SOVIET PHYSICS USPEKHI VOLUME 9, NUMBER 4 JANUARY-FEBRUARY 1967

523.12

Physics of Our Days

THE "HOT" MODEL OF THE UNIVERSE

Ya. B. ZEL'DOVICH

Usp. Fiz. Nauk 89, 647-668 (August, 1966)

THE year 1965 has brought a most important discovery in astronomy. Measurements at short and ultrashort wavelengths ($\lambda = 7$, 3, and 0.25 cm) have demonstrated the presence of an isotropic radiation, i.e., independent of the observation direction, corresponding to a temperature near 3°K. At the time this article was written, the following were available: a brief communication published in July 1965 on measurements at $\lambda = 7.3$ cm, a note on measurements at $\lambda = 3$ cm (March 1966), and a report of unpublished measurements by a different group at $\lambda = 0.25$ cm, also by radio methods*.

A study of the optical absorption spectrum of the cyanide radical (CN) in galactic gas has also confirmed the presence of radiation of 0.254 cm wavelength, the intensity of which corresponds to 3°K. It is curious that the results of measurements pertaining to CN were known already in the forties, but then they remained an unexplained paradox. In the investigated wavelength region, the observed radiation is 10^2—10^5 times larger than expected from the known sources such as stars and radiogalaxies.

The possible existence of such radiation was predicted by Friedmann's theory of the expanding universe. Since 1948, within the framework of this theory, a hot model has been under consideration, in which it is assumed that matter in the prestellar state is characterized by a large entropy. The entropy concept has in this case a simple and intuitive meaning: when thermal equilibrium obtains and the entropy is large, there are many light quanta for each atom, and it turns out that the entropy is directly proportional to the number of quanta. The already mentioned measurements lead to a value of approximately 10^9 quanta/atom. The entropy, meaning also the number of quanta, is conserved during the expansion. However, the quantum energy decreases upon expansion, in accordance with the fact that the wavelength increases in the same proportion in which all distances between any specified pair of particles or pair of galaxies increase during the course of expansion.

Thus, extrapolation to the past leads to the notion of a plasma in which the number of quanta per unit volume exceeds in giant fashion the number of atoms or, more accurately, the number of nucleons and electrons in the same volume. The temperature of this plasma changes during the expansion. Let us assume that there was an instant when the density was infinite, and let us reckon the time from that instant, approximately 10^{10} years ago. From the equations of mechanics and from the presently known relation between the number of quanta and the number of atoms per unit volume, we can find the temperature and write the corresponding time dependence. At the instant $t = 1$ sec, the temperature was approximately 1 MeV, i.e., 10^{10} degrees; in addition to quanta in equilibrium, there were almost as many pairs of electrons and positrons; the nuclei could not exist then, and the number of protons and neutrons was practically equal; collisions with electrons and positrons led to neutral transformations of protons and neutrons.

With increasing expansion, the positrons vanished. A part of the neutrons decayed, and the remainder joined the protons and yielded in final analysis a composition of ~ 70% hydrogen and 30% helium by weight (i.e., approximately 10% of the number of helium atoms). There should also remain from that period neutrinos and antineutrinos in amounts approximately equal to the number of quanta, and with the same average energy, corresponding at the present time to several degrees, i.e., approximately 10^{-3} eV. It was assumed a few years ago that the content of helium in matter that has not been subjected to processing in the stars, is much smaller than 30%, and this is a major difficulty for the hot model.

Also formulated a few years ago was a hypothetical "cold" model, which gives 100% hydrogen and total absence of radio emission of the type recently discovered. It is obvious that in the light of the latest data the cold-model hypothesis should be discarded, although the question of the primary content of helium still remains unclear from the observational point of view and calls for further research*.

Even more paradoxical conclusions follow from the hot model for earlier times and higher temperatures. It follows from the theory that there was a period (when $T \geq 10^{13}$ deg), when there were many nucleons and antinucleons, and from this point of view the present-day nucleons are the result of a small excess of nucleons over antinucleons at this early stage. If heavier particles—quarks—exist, then they should be present in noticeable amounts in equilibrium, and as an es-

---

*Measurements at 20 cm were also made very recently.

*According to private communications, old stars with low helium content have been observed very recently. The reliability of these communications is uncertain.

timate up to 10^{-9}—10^{-10} quarks/hydrogen atom remain whole at the present time; this is more than the average concentration of gold (10^{-12}—10^{-13}) and radium (10^{-18}, likewise per hydrogen atom). It must be emphasized that the conclusion concerning nuclear reactions at $t = 1$ sec and later are practically independent of the assumptions concerning the earlier stage, concerning the nature of quarks and antinucleons, and whether some density higher than 10^6 g/cm^3 and a temperature higher than 10^{10} deg was attained. Even under these conditions, all the processes are rapid; no matter what the initial composition we specify, equilibrium is established at that instant practically instantaneously; the system will forget the initial composition, and further development of events will not depend on the assumptions concerning what had occurred at $t < 1$ sec. We recall that the theory of the hot universe is essentially an extrapolation, to past states, of the universe surrounding us, whose properties are being investigated at the present time. Like any other extrapolation, the closer it is to us, i.e., the less remote the state considered in the past, the more reliable it is.

After the period of nuclear reactions ($1 \text{ sec} < t < 100 \text{ sec}$), the second characteristic landmark is $t = 3 \times 10^6$ years, corresponding to a temperature 3000—4000°. While the hydrogen and helium are more or less uniformly mixed with the radiation, prior to that time they were ionized. The radiation density exceeds the density of ordinary matter. The radiation pressure is high; the "matter," i.e., the electrons and ions (nuclei), interacts strongly with the radiation. This interaction with the radiation prevents formation of stars and galaxies. Gravitational forces that gather matter into the region where the density of matter already exceeds the average density ("gravitational instability") are unable to overcome the radiation pressure, which increases upon compression of the matter. Only after $t = 3 \times 10^6$ years does recombination of the electrons and protons into hydrogen atoms occur, and presumably formation of stars and galaxies begins. This leads to a concept whereby the first generation of the stars constitutes a small fraction of the total mass of matter; perhaps the term "protostars" is more appropriate here, since the conditions for their formation differ radically from the conditions under which stars are formed at the present time; accordingly, their properties can differ, too.

It is thus assumed that the protostars can rapidly release an energy sufficient to heat the remaining, larger portion of matter. A considerable fraction of this matter remains in the form of hot ionized plasma and does not condense into stars. Thus, the concept of the hot model of the universe is related to an independent hypothesis, that of the existence (at the present time!) of hot ionized intergalactic gas of density ($\sim 10^{-29}$ g/cm^3) tens of times larger than the density of matter in the stars, averaged over the entire space. This hypothesis is supported by the recent discovery of unusually bright objects—quasars. The spectrum of one of the most remote of the presently known quasars, 3C-9, has singularities that lead to the conclusion that the intergalactic medium contains 6×10^{-11} atoms of neutral hydrogen per cm^3 (density 10^{-34} g/cm^3)*. This hydrogen absorbs the Lyman α line with wavelength $\lambda = 1216$ Å. The 3C-9 source is so far, that the red shift increases all the wavelengths by a factor 3.012! Because of this, the far ultraviolet region of the spectrum, with wavelength $\lambda = 1216$ Å (which is not transmitted by the atmosphere), is perceived by us as $\lambda = 3662$ Å and can be observed by terrestrial telescopes. The closer to us the considered layer of intergalactic gas, the smaller the red shifts of the absorption line; on the whole, we obtain an absorption band extending for the terrestrial observer from $\lambda = 3.012 \times 1216 = 3662$ Å toward shorter wavelengths. This density of the neutral hydrogen, 10^{-34} g/cm^3, pertains to the vicinity of 3C-9, i.e., not only to a spatially remote object, but also to the remote past. It is assumed that during that period the density of the ionized gas was $(2–5) \times 10^{-28}$ g/cm^3, i.e., the neutral gas amounted to less than 10^{-6} of the entire gas, corresponding to gas temperatures of the order of 10^6 deg. The gas was also quite rarefied, transparent, and far from being in equilibrium with the overall cosmic radiation, whose temperature at that instant was of the order of 10°K. In the period under consideration ($t \sim 2 \times 10^9$ years), the gas was heated only by the energy of such objects as stars and galaxies.

A distinction must be made between two related hypotheses, which essentially are different in scope:

1) The hot model of the universe—temperatures from 10^{10}°K at $t = 1$ sec to 3°K now. Isotropic radio emission at centimeter and millimeter wavelengths is the consequence of the hot model. The radio emission is the direct successor of powerful radiation in the compressed hot plasma, whose density during the earlier stage exceeded by many times the density of matter; it is quite immaterial in this case whether we are dealing with the same individual quanta; in the state of total thermal equilibrium, some quanta are absorbed, others are emitted, still others are scattered, but all these processes are balanced in such a way that neither the total energy of the medium nor the spectrum of the quanta changes. This is the situation in the case of high density. At low density, absorption and emission of quanta becomes insignificant, their scattering plays no role, and it is possible to follow the fate of each individual quantum. In any case, the presently observed radio quanta are either the same quanta or the descendants of the quanta, which, by assumption, had an energy on the order of 1 MeV at $t = 1$ sec and whose en-

---

*More recent measurements of a different source have not confirmed this conclusion, but this does not exclude the presence of ionized hydrogen (see below).

ergy decreased by virtue of the red shift during the course of expansion, i.e., essentially by virtue of the Doppler effect. These quanta are currently referred to as "relict radiation." It is important that neither stars nor radiogalaxies nor the hot interstellar gas can give anything approaching the properties of relict radiation: the energy of the relict radiation is too high, and its spectrum is not similar to either the spectra of the stars or to the spectrum of the radio sources.* This indeed proves the cosmological, relict origin of the radio emission in question.

2) Hot intergalactic plasma. This concept certainly pertains to a period later than $t = 3 \times 10^6$ years; the plasma is heated to 10^5—10^6 degrees and is no longer in equilibrium with the relict radiation. This means that such a plasma needs heat sources.

