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ELECTROMAGNETIC WAVES IN METALS IN A MAGNETIC FIELD

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1. Introduction

A characteristic feature of metals is their high conductivity, which is due to the very large density of carriers—conduction electrons. Because of this, it has been believed for a long time that low-frequency electromagnetic excitations cannot propagate in metals. The reasons were thought to be as follows:

The electromagnetic field in a metal is determined by Maxwell's equations

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{rot} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}. \qquad (1.1)^*$$

Here $\mathbf{E}$ is the high-frequency electric field vector, $\mathbf{H}$ the magnetic field intensity, $\mathbf{j}$ the current density vector, and c the speed of light. Let us eliminate the magnetic field from (1.1). Then for a monochromatic plane wave in an unbounded metal

$$\mathbf{E}(\mathbf{r},\ t) \sim \exp[i(\mathbf{kr} - \omega t)] \qquad (1.2)$$

Maxwell's equations become

$$k^2\mathbf{E} - \mathbf{k}(\mathbf{kE}) = \frac{4\pi i\omega}{c^2}\hat{\sigma}(\omega,\ \mathbf{k})\mathbf{E} + \frac{\omega^2}{c^2}\mathbf{E}. \qquad (1.3)$$

Here $\mathbf{k}$ is the wave vector, ω the wave frequency, and $\hat{\sigma}(\omega, \mathbf{k})$ the Fourier component of the high-frequency conductivity.

At high frequencies ($\omega \gg kv$) the spatial dispersion plays no role and $\hat{\sigma}$ does not depend on $\mathbf{k}$:

$$\sigma = \frac{ne^2}{m(\nu - i\omega)}, \qquad (1.4)$$

where e is the absolute value of the electron charge, m the effective mass, n the conduction-electron density, and ν the frequency of their collisions with the scatterers.

Substituting (1.4) in (1.3) and assuming $\mathbf{E} \perp \mathbf{k}$ we can readily obtain a dispersion equation for the electromagnetic waves

$$k^2c^2 = \omega^2 - \omega_0^2, \quad \omega_0^2 = \frac{4\pi ne^2}{m}, \quad \nu \ll \omega, \qquad (1.5)$$

where ω_0 is the plasma frequency of the metal.

It is obvious that this equation has no real solutions when $\omega < \omega_0$. In other words, for frequencies ω lower than the plasma frequency ω_0 the effective dielectric constant is negative and there are no natural electromagnetic oscillations in the metal (the external wave is totally reflected).

For typical metals $\omega_0 \sim 10^{16}$ sec^{-1}. We, on the other hand, are interested in radio-frequency electromagnetic waves. Their frequencies satisfy the condition

$$\omega \ll \omega_0. \qquad (1.6)$$

We can therefore neglect in Maxwell's equations the displacement current $\omega\mathbf{E}/4\pi i$ compared with the conduction current $\hat{\sigma}\mathbf{E}$. Then (1.3) goes over into

$$k^2\mathbf{E} - \mathbf{k}(\mathbf{kE}) = \frac{4\pi i\omega}{c^2}\hat{\sigma}\mathbf{E}. \qquad (1.7)$$

At room temperatures the frequency ν of the collisions between the conduction electrons and the scatterers is of the order of 10^{13} sec^{-1}. If $\omega \ll \nu$ the conductivity of the metal σ (1.4) is real. This means that the electric field in the metal leads to the appearance of a large dissipative conduction current. The solutions of the dispersion equation

$$k^2c^2 = i\cdot 4\pi\omega\sigma \qquad (1.8)$$

are in this case complex, and again there are no natural electromagnetic oscillations. On the other hand, the external electromagnetic wave incident on the surface of the metal attenuates within a distance of the order of 10^{-3}–10^{-5} cm from the surface. This is known as the skin effect (see, e.g., [1]).

