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# NEW EXPERIMENTS ON THE COMBINING OF MECHANICAL VIBRATIONS AND ON MECHANICAL RESONANCE

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THE suggested demonstration apparatus possesses a certain degree of universality and enables experimental demonstrations of the combining of mechanical vibrations. Included are experiments on the combining of mutually perpendicular vibrations with frequency and phase differences that can be varied over a broad range (Lissajous figures), and experiments on the combining of vibrations occurring in the same direction (interference for various phase differences, beats for all frequency differences, and the combining of multiples of the fundamental frequency).

In addition to these stationary classical experiments, the apparatus permits demonstrations of mechanical resonance. The several methods heretofore used for this purpose do not generally reveal the phase relations that are involved in resonance, whereas the suggested demonstrations emphasize the phase aspect especially distinctly along with the amplitude effect.

The apparatus also makes it possible to demonstrate certain properties of string vibrations such as the dependence of the vibration spectrum on tension and the conditions of activation. Of fundamental importance is the possibility of using the apparatus to demonstrate patterns in which more than two, and particularly three, vibrations are combined.

A fundamental advantage of the proposed scheme, which is based on the principle of an optical oscillograph, is its high luminosity, which permits very large bright patterns that can be seen clearly on a large screen in the largest lecture halls.

## 1. OPTICAL SCHEME

The optical scheme of the apparatus is represented in Fig. 1. The projected object is a brightly illuminated circular aperture S in a small screen A attached to a tuned string. A thin lens $L_1$ rigidly fastened to a prong of an electric tuning fork forms the image of S as a bright spot S′ on the viewing screen B (Figs. 1-2)

It is obvious from the geometry that when the string and the attached screen A oscillate the spot S′ will oscillate similarly with increased amplitude and a $\pi$ phase shift. The amplitude ratio between the oscillations of S′ and S is $k_1 = f/d$. As the lens $L_1$ vibrates the spot S′ will oscillate in directional and phase agreement with the vibrations of the lens center $O_1$ and with the amplification $k_2 = (f + d)/d = k_1 + 1$.

During the simultaneous vibrations of the string and the lens the bright spot S′ will exhibit complex oscillatory motion represented by the vector sum of the two vibrations with an additional $\pi$ phase shift and very close amplifications $k_1$ and $k_2 = k_1 + 1$ ($k_1 \gg 1$).

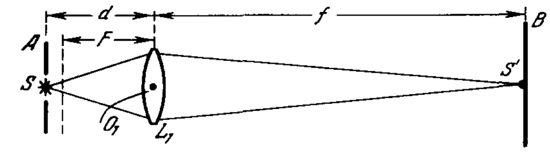


FIG. 1.

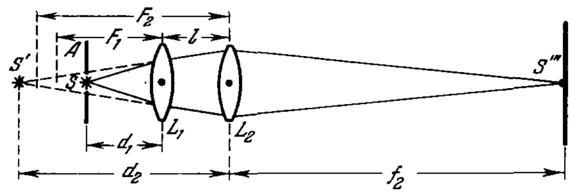


FIG. 2.

If we add a second electric tuning fork and attached lens $L_2$ of suitable optical power ($F_2 > F_1$) having a different natural frequency $\nu_2$ (such as $\nu_2 = 2\nu$), and arrange the instruments as shown in Fig. 2, we can combine the three vibrations with practically no reduction of the luminosity. In this case the amplifications will depend on the focal lengths and positions of the lenses $L_1$ and $L_2$.

## 2. DESCRIPTION OF APPARATUS

One of the two sources of vibrations is a string subject to continuously variable tension and vibrating at its fundamental natural frequency. This string is an iron wire 0.5 mm in diameter and about 100 cm long, mounted on a wooden bench (100 × 30 × 1.5 cm) supported by four legs resting on thick rubber shock absorbers.

The clamping of the string on the bench provides for continuous variation of its tension, and also for rotation of the string about its length whenever necessary. A small opaque screen A is attached to the middle of the string. The screen is held in a lightweight frame consisting of three sewing needles and resembling a letter H lying on its side. The middle of this frame is soldered to the wire; the plane of the frame is vertical. The screen, which is cemented to its frame, is a 4×4-cm piece of black wrapping paper coated with aluminum paint on the side facing the light source. A small circular opening S, 0.5–1 mm in diameter, is located near the middle of the screen, to the upper part of which there is cemented a hori-

