{0e}^2|\mathbf{k}_-|^2}\right]\\ &+\left.\frac{2v_{Te}^2}{k_1^2\omega_{0e}^2}(\mathbf{k}_1\mathbf{k}_2)^2\right\};\end{aligned}\qquad(12.1)$$

$$\mathbf{k}_- = \mathbf{k}_2 - \mathbf{k}_1, \qquad \omega_- = \omega^t(\mathbf{k}_2) - \omega^l(\mathbf{k}_1).$$

The first term of (12.1) describes the polarization effect connected with scattering by a screening cloud. Just as for Langmuir waves, the ions play a very important role in such a screening. The remaining terms of (12.1) describe the Doppler corrections to the Compton scattering. When $m_i \to \infty$ and $k^t \ll k^l$, Eq. (12.1) leads to the result of [20]:*

$$\begin{aligned}\frac{\partial N^t_{\mathbf{k}_2}}{\partial t} = \frac{N^t_{\mathbf{k}_2}}{4m_e n v_{Te}}\int &\frac{N^l_{\mathbf{k}_1}\omega_-\,d\mathbf{k}_1}{(2\pi)^{5/2}|\mathbf{k}_1|^5 k_2^2}\{[\mathbf{k}_1\mathbf{k}_2]^2\\ &\times(k_1^2k_2^2 - 4(\mathbf{k}_1\mathbf{k}_2)^2) + 2k_2^2k_1^2(\mathbf{k}_1\mathbf{k}_2)^2\}.\end{aligned}\qquad(12.2)$$

From (12.2) we get the following estimate for the characteristic conversion time

$$\frac{1}{\tau_{l\to t}} \simeq \frac{W^l}{nT_e}\,\omega_{0e}\cdot\left(\frac{v_{Te}}{v^l_{ph}}\right)\frac{k_2^2}{k_1^2}\,\frac{\omega_-}{\omega_{0e}}.$$

By virtue of

$$\omega_- < k^l v_{Te} \approx \omega_{0e}\frac{v_{Te}}{v^l_{ph}}$$

we have

$$\frac{1}{\tau} \sim \frac{W^l}{nT_e}\,\omega_{0e}\left(\frac{v_{Te}}{v^l_{ph}}\right)^2\frac{v_{Te}^2}{c^2},$$

i.e., it is much longer than the time of nonlinear interaction of the longitudinal waves. Conversion from transverse waves into longitudinal ones is characterized by a time

$$\frac{1}{\tau_{t\to l}} \simeq \frac{W^t}{nT_e}\,\omega_{0e}\frac{|\mathbf{k}_-|^2}{k_1^2}\,v_{Te}^2\,\frac{v_{Te}}{v^l_{ph}}\quad(\omega_- \simeq |\mathbf{k}-\mathbf{k}_1|\,v_{Te},\ k_2 \gg k_1 v_{Te}),$$

which exceeds the time of nonlinear interaction of the transverse waves. The contribution of the ions to the polarization (12.1) comes into play for the $l \to t$ transformation when

$$\frac{v^l_{ph}}{v_{Te}} > \left(\frac{9m_i}{4m_e}\right)^{1/3} v_{Te}^{1/3}.$$

In making this estimate it was assumed that k_2 is of the order of $k_1 v_{te}$. The same criterion is obtained if such an assumption is made for the $t \to l$ transformation.

Thus, the polarization effects due to ions become noticeable for interactions between transverse and longitudinal waves at much lower phase velocities v^l_{ph} than in the interaction of Langmuir waves $(v^l_{ph}/v_{Te} > (9m_i/m_e)^{1/3})$. Therefore (12.2) is valid only in a plasma with either very heavy ions or a high electron temperature. A second consequence of the foregoing inequalities is that the region of applicability of the formulas corresponding to $|\epsilon^l_e| \gg |\epsilon^l_i|$ is broader than for ll interactions, namely, if

$$\frac{v^l_{ph}}{v_{Te}} \ll \sqrt{\frac{9m_i}{m_e}}$$

*A formula for $k^t \gtrsim k^l$ was obtained in [34]. Such a refinement gives only negligibly small corrections to (12.2) at nonrelativistic plasma temperatures $v_{Te} \ll 1$, if one talks of conversion of longitudinal waves into transverse ones with $k^t < k^l\sqrt{3}v_{Te}$, while for generation of plasma waves by transverse ones, by virtue of $k^t > k^l\sqrt{3}v_{Te}$ and $\omega^t - \omega^l \simeq k_2^2/2\omega_{0e} < |k^t - k^l|v_{Te}$, we have $k^t \ll \omega_{0e}v_{Te}$, which is possible only for generation of plasma waves whose phase velocity is much larger than the speed of light, when, as a rule, screening by ions is significant (the first term of (12.1)).