The question of interaction between a hot plasma and the relict radiation is quite interesting. Energy exchange between them is at present small, but it could have been larger earlier. Energy estimates show that the plasma could not produce the radiation which we regard as relict (with its density and spectral composition). On the other hand, the same energy-balance calculations show that the plasma could not have been heated as early as shortly after $t = 3 \times 10^6$ years. The plasma electrons scatter radiation quanta because of the Compton effect. It is curious that an optical thickness of the order of unity is reached in a distance corresponding to a red shift by a factor of 6—8. The isotropic relict radiation does not change its properties during this scattering; the energy of the electrons and of the quanta are such that the scattering is not accompanied by noticeable exchange of energy and does not alter the spectrum. However, observation of remote discrete sources—quasars with large red shift—is made difficult by the electron scattering. At the same time, since the scattering cross section is independent of the frequency, it is very difficult to prove with assurance the presence of Compton scattering and to determine the concentration of the free electrons.

The most important would be observation of helium in the composition of the intergalactic gas and measurement of its temperature by determining the emission of soft x-ray quanta. At the present time measurements yield only the inequality $T < 2 \times 10^6$ for the temperature of the intergalactic gas.

A very difficult problem is the observation of relict neutrinos and antineutrinos. To do this, it is necessary to improve the existing accuracy by a factor 10^6 !

At the same time, we must emphasize the tremendous significance of such an experiment. Observation of neutrinos in the required amount and with the expected spectrum would confirm the hot-model notions concerning the very early stage at $t < 1$ sec, a density larger than 10^6 g/cm^3, and a temperature higher than 1 million volts. Indeed, measurement of relict neutrinos would be the "experiment of the century"!

The hot model poses problems of tremendous importance and difficulty to the theoreticians. These include primarily:

a) The question whether it is possible to construct (using quantum concepts) a theory of transition from compression at $t < 0$ to expansion at $t > 0$.

b) The specific entropy of matter in the hot model (characterized by the number 10^9 quanta/atom) was regarded above as an initially specified characteristic of this model, as one of those initial conditions that must be stipulated for the integration of the equations. Can we ask: Why is the entropy precisely this, not larger or smaller? Only by understanding the origin of the large entropy can we explain satisfactorily how an almost charge-symmetrical state is obtained with a gigantic number of antinucleons and nucleons, but at the same time with a definite small excess of nucleons. In fact, if there were only nucleons when $t < 0$, and these somehow produced a large entropy, then at low density this entropy in turn appeared in the radiation. In such a case the subsequent production of $N + \bar{N}$ pairs upon contraction, and the presence of $N + \bar{N}$ pairs (with conservation of the excess of nucleons N) at the start of expansion at $t > 0$ are consequences of known laws.

c) It is curious that the hot model leads to a reformulation of the problem of existence of superdense bodies within their own gravitational radius. These bodies must gather radiation from their surroundings and with this they increase their mass catastrophically. The hypothesis that superdense bodies have existed initially should in some manner take account of the interaction between these bodies and the radiation surrounding them.

d) We note some concrete problems pertaining to the state of large density: What is the situation with vortical motion in magnetic fields in such a state? What is the degree of homogeneity and fluctuation in this state?

e) The theory of formation and evolution of "ordinary" objects (stars, galaxies, clusters, quasars) should also be developed with allowance for relict radiation.

f) We recall, finally, the most vital question still remains unanswered: Should we imagine the evolution of the universe as 1) a single expansion from a special singular state, or 2) a single contraction from $t = -\infty$, $\rho = 0$ via $t = 0$, $\rho = \infty$ (or 10^{93} g/cm^3) and subsequent expansion, continuing to the present time, or else 3) an infinite sequence of cycles of contraction and expansion? It is impossible to choose from among these hypotheses on the basis of philosophical considerations. Each agrees with the laws of physics known to us; the laws of physics are not connections between sensations experienced by a thinking person,

---

*According to a most recently discussed hypothesis, practically all the nuclear energy was released in the stars during the earlier stage, and the light of the stars was absorbed by dust particles which transformed it into thermal radiation. For many reasons, however, this hypothesis must be regarded as unlikely.

but objectively existing recognizable laws of the external world. In full accordance with this, the answer to our question, i.e., the choice between the different hypotheses, must be based on an objective natural-scientific study of the universe surrounding us, both by observation and by development of a physical theory that accounts for the observational data.

We can advance theoretical evidence (connected with the increase of entropy) against an infinite repetition of such cycles. It is curious that by virtue of the laws of the general theory of relativity, the situation is not at all similar to the ordinary concept of "thermal death." In fact, during the extent of each cycle, the universe can be inhomogeneous, consist of stars, and bear very little resemblance to the constant average density and temperature picture corresponding to the maximum entropy in a system without gravitation. The repetition of these cycles requires that the density at the present time be larger than a definite critical value $\bar{\rho}_c \sim (1-2) \times 10^{-29}$ g/cm^3. With this, the universe should be geometrically closed*. The entropy increases in each cycle, and consequently also the energy per atom in its co-moving coordinate frame; the period and the swing of the cycle, i.e., the maximum radius at the instant of transition from expansion to contraction, also increase.

However, the last word belongs to observational astronomy, namely, measurement of the overall average density of matter (including the most difficultly observed forms—from neutrino and gravitational waves to collapsing stars and galaxies) will determine the properties and the fate of the universe. As already mentioned, expansion gives way to contraction if $\bar{\rho} > \rho_c$. If $\bar{\rho} < \rho_c$, we are left only with the choice between a single expansion or a single contraction that has given way to expansion.

The value of ρ_c itself depends on the Hubble constant $u = Hr$, viz., $\rho_c = 3H^2/8\pi G$, where $H \sim 75$–100 km/sec $\cong 0.3 \times 10^{-17}$ sec^{-1}, and $G = 6.7 \times 10^{-8}$ cm^3/g-sec is the gravitational constant. Therefore ρ_c is known with sufficient accuracy. The main contribution to $\bar{\rho}$ is probably made by the intergalactic plasma, and therefore its study is the most important practical problem.

Let us turn from hypotheses, which have been described in increasing order of boldness (meaning also with decreasing reliability), to the actual observational aspect of the situation.

Is there not a discrepancy between several observations at three or four different wavelengths and the grandiose nature of the conclusions? What consequences can be deduced directly from the experimental data? These two questions are not asked together accidentally. It is understandable that the tremendous resonance and the significance of the experiments referred to depend on their comparison with the entire system of views of evolutional cosmology, with the theory of A. A. Friedmann, and with modern information on the interaction between matter and radiation and on nuclear reactions. We must dwell on this, because quite recently the following point of view was advanced: Friedmann's theory, i.e., the idea of general cosmological evolution, is an approximation. It was useful at a definite stage and led to the discovery of the red shift; but for 30—40 years this theory has no longer given new results, it is not fruitful, further development should follow the line of replacing it with new concepts,—and so on.

However, the development of cosmology in recent and even, more accurately, in the last year, has not confirmed this forecast, meaning that it likewise has not confirmed the estimate given above for the past. New facts (the relict radiation and the chemical composition) can be understood and fitted within the framework of the applicability of Friedmann's theory to an ever more remote past, and this theory is justified for states which are ever closer to singular. This theory is fruitful, it offers guidelines for observations, and without it no fundamental significance would be attached to the observation of the weak radio background, just as in its time no significance was attached to the singularities of the CN spectrum.

Turning to direct consequences of observations that are not connected with the theory, we must emphasize the isotropy of the radiation and the agreement with the form of the equilibrium spectrum (Planck curve) within the limits of the investigated part of the spectrum. The isotropy of the radiation, irrespective of any definite theory and of any assumptions concerning the remote past, is evidence of the identity of the conditions in different directions away from us. The weak interaction of the radiation in question (7—0.25 cm) with dust, neutral atoms, or plasma leads to conclusions pertaining to much larger distances than could be deduced from statistics of remote discrete objects. The presently measured quanta have experienced scattering at a distance corresponding (on the average) to a red shift of the order of $z = 6$ or 8, i.e., to $t \sim t_0/15$, where t_0 is the present-day age. This means that the expansion has been going on isotropically at least since then.

The second consequence is that the earth (sun, galaxy) does not have a large velocity relative to the radiation field; a velocity of 10,000 km/sec would give an anisotropy of 10% at a wavelength 7 or 3 cm and 20% at 0.25 cm. Such an anisotropy could presumably be detected. The Planck spectrum of the relict radiation (which must be verified further to $\lambda \sim 0.05$ cm)

---

*Closure at $\rho > \rho_c$ is deduced as a consequence of the general-relativity equations relating the curvature of space with the density of matter. Additional hypotheses are assumed here, that there is no so-called cosmological term in the equations, that the distribution of matter is homogeneous, and that the expansion is isotropic (equivalence of all directions). These three hypotheses cannot be regarded as rigorously proved, but at any rate they do not contradict the observations, and have recently gained support in connection with measurements of the relict radiation.

can yield indirect confirmation of the existence of a period when light and plasma were in equilibrium, a period pertaining to an earlier stage. The properties of the relict radiation then allow a deeper look into the universe, a check and a direct confirmation of the principles of contemporary cosmological views —isotropy and homogeneity of practically the entire part of the universe that can be presently observed.

The foregoing review was aimed at presenting a general description of the status of cosmology after the sharp turn which has occurred quite recently. This review is aimed primarily at the non-specialist, i.e., the physicist but not the astronomer. To facilitate the perception of the entire picture, we did not interrupt the exposition with technical details, formulas, priority questions, or references to the original literature.

However, a reader interested in this problem or some of its individual parts needs such material. It is all included in the appendices which follow the main text of the review and were purposefully written in dry manner, so as to compensate for the non-rigorous journalistic style of the review.

APPENDICES

1. MEASUREMENTS OF RADIATION AT SHORT WAVELENGTHS

The main measurements were made at 4080 MHz (λ = 7.35 cm) [**A. A. Penzias** and **R. W. Wilson**, Astrophys. J. **142** (1), 419 (1965)].

The work was performed at the Bell Telephone Laboratories. At the indicated frequency, the effective temperature of the antenna (20-foot horn reflector; Fig. 1) was 3.5° higher than assumed. The total measured temperature at the zenith was 6.7°K, of which 2.3° are ascribed to the contribution of the atmosphere, and the calculated contribution of the antenna itself (connected with its ohmic resistance) is 0.8 ± 0.4°K. Reception was with a traveling-wave maser. The authors state with respect to the obtained excess 3.5 ± 1° at 4080 MHz (2×10^{-20} W/m^2 Hz-sr), that, within the limits of observation accuracy, it is isotropic, unpolarized, and not subject to seasonal variations in the observations made from July 1964 through April 1965.

