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HOLOGRAPHY AND INTERFERENCE PROCESSING OF INFORMATION

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INTRODUCTION

THE term holography was introduced by the British physicist D. Gabor 18 years ago [22,24]. Translated from the Greek, the first term stands for whole or entire. At the present time the concept holography covers lenseless methods of obtaining images of objects. Whereas classical photography produces the image directly, holography is technologically a two-step process. The first step is to record the light wave propagating from the object and carrying information concerning the object. In the second step, the scattered-light wave is reconstructed, and this makes it possible to see the object in all its details, including the three-dimensional form and effects of parallax and depth of field. In modern holography both steps are realized by means of laser beams. Whereas classical photography records only the intensity of the scattered wave, and the phase shifts produced by the objects are irretrievably lost, in holography it is possible to record simultaneously yet separately both the amplitude and the phase information contained in the scattered wave.

To assess clearly the place of holography in modern science and technology, attention must be called to the following two main properties of the holography method. First, holography solves the problem of the so-called complete measurement, which in optics is that of simultaneously recording the amplitude and phase information concerning a light wave propagating from the object, including polarization. This problem has something in common with the complete experiment as stipulated in elementary particle physics, where to reconstruct the scattered wave it is necessary to know not only the probability of the process but also different interference effects. Second, the two-step character of the holography affords an unusual opportunity for a posteriori processing of the information contained in the scattered wave, after the end of the experiment. By this is meant the feasibility of focusing, filtration, spectral selection, correction for aberration of the employed optical system, and, finally, changing the point of observation. It is difficult for the time being to estimate fully the opportunities afforded by this perfectly new physical experimentation technique.

The history of holography is quite instructive. Gabor's first work [22] was aimed at improving the resolution of the electron microscope. The problem reduced in fact to the extraction of information concerning a microscopic sample from the blurred part of an image. Gabor's scheme was to record the diffraction pattern produced by individual points of the microsample. His main proposition was to add an intense coherent background to this blurred picture. Such an illumination automatically made the phases of the scattered small and easier to observe. This procedure, however, did not solve the problem completely. The real and virtual images produced during the reconstruction process were superimposed, each acting as a background for the other; furthermore, it was impossible to produce an intense coherent background for all samples [37]. Not until 1962 was the problem of separating the two images completely solved by E. N. Leith and J. Upatnieks [42,43], who used the concepts and principles of the theory of radio communication to eliminate this shortcoming of holography. The ideas of holography were in fact regenerated. This initiated a vigorous development of holography, aided to a considerable degree by lasers, which by that time became widely used. Within a year it became possible to observe three-dimensional objects [44,45,46], in full agreement with Gabor's predictions.

The hologram with the aid of which the wave scattered from the object is registered is similar in its structure to the diffraction grating used in optics to observe an emission spectrum. The "ghosts," i.e., the false lines, which at one time caused so much trouble to spectroscopists, have now, after the invention of holography, become useful and controllable. By now several principally different schemes of obtaining holograms have been investigated [57,63,31,25,59,60]. The elimination of the poor spatial resolution permits a direct approach to the development of an x-ray holographic microscope with a resolution of 1 Å (10^{-8} cm). Such a microscope can reconstruct a three-dimensional image of the molecules of a live cell [58,66]. Holography is gradually being introduced in some branches of science and technology [12,15,23,28,30,52,53].

In parallel with holography, optical methods of information processing have been successfully developed in recent years [7,16,17,18,21,35,48,51,54,55]. The fundamental tasks of this branch of optics are: spatial filtration of the optical image [13,65], separation of a known signal from noise [38,39], image recognition, and rapid processing of information which is two-dimensional in nature. Compared with radio-electronic processing methods, optical methods have that essential advantage that two-dimensional objects are processed with the aid of coherent optics as an entity and practically simultaneously over the entire area, thus eliminating the operations of scanning and sweeping the image. When holography techniques are introduced into optical information-processing methods, new principles of recording and extracting information are revealed, ensuring high information-processing speeds, unusually large capacity, as well as a much higher stability of recording and reading the information. Methods of spatial filtration of the image, developed in recent times, uncover a possibility of broad applications for the processing of the continuously growing volume of information. They have already begun to be used in seismography [35], radio astronomy, and radar [10]. The first models of analog optical computers were constructed [52].

I. FRESNEL-TRANSFORM HOLOGRAPHY

1. Formulation of Problem

Visual perception of surrounding objects consists of several processes that occur in sequence. The object is first illuminated by some light source. The light wave reflected from the object carries into surrounding space information on the object. The pupil of the eye cuts out a small part of the wave front, and the gathering crystalline lens of the eye produces an image which, through photochemical processes, induces with the aid of the nervous system the image of the object in the human brain. The propagation of light from the object to the pupil is an autonomous process, and its property is expressed by the well known Huygens-Fresnel principle, according to which, once a wave is detached from the object it no longer depends on the object and propagates further in accordance with the laws of diffraction. A feature common to all systems used to store or transmit images of objects is that what is ultimately observed is not the light wave itself, but an image obtained from the light wave with the aid of a lens. Until recently it was practically impossible to obtain information concerning the object without the aid of a lens as part of the image-producing system. The principal scheme of any optical image-producing system can be conditionally represented as:

$$\text{Object} \rightarrow \text{Light Wave} \rightarrow \text{Image}. \quad \text{(A)}$$

In addition to this traditional scheme of recording information concerning the object, it is also possible to think of a somewhat unusual scheme, namely:

$$\text{Object} \rightarrow \text{Light Wave}. \quad \text{(B)}$$

In this case no image of the object is produced. What is recorded directly is just the light wave. By somehow imprinting the structure of the light wave, we can subsequently reconstruct it and produce an image of the object as the second step:

$$\text{Light Wave} \rightarrow \text{Image} \quad \text{(C)}$$

Thus, the process consists of two separate steps.

The traditional scheme (A) for obtaining the image is characterized by the fact that the image is obtained by having the photodetector register the average intensity of the wave in the focal plane of the lens; the information concerning the phases of the scattered wave is thereby irretrievably lost. Therefore scheme (A) provides only half the information. The image is thus incomplete. As a consequence of this shortcoming, it is impossible to reconstruct a three-dimensional object directly, and furthermore the entire process is not immune to defocusing of the image. Although the information is not actually destroyed by the defocusing, in fact the information eludes the observer because of this shortcoming. In the two-step process these shortcomings are entirely eliminated. It is the investigation of the two-step photography process that led to the creation of the new branch of optics, subsequently called holography.

In simplified form, the two-step process can be visualized as follows. During the first step the light wave propagating in the space between the object and the detector is "frozen," i.e., the propagation process is interrupted and the instantaneous state of the wave propagation is recorded. During the second step, the wave is "unfrozen" and allowed to propagate further, in accord with the Huygens-Fresnel principle, just as if the period of the "anabiosis" of the light did not occur at all. By placing a pupil in the path of this wave, we see the object in the same form as it existed during the instant when the wave was frozen. Such a observation scheme is a modification of photography: the object is seen at a time when it actually does not exist.

What are the conditions for realizing such a two-step process? It is still difficult to answer this question rigorously, since the study of the holographic methods of observations is still continuing. In the simplest and at the same time most reliable holography scheme, the light source illuminating the object should be monochromatic and coherent. With this, the spatial (transverse) and temporal (longitudinal) coherence should be such that the coherence lengths exceed by many times the dimensions of the object or the depth of the scene. Under these conditions, a stable standing pattern of waves is produced, and if we add to this the requirement that during the exposure time the objects should be securely fixed relative to the remaining parts of the optical system, then both factors will ensure stationarity of the illuminating and reflected light waves.

2. The Holography Scheme

Let an object installed in a coherent system reflect light in all directions, and let part of the reflected light strike the entrance pupil. At each point of the plane of the pupil, the light wave is of the form

$$u(x, y) = a(x, y)\, e^{i\Phi(x, y)}, \quad (1)$$

i.e., it is characterized at each point of the plane of the pupil by an amplitude distribution function $a(x, y)$ and a phase distribution function $\Phi(x, y)$. The common factor $\exp(i\nu t)$, which continuously changes the phase of the light, has been left out, since the system is coherent, meaning that the phase difference between oscillations at two arbitrarily taken points of observation is invariant in time. The field of the light wave is regarded as a scalar, bearing in mind that no changes in the direction of the light polarization take place in the system, so that we can use analogous relations for the remaining components of the electric vector of the light wave [41,67].

Let us place a photographic plate in the plane of the pupil and let us expose it for the required time. The photographic plate, being a square-law detector, records the average intensity of the light wave.

$$I_{av}(x, y) = \frac{1}{2} |a(x, y)|^2.$$

The information concerning the phases will not be recorded in this case. To print also the phase information, it is necessary, as follows from the general rule in physics, to produce an interference picture. To this end, a second light beam is aimed at the plate. It can be obtained with the aid of a flat mirror placed in the beam of light from the coherent source (Fig. 1). This additional beam, called the reference beam, should be directed at an angle to the plane of the photographic plate [24,45]. If θ is the angle of in-

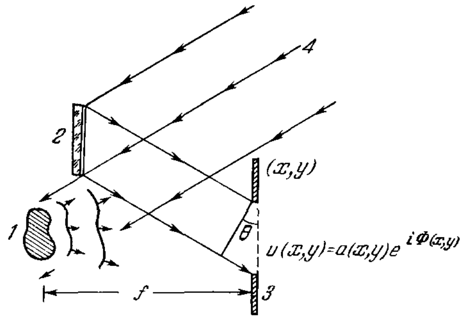


FIG. 1. Scheme for obtaining a hologram. 1–Object, 2–mirror producing a reference beam, 3–plane of the pupil, 4–coherent light source.

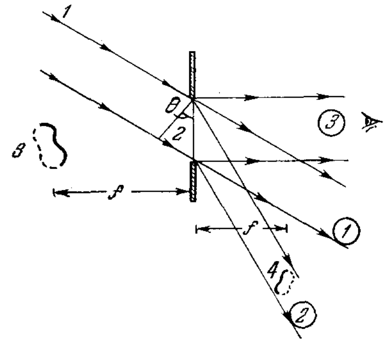


FIG. 2. Scheme for reconstructing the wave. 1–Reference beam, 2–hologram, 3–virtual image of object, 4–real image of object.

cidence of the plane reference beam, then the amplitude of the reference light can be written in the form

$$u_0 = e^{i\omega_x x},$$

where $\omega_x = (2\pi/\lambda)\sin\theta$, and λ is the wavelength of the light. In the presence of a reference beam, the photographic plate will record the intensity

$$I_{av}(x, y) = \langle |u + e^{i\omega_x x}|^2\rangle_{av} = \frac{1}{2}(1 + |u|^2) + \frac{1}{2}[ue^{-i\omega_x x} + u^* e^{i\omega_x x}], \quad (2)$$

where $u = ae^{i\Phi}$.

We see that the intensity registered on the plate includes not only the amplitude a, but also the phase Φ. Thus, we have gathered the complete information on the wave, which in our terminology is "frozen." It now remains to decipher it. To reconstruct the wave, the hologram obtained after developing and reversing the photographic plate is illuminated by the same reference beam from the coherent light source (Fig. 2). The development of the hologram is such, that the intensity transmission of the hologram is proportional to the square of the intensity of the light, i.e., the hologram is developed to a contrast coefficient $\gamma = -2$. When such conditions are fulfilled, the amplitude of the wave directly at the output of the hologram is

$$u(x, y) = e^{i\omega_x x}\cdot I_{av} = e^{i\omega_x x} \times \frac{1}{2}(1 + |u|^2) + \frac{1}{2}u + \frac{1}{2}u^* e^{i2\omega_x x}. \quad (3)$$

It can be divided into three components. The first is in the beam direction, the second propagates at an angle 2θ, and the third is normal to the plane of the hologram. The latter component contains the reconstructed wave $u = ae^{i\Phi}$. It can now be guided to an optical system or observed visually.

3. Real and Virtual Images

The process of propagation of the wave front, in accordance with the Huygens-Fresnel principle, reduces to cascaded transmission of electromagnetic disturbances to neighboring points of space. Such disturbances, naturally, propagate both forward and backward of the wave front, regarded at the given instant of time. However, any disturbance moving backward is cancelled by interaction with its own initial disturbance at the point from which it just emerged. Therefore the front of the wave follows the disturbance that propagates forward. Ahead of the front there is no initial disturbance since the disturbance from the preceding echelon is extinguished by the very echelon that has moved forward.

When the light wave $u = ae^{i\Phi}$ is reconstructed in the plane of the pupil with the aid of the hologram, i.e., when the wave encoded in the hologram is "unfrozen," the initial disturbances are missing, and therefore the light wave regenerated from the hologram propagates on both sides of the hologram, which can be arbitrarily called "future" and "past." Propagating in the "future" is a diverging wave, forming a virtual image of the object. The wave in the "past" is converging and produces a real image. To observe the virtual image, the eye must be placed in the light beam propagating perpendicular to the hologram. The real image will at the same illumination be seen at an angle 2θ. To see it, it is necessary to move back farther than the distance from the object to the hologram. To register the real image, the photographic plate is placed at the same distance as between the object and the hologram during the production of the latter. The real image is pseudoscopic [43,44,47]. A similar effect is produced when the two halves of a stereo photograph are interchanged—hills become valleys and projections become depressions. Therefore, when two objects are situated along the line of sight, the reconstructed picture will have a strange appearance: the nearest object vanishes, and may be by one located farther back. One of the unusual properties of a hologram is that it shows nothing when viewed in simple incoherent light. It looks practically like a uniformly exposed plate. All the information concerning the wave is encoded in an inhomogeneous structure of fringes, which can be

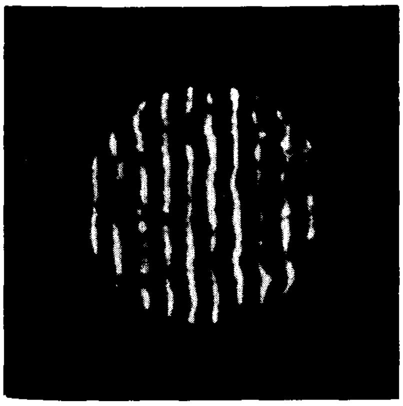


FIG. 3. Microstructure of hologram.

discerned with difficulty under a strong magnifying glass. Only a microscope reveals the details of this structure [24,46] (Fig. 3). The hologram can be broken up into several pieces, and each piece produces the same picture as the entire hologram. With decreasing size of the pieces, the resolution begins to worsen and simultaneously the depth of focus increases. This stability of the hologram becomes understandable if it is recalled that decreasing the dimension of the hologram is equivalent to a decreasing the pupil in ordinary optics with the aid of a diaphragm.