At low temperatures the conduction-electron mean free path l in pure metals reaches values of the order of a centimeter. Therefore in the case of not too low frequencies ω the spatial dispersion becomes appreciable ($kl \gg 1$). The conductivity of the metal is then real but depends on the value of the wave vector k. The main contribution to σ is made in this case by those electrons on the Fermi surface, for which the condition

$$\mathbf{kv} = \omega, \qquad (1.9)$$

where v is the electron velocity, is satisfied. These electrons move in phase with the wave, interact most strongly with it[2], and cause absorption of the electromagnetic wave. This damping mechanism is known in the literature as Landau damping. Because of this mechanism, the electromagnetic waves cannot propagate in the metal in this case. The external wave penetrates only to a depth of the order of 10^{-4}–10^{-5} cm. Inasmuch as the depth of penetration of the field is in this case small compared with the carrier mean free path ($kl \gg 1$), this phenomenon is called the anoma-

*rot ≡ curl.

lous skin effect. Its theory was developed by Reuter and Sondheimer[3].

Thus, no undamped electromagnetic waves exist in metals in the absence of a constant magnetic field.

Konstantinov and Perel'[4], and also Agrain[5], were the first to call attention to the fact that the radio wave can penetrate inside the metal if there exists a strong constant magnetic field **H** directed along the normal to the surface of the crystal. In the magnetic field, the conductivity $\hat{\sigma}$ is a tensor whose elements are essentially of differing natures. If the conduction-electron spectrum is isotropic, then it follows from symmetry considerations that when $\mathbf{k} \parallel \mathbf{H} \parallel Oz$

$$\sigma_{xz} = \sigma_{zx} = \sigma_{yz} = \sigma_{zy} = 0. \tag{1.10}$$

Therefore the propagation of the electromagnetic wave is determined only by the transverse elements of the conduction tensor.

In strong electromagnetic fields H, satisfying the inequalities

$$|\nu - i\omega| \ll \Omega, \tag{1.11}$$

$$kR \ll 1, \tag{1.12}$$

(where $\Omega = eH/mc$ is the cyclotron frequency of the carriers and $R = v/\Omega$ is the Larmor radius), the spatial dispersion does not affect the transverse conductivity even when $kl \gg 1$, since there is no spatial dispersion in the xy plane when the vectors **k** and **H** are parallel.

In the case of one group of carriers, the transverse elements of the tensor $\sigma_{\alpha\beta}$ take the form[4]

$$\sigma_{xy} = -\sigma_{yx} = -\frac{nec}{H}, \tag{1.13}$$

$$\sigma_{xx} = \sigma_{yy} = \frac{ne^2(\nu - i\omega)}{m\Omega^2}. \tag{1.14}$$

It is obvious that the nondissipative Hall conductivity σ_{xy} is much larger than the hermitian part of the conductivity Re σ_{xx}, which determines the absorption of the electromagnetic field.

For circularly-polarized quantities

$$E_{\pm} = E_x \pm iE_y \tag{1.15}$$

Maxwell's equations take the form

$$(k^2 \mp 4\pi\omega c^{-2}\sigma_{\pm}) E_{\pm} = 0, \quad \sigma_{\pm} = \sigma_{xy} \pm i\sigma_{xx}. \tag{1.16}$$

Substituting (1.13) and (1.14) in (1.16) we can easily find the spectrum of the electromagnetic waves[4]:

$$\omega^{(\pm)} = \mp \frac{k^2cH}{4\pi ne}\left(1 \pm i\frac{\nu}{\Omega}\right). \tag{1.17}$$

A positively polarized wave has a negative effective dielectric constant. Consequently, this wave has an imaginary wave vector and attenuates over a length equal to the wavelength. The effective dielectric constant of a negatively polarized wave is positive, and this wave is weakly damped. Its relative damping is equal to ν/Ω and decreases with increasing magnetic field. It is called a helical electromagnetic wave (helicon). Excitations of this type with $kl \ll 1$ are well known in the case of a magnetoactive plasma[6]. Low frequency helicons in metals with $kl \ll 1$ were investigated also in [7,8].