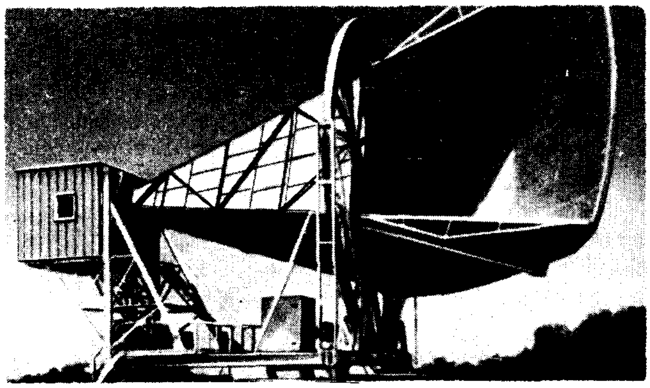

FIG. 1

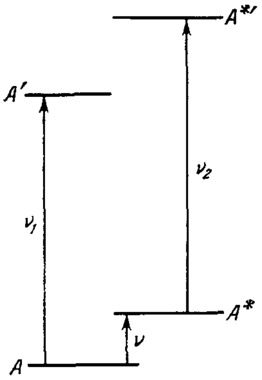

FIG. 2

A brief summary of the proceedings of a conference held in December 1965 in Miami, contained in a preprint by Thronton Paige, mentions the measurements (**P. G. Roll** and **D. T. Wilkinson**, Palmer Physical Laboratory) at wavelengths 3.5 and 0.25 cm, which yielded 3 ± 1° and 3 ± 1°. The same measurements are cited as unpublished by **J. E. Felten**, Phys. Rev. Letts. **15**, 1003 (1965). There are no details concerning the apparatus*. In the conference report, mention is made of the use of the CN radical with a reference to Fields.

The idea of the method is that the molecules experience practically no collisions in interstellar space, and the transitions from the lower state A to the excited state A* are determined by the interaction with the radiation: $h\nu + A \rightleftharpoons A^*$ (Fig. 2),

$$\frac{[A^*]}{[A]} = e^{-h\nu/kT},$$

where T is the effective temperature of radiation with wavelength corresponding to the energy difference between A* and A, in this case 0.254 cm. The populations of the levels A and A* are estimated from the intensities of the absorption lines of optical light from the stars (ν_1 and ν_2) for which the states A and A* are the initial ones.

Thus, the CN molecules assume the role of an infrared (ν) thermometer with optical reading (ν_1, ν_2).

V. I. Slysh has graciously pointed out earlier work on CN [**A. McKellar**, Public Dominion Astrophys. Observ. **7** (15) (1941)]†.

This method was independently proposed by I. S. Shklovskiĭ. The corresponding paper was already published (Astronomic Circular No. 364, 28 March 1966).

Figure 3 shows the spectrum of the relict radiation corresponding to T = 3°K and the spectrum of the background from the radiogalaxies (the points denote the experimental data). The figure shows also the spectra

---

*After our article went to press, we received the paper [**P. G. Roll** and **D. T. Wilkinson**, Phys. Rev. Letts. 16, 405 (1966)], which contains a detailed report of these measurements.

†See the recent papers by **P. Thaddeus** and **I. F. Glauser** [Phys. Rev. Letts. 16, 819 (1966)] and by **G. B. Field** and **J. L. Hitchcock** [ibid. 817].

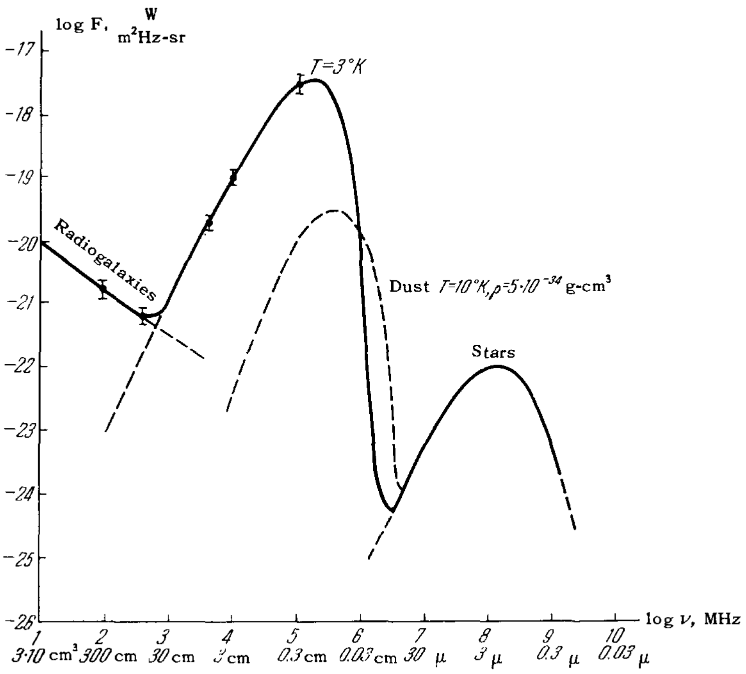

FIG. 3

of the summary radiation of the stars and dust, calculated by I. D. Novikov and A. G. Doroshkevich.

Sheykshaft reported at a symposium in Byurakan on 10 May 1966 measurements made in Cambridge at 20 cm wavelength. These measurements also gave a background intensity (after subtracting the source contribution) corresponding to 3°K.

2. THE HOT MODEL OF THE UNIVERSE

The hot model of the universe was proposed by Gamow in connection with the question of the origin of elements [G. **Gamow**, Phys. Rev. 70, 572 (1946); **74**, 505 (1948); Revs. Modern Phys. **21**, 367 (1949)] and developed in the following papers: **R. A. Alpher, H. A. Bethe,*** and **G. Gamow**, Phys. Rev. **73**, 803 (1948); **R. A. Alpher, J. W. Follin,** and **R. C. Herman**, Phys. Rev. **92**, 1347 (1953). Gamow notes that during the earlier stage the density of the quanta was many times larger than the density of "matter."† In a 1956 review [Vistas in Astron. **2**, 1726 (1956)] it is indicated that the energy density at the present time presumably corresponds to 6°K. In connection with the question of chemical composition (see Appendix 6), Zel'dovich proposed the cold model [JETP **43**, 1561 (1961), Soviet Phys. JETP **16**, 1102 (1962); Voprosy kosmogonii [Problems of Cosmogony] **9**, 232 (1963)].

A discussion and comparison of the cold and hot model (with deductions in favor of the cold one) are given also in the following reviews: **Ya. B. Zel'dovich**, UFN **80**, 357 (1963), Soviet Phys. Uspekhi **6**, 475 (1965); Advances in Astron. and Astrophys. **3**, 241 (1965). For observational choice between the cold and the hot model, **A. G. Doroshkevich** and **I. D. Novikov** [DAN SSSR **154**, 745 (1964)] considered the spectrum of radiation from stars and radio sources. In the hot model, the spectrum of Planck radiation is superimposed on this spectrum. We present a diagram from that paper (Fig. 4)*.

It is concluded in that paper that measurements in the wavelength interval from 0.06 to 30 cm make it possible to assess experimentally the correctness of the hot or the cold model. As seen from Fig. 3, at $\lambda = 0.1$ cm the intensity of the Planck radiation at its maximum is approximately 3×10^5 times larger than the intensity of the radiation of the stars and radio sources.

We present some energy estimates. A temperature of 3°K corresponds to a total relict radiation density of 0.6×10^{-12} erg/cm^3. The total density of radiation of the stars and radio sources, averaged over all of space, is of the order of 10^{-14} erg/cm^3, i.e., smaller by a factor of 60. Assuming the density of matter to be 10^{-29} g/cm^3, we find that the relict radiation at the present time amounts to 6×10^{16} erg/g. This in itself is small compared with the nuclear energy of hydrogen combustion ($\sim 6.5 \times 10^{18}$ erg/g to He^4 and 8×10^{18} erg/g to Fe^{56}). However, there is no mechanism whereby the

---

*Bethe did not take part in the work. His name was included by Gamow to obtain correspondence with the Greek letters: alpha, beta, and gamma, and the work was called "the $\alpha\beta\gamma$ theory."

†We note some predictions concerning the evolution of the universe at the First Soviet Gravitational Conference: V. A. Belokon', Kinematics, Thermodynamics, and Gravitons in Cosmology; Ya. A. Smorodinskiĭ, The Neutrino and the Evolution of the Universe (Moscow State University Press, 1961).

*The Planck spectrum of the relict radiation in Fig. 4. corresponds to 1°K. Of course, the authors could not know then the exact value of the temperature of the relict radiation.

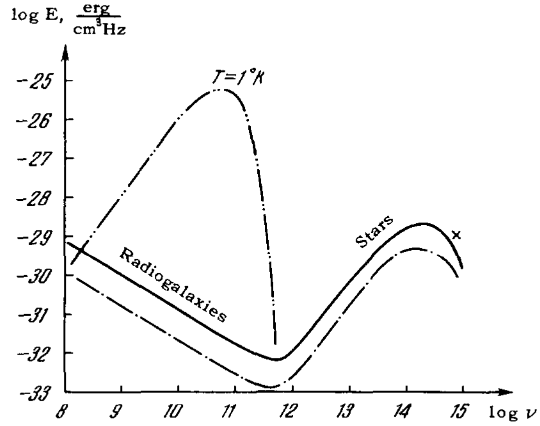

FIG. 4

energy of the nuclear reaction could be transformed into a spectrum corresponding to 3°K. On the other hand, if we assume that the energy was released in stars having surface temperatures of at least 3000°K, and the drop to 3°K occurred during the course of the expansion, then the energy required is already 1000 times as large—6×10^{19} erg/g, which is 10 times more than the entire reserve of nuclear energy. Therefore trivial explanations of the observed radiation have little likelihood. Of decisive significance will be the measurements over the entire spectrum and the determination of the isotropy of the radiation. Hoyle and Tayor [Nature **203**, 1108 (1964)] argued in favor of the hot model on the basis of the chemical composition of gas nebulas (see below). **R. Dicke, P. I. E. Pebbles, P. G. Roll,** and **D. T. Wilkinson,** [Astrophys J. **142**, 414 (1965)] describe how, starting from the hot model, they proceeded to measure the relict thermal radiation at 3 cm wavelength, but even before they started the measurements, they learned of the results of Penzias and Wilson (cited in Appendix I before) and interpreted them from the cosmological point of view. The communications of these two groups are contained in letters published in a single issue of the journal, alongside. These two papers were of decisive significance in the recognition of the correctness of the hot model.