4. Bleached Hologram

If the hologram obtained by developing the photographic plate is placed in a photochemical bleaching solution, the metallic silver is dissolved. If the bleached hologram is then illuminated with a beam of coherent light, then an image brighter than in the case of the unbleached hologram is produced [14]. The information recorded in the hologram does not disappear. The reason for the "immunity" of the hologram is that the transparent bleached hologram still retains pits in the places where the metallic silver was released. The depth arrangement of these pits retains the amplitude and phase information, in the same manner as the picture of alternating strips in the plane of the hologram, the intensities and positions of which retain the complete information on the light wave. The light illuminating the bleached hologram is modulated by the phase shifts of the light passing through the inhomogeneous relief of the hologram. Since there is no light absorption, the image of the object is brighter than in the case when the hologram acts as a light absorber.

5. Dynamic Range of Hologram

From the point of view of scientific photography, great interest attaches to that property of the hologram whereby the range of illumination which can be recorded without distortion is greatly broadened compared with ordinary photography [43]. Whereas the better emulsions have a linear transmission range of approximately 100:1, a hologram can transmit without distortion an intensity differential between the brightest and darkest parts of the object amounting to as much as $(10^4-10^5):1$. The physical reason for this enlarged dynamic range is that the hologram registers an interference pattern that is spread over the entire hologram. Small bright sections of the object spread their energy over the entire hologram, and during the reconstruction this energy is again gathered on to a small area. The advantage of this property of the hologram can hardly be overestimated.

6. Holographic Microscope

The holographic method of obtaining an image of an object was from the very outset connected directly with microscopes [22,8]. After the first research on holography was completed, the efforts of the researchers were aimed, naturally, towards development of a holographic microscope. In the first design, 150-fold magnification was attained. Figure 4 shows the diagram of a two-beam holographic microscope. The microscope lenses produce two diverging beams of coherent light, which overlap in the plane of the hologram. The microsample is so mounted that it is illuminated only by the working beam. The reference beam does not strike the sample. The use of diverging beams causes the diffraction pattern registered on the hologram to increase by an amount equal to the similarity ratio of the corresponding triangles. When the image is reconstructed, the hologram is illuminated by a diverging beam. This creates additional magnification of the image of the microsample. Finally, a third cause of magnification of the holographic microscope is the change of the wavelength of the light during the reconstruction. All these factors make it possible to attain a magnification estimated at approximately one million. An analysis of the correspondence between the microsample and the reconstructed magnified image gives fully defined relations between the geometric parameters of the holographic microscope

The holographic microscope can eliminate the distortion connected with the reconstruction of a three-dimensional object during magnification [47]. Distortion of this type, which occurs when an object is magnified, can be easily discerned in binoculars when the observed object moves at a certain angle to the line of sight. A similar shortcoming is possessed by any optical system. Holography is different, for it solves completely the problem of retaining of the three-dimensional outline of the object during magnification. Analysis shows that to conserve the outline of a three-dimensional object magnified by holograph the longitudinal magnification must equal the square of the transverse magnification. At the present time it is not clear whether a holographic microscope can compete with an ordinary microscope

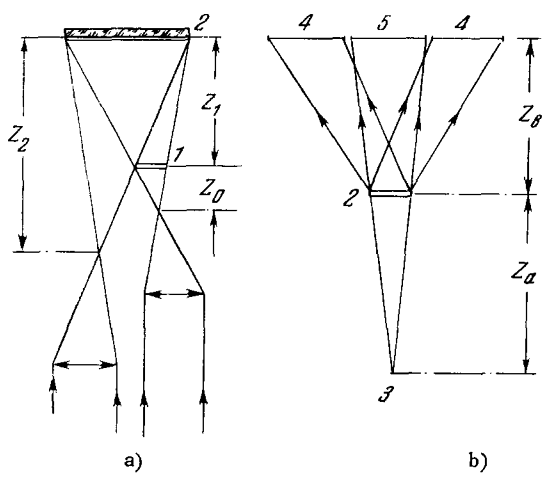


FIG. 4. Diagram of two-beam microscope. a) Scheme for obtaining the hologram, b) scheme for obtaining the magnified image. 1–Microscopic sample, 2–hologram, 3–point source, 4–magnified image, 5–null beam.

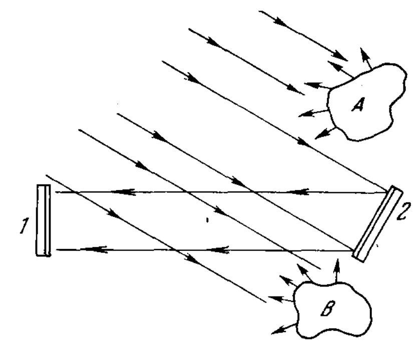


FIG. 5. Hologram in a thick-layer emulsion. 1–Three-dimensional hologram, 2–mirror.

in the visible region of the spectrum. But there is no doubt that it will be indispensable in those spectral regions where an ordinary lens microscope is ineffective. Of particular interest is a holographic microscope for ultraviolet and x-rays. Technical problems delay its development somewhat, but they are not unsurmountable.

7. Hologram in a Thick Emulsion

A hologram can be registered in a thick-layer emulsion, which constitutes a three-dimensional optical medium. The first experiments of this type were carried out by Denisyuk [3-6] and analyzed theoretically by van Heerden [27]. The hologram in a thick emulsion has the structure of a system of planes, diffraction by which is possible if the Bragg relation

$$2d \sin \theta = \lambda \tag{4}$$

holds between the gap d in the reflecting planes and the angle of incidence θ of a light beam having a wavelength λ.

In order that the emulsion act like a three-dimensional medium, it is necessary that the distance between the holographic microstrips be smaller than the thickness of the emulsion. Because the exposed blackened planes of the three-dimensional hologram are inclined at a definite angle, it is possible to obtain either only one virtual image, or, if the orientation of the hologram relative to the beam is changed, only one real image. These properties of a three-dimensional hologram were demonstrated by the following experiment [21] (Fig. 5). The hologram was obtained by the usual procedure. Two objects were placed in a way as to subtend approximately equal angles. Therefore, if the emulsion were thin, then the reconstructed real image of object A would be superimposed on the virtual image of object B, and the real the image of B would fall on the virtual image of A. This would in fact smear out the entire reconstructed picture. However, a hologram in a thick-layer emulsion gives only virtual images of both objects, without any traces of real images. If the hologram is rotated during the reconstruction, then the imaginary images weaken, and finally vanish, and real images appear in their places.

8. Color Holography

It is possible to combine in a single hologram several images, since the hologram is additive with respect to the amplitude of the wave in the pupil. In the first variant, objects which do not overlap one another are used to expose the hologram in sequence. During the reconstruction, all objects are seen together. In the second variant, the objects are placed in sequence in a single place, but the photographic plate is rotated each time through a certain angle after the exposure, so that the hologram acquires the structure of several gratings with different directions [7,43]. The reconstructed objects are seen in different directions corresponding to the orientations of the microstructure of the hologram.

By using the technique of superimposing several holograms, it is possible to obtain a hologram of a multiple-color wave front. The hologram of each color component has not only its own orientation, but also its own lattice spacing. The simplest scheme for obtaining colored holograms is to illuminate the object simultaneously by beams of light of different wavelengths. An appropriate number of mirrors is arranged in line with the object and around it in such a way that each mirror receives light from only one laser. All the reference beams expose a single photographic plate. The multiple-color hologram is then obtained on a black and white emulsion. To reconstruct the colored three-dimensional image, the hologram is illuminated by beams from the same lasers used to illuminate the objects during the exposure. Each beam produces several images after passing through the multi-component hologram. Altogether, n^2 real and n^2 virtual images are produced when n colors are transmitted. Out of the total n^2 images, n coincide, producing in one place a colored three-dimensional image of the object. The remaining images coincide in various other places, not all of them together, producing color-distorted images. For example, the "red" image reconstructed from the "blue" component of the hologram may overlap the "red" image from the "red" component. These incomplete images are produced in other locations without interfering with the main total-color image[44].

The second scheme for obtaining color holograms is based on the use of a thick-layer photographic emulsion, which serves as a three-dimensional recording medium. In this case a three-dimensional lattice is produced, and if the interfering waves contain several spectral lines, then each produces its own system of reflecting planes[53]. When two coherent plane waves interfere, the reflecting planes are directed along the bisector of the angle between the directions of the propagating waves.

A device for obtaining a three-dimensional color hologram is shown schematically in Fig. 6. Collimated beams of coherent light with $\lambda_1 = 4{,}880\,\text{Å}$ (argon laser) and $\lambda_2 = 6{,}328\,\text{Å}$ (neon-helium laser) are mixed in a beam splitting semitransparent mirror. One of the beams, illuminates the object, which is located in one of the arms of the interferometer, while the other is a standard beam. All four components interfere in a thick photographic emulsion having a high volume resolving power. The image reconstruction occurs when the three-dimensional hologram is illuminated with two beams having different wavelengths. Each of the beams interacts with a separate grating. The result is a two-color reconstructed image of the object. If the hologram is bleached in a solution of potassium ferrocyanide, then the brightness of the reconstructed object increases without any deterioration in the image quality. The reconstructed image is seen only if the hologram is illuminated in directions close to the Bragg angle. The spectral resolution of the three-dimensional lattice is 100 Å. Therefore interaction between the first beam and the second grating, which can lead to false crossing components, is practically non-existent.

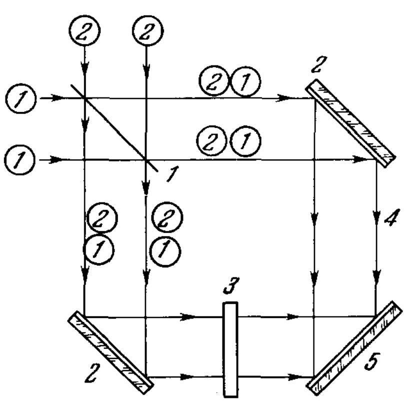


FIG. 6. Scheme for obtaining a two-color hologram. 1–Semitransparent beam splitting mirror, 2–mirrors, 3–colored transparency, 4–two-color standard beam, 5–photographic plate.

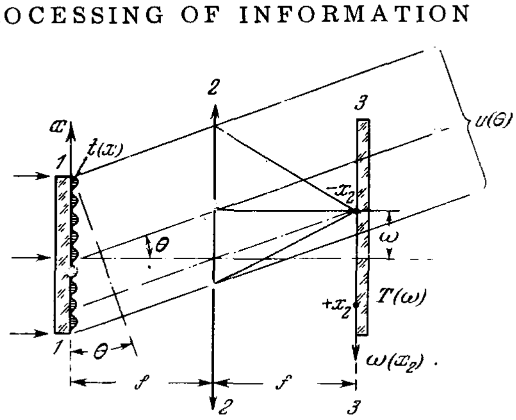


FIG. 7. Diagram of Fourier transformation of the spatial structure of an object. 1–Object, 2–gathering lens, 3–plane of spatial frequencies.

II. INTERFERENCE PROCESSING OF INFORMATION

1. Spatial Filtering of the Structure of an Optical Image

Let us consider a homogeneous parallel light beam. The geometric locus of the points having different phases at a given instant of time forms a system of planes perpendicular to the direction of beam propagation. In any other plane, inclined at an angle θ to the plane of the wave, the phase will vary from point to point, the variation being described by the factor

$$e^{i\omega x} = e^{i\frac{2\pi}{\lambda}\sin\theta x}. \tag{5}$$

We now place in the plane light wave a transparency whose amplitude transmission is equal to (Fig. 7).

$$t(x) = \frac{1}{2}\left(1 + \cos\frac{2\pi}{a}x\right). \tag{6}$$

The quantity $t(x)$ in Fig. 7 is arbitrarily represented by the profile of the transparency: the thicker the profile, the more light is transmitted by the transparency.

The field of the light wave observed in the θ direction at a large distance from the lattice is the result of addition of partial amplitudes of the wave in a plane normal to the line of sight. According to (5), rotation of the line of sight through an angle θ changes $t(x)$ by

$$t(x)\, e^{i\omega_x x}. \tag{7}$$

Summation over the entire plane yields

$$u(\theta) = \int e^{i\frac{2\pi}{\lambda}\sin\theta x}\left[\frac{1}{2} + \frac{1}{4}e^{i\frac{2\pi}{a}x} + \frac{1}{4}e^{-i\frac{2\pi}{a}x}\right]dx$$

$$\sim \frac{1}{2}\delta(\sin\theta) + \frac{1}{4}\delta\left(\sin\theta - \frac{\lambda}{a}\right) + \frac{1}{4}\delta\left(\sin\theta + \frac{\lambda}{a}\right). \tag{8}$$

Thus, we see from (8) that a sinusoidal grating splits a parallel light beam incident on it into three beams. The main beam does not change its direction ($\theta = 0$). The two side beams are deflected at angles

$$\theta = \pm\arcsin\frac{\lambda}{a}.$$

The total separation of the directions of observation of the real and virtual images in holography is based just on this property of a sinusoidal diffraction grating.

