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THE VIOLATION OF CP INVARIANCE

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In the summer of 1964 Christenson, Cronin, Fitch, and Turlay reported that the long-lived neutral K meson, called the K_2^0 meson, decays with a small probability (of the order of 0.2 percent) into two charged π mesons. The existence of the decay $K_2^0 \rightarrow \pi^+\pi^-$ is in contradiction with CP parity conservation, and consequently means that a violation of CP invariance occurs in nature.

We recall that the respective meanings of the operations C and P are charge conjugation and inversion of the space coordinates.

As is well known, K_1^0 and K_2^0 mesons are linear superpositions of K^0 and $\bar{K}^0$ mesons:

$$K_1^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}}, \quad K_2^0 = \frac{K^0 - \bar{K}^0}{\sqrt{2}},$$

where the wave function of the $\bar{K}^0$ meson is defined in the following way: $\bar{K}^0 = CP(K^0)$; therefore $CP(K_1^0) = +K_1^0$ and $CP(K_2^0) = -K_2^0$, i.e., the CP parity of the K_1^0 meson is positive, and that of the K_2^0 meson is negative.

Since the spin of the K meson is zero, the CP parity of a $\pi^+\pi^-$ system arising from decay of a K meson is positive ($CP = +1$, $C = +1$, $P = +1$), and consequently the decay $K_2^0 \rightarrow \pi^+\pi^-$ is forbidden if CP parity is conserved.

The existence of the decay $K_2^0 \rightarrow \pi^+\pi^-$ has been confirmed in a number of experiments made in the last year and a half. Particularly important are the experiments in which interference was observed between the decay $K_2^0 \rightarrow \pi^+\pi^-$ and the well known decay $K_1^0 \rightarrow \pi^+\pi^-$. The existence of interference showed convincingly that what is being observed is actually decay of the neutral K meson, and not of some hitherto unknown particle, and that it is indeed π mesons that appear in the decay. This has served to refute quite a number of hypotheses put forward for the purpose of "saving" CP invariance.

The interference experiments have also made it possible to approach an elucidation of the question as to what interaction is responsible for the violation of CP invariance. At present we know nothing about this interaction. A theoretical analysis has shown that it can either be ultraweak (about ten orders of magnitude weaker than the ordinary weak interaction), or extremely strong (of the order of the ordinary electromagnetic interaction). Versions are of course not excluded in which the constant of the CP-odd interaction lies somewhere within the range 10^{-16}—10^{-2}.

If the CP-noninvariant interaction is ultraweak (constant of order 10^{-16}), its only manifestation will be the CP-odd decays of the K_2^0 meson. Then the ultraweak interaction serves to convert the K_2^0 meson into a virtual K_1^0, which decays with conservation of CP parity. This sort of mechanism of violation of CP conservation is very effective because of the exceptionally small mass difference of the K_1^0 and K_2^0 mesons (of the order of 10^{-5} eV). In all other cases the energy denominators in the matrix elements will be larger by 10 to 14 orders of magnitude, and the effects caused by the ultraweak interaction must be practically unobservable. It follows that a detailed study of the properties of such an ultraweak CP-noninvariant interaction may be a matter of the extremely distant future.

If, on the other hand, the CP-noninvariant interaction had a constant of the order of that of the ordinary electromagnetic interaction, its manifestations must be numerous and varied, and it can be supposed that its properties would be studied in the very near future. Unfortunately, the first experiments made for the purpose of detecting CP-odd effects in strong and electromagnetic processes have given negative results. In particular, they failed to reveal the decay $\eta^0 \rightarrow \pi^0 e^+ e^-$, which, according to theoretical estimates, may have a considerable probability in the case of CP noninvariance. No differences were found in the spectra of π^+ and π^- mesons in the annihilation of antiprotons (there must be a difference if CP invariance is violated). The negative results of these and several other experiments cannot, however, be regarded as a final condemnation of the hypothesis that there is a relatively strong CP-noninvariant interaction, since the accuracy of the experiments is still low (in the best case of the order of several percent).

In this review we discuss a large number of experiments whose performance would make it possible to determine the nature of the CP-noninvariant interaction. Some of these experiments are now in progress. In particular, there will be great interest in the results of the experiment in which one looks for an electric dipole moment of the neutron. The expected accuracy of this experiment is of the order of $e \times 10^{-24}$ cm, which is about 10 orders of magnitude smaller than the magnetic moment of the neutrino and 4 orders of magnitude smaller than the upper limit on the neutron dipole moment which is now known.

Readers uninterested in the details of possible concrete manifestations of CP noninvariance can omit the first four chapters of the review and turn at once to the last chapter, which gives a discussion of the questions of principle which arise in connection with the violation of mirror symmetry in nature.

I. INTRODUCTION

1. The Decay $K_2^0 \rightarrow \pi^+\pi^-$. Basic Experimental Results

The discovery in 1964 of the decay $K_2^0 \rightarrow \pi^+\pi^-$ [1a] cast doubt on the validity of the hypothesis of CP invariance of the equations of physics, since this decay is forbidden if CP parity is conserved.

In fact, the CP parity of the K_2^0 meson is negative, and the CP parity of the $\pi^+\pi^-$ system, which is in an s state (since the spin of the K_2^0 particle is zero) is positive.

Table I. Data on decays $K_2^0 \to \pi^+\pi^-$ *

Experiment	Literature	Medium	Momentum, BeV/c	γ^2**	L***	Matter (Pb) in path of beam, g/cm²	R × 10³
Princeton I	1a	He	1.1	5,5	0,52	45	2.0±0.4
CERN	1b	Vacuum	10.7	460	0.14	55	2.24±0.23
Rutherford Laboratory	1c	Vacuum	3.15	39	0,37	55	2.08±0.35
Princeton II	1d	Vacuum	1.5	10	0,53	45	1.97±0.18

*Table taken from review report[1f].
**$\gamma = E/m$.
***L is the distance from the target, measured in units of the lifetime of the K_2^0 meson.

During the year and a half* since the publication of [1a] it has been confirmed by five experiments[1b,1c,1d,1e] that the decay K_2^0 actually occurs in vacuum. In addition these experiments have shown that the probability of this decay does not depend on the energy of the K_2^0 mesons,[1b] and that there is interference between the decays $K_2^0 \to \pi^+\pi^-$ and $K_1^0 \to \pi^+\pi^-$, if the K_1^0 mesons are produced owing to the coherent regeneration of K_2^0 mesons in matter.[1d,1e] The width of the decay $K_2^0 \to \pi^+\pi^-$ is characterized by the ratio (see Table I)

$$R = \frac{\Gamma(K_2^0 \to \pi^+\pi^-)}{\Gamma(K_2^0 \to \text{all charged particles})} = (2.04 \pm 0.14)\cdot 10^{-3}.$$

If we use the data on the widths for K_1^0 and K_2^0 mesons, we readily find from this that

$$|\eta_{+-}| = \left|\frac{A(K_2^0 \to \pi^+\pi^-)}{A(K_1^0 \to \pi^+\pi^-)}\right| = (2.02 \pm 0.10)\cdot 10^{-3},$$

where the quantities A are the amplitudes for the respective decays.

2. Attempts to Explain the Decay $K_2^0 \to \pi^+\pi^-$ without Violation of CP Invariance

Among the many attempts to explain the decay $K_2^0 \to \pi^+\pi^-$ in the framework of the CP-invariant theory there is not a single one that could be called successful.

The hypothesis[2a,2b,2c] that the transition $K_2^0 \to K_1^0 \to \pi^+\pi^-$ occurs under the action of an external (galactic) field requires that the probability of the decay $K_2^0 \to \pi^+\pi^-$ increase as E^2, where E is the energy of the K_2^0 meson in the laboratory reference system. This dependence is excluded by the experimental data, according to which the probability of decay does not depend on the energy of the K_2^0 meson (cf. Sec. 1). Furthermore, this hypothesis contains serious internal contradictions: long-range forces (corresponding to exchange of particles with zero mass) cannot be introduced into the theory without contradiction if the charge (in this case the hypercharge) which is the source of the forces is not conserved.[2d] Moreover, in this case, as is shown in [2e], the probability of emission of hyperphotons γ' (quanta of the hypothetical long-range field) in the decay $K_2^0 \to \pi^+\pi^-\gamma'$ would be extremely large, and this is also in contradiction with experiment.

Hypotheses according to which in the experiment of [1a] either K_2^0 did not decay into $\pi^+\pi^-$,[2f] or it was not K_2^0 that decayed into $\pi^+\pi^-$,[2g] naturally cannot explain the coherence of the decays $K_2^0 \to \pi^+\pi^-$ and $K_1^0 \to \pi^+\pi^-$ which is experimentally observed (cf. Sec. 1).* This difficulty is avoided by the hypothesis[2k] that there exist "shadow" K_1^0 mesons which are long-lived. As was shown in [2l], however, the existence of "shadow" K_1^0 mesons, which must have a high penetrating power, is excluded by the data obtained in a well known neutrino experiment at CERN, where no anomalous particles were detected beyond a shield (25 m of iron), such as would have properties like those of the "shadow" K_1^0 mesons. (In connection with [2k] see also [2m,2n].)

We shall not discuss hypotheses according to which the observation of the decay $K_2^0 \to \pi^+\pi^-$ is due to: 1) the nonexponential component[2d] of the decay $K_1^0 \to \pi^+\pi^-$,†

*This article was completed in January, 1966. References to some papers published later have been added in proof.

*Besides this, we have as evidence against the hypothesis of [2f] the fact [2i,2j] that the π mesons in the decay of K mesons do not have a spin, as was assumed in [2f].

†We note that the experimental verification of the exponential law in various decays is of interest in itself (cf. the experimental papers [2p], and also the theoretical papers [2q,2r]).

2) violation of the superposition principle,[2s] or 3) failure of the Bose statistics for π mesons.[2t] We shall also not consider various hypothetical mechanisms[2u,2v,2w] of spontaneous breaking of CP invariance in a CP-invariant theory.

It can of course not yet be regarded as proved that no explanation of the results of the experiments[1a-1e] can be found in the framework of a CP-invariant theory.

Nevertheless it follows from what has been said that there is ample present need for a detailed analysis of the ways in which a violation of CP invariance can occur in nature.

3. The Purpose of this Article

There exist at present a number of theoretical schemes in which the existence of the decay $K_2^0 \rightarrow \pi^+\pi^-$ brings with it CP-noninvariant effects in electromagnetic phenomena (radiative decays, dipole moments, and so on), in nuclear reactions at high and low energies, and in slow processes (β decay, decays of strange particles, and so on).

An analysis of the existing experimental data shows that the accuracy with which CP invariance has been verified in the majority of the indicated processes is not high and can be decidedly improved with the present level of experimental methods.

In this review an attempt is made to collect in one place and to discuss the various experimental data, and the suggestions for experiments, which relate to high-energy physics, low-energy nuclear physics, and atomic physics and which could perhaps throw light on the question of violation of CP invariance.

At the same time we make an attempt in this review to collect in one place, classify, and compare with the experimental data the various theoretical schemes of breaking of CP invariance.

There are a great many papers devoted to the theoretical and experimental analysis of the question of the possible CP noninvariance of the equations of physics. Some of these papers have been written in the last year and a half, some in the period from 1956 to 1964, and some still earlier. (The list of literature placed at the end of the article numbers over 200 papers, and is far from anything that could be called complete.)

In analyzing the CP problem, it is natural to raise the following questions:

1. In what sort of interactions can violations of CP invariance originate?
2. In what observable phenomena can violation of CP invariance be manifested?
3. What are the possible causes of violation of CP invariance (owing to what principles is it violated)?
4. How can we reconcile the violation of CP invariance with the absence of any privileged coordinate systems in empty space?
5. Does violation of CP invariance mean that there exists a preferred direction of the flow of time?
6. What sort of consequences does the lack of time reversibility in microscopic processes have for macroscopic processes?

We shall not examine all of these questions in the same amount of detail, since some of them have already been discussed in previously published reviews[3a-3f] and others have not been discussed at all in the literature. We give our main attention to questions 1 and 2.

4. The Hypothesis of CPT Invariance

We take as the basis of the following exposition the hypothesis that the equations of physics are CPT invariant. This construction of the article is to a large extent due to the fact that among the large number of models of the breaking of CP invariance that have been proposed there is in the literature not a single one that has introduced a violation of CPT invariance in a noncontradictory way. (This is evidently a practical consequence of the Lüders-Pauli theorem, according to which CPT invariance is obligatory for a very broad class of theories.[1a,1b,1c])

Possible phenomenological manifestations of CPT noninvariance have been discussed in a number of papers. In particular, neutral K mesons have been discussed from this point of view in [4d,4e,4f], and the decay $\pi^0 \rightarrow 2\gamma$ has been discussed in [4g].

It has been stated in [4g] that detection of a circular polarization of the photons in the decay $\pi^0 \rightarrow 2\gamma$ would mean a violation of CPT invariance. In fact, C parity is conserved in this decay: $C(\pi^0) = C(2\gamma)$, and a correlation of the type $\boldsymbol{\sigma}\cdot\mathbf{p}$ is at first glance T-even and P-odd, and consequently CPT-odd. It is easily verified, however, that the existence of such a correlation would mean a violation of the Hermitian character of the interaction Hamiltonian (in this connection see [4h]). Now for an antihermitian Hamiltonian a correlation of the type $\boldsymbol{\sigma}\cdot\mathbf{p}$ is T-odd. Accordingly, a detection of a circular polarization of the photons in the decay $\pi^0 \rightarrow 2\gamma$ would mean a violation of Hermiticity, but not of CPT invariance.

Owing to CPT invariance the masses and lifetimes of particles and antiparticles must be equal.[4i,8a] The experimental verification of the consequences of CPT invariance is of very great interest precisely because of the fundamental nature of this symmetry. From the equality of the masses of K^0 and $\bar{K}^0$ mesons, which holds to high accuracy, it follows that if the interaction that breaks CPT conserves P (spatial parity) and Y (hypercharge), then it must be at least three orders of magnitude weaker than the square of the weak interaction, and consequently is 17 orders of magnitude weaker than the strong interaction. For possible CPT-noninvariant interactions with change of P and Y, or involving leptons, the limits given by experiment are much less stringent. The accuracies to which equality of lifetimes of particle and antiparticle have been established for μ, π, and K mesons are[4j]

$$\frac{\Delta\tau}{\tau} < \begin{cases} 10^{-3} & \text{for } \mu^{\pm}, \\ 7\cdot 10^{-3} & \text{for } \pi^{\pm\,4\pi}, \\ 15\cdot 10^{-2} & \text{for } K^{\pm}. \end{cases}$$

On the basis of CPT invariance we shall assume in what follows that violation of CP invariance leads to violation of invariance under time reversal. For brevity we shall hereafter make no more mention of this stipulation.

5. A New Interaction

If the decay $K_2^0 \to \pi^+\pi^-$ indeed occurs in vacuum without the action of external fields, its discovery means that CP parity is not conserved. This in turn means that there exists in nature, besides the strong interaction (S), the electromagnetic interaction (E), the weak interaction (W), and the gravitational interaction (G), at least one further interaction, which, unlike the S, E, W, and G interactions, is CP-noninvariant.

Of course the assertion that there is one more interaction is in a certain sense a matter of terminology. As will be seen below, in some cases the CP-noninvariant interaction can be regarded as a "correction" or "admixture" to one of the known types of CP-invariant interaction. It will be more convenient, however, and will call for less in the way of conventional stipulations, if from the start we speak of a new interaction which violates CP invariance.

It is of primary interest to establish the following fundamental properties of this new interaction:

1. Between what particles it acts.
2. Its intensity (the value of its constant).
3. What sort of selection rules it obeys with respect to hypercharge (Y), parity (P), Isotopic spin (I), unitary spin (U), and so on.
4. Its form.

We do not as yet know any of these four things. Detailed information about some of them may be lacking for a long time, owing to the fact that the CP-noninvariant interaction appears not in isolation, but in combinations with the S, E, and W interactions. Nevertheless it can be hoped that within the next few years we shall obtain answers to the first three questions.

As for the fourth question, to obtain the answer to it it is necessary to find other experimental manifestations of the CP-noninvariant interaction. (If the only known manifestation of the weak P-noninvariant interactions were the θ and τ decays, one could scarcely find the V−A form of that interaction.) The determination of the form of the CP-noninvariant interaction might enable us to formulate a symmetry principle which this interaction obeys. (We recall that in the case of the weak interaction this principle is γ_5 invariance. If we construct the Lagrangians for the strong, weak, and electromagnetic interactions and require that they be γ_5-invariant, the strong and electromagnetic currents can conserve P parity, since they are bilinear in operators of the same particle, but the weak current must be V−A.)

It of course cannot be excluded that the principle that lies at the foundation of the violation of CP invariance will be formulated on the basis of purely theoretical arguments before the properties of the CP-noninvariant interaction have been determined experimentally. There is, however, no basis in the history of the study of elementary particles for regarding this possibility as very probable.

In examining the possible answers to the first question, we construct Table II. In this table h, l, γ denote respectively hadrons, leptons, and photons. In the cells of the table we have indicated the possible types of CP-noninvariant interactions:

1) hadrons with hadrons—X
2) hadrons with leptons—Y
3) hadrons with photons—Z
4) leptons with leptons—L
5) leptons with photons—M
6) photons with photons—Γ

We shall discuss these six types of interaction in various degrees of detail. We shall indicate the selection rules with respect to hypercharge Y and spatial parity P that an interaction obeys in the following way. For example,

$X0^+$ denotes a hadron-hadron interaction (X) with $\Delta Y = 0$ and conserving parity ($P = +1$); $X1^-$ denotes an X interaction with $\Delta Y = 1$, $P = -1$; $Z0^+$ denotes a hadron-photon interaction with $\Delta Y = 0$, $P = +1$.