At the symposium in Byurakan, Burbidge raised the question of whether re-radiation by the dust can produce a thermal spectrum. At the present time the dust content is patently inadequate. Its optical thickness at the characteristic length $c/H = 10^{28}$ cm for radiation with wavelength of the order of several centimeters does not exceed 10^{-3}. We can imagine, however, that the density was sufficient in the past, at the instant $t = 10^6$ years. This calls for release of a considerable fraction of the nuclear energy (several times 10%). On the whole, this hypothesis is not very likely [see **Ya. B. Zel'dovich** and **I. D. Novikov**, Astron. zh. **43**, No. 6 (1966), Soviet Astronomy AJ **9**, in press].

Of decisive significance in the choice between the hot model and any variant involving later production of quanta would be a neutrino experiment: relict thermal neutrinos with T = 2° are characteristic of the hot model only.

3. DIMENSIONLESS ENTROPY

In classical theory the entropy is defined in differential form, $dS = dQ/T$, apart from a constant quantity; it has the dimension of cal/deg. Quantum theory defines the absolute value of entropy as $S = k \ln W$, where k is Boltzmann's constant and W is the probability of the state. The use of special thermal units is only a convention; in the rational system T is measured in energy units and $k = 1$. In this system we have for the entropy per nucleon $S_1 = \ln W_1$. If the system consists of nucleons only, then W_1 is the average number of cells in phase volume per nucleon, i.e., the ratio of the number of quantum levels Γ, at which the nucleons are located, to the number of nucleons:

In a nonrelativistic gas

$$\Gamma = \frac{\bar{p}^3 V}{(2\pi\hbar)^3},$$

where V is the volume

$$\bar{p} = (3kTm)^{1/2},$$

$$W_1 = \frac{\Gamma}{N} = \frac{m^{3/2}T^{3/2}}{n} \frac{(3k)^{3/2}}{(2\pi\hbar)^3} = 3\cdot 10^{55} \frac{m^{3/2}T^{3/2}}{n};$$

here N is the total number of particles (nucleons) in the volume V and n is their density, $n = N/V$.

For ionized hydrogen of density $\rho = 10^{-29}$ g/cm^3 ($n = 6 \times 10^{-6}$ cm^{-3}) we obtain at $T = 10^6$ °K (small electron contribution)

$$W_1 = 10^{34}, \qquad S_1 = \ln W_1 = 78.$$

In a star

$$\varrho = 1 \text{ g/cm}^3 \quad (n = 6\cdot 10^{23}\text{ cm}^{-3}), \quad T = 10^8\ ^\circ\text{K},$$
$$W_1 = 10^8, \qquad S_1 = \ln W_1 = 18.$$

Let us take the radiation into account; its energy per unit volume and the corresponding entropy are

$$\varepsilon_\gamma = aT^4, \qquad s_\gamma = \frac{4}{3} aT^3,$$

where

$$a = \frac{\pi^2 k^4}{15(\hbar c)^3} = 7.57\cdot 10^{-15} \text{ erg-cm}^{-3}/\text{deg}$$

The dimensionless entropy per nucleon is

$$S_1 = \frac{s_\gamma}{kn} = 72.5 \frac{T^3}{n}.$$

For comparison, the number of quanta per unit volume is

$$n_\gamma = \frac{8\pi}{(2\pi\hbar)^3} \int_0^\infty \frac{p^2\, dp}{e^{pc/kT} - 1} \approx \frac{\varepsilon_\gamma}{\overline{h\nu}} = \frac{\varepsilon_\gamma}{2.7kT},$$

$$\int_0^\infty \frac{p^2\, dp}{e^{pc/kT} - 1} = \left(\frac{kT}{c}\right)^3 \cdot 2.404,$$

where p is the momentum of the quantum; the number of quanta per nucleon is

$$N_\gamma = \frac{n_\gamma}{n} \approx 20.4\,\frac{T^3}{n} = \frac{S_1}{3.55}\,.$$

A similar calculation for the equilibrium spectrum of the neutrinos and antineutrinos (with chemical potential $\mu = 0$ and with account of the helicity of the neutrino) yields

$$\varepsilon_\nu = \frac{7}{8}\,aT^4, \qquad s_\nu = \frac{7}{6}\,aT^3, \qquad S_2 = \frac{s_\nu}{kn} = \frac{64}{n}\,T^3,$$

$$n_{\nu,\bar\nu} = \frac{4\pi}{(2\pi\hbar)^3}\int_0^\infty \frac{p^2\,dp}{e^{cp/kT}+1} \approx \frac{\varepsilon_\nu}{6.3kT}\,.$$

$$\int_0^\infty \frac{p^2\,dp}{e^{cp/kT}+1} = \left(\frac{kT}{c}\right)^3\cdot 1.83,$$

$$N_{\nu,\bar\nu} = \frac{n_\nu + n_{\bar\nu}}{n} = 15.0\,\frac{T^3}{n} = \frac{S_2}{4.26}\,.$$

From this, rounding off ($3.55 \cong 4$, $4.26 \cong 4$), we get

$$S = S_1 + S_2 \approx 4\,(N_\gamma + N_\nu + N_{\bar\nu}).$$

In this formula everything is clear apart from the factor 4. The thermodynamic probability W_1 is the product of the probabilities (the number of states per particle) of each of the particles (quanta, neutrinos) ascribed to the nucleon.

The average energy of these particles in $h\bar\nu \approx 4kT$; this means that the fraction of occupation of the phase space by the particles is

$$\frac{1}{e^{h\bar\nu/kT}+1} \approx \frac{1}{e^{h\nu/kT}-1} \approx e^{-h\bar\nu/kT} \approx e^{-4},$$

i.e., on the average there are e^4 states per particle. Consequently,

$$W_1 = (e^4)^{N_\gamma + N_\nu + N_{\bar\nu}},$$

from which follows the expression for the entropy.

Without taking the neutrino into account, putting $\bar\rho = 10^{-29}$ g/cm³ ($n = 6 \times 10^{-6}$ cm^{-3}), T = 3°K, we get $S_1 = 72.5\,T^3/n = 3 \times 10^8$. Allowance for the neutrinos may approximately double S_1. The obtained value of S_1 is much larger than the intrinsic entropy of the nucleon and of the electron, which does not reach 100 units. For ideas concerning the origin of the highest entropy and "near symmetry" of the charge, in connection with passage through the singularity $t = 0$ and the preceding period $t < 0$, see **Ya. B. Zel'dovich** and **I. D. Novikov**, JETP Letters 4, 117 (1966), transl. p. 80.

4. MECHANICS OF THE HOT MODEL

In the hot model, the nucleon density during the early stage is low compared with the density of the quanta and other particles with zero rest mass. There are many electron-positron pairs only when the temperature exceeds the energy of the electron rest mass, $T > 6 \times 10^9$ °K ($kT > mc^2$). If this inequality is satisfied, then it is possible to regard both the electrons and the positrons as relativistic particles. The same holds also for heavy particles at an accordingly even higher temperature. Consequently, the following relation is satisfied with high accuracy

$$p = \frac{1}{3}\,\varepsilon = \frac{1}{3}\,\rho c^2 \tag{4.1}$$

(p = pressure, ϵ = energy density, ρ = matter density in g/cm³).

The equation of the mechanics of expansion in the homogeneous and isotropic model in Friedmann's theory, i.e., in accordance with general relativity, is

$$\frac{d^2r}{dt^2} = -\frac{G}{r^2}\,\frac{4\pi r^3}{3}\left(\rho + \frac{3p}{c^2}\right), \tag{4.2}$$

where r is the radius of the world or the distance (proportional to the radius of the world) between a pair of arbitrarily chosen points moving in accord with the Hubble expansion. The equation is linear in r, and consequently multiplication of r by a constant does not change the equation.

The energy equation is

$$\frac{d}{dt}\,(\varepsilon r^3) = -p\,\frac{dr^3}{dt}\,, \tag{4.3}$$

from which, with allowance for (4.1), we get

$$\varepsilon r^4 = \text{const}, \qquad \rho r^4 = B. \tag{4.4}$$

From (4.2) and (4.3) it follows exactly that (even if (4.1) is not satisfied)

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{1}{r}\,G\,\frac{4\pi}{3}\,r^3\rho + A. \tag{4.5}$$

Taking (4.4) into account and neglecting the constant A during the early stage, we obtain

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 = \frac{4\pi GB}{3r^2}\,, \qquad r = \left(\frac{32\pi}{3}\,GBt^2\right)^{1/4},$$

$$\rho = \frac{3}{32\pi Gt^2} = \frac{4.5\cdot 10^5}{t^2}\ \text{g/cm}^3, \quad \varepsilon = \rho c^2 = \frac{4\cdot 10^{26}}{t^2}\ \text{erg/cm}^3. \tag{4.6}$$

If the gas were to consist only of electromagnetic-field quanta, we would have $\epsilon_\gamma = aT^4$. Taking into account the fact that there are different species of particles in equilibrium, giving the coefficient v, we write

$$\varepsilon = \upsilon a T^4 = \rho c^2,$$

where $a = \pi^2k^4/15\hbar^3c^3$, the dimensionless coefficient is $v > 1$;

$$(kT)^4 = \frac{1}{\upsilon}\,\frac{45}{32\pi^3}\,\frac{\hbar^3c^5}{Gt^2}\,,$$

$$T\,(\text{deg}) = t^{-1/2}\upsilon^{-1/4}\cdot 1{,}5\cdot 10^{10}, \quad t\,(\text{sec}) = \frac{2{,}25\cdot 10^{20}}{T^2\,(\upsilon)^{1/2}}\,,$$

$$T\,(\text{MeV}) = t^{-1/2}\upsilon^{-1/4}\cdot 1.3, \qquad t\,(\text{sec}) = \frac{1.7}{T^2\,(\upsilon)^{1/2}}\,.$$

Finally, for the density n of all species of particles we obtain, taking into consideration the fact that the average particle energy is $\approx 4kT \sim \hbar^{3/4}c^{5/4}G^{-1/4}t^{-1/2}\nu^{-1/4}$,

$$n = \frac{\varepsilon}{4kT} = \frac{\nu a T^4}{4kT} \approx \nu^{1/4}t^{-3/2}\,\frac{a\left(\frac{45}{32\pi^3}\,\frac{\hbar^3c^5}{G}\right)^{3/4}}{k^4}$$

$$= t^{-3/2}\nu^{1/4}\cdot 4{,}67\cdot 10^{31}.$$

5. ESTABLISHMENT OF EQUILIBRIUM

There is no doubt that at high temperatures the number of e^+e^- pairs does not differ from the equilibrium value. By way of an example we consider the instant when T = 1 MeV (1.16×10^{10} °K), t = 1 sec, $n_{e^+} \approx n_{e^-} = 10^{30}\ \text{cm}^{-3}$. The annihilation cross section is of the order of $10^{-24}\ \text{cm}^2$ and the particle velocity is of the order of the speed of light; consequently the time of establishment of the equilibrium is of the order of

$$\tau = \frac{1}{\sigma n c} = 10^{-17}\ \text{sec}.$$

Thus, τ is negligibly small compared with t, and complete equilibrium of $e^+e^- \rightleftharpoons \gamma$ is ensured. The situation is exactly the same with the establishment of equilibrium of the muon pairs $\mu^+\mu^-$, and also the mesons and baryons of both species at appropriately higher temperatures.