and we neglect (12.2), we get

$$\frac{\partial N^t_{\mathbf{k}_2}}{\partial t} = -N^t_{\mathbf{k}_2}\int N^l_{\mathbf{k}_1}d\mathbf{k}_1\frac{\omega_{0e}^2|\mathbf{k}_-|^3 v_{Te}}{4\omega_{0e}^3 m_e n(2\pi)^{5/2}}\frac{[\mathbf{k}_1\mathbf{k}_2]^2}{k_1^2k_2^2}\frac{m_e^2}{m_i^2}. \quad (12.3)$$

From (12.3) with $k_2 \sim k_1 v_{Te}$ we get for the characteristic time of the $l \to t$ transformation

$$\frac{1}{\tau} \sim \frac{W^l}{nT_e}\left(\frac{m_e}{m_i}\right)^2\left(\frac{v_{ph}}{v_{Te}}\right)^3.$$

Comparison of (12.3) with the characteristic interaction of longitudinal waves in the region $v_{ph}/v_{Te} \ll (m_i/m_e)^{1/3}$ shows that (12.3) can reach a value of the order of the time of conversion of the longitudinal waves only at the borderline of the applicability of (6.14) and in the region of applicability of (6.17), i.e., when $v^l_{ph}/v_{Te} \gg (m_i/m_e)^{1/3}$. When the inequality $v^l_{ph}/v_{Te} \gg (9m_i/m_e)^{1/2}$ is satisfied, the $l \to t$ conversion is described by the formula

$$\frac{\partial N^t_{\mathbf{k}_2}}{\partial t} = -N^t_{\mathbf{k}_2}\int d\mathbf{k}_1 N^l_{\mathbf{k}_1}\frac{\omega_{0e}^2\omega_-[\mathbf{k}_1\mathbf{k}_2]^2}{4m_e n v_{Te}^3(2\pi)^{5/2}|\mathbf{k}_-|k_1^2k_2^2} \quad (12.4)$$

with a characteristic time (for $k_2 \sim k_1 v_{Te}$)

$$\frac{1}{\tau} \sim \omega_{0e}\frac{v_{Te}}{v^l_{ph}}\frac{W^l}{nT_e},$$

which corresponds to the order of magnitude of the nonlinear interaction (6.18) of Langmuir waves. We thus reach the important conclusion that, approximately, when

$$\frac{v^l_{ph}}{v_{Te}} > \left(\frac{m_i}{m_e}\right)^{1/3}$$

the nonlinear conversion of Langmuir waves into transverse waves is of the same order as the nonlinear conversion of Langmuir waves into each other. This conversion leads to a relatively intense emergence of transverse radiation from a turbulent plasma at frequencies larger than ω_{0e}. It must be borne in mind here that the higher the phase velocity of the exciting Langmuir waves, the smaller the refractive indices for the generated transverse waves.

Let us consider now the effects of nonlinear interaction via a virtual transverse wave. According to [15] we obtain when $(\mathbf{k}\cdot\mathbf{k}_1)\omega_{0e}^2 \gg 2(\mathbf{k}\cdot\mathbf{k}_1)\omega^2\epsilon^t_e$:

$$\frac{\partial N^t_{\mathbf{k}_2}}{\partial t} = -N^t_{\mathbf{k}_2}\int d\mathbf{k}_1 N^l_{\mathbf{k}_1}\frac{\omega_-}{4m_e n v_{Te}|\mathbf{k}_-|(2\pi)^{5/2}}|k_-^2-\omega_-^2\varepsilon^l(\omega_-,\mathbf{k}_-)|^{-2},$$

$$\left\{\frac{[\mathbf{k}_1\mathbf{k}_2]^2}{|\mathbf{k}_-|^2k_1^2k_2^2}[[\mathbf{k}_1\mathbf{k}_2]^2\omega_{0e}^4+2\omega_{0e}^4(\mathbf{k}_2\mathbf{k}_-)(\mathbf{k}_1\mathbf{k}_-) -\omega_{0e}^4(\mathbf{k}_1\mathbf{k}_-)^2]+\frac{2}{k_1^2}\omega_{0e}^4(\mathbf{k}_-\mathbf{k}_1)^2\right\}. \quad (12.5)$$