Comparing the initial and final states in the decay $K_2^0 \to \pi^+\pi^-$ we might conclude that the interaction responsible for this decay is of the type $X1^-$, and that the other types of X interaction, and a fortiori the Y, Z, L, M, and Γ interactions, cannot lead to this decay. It is easily verified, however, that this conclusion is incorrect, since it does not take into account the contributions of virtual strong (S), electromagnetic (E), and particularly weak (W) interactions (the latter can change Y and P). As will be shown below, X0, X1, X2, and X3 interactions could lead to the decay $K_2^0 \to \pi^+\pi^-$. This is also true of Z0, Z1, Z2, and Z3 interactions. For example, an $X1^-$ interaction can arise as the resultant of the following chain of virtual interactions:

$$X1^- = W1^- \times X0^+ \times S,$$
$$X1^- = W1^- \times X2^+ \times S,$$
$$X1^- = W1^- \times Z0^+ \times E \times S \quad \text{etc.}$$

As for the CP-noninvariant interactions involving leptons (Y, L, M), they also, in combination with virtual weak interactions of the leptons, can lead effectively to an interaction of the type $X1^-$.

An upper limit on the constant of any particular interaction is determined by the degree of sensitivity with which experiments on it have so far been done.

Table II. Possible types of CP-noninvariant interaction

	h	*l*	γ
h	*X*	*Y*	*Z*
l		*L*	*M*
γ			Γ

For example, the interaction $Z0^+$ can have a constant of the order of that for the ordinary electromagnetic interaction E. The interaction $X0^+$ must be weaker by a couple of orders of magnitude than the ordinary strong interaction S. Unlike $X0^+$, the interaction $X0^-$ must be no stronger than the weak interaction, since otherwise it would, for example, make a sizable contribution to the dipole moment of the neutron.

The interactions X1 and Z1 cannot be stronger than the weak interaction, since otherwise it would be they, and not the weak interaction itself, that would cause the decays of strange particles.

The maximum possible value of the interaction $X2^-$ is two orders of magnitude smaller than the weak interaction, since it can lead to the hitherto unobserved decays with $|\Delta Y| = 2$.

As for $X2^+$, this interaction, proposed in [5a], can convert a K_2^0 meson into a K_1^0 meson, and because of the very small mass difference of these particles it can lead to the experimentally observed decay $K_2^0 \to \pi^+\pi^-$ by having a constant three orders of magnitude smaller than the square of the constant of the weak interaction; consequently it is of the order 10^{-15}–10^{-17}.

We have considered above the question as to whether particular interactions can give the decay $K_2^0 \to \pi^+\pi^-$. We must allow, however, for the fact that different classes of CP-odd interactions can exist, so to speak, "in parallel," as there exist in parallel the hadron-hadron, lepton-hadron, and lepton-lepton weak interactions, having comparable constants and the same form V – A (at least in theoretical papers).

It must also be pointed out that besides the six types of interaction we have enumerated there are other possible more complicated interactions, either involving several types of particle (for example, $hl\gamma$), or involving new and still unknown particles (for example, a particles).

6. The a-Particles

An examination of the models of CP invariance violations has shown that in a number of cases, owing to additional symmetry properties, the violation of CP invariance does not always manifest itself in the maximum possible degree. A "forced" conservation of CP invariance in the π-meson-nucleon vertex $\pi\bar{p}n$, caused by isotopic invariance, has been pointed out in a number of papers (cf., e.g., [6a,6b]). Analogous conclusions hold for K meson vertices in the framework of SU(3) and SU(6) symmetries.[6c]

An interesting model containing hypothetical a-particles has been suggested in [6d]. In this model a strong interaction of a-particles with ordinary hadrons violates C (and CP), but in such a way that the total Lagrangian is invariant under the operation C_n—charge conjugation of ordinary particles only. An example of a C-odd interaction having this property is the following:

$$(\bar{a}\sigma_{\alpha\beta}\gamma_5 q_\beta a)(\bar{p}\gamma_\alpha\gamma_5 p + \bar{n}\gamma_\alpha\gamma_5 n).$$

If this is the only interaction of a-particles with ordinary particles, then, owing to C_n invariance, in all processes in which real a-particles are not involved there will be no manifestation of the violation of C (and CP) invariance.

If, however, we assume that the a-particles are charged, and consequently that there is an interaction $e(\bar{a}\gamma_\mu a)A_\mu$, then this interaction violates C_n invariance, since the photon is an ordinary C-odd particle. The result is that in all hadron processes there will appear electromagnetically small effects of CP violation.

Ultraweak effects of CP violation of the type $X2^+$ can be obtained[6e] if we assume that the a-particles are neutral but possess an interaction which changes the strangeness of ordinary particles by two units.

7. Summary of Processes in which there Could be Manifestations of Violation of CP Invariance.

Violation of CP invariance has so far been observed in only one decay: $K_2^0 \to \pi^+\pi^-$. As long, however, as we do not know the type of interaction responsible for the CP violation, we can expect manifestations of this interaction in a very broad range of processes.

Some of these are enumerated in Tables III, IV, and V.

Table III contains fast (electromagnetic) decays, which occur with conservation of strangeness. Listed in the table as decaying particles are the neutral mesons (η^0, π^0, and so on), the Σ^0 hyperon, and excited levels of nuclei, denoted by the symbol A*. CP-noninvariant effects could be very important in decays of these particles, if a $Z0^+$ interaction exists; in almost all of them (except the decays $\eta \to 3\pi$) these effects would be weaker by 4 to 6 orders of magnitude if there is an $X0^+$ interaction but no $Z0^+$ interaction.

Table IV contains slow decays (leptonic, nonleptonic, and radiative) of mesons, baryons, and nuclei. In at least some of these decays there should be CP-odd effects for a broad class of interactions (X, Y, Z), except for the ultraweak interaction $X2^+$, which manifests itself only in decays of K^0 mesons.

Table V contains strong, electromagnetic, and weak

Table III. Fast decays

Type of decay \ Particles	π^0	η^0	ω^0	X^0	φ^0	f^0	A_2^0	Σ^0	A^*
	The numbers indicate the sections of this review in which the decays in question are discussed								
Mesonic		19	22	21					
Radiative	17	18, 20	22	21	22	22	22	23	29

Table IV. Slow decays

Type of decay \ Particles (and their anti-particles)	μ^+	π^+	K^+	K^0	n	Λ^0	$\Sigma^\pm$	$\Xi^{\bar{0}}$	Ω	A
	The numbers indicate the sections of this review in which the decays in question are discussed									
Leptonic ..	31	31	32	11, 12	33	33	33			34
Nonleptonic			32	9, 10, 13		35, 37	35, 37	36	36	
Radiative			32	15			38	36	36	

Table V. Reactions

	Beam \ Target	e^-	p	A	Z	E	H
		The numbers indicate the sections of this review in which the reactions in question are discussed					
Mesons	γ		28				
	ν		31				
	$\tilde{\nu}$		31				
	e^+	28	27	27		30	
	e^-		27	27		30	
	μ^+		27	27		30	
	μ^-		27	27		30	
Leptons	π^+						
	π^-		23				
	K^+						
	K^-		23				
	K^0						
	$\tilde{K}^0$						
Baryons	p		25	25, 26		30	
	$\tilde{p}$		24				
	n					30	
	$\tilde{n}$						
	Hyperons						
	Antihyperons						
Nuclei	A (nuclei)			26			
	$\tilde{A}$ (Antinuclei)						

interactions which occur in collisions of beams of various particles with various targets: e —electrons, p —protons, and A —nuclei, and with electromagnetic fields produced by Coulomb fields of nuclei (Z), condensers (E), and magnets (H). In particular, the fifth column of Table V contains dipole moments of particles.

Tables III, IV, and V will serve us as an index of the possible manifestations of the violation of CP invariance. Some of these manifestations will be discussed below (the numbers of the corresponding sections of this article are shown in the tables). Some of the possible effects of violation of CP invariance which an attentive reader can suggest by looking at Tables III, IV, and V have not as yet been discussed in the literature.

We begin the discussion with the possible experiments on the decays of neutral K mesons.

II. NEUTRAL K MESONS*

8. Description of K^0 Mesons

The problem of the description of K^0 mesons when there is violation of CP invariance has been treated in a number of papers.[8a-8e] We shall here expound the main results of these papers as applied to the case in which the degree of CP violation is small.

We shall consider three types of states:

1. K^0 and $\tilde{K}^0$ states with definite strangeness (hypercharge): $Y(K^0) = 1$, $Y(\tilde{K}^0) = -1$, $\tilde{K}^0 = CP(K^0)$. Since hypercharge is not conserved, under the action of the W interaction (in second order in W) the states K^0 and $\tilde{K}^0$ go over into each other.

2. K_1^0 and K_2^0 states with definite CP parity:

$$CP(K_1^0) = +1, \quad CP(K_2^0) = -1.$$

These states go over into each other under the action of the new CP-noninvariant interaction.

3. K_S and K_L states with definite masses (m_S and m_L) and definite lifetimes (τ_S and τ_L). In vacuum these states do not go over into each other. The states K_S and K_L have neither definite values of the strangeness nor definite values of the CP parity.

Decays that violate CP can occur owing to two different mechanisms:

1. Direct transitions of a K_2^0 meson (or a K_1^0 meson) into CP-even (or CP-odd) states.

2. Transitions $K_2^0 \longleftrightarrow K_1^0$ with subsequent CP-conserving weak decay. This second mechanism, which we shall call the pole mechanism, gives amplitudes containing the small denominator $\Delta = m_1 + m_2 + i\gamma_1 - i\gamma_2$, where $m_1(m_2)$ is the mass and $\gamma_1(\gamma_2)$ is the half-width of the $K_1(K_2)$ meson ($\gamma_{1,2} = \Gamma_{1,2}/2$).

In this chapter we discuss experiments which provide possibilities for determining the parameters which characterize these mechanisms of CP invariance violation.

We shall need a description of the behavior of K^0 mesons in vacuum. We consider this in two representations: 1) K^0 and $\tilde{K}^0$; 2) K_1^0 and K_2^0.

The equation describing the transitions $K \to K$, $\tilde{K} \to \tilde{K}$, and $K \longleftrightarrow \tilde{K}$ is of the form[8a]

$$-\frac{d\psi}{dt} = (\gamma + im)\psi = -\Lambda\psi,$$

*In cases in which it does not lead to misunderstanding, we shall use the term K^0 meson to denote the neutral K mesons, both K^0 and $\tilde{K}^0$.

where γ and m are two-rowed Hermitian matrices, the matrix γ corresponding to transitions on the mass shell and the matrix m to transitions off the mass shell, and ψ is a two-component spinor, the upper component being the amplitude for the state $|K\rangle$ and the lower that for $|\tilde{K}\rangle$. If we write a matrix Λ of the form

$$\Lambda = \begin{pmatrix} \lambda_+ & \lambda_\pm \\ \lambda_\mp & \lambda_- \end{pmatrix},$$

where

$$\lambda_+ = -(im_+ + \gamma_+),$$
$$\lambda_- = -(im_- + \gamma_-),$$
$$\lambda_\pm = -(im_\pm + \gamma_\pm),$$
$$\lambda_\mp = -(im_\mp + \gamma_\mp),$$

then CPT invariance requires $\lambda_+ = \lambda_-$, and CP invariance requires $\lambda_\pm = \lambda_\mp$. (In the notations of [8a] $\lambda_\pm = p^2$, $\lambda_\mp = q^2$.)

If we look for the solution of the equation in the form

$$\psi = \begin{pmatrix} C \\ \tilde{C} \end{pmatrix} e^{\lambda t},$$

then it is easy to get for the complex eigenfrequencies $\lambda_{S,L}$ the values

$$\lambda_S = \lambda_+ + \lambda_\pm r\varrho, \quad \lambda_L = \lambda_+ - \lambda_\pm \frac{r}{\varrho},$$

where

$$r^2 = \frac{\lambda_\mp}{\lambda_\pm}, \qquad \varrho = \eta + \sqrt{1+\eta^2}, \qquad \eta = \frac{\lambda_- - \lambda_+}{2\sqrt{\lambda_\pm \lambda_\mp}}.$$

The eigenfunctions are

$$\psi_S = \begin{pmatrix} 1 \\ r\varrho \end{pmatrix} O_S, \qquad \psi_L = \begin{pmatrix} 1 \\ -r/\varrho \end{pmatrix} O_L,$$

where

$$O_{S,L} = e^{\lambda_{S,L} t}, \qquad \lambda_{S,L} = -(im_{S,L} + \gamma_{S,L}).$$

By means of these solutions it is easy to construct the amplitudes for transitions from the states $|K\rangle$ and $|\tilde{K}\rangle$ at the time $t = 0$ to the states $\langle K|$ and $\langle \tilde{K}|$ at time t:

$$\langle K | K(t)\rangle = (1+\varrho^2)^{-1}(O_S + \varrho^2 O_L),$$
$$\langle \tilde{K} | K(t)\rangle = (1+\varrho^2)^{-1}\varrho r(O_S - O_L),$$
$$\langle K | \tilde{K}(t)\rangle = (1+\varrho^2)^{-1}\varrho r^{-1}(O_S - O_L),$$
$$\langle \tilde{K} | \tilde{K}(t)\rangle = (1+\varrho^2)^{-1}(\varrho^2 O_S + O_L).$$

We shall use these amplitudes in the treatment of the leptonic decays of K^0 mesons.

In the treatment of π-mesonic decays of K^0 mesons it is more convenient to start from the description of neutral K mesons in the representation K_1^0, K_2^0. The equation that describes the transitions $K_1^0 \to K_1^0$, $K_2^0 \to K_2^0$, $K_1^0 \longleftrightarrow K_2^0$ is of the form

$$\frac{d\varphi}{dt}=\hat{\Lambda}\varphi,$$

where φ is a two-component spinor, the upper component being the amplitude for the state $|K_1^0\rangle$ and the lower that of the state $|K_2^0\rangle$. If we write

$$\hat{\Lambda}=\begin{pmatrix}\lambda_1 & \lambda_{12}\\ \lambda_{21} & \lambda_2\end{pmatrix},$$

then, using the definitions

$$K_1=\frac{K+\widetilde{K}}{\sqrt{2}},\qquad K_2=\frac{K-\widetilde{K}}{\sqrt{2}},$$

we easily get

$$\begin{aligned}2\lambda_1&=(\lambda_++\lambda_-)+(\lambda_\pm+\lambda_\mp),\\2\lambda_2&=(\lambda_++\lambda_-)-(\lambda_\pm+\lambda_\mp),\\2\lambda_{12}&=(\lambda_+-\lambda_-)-(\lambda_\pm-\lambda_\mp),\\2\lambda_{21}&=(\lambda_+-\lambda_-)+(\lambda_\pm-\lambda_\mp).\end{aligned}$$

If CPT invariance holds, and this is the case we shall be considering, then $\lambda_+=\lambda_-$, and consequently

$$\lambda_{12}=-\lambda_{21}.$$

It is convenient to write

$$\lambda_1=-(im_1+\gamma_1),\quad \lambda_2=-(im_2+\gamma_2),\quad \lambda_{12}=-i(im_{12}+\gamma_{12}),$$

or in matrix form

$$\hat{\Lambda}=-(i\hat{m}+\hat{\gamma})=-\left[i\begin{pmatrix}m_1 & im_{12}\\ im_{21} & m_2\end{pmatrix}+\begin{pmatrix}\gamma_1 & i\gamma_{12}\\ i\gamma_{21} & \gamma_2\end{pmatrix}\right].$$

In this notation m_1, m_2, m_{12}, λ_1, λ_2, and λ_{12} are real numbers. The reason that λ_{12} has an additional factor i is that the expression $K_2=(K-\widetilde{K})/2^{1/2}$ is anti-hermitian, while $K_1=(K+\widetilde{K})/2^{1/2}$ is Hermitian.

It is easy to derive expressions for the complex eigenfrequencies:

$$\lambda_{S,L}=-(im_{S,L}+\gamma_{S,L})=\frac{\lambda_1+\lambda_2}{2}\pm\sqrt{\frac{(\lambda_1-\lambda_2)^2}{4}-\lambda_{12}\lambda_{21}}.$$

If $|\lambda_{12}|\ll|\lambda_1-\lambda_2|$, then

$$m_S\approx m_1,\quad \gamma_S\approx\gamma_1,\quad m_L\approx m_2,\quad \gamma_L\approx\gamma_2.$$

The eigenstates are given by

$$\begin{aligned}K_S&=K_1+\varepsilon K_2,\\K_L&=K_2+\varepsilon K_1,\end{aligned}$$

where

$$\varepsilon=-\frac{\lambda_{21}}{\lambda_2-\lambda_S}=-\frac{\lambda_{12}}{\lambda_1-\lambda_L}\simeq-i\frac{m_{12}-i\gamma_{12}}{m_1-m_2+i\gamma_1-i\gamma_2}.$$

It follows from the equation $\lambda_{12}=\lambda_\pm-\lambda_\mp$ that transitions in which the hypercharge changes by two units contribute to λ_{12}. Such transitions can occur either in second-order perturbation theory—owing to the weak (W) and CP-odd (X1, X2$^-$) interactions—or in first order in a CP-odd interaction (X2$^+$). The quantity ϵ characterizes the pole mechanism mentioned above. It is easily verified that with CPT invariance preserved the parameter r which describes transitions $K^0\longleftrightarrow\widetilde{K}^0$ can be expressed simply in terms of the parameter ϵ which describes the transitions $K_1^0\longleftrightarrow K_2^0$:

$$r\approx1-2\varepsilon\qquad\text{for }\varepsilon\ll1.$$

9. The Decays $K_L^0\to\pi^+\pi^-$ and $K_L^0\to2\pi^0$

The system of two π mesons which are in an s state and have total charge zero has two isotopic states (I = 1, 2):

$$\begin{aligned}\psi_{\pi^+\pi^-}&=\sqrt{\tfrac{1}{3}}\,\psi_2+\sqrt{\tfrac{2}{3}}\,\psi_0,\\\psi_{\pi^0\pi^0}&=\sqrt{\tfrac{2}{3}}\,\psi_2-\sqrt{\tfrac{1}{3}}\,\psi_0.\end{aligned}$$

Here the ψ are wave functions describing so-called standing waves. It is known from experiment that for the K_S meson the dominant state is ψ_0, i.e., $\langle\psi_0|K_1^0\rangle\gg\langle\psi_2|K_1^0\rangle$.