Buchsbaum and Galt[9] have shown that Alfven electromagnetic waves can propagate in bismuth in the direction of a constant magnetic field **H**. The bismuth contains electrons and holes with equal densities ($n_1 = n_2$). Consequently, the Hall conductivity σ_{xy} is equal to zero. On the other hand, each diagonal element of the transverse conductivity is a sum of two terms similar to (1.14) when conditions (1.11) and (1.12) are satisfied, i.e.,

$$\sigma_{xx} = \sigma_{yy} = n(m_1 + m_2) c^2 H^{-2} (\nu - i\omega), \tag{1.18}$$

$$\nu = \frac{m_1\nu_1 + m_2\nu_2}{m_1 + m_2}. \tag{1.19}$$

The subscripts 1 and 2 pertain to electrons and holes, respectively.

Substituting (1.18) in (1.17) we can readily show that if

$$\omega \gg \nu \tag{1.20}$$

there exist in the metal two weakly-damped waves with a spectrum given by the formula[9]

$$\omega = kv_a - \frac{i\nu}{2}, \tag{1.21}$$

where

$$v_a = H[4\pi n(m_1 + m_2)]^{-1/2} \tag{1.22}$$

is the carrier Alfven velocity.

It must be emphasized that the results of Konstantinov and Perel'[4], as well as those of Buchsbaum and Galt[9], are valid only if the vectors **k** and **H** are parallel and are directed along a high-order symmetry axis (particular case—isotropic carrier dispersion). If the vectors **k** and **H** are not parallel or if their direction does not coincide with that of a crystal symmetry axis, then the spatial dispersion becomes appreciable and Landau damping sets in. The situation then becomes more complicated, and the possible existence of weakly-damped waves is no longer obvious.

A number of recent experimental and theoretical investigations have shown that various electromagnetic waves exist in metals at low temperatures and in the presence of a magnetic field. In addition to the various waves with wavelengths much larger than the dimensions of the electron orbits ($kR \ll 1$), short waves with lengths that are small compared with the characteristic orbit dimensions ($kR \gg 1$) can propagate in the metals. In this review we summarize the results of all these investigations.*

From the point of view of modern solid-state theory, the electromagnetic waves are collective oscillations of

*Some results on long-wave excitations are given in Vedenov's review[10].

the electron-hole plasma of the metal. They constitute the electromagnetic branch of Bose-type long-wave excitations. The general cause of these excitation is the fact that the motion of the electrons in the plane perpendicular to the magnetic field H is finite. Rotation of the electrons in the magnetic field leads to a strong decrease of the dissipative currents. As a result, the antihermitian part of the conductivity tensor may turn out to be large compared with the dissipative hermitian part. Then the antihermitial part of the conductivity (the dielectric constant) determines the spectrum of the natural electromagnetic oscillations in the metal, and the Hermitian part characterizes the damping. The smallness of the latter ensures the relatively weak damping of the waves.

It follows from general considerations that the dispersion law $\omega(\mathbf{k})$ of the electromagnetic waves is determined only by the properties of the electron-energy spectrum, i.e., by the shape and topology of the Fermi surface. In the first approximation, the relaxation time does not enter the dispersion law and can influence only the damping of the waves. Since the lengths of all the electromagnetic waves are large compared with the interatomic distances, the phase volume of these waves is small. They therefore make no noticeable contribution to the thermodynamics, but exert a strong influence on the high-frequency properties of the metals. The presence of undamped electromagnetic waves leads to anomalous transparency of the metal and to a number of new resonance effects.

2. General Formulation of the Problem. Dispersion Equation.

As noted in the introduction, in the low-frequency case (Eq. (1.6)) we can neglect the displacement current, and Maxwell's equations for a plane monochromatic wave in an unbounded metal take the form (1.7). The operator $\hat{\sigma}$ in these equations is the conductivity tensor $\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H})$ and takes into account the spatial and temporal dispersion, as well as the dependence on the constant electromagnetic field $\mathbf{H}$. From (1.7) follows the relation

$$\mathbf{kj} = k_\alpha \sigma_{\alpha\beta} E_\beta = 0, \tag{2.1}$$

which is identically equal to the condition for electric quasineutrality of the metal, $\rho' = 0$, where ρ' is the uncompensated volume density of the charge. The repeated vector indices α and β in (2.1) imply summation.