In the particular case when the role of the ions in ϵ^t is negligibly small, (12.5) gives the result of [34]. When

$$v^l_{ph} \ll \min\left\{\frac{1}{v_{Te}}, \sqrt{\frac{m_i}{m_e}}\right\} \text{ or } v^l_{ph} \gg \max\left\{\sqrt{\frac{m_i}{m_e}}, v_{Te}\frac{m_i}{m_e}\right\}$$

and $k_2 \sim k_1 v_{Te}$, the characteristic time (12.5) cannot exceed (6.18), and when $1/v_{Te} \ll v_{ph} \ll v_{Te}(m_i/m_e)$, i.e., in the relatively narrow interval of phase velocities of the waves, in the case when the electron temperature of the plasma is sufficiently high $v_{Te} \gg (m_e/m_i)^{1/2}$, the nonlinear scattering via a virtual transverse wave can reach the same order as scattering via a longitudinal wave.

As a rule, at large phase velocities, when scattering via a transverse wave is possible, the scattering by ions becomes predominant. Under the conditions $\omega_- \ll |k_-|v_{Ti}$, the nonlinear scattering by ions is described by the formula [15,34]*

$$\frac{\partial N^t_{\mathbf{k}_2}}{\partial t} = -N^t_{\mathbf{k}_2}\int N^l_{\mathbf{k}_1}d\mathbf{k}_1\frac{\omega_{0e}^2\omega_- T_e[\mathbf{k}_1\mathbf{k}_2]^2}{4m_e n v_{Te}^2 v_{T_i}(2\pi)^{5/2}T_i\left(1+\frac{T_e}{T_i}\right)^2|\mathbf{k}_-|k_1^2k_2^2}. \quad (12.6)$$

Under the conditions of $l \to t$ conversion, when $k_2 \ll k_1$, and when $\omega_- \sim k_1^2 v_{Te}^2/\omega_{0e}$, we obtain as an estimate of the characteristic time of the process

$$\frac{1}{\tau} \simeq \omega_{0e}\frac{W^l}{nT_e}\frac{v_{Te}^2}{v_{ph}v_{Ti}}\frac{T_e}{T_i\left(1+\frac{T_e}{T_i}\right)^2}$$

The condition $\omega_- \ll |k_-|v_{Ti}$ corresponds to $v^l_{ph} \gg v_{Te}^2/v_{Ti}$. As seen from (12.6), in order for scattering by ions to predominate, it is necessary to satisfy in addition the inequality

$$\frac{m_i}{m_e} \gg \frac{T_e}{T_i} \gg \left(\frac{m_e}{m_i}\right)^{1/3}.$$

b) Nonlinear interaction in the decay of a transverse wave into a Langmuir and ion-sound wave. The decay and coalescence processes $t + s \rightleftarrows l$, and $t \to l + s$ are closest to those considered above, since they give rise to an interaction of transverse waves with frequencies close to ω_{0e}. The first of these processes plays a role in the $l \to t$ conversion and the other in the $t \to l$ conversion. The indicated conversion process was considered in [68] (for the probabilities see also [73c]). By virtue of $\omega_s > 0$ it follows from the energy conservation law for $l \to t + s$ that $k^t < \sqrt{3}v_{Te}k^l \ll k^l$, and since $k^l \ll 1/\lambda_{De}$, we get $k_s \ll 1/\lambda_{De}$ and only the sonic oscillation $\omega_s = k_s v_s$ can occur. From $(k^t)^2 > 0$ it follows that decay is possible only if

