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MULTIPLE ACCELERATION OF PARTICLES IN A POTENTIAL ELECTRIC FIELD BY MEANS OF CHARGE EXCHANGE

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1. INTRODUCTION

A whole discipline of modern nuclear physics, devoted to the study of interactions between complex nuclei, is associated with the use of accelerated heavy ions. The topics covered in this field are most diverse, and include as particular cases large number of problems previously dealt with only with the aid of light nuclei. As the particle energy increases above the Coulomb barrier, reactions peculiar only to interactions between complex nuclei set in, namely multinucleon transfer and total coalescence of two complex nuclei.

Heavy ions have already been used to synthesize new transuranic elements from 102 to 104, to discover nuclear isomers with anomalously short spontaneous-fission periods, to observe the phenomenon of proton radioactivity, and to obtain a number of other new results[1].

The development of this branch of nuclear physics is characterized by a tendency to use accelerated ions of increasingly heavier elements. Whereas a few years ago carbon, nitrogen, oxygen, or neon ions were accelerated, by now several elements up to argon are used, and the prospect of obtaining high-energy ions of bromine, iodine, and even uranium are already under discussion.[1-5]

The use of a beam of accelerated uranium ions would open up to nuclear physics new and broad possibilities of obtaining isotopes situated far from the stability region (neutron-deficient and neutron-excess nuclei), the synthesis of the far transuranic elements, and investigations of nuclear-mechanism reactions.

It is difficult as yet to estimate all the possibilities afforded by the use of uranium ions in nuclear research. One thing is clear—it would permit a qualitatively new approach to the study of interactions between complex nuclei.[1]

However, the problem of accelerating uranium ions may turn out to be utterly fantastic if attempts are made to solve by the presently available means. Indeed, particular interest attaches to the production of ions with energies higher than the Coulomb barrier (for heavy targets), i.e., ions with energies on the order of $\mathcal{E} = 5-10$ MeV/nucleon. Such energies are as yet unattainable for heavy ions. What are the obstacles preventing acceleration of uranium to such energies?

The most important characteristic of heavy ions, which becomes manifest during the acceleration process, is the specific charge, i.e., the ratio of the charge number Z to the mass number A. The specific charge of heavy ions is much smaller than unity. The inequality $Z/A \ll 1$ governs the peculiar behavior of heavy ions during the course of acceleration, compared with light ones ($Z/A \approx 1$ for the lightest ion, the proton).

In all existing charged-particle accelerators, the acceleration can be represented as a single or multiple passage of the particle through a region with a certain potential difference V. During one such passage, the ion energy per nucleon increases by an amount

$$\Delta\mathcal{E} = eV\frac{Z}{A}. \tag{1}$$

To accelerate an ion to a specified energy $\mathcal{E}$, the accelerating gap must be traversed K times:

$$K = \frac{\mathcal{E}}{\Delta\mathcal{E}} = \frac{A}{Z}\frac{\mathcal{E}}{eV}. \tag{2}$$

It is clear therefore that, other conditions being equal, the number of passages through the accelerating section of the trajectory increases in proportion to A/Z. Therefore a linear accelerator for heavy ions must either have A/Z times as many accelerating sections, or an accelerating potential A/Z times larger, or else operate in a mode intermediate between the two.

In a cyclic accelerator it is necessary, in any case, to increase the dimensions. This follows from the relation

$$\mathcal{E} = \text{const}\cdot H^2R^2\frac{Z^2}{A^2}, \tag{3}$$

(R is the ion orbit radius) which determines the energy of the ion (per nucleon), and from the practical impossibility of increasing the magnetic field H.

An isochronous cyclotron can operate with an accelerating potential that does not depend on A/Z. In a classical cyclotron, the accelerating potential, other conditions being equal, increases in proportion to A/Z.

The diameter of a cyclic accelerator capable of accelerating singly-charged uranium ions to an energy $\mathcal{E} \approx 10$ MeV/n, at technically attainable magnetic-field levels, would be of the order of 100 meters. Clearly, this can be only an annular accelerator. However, in

this case the intensity of the beam of accelerated particles will be 3—4 orders of magnitude lower than in the cyclotron.

To reduce the dimensions of the magnetic pole to 10 m, and thus bring the problem to the borderline of technical realizability, the source must deliver 24-fold charged uranium ions. At the present time such an approach is problematic, to say the least, since the yield of multiply-charged ions from a source decreases rapidly with increasing atomic number [1] for a specified Z/A ratio: $^{10}B_5^{+2}$(1.00), $^{16}O_8^{+3}$(0.80), $^{20}Ne_{10}^{+4}$(0.50), $^{28}Si_{14}^{+5}$(0.10), and $^{40}Ar_{18}^{+8}$(0.02) (the parentheses contain the relative intensity).

The difficulties referred to above are connected to a considerable degree with the fact that the hitherto employed acceleration principles, which are effective for light particles, allow only partially for the peculiar features of heavy ions. It is therefore very important to study the possibility of accelerating heavy ions by new methods capable of taking maximum account and making maximum use of the specific features of the problem in question. The distinguishing feature of the problem is that the heavy-element atoms are many-electron systems, i.e., in this case the charge of the ion becomes one more parameter that can be varied within wide limits during the acceleration process.

Variation of the ion charge uncovers new opportunities in accelerator technology. For example, the change of the ion charge at the end of the acceleration process makes it possible to solve relatively simply the problem of extracting heavy ions from a cyclotron with azimuthal variation of the electric field [6]. A change of the ion charge during the course of acceleration is used in accelerators of the tandem-generator type, and also in linear accelerators. Hortig has shown [3] that by varying the charge of the ion during the acceleration it is possible to accelerate ions many times in a potential electric field. This acceleration principle is qualitatively novel and apparently may be more effective than other means of heavy-ion acceleration. We consider below the possibility of accelerating ions in a potential electric field with the aid of multiple charge exchange during the acceleration process.