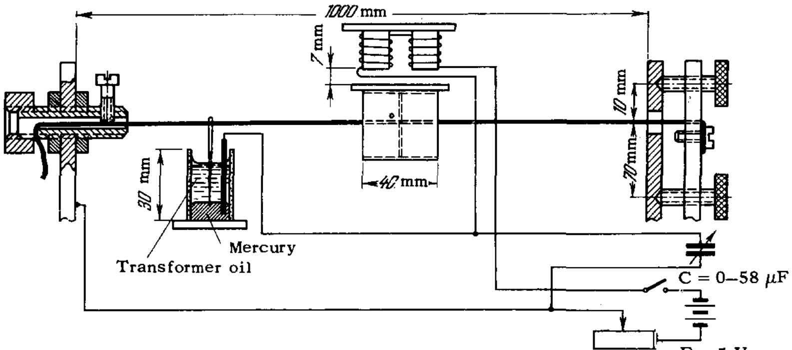


FIG. 3.

zontal strip of sheet metal serving as the armature of an electromagnet. The combined weight of the screen, frame, and armature is about 1.6 g.

Natural vibrations of the string are excited by means of a small removable electromagnet and a mercury interrupter. Figure 3 shows the circuit serving this purpose with indications of some of its parameters and distances separating parts of the apparatus.

The second source of vibrations is a balanced electric tuning fork vibrating at the constant frequency $\nu_0 \cong 51$ cps; attached to one prong is a lens $L_1$ ($F = 3.5$ cm, $D = 3.5$ cm) having the relative aperture $D:F = 1:2$. The electric circuit of this tuning fork is shown in Fig. 4.

## 3. EXPERIMENTAL DEMONSTRATIONS

A light source, condenser, and objective lens are brought close to the bench and string; the image of the lamp filament is projected on the circular hole S in the plane of the small screen. In our experiments a 300-W, 110-V motion-picture projection lamp was used.

For the successful rapid performance of the entire series of experiments we recommend that the following three conditions be observed. Before the experiments are begun it is advisable to insert a new needle into the mercury interrupter that activates the string and to pour a layer of mercury into the pure transformer oil in the cup. In order to prevent any possible coupling of the vibrations the transfer of vibrational energy to the demonstration table should be reduced by placing additional shock absorbers under the legs of the bench and under the base of the stand supporting the electric tuning fork.

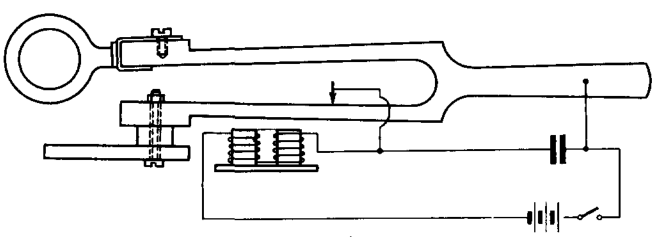

FIG. 4.

Experiment I. Combination of oscillations in a single direction. The electric tuning fork and lens are mounted on a stand; Fig. 5 shows the experimental setup. For nonsimultaneous vibrations of the string and tuning fork the light beam reflected from the rotating mirror traces the appropriate time sweep on the screen. When the string and lens vibrate simultaneously the time sweep of the combined vibration is observed on the motion picture screen. It is useful to perform these experiments with equal amplitudes of the two vibrations.

The following demonstrations are of interest: a) Combining vibrations of identical frequency with slow monotonic variation of the phase shift; b) combining vibrations having slightly different frequencies (beats); c) combining vibrations with the frequency ratio $\nu : \nu_0 = 2:1$. Photographs of the corresponding wave patterns appearing on a motion picture screen located two meters from the apparatus are shown in Fig. 6.

Experiment II. Combination of mutually perpendicular vibrations. The experimental arrangement is shown in Fig. 1. When the string and tuning fork vibrate simultaneously the spot S′ describes a complex figure. By varying the tension of the string the desired frequency ratios (1 : 1, 4 : 3, 3 : 2, 5 : 3, 2 : 1 etc.) are easily achieved, producing the corresponding Lissajous figures. It is interesting to produce a stationary figure for the desired phase difference. This is accomplished successfully by means of a screw that varies the string tension continuously. The Lissajous figures in Fig. 6 were projected on the screen B (0.8 × 0.8 m) placed 4 m from the apparatus in a darkened lecture hall.

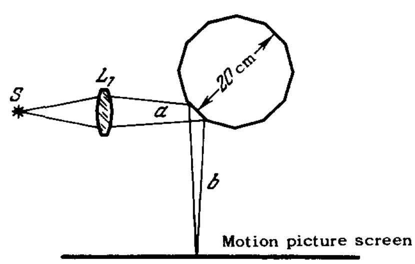


FIG. 5

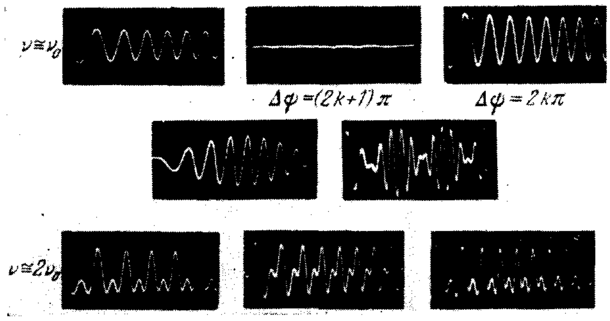


FIG. 6. For $a + b \cong 2$ m the amplitude of the resultant oscillation is 0.5 m (obtained with a hole S of about 0.4-mm diameter).