The question of establishment of equilibrium should be considered in only two cases:

a) Weak interaction, when the cross section of the reaction is many times smaller than $10^{-24}\ \text{cm}^2$; this includes the question of the neutrinos and gravitons.

b) For particles with non-zero rest mass at low temperature, when the equilibrium concentration becomes small; this includes the question of the remaining antiparticles—antiprotons and positrons—and the question of the possible concentration of quarks. We shall consider these questions in turn.

The creation of electronic neutrinos and antineutrinos proceeds essentially via the reaction $e^- + e^+ = \nu_e + \bar{\nu}_e$. Its cross section for relativistic electrons and positrons is

$$\sigma_\nu \approx \frac{g^2E^2}{\hbar^4c^4},$$

where g is the weak-interaction constant, $\sim 10^{-49}$ erg-cm^3. We substitute kT for E in order of magnitude, and set up, using the expression for T, the condition

$$t \approx \tau = \frac{1}{\sigma_\nu n c} \sim \frac{G^{5/4}\hbar^{13/4}}{g^2c^{1/4}}\,t^{5/2}, \qquad t = 10\ \text{sec}.$$

This estimate is quite crude and is presented only to show how the constants G, g, and others enter into the sought expression.

For a more accurate estimate we shall use the expression for the rate of transfer of energy from e^+e^- to $\nu\bar{\nu}$, as calculated by Chiu (in the next two formulas T is in °K):

$$W = 4{,}6\cdot 10^{-66}T^9\ \text{erg/cm}^3\text{sec}, \qquad T > 3\cdot 10^9,$$

and compare it with the equilibrium energy density of ν and $\bar{\nu}$:

$$\varepsilon = 6{,}8\cdot 10^{-15}T^4\ \text{erg/cm}^3,$$

whence the time for establishment of equilibrium density is

$$\tau = \frac{\varepsilon}{W} = 1{,}5\cdot 10^{54}/T^5 = \frac{7}{T^5(\text{MeV})}\,(\text{sec}).$$

We use the connection between T and t:

$$t = \frac{1{,}7}{T^2\,(\text{MeV})\,\sqrt{\nu}}.$$

Equating t = τ, we obtain (for ν = 4.5)

$$T = 2\ \text{MeV} = 4m_ec^2, \qquad t = 0.2\ \text{sec}.$$

It is important that the equilibrium between $\nu\bar{\nu}$ and e^+e^- is maintained only during that period when $T > m_ec^2$.

It is known that there are two types of neutrinos, electronic ν_e and muonic ν_μ. The foregoing calculation pertained to electronic neutrinos.

Muonic neutrinos are created in the reactions

$$\mu^+ + \mu^- = \nu_\mu + \bar{\nu}_\mu,$$
$$\mu^+ = e^+ + \bar{\nu}_\mu + \nu_e, \qquad \mu^- = e^- + \nu_\mu + \bar{\nu}_e.$$

The lifetime of the muon is 2×10^{-6} sec. It follows therefore that the time of establishment of equilibrium is approximately 2×10^{-6} sec when $T > m_\mu c^2$ and there are many muons; on the other hand, if $T < m_\mu c^2$ and the number of muons decreases, the time of establishment of equilibrium decreases accordingly:

$$\tau = 2\cdot 10^{-6}e^{m_\mu c^2/T} \approx 2\cdot 10^{-6}e^{107/T}$$

(T is in MeV). Comparing with the expression for T(t), we obtain from the condition t = τ, T = 12 MeV, t = 0.01 sec, and $\exp(-m_\mu c^2/T) \sim 10^{-4}$.

Thus, a period exists when there is complete thermodynamic equilibrium of all types of particles, including both species of neutrinos and antineutrinos. With this, $m_\mu c^2 > T_1 > m_ec^2$, meaning that the ratios of the equilibrium energy densities are (at the instant $T = T_1$)

$$\gamma : e^\pm : \nu_e, \bar{\nu}_e : \nu_\mu, \bar{\nu}_\mu = \varepsilon_1 : \frac{7}{4}\varepsilon_1 : \frac{7}{8}\varepsilon_1 : \frac{7}{8}\varepsilon_1,$$
$$\varepsilon_1 = aT_1^4.$$

These ratios can be found, for example, in "Statistical Physics" by Landau and Lifshitz.

For brevity we write in lieu of ϵ_γ simply γ... There are practically no other particles (we shall discuss gravitons later). The baryon density at that instant is denoted by ρ_1. Before this period, particles of all types are also in equilibrium; at higher temperatures, strongly interacting mesons and baryon-antibaryon pairs are added. The equilibrium between the strongly interacting particles, the electromagnetic quanta, and the charged leptons is established practically instantaneously. Consequently, above T_1 there is total equilibrium, and below T_1 the neutrinos (both ν_e and ν_μ) expand without interacting with other types of particles.

It is easy to find the specific entropy at the instant T_1 separately for γ and pairs, and separately for neutrinos:

$$S_\gamma + S_{e+,\, e-} = \frac{4}{3}\,\frac{aT_1^3 + \frac{7}{4}\,aT_1^3}{\varrho_1}\,,$$

$$S_{\nu_e,\, \bar{\nu}_e} = S_{\nu_\mu \bar{\nu}_\mu} = \frac{4}{3}\,\frac{7}{8}\,\frac{aT_1^3}{\varrho_1}\,.$$

During the course of expansion following t_1, when $T < T_1$, the parts of the entropy dependent on the neutrinos and dependent on γ and $e^\pm$ are individually conserved. However, γ and $e^\pm$ are in equilibrium with each other and exchange energy and entropy.

We denote by the subscript 0 the situation at the present time. We obtain, taking into account the fact that there are already no more $e^\pm$ pairs at low temperature

$$S_0(\gamma) = S_1(\gamma) + S_1(e^+e^-) = \frac{4}{3}\,\frac{a(T_{0\gamma})^3}{\varrho_0} = \frac{4}{3}\,\frac{11}{4}\,\frac{aT_1^3}{\varrho_1}\,,$$

$$S_0(\nu_e\bar{\nu}_e) = S_0(\nu_\mu\bar{\nu}_\mu) = \frac{4}{3}\,\frac{7}{8}\,\frac{a(T_{0\nu})^3}{\varrho_0}\,.$$

Here $T_{0\gamma}$ is the present-day quantum temperature, 3°K, and ρ_0 is by assumption 10^{-29} g/cm^3. Regardless of the numerical value of ρ_0, it follows from the foregoing calculations that

$$T_{0\nu} = \sqrt[3]{\frac{4}{11}}\,T_{0\gamma} = 0.7 \cdot 3^\circ\ \mathrm{K} = 2^\circ\ \mathrm{K},$$

$$\varepsilon_0(\nu_e\bar{\nu}_e) = \varepsilon_0(\nu_\mu\bar{\nu}_\mu) = \frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\varepsilon_0(\gamma) = 0{,}35\varepsilon_0(\gamma).$$

The question of the density of the neutrino in the hot model was considered briefly in an article by Zel'dovich [Atomnaya énergiya **14**, 92 (1963)] and in detail in the already-mentioned reviews; see also the cited paper by R. Dicke and his co-workers*.

Experimental observation of the Planck spectrum of neutrinos would be of tremendous interest. If the ratio $T_{0\nu} : T_{0\gamma}$ were confirmed, this would yield proof of the correctness of our concepts concerning the earliest stages of the expansion ($T \sim 10^{10}$ deg, $\rho_r \approx 10^5$ g/cm^3). In principle the presence of cosmological neutrino and antineutrino backgrounds changes the form of the spectrum of the β-decay electrons near the maximum energy [**S. Weinberg**, Phys. Rev. **128**, 1457 (1962)], which, as commented by A. D. Sakharov, could be used to detect relict neutrinos. However, as noted by B. M. Pontecorvo, such a change in the spectrum also implies a small rest mass for the neutrino. Experiment yields $m_\gamma < 100$ eV, the cosmic background contained neutrinos with energies of the order of 10^{-2} eV. This means that for the observation the experimental accuracy must be increased by a factor $\sim 10^5$!

We have seen above that relatively early detachment (cessation of interaction) of the neutrinos from the other particles decreases their energy, since during the early stages the energy was distributed among a larger number of particle species.

This reasoning imparts high stability to the entire theoretical picture: If there exist some unknown types of particles which interact even more weakly with the known ones than neutrino does, then they are detached even earlier, and consequently make a much lower contribution to the energy density than the neutrino*.

Obviously, unknown strongly-interacting particles could conceivably exist only in the region of large masses. Otherwise they would have already been discovered! A particular case of unknown hypothetical particles are quarks.

Such heavy strongly-interacting particles change the relation between T and ϵ and $T > mc^2$, but once the instant when $T = T_1$ is reached, they die out and exert no influence from then on.

Particles capable of spontaneous decay vanish exponentially as a function of time after the temperature is reduced (i.e., when their production ceases), and observation of relict particles of this kind is impossible.

The situation is different with particles which are stable in vacuum and which vanish only by interaction with other particles. These include antiprotons $\bar{p}$, positrons e^+, some species of quarks (if they exist), and antiparticles of this species.

Assuming homogeneity of the initial distribution of matter, there exists everywhere an excess of nucleons and electrons; under these conditions the residual concentrations of $\bar{p}$ and e^+ are negligible, much lower than those generated at the present time in cosmic rays.