Since strangeness is conserved in strong interactions, the state vector $|K\rangle$ can be multiplied by an arbitrary phase factor: $|K\rangle\to|K'\rangle=e^{i\chi}|K\rangle$, whereupon $|\widetilde{K}''\rangle=e^{-i\chi}|\widetilde{K}\rangle$, so that

$$CP|K'\rangle=e^{2i\chi}|\widetilde{K}'\rangle.$$

By assigning various values of the phase χ we shall be assigning the phases of the matrix elements for transitions of K and $\widetilde{K}$ into other particles (π mesons, leptons, and so on). It is convenient to choose χ in such a way [8b-8e] that

$$\langle\psi_0|K'\rangle=\langle\psi_0|\widetilde{K}'\rangle.$$

Then the state $K_2''=(K'-\widetilde{K}')/2^{1/2}$ will not make transitions to ψ_0:

$$\langle\psi|K_2'\rangle=0.$$

Accordingly the direct CP-odd transition $K_2'\to(2\pi)_{I=0}$ can be transformed away.

In what follows we shall omit the prime, assuming that the condition $\langle\psi_0|K_2'\rangle=0$ is satisfied. The result is that K_L can make transitions to ψ_0 only owing to transitions $K_2\longleftrightarrow K_1$. If we write

$$A_0=\langle\psi_0|K_1^0\rangle,$$

then, because $K_L=K_2+\epsilon K_1$, we get

$$\langle\psi_0|K_L\rangle=\varepsilon A_0.$$

Let us now consider transitions to the state with I = 2. Suppose that under the condition that $\langle\psi_0|K_2\rangle=0$ both amplitudes are nonvanishing,

$$\langle\psi_2|K_1\rangle\neq0,\qquad\langle\psi_2|K_2\rangle\neq0.$$

If we define

$$\langle\psi_2|K\rangle=\frac{A_2}{\sqrt{2}},\qquad\langle\psi_2|\widetilde{K}\rangle=\frac{A_2^*}{\sqrt{2}},$$

then

$$\langle\psi_2|K_1\rangle=\frac{A_2+A_2^*}{2}=\operatorname{Re}A_2,$$

$$\langle\psi_2 \mid K_2\rangle = \frac{A_2 - A_2^*}{2} = i\,\mathrm{Im}\,A_2.$$

If we consider transitions not to standing waves but to diverging waves, then the amplitudes must be multiplied by the appropriate phase factors:

$$\langle\psi_0^{in} \mid K_1^0\rangle = A_0 e^{i\varphi_0},$$
$$\langle\psi_0^{in} \mid K_2^0\rangle = 0,$$
$$\langle\psi_2^{in} \mid K_1^0\rangle = \mathrm{Re}\,A_2 e^{i\varphi_2},$$
$$\langle\psi_2^{in} \mid K_2^0\rangle = i\,\mathrm{Im}\,A_2 e^{i\varphi_2}.$$

Here $\varphi_0(\varphi_2)$ is the phase for $\pi\pi$ scattering in the state I = 0 (2) at an energy in the center-of-mass system equal to the mass of K^0 mesons. We can now write in explicit form the amplitudes for the decays $K_{L,S} \to 2\pi$, neglecting higher-order terms:

$$\langle\pi^+\pi^- \mid K_S\rangle = \sqrt{\tfrac{2}{3}}\,A_0 e^{i\varphi_0} + \sqrt{\tfrac{1}{3}}\,\mathrm{Re}\,A_2 e^{i\varphi_2},$$
$$\langle\pi^0\pi^0 \mid K_S\rangle = -\sqrt{\tfrac{1}{3}}\,A_0 e^{i\varphi_0} + \sqrt{\tfrac{2}{3}}\,\mathrm{Re}\,A_2 e^{i\varphi_2},$$
$$\langle\pi^+\pi^- \mid K_L\rangle = \varepsilon\left(\sqrt{\tfrac{2}{3}}\,A_0 e^{i\varphi_0} + \sqrt{\tfrac{1}{3}}\,\mathrm{Re}\,A_2 e^{i\varphi_2}\right) + \sqrt{\tfrac{1}{3}}\,\mathrm{Im}\,A_2 e^{i\varphi_2},$$
$$\langle\pi^0\pi^0 \mid K_L\rangle = \varepsilon\left(-\sqrt{\tfrac{1}{3}}\,A_0 e^{i\varphi_0} + \sqrt{\tfrac{2}{3}}\,\mathrm{Re}\,A_2 e^{i\varphi_2}\right) + \sqrt{\tfrac{2}{3}}\,i\,\mathrm{Im}\,A_2 e^{i\varphi_2}.$$

By means of these amplitudes we can determine in the following way the ratios of the amplitudes for decays of K_S and K_L mesons:

$$\eta_{+-} = \frac{\langle\pi^+\pi^- \mid K_L\rangle}{\langle\pi^+\pi^- \mid K_S\rangle} = \varepsilon + \theta, \quad \eta_{00} = \frac{\langle\pi^0\pi^0 \mid K_L\rangle}{\langle\pi^0\pi^0 \mid K_S\rangle} = \varepsilon - 2\theta.$$

Here the quantity $\epsilon = -\Lambda_{12}/\Delta$ has been defined earlier. The quantity

$$\theta = i\left(\mathrm{Im}\,A_2/\sqrt{2}A_0\right) e^{i(\varphi_2 - \varphi_0)}$$

characterizes the violation of CP invariance in the transition to the state I = 2. In the derivation of these relations we have used the fact that $\mathrm{Re}\,A_2 \ll A_0$.

Experimentally (cf. Sec. 1)

$$R_{+-} = |\eta_{+-}|^2 = |\varepsilon + \theta|^2 = |2.02 \pm 0.10|^2 \cdot 10^{-6}.$$

If we measure the width of the decay $K_L^0 \to 2\pi^0$ we can determine another combination of the constants ϵ and θ:

$$R_{00} = |\eta_{00}|^2 = |\varepsilon - 2\theta|^2.$$

The various models of the violation of CP invariance give different predictions for the quantities ϵ and θ. For example, the $X2^+$ model gives $\theta = 0$, $\gamma_{12} = 0$ [we recall that $\epsilon = -i\lambda_{12}/\Delta$, where $\lambda_{12} = -i(\mathrm{im}_{12} + \gamma_{12})$, $\Delta = (m_1 - m_2) + i(\gamma_1 - \gamma_2)$]. The quantity θ is not small in a model [9a] in which an interaction with $\Delta T = 3/2$ is responsible for the violation of CP invariance.

If the X1 interaction with $\Delta T = 5/2$ is responsible for the violation of CP invariance, as assumed in [9b], then $\epsilon \ll \theta$; furthermore $K^0 \to (2\pi)_{I=0}$, $K_2^0 \to (2\pi)_{I=2}$, and the transitions $K_1^0 \longleftrightarrow K_2^0$ in the mass matrix are very small (if there were no virtual photons, these elements would be absent, since the states with T = 0 and with T = 2 are orthogonal, and we would have $\epsilon = 0$; see also [9c,9d]).

10. Interference Experiments with 2π Decays

Important information about the phenomenological parameters which describe the violation of CP invariance in the decays $K_2^0 \to 2\pi$ have already been obtained [1d,1e] and undoubtedly will be obtained in the future from experiments in which one observes the interference of the decays $K_S \to 2\pi$ and $K_L \to 2\pi$. Such experiments have been suggested in a number of theoretical papers. [10a-10d] We shall consider three types of experiments of this kind:

1. The observation of 2π decays in vacuum in a beam which has a definite strangeness at t = 0 (for example, contains only K^0). In this case the probabilith of 2π decays as a function of time is proportional to

$$|O_S + \eta O_L|^2 \sim e^{-\Gamma_S t} + |\eta|^2 e^{-\Gamma_L t} + 2\mathrm{Re}\,\eta e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos\Delta m\,t - 2\,\mathrm{Im}\,\eta e^{-\frac{\Gamma_S + \Gamma_L}{2}t}\sin\Delta m\,t,$$

where $\eta = \eta_\pm$ or η_{00}, and $\Delta m = m_S - m_L \approx m_1 - m_2$. It is easy to see that this experiment allows one to determine Re η, Im η, and Δm (the last two up to a possible simultaneous change of signs).

2. The observation of 2π decays in the passage of a K_L beam through an extended homogeneous medium in which the amplitude for regeneration of K_S from K_L is comparable with the quantity ϵ. In this case the ratio of the probabilities of 2π decays in the medium and in vacuum is given by the expression

$$\left|\eta + \frac{2\pi f_{21} N}{m_K \Delta}\right|^2,$$

where $\Delta = m_1 - m_2 + i(\gamma_1 - \gamma_2)$. It is easy to understand this expression if we note that in a homogeneous medium the transitions $K_2 \to K_1$ at the nuclei combine coherently with the vacuum transitions $K_2 \to K_1$ caused by nonconservation of CP.

As is well known (cf., e.g., [10e]), the regeneration in a layer of thickness dx is described by the equation

$$\frac{d\psi_1}{dx} = i\lambda f_{12} N \psi_2,$$

where $\lambda = 2\pi/k_1$, and k_1 is the momentum of the K_1 meson in the laboratory system, N being the number of nuclei per cm^3. If we go over to the proper time τ of the K meson, this can be rewritten in the form

$$\frac{d\psi_1}{d\tau} = i\,\frac{2\pi}{m}\,f_{12} N \psi_2.$$

When there is nonconservation of CP we must add the quantity $\lambda_{12}\psi_2$ to the right member of this equation. Accordingly, in the medium the quantity $\epsilon = -i\lambda_{12}/\Delta$

$$\varepsilon' = -i\left[\lambda_{12} + i\,\frac{\pi}{m}\left(f_K^{\mathrm{lab}} - f_{\bar{K}}^{\mathrm{lab}}\right) N\right] \Big/ \Delta.$$

The amplitudes f are defined so that

$$f^{\text{lab}}/k^{\text{lab}} = f^{\text{c.m.}}/k^{\text{c.m.}},$$
$$f^{\text{c.m.}} = i\,\text{Im}\, f^{\text{c.m.}} + \text{Re}\, f^{\text{c.m.}}$$
$$\text{Im}\, f^{\text{c.m.}} = k^{\text{c.m.}}\sigma/4\pi.$$

It has been shown experimentally [1d] that the phases of f_{12} and $\eta_{\pm}\Delta$ are nearly equal. Since evidently Im f $\gg$ Re f, we can conclude from this that models in which $\theta = 0$ and $\gamma_{12} = 0$ are not in contradiction with experiment. A particular model of this sort is $X2^{+}$.

3. The observation of 2π decays in vacuum beyond a plate through which a beam of K_L mesons is passing. The spatial distribution of these decays can be calculated easily if we note that immediately beyond the plate the amplitude of the beam is of the form

$$|K_L\rangle + A\,|K_S\rangle,$$

where

$$A = \frac{i f_{12}\lambda N \Lambda_S}{i\delta + 1/2}\left[1 - \exp\left(-i\delta l - \frac{l}{2}\right)\right],$$
$$l = \frac{L}{\Lambda_S}, \quad \Lambda_S = v\gamma\tau_S, \quad \delta = \left(m_1 - m_2\right)\tau_S.$$

This experiment, [1e] like the preceding one, [1d] showed that the phases of f_{12} and $\eta_{\pm}\Delta$ are nearly the same. The interpretation of both experiments is hindered by the fact that at present there are no reliable data on the quantities f_K and $f_{\tilde{K}}$.

11. Leptonic Decays of K^0 Mesons and the Rule $\Delta Q = \Delta S$

Let us consider the K_{e3} decays, which are simpler than the $K_{\mu 3}$ type, having amplitudes containing only one form-factor. We assign symbols to the amplitudes for the decays:

$$K^0 \to e^+\nu\pi^- \quad f, \qquad \tilde{K}^0 \to e^-\tilde{\nu}\pi^+ \quad f^*,$$
$$\tilde{K}^0 \to e^+\nu\pi^- \quad g, \qquad K^0 \to e^-\tilde{\nu}\pi^+ \quad g^*.$$

If CP is conserved, then $f = f^*$, $g = g^*$. If the rule $\Delta Q = \Delta S$ holds, then $g = g^* = 0$. The question as to whether the rule holds is not yet settled. To settle it it is most convenient to study the K_{e3} decays in the initial stages of the life of a K^0 beam, close to the place where the K^0 mesons are produced. The slow transitions $K_2 \longleftrightarrow K_1$ described by the parameter ϵ are then of practically no importance, and the amplitudes for positron and electron decays are proportional to the following respective quantities:

for a beam which at $t = 0$ contained only K^0 mesons,

$$A^+ = O_S(1+x) + O_L(1-x),$$
$$A^- = O_S(1+x^*) - O_L(1-x^*);$$

for a beam which at $t = 0$ contained only $\tilde{K}^0$ mesons,

$$A^+ = O_S(1+x) - O_L(1-x),$$
$$A^- = O_S(1+x^*) + O_L(1-x^*).$$

Here

$$O_S = e^{-(im_S+\gamma_S)t}, \quad O_L = e^{-(im_L+\gamma_L)t}, \quad x = \frac{g}{f}.$$

It follows from this that the behaviors of the corresponding decays as functions of time are as follows [3c] *:

$$N^+ = N_a + N_b - N_c,$$
$$N^- = N_a - N_b - N_c,$$
$$\bar{N}^+ = N_a - N_b + N_c,$$
$$\bar{N}^- = N_a + N_b + N_c,$$

where

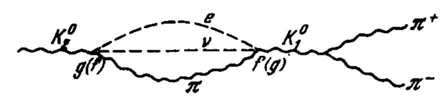
$$N_a = |1+x|^2 e^{-\Gamma_S t} + |1-x|^2 e^{-\Gamma_L t},$$
$$N_b = 2(1-|x|^2)\cos \Delta m\, t e^{-(\Gamma_S+\Gamma_L)t/2},$$
$$N_c = 4\,\text{Im}\, x \sin \Delta m\, t e^{-(\Gamma_S+\Gamma_L)t/2}.$$

The experimental results [11a, 11b, 11c] give no clear indications that the rule $\Delta Q = \Delta S$ is violated. On the basis of these experiments it can be concluded that

$$|\text{Re}\, x| \leqslant 0.1, \quad |\text{Im}\, x| \leqslant 0.25.$$

This agrees with the results of other experiments, in which attempts were made to observe transitions with $\Delta Q = -\Delta S$ (see, e.g., [11d]). In particular, it has been shown that decays $\Sigma^+ \to n e^+ \nu$ are less probable than decays $\Sigma^- \to n e^- \nu$ by at least a factor 25. At present there are 69 known cases of the decay $K^+ \to \pi^+\pi^- e^+\nu$ (with $\Delta Q = \Delta S$) and not a single case of the decay $K^+ \to \pi^+\pi^- e^-\tilde{\nu}$ (sic) (with $\Delta Q = -\Delta S$).

Decays with $\Delta Q = -\Delta S$ could arise from the CP-odd Y1 interaction. The hypotheses has been put forward in [11e, 11f, 11g] that it is precisely a Y1 interaction that is the primary CP-odd interaction. Here, as in the X2 model, decays $K_2^0 \to \pi^+\pi^-$ would occur owing to pole transitions.



Since the limiting momenta of the virtual leptons can be very large, and consequently the contribution of the lepton loop can be large, [11i] this sort of mechanism could be important even if g is many orders of magnitude smaller than f.

12. Leptonic Decays of the K_L^0 Meson

Leptonic decays of the K_L^0 meson are determined both by the quantity ϵ and by the quantity x. Using the fact that $K_L = K_2 + \epsilon K_1 = 2^{-1/2}[(1+\epsilon)K - (1-\epsilon)\tilde{K}]$, we can easily derive the result [8c]

$$R = \frac{W(K_L \to e^-\tilde{\nu}\pi^+)}{W(K_L \to e^+\nu\pi^-)} = \left|\frac{(1+\varepsilon)g^* - (1-\varepsilon)f^*}{(1+\varepsilon)f - (1-\varepsilon)g}\right|^2 \simeq \left|\frac{(1+2\varepsilon)x^* - 1}{1+2\varepsilon - x}\right|^2$$
$$\simeq 1 - 4\,\text{Re}\,\varepsilon \frac{1-|x|^2}{1+|x|^2 - 2\,\text{Re}\, x}.$$

Accordingly, a measurement of the charge asymmetry of the leptons in decays of the K_L^0 meson allows us to

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*The time dependence of the polarization of the muons in K_{μ_3} decays are discussed in [11h].

determine the quantity Re ϵ if the quantity x is known. In X2 models the magnitude of the asymmetry $R - 1$ is of the order of 0.6 percent, if $x = 0$ and if $|m_1 - m_2| \sim \Gamma_1/2$.