$$\frac{v^l_{ph}}{v_{Te}} < \sqrt{\frac{9m_i}{4m_e}},$$

i.e., $k^l > 2k_0$ Let us consider the initial stage of the decay, when the numbers N^l and N^s are small, and the characteristic time of t-wave generation can be estimated from

$$\frac{1}{N^t}\frac{\partial N^t}{\partial t} \simeq \int u_l^{ts}N^l\frac{d\mathbf{k}^l\,d\mathbf{k}_s}{(2\pi)^6} \simeq \frac{W^l}{nT_e}\left(\frac{4m_e}{9m_i}\right)^{1/2}\omega_{0e}\frac{v^l_{ph}}{v_{Te}}\frac{k^l}{\Delta k^l}. \quad (12.7)$$

Here $\Delta k^l < k^l$ is the width of the l-wave spectrum and u_l^{ts} is the decay probability. Conversion of the waves into transverse ones is most effective when $k^l \sim 2k_0$, i.e., at the limit permissible by the conservation laws, where the indicated conversion can greatly exceed the nonlinear scattering by electrons (12.4), but conver-

*This formula takes into account only nonlinear scattering, since the Compton scattering is negligibly small and is analogous to ll scattering.

sion by the ions can become comparable with (12.7).

Let us consider further a process in which a transverse wave with frequency close to ω_{0e} decays into a Langmuir and ion-sound wave. With this, $k^t > \sqrt{3}k^l v_{Te}$. If we assume that $k^t \ll k^l$, then we have for the momentum of the excited l-wave

$$k^l = \sqrt{k_0^2 + \frac{(k_t)^2}{2v_{Te}^2}} - k_0,$$

and the characteristic time of generation of the l-waves is of the order

$$\frac{1}{N^l}\frac{dN^l}{dt} \simeq \int u_t^{ls} N_{\mathbf{k}^t}^t \frac{d\mathbf{k}^t\, d\mathbf{k}_s}{(2\pi)^6} \simeq \frac{W^t}{nm_e c^2}\,\omega_{0e}\,\frac{k^l k_0}{k^t \Delta k^t}\,. \quad (12.8)$$

An estimate of the intensity of generation of l-waves for the process $t + s \to l$ can be obtained from

$$\left.\begin{aligned} \frac{\partial N^l}{\partial t} &\simeq \int u_{ts}^l N^t N^s\, d\mathbf{k}^t\, d\mathbf{k}_s\,(2\pi)^{-6}, \\ \frac{dW^l}{dt} &\simeq \frac{W^t W^s \omega_{0e}}{nT_e}\,\frac{\omega_{0e}^2}{k^t \Delta k^t}, \qquad \omega_{0e} \gg k^t;\ \Delta k^t \lesssim k^t. \end{aligned}\right\} \quad (12.9)$$

At a considerable low-frequency sonic turbulence, (12.9) corresponds to more effective generation of l waves than (12.8). For (12.9), however, the growth of W^l is not exponential as for (12.8).

c) Generation of transverse waves upon coalescence of Langmuir waves and of Langmuir and ion-sound waves. We consider here two processes, $l + s \to t$ and $l + l \to t$ These processes occur in a turbulent plasma and their intensity is proportional to the turbulence energy. The process $l + s \to t$ can serve as a source of transverse waves in a plasma in which both l- and s-waves are excited, and its estimate is

$$\frac{dW^t}{dt} \simeq \frac{W^l W^s}{nT_e}\,\omega_{0e}\,\frac{\omega_{0e}^2}{k^t \Delta k^t}\,. \quad (12.10)$$

The coalescence process $l + l \to t$ leads to generation of transverse waves whose frequencies are close to $2\omega_{0e}$. The probability of this process can be readily obtained from the hydrodynamic nonlinear current (4.8) [31a]

$$\begin{aligned} u_t^{ll} = (2\pi)^6 \frac{e^2}{16\pi m_e^2}\,\frac{1}{k^2\omega_{0e}}\,(k_1^2 - k_2^2)^2\,\frac{[\mathbf{k}_1^2\mathbf{k}_2^2]^2}{k_1^2 k_2^2} \\ \times\ \delta(\omega^t - \omega_1^l - \omega_2^l)\,\delta(\mathbf{k}^t - \mathbf{k}_1 - \mathbf{k}_2). \end{aligned} \quad (12.11)$$