A theory exists, according to which the world has on the average charge symmetry, i.e., it contains an equal amount of baryons and antibaryons. Annihilation does not occur because they are spatially separated. Moreover, the hypothesis has been advanced that during the early stage the baryons and antibaryons were situated together and were subsequently separated by the combined action of gravitational and magnetic forces. [**H. Alfven**, Astron. zh. **42**, 873 (1965), and H. Alfven and O. Klein, Ark. Phys. **23**, 187 (1962)]. This theory is unsatisfactory, since it is necessary to start the analysis with a rarefied low-density plasma, and not with $\rho = \infty$. Adhering henceforth to the conservative view concerning the excess of baryons and

---

*See also the paper by Peebles, referred to in the footnote on p. 613.

*According to a remark by Gershteĭn and Zel'dovĭch (JETP Letters 4, 174 (1966), transl. p. 120) it follows from cosmological considerations that the rest mass of muonic neutrinos is smaller than 5 eV/c^2. All that has been proved experimentally is that this mass is smaller than 2 MeV/c^2).

the absence of separation, let us turn to quarks.

Quarks can vanish only by collisions with other quarks ($q + q = N + \bar{q}$) or with antiquarks ($q + \bar{q} = \pi, \rho, \ldots$). Therefore when their concentration is decreased, the rate of quark loss decreases. This question was analyzed quantitatively in the following papers: **Ya. B. Zel'dovich, L. B. Okun', and S. B. Pikel'ner**, UFN **87**, 113 (1965), Soviet Phys. Uspekhi **8**, 702 (1966); **L. B. Okun, S. B. Pikelner, and Ya. B. Zeldovic**, Phys. Letts. **17**, 164 (1965). To the analysis given there we can only add that the article was written prior to observations which have confirmed the hot model. At the present time, out of the range 10^{-9}—10^{-18} indicated in these articles (10^{-18} pertained to the cold model), one must choose a figure close to the largest. When considering burnout in the stars, it must be borne in mind that if the p-quark with charge $+2e/3$ is stable, then the $\bar{p}$-quark with charge $-2e/3$ is also stable, and should by virtue of Coulomb interaction already stick to the heavy nuclei. Strong interaction can also play a role in the adhesion. On the whole, allowance for adhesion decreases the degree of burnout of the quarks in stars. At present, the most probable ratio is $\sim 10^{-10}$—10^{-13} quarks per nucleon.

6. NUCLEAR REACTIONS IN THE HOT MODEL

In addition to the already cited papers by Gamow, Alfven, and Herman and the papers of Fermi and Turkevich cited there but unpublished, we call attention to a paper by C. Hayashi [Progr. Theor. Phys. (Japan) **5**, 224 (1950)] dealing with the time variation of the ratio between protons and neutrons.

At high temperatures the process is much faster than spontaneous decay of neutrons ($n \to p + e^- + \tilde{\nu}$, $\tau = 11$ min), owing to the induced transformation*

$$\nu + n \rightleftarrows p + e^-, \quad e^+ + n \rightleftarrows p + \tilde{\nu}. \tag{6.1}$$

At temperatures higher than $T_2 = 1$ MeV, there is thermodynamic equilibrium:

$$\frac{n}{p} = e^{-\frac{\Delta mc^2}{kT}},$$

where Δm is the mass difference, equal to 1.3 MeV.

It was necessary to revise the calculations of Hayashi [the calculations are given in Advances Astron. and Astrophys. **3**, 241 (1965)], with account taken of the Pauli principle: for example, the rate of the process $e^- + p \to n + \nu$ depends not only on the number of electrons having the required energy, but also on the number of free places in the neutrino distribution. Inasmuch as the overall concentration of the baryons is negligible compared with the concentration of $e^{\pm}$, ν, and $\tilde{\nu}$, the n/p ratio depends only on the time and does not depend on the baryon concentration itself, i.e., does not depend on the specific entropy. We obtain finally $n = 0.165(n + p)$ and $p = 0.835(n + p)$.

At higher temperatures, the reactions (6.1) practically stop, and the spontaneous decay of the neutrons competes with the nuclear reactions, which start with neutron capture $p + n = D + \gamma$ and end with formation of He^4. The process competing with the neutron decay calls for double collisions, and therefore the higher the density of baryons at a given temperature, i.e., the less the specific entropy, the more strongly is this process represented. At low entropy, all the neutrons are transformed into He^4 and consequently we get 33% He^4 and 67% hydrogen. The approximate dependence of the chemical composition of the universe on the time is shown in Fig. 5.

A measure of the entropy is the ratio T^3/ρ_m. We present results of calculations by Yu. N. Smirnov [Astron. zh. **41**, 1084 (1964), Soviet Astronomy AJ **8**, (1964)] of the final composition of matter in the universe in per cent, for three values of the entropy, corrected with allowance for the statements made above (table).

$\frac{T^3 \rho_m}{10^{39}}$	H	He^4	D	T	He^3
7000	92	5	3	0,01	0,03
70	70	30	0,1	$\sim 2 \cdot 10^{-5}$	$\sim 2 \cdot 10^{-6}$
0,7	67	33	$0,3 \cdot 10^{-3}$	$\sim 10^{-7}$	$\sim 2 \cdot 10^{-7}$

We recall that by assuming $T = 3°$, and $\rho_m = 10^{-29}$, we obtain $T^3/\rho_m = 3 \times 10^{30}$, and at the extreme value of the density $\rho_m = 3 \times 10^{-31}$ we obtain $T^3/\rho_m = 100 \times 10^{30}$. Consequently, the expected content of helium in the prestellar matter ranges from 28 to 32%.

The question of the actual amount of helium in different objects was discussed from the point of view of the hot model in a remark by Hoyle and Tayler (Nature **203**, 1108 (1964)). They assume that good agreement exists with the expected value of 30%. It was assumed earlier that there is much less helium. Thus, for example, in Bethe and Salpeter's book "Quantum Mechanics of Atoms with One and Two Electrons" (Chap. IV, Sec. 3, Par. 74, B, p. 317 of the English edition of 1957)

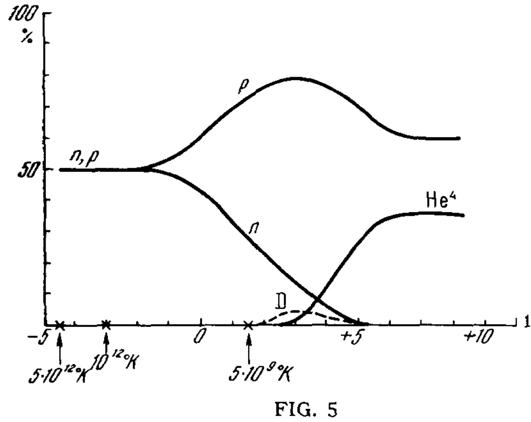

FIG. 5

---

*We are considering electronic neutrinos ν_e throughout, and omit the subscript e.

it is stated, without indicating the source, that typical stellar matter consists principally of hydrogen plus approximately 10% helium (by mass).

Several authors believe even now that the content of the primordial helium is less than 30% (of the order of 13--20%) and regard this as a difficulty of the proposed theory of the hot model of the universe (see the remarks by Woolf and Peebles in Paige's preprint; see also the cited paper by Dicke and co-workers).

The determination of the content of helium in old stars is made especially difficult precisely because *the old stars have a low surface temperature.*

The question of methods of observing helium in intergalactic gas is considered by R. A. Syunyaev (Astron. zh. No. 5 (1966)). These methods are quite difficult. In principle it would be quite interesting to measure the content of primordial deuterium and He^3 in prestellar matter; their content depends on the entropy in that region where the amount of He^4 reaches saturation. But this is an even more difficult problem. Furthermore, nuclear reactions induced by cosmic rays easily mask the content of relict He^3, and perhaps also deuterium.*

7. GRAVITATIONAL UNITS, CREATION OF GRAVITONS

At present there is no consistent quantum theory of gravitation. The quantum theories of the electromagnetic field, electrons, and other fields and particles are constructed in a specified space and time; in accordance with special relativity theory, this is the Minkowski space-time, i.e., the Lorentz transformations entangle time with the spatial coordinates. So long, however, as gravitation is not taken into account, there is no influence of the particles or fields on the space-time itself; in particular, the coordinates x and t can be regarded as numbers and not as operators.

In gravitation theory the metric of space itself becomes dependent on the presence of the fields and particles. The theory of weak gravitational fields can be regarded as the theory of a tensor field of small additions h to the metric coefficients g^0 of the Minkowski space. We can then apply to these additions, which are considered in flat space, the usual methods of the theory; this yields, in particular, a theory of gravitational waves, and quantization of this theory leads to the concept of the gravitational-wave quantum—the graviton. Just as in the case of the Coulomb field, quantization does not influence at all the static gravitational interaction.

The statement "gravitation is effected by graviton exchange" means nothing and can only confuse the nonspecialist. Introduction of gravitons does not change the picture of gravitational interaction of macroscopic bodies at all.

A description of gravitational radiation from double stars and similar macroscopic bodies with the aid of the concept of gravitons is unnecessary in the same sense that a quantum description of the operation of the radio station is unnecessary: when there are many gravitons, they all have the same frequency and are coherent in phase, so that we are justified in speaking of a classical field.

We deal with individual gravitons at the atom and particle level*. The probability of graviton emission is quite small, not only compared with the emission of an electromagnetic quantum, but compared with the emission of a $\nu\bar{\nu}$ pair. This was noted in a remark to the first article on the emission of $\nu\bar{\nu}$ pairs, by G. M. Gandel'man and V. S. Pinaev [JETP **37**, 1072 (1959), Soviet Phys. JETP **10**, 764 (1960)]. Therefore the emission of gravitons from stars is negligibly small. We note that the emission of low-frequency gravitational waves as a result of macroscopic motions can reach values of the order of the optical luminosity and carry away an energy of the order of several per cent of mc^2 [see **V. B. Braginskiĭ**, UFN **86**, 433 (1965), Soviet Phys. Uspekhi **8**, 513 (1966); **Ya. B. Zel'dovich** and **I. D. Novikov**, DAN SSSR **155**, 1033 (1964), Soviet Phys. Doklady **9**, 246 (1964)].

What is the situation with emission of gravitons during the earliest stage of the hot universe? A calculation similar to that presented above for the neutrino shows that establishment of equilibrium calls for a density that lies on the borderline of applicability of non-quantum gravitation theory to the mechanics of the universe.