2. PRINCIPAL SCHEME OF MULTIPLE ACCELERATION IN A POTENTIAL ELECTRIC FIELD

All the known methods of accelerating charged particles are based on the property that the energy of the moving charge is altered by an electric field:

$$\Delta W = W_2 - W_1 = \int_{r_1}^{r_2} Z\,(\mathbf{E}\,d\mathbf{r}) \qquad (e=1). \tag{4}$$

If a constant charge moves in a potential electric field $\mathbf{E} = -\nabla\varphi$, then the change of energy on a closed loop (or on a section of the trajectory between points of equal potential) is equal to zero. This fact evidences that multiple acceleration of a fixed-charge particle is impossible in a potential electric field. On the other hand if the electric field is not potential, then the work done on a closed loop is not equal to zero. This property of nonpotential electric fields is used to accelerate particles of constant charge in resonant accelerators.

However, as follows from (4), to accelerate a charged particle there is no need for the electric field to be nonpotential. The necessary condition for the acceleration is that the product ZE be nonpotential. This can be realized also in potential electric fields if the particles have a variable charge Z.

Assume that the electric field in some region is characterized by a potential φ, and let some surface Φ divide the entire region into two parts: to the left of the surface Φ all particles have a charge Z_1, and to the right a charge Z_2 (Fig. 1). Thus, the surface Φ changes the charge of a particle: $Z_1 \to Z_2$ when it moves from left to right, and $Z_2 \to Z_1$ when it moves from right to left.

Let us consider the motion of a particle along a closed curve $\varphi_1\varphi_2\varphi_3\varphi_1$ crossing the surface Φ. In this case the particle with acquire in one revolution an energy increment $\Delta W = (Z_1 - Z_2)(\varphi_2 - \varphi_3)$.

Consequently, for multiple particle acceleration in a potential electric field it is necessary to have a system of charge-exchange devices; the system must have the properties of the surface Φ. Such a system can be realized, in particular, by using the particle charge exchange produced when a beam of ions passes through matter.

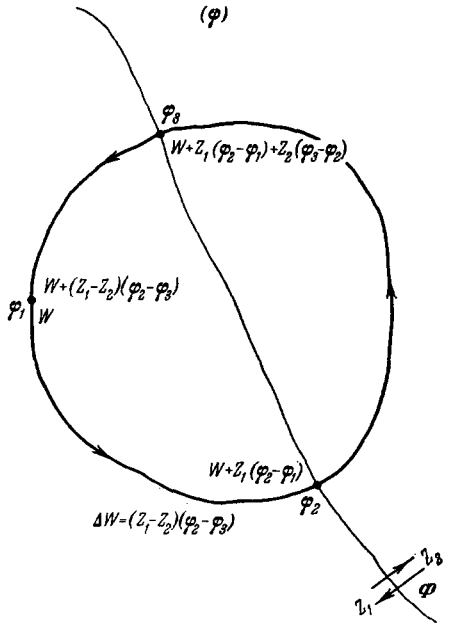

FIG. 1. Illustration of the principle of acceleration of a particle with variable charge in a potential electric field.

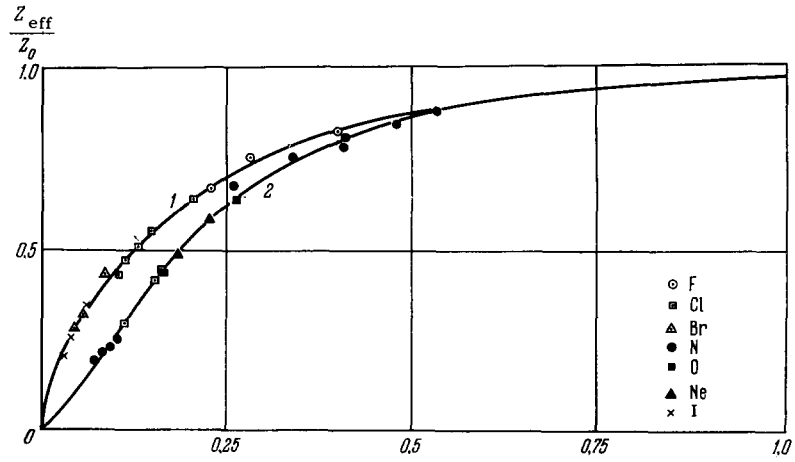

FIG. 2. Effective charges of ions after passing through condensed (1) and rarefied (2) targets. The abscissas represent the quantity $\sqrt{2\mathscr{E}}Z_0^{2/3}$.

3. CHARGE-EXCHANGE EFFECTS WHEN HEAVY PARTICLES PASS THROUGH MATTER*

The charge of a heavy ion moving through a medium changes as a result of interaction with the atoms of the medium. After passing through a sufficiently thick layer of matter, no further change in the statistical stationary charge distribution takes place, i.e., a statistical equilibrium distribution sets in.

In this case, the charge of each ion fluctuates about an equilibrium value determined by two opposing processes, electron loss and electron capture.

Experiments show that in order for a statistical distribution with respect to the charge states to be established, it is sufficient to have a layer of matter of 10^{16}–10^{17} at/cm^2, which yields charge-exchange cross sections on the order of $\sigma_0 \approx 10^{-17}$–$10^{-16}$ cm^2. The equilibrium distribution is then reached before a noticeable change takes place in the ion energy.