In the case of a small number N^t, the change in the number of longitudinal waves due to the decay is of the form [34]

$$\begin{aligned} \frac{\partial N_{\mathbf{k}_1}^l}{\partial t} = -\frac{N_{\mathbf{k}_1}^l}{12(2\pi)^2\omega_{0e}^3 nm_e}\int N_{\mathbf{k}_2}^l \frac{[\mathbf{k}_1\mathbf{k}_2]^2}{k_1^2 k_2^2} \\ \times\ (k_1^2 - k_2^2)^2\,\delta\left(3 - \frac{(\mathbf{k}_1 + \mathbf{k}_2)^2}{\omega_{0e}^2}\right) d\mathbf{k}_2. \end{aligned} \quad (12.12)$$

Since the sum of $|\mathbf{k}_1|$ and $|\mathbf{k}_2|$ is of the order of ω_{0e}, we find for $v_{ph}^l \gg 1$ that $k_1^2 - k_2^2$ is of the order of ω_{0e}^2, and $[\mathbf{k}_1 \times \mathbf{k}_2]^2/k_1^2 k_2^2 \sim \omega_{0e}^2/k_1^2$ and the characteristic decrement time due to the process (12.22) is estimated at

$$\frac{1}{\tau} \simeq \omega_{0e}\,\frac{W^l}{nT_e}\,v_{ph}^4.$$

When $v_{ph} \sim 1$, this estimate corresponds to that obtained in [34]. The time (12.12) is small compared with the characteristic time of conversion due to scattering and to the decays considered above. It must be borne in mind that it leads to an effect that can be registered experimentally, namely plasma radiation at the frequency $2\omega_{0e}$. The possibility of this spontaneous radiation of a turbulent plasma was pointed out in [31a]. (See [87] concerning astrophysical applications to emission from the sun.) When N^t is small, we can estimate the power of the transverse waves generated as a result of the process $l + l \to t$

$$\frac{dW^t}{dt} \simeq \frac{W^l}{nm_e c^2}\,W^l\omega_{0e} \quad (12.13)$$

for the most effective case $v_{ph}^l \simeq 1$.

d) Nonlinear interactions of transverse and Langmuir waves in stimulated scattering by superthermal plasma particles. This question is of interest because a turbulent plasma is characterized by the presence of superthermal particles in the distribution tails. The appearance of such particles is due to the accelerating mechanisms that come into play when the particles interact with the plasma turbulent pulsations [86]. Accelerated particles are observed experimentally in almost any turbulent plasma [86]. Beams of charged particles interacting with a plasma are likewise natural systems containing superthermal particles. We confine ourselves here to the analysis of effects of nonlinear interaction for isotropically distributed superthermal particles*. It should be noted that as a rule the number of superthermal particles is small. Their contribution to effects that occur with thermal particles is small†. The interaction of transverse and Langmuir waves scattered by superthermal particles, on the other hand, leads to new qualitative effects - possible nonlinear interaction for high-frequency transverse waves, $\omega^t \gg \omega_{0e}$ (so far, the frequencies of the waves participating in the interaction were of the order of ω_{0e}).

In order to prove the foregoing statement, it is sufficient to consider the energy conservation laws

$$\omega^t - \omega^l \simeq \omega^t = (\mathbf{k}^t - \mathbf{k}^l)\,\mathbf{v},$$

since $k^t \sim \omega^t$, we get

$$\frac{\omega^t}{\omega_{0e}} \sim \frac{|\mathbf{k}^l|\,v}{\omega_{0e}} > \frac{v}{v_{T_e}} \gg 1.$$

The scattering of transverse waves into longitudinal ones by superthermal particles is remarkable also because the main contribution is made by nonlinear scattering via a virtual transverse wave, unlike scattering by thermal scattering, when scattering via a

*An analysis of nonlinear effects in the presence of particle beams is beyond the scope of the present review. The qualitative estimates of the efficienty of nonlinear interactions, however, are the same for isotropically and anisotropically disturbed superthermal particles.