We choose a coordinate frame xyz such that the z axis is parallel to the magnetic field $\mathbf{H}$ and the x axis is transverse to $\mathbf{k}$ and $\mathbf{H}$. We shall also need the frame $x\eta\zeta$, in which the ζ axis is parallel to the vector $\mathbf{k}$. The angle between the vectors $\mathbf{k}$ and $\mathbf{H}$ is Φ:

$$k_x = 0, \quad k_y = k \sin\Phi, \quad k_z = k\cos\Phi. \tag{2.2}$$

The spectrum, damping, and polarization of the natural oscillations of the field are determined from the homogeneous system (1.7). The dispersion equation is obtained by equating to determinant of this system to zero. It can be written in the form

$$\left(\frac{k^2c^2}{4\pi\omega}\right)^2 - \mathrm{Det}\,\tilde{\sigma}_{\alpha\beta} - i\frac{k^2c^2}{4\pi\omega}\,\mathrm{Sp}\,\tilde{\sigma}_{\alpha\beta} = 0, \tag{2.3}$$

where

$$\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta} - \frac{(\sigma_{\alpha\alpha'}k_{\alpha'})(k_{\beta'}\sigma_{\beta'\beta})}{k_\gamma\sigma_{\gamma\delta}k_\delta} \tag{2.4}$$

is a "renormalized" two-dimensional conductivity tensor.

The elements of the tensor $\sigma_{\alpha\beta}$ should be obtained with the aid of the kinetic equation for the electronic distribution function

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} - e\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial \mathbf{p}}\right\} F(\mathbf{r}, \mathbf{p}, t) + \hat{I}(F) = 0. \tag{2.5*}$$

Here $\mathbf{p}$ is the electron quasimomentum and $\hat{I}(F)$ the collision operator.

We seek the solution of (2.5) in the form

$$F(\mathbf{r}, \mathbf{p}, t) = f_0(\varepsilon) - \chi(\mathbf{r}, \mathbf{p}, t)\frac{\partial f_0}{\partial \varepsilon_F}, \tag{2.6}$$

where

$$f_0(\varepsilon) = \left[\exp\left(\frac{\varepsilon - \varepsilon_F}{T}\right) + 1\right]^{-1} \tag{2.7}$$

is the Fermi distribution function, $\epsilon = \epsilon(\mathbf{p})$ is the energy of an electron with quasimomentum $\mathbf{p}$, ϵ_F is the Fermi energy, and T is the temperature in energy units.

In the approximation linear in $\mathbf{E}$, the function χ satisfies the equation

$$\left\{i(\mathbf{kv} - \omega) + \nu + \Omega\frac{\partial}{\partial\tau}\right\}\chi = e\mathbf{Ev}, \tag{2.8}$$

in the derivation of which from (2.5) we took into account the fact that, according to Lifshitz, Azbel', and Kaganov [11],

$$\frac{e}{c}\left[\mathbf{v}\frac{\partial}{\partial \mathbf{p}}\right]\mathbf{H} = \Omega\frac{\partial}{\partial\tau}.$$

Here $\tau = \Omega t$ is the dimensionless time of motion (phase) of the electron on the orbit in the magnetic field, $\Omega = eH/mc$ is the cyclotron frequency,

$$m = (2\pi)^{-1}\frac{\partial S(\varepsilon, p_z)}{\partial\varepsilon}$$

is the effective mass, and $S(\epsilon, p_z)$ is the area of the intersection of the equal-energy surface $\epsilon(\mathbf{p}) = \epsilon$ with the plane p_z = const. We have taken the collision integral in the form

$$\hat{I}(F) = \nu(F - f_0(\varepsilon)). \tag{2.9}$$

*$[\mathbf{vH}] \equiv \mathbf{v}\times\mathbf{H}$.