Each of the produced light beams corresponds to a definite frequency of the spatial structure of the grating. The main beam corresponds to zero frequency, i.e., to constant values of the amplitude transmission of the grating (6). The side beams correspond to frequencies

$$+\omega_0=\frac{2\pi}{a} \text{ and } -\omega_0=-\frac{2\pi}{a}.$$

A gathering cylindrical lens placed past the diffraction grating at a distance f—the focal distance of the lens—transforms the space of directions into a space of coordinates on the rear focal plane 3. An image of three bright lines, shown arbitrarily in Fig. 7 in the form of dots, appears in this plane. The coordinates of these dots are, in the small-angle approximation,

$$\pm x_2=\pm\frac{\lambda f}{a}.$$

We see from this example that lens 2 produces in plane 3 the spectrum of the signal t(x), which is specified in plane 1. Inasmuch as t(x) is real, both positive and negative spatial frequencies of equal intensity are produced. Figure 8 shows the Fourier transform of a long homogeneous transparency in the form of a strip of width a, obtained with the aid of a cylindrical gathering lens. After this Fourier transform of the images obtained, it is possible to carry out different filtering operations, similar to those used in radio engineering.

2. Optical Multiplication Scheme

Let us consider a semi-transparent diapositive illuminated by a coherent beam of light. Assume that it attenuates the intensity by a factor $r^2(x, y)$ ($0 < r < 1$), and that the optical thickness is $\alpha(x, y)/2\pi(n-1)$ in wavelength units, where n is the refractive index of the transparency. It can then be

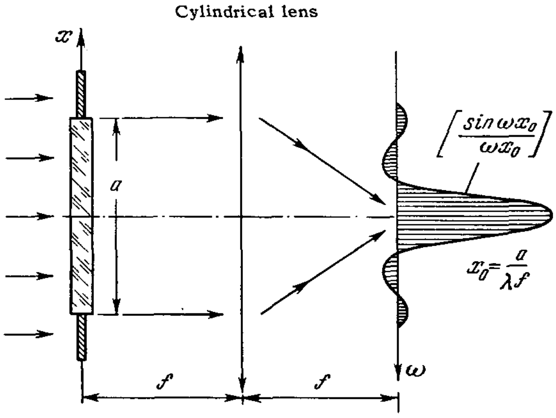


FIG. 8. Fourier transform of homogeneous band.

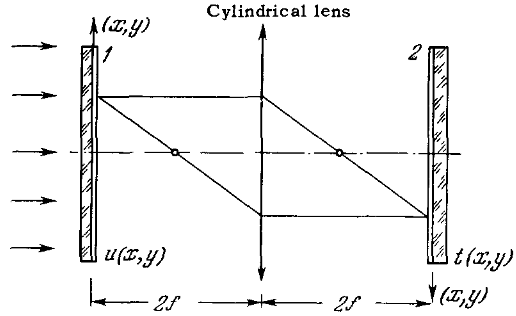


FIG. 9. Diagram of optical multiplication. 1—First transparency, 2—second transparency.

stated that the transparency has an amplitude transmission function

$$t(x, y)=r(x, y)e^{i\alpha(x, y)}. \tag{9}$$

the transparency is illuminated with an inhomogeneous wave of coherent light

$$u(x, y)=a(x, y)e^{i\Phi(x, y)},$$

then the light wave at the output of the transparency is given by

$$\psi(x,y)=u\cdot t=are^{i(\alpha+\Phi)}. \tag{10}$$

Thus, the amplitude of the wave $\Psi(x, y)$ at the output is the product of two complex functions, u(x, y) and t(x, y) [16 51]. This indeed is the operation of complex multiplication in optics. In order to realize it, it is necessary to place two transparencies one behind the other in such a way that the first is projected on the second (Fig. 9). The permissible values of the complex number t(x, y), which represent the amplitude-transmission function, are bounded by the unit circle in the complex variable plane [16] (Fig. 10). A transparency with homogeneous phase but with different density, corresponds to the points of the segment OA. If there are only phase inhomogeneities, then this case corresponds to the unit circle. It is precisely on this circle that the points describing the bleached hologram are located.

Until recently [13,65] it was very difficult to construct an optical transparency having a specified

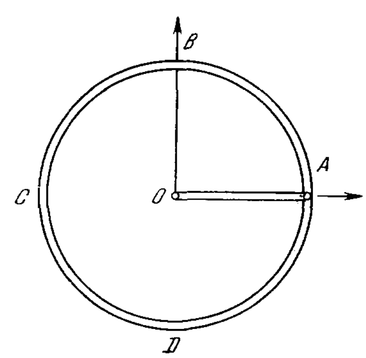


FIG. 10. Plane of complex transmission amplitude. OA—Transparency with homogeneous phase, ABCD—bleached hologram.

transmission function $t(x, y)$. Only a combination method existed, wherein inhomogeneous transmission $r^2(x, y)$ was first produced, followed by preparation of a set of phase plates to imitate $\Phi(x, y)$. Schemes for preparing any complex transparency have been devised only recently.

3. Integral Operations in Optics

The simplest integral operation is the Fourier transformation, which in coherent optics is realized by means of a gathering lens (Fig. 7). More complicated integral operations are: convolution, cross correlation, autocorrelation, Laplace transformation, [16] and others. To explain a scheme ensuring any particular type of integral operation, we use relations that are known from radio engineering and mathematics and pertain to systems satisfying the superposition principle [1,2,69]. We first find the Fourier transform of a product $T_1(\omega) \cdot T_2(\omega)$ of two complex spectra. By definition

$$T(\omega) = \int t(x) e^{-i\omega x} dx, \tag{11}$$

where $t(x)$ is the initial function, specified in the coordinate plane. The operation of multiplication $T_1 \cdot T_2$ is represented in the following form:

$$T_1(\omega) \cdot T_2(\omega) = \int e^{-i\omega x_1} t_1(x_1) dx_1 \int e^{-i\omega x_2} t_2(x_2) dx_2.$$

We transform it to standard form (11). We have

$$T_1(\omega) \cdot T_2(\omega) = \int\int e^{-i\omega(x_1+x_2)} t_1(x_1) t_2(x_2) dx_1 dx_2$$
$$= \int\int e^{-i\omega x} t_1(x_1) t_2(x - x_1) dx_1 dx$$
$$= \int e^{-i\omega x} dx \int t_1(x_1) t_2(x - x_1) dx_1. \tag{12}$$

An integral of the form

$$\int t_1(x_1) t_2(x - x_1) dx_1 = t_1 \circledast t_2$$

is called a convolution integral and is designated by the symbol $\circledast$. It can be represented geometrically as the limit of the sum of the products of the object $t_1(x)$ by the inverted object $t_2(x)$ for all possible relative shifts. Thus, it is seen from (12) that the operation of multiplying the Fourier transforms $T_1 \cdot T_2$ in the frequency plane corresponds to the integral operation of the convolution of the initial transforms t_1 and t_2 in coordinate plane. Similarly, the operations of multiplying the original images t_1 and t_2 in coordinate plane corresponds to the integral operation of convolution of the Fourier transforms T_1 and T_2 in the frequency plane:

$$t_1(x) \cdot t_2(x) = \int e^{i\omega_1 x} T_1(\omega_1) d\omega_1 \int e^{i\omega x_2} T_2 \omega_2 d\omega_2$$
$$= \int e^{i\omega x} (T_1 \circledast T_2) d\omega. \tag{13}$$

On the other hand, if the operation $T_1 \cdot T^*_2$ is performed in the frequency plane, then

$$T_1(\omega) \cdot T_2^*(\omega) = \int e^{-i\omega x_1} t_1(x_1) dx_1 \int e^{i\omega x_2} t_2^*(x_2) dx_2$$
$$= \int e^{-i\omega x} dx \int t_1(x_1) t_2^*(x_1 - x) dx_1 = \int e^{-i\omega x} (t_1 \ast t_2^*) dx, \tag{14}$$

where

$$t_1 \ast t_2^* = \int t_1(x) t_2^*(x - x'') dx$$

is the cross-correlation integral.

Geometrically, the cross-correlation operation can be represented as the limit of the sum of the products of the object $t_1(x)$ by the object $t_2(x)$ for all possible relative shifts between them.

The table gives a summary of the correspondences between the operations in the object plane and the operations in the Fourier plane [68].

4. Complex-conjugate Filter

There is a theorem in communication theory, which establishes the conditions under which the maximum intensity ratio of a known signal to noise is attained [1,2,16,51]. The need for using such a theorem arises under conditions of relatively large noise, when observation of the signal is impossible without preliminary noise suppression. Let a useful signal $s(x, y)$ appear against a background $n(x, y)$, so that the input signal is

$$n(x, y) = s(x, y) + n(x, y).$$

To ensure a maximum of the ratio s_{av}^2/n_{av}^2, it is necessary to pass the Fourier transform of the initial signal through an optimal amplitude filter [39]

$$T_f^{opt}(\omega_x, \omega_y) \approx \frac{S^*(\omega_x, \omega_y)}{N(\omega_x, \omega_y)}. \tag{15}$$

Since N is usually homogeneous, (15) means in practice that the optimal filter $T_f^{opt} \approx S^*$, i.e., it is the complex conjugate function of the Fourier transform of the useful signal $s(x, y)$.

An optical complex-conjugate filter is equivalent to a matched filter in radio [1,2,36,38]. It is obvious that this filter should transmit only those frequencies which are contained in the useful signal. Complex conjugation is necessary in order that the useful signal be transformed as a result of the filtration into a positive real amplitude. In first approximation a quasi-plane wave with a certain averaged amplitude,

Object	Fourier Transform	Object	Fourier Transform
$u(x)$	$U(\omega)$	$u \circledast v$	$U \cdot V$
$u(-x)$	$U(-\omega)$	$u \cdot v^*$	$U \ast V^*$
$u^*(x)$	$U^*(-\omega)$	$u \ast v^*$	$U \cdot V^*$
$U(x)$	$u(-\omega)$	$\delta(x)$	1
$u \cdot v$	$U \circledast V$	$e^{-\pi x^2}$	$e^{-\pi\omega^2}$

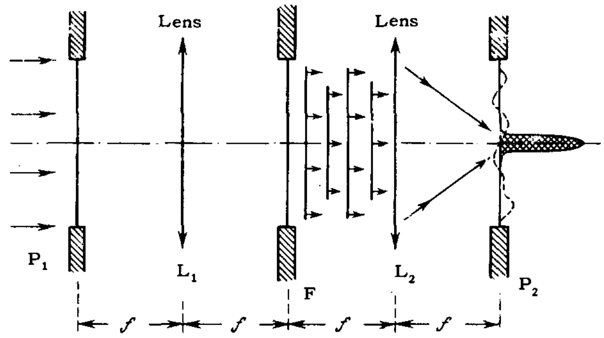


FIG. 11. Diagram of filtration with the aid of a complex-conjugate filter.

which the lens L_2 (Fig. 11) transforms into a small spot on the P_2 plane.

In order to prepare a complex-conjugate filter (CCF), we place the useful signal s in the form of a transparency in the plane P_1. Its Fourier transform appears in the plane F_1 (Fig. 12):

$$S(\omega)=\int s(x)\,e^{-i\omega x}\,dx.$$

In order to record S completely, i.e., to register both the amplitude and the phase, it is necessary to aim on the plane F_1 an oblique standard beam from the same source of coherent radiation. Then the photographic plate will register the intensity

$$I(\omega)=|S(\omega)+e^{i\omega x_0}|^2=1+|S|^2+S^*e^{i\omega x_0}+Se^{-i\omega x_0}. \quad (16)$$

This completes the preparation of the complex-conjugate filter. Let us now pass through this filter-hologram the Fourier transform of the input signal u(x) (Fig. 13). The output of the filter is the signal

$$R(\omega)=U(\omega)I(\omega)=U[1+|S|^2]+US^*e^{i\omega x_0}+USe^{-i\omega x_0}. \quad (17)$$

Just as in holography, three beams are produced. The main beam, represented by the first term of (17), proceeds in the direction of the optical axis of the system and is close to the Fourier wave of the signal u(x). The second term of (17) describes a beam traveling in the same direction as the standard beam

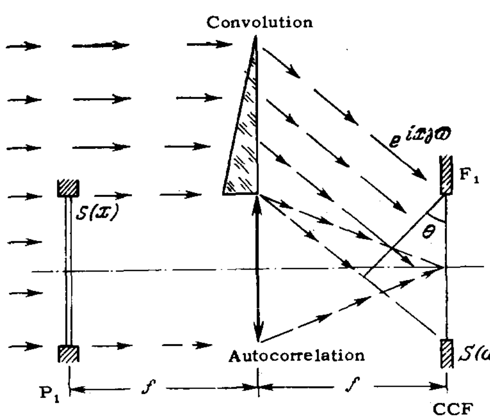


FIG. 12. Scheme showing the preparation of a complex-conjugate filter.