13. Decays of K^0 Mesons into Three π Mesons

As is well known, with conservation of CP parity K_L^0 can decay into three π mesons which are in a state with CP = −1 [we shall call this state $(3\pi)^-$], and the K_S^0 meson can decay into three π mesons in a state with CP = +1 [which we shall call $(3\pi)^+$]. The state $(3\pi)^-$ corresponds to even orbital angular momenta of the π mesons: $l = L = 0, 2, 4, \ldots$, and exists for the system $3\pi^0$, as well as for the system $\pi^+\pi^-\pi^0$. The state $(3\pi)^+$ corresponds to odd angular momenta: $l = L = 1, 3, 5, \ldots$, and exists only for the system $\pi^+\pi^-\pi^0$.

The decays we call CP-odd are those of the K_S meson into $3\pi^0$ and $\pi^+\pi^-\pi^0$ with CP = −1 and those of the K_L meson into $\pi^+\pi^-\pi^0$ with CP = +1. The CP-odd decays of K^0 mesons must be very small if the effective mechanism is of the type $X2^{\pm}$. For example, the decay $K_S^0 \to 3\pi^0$, which is forbidden by CP conservation, would for this case have a width given by

$$\Gamma_S(3\pi^0) = |\varepsilon|^2 \Gamma_L(3\pi^0) \sim 4 \cdot 10^{-6} \cdot 5 \cdot 10^6 \text{ sec}^{-1} \sim 20 \text{sec}^{-1}.$$

The same applies also to the decay of K_S^0 into the system $\pi^+\pi^-\pi^0$ with I = 1 and CP = −1. The amplitudes for CP-odd 3π decays could be larger in the case of the $X1^+$ interaction. In this case

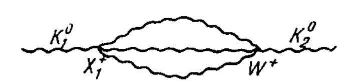
$$\eta_{000} = \frac{\langle \pi^0\pi^0\pi^0 \mid K_S \rangle}{\langle \pi^0\pi^0\pi^0 \mid K_L \rangle} = \varepsilon + \tau_1 + 2\tau_3,$$

$$\eta_{+-0} = \frac{\langle \pi^+\pi^-\pi^0 \mid K_S \rangle}{\langle \pi^+\pi^-\pi^0 \mid K_L \rangle} = \varepsilon + \tau_1 + 3\tau_3,$$

where

$$\tau_1 = \frac{\langle 3\pi cI = 1 \mid K_1^0 \rangle}{\langle 3\pi cI = 1 \mid K_2^0 \rangle}, \qquad \tau_3 = \frac{\langle 3\pi cI = 3 \mid K_1^0 \rangle}{\langle 3\pi cI = 1 \mid K_2^0 \rangle}.$$

The quantity τ_1 cannot, however, be many orders of magnitude larger than ϵ. This is due to the fact that in second-order perturbation theory the CP-odd 3π decays themselves make a contribution to ϵ:



We have no reason to suppose that the integral corresponding to this diagram is small. More detailed discussions of $K_{3\pi}^0$ decays can be found in [13a–13d].

14. The Contribution of Real States to the Transitions $K_1^0 \longleftrightarrow K_2^0$

As was first pointed out in [8b] an upper limit can be found on the parameter γ_{12} [we recall that $\epsilon = -(im_{12} + \gamma_{12})/(m_1 - m_2 + i\gamma_1 - i\gamma_2)$] from the existing experimental data on $K_{2\pi}$, $K_{3\pi}$, K_{e3}, and $K_{\mu 3}$ decays of $K_{L,S}^0$ mesons. The quantity γ_{12} is the sum of the contributions of these processes on the mass shell to the transition $K_2^0 \longleftrightarrow K_1^0$.

If the rule $\Delta Q = \Delta S$ is satisfied, the contribution of the K_{e3} and $K_{\mu 3}$ processes is zero. If Re x = 0, Im x = 0.25, then

$$\gamma_{12}(l_3) \approx \operatorname{Im} x\Gamma(K_L \to e_3) \approx 3 \cdot 10^6 \text{ sec}^{-1}.$$

The contribution of the 3π processes, $\gamma_{12}(3\pi)$ is surely smaller than

$$\frac{1}{2}\Gamma(K_L \to 3\pi) \approx 3 \cdot 10^6 \text{ sec}^{-1}.$$

The contribution of the 2π decays is determined by the magnitude of the CP-odd amplitude with T = 2:

$$\gamma_{12}(2\pi) = \frac{1}{2} \operatorname{Im} A_2 \operatorname{Re} A_2.$$

If we assume that $\operatorname{Im} A_2/A_0 \lesssim \epsilon$, and use the fact that $\operatorname{Re} A_2/A_0 \ll 1$ (the latter inequality holds provided that $\varphi_2 - \varphi_0 \neq \pi/2$), then we can neglect the contribution of the 2π states to the quantity γ_{12}: $\gamma_{12}/A_0^2 \ll \epsilon$. Using the fact that the denominator of the quantity ϵ is of the order of 10^{10} sec^{-1}, and assuming that in order of magnitude $\epsilon \sim \eta_{+-}$, we arrive at the conclusion that $|\gamma_{12}| \ll |m_{12}|$.

15. Radiative Decays of K^0 Mesons

The CP-odd effects in radiative decays of neutral K mesons, which have been discussed in a number of papers,[15a–15e] could be large and consequently interesting in the case of the $Z0^{\pm}$ and $Z1^{\pm}$ interactions. This applies in particular to the decays $K_{L,S}^0 \to 2\pi^0\gamma$, $K_{L,S}^0 \to \pi^+\pi^-\gamma$, $K_{L,S}^0 \to \pi^0 e^+ e^-$, $K_{L,S}^0 \to \pi^0\mu^+\mu^-$, and $K_{L,S}^0 \to 2\gamma$. If there are large CP-odd effects in these decays, then in treating them it will be legitimate to neglect the pole amplitude, since $\epsilon \ll 1$.

Let us first consider the decays $K_{1,2}^0 \to 2\pi\gamma$, confining ourselves to the lowest values of the orbital angular momentum of the mesons. Since the state $2\pi^0\gamma$ has C = −1, with conservation of CP the decay $K_1^0 \to 2\pi^0\gamma$ goes with conservation of P parity (a magnetic quadrupole or M2 transition), and the decay $K_2^0 \to 2\pi^0\gamma$ goes with nonconservation of P parity (an electric quadrupole or E2 transition). In the decay $K_1^0 \to \pi^+\pi^-\gamma$ ($K_2^0 \to \pi^+\pi^-\gamma$) with conservation of CP parity the dipole transition E1 (M1) is also allowed. Table VI shows the amplitudes allowed (CP = +1) and forbidden (CP = −1)

Table VI. Classification of amplitudes in decays

Decay \ Type of Interaction	$CP=+1$	$CP=-1$
$K_1^0 \to 2\pi^0\gamma$	$M2$	$E2$
$K_1^0 \to \pi^+\pi^-\gamma$	$E1,\ M2$	$M1,\ E2$
$K_2^0 \to 2\pi^0\gamma$	$E2$	$M2$
$K_2^0 \to \pi^+\pi^-\gamma$	$M1,\ E2$	$E1,\ M2$

by conservation of CP parity. If we write the amplitudes for $K_{2\pi\gamma}$ decays in relativistically invariant form, the resulting expressions will contain contributions also from higher angular momenta:

$$E1 \to p_\alpha q_\beta F_{\alpha\beta} e^{i\varphi_{E1}}, \qquad M1 \to p_\alpha q_\beta \widetilde{F}_{\alpha\beta} e^{i\varphi_{M1}},$$
$$E2 \to i(pq)\, p_\alpha q_\beta F_{\alpha\beta} e^{i\varphi_{E2}}, \qquad M_2 \to i(pq)\, p_\alpha q_\beta \widetilde{F}_{\alpha\beta} e^{i\varphi_{M2}},$$
$$F_{\alpha\beta} = k_\alpha A_\beta - k_\beta A_\alpha, \qquad \widetilde{F}_{\alpha\beta} = \varepsilon_{\alpha\beta\gamma\delta} F_{\gamma\delta};$$

p is the four-momentum of the K meson, and q is the difference of the four-momenta of the π mesons. The factors i in the expressions for E2 and M2 are due to the requirement that the effective interaction Hamiltonian be Hermitian. The interference of amplitudes with different CP parities but the same P parity (for example, M1 and M2) must lead to an asymmetry of the distribution on the Dalitz plot (cf. the decay $\eta \to 2\pi\gamma$, Sec. 20). For the decays of K_2^0 the phases φ of the amplitudes are mainly determined by $\pi\pi$ scattering in the dominant angular states p and d: φ_{E1} (φ_{M1}) $\approx \varphi_p$, φ_{E2} (φ_{M2}) $\sim \varphi_d$. For the decays $K_1^0 \to 2\pi\gamma$ an additional source of imaginary character is in transitions of the type

$$K_1^0 \xrightarrow{W} 2\pi \xrightarrow{E,\, Z0+} 2\pi\gamma.$$

In the decays $K_{L,S} \to 2\pi\gamma$ there must occur interference effects and the associated characteristic oscillations in time, similar to effects discussed above for other decays. Since in the decay $K_1^0 \to 2\pi\gamma$ soft accompanying bremsstrahlung is dominant, but harder radiation is dominant in the decays $K_2^0 \to 2\pi\gamma$, there will be maximum interference in the decays of K_L mesons at intermediate photon energies. An experimental search for the decay $K_L^0 \to 2\pi\gamma$ is reported in [15k].

It can easily be seen that the ordinary weak CP-invariant interaction W combined with the ordinary CP-invariant electromagnetic interaction E can lead to the decay $K_1^0 \to \pi^0\gamma \to \pi^0 e^+e^-$, but cannot lead to the decay $K_2^0 \to \pi^0\gamma \to \pi^0 e^+e^-$. The latter statement follows from the fact that the P-odd (and owing to the CP invariance, C-odd) component of the W interaction cannot contribute to the vertex $K_2^0 \to \pi^0\gamma$ owing to the fact that $P(\pi^0) = P(K_2^0)$, and the P-even (and consequently C-even) component cannot contribute owing to the fact that $C(\pi^0) = C(K_2^0)$ and $C(\gamma) = -1$. Accordingly the experimental observation of a decay $K_L^0 \to \pi^0 e^+e^-$ going with large probability would mean a sizable nonconservation of CP parity in the direct transition $K_2^0 \to \pi^0 e^+e^-$. If furthermore it should turn out that the nonconservation of CP parity in radiative processes that conserve strangeness is small (so as to exclude the $Z0^+$ interaction), this would mean that a primary Z1 interaction exists. This applies also to the observation of a charge asymmetry in decays $K_{L,S}^0 \to \pi^+\pi^-\gamma$.

Unfortunately, the expected probability of the decay $K_L^0 \to \pi^0 e^+e^-$ cannot be very large, since the Z1 interaction could manifest itself not only in this decay but also in the decay $K^+ \to \pi^+e^+e^-$, for which there is a known upper limit.[15g] Using the fact that $\tau_{K_2^0} \sim 5\tau_{K^+}$, we can expect that

$$\frac{\Gamma(K_L^0 \to \pi^0 e^+e^-)}{\Gamma(K_L^0 \to \text{all particles})} \leqslant 10^{-5}.$$

We must, however, allow for the fact that if there are no $Z0^+$ and Z1 interactions and the decay $K_L^0 \to \pi^0\gamma \to \pi^0 e^+e^-$ goes through the pole transition $K_2^0 \longleftrightarrow K_1^0$ it can then be expected that

$$\frac{\Gamma(K_L^0 \to \pi^0 e^+e^-)}{\Gamma(K_L^0 \to \text{all charged particles})} \approx \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_L^0 \to \text{all charged particles})}$$
$$\times \frac{\Gamma(K_S^0 \to \pi^0 e^+e^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)} \approx 2\cdot 10^{-3}\, \frac{\Gamma(K^+ \to \pi^+e^+e^-)}{\Gamma(K^+ \to \text{all})}$$
$$\times \frac{\Gamma(K^+ \to \text{all})}{\Gamma(K_S^0 \to \pi^+\pi^-)} \leqslant 2\cdot 10^{-3}\cdot 5\cdot 10^{-6}\cdot 10^{-2} = 10^{-10}.$$

The rare and as yet unobserved decays $K_{L,S}^0 \to 2\pi e^+e^-$ [15f] and $K_{L,S} \to 2\gamma$ [15h, 15j, 15i] are of interest from the point of view of studying the mechanism of CP nonconservation.

III. FAST PROCESSES

16. Properties of the $X0^+$ and $Z0^+$ Interactions

The hypothesis that the interaction $X0^+$ is a possible cause of the decays $K_2^0 \to \pi^+\pi^-$ has been suggested in [16a, 16b, 16c], and the hypothesis of the $Z0^+$ interaction has been suggested in [16d, 16e].

According to the hypothesis of [16d] the electromagnetic current of the hadrons is the sum of two currents: the ordinary C-odd current J and a C-even current K. It is the interaction of the K current with photons that gives the $Z0^+$ interaction.

The introduction of the K current calls for an examination of the principle of minimal electromagnetic interaction. It was shown in [16f] that CP symmetry can be broken in the electromagnetic interaction without its being broken in the strong interaction if one adds to the Lagrangian a CP-noninvariant term which is a four-divergence. This term does not itself contribute to the equations of motion, but when the electromagnetic field is included through the standard replacement $\partial/\partial x \to \partial/\partial x - ieA$ it gives a CP-noninvariant interaction $Z0^+$ involving photons.

The interactions $Z0^+$ and $X0^+$ can lead to CP-nonconserving processes which can have such large amplitudes that in principle they could be observed against the background of the CP-invariant strong and electromagnetic interactions. In this chapter we consider some of these processes: fast decays of mesons and of the Σ^0 hyperon (Table III), reactions belonging to the domain of high-energy physics (Table V), and, finally, electromagnetic decays of nuclei and other fast processes in the domain of low-energy nuclear physics (Tables III and V).

17. The Decay $\pi^0 \to 3\gamma$

Experimental searches for this decay, which is forbidden by CP conservation, have given [17a,17b]

$$R = \frac{\Gamma(\pi^0 \to 3\gamma)}{\Gamma(\pi^0 \to 2\gamma)} < 5 \cdot 10^{-6}.$$

The theory of the decay $\pi^0 \to 3\gamma$ has been treated in [17c] and [17d] (see also [17e]). As is shown in these papers, the amplitude for the decay is proportional to $(kr)^7$, where k is the mean momentum of the photons in the decay and r is the radius of the emitting region. For $r^{-1} \sim 300$ MeV/c the expected value of the ratio is $R \sim 10^{-11}$.

We note that the CP-odd decay $\pi^0 \to \gamma \to e^+e^-$ involving a single virtual photon is forbidden owing to conservation of electric current.

18. The Decays $\eta^0 \to \pi^0 e^+ e^-$ and $\eta^0 \to \pi^0 \mu^+ \mu^-$

Searches for the decay $\eta^0 \to \pi^0 e^+ e^-$ have established an upper limit on the ratio $R = \Gamma(\eta^0 \to \pi^0 e^+ e^-)/\Gamma(\eta^0 \to \pi^0 \mu^+ \mu^-)$:

$$R = (1.1 \pm 1.1)\%\ ^{18a}, \quad R < 2.8\%\ ^{18b}, \quad R < 7\%\ ^{18c}.$$

A theoretical estimate of the probability of this decay owing to a $Z0^+$ interaction gives [16d] $R \approx 1$ if the decay goes according to the scheme $\eta^0 \to \pi^0\gamma \to \pi^0 e^+ e^-$ and is not specially inhibited for any reason. [We note that the vertex $\eta^0 \to \pi^0\gamma$ is C (and also CP) odd. The decay $\eta^0 \to \pi^0\gamma$ with emission of a real photon is forbidden, as a 0—0 transition.] A value $R < 0.01$ does not, however, exclude a $Z0^+$ interaction with $\Delta I = 0$, since $\Delta I = 1$ at the vertex $\eta^0 \to \pi^0\gamma$ and the isoscalar interaction cannot in itself give such a vertex.

If we turn to possible causes of forbiddenness associated with SU(3), one can get [18d] a value $R \sim 0.01$ by assuming that the $Z0^+$ interaction with $\Delta I = 1$ is a component of a unitary octet. [18d] (If this is the case and if unitary symmetry held rigorously the vertex $\eta^0 \to \pi^0\gamma$ would be forbidden.) Further lowering of the experimental limit on R is a matter of great interest. It must be remembered, however, that in fourth order in the electromagnetic interaction the decays $\eta^0 \to \pi^0 e^+ e^- (\mu^+\mu^-)$ can go with conservation of CP parity:

$$\eta^0 \to \pi^0\gamma\gamma \to \pi^0 e^+ e^- (\mu^+\mu^-).$$

If the decays $\eta^0 \to \pi^0 e^+ e^-$ and $\eta^0 \to \pi^0 \mu^+ \mu^-$ are indeed observed, it will be possible to get an indication as to whether CP invariance is violated by measuring the ratio of their widths. In the case of single-photon exchange this ratio must have the value

$$\frac{\Gamma(\eta^0 \to \pi^0 e^+ e^-)}{\Gamma(\eta^0 \to \pi^0 \mu^+ \mu^-)} = 3.4.$$

19. The Decay $\eta^0 \to \pi^+\pi^-\pi^0$

The decay $\eta^0 \to \pi^+\pi^-\pi^0$ is especially favorable for the search for the $X0^+$ and $Z0^+$ interactions, since the main CP-even amplitude for this decay is due not to the strong interaction, but apparently to a virtual electromagnetic interaction [the work "apparently" reflects the fact that the experimental ratio of the widths of the decays, $\Gamma(\eta^0 \to 3\pi)/\Gamma(\eta^0 \to 2\pi)$, is too large as compared with theoretical estimates; both probabilities are of order α^4, but the phase volume for the first process is much smaller].