Let us define the effective ion charge Z_{eff} as the average over the statistical equilibrium distribution. The effective charge of the ions can be obtained also with the aid of a dynamic description of the interaction between the ion and the atoms of the medium [8]. The ratio of the cross sections for the loss and capture of electrons by the ion is determined by the ratio of the velocity of the electron on the ion orbit and the velocity of translational motion of the ion as a whole. The cross section for electron loss exceeds the capture cross section if the ion velocity is larger than the corresponding orbital velocity of the electron, and vice versa. In the case when these two velocities are equal, the cross sections are approximately the same. Equality of the cross section defines the effective ion charge.

In atoms with a large number of electrons, the electron velocity distribution can be obtained by the Thomas-Fermi method. Then we obtain for the mean velocity of the electrons in an atom with a nuclear charge Z_0 a value on the order of $v_0 Z_0^{2/3}$, where $v_0 = e^2/\hbar = c/137$ is the velocity of the electron in the ground state of the hydrogen atom.

We can expect the effective charge $Z_{eff}/Z_0 = q$ to be, for all ions, a universal function of the "reduced" ion velocity $v/Z_0^{2/3}$ [9]:

$$\frac{Z_{eff}}{Z_0} = f_1\left(\frac{v}{Z_0^{2/3}}\right) = f\left(\frac{\sqrt{2\mathscr{E}}}{Z_0^{2/3}}\right). \tag{5}$$

The experimental data [2,9,10] show that this is indeed the case (Fig. 2). Curve 1 gives the effective ion charge after passing through a carbon foil and curve 2, after passing through different gases at pressures of the order of 10 mm Hg. The effective ion charge depends on the density of the matter through which the ion beam passes, increases with increasing gas pressure, and is much higher in solids than in gases.

The dependence of the effective charge on the density of the medium can be explained, following Bohr and Lindhard [11] as follows: When the exciting ion moves in a relatively rarefied medium between two successive collisions, the ion has time to become deexcited, so that further ionization is from the ground state of the ion. With increasing density of the medium, the collision frequency increases and the time between two successive collisions becomes shorter than the lifetime of the excited state of the ion. This brings about a situation wherein the ionization in the denser media is from the excited states, thus making detachment of the electrons easier.

After the ion beam passes through the target, the ion-charge distribution function does not depend on the form of the initial normalized distribution function $N(Z, W)$ in charge space, and depends only on the particle energies and on the character of the target, so that for a condensed target (L) and a rarefied target (C) we have the following relations

$$LN(W, Z) = f_l\left[\frac{Z - Z_l(W)}{\Delta_l}\right], \quad CN(Z, W) = f_c\left[\frac{Z - Z_c(W)}{\Delta_l}\right],$$
$$Z_l(W) = \int Z f_l\, dZ, \qquad Z_c(W) = \int Z f_c\, dZ, \tag{6}$$
$$\Delta_l^2 = \int (Z - Z_l)^2 f_l\, dZ, \qquad \Delta_c^2 = \int (Z - Z_c)^2 f_c\, dZ.$$

*A detailed analysis of the latest papers can be found in the review [7].

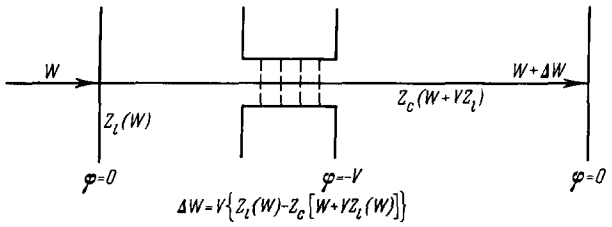

FIG. 3. Scheme of accelerating element.

4. SCHEME OF ACCELERATING ELEMENT

In [3] it is proposed to accelerate heavy ions by charge-exchanging them as they pass through thin targets of different density. Since for a given ion energy W the average charge is larger when the ions move in a condensed medium than in a rarefied one, they can be accelerated by means of the scheme illustrated in Fig. 3.

After the ions with energy W pass through a dense target L (foil), their average charge is $Z_l(W)$. After being accelerated by the potential V to an energy $W + VZ_l(W)$, the ions pass through a second charge-exchange device C (gas target), where they acquire an average charge $Z_c(W_1)$ and are decelerated by the potential V.

The average increment of ion energy after passing through such an accelerating element V_-CV_+L (V_+ and V_- denote the accelerating and decelerating actions of the electric potential V) is equal to

$$\Delta W(V, W)=V\{Z_l(W)-Z_c[W+VZ_l(W)]\}. \quad (7)$$

The resultant energy increment ΔW can be either positive or negative, depending on the ratio of $Z_l(W)$ and $Z_c(W_1)$.

It is of interest to consider the dependence of the energy increment ΔW, for a specified ion energy W, on the accelerating potential V. It is clear that ΔW has a maximum as a function of V, since ΔW = 0 when V = 0, ΔW > 0 when V is small, and ΔW becomes negative at sufficiently large V, when $Z_l(W) < Z_c[W + VZ_l(W)]$. The energy increment ΔW for the uranium ions was calculated under the assumption that the effective charges $Z_l(W)$ and $Z_c(W)$ can be determined with the aid of the curves shown in Fig. 2.

The function $\Delta W(V, \mathcal{E})$ is represented in Figs. 4 and 5 by families of plots of $\Delta W(V, \mathcal{E})|_{\mathcal{E}=\text{const}}$ and $\Delta W(V, \mathcal{E})|_{V=\text{const}}$. It follows from these plots that near its maximum the function $\Delta W(V, \mathcal{E})|_{\mathcal{E}=\text{const}}$ depends little on V. In addition, the optimal potential, which ensures the maximum energy increment, changes slightly in a large region of the energy interval. This means that in this energy region it is possible to ensure practically the maximum energy increment for a single chosen value of the potential.