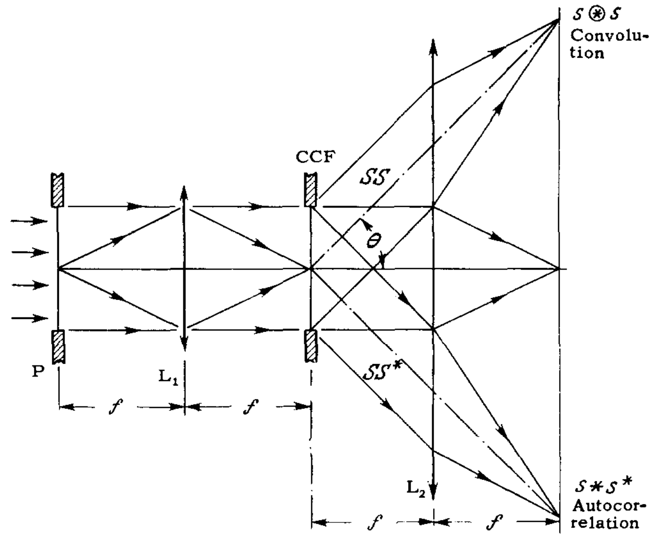


FIG. 13. Diagram showing the operation of a complex-conjugate filter.

used during preparation of the filter. The third term describes a beam deflected away from the axis in the opposite direction. The signal transmitted by the lower side beam

$$US^*=(S+N)S^*=|S|^2+NS^*,$$

consists of two terms, $|S|^2$ and NS^*. By Fourier transformation (with the aid of lens L_2) from the frequency plane, where the filtration operation was carried out, to the coordinate plane, where the output signal is observed, the function $|S|^2$ is transformed into $s \ast s^*$, i.e., into the autocorrelation signal of the sought transform s(x). This signal appears at the center of the upper side image. On the other hand, if the analyzed signal is shifted a distance x_1, relative to the position of the calibration signal used to prepare the filter, so that the signal at the input is

$$s_1=s(x+x_1), \quad (18)$$

then

$$S_1(\omega)=\int s(x+x_1)\,e^{-i\omega x}\,dx=\int s(x+x_1)\,e^{-i\omega(x+x_1)}\,e^{i\omega x_1}\,dx$$
$$=e^{i\omega x_1}S(\omega), \quad (19)$$

i.e., the spectrum S_1 differs from S only in the factor $\exp(i\omega x_1)$, which leads to an additional inclination of the side beam. The filtering operation yields

$$S_1(\omega)S^*(\omega)e^{i\omega x_0}=SS^*e^{i(x_0+x_1)\omega}. \quad (20)$$

Therefore, if the signal SS^* produces a wave directed at an angle x_0, then the angle of the wave of the signal S_1S^* is (x_0+x_1). The increase of the angle in the Fourier-transform plane relative to the axial direction x_1 changes the coordinate in the plane of the output signal without distorting the latter at all. Since by definition $|x_1|$ is always smaller than $|x_0|$, the side images never overlap. This leads to the following important conclusion: if we direct several signals s(x) located in different places to the input of a complex-conjugate filter, then the bright spots will appear in the plane of the output signal at the same places where the signals are located. These will not be the images of the objects themselves, but of their autocorrelations $s \ast s^*$. The signal making the angle (−x) will be SS, i.e., the convolution signal $s \circledast s$ will appear in the output plane. Signals of the type NS and NS* will, to be sure, be propagated entirely in the directions of SS and SS*, but by virtue of the random nature of the Fourier transform of the noise, the values of their convolution and of their cross correlation will be quite small. No quasiplane wave is produced in the filter. The noise is suppressed because of scattering over the entire image. With this, as a result of such a filtration, maximum suppression of noise is attained. If we change the scale of the object s(x), i.e., if we take an object $s_2(x) = s(x/\beta)$ magnified by a factor β, then its spectrum

$$S_2(\omega) = \int s\left(\frac{x}{\beta}\right) e^{-i\omega x}\, dx = \beta \int s(x')\, e^{-i\beta\omega x'}\, dx' = \beta \int s(x')\, e^{-i\omega x'}\, dx' \cdot e^{-i(\beta-1)\omega x'} \tag{21}$$

will be stretched by a factor $1/\beta$, and the amplitude of the spectrum will then be increased β times. At the output of the complex-conjugate filter in the object plane there will appear in place of the convolution the signal

$$\int s(x)\, s(x'-x)\, e^{-i(\beta-1)\omega x}\, dx, \tag{22}$$

whose integrand contains an attenuation factor $\exp[-i(\beta - 1)\omega x]$. The signal in the outer plane will be weakened as a result of such an operation. Thus, translational motion of the object in the plane P_1 leads to a similar translational motion of the convolution signal $s \circledast s$ in the object plane P_2. Changes in the scale attenuate the signal $s \circledast s$. Analogously, the filtration is sensitive to an equal degree to rotation of the object relative to the optimal complex-conjugate filter.

FIG. 14.

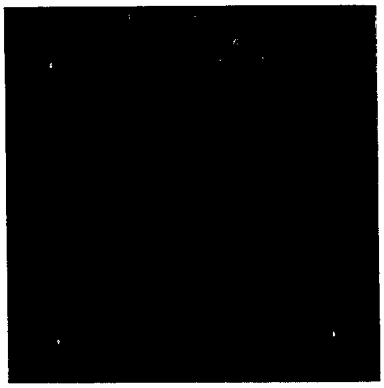


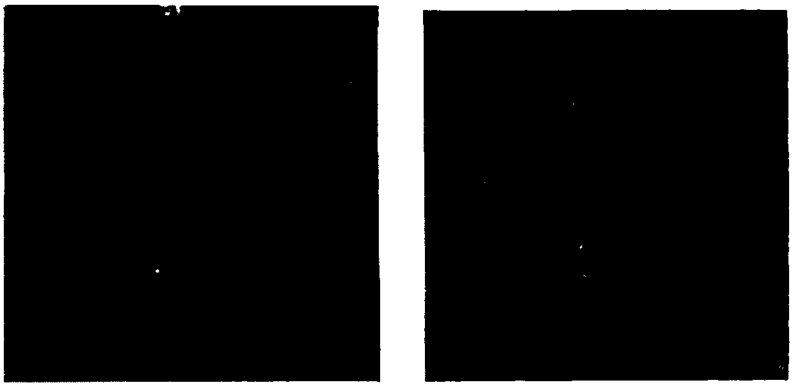
FIG. 15

The first example of the use of a complex-conjugate filter pertains to the recognition of a small rectangle from a set of different geometrical images (Fig. 14). By preparing a complex-conjugate filter, which in this case is real by virtue of the symmetry of direct angle, and by placing the filter in the plane F, we obtain at the output the side image (Fig. 15), corresponding to the autocorrelations of the small rectangle. We see that the complex conjugate filter simultaneously picks out three other rectangles, regardless of their location.

The next figures show the result of extraction of the signal L from among the geometrical figures shown in Fig. 14. Figure 16a shows the operation US*, i.e., cross correlation. The signal has a symmetrical form, as it should in accord with the properties of this operation. The convolution is shown in Fig. 16b. The remaining figures L, having different orientations, give no noticeable signal at the output. However, rotation of the filter relative to the figures leads to a successive appearance of the figures L with other inclinations. The foregoing examples pertain to the case when the spectral composition of the noise is practically homogeneous. If this is not the case, then the complex-conjugate filter should be composite, in accordance with the general expression given in [15]. An installation in which a complex-conjugate filter is used is sometimes called a correlator. Correlators are the main units of the optical analog computers, several models of which were constructed in 1965 [10,28,52].

5. Optical Correlator—Prototype of the Computer of the Future

In the interference method of information recording, all the operations in the coherent optical system are performed in accordance with the program of simultaneous-action computers. The operations are performed here on functions specified in two dimensions. The optical correlator can serve as the initial element of a possible computer in which the main operation will be to switch rapidly a large number of various information-containing pictures from one



a) b)

FIG. 16

block to another[19]. The interference system of recording and transmitting information is most suitable for realization of rapid transmission of information pictures, and the integral character of the processing of the interference information ensures in this case the necessary stability of all the operations.

At the present time the operating speed of optical analog-computers is determined by the time of transformation of the signals in the input and output blocks, where the electric signals are transformed into optical ones and vice versa. However, owing to the program of simultaneous action and to utilization of a beam of coherent light in the transmission of the information, even the first models of the optical correlators can solve quite rapidly the most delicate cybernetic problems, of the type of recognition of images in the presence of noise. The information is usually introduced into an optical correlator in the form of photographs on diapositives or text on microfilm, i.e., the transformation of the recorded information occurs outside the optical machine. This gives rise to the concept of operating speed of an optical-correlator block. Thus, for example, the operation of multiplying a vector having 10^4 components by a quadratic matrix with $10^4 \times 10^4$ elements takes approximately 1 millisecond, i.e., is performed at a rate of 10^{11}—10^{12} operations per second[52]. This is approximately 10^4—10^5 faster than the most rapid of modern electronic computers[19].

A block diagram of an optical correlator, the basic element of an optical analog computer, is shown in Fig. 17. The input converter 1 transforms the incoming electric signals, which vary both in time and in frequency, into optical inhomogeneities of the input transparency-block. A coherent light beam passes through the input transparency, is modulated in space and in time, and proceeds through a system of optical lenses and complex-conjugate filters 2, in which the specified program is realized. The output converter 3 picks off the information from the output beam and directs the answer to a recorder. The operating program is executed simultaneously in the entire depth of the optical correlator. The elementary operations are: addition and multiplication of complex functions, integral operations, and the filtration referred to above. The operations can be carried out in several channels simultaneously and, owing to the presence of an entire library of complex-conjugate filters in the correlator, the program can be a branched one.

III. FOURIER-TRANSFORM HOLOGRAPHY

1. Resolving Power of the Hologram

In projection or Fresnel-transform holography, which was considered in Sec. I, no lenses are used to obtain the hologram or to reconstruct the image. The resolving power of such a holographic method is determined by the resolving power N (is in lines per millimeter) of the photographic emulsion used to record the hologram, and by the dimension Γ of the source of coherent radiation:

$$\frac{1}{\varepsilon} < \left(\frac{1}{\Gamma} + N\right). \qquad (23)$$

However, the resolving power can be increased by several ten or even a hundred times by going over to a somewhat different scheme of obtaining the hologram. Besides the original projection or Fresnel holography, the following are possible:

1) The Fourier-Fraunhofer holography method,

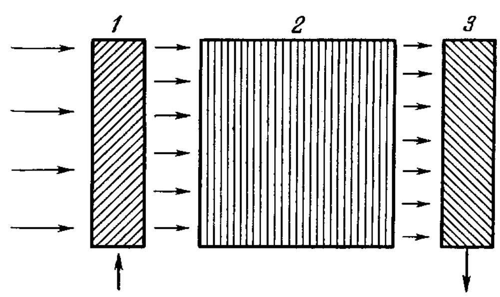


FIG. 17. Block diagram of optical correlator. 1—Input converter 2—system of complex-conjugate filters 3—output converter.

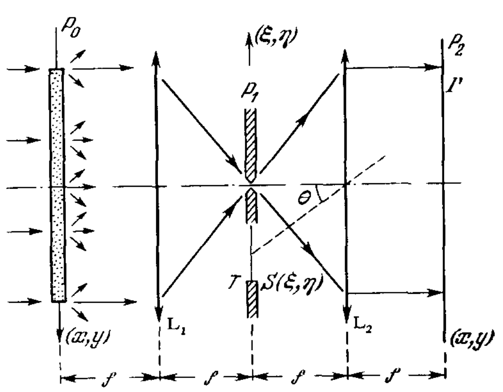


FIG. 18. Diagram of Fraunhofer holography.

using a pointlike standard beam to obtain the hologram and a gathering lens to reconstruct the object.

2) The method of multidirectional illumination.

3) The method of lenseless Fourier hologram.

Each has definite advantages over the original scheme.

2. Fourier-Fraunhofer Hologram

In Fraunhofer holography, a pointlike standard beam is used. A diagram of the first variant of this method is shown in Fig. 18. An opaque screen P_1 has pinpoint holes on the axis and a large opening off the axis, in which the transparency to be registered is mounted. The hologram is obtained in the plane P_2, where the Fourier transform of both the transparency and of the pinpoint hole are produced. The screen P_1 is illuminated with diffuse light from a scatterer placed in the plane P_0. The amplitude of the light wave past the scatterer is equal to

$$t(x, y) = a_0 + n(x, y), \qquad (24)$$

where a_0 describes the unscattered wave and $n(x, y)$ the diffusely scattered wave, which is close in character to white noise. The light wave in the plane P_1 also consists of two components:

$$T(\xi, \eta) = a_0\delta(\xi, \eta) + N(\xi, \eta), \qquad (25)$$

where $N(\xi, \eta)$ is the Fourier transform of the noise. After passing through the transparency, the light wave is described by the amplitude

$$T_1(\xi, \eta) = a_0\delta(\xi, \eta) + N(\xi, \eta) S(\xi, \eta), \qquad (26)$$

where $S(\xi, \eta)$ is the amplitude transmissivity of the investigated transparency-object. The Fourier transform of this signal, obtained with the aid of the second gathering lens L_2, is given by

$$\chi(x, y) = a_0 + n \circledast s, \qquad (27)$$

where $s(x, y)$ is the Fourier transform of the transparency $S(\xi, \eta)$, and $\circledast$ denotes an integral of the type of convolution of two quantities s and n.

The hologram registers the intensity

$$I(x, y) = |\chi|^2 = a_0^2 + |n \circledast s|^2 + a_0(n \circledast s) + a_0(n \circledast s)^*. \qquad (28)$$

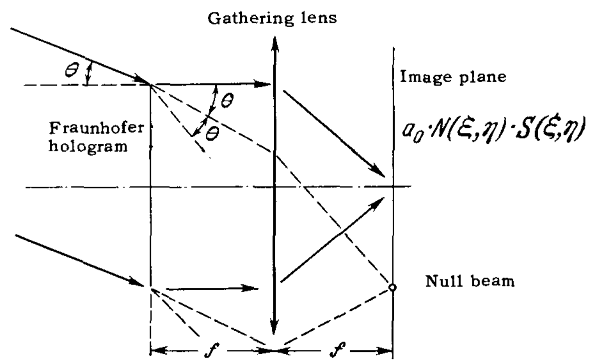


FIG. 19. Diagram showing the reconstruction of the wave for Fraunhofer holography.