†This is the case, for example, for the interaction of Langmuir waves with each other (calculation of the scattering cross sections for superthermal resonant particles and corrections for Cerenkov radiation are given in[18b]).

transverse wave can be significant only under exceptional conditions when very stringent requirements are satisfied (this was discussed in detail above). The cross section of $l \to$ t scattering for superthermal particles and for $\omega^t \gg \omega_{0e}$ can be readily obtained from the general expressions given above [18b]:

$$w_e^t = (2\pi)^3 \frac{e^4\omega_{0e}k_1^2}{m_e^2 k_2^3} |[\beta \mathbf{k}_2]|^2 \delta(|\mathbf{k}_2| - \omega - \mathbf{k}_2\mathbf{v} + \mathbf{k}_1\mathbf{v}),$$

$$\beta = \frac{\mathbf{k}_1}{|k_2|} \frac{\sqrt{1-v^2}}{k_1^2\left(1-\frac{\mathbf{k}_2\mathbf{v}}{|k_2|}\right)} + \frac{\mathbf{v}\sqrt{1-v^2}}{(\mathbf{k}_1\mathbf{v})^2 k_1^2}((\mathbf{k}_1\mathbf{k}_2) - |\mathbf{k}_2|(\mathbf{k}_1\mathbf{v}))$$

$$-\frac{\mathbf{k}_1}{|\mathbf{k}_2|} \frac{1}{(\mathbf{k}_1-\mathbf{k}_2)^2-\mathbf{k}_2^2} - \frac{\mathbf{v}}{(\mathbf{k}_2-\mathbf{k}_1)^2-\mathbf{k}_2^2}. \qquad (12.14)$$

The second term β describes nonlinear scattering via a virtual transverse wave. (12.14) was first obtained for $\omega^t \gg \omega_{0e}$ in [18a] by using an intuitive analogy between nonlinear scattering and transition radiation from density inhomogeneities produced by the plasma wave. This calculation of [18a] is closely related to papers in which transition radiation in a medium with periodic structure is considered [63,64]. In this case the plasma wave produces an alternating electron concentration, and consequently modulates the dielectric constant in space and in time

$$\varepsilon^t = 1 - \frac{\omega_{0e}^2}{\omega^2} - \frac{\omega_{0e}^2}{\omega^2}\frac{\delta n}{n} \simeq 1 - \frac{\omega_{0e}^2}{\omega^2} - Ae^{-i\omega t + i\mathbf{kr}}.$$

Such an intuitive interpretation is permissible only by virtue of $\omega^t \gg \omega_{0e}$ (see [18b]). An analysis of (12.14) (see [18b]) shows that scattering via a virtual transverse wave cancels out in first approximation the Compton scattering if $v_{ph}^l \ll 1$ and $v \ll 1$; for $v_{ph}^l \sim 1$, the cancellation disappears if at the same time $v \ll 1$; for relativistic particles with $v \to 1$ the principal role is played only by Compton scattering. A detailed numerical analysis of the spontaneous scattering produced in scattering of fast particles by isotropically distributed plasma waves is contained in [18a]. Here we consider nonlinear effects arising at sufficiently high transverse-wave intensity and $\omega \ll \omega_{0e}$, when induced scattering plays an important role. It is easy to find that the induced scattering of transverse waves leads for isotropically distributed thermal particles to additional absorption of the transverse waves [19a]. For $\omega \gg \omega_{0e}$, $v_{ph}^l \ll 1$, and $v \ll 1$ we get

$$\frac{\partial N_{\mathbf{k}_2}^t}{\partial t} = -\frac{1}{4(2\pi)^2} N_{\mathbf{k}_2}^t \int \frac{\omega_{0e}^5}{k_2^2|\mathbf{k}_1|} N_{\mathbf{k}_1}^l \frac{d\mathbf{k}_1}{n_0 m_e}\left(\frac{n_1}{n_0}\right)\overline{v^{-1}}$$

$$\times \left\{\left(1 - 2\frac{(\mathbf{k}_1\mathbf{k}_2)^2}{k_1^2k_2^2}\right)^2 + \frac{(\mathbf{k}_1\mathbf{k}_2)^2}{k_1^2k_2^2}\right\}, \qquad (12.15)$$

where n_1 is the total number of superthermal particles capable of participating in the scattering

$$\overline{v^{-1}} = n_1^{-1} \int_{v > \frac{\omega^t}{k_l}} f_{\mathbf{p}} \frac{d\mathbf{p}}{(2\pi)^3} v^{-1}.$$