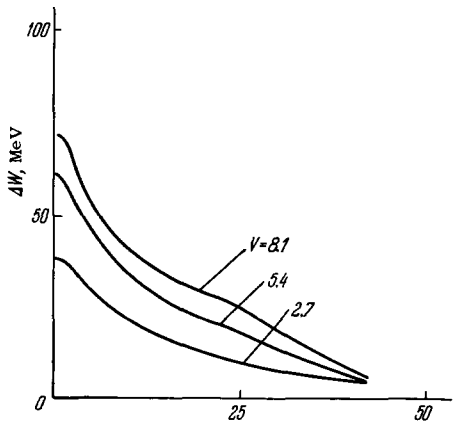

FIG. 5. Average increment of uranium ion energy in the accelerating element (for different accelerating potentials V in MV). The abscissas represent $\mathcal{E}$ in MeV.

The energy increment $\Delta W(V, \mathcal{E})|_{V=\text{const}}$ in one accelerating element, for a specified value of the potential, depends quite weakly on the energy and, up to appreciable energies, can be made not smaller than a definite value. For example, in an accelerating potential V = 5.4 MV, the energy increment remains not lower than 30 MeV up to energies $\mathcal{E} \approx 12.5$ MeV/n (W ≈ 3 GeV).

By constructing a system in which the periodicity element is the described accelerating element, we can realize multiple particle acceleration.

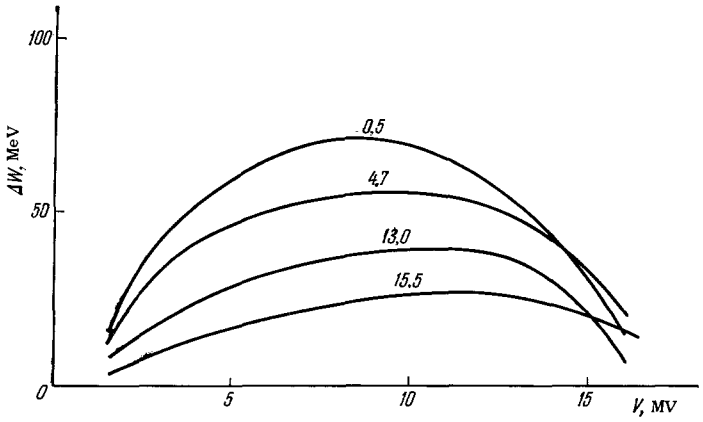

FIG. 4. Average increment of uranium-ion energy in the accelerating element (for differential initial ion energies $\mathcal{E}$ in MeV/n).

In particular, to accelerate uranium ions to $\mathcal{E} \approx 10$ MeV/n ($W \approx 2.5$ GeV) at an accelerating potential $V = 5.4$ MV, about 50 passages through the accelerating elements are necessary.

Acceleration is possible everywhere in the region $\Delta W > 0$, i.e., at least up to the energy W_f for which $Z_l(W_f) \approx Z_c(W_f)$. This determines the limiting ion energy attainable in the accelerator of this type:

$$\mathcal{E}_f \approx 0.125 Z_0^{4/3} \left(\frac{\text{MeV}}{n}\right), \quad W_f \approx 0.125 A_0 Z_0^{4/3} \text{ (MeV)}. \quad (8)$$

For uranium ions this amounts to $\mathcal{E}_f \approx 50$ MeV/n and $W_f \approx 12$ GeV.

As to acceleration of other heavy ions, we can conclude that all heavy ions, starting with $Z_0 \gtrsim 20$, can be accelerated to energies exceeding the Coulomb barrier for heavy targets.

5. ION SCATTERING IN MULTIPLE ACCELERATION

When a beam of ions passes through the accelerating element, the particles are scattered by the charge-exchanging targets, leading to loss of beam intensity.

Nuclear processes occur only at sufficiently high energies, with a cross section $\sigma_N \lesssim 10^{-24}$ cm^2, so that the scattering is determined essentially by the Coulomb interaction [4].

a) Small-angle scattering. Small-angle scattering takes place during the multiple scattering of the particle in the target material. The rms multiple-scattering angle $\overline{\vartheta_m^2}(W)$ in Coulomb interaction decreases rapidly with the energy of the scattered particles:

$$\overline{\vartheta_m^2}(W) = \frac{\text{const}}{W^2}. \quad (9)$$

For a particle injection energy W_0, the rms scattering angle after K passages through the accelerating elements is obtained by summing (9) over all energies and is equal to

$$\overline{\vartheta_{mK}^2} = \frac{\text{const}}{\Delta W \cdot W_0} \frac{K}{\frac{W_0}{\Delta W} + K} \quad (10)$$

provided the energy increment ΔW is independent of the particle energy W. Assuming that ΔW is constant, we obtain for large K

$$\overline{\vartheta_m^2} = \frac{\text{const}}{\Delta W \cdot W_0}, \quad (10a)$$

i.e., the average angular dimensions of the beam do not exceed the constant.*

Assuming that the scattering in the accelerating element is equivalent to the scattering in a carbon foil 14 μg/cm thick, we find that for uranium ions with injection energy $W_0 = 25$ MeV and $\Delta W = 30$ MeV the rms of the multiple-scattering angle is $\overline{\vartheta_m^2} \approx 1.5 \times 10^{-5}$.

b) Large-angle scattering. Large-angle scattering in thin targets is produced by single collisions, and is governed by the Rutherford law. Although the cross section for large-angle Rutherford scattering is small, any scattering through an angle larger than the accelerator aperture θ takes the particles out of the acceleration mode, and therefore can serve, in the case of multiple repetition, as a source of intensity loss. Since the cross section for scattering through angles $\vartheta \gtrsim \theta$ decreases with increasing energy like $1/W^2$, we can calculate in similar fashion the probability of scattering the particle through angles larger than the angular aperture of the accelerator in the case of multiple acceleration [4]

$$P(\theta) = \frac{\text{const}}{\Delta W \cdot W_0} \frac{\cos\theta}{\sin^2\theta} \sqrt{1 - \frac{A_0^2}{A_1^2}\sin^2\theta}, \quad (11)$$

where A_0 and A_1 are the mass numbers of the scattered and scattering particles.