We start with defining this borderline. As is well known, we can construct a quantity of any dimensionality from the world constants G (6.7×10^{-8} cm^3/g-sec^2), $\hbar$ (1.05×10^{-27} g-cm^2/sec), and c (3×10^{10} cm/sec). In particular [see, for example, **A. D. Sakharov**, JETP **49**, 345 (1965), Soviet Phys. JETP **22**, 241 (1966)] we

---

*After this article was written, we received the paper by **P. J. E. Peebles**, [Phys. Rev. Letts. 16, 410 (1966)], in which Smirnov's calculations are repeated and the importance of measuring the primordial content of helium is again indicated. In a preprint by Sargent and Cyril it is indicated that the content of helium on the surface of stars of the spherical component of the galaxy (halo) is 100 times smaller than on the surface of the stars of the disc (type I population). The data pertain to stars of the main sequence. Tayler and Hawking note that in non-isotropic expansion during the early stage, the content of helium can be either larger or smaller than 30% at the given entropy (Nature 209 (5030), 1278 (1966)). The anisotropy of the expansion can be connected with the primordial magnetic field [see, for example, **Ya. B. Zel'dovich**, JETP 48, 986 (1965), Soviet Phys. JETP 21, 656 (1965)].

*We note rather interesting papers not pertaining directly to our topic, dealing with the possibility of coherent emission and detection of gravitational waves: **V. I. Pustovoit** and **M. E. Gertsenshtein**, JETP 42, 163 (1962), Soviet Phys. JETP 15, 116 (1962); **U. Kh. Kopvilem** and **V. R. Nagibarov**, JETP Letters 2, 529 (1965), transl. p. 329.

obtain the length l_g, the mass m_g, and the unit of time t_g:

$$l_g=\sqrt{\frac{G\hbar}{c^3}}=1{,}7\cdot 10^{-33}\text{ cm},\qquad m_g=\sqrt{\frac{\hbar c}{G}}=2\cdot 10^{-5}\text{ g},$$

$$t_g=\frac{l_g}{c}=0{,}6\cdot 10^{-43}\text{ sec}.$$

From these quantities we readily obtain also the characteristic density

$$\varrho_g=\frac{m_g}{l_g^3}=4\cdot 10^{93}\text{ g/cm}^3$$

In classical theory of expansion, this density is attained at the instant when $t \approx t_g$. In fact, the general formula is

$$\varrho=\frac{3}{32\pi G t^2},$$

whence

$$t_1=6.5\sqrt{\frac{\varrho_g}{G}}=6.5 t_g.$$

The existing theory is not applicable to the earlier period. If a transition from contraction to expansion is possible, then it can be assumed that the maximum attainable density is of the order of ρ_g.

The convenience of using "gravitational" units for the measurement of length, mass, and all other quantities lies in the fact that in all calculations we can put in the formulas $G = \hbar = c = 1$.

Thus, let us consider the creation of gravitons, using these units and starting the integration from $\rho = 1$ and $t = 6.5$. We recall that in these units the proton mass is negligibly small: $m_p = 10^{-19}$.

The expression for the density of electromagnetic radiation is

$$\varrho_\gamma=\frac{\pi^2}{15}T^4=0{,}65T^4$$

(the density of the mass and energy densities coincide, since $c = 1$). Obviously, T is here in energy units; one energy unit corresponds to $kT_g = m_g c^2$; $T_g = 1.3 \times 10^{32}$ deg.*

The equilibrium density of gravitons is the same as that of quanta. Owing to the presence of particles of different species, which under these conditions are all ultrarelativistic†, we get

$$\varrho=v\cdot 0{,}65T^4=\frac{3}{G\cdot 32\pi t^2}=\alpha t^{-2},\qquad \varrho_i=\frac{\varrho}{v},\qquad n_i=\frac{\varrho}{\eta\cdot 4T},$$

where v can be* of the order of 20 or 50. The index i pertains to the individual species of particles.

Yu. S. Vladimirov [JETP **45**, 251 (1963), Soviet Phys. JETP **18**, 176 (1964)]† considers the creation of gravitons by annihilation. Obviously, at high temperatures this is just the process which should predominate over bremsstrahlung of gravitons, just as the process $e^- + e^+ = \nu_e + \bar{\nu}_e$ predominates for neutrinos. However, the graviton is not charged and therefore not only pairs but also single gravitons can be created:

$$A+\bar{A}=2g\quad\text{and}\quad A+\bar{A}=g+B,$$

where B should also be neutral.

The second process contains a small quantity G raised to a lower power. Assuming that B is the electromagnetic quantum, Vladimirov finds for ultrarelativistic charged A and $\bar{A}$

$$\sigma=\frac{e^2}{\hbar c}l_g^2.$$

However, if the quantum is replaced by a strongly interacting particle and if A and $\bar{A}$ are also taken to be strongly interacting, then we can expect

$$\sigma\sim l_g^2,$$

i.e., $\sigma \sim 1$ for ultrarelativistic strongly interacting particles (in gravitational units of area). On the other hand, if the average energy of the particles approaches unity (i.e., m_g), then the cross section for the emission of two gravitons should become of the same order.

Consequently, the relaxation time for the formation of gravitons is given by the expression

$$\tau=\frac{n_g}{\sum\sigma_i n_i^2}\cong\frac{1}{\sum\sigma_i n_i}=\frac{4t^{3/2}}{v}\left(\frac{v\cdot 0{,}65\cdot 32\pi}{3}\right)^{3/4}=t\,\frac{4{,}5\sqrt{t}}{v^{1/4}}.$$

According to this formula, the time of establishment of equilibrium would become comparable with t at $t \sim \sqrt{v}/20 \sim 0.5$ (at $v \sim 100$), but we are not justified in putting $t < t_1 = 6.5$ (all in gravitational units). Thus, establishment of equilibrium is not guaranteed in this case, since $\tau > t$ when $t > t_1$.

---

*__A. D. Sakharov__ (JETP Letters 3, 439 (1966), transl. p. 288) believes that T_g is the upper limit of the temperature of thermal radiation due to gravitational interaction between particles.

†We note that **M. A. Markov** (Suppl. Progr. Theor. Phys., Comm. Iss. Yukawa, 1965, p. 85); JETP 51, 878 (1966), Soviet Phys. JETP 24, (1967) in press) proposes that there exist elementary particles with mass of the order of m_g—maximons—and raises the question whether quarks are such particles.

*__R. Hagedorn__ (Nuovo Cimento Suppl. 3 (2), 147 (1965)) considers excited states of nucleons and mesons as statistical independent particles and arrives at the same time at the conclusion that v increases without limit with increasing T. It follows therefore that the growth of T slows down asymptotically. He assumes that the maximum temperature T is of the order of 150 MeV (1.5×10^{12} °K). These considerations were developed by him in connection with the theory of collision of cosmic rays of maximal energy. In referring to this article for the record, the author does not believe its results to be convincing, since the assumption of statistically independent and non-interacting particles can hardly be made at a high energy density.

†This paper contains also references to earlier papers on the theory of graviton creation.

In principle, it is not excluded here that the energy density of the gravitons can also be larger than equilibrium during the earlier stage; the fact that equilibrium is not established in this case means that the density will always remain higher than the equilibrium value. Such a situation is possible, in particular, in collective graviton-creation mechanisms by macroscopic motion of matter.

If we assume nevertheless that the gravitons are in equilibrium at a certain early stage, and are only later "detached" from the particles, then their present-day density is connected with the density of the electromagnetic radiation via the value of ν at the instant of detachment. It can be shown (with account of the succeeding partition of the energy between the neutrinos, gamma rays, electrons, and positrons, see above), that at the present time

$$\varepsilon_g = \left[\frac{18}{11(\nu-1)}\right]^{3/4} \varepsilon_\gamma < (0{,}1 \div 0{,}02)\, \varepsilon_\gamma .$$

8. INTERGALACTIC MEDIUM

The most important paper on the physics of the intergalactic medium is "On the Temperature of Intergalactic Gas" by **V. L. Ginzburg** and **L. M. Ozernoĭ** [Astron. zh. 42, 943 (1965), Soviet Astronomy AJ **9**, 726 (1966)]. Assuming the contemporary density of matter to be known at $(1-2) \times 10^{-29}$ and knowing that the density of matter in the galaxies, averaged over the entire volume, is much lower, the authors analyze the situation in which the main mass of the matter is located precisely in the intergalactic gas. The article contains references to earlier papers on this question.

The main conclusions of the article are as follows: The gas is almost fully ionized, the kinetic temperature of the electrons and nuclei ranges between 10^5 and 10^6 °K; the main heating mechanisms are explosions of galaxies and the dissipation of cosmic-ray energy via plasma oscillations; the main cooling mechanism at present is not radiation but adiabatic cooling accompanying the general cosmological expansion.

A confirmation of these views was the observation of neutral hydrogen by means of the absorption of the line $L_\alpha(1S \rightarrow 2P)$ in the spectrum of the remote quasar 3C-9 [see **J. E. Gunn** and **B. A. Peterson**, Astrophys. J. **142**, 1634 (1965)]*. According to the data of the authors, the absorption corresponds to a neutral-hydrogen density 6×10^{-34} g/cm^3 during the period when the total density was 27 times larger than the contemporary one, i.e., $\sim 5 \times 10^{-28}$ g/cm^3.

The fraction of neutral hydrogen $(1-2) \times 10^{-7}$ of the ionized hydrogen agrees with the notions of Ginzburg and Ozernoĭ.

An x-ray study of hot ionized gas was considered in the paper by **G. B. Field** and **R. C. Henry** [Astrophys. J. **140**, 1002 (1964)].

Comparison of the results with measurements of the x-ray background shows that the gas temperature does not exceed $(2-3) \times 10^6$ °K. New measurements by Friedman are available only in a preprint, but they change this estimate little. Measurements are needed in the region of the longer wavelengths (40–100 Å).

We note several papers containing hypotheses concerning the possible further investigation of intergalactic medium.

N. S. Kardashev and G. B. Sholomitskiĭ (Astron. tsirkulyar No. 336 (1965)) note that at the proposed electron density in the ionized gas the Compton scattering yields an optical thickness of the order of unity for objects with a red shift $z = \Delta\lambda/\lambda_0 = 6$. Unfortunately, Compton scattering is spectrally nonspecific, and it is not easy to prove the presence of such a scattering.