The characteristic decrement γ for such nonlinear absorption is estimated at

$$\gamma = \frac{1}{\tau} \simeq \pi \frac{W^l}{n_0 m_e c^2} \frac{v_{ph}^l}{v}\left(\frac{\omega_{0e}}{\omega^t}\right)^2 \frac{n_1}{n_0}.$$

It is possible to find in similar fashion the change in the number of longitudinal waves. This can be accompanied by buildup of longitudinal waves [19a].

e) Decays of transverse waves into transverse and Langmuir waves. This type of interaction is also possible for transverse high-frequency waves $\omega^t \gg \omega_{0e}$. It is easy to understand that in these processes the total number of transverse waves is conserved, and consequently the process under consideration leads to a redistribution of the t-waves over the spectrum. The probability of such a process was obtained in [19b]:

$$u_t^{tl} = (2\pi)^6 \frac{e^2(\mathbf{k}^l)^2\omega^l}{16\pi m_e^2\omega_1^t\omega_2^t}\left(1 + \frac{(\mathbf{k}_1^t\mathbf{k}_2^t)^2}{(k_1^tk_2^t)^2}\right)$$

$$\times \delta(\omega_1^t - \omega_2^t - \omega^l)\,\delta(\mathbf{k}_1^t - \mathbf{k}_2^t - \mathbf{k}^l). \qquad (12.16)$$

This process is similar in structure to $l \to$ s $+ l$ decay and many of the facts revealed for this decay can be qualitatively extended to the t $\to$ t $+ l$ decay.

It is significant that this decay can likewise not proceed without limit, since the decay results into two waves whose frequencies are larger than ω_{0e}. Consequently, the frequency of the decaying wave should be larger than $2\omega_{0e}$. This imposes an upper limit $v_{ph}^t < 2/\sqrt{3}$ on the possible phase velocities of the t-wave during such a conversion. It is essential that the decay processes convert the transverse waves in that frequency region where the induced scattering is not effective. The result of this conversion are plasma waves, which were investigated in detail in [15, 74c, 98]. The energy generated by the plasma waves will be of the same order as the energy of the initial transverse waves, i.e., the transverse waves become noticeably dissipated in the plasma as a result of the nonlinear processes if the frequencies ω^t are decreased during the conversion by an amount of the order of ω^t. In the case of narrow t-wave spectra ($\Delta\omega^t \ll \omega_{0e}$), the conversion leads to the appearance of satellites. The characteristic conversion time for one satellite at [98] $\Delta\theta^t \ll (\omega_{0e}/\omega^t)^{3/2}$ is of the order of

$$\frac{1}{\tau} \simeq \frac{\pi}{2}\,\omega_{0e}\frac{\omega_{0e}}{\Delta\omega^t}\frac{W^t}{nm_ec^2}.$$

For a broad wave spectrum $\Delta\omega^t \gg \omega_{0e}$, the estimated characteristic generation time is

$$\frac{1}{\tau} \simeq \frac{\pi}{4}\,\omega_{0e}\left(\frac{\omega_{0e}}{\Delta\omega^t}\right)^2\frac{W^t}{nm_ec^2}. \qquad (12.17)$$

In analogy with the $l \to$ s $+ l$ decay, in this case the conversion at high l-wave energy proceeds in a direction such as to increase the frequency of the transverse waves.

13. Nonlinear Interaction of Transverse and Ion-sound Waves

There are two types of interactions of t- and s-waves.