For the foregoing accelerator parameters, with aperture $\theta = 3 \times 10^{-2}$, this probability is of the order of 10^{-3}.

We see therefore that the particle scattering is determined by the injection energy W_0 and by the energy increment ΔW in one passage through the accelerating element. Therefore beam intensity loss occurs essentially only during the first passages through the accelerating element, while the particle energy is low, i.e., it is determined by the initial acceleration region.

6. FOCUSING AND INTENSITY OF CHARGED PARTICLE BEAM

As a result of scattering by the targets of the accelerating element, the particle beam becomes defocused. Let us consider the equation for small oscillations of the particle about the equilibrium trajectory (the length of the periodicity element is assumed equal to unity):

$$x'' + \nu^2 x = \frac{1}{2\pi}\vartheta_x \delta(\psi - \psi_0); \quad (12)$$

x is the small deviation of the particle, ν the number of betatron oscillations in the periodicity element, ψ the (longitudinal) coordinate of the particle along the periodicity element, and ψ_0 the coordinate of the accelerating element.

Solving (12) and averaging over ψ, we obtain the rms deviation of a particle with energy W:

$$\overline{x^2}^{\psi} = \frac{1}{8\pi^2\nu^2}\vartheta_x^2(W), \quad \overline{y^2}^{\psi} = \frac{1}{8\pi^2\nu^2}\vartheta_y^2(W). \quad (13)$$

These expressions must be averaged over the scattering angles and summed over the particle energies W:

$$\langle x^2\rangle = \frac{1}{8\pi^2\nu^2}\langle\vartheta_x^2\rangle, \quad \langle y^2\rangle = \frac{1}{8\pi^2\nu^2}\langle\vartheta_y^2\rangle, \quad (13a)$$

with $\langle\vartheta_x^2\rangle = \langle\vartheta_y^2\rangle = \frac{1}{2}\langle\vartheta^2\rangle$ for the axially-symmetrical problem. It is shown in [4] that

*The estimate of $\overline{\vartheta_m^2}$ is too high, since it is based on the maximum charges of the Coulomb centers (charges of the nuclei). The actual ion scattering is apparently determined by the effective charge.

$$\langle\vartheta^2\rangle=\frac{td}{A_1}N_0\frac{\pi e^4Z_0^2Z_1^2}{\Delta W\cdot W_0}\left[1+\ln\frac{\theta}{\alpha^2Z_0Z_1(Z_0^{2/3}+Z_1^{2/3})^{1/2}}\frac{W_0}{m_0c^2}\right], \quad (14)$$

t is the thickness, d the density, Z the atomic number, A_1 the atomic weight of the target; Z_0 and A_0 are the atomic number and the atomic weight of the scattered particle; N_0 is Avogadro's number, m_0 the electron mass, and $\alpha = 1/137$.

Assuming that the particle distribution relative to the x and y axes is Gaussian, we obtain the beam intensity after a large number of passages through the accelerating element:

$$\frac{I}{I_0}=1-\exp\left(-8\pi^2\nu^2\frac{\theta^2}{\langle\vartheta^2\rangle}\right)-P(\theta). \quad (15)$$

For the indicated accelerator parameters, at a focusing hardness $\nu = 0.2$, the beam intensity of the accelerated uranium ions is $I/I_0 \approx 0.95$.

7. CHARGE AND ENERGY DISTRIBUTION FUNCTION OF THE PARTICLES IN THE ACCELERATOR

When considering the ion beam intensity loss in multiple acceleration, we used the average energy increment after the particles pass through the accelerating element (Eq. (7)). This means that the charge-state distribution function of particles with energy W was defined essentially as $\delta[Z - Z_{eff}(W)]$, where

$$Z_{\text{eff}}(W)=\begin{cases}Z_l(W) & \text{—effective charge of ions after passage through a condensed target,}\\ Z_c(W) & \text{—effective ion charge after passing through a rarefied target.}\end{cases}$$

Since the charge distribution of the particles has actually a certain width, not all particles can be captured into the acceleration mode at a specified injection energy W_0[5].

If the particle distribution in the energy and charge space is described by the function N(W, Z) on entering the basic accelerating element LV-CV+, then the particle distribution function $N_1(W, Z)$ on leaving the element is given by

$$N_1(W,\ Z)=LV\text{-}CV_+N(W,\ Z), \quad (16)$$

from which we can easily obtain with the aid of relations (6) (e = 1):

$$N_1(W,\ Z)=f_l\left[\frac{Z-Z_l(W)}{\Delta_l}\right]\int\int dZ_1\,dZ_2$$
$$\times f_c\left[\frac{Z_2-Z_c(W+VZ_2)}{\Delta_c}\right]N(W-VZ_1+VZ_2,\ Z_1). \quad (16a)$$

It is seen from (16a) that the particle distribution function can be represented in the form $N = f_l n(W)$; hence

$$n_1(W)=\int\int dZ_1\,dZ_2\,f_c\left[\frac{Z_2-Z_c(W+VZ_2)}{\Delta_c}\right]$$
$$\times f_l\left[\frac{Z_1-Z_l(W+VZ_2-VZ_1)}{\Delta_l}\right]n(W-VZ_1+VZ_2). \quad (17)$$

Integrating (17), we can find the variation of the particle distribution function after Δk passages through the accelerating element. In addition, the distribution function of the particles is altered by their injection and extraction from the accelerator, and also by losses during the acceleration process.