The three π mesons can be either in C-even states with I = 1 and 3, or in C-odd states with I = 0 and 2. The transition to these latter states is possible owing to the $X0^+$ interaction or to the combined action of the $Z0^+$ interaction and the ordinary electromagnetic interaction (E). [The connection between the C parity and the isotopic spin of the system of three π mesons follows from the fact that their G parity is negative: $(-1)^I C = -1$.] Let us consider, for example, the interference of two matrix elements: the CP-even element, with constant c, and the CP-odd element that arises as a result of the transition $\eta^0 \to \pi^0\gamma \to \pi^+\pi^-\pi^0$

$$\varphi_\eta\varphi_+\varphi_-\varphi_0 (c + i4\pi\alpha f (p_\eta + p_0)_\mu (p_+ - p_-)_\mu)$$

$$= c(1 + i8\pi\alpha f m_\eta c^{-1} (E_+ - E_-))\, \varphi_\eta\varphi_+\varphi_-\varphi_0,$$

where icf is a dimensional constant ($[f] = [m]^{-2}$) which characterizes the vertex $\eta^0 \to \pi^0\gamma$. Owing to Hermiticity, this constant is pure imaginary. Therefore there is no interference of the C-even and C-odd terms. If, however, we allow for the fact that in different states the mesons interact differently with each other, an additional phase difference arises between the first and second terms, and the square of the absolute value of the matrix element is proportional to

$$1 + \frac{16\pi\alpha \sin\delta f m_\eta}{c} (E_+ - E_-) + O(\alpha^2).$$

The result is that there is a charge asymmetry in the energy distribution of the π mesons, and in the Dalitz diagram for the η decay the distribution is unsymmetrical relative to the vertical axis.

Data collected from various laboratories possibly indicate the existence of an asymmetry of about 10 percent.* If this result is confirmed, it will mean that CP parity is not conserved in the decay $\eta^0 \to \pi^+\pi^-\pi^0$.

The CP-odd matrix element we have considered corresponds to a transition to a state with I = 2. The state with I = 0 is completely antisymmetric in the isotopic variables ($\mathbf{a}\times\mathbf{b}\cdot\mathbf{c}$) and therefore must be completely antisymmetric in the energy variables. This means a matrix element of the type

$$(E_+ - E_-)(E_- - E_0)(E_0 - E_+)\frac{1}{m^3},$$

where m^{-1} is the range of the interaction. It can be supposed that $m \gg E$. Therefore this matrix element is kinematically inhibited. We thus see that the decay $\eta^0 \to \pi^+\pi^-\pi^0$ is sensitive to the existence of an $X0^+$ interaction with $\Delta I = 2$ and insensitive to an $X0^+$ interaction with $\Delta I = 0$.

*M. Schwartz, private communication (May, 1965).

If it is demonstrated that there is violation of CP invariance in the decay $\eta^0 \to \pi^+\pi^-\pi^0$ but the CP-noninvariant decays $\eta^0 \to \pi^0\gamma \to \pi^0 e^+e^-$ are not observed, this will mean that the CP-noninvariant $X0^+$ interaction which gives the decay $\eta^0 \to \pi^+\pi^-\pi^0$ is not a secondary consequence of the $Z0^+$ interaction: $X0^+ \neq Z0^+ \times E$. CP nonconservation in $\eta \to 3\pi$ decays has been discussed in a number of papers.[19a-19f]

20. The Decays $\eta^0 \to \pi^+\pi^-\gamma$ and $\eta^0 \to 2\pi^0\gamma$

The two π mesons appearing in these decays can be either in a state with $C = -1$, $I = 1$ or in states with $C = +1$, $I = 0, 2$. The transitions into these latter states violate conservation of CP and can be caused by the $Z0^+$ interaction. Two π^0 mesons can be only in a state with $C = +1$, and therefore the observation of the decay $\eta^0 \to 2\pi^0\gamma$ would mean nonconservation of CP parity in an interaction of the type $Z0^+$ or $X0^+$. Furthermore the expected probability of decay in the latter case must be 4–6 orders of magnitude smaller than in the former. (We recall that an interaction of the type $Z0^-$ is excluded by the data on the dipole moment of the neutron.)

Interference of the CP-even (A^+) and CP-odd (A^-) amplitudes in the decay $\eta^0 \to \pi^+\pi^-\gamma$ must lead to an asymmetry of the distribution in the Dalitz diagram for this decay. If we confine ourselves to only the lowest values of the angular momenta, the state with $C = -1$ corresponds to a p wave of the two π mesons, and the state with $C = +1$ to a d wave. The covariant amplitudes are of the form

$$A^+ \sim p_\alpha q_\beta \tilde{F}_{\alpha\beta} e^{i\varphi_+}, \qquad \varphi_+ \approx \varphi_p,$$
$$A^- \sim i(pq)\, p_\alpha q_\beta \tilde{F}_{\alpha\beta} e^{i\varphi_-}, \qquad \varphi_- \approx \varphi_d.$$

Here p is the four-momentum of the η meson, and $q = p_+ - p_-$ is the difference of the four-momenta of the π mesons. $\tilde{F}_{\alpha\beta} = \epsilon_{\alpha\beta\gamma\delta}F_{\gamma\delta}$, and $F_{\gamma\delta} = k_\gamma A_\delta - k_\delta A_\gamma$ is the intensity of the electromagnetic field. In the rest system of the η meson

$$A^+ \sim \mathbf{qH} e^{i\varphi_p},$$
$$A^- \sim i(E_+ - E_-)\,\mathbf{qH} e^{i\varphi_d},$$

where **H** is the magnetic field of the photon.

The interference term is proportional to $(E_+ - E_-)\sin(\varphi_p - \varphi_d)$. Even for $Z0^+$ the expected value of the asymmetry is small, since the d wave is suppressed in comparison with the p wave owing to the centrifugal barrier.

The possible properties of the $Z0^+$ interaction and its manifestations in various decays of the η^0 meson are summarized in Table VII. The crosses indicate the $Z0^+$ interactions with the values of ΔI that manifest themselves in the best way in the decays listed.

Table VII. Change of isotopic spin in CP-forbidden amplitudes for decays of η^0 mesons

ΔI \ Decay	$\eta \to \pi e^+e^-$	$\eta \to 3\pi$	$\eta \to 2\pi\gamma$
0			×
1	×	×	
2		×	×

21. Decays of the X^0 Meson

Possible CP-odd decays of the X^0 meson ($m_X \approx 960$ MeV, $J^P = 0^-$, $I = 0$) are of interest from several points of view. Firstly, the energy released in the decay $X^0 \to 2\pi\gamma$ (which makes up about 20 percent of all decays of the X^0 meson) is much larger than that released in the decay $\eta^0 \to 2\pi\gamma$. Therefore in the former decay there may not be much suppression of the d wave. Accordingly it can be expected that a $Z0^+$ interaction with $\Delta I = 0, 2$ should give a sizable probability for the decay $X^0 \to 2\pi^0\gamma$ and would produce a considerable charge asymmetry in the decay $X^0 \to \pi^+\pi^-\gamma$. The corresponding experimental data are: 200 cases of the decay $X^0 \to \pi^+\pi^-\gamma$ showed no asymmetry.[18b]

Secondly, the decays $X^0 \to \pi^0 e^+e^-$ and $X^0 \to \pi^0\mu^+\mu^-$, unlike the decays $\eta^0 \to \pi^0 e^+e^-$ and $\eta^0 \to \pi^0\mu^+\mu^-$, are not forbidden by SU(3) invariance in the case in which the $Z0^+$ interaction has $\Delta I = 1$ and is a component of an octet. Using the facts that the width in question increases as M^5 and that the SU(3) suppression for the decay $\eta^0 \to \pi^0 e^+e^-$ amounts to about a factor $1/30$, we can conclude that

$$\frac{\Gamma(X^0 \to \pi^0 e^+e^-)}{\Gamma(\eta^0 \to \pi^0 e^+e^-)} \sim 10^3.$$

To reach further conclusions we need to know the total widths of the η^0 and X^0 mesons. Experimentally $\Gamma_{X^0} < 4$ MeV. Theoretical estimates give $\Gamma_{X^0} \cong 0.1$–0.5 MeV,[21a] $\Gamma_{\eta^0} \cong 300$ eV. On the basis of these estimates it can be expected[20a] that $\Gamma(X^0 \to \pi^0 e^+e^-)/\Gamma(X^0) \sim 1$ to 3 percent. Experiment gives for this quantity the upper limit 1.3 percent. It also gives $\Gamma(X^0 \to \eta^0 e^+e^-)/\Gamma(X^0) < 1.1$ percent. According to estimates based on SU(3) symmetry[18d] the expected value of this ratio is about 0.1 percent.

With CP nonconservation the decays $X^0 \to \rho^0\pi^0$ and $X^0 \to \omega^0\pi^0$ are also allowed.

22. Decays of Mesons with $J \geq 1$

We first of all point out that a number of decays forbidden by conservation of CP parity are also forbidden for other reasons. This is true of decays of the type of $\omega^0 \to 2\pi^0$, $\rho^0 \to 2\pi^0$, $\varphi^0 \to 2\pi^0$, $\varphi^0 \to 2K_1^0$, which are forbidden because of the Bose-Einstein statistics of π^0 and K_1^0 mesons, and also of decays of the type ω^0, ρ^0, $\varphi^0 \to 2\gamma$, which are forbidden because of the transverse character of the photon.

If CP is not conserved, the following decays are allowed: $\varphi^0 \to \eta^0\pi^0$, $\omega^0 \to \eta^0\pi^0$, $\omega^0 \to 3\pi^0$, $\varphi^0 \to \omega^0\gamma$, $\varphi^0 \to \rho^0\gamma$, $\omega^0 \to \rho^0\gamma$. According to a theoretical estimate,[20a] for example, $\Gamma(\varphi^0 \to \omega^0\gamma)/\Gamma(\varphi^0) \sim 0.02$. If CP invariance is violated, there can be asymmetries in the Dalitz distributions for the decays $\varphi^0 \to \pi^+\pi^-\gamma$, $\omega^0 \to \pi^+\pi^-\gamma$, $\omega^0 \to \pi^+\pi^-\pi^0$.[22a,19c,19e,16d]

An asymmetry in the decay $\omega^0 \to \pi^+\pi^-\pi^0$ can occur only if the $X0^+$ interaction (primary or secondary) satisfies the condition $\Delta I = 1, 3$. This asymmetry becomes zero if we neglect the interaction of the π mesons in the final state, which is as it were the "developer" for this case of CP nonconservation. No CP-odd effects have been found experimentally[22b,22c] in the decays of vector mesons:

$$\Gamma(\varphi \to \varrho\gamma) \leqslant 0.03\Gamma(\varphi \to K\tilde{K}), \quad \Gamma(\varphi \to \omega\gamma) \leqslant 0.09\Gamma(\varphi \to K\tilde{K}),$$
$$\Gamma(\varphi \to \pi^+\pi^-\gamma) \leqslant 0.05\Gamma(\varphi \to K\tilde{K}), \quad \Gamma(\varphi \to \eta\pi) \leqslant 0.15\Gamma(\varphi \to K\tilde{K}).$$

According to the theoretical estimates in [22d] the CP-odd decays $f^0 \to \pi^0\gamma$ and $A_2^0 \to \pi^0\gamma$ can amount to several percent of the corresponding total widths. Among other CP-odd decays we mention $B^0 \to \rho^0\gamma$, $\omega^0\gamma$, $\varphi^0\gamma$, $f^0 \to \eta^0\gamma$, $A_2^0 \to \eta^0\gamma$, $f^0 \to X^0\gamma$, $A_2^0 \to X^0\gamma$.

23. The Decay $\Sigma^0 \to \Lambda^0 e^+e^-$ and the Reactions $\pi^- p \to ne^+e^-$ and $K^-p \to \Lambda^0 e^+e^-$

In the decay $\Sigma^0 \to \Lambda^0 e^+e^-$ nonconservation of CP manifests itself in the nonzero relative phase φ of the two terms in the electromagnetic vertex $\Sigma^0 \to \Lambda^0\gamma$: the magnetic moment $\bar{u}_\Lambda \sigma_{\mu\nu} q_\nu u_\Sigma$ and the monopole $u_\Lambda[(m_\Sigma - m_\Lambda)q_\mu + q^2\gamma_\mu]u_\Sigma$. The interference of these terms must lead[16d,23a] to a correlation of the type $\sin\varphi\,\boldsymbol{\zeta}_\Sigma\cdot\mathbf{N}$ and $\sin\varphi\,\boldsymbol{\zeta}_\Lambda\cdot\mathbf{N}$, where $\boldsymbol{\zeta}_\Sigma$ and $\boldsymbol{\zeta}_\Lambda$ are unit vectors in the respective directions of polarization of the Σ and Λ particles, $\mathbf{N} = \mathbf{n} \times (\mathbf{n}_+ + \mathbf{n}_-)$, and $\mathbf{n}$, $\mathbf{n}_+$, and $\mathbf{n}_-$ are unit vectors in the respective directions of the momenta of the Λ particle, the positron, and the electron. The fact that the vector $\mathbf{N}$ does not change when we interchange $\mathbf{n}_+ \longleftrightarrow \mathbf{n}_-$ is due to the fact that the interaction of electron and photon is C and CP invariant. As for the expected size of the asymmetry, *there is a nonzero phase* φ *only if for the* $Z0^+$ *interaction* $\Delta I = 1$ [and the $Z0^+$ is not a component of a SU(3) octet]. This last selection rule is due to conservation of U-spin. In fact, from conservation of U-spin it follows that $\langle \Sigma_U | \Lambda_U \rangle = 0$. Going over from Σ_U and Λ_U to Σ and Λ, we get

$$\langle \Sigma_U | \Lambda_U \rangle = -\frac{\sqrt{3}}{4}\langle \Lambda | \Lambda \rangle + \frac{\sqrt{3}}{4}\langle \Sigma | \Sigma \rangle + \frac{1}{4}\langle \Sigma | \Lambda \rangle - \frac{3}{4}\langle \Lambda | \Sigma \rangle = 0.$$

It follows from this that the vertex $\langle \Sigma | \Lambda \rangle$ *must be* real, like the vertices $\langle \Lambda | \Lambda \rangle$ and $\langle \Sigma | \Sigma \rangle$, which are real because the electric current is Hermitian. An experiment[23b] on the measurement of the $p_{\Lambda\perp}$ polarization normal to the plane of the decay was not accurate enough to give an unambiguous answer.

A correlation of the type $\boldsymbol{\zeta}_n \cdot \mathbf{N}$ should be observed in the capture of slow π^- mesons in hydrogen: $\pi^- p \to ne^+e^-$. The expected asymmetry in this case must be small, even if there is a large violation of CP symmetry (see [23a]) An analogous correlation $\boldsymbol{\zeta}_\Lambda \cdot \mathbf{N}$ can be studied in the capture of slow K^- mesons in hydrogen: $K^-p \to \Lambda e^+e^-$. In this process, however, there should be a correlation $\boldsymbol{\zeta}_\Lambda \mathbf{N}$ even if CP parity is conserved. The reason is that the process K^-p can go through real intermediate states: $K^-p \to \Lambda\pi \to \Lambda\gamma \to \Lambda e^+e^-$ and $K^-p \to \Sigma\pi \to \Lambda\gamma \to \Lambda e^+e^-$. The presence of real intermediate states can have the consequence that the coefficients in the two terms of the electromagnetic vertex (dipole and anapole) will in general have different phases.

24. Reactions in Antiproton Beams

In each of the channels of $\bar{p}p$ annihilation the spectra of the positive and negative particles (π^+ and π^-, K^+ and K^-) must be identical if CP parity is conserved. If it is not, there should be an asymmetry similar to that discussed above in connection with the decays $\eta^0 \to \pi^+\pi^-\gamma$, $\eta^0 \to \pi^+\pi^-\pi^0$, and so on. Experimental data[24a] obtained in the study of 40,000 cases of $\bar{p}p$ annihilation have shown no differences in the spectra of π^+ and π^- mesons to an accuracy of the order of one percent, nor in the spectra of K^+ and K^- mesons to an accuracy of several percent (see also [24b]). Since this result relates to a set of different channels (different both in the number of particles and in the isotopic spin and total angular momentum), "accidental" selection effects in individual channels (of the type of those we have discussed for the η^0 meson) could not have any serious effect on the result. On the other hand, the larger energy release makes it possible for states with high angular momenta to compete successfully with states with smaller angular momenta. Finally, at high energies the $\pi\pi$ interaction (and also the πK and KK interactions) necessary for the "development" of the CP-odd effects is not small, and there is no reason to expect that these effects will not appear.

Accordingly, we may suppose that the data collected on the decays of neutral mesons and on $\bar{p}p$ annihilation indicate that the $X0^+$ interaction, if it exists, is at least two orders of magnitude weaker than the strong interaction: $g^2(X0^+)/g^2(S) < 10^{-2}$. It must be pointed out once more that the natural size of this ratio, which can be derived on the basis of the ratio $\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)$ must be of the order of 10^{-3}.

In conclusion we mention theoretical suggestions for studies of the asymmetry in the channel $\bar{p}p \to \pi^+\pi^-\gamma$[16d] and of the polarization of Λ and $\bar{\Lambda}$ in the channel $p\bar{p} \to \Lambda\bar{\Lambda}$.[24c] The elastic scattering $p\bar{p} \to p\bar{p}$ is of interest from the point of view of testing CPT. (cf. the theoretical papers [24d] and [24e]).