The interference terms of this expression, as usual, carry information on the object $S(\xi, \eta)$, imprinted in the form of a Fourier transform $n \circledast s$. The process of reconstruction is in accordance with the scheme shown in Fig. 19. The hologram is placed in a coherent-light beam. To go from the Fourier transform to the object, it is necessary to introduce a gathering lens, which effects the reconstruction. The term a_0^2, describing the uniformly distributed background, is transformed into the δ-function of the standard beam. The images:

$$a_0 N(\xi, \eta) S(\xi, \eta) \text{ and } a_0 N^*(-\xi, -\eta) S(-\xi, -\eta)$$

appear in the plane. The first of them is the reconstructed image, which is produced in the case of diffuse illumination. The second term also gives the image of the object but spatially reversed.

In the first experiments with the Fourier-Fraunhofer hologram [44], it was not noted that it gives a higher resolution than the projection Fresnel holography.

3. Lensless Fourier Hologram

The introduction of the Fourier-transformation operation during one of the stages results in important advantages of this type of holography, particularly high spatial resolving power over the object if the photographic film used to record the hologram has a insufficiently high resolution. Naturally, this suggests that it is preferable to register not the Fresnel picture in the pupil, but the Fourier transform of the object. A second Fourier transformation is effected during the reconstruction.

Let us consider Fourier holography once more. A point-like reference source 2, located in the plane of the object, produces near the plane of the hologram a lateral reference beam (Fig. 20). Each point of the object P_n produces near the hologram plane waves with inclination angle θ_n determined by the coordinate of the point P_n. Each point produces a system of holographic strips, the spaces between which differs for the different points P_n. During the reconstruction, a system of plane waves traveling at dif-

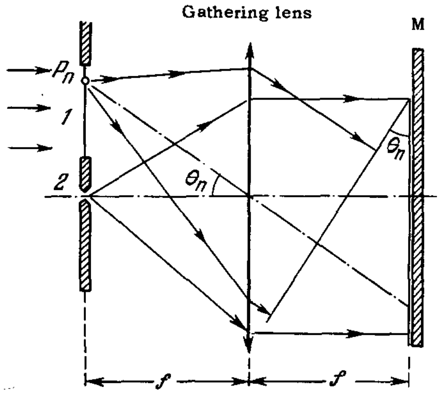


FIG. 20. Fourier-transform hologram. 1—Object, 2—pointlike standard source, 3—hologram, 4) gathering lens.

ferent angles θ_n is produced in the hologram. The lens that reconstructs the image of the object transforms the direction space into a coordinate space, and a reconstructed system of points P_n, forming the original object 1, appears on the output plane. During the succeeding development stage, it was found possible to eliminate the gathering lens and replace it with a photographic plate [57] (Fig. 21). Now interference is produced in the hologram plane between spherical waves that originate both at the object points P_n and the pointlike reference source. All the spherical waves have in this case the same curvature. The reference source 2 in the hologram plane produces a wave

$$u_0 = e^{ik\sqrt{f^2+(x+a)^2}},$$

where $k = 2\pi/\lambda$, f is the distance from the object to the hologram plane, and x is the coordinate of the point on the hologram. The factor $1/\Gamma$ has been left out, since $f \gg x_{max}$. Then

$$u_0 = e^{ikf} e^{ik\frac{(x+a)^2}{2f}}. \tag{29}$$

The scattering points of the object P_n produce in the hologram plane a wave amplitude

$$u(x, \xi) = t(\xi)\, e^{ik\sqrt{f^2+(x-\xi)^2}} \approx t(\xi)\, e^{ikf} e^{i\frac{k}{2f}(x-\xi)^2}. \tag{30}$$

Omitting the common phase kf, we obtain the interference pattern. Its intensity is equal to

$$I(x) = \left|\int u(x, \xi)\, d\xi + u_0(x)\right|^2,$$

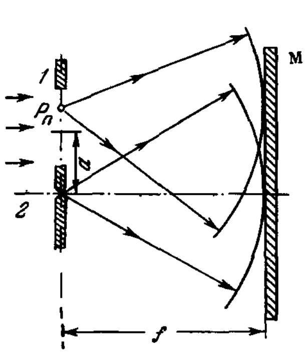


FIG. 21. Diagram of Fourier holography without lenses. 1—Object, 2—pointlike reference source, 3—hologram.

where

$$\int u(x, \xi)\, d\xi = \int t(\xi)\, e^{i\frac{k}{2f}(x-\xi)^2} d\xi$$

$$= e^{i\frac{k}{2f}x^2} \int t(\xi) e^{i\frac{k}{2f}\xi^2} e^{-i\frac{k}{f}x\xi} d\xi = e^{i\frac{k}{2f}x^2} \int v(\xi)\, e^{-i\frac{k}{f}x\xi} d\xi,$$

$$v(\xi) = t(\xi)\, e^{i\frac{k}{2f}\xi^2}. \tag{31}$$

The difference between $v(\xi)$ and $t(\xi)$ reduces to an additional phase factor $\exp(ik\xi^2/2f)$. From (31) we see that

$$\int u(x, \xi)\, d\xi = e^{i\frac{k}{2f}x^2} V(x),$$

where $V(x)$ is the Fourier transform of $v(\xi)$. The structure of the hologram is therefore

$$I(x) = \left| e^{i\frac{k}{2f}x^2} V(x) + e^{i\frac{k}{2f}(x+a)^2} \right|^2 = |ce^{i\omega x} + V(x)|^2$$

$$= 1 + |V(x)|^2 + cV(x)\, e^{-i\omega x} + c^* V^*(x)\, e^{i\omega x},$$

$$c = e^{i\frac{k}{2f}a^2}, \quad \omega = \frac{ka}{f}. \tag{32}$$

Thus, if the curvature radii of all the waves are identical, the spherical waves interfering with the spherical reference beam actually produce a Fourier transformation without the aid of lenses. The only difference from the Fourier transformation obtained with the aid of a lens is that the reconstruction yields the object $t(\xi)$ with an additional phase factor. To remove this factor during the reconstruction, it is necessary to install directly in front of the photographic plate a scattering lens which suppresses the phase $\exp(ik\xi^2/2f)$. A gathering lens located behind the hologram at a distance f produces in its real focal plane two symmetrical images. These reconstructed images are situated on both sides of the null image without superposition, as is usual in holography [44,63].

4. Amplitude Addition and Subtraction with the Aid of a Fourier Hologram

In addition to filtering with the aid of the complex-conjugate filter, which performs the integral operations of convolution and correlation, it is possible to realize in holography additive operations, namely addition and subtraction of complex amplitudes of objects [26]. This is simplest to realize with the aid of a Fourier hologram. A diagram of the synthesizing equipment is shown in Fig. 22.

As usual, the intensity recorded by the photographic plate is

$$I(x) = |S(x) + e^{i\omega x}|^2 = 1 + |S|^2 + Se^{-i\omega x} + S^* e^{i\omega x},$$

where $S(x)$ is the Fourier transform of the object $s(\xi)$.

It is possible to place in front of the lens producing the reference beam a flat phase plate, which shifts the

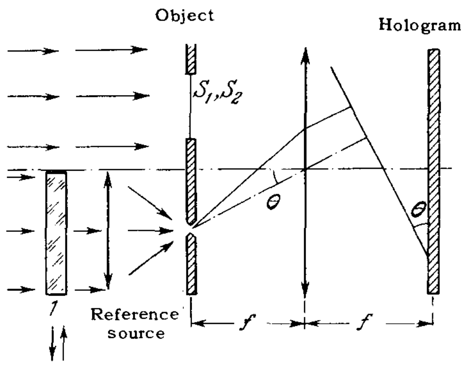


FIG. 22. Synthesis of the amplitudes of two objects with the aid of a Fourier hologram; 1–phase plate.

phase by π. If the object S_1 is illuminated without the phase plane, then

$$I_1 = 1 + |S_1|^2 + S_1 e^{-i\omega x} + S_1^* e^{i\omega x}. \quad (33)$$

We now leave only the object S_2, which is contained in S_1 and whose amplitude must be subtracted from that of S_1. A second illumination of the same hologram is effected with the π-phase plate. The intensity of the second exposure is

$$I_2 = 1 + |S_2|^2 + S_2 e^{-i(\omega x + \pi)} + S_2^* e^{i(\omega x + \pi)}, \quad (34)$$

and its intensity is added to that of the first exposure, so that we have after the two exposures

$$I = I_1 + I_2 = 2 + |S_1|^2 + |S_2|^2 + (S_1 - S_2) e^{-i\omega x} + (S_1 - S_2)^* e^{i\omega x}. \quad (35)$$

Expression (35) is equivalent to the hologram of the object $(S_1 - S_2)$, to a difference of the amplitudes of S_1 and S_2. Thus the intensity-additivity properties of the photographic film make it possible to realize with the aid of the hologram the operation of amplitude addition and subtraction of two objects described by complex amplitudes S_1 and S_2. With the aid of this amplitude-subtraction operation it is possible to suppress an undesirable background accompanying the observed phenomenon. To this end it is necessary not only to use equal light fluxes in the two exposures, but also produce the minimum noise level which is inherent in the amplitude synthesis process itself[26].

5. Extended Source

Experiments with Fourier holography have fully confirmed that the spatial resolution of a Fourier hologram is actually increased by approximately 100 times[63]. However, to obtain a Fourier hologram without lenses it is necessary to use a pointlike reference source, and this leads to prolonged exposures. It was possible to eliminate this difficulty.

It has been proved that the finite size of the source in Fourier holography is compensated for if the reconstructing source and the source used to obtain the hologram are identical and have certain symmetry properties[64]. In order to verify this, we turn again to the scheme for obtaining a Fourier hologram with the aid of a lens (Fig. 20). Let $t_0(\xi)$ be the object and $t_s(\xi - a)$ an off-axis extended source of diverging waves (Fig. 23). In the Fourier transform plane these correspond to the functions $T_0(x)$ and $T_s(x)$. The intensities registered by the Fourier hologram is

$$I(x) = |T_0(x) + T_s(x) e^{i\omega_0 x}|^2 = T_0^2 + T_s^2 + T_0^* T_s e^{i\omega_0 x} + T_0 T_s^* e^{-i\omega_0 x}. \quad (36)$$

The factor $\exp(i\omega_0 x)$ is due to the shift of the source relative to the object by an amount $a = \omega_0/2\pi$. The interference terms have the structure of the product of the Fourier transforms T_0 and T_s, which after illumination by a pointlike reference source produce the well known operations of cross correlation

$$\int T_0(x) T_s^*(x) e^{i\omega_0 x} e^{i\omega x} dx = (t_0 \ast t_s^*)_{\omega_0+\omega} \quad (37)$$

and

$$\int T_0^*(x) T_s(x) e^{i\omega_0 x} e^{-i\omega x} dx = (t_0^* \ast t_s)_{\omega_0-\omega}. \quad (38)$$

Thus, illumination of the hologram (36) with a point source leads to a smearing of the image of the object, and not to its reconstruction. If the hologram is illuminated not by a point source, buy by a source $t_{s'}$ Fourier transform is $T_{s'}$, then we obtain

$$T_{s'}(T_0 T_s^* e^{i\omega_0 x}) = T_0 e^{i\omega_0 x}(T_{s'} T_s^*) \quad (39)$$

for the upper side beam and

$$T_0^* e^{-i\omega_0 x}(T_{s'} T_s) \quad (40)$$

for the lower side beam.

The gathering lens carries out the inverse Fourier transformation, and at the output we obtain the signal

$$[t_0 \circledast (t_{s'} \ast t_s^*)]_{\omega_0+\omega} \quad (41)$$

from the upper side beam and

$$[t_0^* \ast (t_{s'} \circledast t_s)]_{\omega_0-\omega} \quad (42)$$

from the lower side beam.

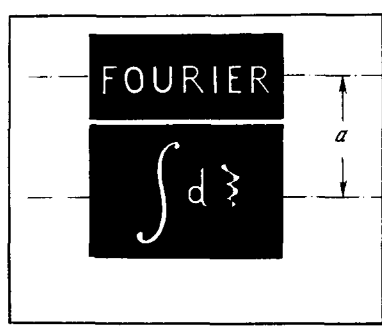


FIG. 23. Appearance of the object (FOURIER) and the extended source of diverging waves ($\int d\xi$).

If ($t_S' \ast t_S^*$) is a δ-function, i.e., if the cross correlations of the illuminating and reconstructing sources are very narrow functions, then the object is reconstructed without distortion. If $t_S' = t_S$, then the requirement that the autocorrelation function be very narrow is equivalent to having t_S contain a very broad spectrum of spatial frequencies. The lower side beam contains the convolution, which can be equal to the δ-function only if the source has 180° rotational symmetry relative to the optical axis. This result indicates the additional condition under which two complex-conjugate images—virtual and real—cannot be present simultaneously. To this end it is sufficient to use an extended reconstructing source t_S' which has no symmetry whatever against rotation through 180°.

IV. HOLOGRAPHY OF INCOHERENT OBJECTS

1. Is It Possible?

The requirement of coherence with respect to the system of the illuminating source and the object was until recently regarded as a fundamental condition, without which no holography could be obtained. This condition is indeed essential in the schemes considered in the preceding sections. However, research carried out last year has shown that a hologram can actually be obtained from incoherently illuminated or self-illuminated objects. The first indications of the feasibility of this desirable holography were obtained by Mertz and Young[50], who used for this purpose a holography technique based on the shadow picture of a Fresnel zone plate. Lohmann[40] analyzed the feasibility of holography of incoherent objects and described several principal schemes for obtaining such holograms. The holography of incoherent objects is based on the ability of the hologram to register interference patterns from several objects by successive exposures of identical objects. This property of the hologram makes it possible to regard the object as a system of elements and a corresponding system of interference patterns, produced independently by each element. The individual pictures add up incoherently, i.e., additively in intensity. Such a scheme of subdividing the object into a system of independent elements does not require spatial coherence, i.e., the object can be incoherent or self-illuminated, for example a star in outer space or a plasma in a laboratory. In order to obtain an interference pattern from each element separately, the light wave diverging from the given element must be split into two mutally coherent parts, which will then produce the interference picture. One of the conditions for the interference pattern to be sharp is monochromaticity of the light. The monochromaticity required depends on the dimensions of the pupil where the hologram is placed. With this, the larger the dimension of the pupil, the larger the necessary monochromaticity of the beam.