Equation (2.8) has a solution periodic in τ, with period 2π, in the form

$$\chi(\varepsilon, p_z, \tau) = \frac{e\mathbf{E}}{\Omega(\varepsilon, p_z)} \int_{-\infty}^{\tau} d\tau' \mathbf{v}(\varepsilon, p_z, \tau')$$
$$\times \exp\left[\frac{(\nu - i\omega)(\tau' - \tau)}{\Omega(\varepsilon, p_z)} + \frac{i\mathbf{k}}{\Omega(\varepsilon, p_z)} \int_{\tau}^{\tau'} \mathbf{v}(\varepsilon, p_z, \tau'')\, d\tau''\right]. \quad (2.10)$$

The conduction-current density is given by

$$\mathbf{j} = -\frac{2e}{(2\pi\hbar)^3}\int d^3p\mathbf{v}F = \frac{2e}{(2\pi\hbar)^3}\int d^3p \frac{\partial f_0}{\partial \varepsilon_F}\mathbf{v}\chi, \quad (2.11)$$

where $\hbar$ is Planck's constant divided by 2π. Making the following change of integration variables in (2.11):

$$\int d^3p \ldots = \int_0^{\infty} d\varepsilon \int_{-p_{z\max}(\varepsilon)}^{p_{z\max}(\varepsilon)} m(\varepsilon, p_z)\, dp_z \int_0^{2\pi} d\tau \ldots, \quad (2.12)$$

we obtain the following formula for the conductivity tensor:

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) = -\frac{2e^2}{(2\pi\hbar)^3}\int_0^{\infty} d\varepsilon \frac{\partial f_0}{\partial \varepsilon_F}\int_{-p_{z\max}}^{p_{z\max}} dp_z$$
$$\times \frac{m(\varepsilon, p_z)}{\Omega(\varepsilon, p_z)}\int_0^{2\pi} d\tau\, v_\alpha(\varepsilon, p_z, \tau)\int_{-\infty}^{\tau} d\tau' v_\beta(\varepsilon, p_z, \tau')$$
$$\times \exp\left\{\frac{1}{\Omega(\varepsilon, p_z)}\int_{\tau}^{\tau'}[\nu - i\omega + i\mathbf{k}\mathbf{v}(\varepsilon, p_z, \tau'')]\, d\tau''\right\}. \quad (2.13)$$

The dependence of the conductivity tensor $\sigma_{\alpha\beta}$ (2.13) on $\mathbf{k}$, ω, and $\mathbf{H}$ is much more complicated, and the dispersion equation cannot be solved in general form. We shall investigate later the asymptotic expressions for the conductivity tensor (2.13) in different limiting cases.

I. LONG WAVES ($kR \ll 1$)

3. Helicons (Isotropic Case)

a) Weak spatial dispersion. We consider first the simplest case of low frequencies and strong magnetic field, when the spatial dispersion plays no role, i.e.,

$$kv \ll \nu \ll \Omega. \quad (3.1)$$

When these inequalities are satisfied, the temporal dispersion is also negligible ($\omega \ll \nu$). Therefore the conductivity tensor of a metal with spherical Fermi surface will take the form [11] (in the xyz coordinate frame)

$$\sigma_{\alpha\beta} = \frac{nec}{H}\begin{pmatrix} \frac{\nu}{\Omega} & -1 & 0 \\ 1 & \frac{\nu}{\Omega} & 0 \\ 0 & 0 & \frac{\Omega}{\nu}\end{pmatrix}. \quad (3.2)$$

After simple transformations, the two-dimensional tensor of transverse conductivity $\tilde{\sigma}_{\alpha\beta}$ is reduced to the form

$$\tilde{\sigma}_{\alpha\beta} = \frac{nec}{H\cos\Phi}\begin{pmatrix} \frac{\nu}{\Omega\cos\Phi} & -1 \\ 1 & \frac{\nu}{\Omega\cos\Phi}\end{pmatrix}. \quad (3.3)$$