As shown above, the particle loss in multiple acceleration is small and occurs essentially in the initial acceleration region, i.e., in a rather narrow energy interval. It can be approximately assumed that the loss occurs at a certain specified ion energy W_i determined, for example, from the condition that the rms scattering angle be equal to the angular aperture of the accelerator.

The injection and extraction of the particles occur also in sufficiently narrow intervals, and the number of particles injected or extracted during a single acceleration is small compared with the total number of particles in the accelerator.

Under these conditions and for sufficiently small accelerating potentials V, when the integrands can be expanded in Taylor series, we obtain in second order in V the following equation for the distribution function[5]

$$\frac{\partial n}{\partial k}=\frac{\partial}{\partial W}\left\{\left[(V(Z_c-Z_l)+\frac{1}{2}V^2(Z_c'Z_c-Z_l'Z_c+Z_l'Z_l+Z_c'Z_l)\right]n\right.$$
$$\left.+\frac{1}{2}V^2(\Delta_l^2+\Delta_c^2)\frac{\partial n}{\partial W}\right\}+n_0\delta(W-W_0)-n_i(k)\,\delta(W-W_i)$$
$$-n_f(k)\,\delta(W-W_f), \quad (18)$$

where $n_0\Delta k$, $n_f\Delta k$, and $n_i\Delta k$ are the number of injected, extracted, and lost particles after Δk accelerations.

The result has a clear physical meaning. Since the operator L is the last to be applied in our chain, the charge distribution of the particles is a known function, determined for a specified particle energy by the operator L. This is indeed the case for all particles outside the accelerating element, i.e., for the majority of the particles in the accelerator, if the dimensions of the accelerating element are sufficiently small.

In the derivation of the kinetic equations we used a "universal time," namely the number of passages of all the particles through the accelerating element. Naturally, for particles with different energies W (and accordingly velocities v), the number of passages through the accelerating element in a time Δt will be different.

The variables k and t are connected by the simple relation $l\Delta k = v\Delta t$, where $l(v)$ is the length of the periodicity element. The function $l(v)$ is obtained from simple kinematic considerations and depends on the concrete accelerator variant. Introducing the time in which a particle of energy W traverses the periodicity element, or the "revolution time" $T(W) = l/v$, we obtain an equation for the time variation of the particle distribution function:

$$T(W)\frac{\partial n}{\partial t}=\frac{\partial}{\partial W}\left\{\left[V(Z_c-Z_l)+\frac{1}{2}V^2(Z_c'Z_c-Z_l'Z_c+Z_l'Z_l+Z_c'Z_l)\right]n\right.$$
$$\left.+\frac{1}{2}V^2(\Delta_c^2+\Delta_l^2)\frac{\partial n}{\partial W}\right\}+n_0\delta(W-W_0)-n_i(t)\,\delta(W-W_i)$$
$$-n_f(t)\,\delta(W-W_f). \quad (18a)$$

8. DETERMINATION OF THE COEFFICIENT OF PARTICLE CAPTURE IN THE MULTIPLE ACCELERATION MODE

We define the coefficient for the capture of the particles in the multiple acceleration mode as the ratio n_f/n_0 for the stationary acceleration mode as $t \to \infty$.

Since it can be assumed (see (5)) that the relative charges $Z_l/Z_0 = q_l$ and $Z_c/Z_0 = q_c$ are universal functions of the quantity $\epsilon = 2\mathscr{E}/Z_0^{4/3}$ for all elements, it is convenient to introduce the variable ϵ. Putting

$$-a = U(q_c - q_l) + \frac{1}{2}U^2(q_c'q_c - q_l'q_c + q_l'q_l + q_c'q_l),$$

$$b = \frac{1}{2}U^2(\delta_c^2 + \delta_l^2),$$

$$\delta_c = \frac{\Delta_c}{Z_0},\ \delta_l = \frac{\Delta_l}{Z_0},\ U = \frac{2V}{A_0 Z_0^{1/3}},$$

we obtain the following stationary equation for the determination of the coefficients n_f and n_l

$$\frac{d}{d\varepsilon}\left(-an + b\frac{dn}{d\varepsilon}\right) + \frac{2}{A_0 Z_0^{4/3}}[n_0\delta(\varepsilon-\varepsilon_0) - n_f\delta(\varepsilon-\varepsilon_f) - n_i\delta(\varepsilon-\varepsilon_i)] = 0 \quad (19)$$

with boundary conditions

$$j = an - b\frac{dn}{d\varepsilon} \to 0 \text{ for } \varepsilon \to 0,\ \varepsilon \to \infty,$$
$$n = 0 \text{ for } \varepsilon \leqslant \varepsilon_i,\ \varepsilon \geqslant \varepsilon_f.$$

The solution of (19) determines the energy distribution of the particles in the stationary acceleration mode, and the particle capture and loss coefficients

$$n(\varepsilon) = \frac{2}{A_0 Z_0^{4/3}} \exp\left(\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right) \times \left[n_i\theta(\varepsilon-\varepsilon_i)\int_{\varepsilon_i}^{\varepsilon} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right) + n_f\theta(\varepsilon-\varepsilon_f)\int_{\varepsilon_f}^{\varepsilon} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right) - n_0\theta(\varepsilon-\varepsilon_0)\int_{\varepsilon_0}^{\varepsilon} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right)\right], \quad (20)$$

$$\frac{n_f}{n_0} = \frac{\int_{\varepsilon_i}^{\varepsilon_0} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right)}{\int_{\varepsilon_i}^{\varepsilon_f} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right)},\quad \frac{n_i}{n_0} = \frac{\int_{\varepsilon_0}^{\varepsilon_f} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right)}{\int_{\varepsilon_i}^{\varepsilon_f} d\varepsilon \exp\left(-\frac{1}{b}\int_{\varepsilon_i}^{\varepsilon} a\,d\varepsilon\right)}. \quad (21)$$

Extrapolating the experimental data for the effective charges and the widths of the ion distributions over the charge states during stripping in solid and gaseous targets, of atoms with atomic numbers $Z_0 \gtrsim 50$ towards the heavier elements, we estimate the coefficient of capture of uranium ions in the acceleration mode.