25. Polarization and Asymmetry in Elastic Scattering of Protons

When unpolarized protons are scattered by nuclei through some angle ϑ they acquire a polarization in the direction normal to the plane of the scattering. The degree of polarization $P(\vartheta)$ is defined in the following way:

$$P(\vartheta)=\frac{I^+(\vartheta)-I^-(\vartheta)}{I^+(\vartheta)+I^-(\vartheta)}.$$

Here $I^+(I^-)$ is the number of protons with spin directed upward (downward).

When completely polarized protons are scattered by nuclei their angular distribution is asymmetrical. The asymmetry is defined in the following way:

$$A(\vartheta)=\frac{I(\vartheta)-I(-\vartheta)}{I(\vartheta)+I(-\vartheta)},$$

where $I(\vartheta) = I^+(\vartheta) + I^-(\vartheta)$.

If there is invariance with respect to time reversal, then $A(\vartheta) = P(\vartheta)$. It is easy to verify that this equation will be satisfied automatically (independently of CP conservation) for scattering by nuclei with spin zero. For nuclei with $J \neq 0$, however, this equation will in general be satisfied only if there are no T-noninvariant terms in the amplitude. In pp scattering a term in the amplitude which is P-invariant but not T-invariant is of the form

$$(\sigma_1 \mathbf{l})(\sigma_2 \mathbf{m})+(\sigma_1 \mathbf{m})(\sigma_2 \mathbf{l})$$

(σ is the Pauli matrices, $\mathbf{l} = (\mathbf{p}_{1i} + \mathbf{p}_{1f})/|\mathbf{p}_{1i} + \mathbf{p}_{1f}|$, $\mathbf{m} = (\mathbf{p}_{1f} - \mathbf{p}_{1i})/|\mathbf{p}_{1f} - \mathbf{p}_{1i}|$. The indices 1 and 2 refer to the two particles, and the indices i and f to the initial and final states. The interference of this term with the T-invariant terms gives a correlation of the type $\zeta_f \cdot \mathbf{n} - \zeta_i \cdot \mathbf{n}$, where $\zeta_f(\zeta_i)$ is the polarization of the incident (scattered) proton, and $\mathbf{n}$ is the normal to the plane of the scattering. By themselves the T-invariant terms of the amplitude—1, $(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n})$, $(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l})$, $(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m})$, and $\sigma_1 \cdot \mathbf{n} + \sigma_2 \cdot \mathbf{n}$ —can give correlations of the type $\zeta_f \cdot \mathbf{n} + \zeta_i \cdot \mathbf{n}$, but not $\zeta_f \cdot \mathbf{n} - \zeta_i \cdot \mathbf{n}$. The considerations we have given were first formulated in [25a,25b] and stimulated a number of experimental researches. The experiments [25c-25f] showed that to an accuracy of a few percent $P = A$. In one of the latest experiments [25f] it was shown that the T-noninvariant contribution to the transition $^3P_2 \longleftrightarrow {}^3F_2$ is not more than 7 percent of the maximum attainable value in the energy range 140—210 MeV. T invariance requires that the amplitudes for the transitions $^3(J-1)_J \rightarrow {}^3(J+1)_J$ and $^3(J+1)_J \rightarrow {}^3(J-1)_J$ be equal. The comparison of P and A for scattering of particles with arbitrary spin is discussed in [25g].

26. Comparison of the Cross Sections of Direct and Inverse Reactions

If the interaction is T-invariant, the cross sections σ_{if} and σ_{fi} of the direct and inverse reactions satisfy the condition of detailed balancing

$$p_i^2 g_i \sigma_{if} = p_f^2 g_f \sigma_{fi}.$$

Here g_i and g_f are the statistical weight factors, and p_i and p_f are the relative momenta in the initial and final states. Sometimes, however, the condition of detailed balancing can be satisfied even when T invariance is violated. This is the case, for example, when the cross section can be calculated in the Born approximation or when the S matrix breaks up into 2×2 matrices which connect only two channels a and b. In the latter case it follows from the condition that the S matrix be unitary that $|\langle a|s|b\rangle| = |\langle b|s|a\rangle|$, in spite of the fact that when T invariance is violated $\langle a|s|b\rangle \neq \langle b|s|a\rangle$. In this case the cross sections of the direct and inverse reactions will satisfy the condition of detailed balancing. Accordingly, for the "developing" of irreversibility in channels a and b it is necessary that other channels be open connecting with a and b.

A detailed theoretical discussion of the cross sections of direct and inverse reactions is given in [26a], in which the processes

$$np \longleftrightarrow d\gamma, \quad pH^3 \longleftrightarrow He^4\gamma, \quad pp \longleftrightarrow \pi+d,$$
$$pLi^7 \longleftrightarrow dLi^6, \quad pHe^4 \longleftrightarrow dHe^3, \quad pH^3 \longleftrightarrow nHe^3,$$
$$dN^{14} \longleftrightarrow \alpha C^{12}, \quad dC^{12} \longleftrightarrow \alpha B^{10}.$$

are examined. In [26c] experimental data are given relating to the reactions pt $\longleftrightarrow$ dd at proton energy $E = 3.8$ MeV; from these data it follows that the ratio of the T-noninvariant and T-invariant amplitudes is not larger than 2 percent. According to [26d] this ratio is not larger than 3 percent for the reactions $\alpha C^{12} \longleftrightarrow dN^{14}$ for deuterons with energy 20 MeV.

27. Scattering of Electrons by Protons and Nuclei

A CP-odd but P-even interaction cannot manifest itself in elastic ep scattering. In the expression

$$\bar{u}\,[\gamma_\mu F_1(q^2)+\sigma_{\mu\nu}q_\nu F_2(q^2)+q_\mu F_3(q^2)]\,uA_\mu$$

owing to the Hermitian character of the electromagnetic current the form-factors F_1 and F_2 must be real, and F_3 must be imaginary. Since the terms γ_μ and $\sigma_{\mu\nu}q_\nu$ on one hand and the term q_μ on the other have different C-parities, the vertex for interaction of a photon with a proton would be C-noninvariant if $F_3 \neq 0$. It is easy to see, however, that $F_3(q^2) \equiv 0$ owing to the conservation of the electromagnetic current.

The result is that in the scattering of electrons by protons correlations of the T-odd types $\zeta_1 \cdot \mathbf{n}$ or $\zeta_2 \cdot \mathbf{n}$ can appear only owing to the exchange of two or more photons (here ζ_1 is the polarization of the initial proton, ζ_2 is that of the final proton, and $\mathbf{n}$ is the normal to the plane of the scattering.

As for the P-odd correlations of the types $\zeta_2 \cdot \mathbf{p}_2$, $\zeta_2 \cdot \mathbf{p}_1$, $\zeta_1 \cdot \mathbf{p}_2$, or $\zeta_1 \cdot \mathbf{p}_1$, they are rigorously forbidden by conservation of P-parity. Such correlations would appear if the proton had an anapole moment $(q^2\gamma_\mu - 2mq_\mu)\gamma_5$ (P-odd, but CP-even). If the proton had an electric dipole moment, then there would be a correlation of the type $\boldsymbol{\zeta}_1 \times \boldsymbol{\zeta}_2 \cdot \mathbf{p}$. Experimentally,[27a] the polarization of the recoil proton normal to the plane of the scattering is 0.038 ± 0.038, and that in the plane of scattering is -0.014 ± 0.031.

For the case of elastic scattering of electrons by nuclei with spin $J \geq 1$, unlike the case of ep scattering, the CP-noninvariant contribution of an $X0^+$ or $Z0^+$ interaction to the vertex is not zero.[27b-27f] For a nucleus with $J = 1$ (for example, the deuteron), besides the CP-invariant terms (charge, magnetic moment, quadrupole moment)

$$(\varphi^+\varphi)(pA), \qquad (\varphi^+q)(\varphi A) - (\varphi^+A)(\varphi q), \qquad (\varphi^+q)(\varphi q)(pA),$$

there can be a CP-noninvariant term of the type

$$i\,[q^2(\varphi^+q)(\varphi A) + q^2(\varphi^+A)(\varphi q) - 2(\varphi^+q)(\varphi q)(qA)].$$

Here $p = p_1 + p_2$, $q = p_2 - p_1$; p_2 and p_1 are the four-momenta of the initial and final deuterons; φ^+ and φ are their wave functions; $(\varphi q) \equiv \varphi_\alpha q_\alpha$; and the factor i is due to the requirement of Hermiticity. It is easy to see that the CP-noninvariant term is zero for real photons. Because of the factor q^2 it describes a contact interaction of the electron with the deuteron. The presence of this term must lead to CP-odd correlations of the types $\boldsymbol{\zeta}_1 \cdot \mathbf{n}$ and $\boldsymbol{\zeta}_2 \cdot \mathbf{n}$, where $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ are the polarization vectors of the initial and final deuterons, and $\mathbf{n}$ is the normal to the plane of the reaction.

A correlation of this type can also arise when CP is conserved, if we include, besides the single-photon exchange, also exchange of two or more photons between e and d. Correlations caused by exchange of an even number of photons must change sign when the electron is replaced by a positron, whereas correlations caused by a CP-odd term in the vertex or by the exchange of $2n+1$ photons do not change sign for $e^+ \longleftrightarrow e^-$. Since the contribution of two-photon exchange is much larger than that of the other many-photon diagrams, it is in principle a simple matter to determine a CP-odd term in the vertex, if its magnitude in comparison with the other terms in the vertex is larger than 10^{-4}.

There must also be CP-odd (P-even) moments for nuclei with larger spins. The number of such moments is J for integer J and $J - 1/2$ for half-integer J. [We recall that the number of CP-even (and P-even) moments is $2J+1$.]

A detailed discussion of CP-odd effects in the inelastic scattering of electrons by nucleons is given in [23a]. In the process $e^\pm + N \to e^\pm + \Gamma$, where Γ is a system of strongly interacting particles, $(\Gamma \neq N)$, the following correlations are of interest:

1. Right-left asymmetry in the scattering of electrons by nucleons polarized along the normal to the plane of the interaction (correlation of type $\boldsymbol{\zeta}_1 \cdot \mathbf{n}$).

2. Polarization of the nucleon N arising from the decay of the resonance N^* in the reaction $e^\pm + N \to e^\pm + N^*$, $N^* \to N + \pi$. (Correlation of the type $\boldsymbol{\zeta}_2 \cdot \mathbf{n}$.) This sort of correlation can also occur with CP conserved, owing to interference of a resonance channel with the nonresonance background, and therefore its interpretation requires care.

28. Other Electromagnetic Reactions

Possible effects of CP noninvariance in colliding electron-positron beams have been considered in [28a,28b], and such effects in the photoproduction of π mesons have been considered in [28c,28d].

The decay of parapositronium into three photons has been discussed theoretically in [28e]. The authors of [28f] assert that they have observed this CP-forbidden decay. Violation of CP-invariance in Compton scattering by protons has been discussed in [28g].

29. Electromagnetic Transitions in Nuclei

Let

$$\delta = |\delta|\,e^{i\eta} = \frac{\langle J_b \| L+1 \| J_a \rangle}{\langle J_b \| L \| J_a \rangle}$$

be the ratio of the reduced matrix elements of a mixed transition between nuclear states with angular momenta J_a and J_b. As was first pointed out in [29a], δ must be real if there is T invariance.

If T invariance is violated in the electromagnetic interaction ($Z0^+$) or in nuclear forces ($X0^+$ interaction), then δ must be complex: $\eta \neq 0$.

Possible experiments for the measurement of the quantity η can be divided into two groups: a) experiments to measure $\cos\eta$; b) experiments to measure $\sin\eta$. An experiment of type a) is the simple measurement of the angular correlation of two γ-ray quanta, the first of which is mixed. It is best to use for the experiment nuclei with $|\delta| \sim 1$. One such nucleus is Hg^{198}, and measurements with it have given[29b] $\cos\eta = -1.037 \pm 0.079$. This corresponds to $|\sin\eta| \leq 0.3$.

The quantity $\sin\eta$ can be measured with experiments which measure T-odd correlations in a mixed transition[29c]:

$$(\mathbf{kj}_b)(\mathbf{kj}_a \times \mathbf{j}_b),$$
$$(\mathbf{kj}_a)(\mathbf{kj}_a \times \mathbf{j}_b),$$
$$(\mathbf{k}\boldsymbol{\sigma})(\mathbf{kj}_a \times \mathbf{j}_b),$$
$$(\mathbf{k}\boldsymbol{\sigma})(\mathbf{kj}_a \times \mathbf{j}_b)(\mathbf{j}_a\mathbf{j}_b),$$
$$(\mathbf{kj})(\mathbf{kj} \times \boldsymbol{\varepsilon})(\boldsymbol{\varepsilon}\mathbf{j}).$$

Here $\mathbf{k}$ is the momentum of the photon, σ is its circular and ϵ its linear polarization, and the vector $\mathbf{j}_a(\mathbf{j}_b)$ characterizes the initial (final) state of polarization of the nucleus. If $\mathbf{j}_a$ does not appear, the nucleus is completely unoriented. If $\mathbf{j}_a$ appears to the first power, the nucleus is polarized, if to the second power it is aligned, and so on. In the last expression $\mathbf{j}$ means $\mathbf{j}_a$ and $\mathbf{j}_b$.

The polarization state of the nucleus b can be determined by measuring T-invariant correlations in its decay:

$$(\mathbf{k}'\mathbf{j}_b)^{2n},$$
$$(\mathbf{k}'\boldsymbol{\sigma}')(\mathbf{k}'\mathbf{j}_b)^{2n+1},$$
$$(\mathbf{j}\boldsymbol{\sigma}')(\mathbf{k}'\mathbf{j}_b)^{2n},$$
$$(\boldsymbol{\varepsilon}'\mathbf{j}_b)(\mathbf{k}'\mathbf{j}_b)^{2n}.$$

The polarization state of the nucleus a can be prepared by means of a magnetic field, fields in crystals, allowed β transitions of the Gamow-Teller type, forbidden β transitions, and so on.

If it is a β transition that serves as the polarizer of nucleus a (a P-odd correlation of the type $\mathbf{p}_e\cdot\mathbf{j}_a$, where $\mathbf{p}_e$ is the momentum of the electron), the analyzer for nucleus b is a γ transition (a correlation of the type $(\mathbf{k}'\cdot\mathbf{j}_b)$, and the T-noninvariant transition is described by a correlation of the type $(\mathbf{k}\cdot\mathbf{j}_b)(\mathbf{k}\cdot\mathbf{j}_a\times\mathbf{j}_b)$, then the correlation experimentally measurable will be of the form

$$(\mathbf{k}\mathbf{k}')(\mathbf{p}_e\mathbf{k}\times\mathbf{k}').$$

This type of correlation has been measured in the decay

$$\mathrm{Ca}_{21}^{47}\left(\tfrac{7}{2}-\right)\xrightarrow{\beta^-}\mathrm{Sc}\left(\tfrac{5}{2}-\right)\xrightarrow{M1+E2}\mathrm{Sc}\left(\tfrac{3}{2}-\right)\xrightarrow{E2}\mathrm{Sc}\left(\tfrac{7}{2}-\right).$$

When the counting rate was measured for two directions of $\mathbf{k}'$ (L and R) with fixed directions of $\mathbf{p}_e$ and $\mathbf{k}$, the result obtained was

$$\varepsilon=\frac{R-L}{R+L}=-(0.44\pm 5.7)\,10^{-4},$$

from which a value of η could be derived:

$$\sin\eta=(-2\pm 29)\cdot 10^{-2}\ [29]$$

A much more stringent restriction on the value of η has been obtained from a measurement of the $\beta\gamma\gamma$ correlation in ruthenium → rhodium → palladium decays:

$$\mathrm{Ru}_{44}^{106}\xrightarrow{\beta^-}\mathrm{Rh}_{45}^{106}(1^+)\xrightarrow{\beta^-}\mathrm{Pd}_{46}^{106}(2^+)\xrightarrow{M1+E2}\mathrm{Pd}\,(2^+)\xrightarrow{E2}\mathrm{Pd}\,(0^+).$$

The value found for ϵ was $\epsilon = (5 \pm 7)\times 10^{-4}$, from which we have for the magnitude of η:

$$\sin\eta=(3\pm 4)\cdot 10^{-2}\ [29f].$$

In [29g] observations were made on $\gamma\gamma$ correlation in the decay of a polarized excited state of the nucleus Ti^{49}, which was produced in the capture of a polarized neutron by the nucleus Ti^{48}:

$$\mathrm{Ti}_{22}^{48}+n\longrightarrow\mathrm{Ti}_{22}^{49}\left(\tfrac{1}{2}-\right)\xrightarrow[0.31\ \mathrm{MeV}]{M1+E2}\mathrm{Ti}\left(\tfrac{3}{2}-\right)$$
$$\times\xrightarrow[1.38\ \mathrm{MeV}]{E2}\mathrm{Ti}\left(\tfrac{7}{2}-\right).$$

(This experiment had been suggested in [29h].) If the magnitude of δ for the mixed transition is 2.2, the result found in [29g] for the magnitude of η is

$$\eta=(-0.4\pm 2)\cdot 10^{-2}.$$

Unfortunately the restrictions that these results impose on the sizes of the $X0^+$ and $Z0^-$ interactions are not very severe. This is due to the fact that C-noninvariant (and CP-noninvariant) effects in nuclei must be small, to the extent that these transitions can be reduced to the interaction of individual free nucleons with photons. The nucleon vertex is indeed "automatically" CP-even (cf. Sec. 27).