Formula (3.3) holds true for all values of the angle Φ, except a small vicinity of $\Phi = \pi/2$ ($\cot^2\Phi \gg \nu^2/\Omega^2$). Substituting (3.3) in the dispersion equation (2.3) we obtain the spectrum and damping of the electromagnetic waves in this wase. Just as in the case when $\mathbf{k}$ is parallel to $\mathbf{H}$, one solution has an imaginary wave vector. The second solution gives a weakly-damped wave

$$\omega = \frac{k^2cH\cos\Phi}{4\pi ne}(1 - i\Gamma), \quad (3.4)$$

$$\Gamma = \frac{\nu}{\Omega\cos\Phi}. \quad (3.5)$$

The spectrum of the wave is determined by the nondissipative Hall conductivity, and its damping by the dissipative one ($\tilde{\sigma}_{xx} + \tilde{\sigma}_{yy}$). Using Maxwell's equations (1.7), we can easily show that the electric-field vector in the wave is elliptically polarized in a plane perpendicular to the magnetic field $\mathbf{H}$:

$$E_y = iE_x\sec\Phi, \quad E_z = 0. \quad (3.6)$$

The part of the field $\mathbf{E}$ which is transverse to the wave vector $\mathbf{k}$ is circularly polarized. This is why the wave (3.4)–(3.6) is called a helicon. The existence of these waves in a magnetoactive plasma was discovered by Ginzburg [6] and Piddington (helicons in the ionosphere are known as "whistlers"). The possibility of their propagation in metals was indicated by Konstantinov and Perel' [4] and by Agrain [5].

The existence of the undamped helicon (3.4) in the limit as $\nu \to 0$ is due to the fact that the Hall current

$$\mathbf{j}_H = \frac{nec}{H^2}[\mathbf{EH}] \quad (3.7)$$

is orthogonal to the electric field $\mathbf{E}$. Consequently, the Joule loss, i.e., the wave damping, vanishes when $\nu \to 0$.

For typical metals with electron density on the order of unity per atom at low temperatures, when $\nu \sim 10^9\ \mathrm{sec}^{-1}$, the conditions (3.1) are well satisfied in fields $H \gtrsim 10^3$ Oe at frequencies $\omega \lesssim 10^2\ \mathrm{sec}^{-1}$.

b) Strong spatial dispersion. At higher frequencies, the length of the electromagnetic wave becomes smaller than the mean free path of the electrons, and the spatial dispersion begins to play an important role. In the limiting case

$$|\nu - i\omega| \ll kv \ll \Omega \quad (3.8)$$

there appears Cerenkov absorption of the wave by the electrons moving in phase with the wave (Landau damping). The asymptotic behavior of the elements of the conduction tensor differs significantly in this case from (3.2). According to [12], the expression for $\sigma_{\alpha\beta}$ takes the form

$$\sigma_{\alpha\beta} \atop (\alpha,\,\beta=x,\,y,\,z) = \frac{nec}{H}\begin{pmatrix} \frac{\nu-i\omega}{\Omega}+\frac{3\pi}{8}k_zR\,\mathrm{tg}^2\Phi & -1 & \mathrm{tg}\,\Phi \\ 1 & \frac{\nu-i\omega}{\Omega} & 0 \\ -\mathrm{tg}\,\Phi & 0 & 3\frac{\nu-i\omega}{\Omega(k_zR)^2} \end{pmatrix}. \quad (3.9)$$

The difference from the case (3.2) consists primarily in the change of the magnitude and direction of the high-frequency Hall current

$$\mathbf{j}_H=\frac{nec}{\mathbf{kH}}[\mathbf{kE}], \quad (3.10)$$

which now is perpendicular to the vectors **k** and **E**, Further, owing to the strong spatial inhomogeneity of the wave field in the **H** direction, the longitudinal conductivity σ_{zz} decreases by a factor $|k_z v/(\nu-i\omega)|^2$. Finally, the third difference from (3.2) lies in the appearance of Cerenkov absorption of the wave. The electrons for which the condition

$$\mathbf{k}\bar{\mathbf{v}}=\omega \quad (3.11)$$

is satisfied move on the average in phase with the wave and interact with it most effectively (the bar over **v** in (3.11) denotes averaging over the period of revolution of the electron in the magnetic field). The change in the momentum of the electrons under the influence of the wave causes them to drift transversely to the vectors **k** and **H**, i.e., in the direction of the x axis. As a result, the Hermitian part of the conductivity σ_{xx} acquires an additional component