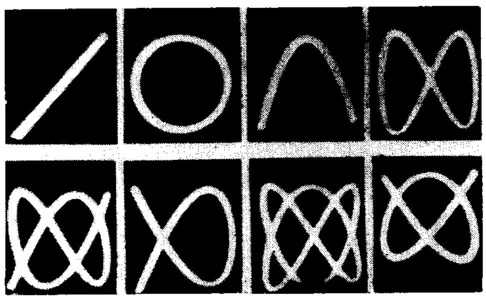

FIG. 7. $f \cong 4$ m; diameter of hole S about 1 mm; darkened lecture hall; photographic exposure time $t \cong 1$ sec.

Experiment III. Resonance at $\nu = \nu_0$ and $2\nu_0$. Amplitude and phase relations at resonance. A more convincing demonstration is performed when the electromagnet and mercury interrupter are removed from the bench. The electric tuning fork is fastened to a special stand of suitable height supported on three legs. The stand is positioned close to the string. To achieve the largest effect one of the legs of the stand should rest on the bench under the middle of the string, while the other two legs touch a straight line parallel to the string.

At $\nu = \nu_0 = 51$ cps the string vibrations are only weakly damped and the resonance amplitude can be excessively large; the resonating system is then unstable. For resonance at $\nu = \nu_0$ it is therefore advisable to reduce considerably the amplitude of the tuning-fork vibrations and at the same time to weaken the coupling between the tuning fork and the string. This can be accomplished by placing a wooden block under the leg of the tuning-fork stand that is located below the string while rubber spacers of the same thickness are placed under the other legs. The resonant frequency should be approached very slowly through successive slight changes of string tension. A clear stable resonance effect is easily produced when the foregoing details and precautions are observed.

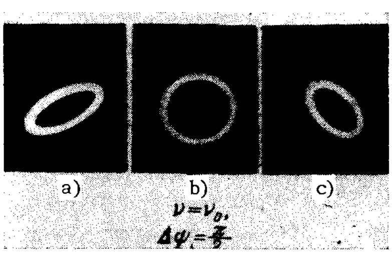


FIG. 8. $f \cong 3$ m; diameter of hole S about 1 mm.

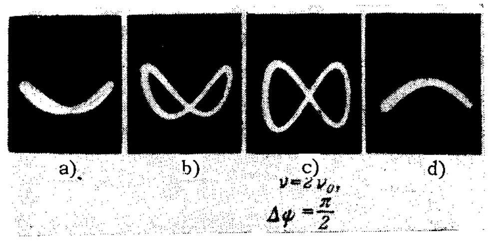


FIG. 9. $f \cong 3$ m; diameter of hole S about 1 mm.

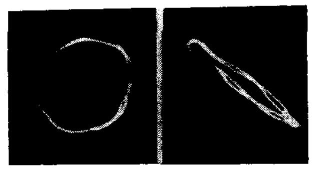

FIG. 10. Diameter of hole S about 0.4 mm.

The photographs in Fig. 8 ($f = 3$m) show how the amplitude and phase differences change as $\nu_0$ is approached. Figure 8b ($\nu = \nu_0$) shows the characteristics of resonance,—the string vibrations have maximum amplitude and the phase difference is $\pi/2$.

Resonance at $\nu = 2\nu_0$ can be observed with equal success. Because of greater damping of the resonating system at this frequency the resonance curve is less sharp. Consequently a stable effect can be achieved as the resonant frequency is approached and passed, without observing the precautions required in the preceding experiment. For the demonstration of this resonance the inserts should be removed from under the legs of the stand, and the amplitude of tuning-fork vibrations should be quite large. Figure 9 illustrates convincingly the amplitude and phase relations existing in resonance at $2\nu_0$.

Experiment IV. Dependence of string vibration spectrum on conditions of excitation. This experiment should be demonstrated immediately following the experiment on resonance at $\nu_0$, without altering the experimental arrangement or tuning of the string. After the resonant vibrations of the string are stopped by a hand they should then be excited again by plucking the string. When the string is excited at a point near a clamp it is clearly observed (with a hole S of about 0.5-mm diameter) that higher harmonics are superimposed on the fundamental vibrations at $\nu_0$. Photographs of these effects are seen in Fig. 10.

Translated by I. Emin