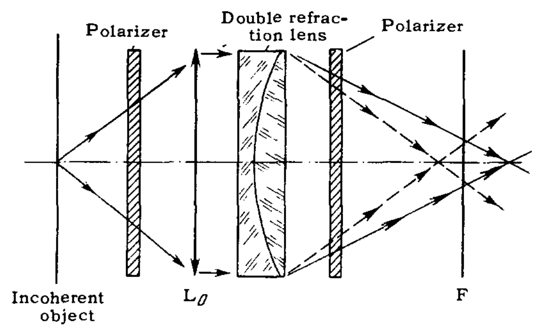


FIG. 24. Amplitude splitting of a light wave from an incoherent object with the aid of a double-refraction lens.

If this condition is not satisfied, the interference patterns will become smeared out on the edge of the hologram.

Several schemes were proposed for splitting the light wave from an individual point. The amplitude splitting is either with a two-beam Michelson or Mach-Sender interferometer, or with the end of a two-focus double refraction lens (Fig. 24) and a system of two polarizers, one of which is placed ahead of the two-focus lens and the other behind it. These polarizers should be oriented parallel to each other, but at 45° to the crystal axis of the two-focus lens. The Fresnel zone plane also makes it possible to produce a two-focus lens. In this case the neighboring rings should differ stepwise in phase by π (Fig. 25).

It is possible to split the wave with the aid of an inhomogeneous aperture, each half of which has a different focal distance (Fig. 26). In the latter case the pinpoint at the top gives an almost homogeneous spherical standard wave, but this, of course, entails a great loss of light.

As in any type of holography, the reconstruction of the incoherent image is produced in a beam of spatially-coherent light.

2. The First Experiment

A hologram of an incoherent object was first obtained experimentally with the aid of Fourier holography[62]. A diagram of the installation is shown in

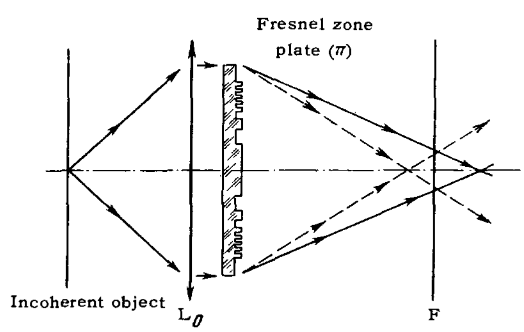


FIG. 25. Amplitude splitting of a light wave from an incoherent object with the aid of a Fresnel π-zone plate.

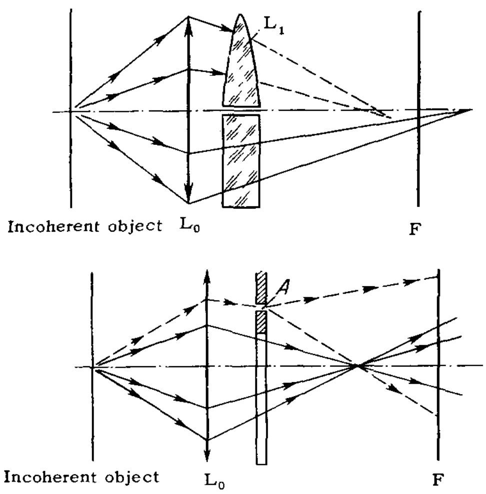


FIG. 26. Amplitude splitting of light wave from an incoherent object with the aid of a half-lens L_1 and a pinpoint A.

Fig. 27. The object in the form of a transparency was illuminated by a spatially-incoherent monochromatic source. The splitting in this experiment is by a sinusoidal diffraction grating, which forms two images of equal intensity $I(\xi)$ and $I(-\xi)$, with mirror-image symmetry about the Z axis. These images, upon reflection from mirrors 1 and 2, overlap at distances z = f from the diffraction grating and produce an interference pattern. Each point i of the object forms two mutually coherent waves $I(\xi_i)$ and $I(-\xi_i)$. The interference is effected without lenses by the Fourier-holography scheme. The intensity of the light registered on the Fourier hologram is

$$I(\omega)=\int I(\xi)\,[e^{i\omega\xi}+e^{-i\omega\xi}]^2\,d\xi. \tag{43}$$

Reconstruction of the images is with the aid of a coherent light source. Two images $I(\xi)$ are produced, symmetrically located on both sides of the null image, which is concentrated on the optical axis. In the described experiment, the spatial incoherence of the source was produced by placing a rapidly moving diffusor in the beam of a low-power laser. The image of the diffuser was projected on the object. The motion actually destroyed the coherence completely. This was demonstrated by a control experiment, in which the hologram was obtained with a stationary diffusor. The spatial coherence remained in this case. It was impossible to reconstruct the image in this experiment (!). The reason was that in this case the reconstructed wave, in accordance with (37) and (38), was a convolution of the image $I(\xi)$ with itself. The output was a certain smeared picture, and not a sharp image of the object itself. This most important experiment not only proved the feasibility of holograms of incoherent objects, but also uncovered a way of producing a three-dimensional holographic x-ray microscope, and also of using Mossbauer radiation to observe microscopic objects. To split the x-rays it is necessary to employ a single crystal which, in analogy with the diffraction grating in the described experiment, has the same splitting property with respect to x-rays as an ordinary diffraction grating in visible light.

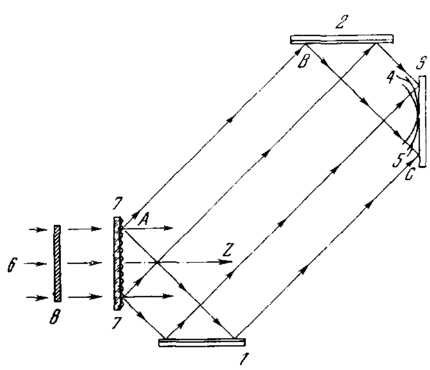


FIG. 27. Fourier holography of an incoherent object. 1, 2–Mirrors, 3–hologram, 4–light wave of left-hand point of the object, 5–light wave of right-hand point, focal distance f = ABC. 6–monochromatic incoherent source, 7–sinusoidal diffraction grating, 8–incoherent object.

V. ASSOCIATIVE MEMORY

1. Interference Memory and Phantom Images

The brilliant advantages of holography over photography are due to fact that the hologram is an interference pattern. In other words, information on the object or its Fourier transform is registered in the form of a system of interference patterns. If the information in the broad sense of this word is also recorded in interference form, then, like holography, we should obtain noticeable advantages over recording the intensity only. This is indeed the case. When information is recorded in interference form, the process of searching for the necessary information contained in the memory block is carried out practically simultaneously in the entire memory, no matter how cumbersome and multidimensional it may be. Like in optical analog computers, this operation occurs in accordance with a real-time program. Information recorded in the form of interference patterns turns out to be more stable than element by element recording. Loss of part of the sought object, or wear or destruction of some part of the memory, does not prevent a search from being made in the entire library of information transferred to the memory block. Finally, a most important factor from the practical point of view, when the interference form of recording is used the capacity of the memory block turns out to be equal to the capacity of a block in which the information is recorded element by element[27,29,33]. Among the most unusual properties of the interference form of recording information is the possibility of obtaining a phantom image of a lost part of a sought object.

Let us examine the interference information

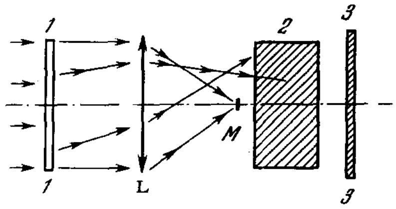


FIG. 28. Transfer of information from an object to a memory block by interference. 1–Object, 2–memory block, 3–plane of reconstructed image.

scheme (Fig. 28). Let it be required to transfer information from an object (transparency) to three-dimensional block 2. To this end, the object is illuminated by a powerful beam of spatially-coherent light, which is diffracted by the elementary diffraction gratings contained in the object. The mask M blocks the direct beam due to the average illumination of the object. Incident on the memory block is a wave $u = \sum_i A_i$, where A_i is the diffraction wave going in the i direction and produced by the i-th diffraction grating of the object. The intensity registered in the block will be

$$I = |\sum_i A_i|^2 = \sum_i \sum_j (A_i A_j^* + A_i^* A_j). \quad (44)$$

If we photographically develop the block and then illuminate it with the initial wave $u = \sum_i A_i$, then the interference pattern of the intensity (44) registered in the block will produce in each of the directions i a wave

$$\sum_j (A_i A_j^* + A_i^* A_j) A_j. \quad (45)$$

The entering wave (45) has a component which is equivalent to the wave from the hologram and which forms a virtual image. This component is equal to $A_i I_0$, where $I_0 = \sum_i |A_i|^2$ is the non-interfering intensity. Summing over all the directions i, we obtain the reconstructed object wave $I_0 \sum_i A_i$. Thus, the signal at the output of the block confirms that a wave $u = \sum_i A_i$ is present in the block; this confirmation is in the form of the reconstructed picture of the object. We now replace the entire object 1 by a small fragment of the object, i.e., a small part of the entire object, so that the wave illuminating the memory block is

$$u_f = \sum_i f_i, \text{ where } A_i = f_i + \Gamma_i.$$

$\sum_i \Gamma_i$ is the lost part of the object. In this case the wave travelling in the direction i is

$$\sum_j (A_i A_j^* + A_i^* A_j) f_i = A_i \sum_j |f_j|^2 + A_i \sum_j \Gamma_j^* f_j + A_i^* \sum_j (f_j + \Gamma_j) f_j. \quad (46)$$

The "imaginary" wave in (46) will produce the same wave A_i as produced by the object 1, but with a reduced intensity

$$A_i I_f,$$

where $I_f = \sum_i |f_i|^2$ is the partial intensity of the fragment. The second term gives a bipolar background that makes the search difficult. Thus interference memory can raise the ghost of the lost part of the object and carry out without hindrance an associative search in the memory block. The block will provide an answer confirming the presence of the object even if the search information is incomplete. Just as in the operation of filtration with a complex-conjugate filter, the process of searching for information in the block is sensitive to displacements of the object and to changes in the focal distance of the lens, i.e., to changes in the scale of the image [87]. Calculation shows [27] that a photographic plate of standard size, 70 × 70 mm, can hold approximately 10^8 bits, which is equivalent to a library of 300 books of 200 pages each. With the aid of such a memory block one can ascertain whether a given page is present in the library. Moreover, because of the interference memory system, all we need to obtain a definite answer is to have only 1/30-th of the page. The page will be recognized and located, and the contents of the missing part of the page can be read. Moreover, the ghost image will be quite distinguishable, since the ratio of the signal to the noise will be approximately 20:1.

2. Three Dimensional Storage Technique

As the first stage in the development of the technique for the interference method of information storage, we consider an alkali-halide crystal [27, 56] containing F-centers uniformly distributed through its volume. Light waves of suitable wavelength bleach the F centers. At low light intensities such a bleaching process is proportional to the absorbed light and is sufficiently effective [56]. Let the crystal be bleached simultaneously by two waves of coherent light A_0 and A_1 of identical length (Fig. 29). After the bleaching, the color centers will no longer be uniformly distributed, and the picture will consist of equidistant planes. These planes are oriented in such a way that the direction of propagation of A_1 is a reflection of the wave A_0 in these planes, and vice versa. If we now illuminate the bleached crystal with wave A_0 only, and assume that the effect of bleaching during the time of the second exposure can be neglected, then a plane wave propagating in the A_1 direction will appear in addition to the direct ray A_0. We see from this example that the three-dimensional picture memorizes the direction of the bisector of the angle between the rays A_0 and A_1, and only the wave producing the virtual image of the object appears upon reconstruction. This effect was first ob-

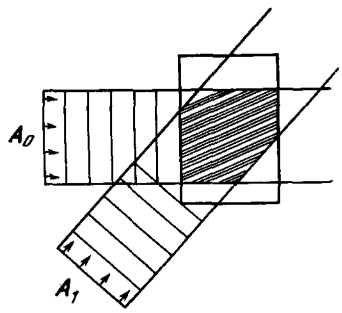


FIG. 29. Scheme for bleaching a crystal by two waves A_0 and A_1.

served experimentally by Denisyuk [3-6]. Let us estimate the capacity of a three-dimensional interference memory. Let the dimensions of the crystal be a × a cm and let the depth be d cm. The light wave diffracted by such a crystal produces a narrow pattern in a solid angle $\Omega \approx (\lambda/a)^2$. Therefore it is possible to distinguish on such a crystal approximately $(a/\lambda)^2$ different directions of planes produced in the interference between two waves A_0 and A_1. The number of different wavelengths which can be stored without mutual distortion and the crystal is (d/λ_0), where λ_0 is the average wavelength of the employed spectrum. The total number of independent interference memory elements which can be stored in the three-dimensional interference pattern is thus equal to

$$n_0 \approx \left(\frac{a}{\lambda_0}\right)^2 \left(\frac{d}{\lambda_0}\right) = \frac{V}{\lambda_0^3}, \qquad (47)$$

where V is the volume of the crystal. This number is equal to the number of local memory elements which can be placed in a three-dimensional photographic recorder such as a nuclear emulsion, where the information is stored element by element. To estimate the capacity of the memory it is necessary to know, besides the number of the independent memory elements, also the signal-to-noise ratio in each cell. The noise produced by all the color centers is determined by the total number of the occupied elements and is equal to $\sqrt{N_0\lambda_0^3}$. The capacity E in bits is equal to the number of independent elements multiplied by the logarithmic signal-to-noise ratio:

$$E = n_0 \ln \sqrt{N_0\lambda_0^3}. \qquad (48)$$

For $N_0 = 10^{15}$ color centers per cm^3, which gives 25% absorption, and $\lambda \approx 1 \times 10^{-4}$ cm, we get $E \approx 10^{13}$ bits. In practice it is possible to use at sufficiently low noise level, a capacity $E_0 = 3 \times 10^{11}$ bits.