$$\frac{3\pi}{8}\frac{nec}{H}k_zR\,\mathrm{tg}^2\Phi. \quad (3.12)^*$$

The magnitude of this component is of the order of $\sigma_0(kR)^2$, where

$$\sigma_0=\frac{ne^2}{mkv}$$

is the conductivity connected with the Landau damping in the absence of a magnetic field. Such a decrease of the Landau damping is explained by the fact that the rotation of the electrons in the magnetic field does not allow them to move strictly in phase with the wave. The electric field acts on the electron in opposite directions when the electron is on opposite sides of the orbit. However, owing to the small phase shift, the values of the field **E** within the span of the orbit differ by an amount on the order of $kR \ll 1$. The net action of the electric field on different sections of orbit cancel out accurate to terms of the order of $(kR)^2$. So large a decrease in the Cerenkov absorption causes the dissipative conductivity σ_{xx}, which determines the damping of the wave, to be much smaller than the nondissipative Hall conductivity, which characterizes the spectrum. This indeed makes it possible for a weakly damped wave to propagate under the conditions of the strong spatial dispersion (3.8).

It follows from the asymptotic formulas (3.9) that $\sigma_{x\zeta}=0$ and $\tilde{\sigma}_{xx}=\sigma_{xx}$. The expressions for the elements $\tilde{\sigma}_{x\eta}$ and $\tilde{\sigma}_{\eta\eta}$, as shown by calculation, are determined as before by (3.3). Therefore the spectrum and the polarization of a helicon do not change, and its damping Γ now takes the form[12]

$$\Gamma=\frac{\nu}{\Omega}\sec\Phi+\frac{3\pi}{16}kR\sin^2\Phi. \quad (3.13)$$

We emphasize that the Landau damping depends strongly on the angle between the vectors **k** and **H** and vanishes when $\Phi=0$. In transverse propagation, $\Phi=\pi/2$, ($\cos\Phi \lesssim \nu/\Omega$), there are no helicons.

Thus, the region of existence of the helicon is determined by the conditions

$$\omega,\ \nu,\ kv \ll \Omega. \quad (3.14)$$

Its spectrum and polarization are determined by (3.4) and (3.6) regardless of the ratio of the wavelength $2\pi/k$ and the electron mean free path $l=v/\nu$. The case of strong spatial dispersion (3.8) differs from (3.1) in the appearance of Landau damping.

Expressing k in terms of ω, we can rewrite the most stringent condition $kR \ll 1$ in the form

$$\frac{\omega}{\Omega} \ll \frac{v_a^2}{v^2}. \quad (3.15)$$

It follows from this that the maximum frequency of the helicon increases like the cube of the magnetic field.

4. Magnetohydrodynamic Waves (Isotropic Case)

We shall investigate whether electromagnetic waves can propagate in metals with equal electron and hole densities ($n_1=n_2=n$). Several different cases can be realized here.

a) Weak spatial dispersion. We consider first the simplest limiting case of strong magnetic field, when the following conditions are satisfied:

$$kv_s \ll \omega \ll \Omega_s \quad (s=1,\ 2). \quad (4.1)$$

In this case spatial dispersion plays no role, and in the general expression (2.13) for the conduction tensor we can put $k=0$. Because the compensation condition $n_1=n_2$ is satisfied, the Hall conductivity is equal to zero and the tensor $\sigma_{\alpha\beta}$ is diagonal. The elements of the transverse conductivity are determined by formulas (1.18) and (1.19), while the longitudinal conductivity

$$\sigma_{zz}=\sum_{s=1}^{2}\frac{ne^2}{m_s(\nu_s-i\omega)} \quad (4.2)$$

has the same form as in the absence of a magnetic field.

In our case, the conductivity tensor has a rather simple form, and we can use Maxwell's equations (1.7) directly to determine the spectrum of the natural electromagnetic oscillations. Since the longitudinal con-

*tg ≡ tan.

ductivity σ_{zz} is much larger than the transverse one, we can put $E_z = 0$ in (1.7). Substituting (1.18) in (1.7) we obtain

$$\left[k^2 - \frac{\omega^2}{v_a^2}\left(1 + \frac{i\nu}{\omega}\right)\right] E_x = 0,$$
$$\left[k_z^2 - \frac{\omega^2}{v_a^2}\left(1 + \frac{i\nu}{\omega}\right)\right] E_y = 0, \tag{4.3}$$

where v_a is determined by formula (1.22).