At the chosen value of U, the energy increment $a(\epsilon, U) > 0$ in the energy interval $(\epsilon_i', \epsilon_f)$ and vanishes on its boundaries. The minimal energy of the ions in the accelerator should be taken to be the larger of the two quantities ϵ_i and ϵ_i' (ϵ_i is the ion energy at which the rms scattering angle is equal to the accelerator angular aperture).

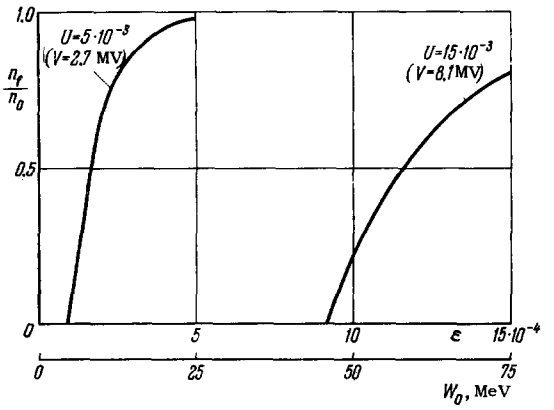

FIG. 6. Coefficient of capture of uranium ions in the multiple acceleration mode.

Figure 6 shows the coefficient of capture of uranium ions in the multiple-acceleration mode, as a function of the injection energy for accelerating potentials V = 2.7 MV and V = 8.1 MV,* (the potential V = 8.1 ensures maximum energy increment in the greater part of the energy interval ϵ_0, ϵ_f) in the accelerator with the parameters given above.

If we regard the capture coefficient as a function of the accelerating potential for a specified injection energy, then this function has a maximum at a potential which in general differs from that ensuring the optimal multiple-acceleration mode. Consequently, different accelerating potentials must be chosen in the region of the injection energy and in the region of multiple acceleration, or else a single potential which is optimal from the points of view of both capture and acceleration.

9. POSSIBLE ACCELERATOR VARIANTS

To realize an accelerator using the foregoing accelerating element, it is necessary to satisfy the following requirements: 1) the energy increment in the accelerating element should exceed the energy loss; 2) the intensity loss in multiple acceleration should be relatively low.

Estimates show that for uranium ions with an injection energy W_0 = 25 MeV and an accelerating potential V = 2.7 MV, the per-pass loss in the accelerating element (the equivalent thickness of the charge-exchange targets is 14 $\mu g/cm^2$ of carbon) will be of the order of 5% of the energy increment, Consequently, the energy loss does not greatly affect the acceleration process.

*It is of interest to consider an acceleration variant with minimum potential V capable of ensuring the operating reliability of the accelerating element.

The beam-intensity loss, as already seen, is determined by the initial energy region of the accelerator. However, in view of the large difference $\Delta Z = Z_l(W) - Z_c(W)$ at low energies it is possible to ensure a sufficiently large energy increment, so that the particles go rapidly out of the dangerous region.

For multiple ion acceleration one can use either a linear accelerator with a large number of accelerating elements, or a cyclic accelerator with one or several accelerating sections.

If 100 or more passages through the accelerating element are needed to attain the required energy, then the linear accelerator must consist of as many accelerating elements. If these elements are cheap and reliable, then the simultaneous use of many of them entails no economic or technical difficulties. The rms ion-energy dispersion on leaving the K-th stage of such an accelerator is of the order of

$$\frac{\sqrt{\overline{W^2}-\overline{W}^2}}{\overline{W}} \cong \frac{\Delta}{\langle Z_l - Z_c\rangle}\sqrt{\frac{2}{K}}, \quad \Delta = \Delta_l \approx \Delta_c, \qquad (22)$$

which yields for uranium ion a value of the order of $0.6/\sqrt{K}$.

It is possible in principle to realize multiple ion acceleration with the aid of at least one reliable accelerating element, i.e., to use a cyclic accelerator design. As seen from Fig. 2, $Z_{eff} \sim \sqrt{\mathcal{E}}$ on an appreciable section of the curve. Taking (3) into account, we can expect the dimensions of the magnetic system not to increase rapidly with increasing particle energy. In other words, an accelerator can be constructed with a magnetic track whose width is determined by the statistical scatter of the particle charges about the equilibrium value.

At a uranium-ion energy $\mathcal{E} \cong 50$ MeV/n, the effective charge is $Z_{eff} \cong 82$, and the number of ions with charges $Z \leq 72$ will not exceed one per cent (for a Gaussian charge distribution obtained by extrapolating the experimental data). It can therefore be assumed that at maximum uranium-ion energy the minimum specific charge is $Z/A \cong 0.3$. This means that the maximum hardness of the ions in the accelerator, corresponding to maximum energy at minimum charge, will be of the order of 3.5×10^6 G-cm. At a magnetic-field level $H = (1.5-1.7) \times 10^4$ G the radius of the magnetic system amounts to a value of the order of $R \approx 2-2.5$ m.

A cyclic accelerator of this type should preferably be one with tangent orbits, the accelerating element being located in the section where the orbits come in contact.

The ions can be focused by slotting the magnets and introducing focusing lenses between the accelerating element and the magnets. This creates a number of difficulties connected with the large scatter of the particle hardness and the presence of linear sections. A special study must be made to ensure stability of particle motion under such conditions.