An interesting proposal was advanced by **F. T. Haddock** and **D. W. Sciama** [Phys. Rev. Letts. **14**, 1007 (1965)]. Free electrons in intergalactic space influence the velocity of radio wave propagation. (In principle they influence also the velocity of light, but the influence is inversely proportional to the square of the frequency of the wave.)

If the quasar produces a burst of radiation simultaneously at all wavelengths, then we shall receive a wave train that is stretched out, with the low frequencies lagging. This makes it possible in principle to find the electron concentration. The variability of the remote source CTA-102 (with red shift $\Delta\lambda/\lambda_0 = 1$) was observed by G. B. **Sholomitskiĭ** (Astron. tsirk. No. 359, 5 March 1966). According to his calculations, however, the ratio of the wavelength to the period is not suitable here for determination of the dispersion of the radio waves.

In a remark contained in the preprints of Bacall and Salpeter [Astrophys. J. **142** (4), 1677 (1965)], the possibility is considered of observing different atoms and ions in the intergalactic space by measuring the absorption. In a recent paper by R. A. Syunyaev (Astron. zh. **43** (5) (1966), Soviet Astronomy AJ, in press) there is a special analysis of the question of observing helium, whose contents should amount to 30% that of hydrogen.

Helium also influences noticably the heat balance of the gas, as noted earlier qualitatively by L. M. Ozernoĭ and investigated by R. A. Syunyaev.

The radiation of a hot gas and its interaction with quanta was considered by Kaufman [Nature **207**, 736 (1965)]. It is dealt with in greater detail in a paper by R. Weyman, available in the form of a preprint. By the time this article is published, articles by Weyman will already have been printed in "Astrophysical Journal" and "Physics of Fluids." The latter paper considers the emission of quanta and the change of their frequency when scattered by moving electrons, and duplicates to a considerable degree the work of a Soviet writer (apparently unnoticed by Weyman), A. S. Kompaneets "Establishment of Equilibrium between Radiation and Ionized Gas" [JETP **31**, 877 (1956), Soviet Phys. JETP **4**, 730 (1957)].

Weyman's general conclusion reduces to the fact that the ionized hot gas cannot noticeably change the spectrum of the relict radiation in that region of the spectrum where the relict radiation exceeds the radiation of the discrete sources—stars and radiogalaxies.

We emphasize that energy estimates exclude the possibility of an appreciable difference between the electron temperature and the temperature of relict radiation during the early stage, when the density of the latter was large.

Relict quanta should interact with fast electrons from cosmic rays and slow down the latter; owing to the "inverse Compton effect" this should give rise to optical and x-ray quanta (depending on the electron energy).

A. I. Nikishov [JETP **41**, 549 (1961), Soviet Phys. JETP **14**, 393 (1962)] and P. G. Goldreich and P. Morrison [JETP **45**, 344 (1963), Soviet Phys. JETP **18**, 239 (1964)] consider pair production in the interaction of cosmic γ rays with photons, $\gamma + \gamma = e^+ + e^-$. As shown by **J. V. Jelley** [Phys. Rev. Letts. **16**, 479 (1966)], and also by **R. I. Gould**, and **G. Schreder** [Phys. Rev. Letts. **16**, 252 (1966)], the relict radiation interacts quite effectively with quanta of energy 10^{15}—10^{16} eV, decreasing their range to 3000 psec, which is much smaller than the dimension of our galaxy. Finally, in an article "End to the Cosmic Ray Spectrum" [**K. Greisen**, Phys. Rev. Letts. **16**, 748 (1966)] it is shown that the interaction of protons with relict quanta cuts off steeply the spectrum of cosmic rays at an energy of 10^{20} eV. Independently, although later, detailed calculations were made by **G. T. Zatsepin** and **V. A. Kuz'min** ("On the Upper Limit of the Spectrum of Cosmic Rays," JETP Letters **4**, 114 (1966), transl. p. 78).

9. INSTABILITY AND DEVELOPMENT OF FLUCTUATIONS

For the general concepts involved in instability of the homogeneous distribution of matter in an expanding universe we refer the reader to the following papers: **J. Jeans**, Astronomy and Cosmology, Cambridge Univ. Press, 1929, p. 345; Phil. Trans. Roy. Soc. **A199**, 1 (1929); **E. M. Lifshitz**, JETP **16**, 587 (1946); **W. B. Bonnor**, Month. Not. 117, 104 (1957); **Ya. B. Zel'dovich**, Voprosy kosmogonii **9**, 240 (1965); UFN **80**, 357 (1963), Soviet Phys. Uspekhi **6**, 475 (1964); **I. D. Novikov**, JETP **46**, 686 (1964), Soviet Phys. JETP **19**, 470 (1964); **Ya. B. Zel'dovich**, Advances Astron. and Astrophys. **3**, 241 (1965).

This general conception consists in the fact that depending on the scale of the perturbation (its wavelength l or the mass subtended by its M) there is either predominance of the elasticity of the matter, which equalizes when $l < l_D$ and $M < M_D$ the perturbation of the density, or predominance of the gravitation force, which intensifies the perturbation at $l > l_D$ and $M > M_D$. The critical dimensions and the mass l_D and M_D are termed "Jeans" dimensions.

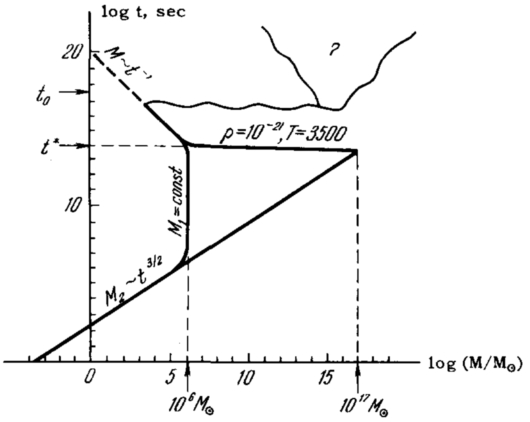

FIG. 6

In the hot model, the most important role is played by the radiation pressure. Taking into account this dependence of the temperature and density on the time, which follows from the mechanics of the expansion, we can set up the diagram of Fig. 6, which shows the dependence of M_D on the time. This diagram calls for some comments.

By "mass" we mean henceforth the mass of the baryons and electrons in a given volume, i.e., a quantity which is conserved in spite of the change in the quantum energy or their drift from the considered region of space containing the given baryons. In setting up the diagram, we have assumed for the present time $T_0 = 3°$ and $\rho_0 = 10^{-29}$ g/cm^3.

The line $M_2 \sim t^{3/2}$ corresponds to compression of "matter" together with the radiation; the resultant pressure drop is maximal. The equations of motion take into account the weight of the radiation. The critical mass M_2 increases, reaching 10^{17} $M_\odot$ and $\rho = 10^{-21}$ g/cm^3, at $t = t^* = 10^{14}$ sec, after which it remains constant.

There is no need to continue this line beyond $t > 10^{14}$ sec, since during the course of the expansion a temperature lower than 3500° is reached, recombination of hydrogen takes place, and the neutral atoms do not interact with the radiation. Therefore the solid line is continued in such a way that the region of stability (with allowance for radiation) is closed. There exists another type of perturbation, in which the density of "matter" in space is inhomogeneous, whereas the density of the radiation remains constant. Such perturbations can be called isothermal, and better still entropy perturbations, since $S \sim T^3/\rho_m$ and when ρ_m is variable, so is S. Counteracting the growth of the perturbations is only the gradient of the gas pressure $p_m = RT\rho_m$, which is much lower than the radiation pressure*.

---

*We note that in a preprint by the Burbidges and Linds (Kitt Peak Observatory, 1 March 1966), only absorption by a cloud (or shell) of gas around the quasar was observed in the spectrum of the similar object, 3C-191. The authors note the great difficulty of observations of this type and cast doubts on the foregoing results.

Accordingly, the critical Jeans mass M_1, which does not depend on the time at 10^4 sec > t > 10^6 sec, is also much smaller.

However, so long as the matter is ionized and the radiation density is large, the entropy type perturbations with $M > M_1$, while increasing, do so very slowly; to increase the perturbation, the matter must move relative to the quanta, and the interaction with quanta hinders this motion.

In practice we can also regard the perturbation as frozen. This is noted by **P. J. E. Peebles** [Astrophys. J. **142**, 1317 (1965)] and L. M. Ozernoĭ (Trans. of Symposium "Variable Stars and Stellar Evolution," Moscow, 1966, Dissertation, Shternberg Astronomical Institute, 1966). A rapid growth occurs after $t^* = 10^{14}$ sec.

The complete picture of development of inhomogeneities, creation of galaxies, etc., is still being established at present. The diagram of Fig. 6 is exceedingly important for this picture. Notice should also be taken on this diagram of the decrease of M_D after new heating and ionization of hydrogen (the wavy line above $t = 10^{14}$ sec). We note in this connection the papers by **L. M. Ozernoĭ** (Trans. of the cited symposium) and **A. D. Doroshkevich, I. D. Novikov,** and **Ya. B. Zel'dovich** (Astron. zh. **43** (4) (1966), Soviet Astronony AJ in press).

However, besides the question of the regions of instability, assumptions are needed concerning the initial amplitudes of the perturbations of different types and different wavelengths. The papers cited above can be called phenomenological: they choose more or less likely functions for the amplitude by comparison with experiment.

The first attempt at presenting a theory of the initial amplitudes on the basis of "first principles" belongs to **A. D. Sakharov** [JETP **49**, 345 (1965), Soviet Phys. JETP **22**, 241 (1966)]. The paper pertains to the cold model and is hardly correct literally, but the method of the approach may possibly be significant in the future.

So far there are no real attempts to consider the transition of the universe through the singularity. The difficulties noted in the text, connected with the assumption of earlier formation of large perturbations (bodies under their own gravitational radius) are discussed in the paper by **Zel'dovich** and **Novikov** (Astron. zh. **43** (4) (1966), Soviet Astronomy AJ, in press).

Translated by J. G. Adashko

---

*A general classification of adiabatic and isothermal perturbations is due apparently to **A. I. Lebedinskii** [Voprosy kosmogonii 2, 5 (1945)] and **L. E. Gurevich** [ibid. 3, 94 (1954)]. Considerations concerning the different rates of their growth are found in an article by **Ya. B. Zel'dovich** and **I. D. Novikov** [UFN 86, 447 (1965), Soviet Phys. Uspekhi 8, 522 (1966)].