30. Dipole Moments of Particles

A dipole moment d of a particle (a vertex of the type $d\sigma\mathbf{E}$) can arise only if there is violation not only of CP invariance, but also of P invariance.[30a]

If we write d in the form el, where e is the electric charge of the electron, then the upper limits on the values of l given by experiments already done are

for the neutron $l_n < 5\cdot 10^{-20}$ cm [30b],
for the proton $l_p < 1.3\cdot 10^{-13}$ cm [30c],
for the electron $l_e < 2\cdot 10^{-21}$ cm [30d],
for the muon $l_\mu < 1\cdot 10^{-17}$ cm [30e].

We remark that the most exact data on the quantity d_e are obtained from experiments on the measurement of the dipole moments of atoms in atomic beams,[30d] and not from atomic spectra[30f,30g] or from experiments on the scattering of electrons.[30h,30i]

The upper limit $l_p < 1.3\times 10^{-13}$ is obtained[30d] from the experimental data on the Lamb shift between the levels $^2S_{1/2}$ and $^2P_{1/2}$ in the hydrogen atom. (A dipole moment d_p would produce a contribution to this shift in second order in d_p.)

The upper limits on the quantity l_p determined from experiments on the measurement of relaxation times of nuclear spins in monatomic gases are several times as large: $l_p \le 6\times 10^{-13}$ cm for He^3; $l_p \le 4\times 10^{-13}$ cm for Xe^{129} (see the discussion of this question in [30n,30o,30p]).

The effect of the screening, which hinders the measurement of the dipole moment of a proton in an atom, is discussed in [30j].

In the case of the $Z0^+$ interaction a dipole moment of the neutron arises owing to the $Z0^+$ and $W0^-$ interactions. The expected order of magnitude is 10^{-20}–10^{-21} cm. A value of this same order can be expected if there exists a $Z0^-$ interaction a factor e weaker than the $W0^-$ interaction.

In the case of the $X0^+$ interaction, d_n is the result of E, E, $X0^+$, and W interactions and its value is of the order of 10^{-24} cm. The same value would be given by an $X0^-$ interaction weaker by three orders of magnitude than the $W0^-$ interaction.

Estimates of the quantity d_n in a model with an intermediate W boson are given in [30k], and an estimate without the W boson in [30l].

We note that if the W boson has an electric dipole moment, as is assumed in [30m] (see also [30q]), then there must exist dipole moments of both baryons and leptons (e and μ). The search for d_e and d_μ is interesting in connection with possible interactions of types L, M, and Γ (see Sec. 5).

If it is shown experimentally that $l_n \ll 10^{-21}$ cm, and a weak interaction $W0^-$ exists, then this will unambiguously exclude the hypothesis of the $Z0^+$ interaction.

IV. SLOW PROCESSES

31. General Remarks on Weak Currents

Violation of CP invariance in weak interactions can in general arise from a great many mechanisms. We shall consider only a few of them, confining ourselves to the framework of the V – A theory.

One can imagine, for example, that besides the term $e\gamma_\alpha(1+\gamma_5)\nu$ the lepton currents contain terms of the types $g_T\sigma_{\alpha\beta}q_\beta$, $g_{T'}\sigma_{\alpha\beta}q_\beta\gamma_5$, $g_S q_\alpha$, $g_P q_\alpha\gamma_5$, with complex coefficients g. From the data on K_{e2} and $K_{\mu2}$ decays[31a] it can be concluded that $g_S, g_P < 1/100\, m_K$. From the data on K_{e3} and $K_{\mu3}$ decays[31b] it follows that $g_T, g_{T'} < 1/3m_K$. The restrictions that follow from the μ decay are weaker ($g_T < 1/600$ MeV), since the energy released in this decay is smaller (cf. [31c]).

Violation of CP invariance in hadron currents can be due to a difference of the phases of the vector and axial-vector constants. Such a phase difference arises, for example, if we make the Cabibbo rotation[31d] not around the y axis in the U-spin space, but around some axis in the xy plane, and also choose the angles of rotation somewhat different for the vector and axial-vector currents, as is proposed in [31e]. Unfortunately, the predictions on the basis of this scheme which are made in [31e] are based on a definite choice of the current subjected to the rotation. There are as yet no physical grounds for the idea that it is necessary to choose the current $\bar{n}p$ for this role. A difference of the phases of the V and A currents should give CP- and P-odd correlations of the type $[\boldsymbol{\zeta}_1 \times \boldsymbol{\zeta}_2]\cdot \mathbf{p}$.

As has been pointed out in [31f] a nonconservation of CP can be due to complex values of the coefficients of terms caused by so-called currents of the second kind.[31g] A phase difference between currents of the first and second kinds should give CP-odd but P-even correlations of the type $[\mathbf{p}_1 \times \mathbf{p}_2]\cdot\boldsymbol{\zeta}$, even in cases in which only one of the currents, V or A, is effective.

A difference of the phases of the individual terms of the total V – A current, for example of the hadron and lepton currents (or of the hadron currents with $\Delta S = 0$ and with $\Delta S = 1$) does not give any nonconservation of CP, nor in general any observable effects, since these phases can be transformed away by multiplying the ψ operators of the particles involved in the currents by phase factors.

If the violation of CP invariance is due to differences of the phases of individual terms of the weak Lagrangian (so that the Lagrangian is not the square of the total current), then this will not manifest itself in first order in the weak interaction. There will, however, be observable CP-odd effects in second order in the weak interaction.[31h,31i]

32. Decays of $K^\pm$ Mesons

A violation of CP invariance will appear in $K^\pm_{\mu3}$ decays if there exist two types of vector currents with $\Delta S = 1$ and with relative phase different from zero (cf., e.g., [31d]). In this case the parameter ξ in the amplitude for $K_{\mu3}$ decay

$$[(p_K + p_\pi)_\alpha + \xi (p_K - p_\pi)_\alpha]\, u_\mu \gamma_\alpha (1+\gamma_5)\, u_\nu$$

will be complex and there will be a correlation of the type $\xi_\mu\cdot[\mathbf{p}_\pi \times \mathbf{p}_\mu]$, proportional to Im ξ. Experimentally we have values Im $\xi = 0.15 \pm 0.75$[32a] and Im $\xi = 0.15 \pm 0.4$.[32b] It is easily verified that if there is nonconservation of CP the polarization of the muons normal to the plane of the reaction will be of opposite signs for $K^+_{\mu3}$ and $K^-_{\mu3}$ decay.

In $K^\pm_{2\pi}$ decays nonconservation of CP can appear in the failure of the relation $\Gamma(K^+ \to \pi^+\pi^0) = \Gamma(K^- \to \pi^-\pi^0)$. The discrepancy will, however, be electromagnetically small, even if there is a relatively strong violation of CP invariance by an $X1^-$ interaction. We note that experimentally $\Gamma(K^+_{\pi2}) = 20.9 \pm 0.4$[32c] and $\Gamma(K^-_{\pi2}) = 25 \pm 3.3$ percent.[32d]

In the case of the Z1 interaction a considerable CP-odd effect can be expected in decays $K^\pm \to \pi^\pm\pi^0\gamma$. The photons in these decays can be emitted either by the Z1 interaction or by the ordinary electromagnetic interaction E. Since the contributions of real intermediate states are different in these two mechanisms, $\Gamma(K^+ \to \pi^+\pi^0\gamma) \neq \Gamma(K^- \to \pi^-\pi^0\gamma)$. The question of the effects of a Z1 interaction has been discussed in [32h].

A nonconservation of CP in the decays $K^\pm \to \pi^\pm\pi^0\pi^0$ and $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ can manifest itself both in inequality of the corresponding partial widths for K^+ and K^- mesons and in different π-meson spectra. This question has been treated theoretically in [32e].

The violation of T invariance in the decays $K \to \mu\nu\gamma$ and $\pi \to e\nu\gamma$ has been discussed in [32f] and [32g].

33. Leptonic Decays of Baryons and Neutrino Reactions

Because of its small energy release the β decay of the neutron is insensitive to possible modifications of the lepton current and to the presence of currents of the second kind (see [33a,33b]). CP-odd correlations can appear in this decay if there is a phase difference φ between the vector and axial-vector currents which is different from zero or π. The angular distribution of the electrons and protons in the decay of polarized neutrons has been measured in [33c]. The coefficient D of the term $\boldsymbol{\zeta}_n\cdot[\mathbf{v}_e \times \mathbf{v}_\nu]$ was found to be 0.04 ± 0.05. The authors concluded from this that $\varphi = 175° \pm 10°$.

Leptonic decays of hyperons can be used to look for CP-odd correlations of the type $\boldsymbol{\zeta}\cdot[\mathbf{P}_e \times \mathbf{P}_\nu]$, where $\boldsymbol{\zeta}$ is a unit vector in the direction of polarization of the incident hyperon or of the baryon produced in the decay (cf., e.g., [33d]). There are as yet no experimental data of this sort on leptonic decays (cf., e.g., [33e]).

The possibility of using neutrino reactions and looking for analogous CP-odd correlations in them has been discussed in [33f–33j].

34. Weak Decays of Nuclei

Possible ways of looking for violations of CP invariance in β decays of nuclei have been discussed in a number of papers. T-odd spin correlations have been calculated in [34a]. $\beta\gamma$ and $\beta\alpha$ correlations have been treated in [34b,34c]. A measurement[34d] of the $\beta\gamma$ correlation in the decay of polarized Mn^{52} nuclei did not reveal any violation of T invariance, but the accuracy of the experiment was low ($120° < \varphi < 270°$). The use of the Mössbauer effect to determine T-odd $\beta\gamma$ correlations has been discussed in [34e].

An interesting case is that of the decay of RaE, in which there is a $1^- \to 0^+$ transition. The anomalous character of the spectrum of RaE indicates that the first-forbidden axial-vector and vector matrix elements cancel to very high accuracy. As has been pointed out in [34f,34g], this can be the case only if their phase difference φ is close to 180° ($\varphi = 180°$ if T invariance holds). In [34h] it is concluded, on the basis of measurements of the spectrum of RaE, that $\varphi = 175.5° \pm 1°$. The accuracy 1° is evidently exaggerated, if we remember that in the interpretation of the spectrum one needs a knowledge of second-forbidden matrix elements. Experimental[34i] and theoretical[34j] studies of the longitudinal polarization of the electrons in the decay of RaE have not revealed any violation of T invariance, to an accuracy of 5 percent. A detailed discussion of the experiments mentioned in this section is contained in the book [34k].

35. Nonleptonic Decays of Λ^0 and $\Sigma^\pm$ Particles

As is well known, each channel of nonleptonic decay of a hyperon with $J = 1/2$, for example $\Sigma \to p\pi^0$, is characterized by four numbers: the partial width $\Gamma = |s|^2 + |p|^2$ and the correlation parameters

$$\alpha = \frac{2\,\mathrm{Re}\, s^*p}{|s|^2+|p|^2}, \quad \beta = \frac{2\,\mathrm{Im}\, s^*p}{|s|^2+|p|^2}, \quad \gamma = \frac{|s|^2-|p|^2}{|s|^2+|p|^2},$$

where s and p are the respective amplitudes of the waves with $l = 0$ and $l = 1$. (We remark that $\alpha^2 + \beta^2 + \gamma^2 = 1$.)

If CP parity is conserved and if we neglect the interaction of the particles in the final state, the amplitudes s and p must be real, and consequently $\beta = 0$. To include the interaction in the final state, we must multiply each amplitude with a given isospin I by the quantity $e^{i\varphi}$, where φ is the phase shift for the scattering of a π meson by the baryon in question with given J, l, and I and with total energy equal to the mass of the decaying hyperon.

There is experimental information on the phases for πN scattering (cf., e.g., [35a]), which are necessary for the analysis of the decays $\Sigma^+ \to p\pi^0$, $\Sigma^+ \to n\pi$, $\Sigma^- \to n\pi^-$, $\Lambda^0 \to p\pi^-$, and $\Lambda^0 \to n\pi^0$. These phases are given in Table VIII.

Table VIII. Phase shifts for πN scattering

Momentum \ Phase Shift	φ_1 ($I=\frac{1}{2}$, $l=0$)	φ_3 ($I=\frac{3}{2}$, $l=0$)	φ_{11} ($I=\frac{1}{2}$, $l=1$)	φ_{31} ($I=\frac{3}{2}$, $l=1$)
$99.5\,\frac{\text{MeV}}{c}$ (Λ)	$\sim+7°$	$\sim-4°$	$\sim0°$	$\sim0°$
$189.0\,\frac{\text{MeV}}{c}$ (Σ)	$\sim+9°$	$\sim-11°$	$\sim0°$	$\sim-3°$

As for the parameters Γ, α, β, γ, they are given in Table IX, which is compiled on the basis of data given in [35b,35c].

A violation of CP invariance would have to have the consequence that the angle determined from the relation $\tan(\varphi_p - \varphi_s) = \beta/\alpha$ differs from the angle of scattering of the π meson by the nucleus. The comparison can actually be made only for the decay $\Lambda^0 \to p\pi^-$. If we use the fact that for this decay the rule $\Delta T = 1/2$ is satisfied, we have $\varphi_p - \varphi_s = -6.5° \pm 0.5°$. From the ratio $\beta/\alpha = -0.29 \pm 0.39$[35d] it follows that $\varphi_p - \varphi_s = -16° \pm 20°$.

In the decay of Σ hyperons there are amplitudes with $I = 1/2$ and $I = 3/2$. If the $\Delta T = 1/2$ rule holds, then

$$A(\Sigma^- \to n\pi^-) = s^- + p^- = s_{3/2} + p_{3/2},$$

$$A(\Sigma^+ \to n\pi^+) = s^+ + p^+ = \frac{1}{3}(s_{3/2} + 2s_{1/2}) + \frac{1}{3}(p_{3/2} + 2p_{1/2}),$$

$$A(\Sigma^+ \to p\pi^0) = s^0 + p^0 = \frac{\sqrt{2}}{3}(s_{3/2} - s_{1/2}) + \frac{\sqrt{2}}{3}(p_{3/2} - p_{1/2}).$$

From the smallness of the parameters α^- and α^+ for the decays $\Sigma^- \to n\pi^-$ and $\Sigma^+ \to n\pi^+$ and the large value of α^0 we can conclude that either $s_{3/2} \simeq 0$ and $p_{3/2} + 2p_{1/2} \simeq 0$, or else $p_{3/2} \simeq 0$ and $s_{3/2} + 2s_{1/2} \simeq 0$. In either case the relative phase of the amplitudes s^+ and p^+ is not small, even if CP is conserved (because of the cancellation of the real parts), and the phases of the amplitudes s^0 and p^0 are expressed to good accuracy in terms of the phase shifts for πN scattering.

In the first case

$$s^0 \sim e^{i\varphi}, \quad p^0 \sim 2e^{i\varphi_{31}} + e^{i\varphi_{11}} \sim e^{i\frac{2\varphi_{31}+\varphi_{11}}{3}},$$

$$\varphi_p^0 - \varphi_s^0 \approx \frac{2\varphi_{31}+\varphi_{11}}{3} - \varphi_1 \approx -11^0, \quad \beta^0 = \alpha^0 \tan(\varphi_p - \varphi_s) \approx +0.17.$$

In the second case

$$s^0 \sim 2e^{i\varphi_3} + e^{i\varphi_1} \sim e^{i\frac{2\varphi_3+\varphi_1}{3}}, \qquad p^0 \sim e^{i\varphi_{11}},$$

$$\varphi_p^0 - \varphi_s^0 \simeq \varphi_{11} - \frac{2\varphi_3+\varphi_1}{3} \approx 4^0, \quad \beta^0 = \alpha^0 \tan(\varphi_p - \varphi_s) \approx -0.06.$$

These are tentative values because of the inaccuracies of the phase-shift analysis of πN scattering (cf., e.g., [35a,35f,35g]). Accordingly, a measurement of the parameter β for the decay $\Sigma^+ \to p\pi^0$, together with a sufficiently accurate determination of the phases from

Table IX. Parameters for nonleptonic decays of hyperons

Decay	10^{10} sec^{-1}	α	β	γ
$\Lambda^0 \to p\pi^-$	0.262 ± 0.028	$+0.62 \pm 0.05$	-0.18 ± 0.24	0.78 ± 0.06
$\Lambda^0 \to n\pi^0$	0.130 ± 0.060	$+0.68 \pm 0.17$		
$\Sigma^+ \to p\pi^0$	0.646 ± 0.030	-0.84 ± 0.15		
$\Sigma^+ \to n\pi^+$	0.584 ± 0.030	-0.05 ± 0.09		
$\Sigma^- \to n\pi^-$	0.606 ± 0.015	-0.16 ± 0.21		
$\Xi^- \to \Lambda^0\pi^-$	$0.575^{+0.36}_{-0.17}$	-0.48 ± 0.08	-0.06 ± 0.3 *)	0.90 ± 0.03 *)
$\Xi^- \to \Lambda^0\pi^0$	$0.35^{+0.06}_{-0.04}$	-0.39 ± 0.17	~ 0.33	~ 0.91

*These averages do not include the data from the laboratory of the University of California at Los Angeles: $\beta = -0.63 \pm 0.16$ and $\gamma = +0.46 \pm 0.22$.

experiments on πN scattering, would[35h] enable us not only to check the conservation of CP parity in this decay, but also to choose between the two solutions allowed by the rule $\Delta T = 1/2$.