3. Associative Memory

Assume that we wish to construct a cybernetic machine [27] that is confronted with many situations, in each of which it must produce a definite instruction. Both the situations and the instructions are recorded in transparency form. For example this may be a *machine-translation* computer. The situations for such a machine are phrases in a foreign language, in standard form, and the instructions are to translate these phrases into the native language. In this example all the situations can be arranged in alphabetical order, and the instructions can be written on the back. This is a catalog memory. In most cases, such a memory is either undesirable or even impossible. An associative memory would make it possible to search for the instruction within a very short time, regardless of the order in which the situation-plus-instruction pair is arranged. Let us examine Fig. 30. The situation is introduced in the form of a transparency S_i in the object plane P. The transparency is illuminated by a converging beam of light L_1. The mass M blocks the illuminating beam. The diffracted light, which carries information concerning the instruction S_i, illuminates the memory-block crystal. A second converging beam L_2, coherent with L_1, simultaneously illuminates the crystal. It is focused in a point directly ahead of the transparency T, which is divided into n^2 squares, in each of which is written an instruction I_i. The beam L_2 illuminates the entire instruction I_i. We place alternately the situations S_i in the beam L_1, and we let the beam L_1 move each time in the corresponding direction I_i. In this manner the associative memory will be recorded in the block by interference. If now we are given an arbitrary situation S_i, which is inserted in the plane T, then the beam L_i, illuminating S_i, will produce a phantom image of the corresponding instruction in the form of a beam L_{2i}, which illuminates the sought instruction I_i on the transparency. All that remains is to read this instruction.

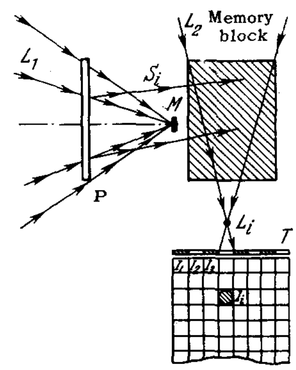


FIG. 30. Associative memory using two coherent beams L_1 and L_2.

We can present also a simpler associative memory (Fig. 31), in which the situation S_i and the instruction

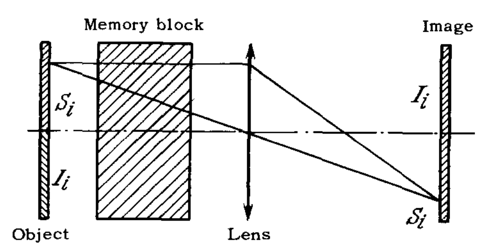


FIG. 31. Simplified diagram of associative memory.

I_i are written in the memory block simultaneously by illuminating the object $S_i + I_i$. The instruction S_i appears in the form of a phantom image in the image plane. However, such a scheme has a major shortcoming in that different situations S_i and S_k can have common parts which are identical to each other. In this case the appearance of S_i in the plane of the object will produce phantom images I_i and I_k, which simultaneously appear in the image plane and cause confusion. In the earlier scheme (Fig. 30), two situations would likewise appear simultaneously in such a case, but they will not be superimposed on each other. Therefore a stable optical associative memory must always have two separated storage devices, one where all the information is mixed together, and another where individual cells of the information remain separated, thus ensuring readout unaffected by noise.

Beurle[9] proposed a hypothetical mechanism, based on the interaction of propagating waves, storing information in a brain. There exists, obviously, a rather full analogy between associative memory and Beurle's principle of storage of information with the aid of waves transmitted by the brain cells, whose properties are altered by the waves in a manner similar to the bleaching of a crystal by a coherent light wave. It is obvious that the scheme of three dimensional interference storage of information in a crystal is essentially applicable to any quasi-wave phenomenon satisfying the Huygens principle. One can suggest, apparently, a system of neurons in the brain such that each cell of one layer transmits an electric excitation pulse it receives from the nerve to several cells of the next layer of neurons (Fig. 32). The main fact which makes Beurle's hypothesis believable is that it is possible in this manner to store a large amount of information in very stable form. If the duration of the electric pulses is shorter than the time of propagation between the cells, then the amount of information which can be accumulated is equal to the number of brain cells. On the other hand, the theory of associative memory calls for satisfaction of exact phase relations in the wave over a large signal-propagation path. This makes it necessary to assume that the brain possesses a calibrating system that monitors continuously and compensates the rate of propagation when necessary. In order for the search process to be stable, optical associative memory must contain two different memory regions. In one region the information of the observations and experiment are mixed together to carry out a rapid search and recognition. Information concerning different situations is stored separately in the other region.

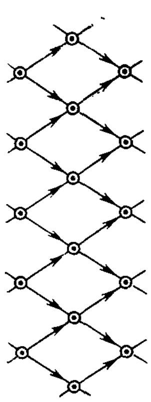


FIG. 32. Chain of neurons operating in accordance with the Huygens principle.

VI. APPLICATIONS OF HOLOGRAPHY

1. Certain Schemes of Holographic Installations

Although the inventors of holography Leith and Upatnieks' did not themselves proposed a concrete application of holography, other than for a microscope, the publication of their papers was immediately followed not only by an intensive study of the holography processes described in the preceding sections, but also by extensive utilization of holography in different branches of science and technology.

Let us examine some characteristics of holography installations. The angle between the wave incident on the hologram and the reference beam is usually chosen small, 3—5° on the average. At large angles, the insufficient resolution of the emulsion comes into play in the reconstruction process. Whereas in the first investigations the mirror producing the reference beam was placed near the object, an inverted reference beam technique was proposed later[32], which made it possible to aim the reference beam on the photographic plate from the direction of the base (Fig. 33). This procedure is convenient when the object is very far, and makes the holography installation more compact. The reference beam of such a scheme is produced by a wedgelike glass plate with an 8° angle, which splits the diverging laser beam into two: the standard beam and the beam illuminating the object.

Two toy soldiers, placed on a white background at a distance 1 meter from the source, were photographed. Kodak 647-f spectrometric photographic

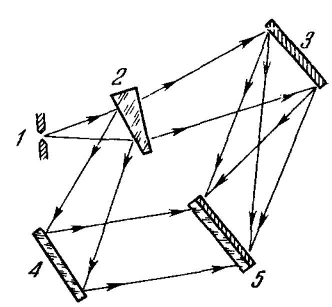


FIG. 33. Holography with inverted reference beam. 1—Coherent source, 2—wedge-like plate, 3—object, 4—mirror, 5—hologram.

plates were used, with glass thicknesses 1 mm and 6.35 mm. Equally successful holograms were obtained regardless of whether the photographic emulsion faced one beam or the other. Such a holographic installation can if necessary introduce optical delays to equalize the optical paths of the two beams. If the light source employed is a cw gas laser, the power of which is usually 10 mW, then rather long exposures are necessary. The object should not move or oscillate by more than 0.03 μ, i.e., $\lambda/20$, during the entire time of exposure. If oscillations are produced with an amplitude commensurate with the wavelength of light, the hologram becomes blurred. This property of the hologram is used to detect vibrations of minute or very light details. In all other cases, this property of the holograms creates great inconvenience, since it makes observation of any moving object impossible. The use of specially developed [34] powerful pulsed lasers in holography has eliminated this shortcoming. The required coherence and power were obtained by using a special system for selecting longitudinal and transverse oscillations modes. The energy of the flash was 60 MJ at ~ 30 nsec duration. Under these conditions, the hologram and the object can be held by hand. The laser power has been increased to such an extent, that an attenuating filter has to be used when exposing with a sensitive film. The object is reconstructed in a cw gas-laser beam. To check on the resolving power of pulsed-laser holography, holograms of letters on microfilm, 0.1—0.2 mm high and merely 0.03 mm thick, were successfully produced. The resolution of the emulsion was 200—2,000 lines/mm. The signal/noise ratio was worse for a thick sensitive emulsion.

The ability of holography to record the entire depth of a scene makes it possible to carry out observations of suspended dust particles or fog droplets. The holograms are then viewed in the laboratory successively over their depth, and each individual particle is carefully examined. Several holograms made in succession make it possible to trace the evolution of a process in time and in space. A program of holographic investigations of aerometeorological processes is now being planned [45].

2. High Speed Holography

Holography in which the object is illuminated by a powerful short-duration beam from a pulsed laser was developed for the observation of rapid processes, such as photography of a bullet moving at 375 m/sec [12]. The moving bullet breaks a wire, triggering a laser illumination system. A diagram of the holographic installation is shown in Fig. 34. The primary laser beam is made divergent by a negative lens. The beams produced in the splitter again overlap in the plane of the photographic film. Both beams experience an equal number of reflections and cover

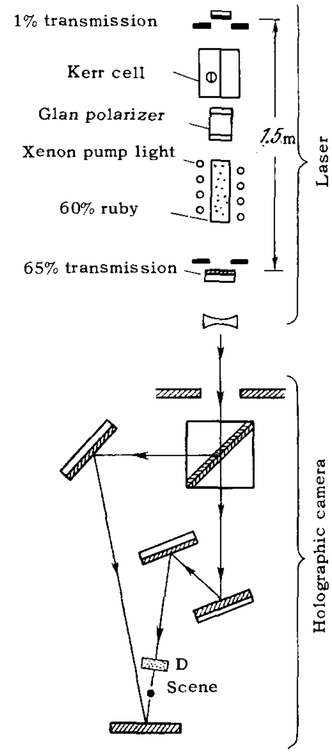


FIG. 34. Holographic installation for the observation of rapid processes.

equal light paths. This gives the required spatial and temporal coherence of the beams.

The photographic film is mounted with its plane perpendicular to the bisector of the angle between the optical axes of the beams. The distance from the virtual point of the source to the film is approximately 1.5 meters, and the angle between the beam axes is 20—30°. To obtain photographs against a light background, a diffusion screen was placed behind the scene. An ordinary ruby laser was used in the photography. The mirrors made up a 1.5-meter cavity. This mirror spacing is the simplest means of suppressing the off-axis modes in the laser. A Glan polarizer and a Kerr cell with nitrobenzene were placed between the mirrors. When operating with short flashes, of 60 nsec duration, the Kerr cell was triggered by a hydrogen thyratron (≈ 19 kV) which was ignited approximately 1.8 msec after ignition of the lamp. The reconstruction from the hologram was in the beam of a cw laser. The depth of field of the virtual image was approximately 1—2 cm, so that the test image was completely smeared into a uniform white background when the focus was displaced by 5 cm away from the plane of the bullet,

3. Holographic Spectroscope

The lensless holographic method of obtaining images can be readily used in a slitless spectrometer [61]. The spectral Fourier hologram is registered in a two-beam interferometer with a small angle θ

between wave fronts. The system of interference fringes is registered by a photographic plate mounted symmetrically relative to the two beams. Each wavelength λ produces its own system of fringes. These systems add up on the hologram incoherently. If the object is uniformly illuminated, then each monochromatic line produces its own uniform system of interference fringes. The equation of the system of fringes in the plane of the photographic plate is

$$I(x) = \sum I(\sigma_i)\,[e^{i\pi\sigma_i\theta x} + e^{-i\pi\sigma_i\theta x}], \qquad (49)$$

where $\sigma_i = 1/\lambda_i$, and $I(\sigma_i)$ is the intensity of the i-th spectral line.

When the hologram is illuminated by a spatially-coherent monochromatic plane wave, each grating produces two plane side beams. The Fourier transform of these plane waves is obtained in the focal plane of the gathering lens in the form of pointlike images. The points are arranged symmetrically relative to the null beam. The wavelength of each line is determined directly from the distance between them, with due allowance for the geometry of the experiment. No recalculation or calibration is necessary here. In the first holographic double-beam spectroscope, a compensated Michelson interferometer was used. The radiation from a low-temperature mercury arc illuminated uniformly a diffuser located at the entrance to the interferometer. Five spectral lines from 4047 Å to 6234 Å, were separated in the mercury spectrum obtained by illuminating the hologram with a laser having λ_0 = 6328 Å. The resolution was approximately 50 Å.

4. Processing of Geophysical Data

Seismograms, isomaps, and other geophysical data are processed in practice by means of spatial filtration of the data, which should initially be plotted on isomaps or on transparencies [35,28]. A coherent processing system makes possible visualization of a tremendous amount of data gathered during the investigation of geophysical phenomena. The simplest form of processing is a survey of the spectral composition of the seismogram. A Fourier analyzer with a cylindrical lens makes it possible to process multichannel information. The spatial frequencies of all the channels are produced simultaneously in the focal plane of the analyzer. The exposure time with a gas (He-Ne) laser is 1/125 sec. The system makes it possible to investigate the spectra of the spatial frequencies of even small sections of the seismogram, and thus observe the time variation of the spectrum. The form of the spectrum turns out to be stable to different shapes of the input aperture of the analyzer. The result is plotted in (ω, t) coordinates, where ω is the signal frequency and t is the time. These data can be scanned in three principal variants: t = const, ω = const, and ωt = const. The latter corresponds to constancy of the number of wavelengths subtended by the path from the center of the earthquake to the seismograph. The Fourier spectrum of the seismogram makes it possible to observe clearly the effect of dispersion due to the frequency dependence of the propagation velocity. If the velocity of the wave increases with frequency, then the spatial picture becomes denser in time, and this decreases the distance to the plane where the frequency spectrum is located. In the opposite case, the distance to the focus increases. If the photographic plate is placed in the plane of the figure, the picture produced is a caustic whose form and size can be readily related to the character of the dispersion of the seismic waves.