The beam can be extracted from the accelerator by disturbing the symmetry of the magnet near the final orbit. It can be shown here that a sufficiently monoenergetic beam can be extracted near the point $Z_l \cong Z_c$ as a result of accumulation of particles with limiting energy.

10. CONCLUSION

The acceleration of heavy-element ions (uranium and others) raises a new physical problem having a number of distinct features compared with the acceleration of light particles. The method of accelerating heavy ions with variable charge in a potential electric field, proposed by G. Hortig, is therefore of interest from this point of view.

A multiple-action accelerator based on this principle differs greatly from resonant accelerators. The usual concept of synchronism and the concomitant concept of phase motion is not applicable to such an accelerator. A unique synchronism between the frequency at which the ion crosses the accelerating gap and the frequency of variation of the ion charge is maintained in the accelerator automatically. This makes it possible to accelerate ions continuously. A characteristic feature of the method is a jumplike change in the ion charge during the acceleration process. This change of charge plays in this case a role similar to that of sign reversal of the electric field in resonant accelerating gaps. It is effected by using the density dependence of the charge of the ion moving in matter. Owing to the appreciable magnitude of the charge-exchange cross sections, $\sigma_0 \sim 10^{-16}$ cm^2, the equilibrium charge is established in the ion beam by means of sufficiently thin targets ($t \sim 10^{+16}-10^{17}$ at/cm^2). Therefore the energy loss by the ion can be made smaller than the increment, or even negligibly small.

However, the energy loss in the target may become dangerous to the target itself; for example, it may damage a solid target. The need for producing a charge-exchange device having high reliability at sufficiently high accelerated-beam intensities is a separate technical problem.

The next most important problem is connected with the accelerating element as a whole. Structurally this element should apparently be similar to or identical with the tandem generator. However, the existing tandem generators have, in the main, low power and are designed for relatively small current loads. In addition they have, generally speaking, rather large dimensions. Therefore the question of the choice of an accelerating element and, possibly, the development of a special compact strong-current tandem generator, deserves special attention.

Special consideration should be given to problems connected with development of an ion-optical system ensuring stable operation of the accelerator. A distinguishing feature of such an accelerator is that ions

with quite high charge and velocity scatter can be situated simultaneously at a given point (scatter of the order of 15% and more). To avoid additional intensity loss, the ion-optical system must ensure stability of ion motion in a wide interval of charges and velocities.

Independently of the results of the detailed analysis of these problems, it can be asserted that intense beams of heavy ions can in principle be accelerated to high energies in a potential electric field, using the dependence of the ion charge on the density of the charge-exchange target.

The maximum attainable ion energy in such an accelerator is quite high, but not unlimited. With increasing energy, the difference between the equilibrium charges $Z_l(W)$ and $Z_c(W)$ in targets of different density tends to zero. Accordingly, the energy increment ΔW in the accelerating element vanishes and may even become negative. The point where ΔW vanishes indeed determines the energy limit. If we neglect the energy loss and assume that the potential V is sufficiently small, then the limiting energy W_f can be obtained from the approximate relation $Z_l(W_j) \cong Z_c(W_f)$ and turns out to be $W_f \simeq 0.125\, A_0 Z_0^{4/3}$ (MeV).

The $Z_l(W)$ and $Z_c(W)$ curves for uranium were obtained by extrapolating experimental data on the average charges of ions of the elements with $Z_0 \gtrsim 50$. The reliability of such an extrapolation is still in doubt. We have mentioned above the Bohr-Lindhard hypothesis, which ensures the existence of a nonzero difference $\Delta Z = Z_l - Z_c$. As noted by G. N. Flerov, this effect may be missing for very heavy ions with large effective charge (for example, uranium ions with $Z_{eff} \gtrsim 50$), since the lifetime τ of the ion in the excited state decreases with increasing Z_{eff}. It may turn out that the dependence of τ on Z_{eff} alters greatly both the value of ΔZ obtained from the extrapolation, and the estimate of the limiting energy. Since this effect cannot be verified experimentally as yet for heavy ions (there are no ions of appropriate energy), theoretical and semi-empirical calculations of ΔZ as a function of the energy become particularly important.

The estimate of the coefficient for capture of the particles in the acceleration mode was also based on the extrapolation of the experimental data on the equilibrium charge distribution of the ions in the low-energy region. This question is of great importance for the assessment of the beam intensity attainable in the accelerator, and the results need refining. The answer to this question influences the choice of the minimal initial ion energy, the minimum acceleration potential, and also decides whether pre-acceleration of the ions is necessary. The presence of reliable experimental data on the charge distribution functions of the heavy ions at low energies would make possible more accurate numerical calculations. Such calculations would serve as an important basis for the determination of the concrete design variant of the accelerator. Special notice should be taken of the fact that the energies needed to investigate charge exchange in this region are presently attainable even for the heaviest ions.

An impression is gained that the discussed acceleration principle is a natural solution to the problem of obtaining high-energy ions of heavy elements such as uranium. This follows, in particular, from the estimate of the limiting ion energy, which increases with the atomic and mass numbers of the element like $A_0 Z_0^{4/3}$. For uranium, for example, this factor is equal to $\sim 10^5$. Even of the dependence of the lifetime τ on Z_{eff} changes strongly the $\Delta Z(W)$ curve and lowers appreciably the estimated limiting energy $\mathcal{E}_f \cong 50$ MeV/n for uranium ions, the margin is larger by approximately one order of magnitude compared with the energy required in experiments (5–10 MeV/n), and therefore gives grounds for assuming that the principle of acceleration through the use of charge exchange of ions in matter is suitable for the solution of the problem and has a wide range of practical application. In addition, the growth of the charge during the acceleration process makes it possible to construct an accelerator with reasonable dimensions. These considerations give grounds for believing that this method makes considerable allowance for the specific nature of the problem and has a promising future.

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