a) Stimulated scattering of transverse waves by electrons with transformation into ion-sound waves. This interaction leads to absorption of the transverse waves and is in many respects analogous to the absorption of l-waves in $l \to s$ scattering. If $\omega^t \ll |k^s| v_{Te}$ and $\omega^t \gg \omega_{0e}$, which corresponds to the conversion of transverse waves into ionic oscillations $\omega_s \sim \omega_{0i}$, then the probability of the scattering process takes the form

$$w_t^{ts} = (2\pi)^3 \frac{e^4}{m_e^2} \frac{\omega_{0i}}{(\omega^t)^3} \frac{[\mathbf{k}_s \mathbf{k}^t]^2}{(k_s k^t)^2} \delta(\omega^t - \omega_s - (\mathbf{k}^t - \mathbf{k}_s)\mathbf{v}),$$

and the nonlinear absorption is described by the formula

$$\frac{\partial N_{\mathbf{k}_2}^t}{\partial t} = -N_{\mathbf{k}_2}^t \int \frac{N_{\mathbf{k}_1}^s \left(\frac{m_e}{m_i}\right)^{1/2} \omega_{0e}^3 d\mathbf{k}_1}{2(2\pi)^{5/2} n_0 m_e v_{Te}^2 |\mathbf{k}_s| v_{Te}} \left(\frac{\omega_{0e}}{\omega^t}\right)^2 \frac{[\mathbf{k}_1 \mathbf{k}_2]^2}{k_1^2 k_2^2} \quad (13.1)$$

b) Decay of transverse waves into ion-sound waves, described by the probability [19d]

$$u_t^{ts} = (2\pi)^6 \frac{1}{64\pi} \frac{e^4}{m_e^2 v_{Te}^4} \frac{|\omega_s|^3 \omega_{0e}^4}{\omega_1^t \omega_2^t k_s^2 \omega_{0i}^2}$$
$$\times \left(1 + \frac{(\mathbf{k}_1^t \mathbf{k}_2^t)^2}{(k_1^t k_2^t)^2}\right) \delta(\omega_1^t - \omega_2^t - \omega^s)\, \delta(\mathbf{k}_1^t - \mathbf{k}_2^t - \mathbf{k}_s). \quad (13.2)$$

An analysis shows that a multiple-step decay, in which the transverse wave first decays into a Langmuir wave, after which the Langmuir waves decay into ion-sound waves [74c] is more probable than direct decay of the transverse waves into sonic waves.

CONCLUSIONS

A few words should be said in conclusion concerning unsolved problems in the theory of nonlinear interaction. The solution of the system of nonlinear equations is quite complicated even in the simplest case, but one should hope that it is precisely the solution of such problems which will lead to important information on the stationary-turbulence spectra. Notice should be taken of the papers [21,61,85], which are the first to attempt to solve such problems. Another problem, no less important, is the role of nonlinear effects in the development of two-stream instability. Notice should be taken here of [36], where it is shown that the nonlinear effects can lead to a suppression of two-stream instability. Further, no less important is the question of radiation from a turbulent plasma, which is produced when longitudinal waves are converted into transverse ones, and which can serve as the source of energy loss and plasma cooling [31]. On the other hand, various energy conversions can transfer the oscillations to the region of absorption and by the same token raise the plasma temperature. This is particularly important for a plasma situated in magnetic fields [35]. An investigation of these questions can present the picture of the dynamics of a turbulent plasma. Therefore an investigation of nonlinearities of a magnetoactive plasma is of considerable interest. At the present time we cannot regard the investigation of these problems as complete.

Nonlinear effects exert a great influence on the spectra of the particles accelerated in a turbulent plasma, on the isotropization of cosmic rays [38,39], the generation of turbulence in cosmic plasma by gravitational instability, etc.

Finally, nonlinear effects of plasma inhomogeneities determine the particle-diffusion processes, knowledge of which is essential for the problem of plasma containment.

Note added in proof In Ch II we considered nonlinear interactions of waves scattered by plasma particles whose distribution is Maxwellian The general equations of Ch I take also into account the reciprocal influence of the stimulated scattering on the particle distribution function, which may reduce the nonlinear interaction of the intense waves in a plasma with small ratio ν/ω_{0e} (ν – average frequency of electron-electron collisions) For a one-dimensional interaction of Langmuir waves, in accordance with the estimates of A. S Chikhachev and the author, the decrease is by a factor η^{-2} only if $\eta > 1$

$$\eta_e \approx \frac{nT_e}{W^l}\left(\frac{\nu}{\omega_{0e}}\right)^{1/2}; \qquad \eta_i \approx \frac{nT_e}{W^l}\left(\frac{\nu V_{Te}}{\omega_{0e} V_{Ti}}\right)^{1/2}\left(1 + \frac{T_e}{T_i}\right)$$

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