Two-dimensional Fourier analysis with the aid of a spherical lens produces a plot in terms of the coordinates ω and k, where k is the wave number. The following operations are then possible: filtration with respect to frequency independently of the orientation of the signal wave (opaque ring), filtration of the orientations independently of the frequency (wedge-like sector mask), and, finally, any combination of ω and k. It is convenient here to use a nonmonochromatic light source, to filter the orientations. Finally, a specially prepared complex-conjugate filter can be used to separate any form of the elementary component of the process with different models. The inclination of the slits that separate the spectrum in the (ω, k) plane provides direct information on the wave propagation velocity. The most effective example demonstrating the efficacy and operating convenience of spatial filtration of the image is the separation of different specified orientations of curves on isomaps, such as isotherms and isobars on meteorological maps (Fig. 35). The presence of any desired direction on the isomap corresponds to a gradient in the perpendicular direction. With the aid of a complex conjugate filter, this operation, hitherto performed manually, can now be completely automatized. Cloud formation photographs obtained with meteorological satellites can be similarly filtered out. An optical installation for spatial filtration of seismograms, using 70 mm film, is described in [35].

5. Holographic Interferometer

The holographic interferometer [11] operates on the Fresnel holography principle (Fig. 36). If a hologram obtained in a homogeneous operating beam is illuminated by a standard beam, then a second homogeneous beam appears as a result of diffraction. If the idle hologram is illuminated by two beams, working and reference, then the fronts of the waves of these beams fully coincide, even if these fronts are not perfectly flat as a result of imperfections in the optical systems. Two identical fronts will, of course produce no interference pattern whatever. Placing the investigated object in the working beam distorts

FIG. 35. Filtration of the directions on topographical contour maps, geomagnetic and gravitational survey maps, or meterological maps. The identification of contour lines that are parallel to a specified direction is equivalent to the problem of finding the geometric locus of the points at which the field gradient has a perpendicular direction. Until recently this laborious operation was performed by hand.

The upper figure shows a contour map. In the middle is shown the result of filtration by removal of the lines in the angle range 45° ± 20°. The lowest figure shows the result of filtration in accordance with an additional program, wherein the lines in the range 45° ± 20° were left in.

one of the fronts, and an interference pattern is produced. Illumination of the object by scattered light provides more complete information on the object and facilitates at the same time the visual observation of the object.

The sequence of operations with such an interferometer is as follows: The investigated object is inserted in one of the beams, and the hologram of the

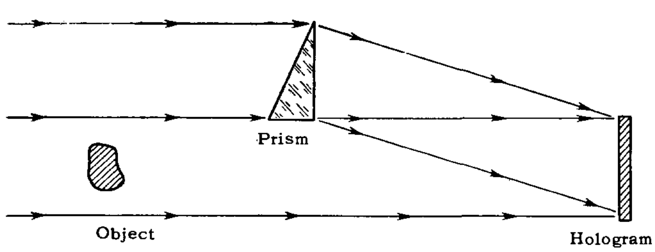


FIG. 36. Diagram of holographic differential interferometer.

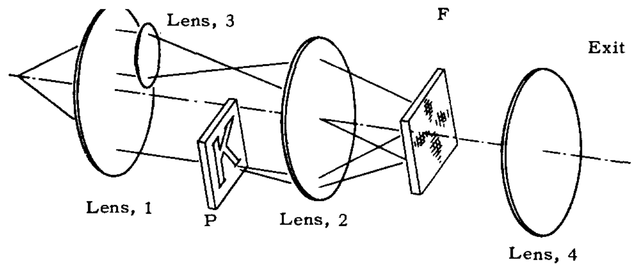


FIG. 37a. Diagram of optical "reader."

unperturbed picture is recorded. The developed hologram is then mounted in its previous position. The experiment is then performed and results in disturbances that produce a differential interference pattern. This is the most important advantage of the holographic interferometer compared over the ordinary one. Another important advantage is that the holographic differential interferometer can tolerate noticeable imperfections in the optical system. The differential pattern can be observed in higher orders. The picture is sharper, but at the expense of considerable loss of intensity. In [28] it was possible to go as far as the 23rd order (!) in the observation of side beams of higher overtones of the hologram. Under those conditions, the exposure lasted many hours, so that this record accomplishment has no practical significance. The optimum is the third order, in which high discrimination is obtained but the intensity loss is still tolerable.

6. Optical "Reader"

The General Electric Company [28] is readying for production a commercial holographic apparatus called an "optical reader." Such an installation is equivalent to the optical correlator described in Ch. II, Sec. 4, and operates on the principle of the optical complex-conjugate filter. Owing to the use of 16-mm film, the length of the apparatus is only 120 cm. A diagram of the General Electric scheme is shown in Fig. 37a. The selected object-transparency, for which the complex-conjugate filter is prepared, is mounted in the entrance plane P. The lens L_1 produces a plane beam, and lens L_2 carries out the Fourier transformation. Lens L_3 focuses the beam on the exit plane, forming a pointlike reference beam. The inclined standard wave is incident on the frequency plane F, and is used to record the transparency in the form of a Fourier hologram. The "optical reader" can pick out a single letter, a single word, or choose any object on a photograph (Fig. 37b). The system admits of 5% variation of the scale, and the signal amplitude in the third order of diffraction is reduced by merely 20% (Fig. 38). In all other respects it operates like a complex-conjugate filter.

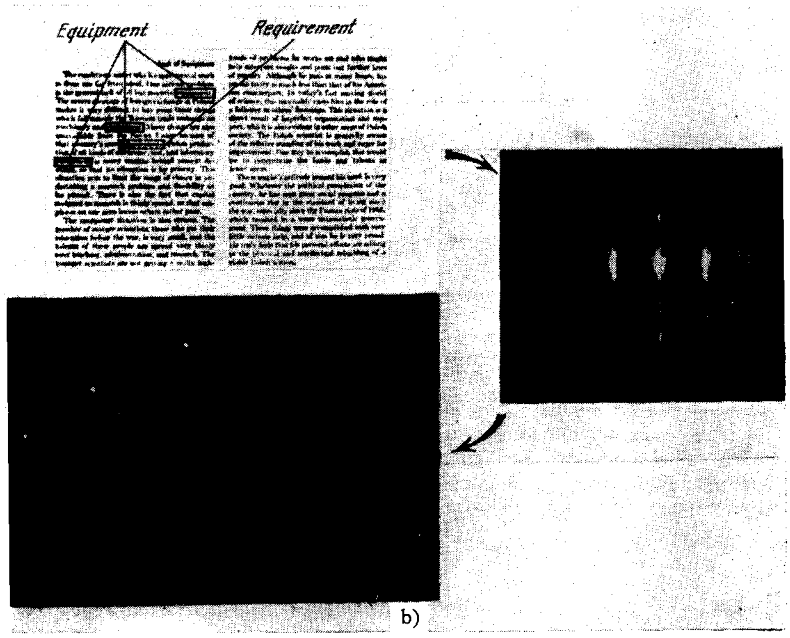


FIG. 37b. Top left: text in which the word "equipment" is picked out. Top right: filter for the word "equipment." Lower left: three bright points–cross correlation signals of the word "equipment" with a text containing this word in three places: two weak double points–cross correlation of the word "equipment" with the word "requirement."

7. Electro-optical Converter in Radar

The optical method of filtration of two-dimensional signals has recently found extensive use in radar[10] for the analysis of radio signals received from multielement antennas. With increasing accuracy and angular resolution of such systems, the number of elements of the observation system network increases rapidly. The volume of information that must be processed increases to such an extent, that the electronic signal-processing systems become extremely complicated. Further progress in this direction is possible by replacing the traditional methods of signal analysis by a method of optical processing based on a coherent illumination system. This example demonstrates most clearly the advantages of optics over electronics in the processing of information obtained from a phased array having a two-dimensional nature. The simplest phased system consists of two dipole antennas (Fig. 39) arranged in tandem[10]. The signals from the radio source 1 strike the antennas simultaneously and are added in a common network. For source 2, a certain phase shift is produced, and the summary signal is weaker. The delay in the instant of arrival of the signal makes the system directional. The operation of the electronic phase search is carried out by a scanning method in which only one point on the celestial sphere is observed at any given instant of time. With increasing number of elements of the antenna array, the processing of the signals by usual means of radio electronics becomes very slow and cumbersome. Here is where optics comes to the rescue. The radio signals are transformed into an optical inhomogeneity of a transparent medium. Ultrasound is employed for such a conversion. The regions of compression or rarefaction in a liquid form a spatial diffraction grating. The scheme of the first model of such a device[10] is shown in Fig. 40. A broad laser beam passes through a system of channels, after which it is fed to a light modulator. Each channel separates a vertical column in the water

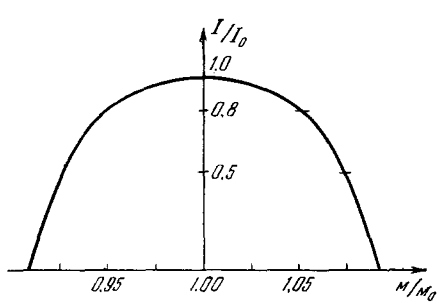


FIG. 38. Attenuation of the signal when the scale of the picked object deviates from the scale of the sample.

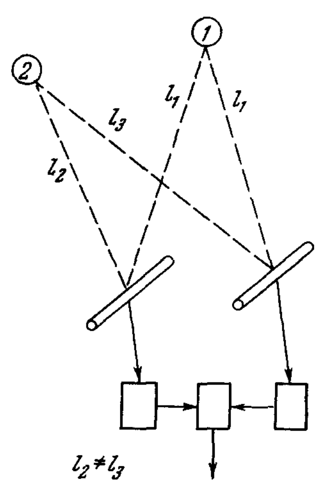


FIG. 39

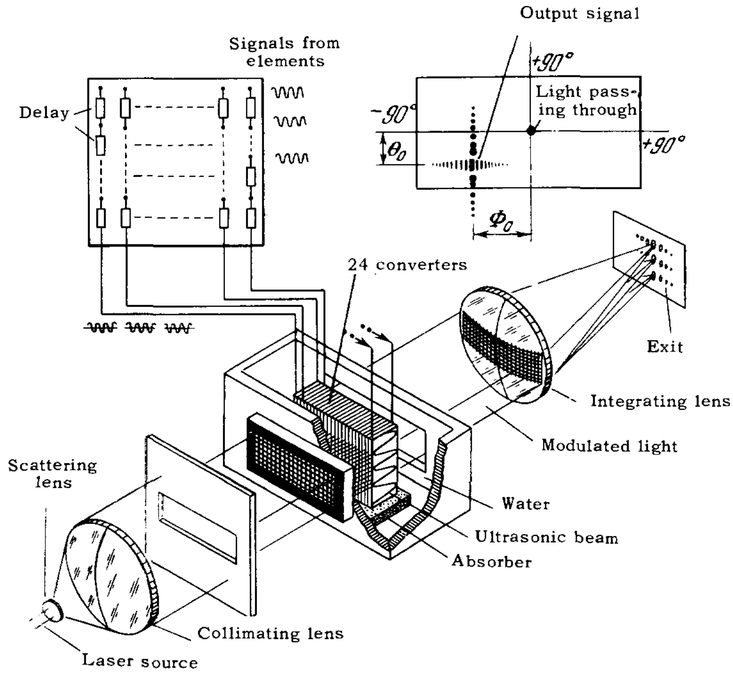


FIG. 40

and ultrasonic converters are placed above each column. The signals from the antenna elements are sequentially delayed in time relative to one another and are fed separately to the lower end of the grating. The signal from the lowest element of the antenna is not subject to delay, and the signal from the highest element of the antenna is delayed by the maximum amount of time. Thus, the output signal from the lower end of each antenna consists of a series of signals, each of which begins after the preceding one has ended. Each series of signals excites a piezoelectric converter. The ultrasonic oscillations propagate from the converter and, after reaching the bottom, are completely absorbed. The laser beam passing through the space filled with the signal from the antenna array contains the complete information concerning the times of arrival of the signals at each element of the antenna. A light spot appears in the focal plane of the gathering lens, and indicates the position of the radio source on the celestial sphere. With the aid of such a device it is possible to observe simultaneously, with high degree of resolution, all the radio sources which fall in the operating range of the antenna array. According to calculations, such a visualizing device can process signals received from a system of 10^4 dipoles. This uncovers wide prospects for increasing the resolving power of radio telescopes and radar systems.

CONCLUSION

Holography, optical correlators, and optical analog computers constitute the basis of a new system of information processing, which should be called optics of interference recording and processing of information. With the aid of interference optics it becomes possible to carry out rapidly all mathematical operations on two-dimensional functions encountered in information theory, such as addition, subtraction, and multiplication as well as integral operations with complex amplitudes. An interference system of information recording solves the problem of performing complete measurement in optics, a problem that could not be solved directly, owing to the quadratic character of the light detectors. The exceedingly rapid operating speed of optical machines and the unusual stability of all the operations in optics with respect to the loss of part of the sought image or a loss of part of the memory block, predicts a promising future for this new branch of optics. There is no doubt that the catastrophic growth of the amount of information subject to processing and faced by humanity will be offset by the new qualities of interference optics, so that the problem of information processing will become stabilized in the nearest future. It is also obvious that the a posteriori properties of the interference "observer" will lead to a radical review of traditional schemes of performance of physical experiments.

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