The effects we are here considering, in which CP-noninvariance is manifested in decays of Λ and Σ hyperons, must occur if the interaction responsible for the violation of CP invariance is of type $X0^+$ or $X1^\pm$. In both cases the expected magnitude of the effect is in general very small: $\Delta\varphi \sim 0.1°$. As has been pointed out in [35e], this quantity can be some tens of times larger if an $X1^+$ interaction changes the isotopic spin by $5/2$. An $X1^+$ interaction with $\Delta I = 5/2$ would give transitions $K_2^0 \longleftrightarrow K_1^0$ by acting along with a weak CP-invariant interaction $W1^+$ with $\Delta I = 3/2$ or $5/2$. The strength of this $X1^+$ interaction would then be about an order of magnitude weaker than that of the ordinary weak interaction. An $X1^+$ interaction with $\Delta I = 5/2$ could not manifest itself in the decays $\Lambda^0 \to p\pi^-$ and $\Lambda^0 \to n\pi^0$, since the maximum possible ΔI for these decays is $3/2$. This sort of interaction could, however, manifest itself in the decays of Σ particles. It could also manifest itself in $K_{\pi 3}$ decays.

36. Nonleptonic and Radiative Decays of Ξ and Ω Particles

Since the phase shift for scattering of π mesons by Λ particles is not known, more accurate determination of the parameter β in the decays $\Xi^- \to \Lambda^0\pi^-$ and $\Xi^0 \to \Lambda^0\pi^0$ could not in itself give an answer to the question as to whether CP parity is conserved in these decays. The same is true of the decays of the Ω^- particle: $\Omega^- \to K^-\Lambda$, $\Omega^- \to \pi^-\Xi^0$, and $\Omega^- \to \pi^0\Xi^-$. The study of the hyperons is of interest from another point of view. The point is that in the decays of these hyperons we can test whether there exist interactions with $|\Delta S| > 1$.

The interaction $X2^-$ together with $W0^-$ could lead to the decay $K_2^0 \to 2\pi$ if it were weaker than the weak interaction by a factor of 100 to 1000. It is easy to see that such an interaction would give decays with $|\Delta S| = 2$:

$$\Xi^- \to n\pi^-,\quad \Xi^0 \to p\pi^-,\quad \Xi^0 \to n\pi^0,\quad \Xi^- \to 2\pi^- p \text{ and so on}$$
$$\Omega^- \to \Lambda^0\pi^-,\quad \Omega^- \to \Sigma^0\pi^-,\quad \Omega^- \to \pi^0\Sigma^-,\quad \Omega^- \to K^- n,$$
$$\Omega^- \to \pi^-\Sigma^{0*} \to \pi^-(\pi^-\Sigma^+) \text{ or } \pi^-(\pi^+\Sigma^-) \text{ and so on}$$

In this case the fractional widths of the two-particle decays would be of the order 10^{-4} to 10^{-6}. Experimentally[36a]

$$\Gamma(\Xi^- \to n\pi^-)/\Gamma(\Xi^- \to \Lambda^0\pi^-) < 0{,}5\%.$$

No upper limits on the widths of the other decays listed above are given in the literature. It is relatively easy to find such a limit for the decay $\Xi^0 \to p\pi^-$.

The interaction $X3^\pm$ in combination with the weak interaction $W1^\pm$ would lead to transitions $K_2^0 \longleftrightarrow K_1^0$. This interaction would give decays with $|\Delta S| = 3$:

$$\Omega^- \to n\pi^-,\quad \Omega^- \to p2\pi^- \text{ and so on}$$

It could be expected that the fractional widths of these decays would be of the order 10^{-4} to 10^{-6}.

Let us also point out here that the Ξ^- and Ω^- hyperons are exceptionally suitable objects for settling the question as to whether there exist $Z2^-$ and $Z3^\pm$ interactions of hadrons with photons, changing the strangeness of the hadrons by two or three units. If one of these interactions is responsible for the observed decay $K_2^0 \to 2\pi$, the effective $X2^+$ interaction converting K_2^0 into K_1^0 is proportional to

$$X2^+ \sim Z2^- \times W0^- \times E \quad\text{or}\quad X2^+ \sim Z3^\pm \times W1^\pm \times E,$$

where E is the ordinary electromagnetic interaction, which absorbs a virtual photon emitted by the $Z2^-$ or $Z3^\pm$ interaction. It follows from this that the expected strength of the $Z2^-$ or $Z3^\pm$ interaction must be all told 10 to 100 times smaller than that of the weak W interaction. Therefore it can be expected that the fractional widths of the decays

$$\Xi^- \to \Delta^-\gamma \to (\pi^- n)\gamma,\quad \Omega^- \to \Delta^-\gamma \to (\pi^- n)\gamma,$$
$$\Xi^0 \to n\gamma,\quad \Omega^- \to \Sigma^-\gamma$$

will in this case be of the order 10^{-2}—10^{-4}.

37. Nonleptonic Decays of Hyperons and Antihyperons

As was first pointed out in[37a], if CP invariance is violated the partial widths of the decays of hyperons and antihyperons are not equal (whereas the total widths are of course equal). Let us consider, for example, the decays

$$\Sigma^+ \to p\pi^0, \quad \Sigma^+ \to \pi^+ n \text{ and } \tilde{\Sigma}^+ \to \tilde{p}\pi^0, \quad \tilde{\Sigma}^+ \to \tilde{n}\pi^-.$$

Nonconservation of P parity is of no importance for the effects in question, and therefore we shall consider only one of the waves, for example the p wave (there is no interference between the contributions of the s and p waves to the partial widths). As has been pointed out above, the amplitudes p^0 and p^+ can be expressed in terms of the amplitudes with definite isotopic spin, $p_{3/2}$ and $p_{1/2}$, in the following way:

$$p^0 = \frac{\sqrt{2}}{3}[|p_{3/2}|e^{i(\varphi_{13}+\Delta_{13})} - |p_{1/2}|e^{i(\varphi_{11}+\Delta_{11})}],$$

$$p^+ = \frac{1}{3}[|p_{3/2}|e^{i(\varphi_{13}+\Delta_{13})} + 2|p_{1/2}|e^{i(\varphi_{11}+\Delta_{11})}].$$

Here the phases φ are determined by the scattering of the π meson by the nucleon, and the phases Δ by the violation of CP invariance.

The amplitudes for decay of the antihyperon are of the form

$$\bar{p}^0 = \frac{\sqrt{2}}{3}[|p_{3/2}|e^{i(\varphi_{13}-\Delta_{13})} - |p_{1/2}|e^{i(\varphi_{11}-\Delta_{11})}],$$

$$\bar{p}^+ = \frac{1}{3}[|p_{3/2}|e^{i(\varphi_{13}-\Delta_{13})} + 2|p_{1/2}|e^{i(\varphi_{11}-\Delta_{11})}].$$

It is easy to understand this form of the amplitudes $\bar{p}^0$ and $\bar{p}^+$ if we note that, first, the phase shifts for scattering of the π meson by the nucleon and the antinucleon in states with the same T are equal, and second, that the terms of the Lagrangian responsible for the decay of the hyperon and antihyperon are each other's Hermitian adjoints. It is easy to see that if $\Delta_{11} \neq \Delta_{13}$, then $|\bar{p}^0| \neq |p^0|$ and $|\bar{p}^+| \neq |p^+|$.

If the π meson and nucleon are in a state with definite isotopic spin, as is the case, for example, in the decays $\Lambda^0 \to p\pi^-$ and $\Lambda^0 \to n\pi^0$, then, as can be seen from the foregoing discussion, the partial widths for decay of the hyperon and antihyperon are equal (to the accuracy with which the $\Delta I = 1/2$ rule holds).

As for the asymmetry coefficients α and β, because of the difference of the relative phases of the s and p waves these coefficients will be different for particles and antiparticles even in the case of a single isotopic channel. The experimental data on the partial widths of the Λ and Σ hyperons have an accuracy of the order of 5 percent

$$\left(B_\Lambda = \frac{\Gamma(\Lambda \to \pi^- p)}{\Gamma(\Lambda \to \pi^- p) + \Gamma(\Lambda \to \pi^0 n)} = 0.675 \pm 0.028,\right.$$

$$\left. B_\Sigma = \frac{\Gamma(\Sigma^+ \to \pi^0 p)}{\Gamma(\Sigma^+ \to \pi^0 p) + \Gamma(\Sigma^+ \to \pi^+ n)} = 0.525 \pm 0.02\right).$$

There are no corresponding data for the antihyperons.

38. Radiative Decays of Hyperons and Antihyperons

A detailed theoretical treatment of the decays $\Sigma^+ \to p\gamma$ and $\Lambda^0 \to n\gamma$ has been given in [38a-38c]. With the nonconservation of P parity taken into account, the amplitudes for these decays are of the form[38c]

$$M = u_2(A + B\gamma_5)\hat{k}\hat{e}u_1,$$

where u_1 and u_2 are the wave functions of the initial and final baryons, $\hat{k} = k_\mu\gamma_\mu$, $\hat{e} = e_\mu\gamma_\mu$, k is the four-momentum of the photon and e is its wave function (sic), and A and B are complex numbers. The imaginary parts of these numbers are different from zero even if CP parity is conserved. This is due to the fact that in these decays there are intermediate states lying on the mass shell:

$$\Sigma^+ \to \pi^0 p \to p\gamma, \qquad \Sigma^+ \to \pi^+ n \to p\gamma,$$

$$\Lambda^0 \to \pi^0 n \to n\gamma, \qquad \Lambda^0 \to \pi^- p \to n\gamma.$$

The imaginary parts of the amplitudes A and B caused by real intermediate states can be calculated by using the experimental data on the amplitudes for photoproduction of π mesons and on π-mesonic decays of hyperons. The parameters characterizing a radiative decay can be expressed in terms of the numbers A and B in the following way:

$$\Gamma_\gamma = (|A|^2 + |B|^2)\frac{1}{\pi}\left(\frac{m_1^2 - m_2^2}{m_1}\right)^3,$$

$$\alpha_\gamma = \frac{2\,\mathrm{Re}\,AB^*}{|A|^2 + |B|^2}, \quad \beta_\gamma = \frac{2\,\mathrm{Im}\,AB^*}{|A|^2 + |B|^2}, \quad \gamma_\gamma = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2}.$$

The parameter α characterizes the asymmetry of the angular distribution of the photons in the decay of completely polarized hyperons; this distribution is of the form $1 - \alpha\cos\chi$, where χ is the angle between the unit vectors in the directions of the hyperon polarization $\boldsymbol{\zeta}$ and the photon momentum **k**.

The parameter α also characterizes the degree of circular polarization of the photons. Data on the relative phase of A and B can be obtained by measuring the parameter β (or γ). (We recall that $\alpha^2 + \beta^2 + \gamma^2 = 1$.) To determine the parameter β it is necessary to measure the angular distribution of linearly polarized photons either in the decay of polarized hyperons or together with a measurement of the polarization of the nucleons. Even a measurement of β, however, does not by itself make it possible to settle whether or not CP parity is conserved in the decay, since one cannot from this determine the imaginary parts of A and B.

One can settle whether CP parity is conserved in radiative decays of hyperons if one compares the radiative decays of hyperons and antihyperons.[38d] As has already been pointed out, imaginary parts of amplitudes caused by violation of CP invariance change sign when we go from particles to antiparticles, while the imaginary parts caused by real intermediate states do not change sign. The result is that for the Z1± interactions, for example, the partial widths of chargemirrored radiative decays of hyperons and antihyperons will be different.

Experimental results are $\Gamma(\Sigma^+ \to p\gamma)/\Gamma(\Sigma^+ \to p\pi^0) = 0.37$ percent[38e]; 0.17 percent[38f] Radiative decays of antihyperons have not been observed.

V. CONCLUDING REMARKS

39. The Absolute Difference Between Particles and Antiparticles

Although so far only the decay $K_2^0 \to \pi^+\pi^-$ has been observed, it is unlikely that the violation of CP invariance would not lead to other effects. Of course these effects could be extraordinarily small. For example, if the CP-noninvariant interaction has a constant of the order of 10^{-17} and changes the hypercharge by two units (an $X2^+$ interaction), it will manifest itself practically only in decays of neutral K mesons. In principle, however, even if with a very small and practically unobservable probability, even such an ultraweak interaction gives other effects which we have discussed: the asymmetries in the decays of particles and antiparticles will be different not only in sign, but also in magnitude, the partial widths of decays of hyperons and antihyperons will be different, and so on. Such effects will be quite observable if the constant of the CP-noninvariant interaction is large (only two or three orders of magnitude smaller than the constant of the weak interaction or of the strong interaction).

The existence of CP-noninvariant effects allows us to establish an absolute, not merely relative, difference between particles and antiparticles.

It is easy to indicate an appropriate thought experiment even in the case of the ultraweak $X2^+$ interaction. In fact, if the oscillation of a beam of K^0 mesons produced in the reaction $\pi^- p \to K^0\Lambda^0$ is described by the parameter r, then the behavior of a beam of $\widetilde{K}^0$ mesons produced in the reaction $\pi^+\widetilde{p} \to \widetilde{K}^0\widetilde{\Lambda}^0$ is described by the parameter 1/r (see Sec. 8). Accordingly, by observing the leptonic decays of neutral K mesons or their absorption in matter and exchanging information, two experimenters, one among us here on the Earth, and the other a representative of the nearest stellar civilization, can find out whether a prospective meeting between them will be safe or result in annihilation.

We emphasize that if one has available only CP-invariant interactions it would be impossible to get the answer to this question (cf. [39a]).

40. Absolute Helicity

The violation of CP invariance enables us to introduce an absolute concept of right and left. Even before the discovery of the decay $K_2^0 \to \pi^+\pi^-$ one could, for example, define as left the direction of polarization of the μ^+ in the decay $\pi^+ \to \mu^+\nu$, and as right the direction of polarization of the μ^- in the decay $\pi^- \to \mu^-\widetilde{\nu}$. An isolated observer could not, however, find out what he was dealing with, π^+ or π^-, since he would have no way to find out what his laboratory and he himself were made of—of matter or of antimatter.

If CP invariance is violated, this last question is easily settled, and consequently helicity takes on an absolute meaning.

Whereas with conservation of CP and nonconservation of P the helicities of particles and antiparticles were equal in magnitude and opposite in sign, there is now some difference in the actual magnitudes of the "screws" possessed by particles and antiparticles. Accordingly, the elementary particles have become carriers of handedness, like living beings.[40a,40b]

This result changes radically the idea which existed up to 1964, that the total Lagrangian describing the interaction of elementary particles has the same symmetry properties as the Lorentz interval $t^2 - x^2$. It was believed that this Lagrangian is invariant with respect to space and time translations, space and Lorentz rotations, and also reflections of the space and time axes. The conviction that it is invariant under reflections was not shaken even in 1956, when nonconservation of P parity was discovered. According to the hypothesis of combined invariance[40c,d,e] the physical operator for space inversion is the operator CP. Accordingly, by sacrificing invariance under charge conjugation, one could save the principle according to which the Lagrangian is invariant not only with respect to translations and rotations of the axes of the four-dimensional space, but also with respect to reflections of these axes.

If CP invariance is violated, as apparently follows from the existence of the decay $K_2^0 \to \pi^+\pi^-$, then there is no invariance with respect to reflections. At first glance one might again try to generalize the concept of reflection, by replacing the operator CP by CPT. This is, however, very different from the generalization from P to CP. The charge conjugation C, with which the operation P was supplemented, is not a geometrical operation, but lies outside geometry. The time reversal T, with which we can supplement CP, is a geometrical operation. If there is only CPT invariance, and no CP invariance, this means that we are obliged to accompany a reflection of the space axes with a reflection of the time axis, if we wish to leave the Lagrangian invariant. In a certain sense this is just as surprising as if it suddenly turned out that it was necessary to accompany a translation along the z axis with, say, a rotation around this axis, and there were no invariance with respect to either of these operations taken separately. Of course this analogy is farfetched; in the latter case we have to do with continuous operations, while the transformations P and T are discrete. Nevertheless there remains the fact, which is that the t and x axes are "connected" with each other; we cannot reflect only one of them and leave the Lagrangian invariant, we can only reflect both at once.

With a fixed direction of t the concepts "right" and "left," unlike, for example, the concepts "up" and "down," take on an absolute, not a relative, meaning (cf. [39a]). If all of the symmetry properties of particles were determined by the geometry of space, this would mean that our space has a definite helicity.[40f]

41. "Mirror" Particles

If we try to keep for the concept of helicity a merely relative meaning, it is necessary to put forward the hypothesis that there exist besides the ordinary particles so-called "mirror" particles. Then to each ordinary particle there must correspond a "mirror" particle with the same mass, spin, charge, and so on, but with opposite helicity. If we designate the transition from ordinary particles to "mirror" particles as operation A, we can require that the complete Lagrangian be CPA invariant and TA invariant (and consequently CPT invariant). The hypothesis of the existence of mirror particles was put forward in [41a] (see also [41b]) and discussed in detail in [41c].

As was shown in [41c], it follows from experiments already done that the interaction of "mirror" particles with ordinary particles cannot be either strong or electromagnetic. The gravitational interaction between "mirror" and ordinary particles must necessarily be the same as between ordinary particles. (It follows from this that there are no large numbers of "mirror" particles within the limits of the solar system.) An interesting possibility is that the same neutrinos interact with both "our" particles and the "mirror" particles.

If "mirror" particles exist, there must be some cosmological reason for the prevalence of particles of a definite helicity in our part of the world (just as there must be a reason for the prevalence here of baryons in comparison with antibaryons).

As has been pointed out, the interaction of left-handed and right-handed particles under the conditions that have been studied experimentally is evidently very weak. It is not excluded, however, that it may increase with increase of the energy of the interacting particles, and at high energies a search for "mirror" particles might be more promising.

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