These equations show that two weakly damped electromagnetic waves exist in the metal when $\nu \ll \omega$. The electric field is directed along the x axis in one of the waves and along the y axis in the other.

The spectrum and damping of the first wave are determined by (1.21), while the dispersion law of the second wave is of the form [12]

$$\omega = k_z v_a - i\frac{\nu}{2}. \tag{4.4}$$

The wave (4.4) with phase velocity $v_a \cos\Phi$ is similar to the Alfven wave in a plasma. On the other hand, the wave (1.21) in which the electric field is transverse to the vectors **k** and **H** (along the x axis) is the analog of the fast magnetic-sound wave. It must be emphasized that waves (1.21) and (4.4) can propagate in metals only when $\nu \ll \omega$, i.e., under conditions directly opposite to those prevailing in magnetohydrodynamics:

$$\omega \ll \nu, \quad \Omega \ll \nu. \tag{4.5}$$

However, in spite of the difference in the conditions for the existence of waves in metals and in magnetohydrodynamics, in view of the similarity of their spectra we shall call waves (1.21) and (4.4) magnetohydrodynamic.* They are sometimes also called magnetoplasma waves.

Besides the Alfven and fast magnetic-sound wave, there is also a slow magnetic-sound wave in magnetohydrodynamics. In the case of a strong magnetic field, when the Alfven velocity is much larger than the speed of sound, the phase velocity of this wave is of the order of that of sound. Owing to the Fermi statistics of the carriers, the role of the speed of sound in an electron-hole plasma of a metal can be assumed only by the Fermi velocity. Therefore in the considered limiting case $kv_s \ll \omega$ this wave does not exist (in the case (4.1) the asymptotic value of the conductivity tensor cannot be used to investigate the spectrum of this wave).

Expressing the wave vector k with the aid of (4.4) in terms of ω, we rewrite conditions (4.1) and (1.20), which determine the region of existence of the weakly-damped waves (1.21) and (4.4), in the form

$$\nu_s \ll \omega \ll \Omega_s, \tag{4.6}$$

$$v_s \ll v_a \quad (s = 1, 2). \tag{4.7}$$

In the case of transverse propagation, $\Phi = \pi/2$, there is no Alfven wave.

In ordinary metals ($n \sim 10^{22}$ cm^{-3}) condition (4.7) can be satisfied only in very strong magnetic fields—of the order of several million Oe. In bismuth $n \sim 10^{17}$ cm^{-3} and condition (4.7) is already satisfied when $H \gtrsim 10^3$ Oe. The first to indicate that Alfven waves can propagate in bismuth when **k** ∥ **H** were Buchsbaum and Galt [9].

b) Strong spatial dispersion. Let us consider the possible propagation of magnetohydrodynamic waves in metals with $n_1 = n_2$ in the case of not too strong magnetic field, satisfying the conditions:

$$v_a \ll v_s, \tag{4.8}$$

$$\nu_s \ll \omega \ll kv_s \ll \Omega_s. \tag{4.9}$$

In a plasma with

$$v_a \ll w_0 \tag{4.10}$$

(w_0 is the speed of sound), magnetic-sound waves have the following spectrum:

$$\omega_+ = kw_0, \quad \omega_- = kv_a, \tag{4.11}$$

where the plus and minus pertain to fast and slow waves respectively.

Inasmuch as the role of the speed of sound in a degenerate electron-hole plasma of a metal can be played only by the Fermi velocity, the fast magnetic-sound wave does not satisfy conditions (4.9) ($\omega \sim kv$ for this wave). The asymptotic expressions for $\sigma_{\alpha\beta}$ can therefore yield in the case (4.8) and (4.9) only the Alfven and the slow magnetic-sound waves.

The asymptotic expression for the electronic part of the tensor $\sigma_{\alpha\beta}$ in the case (4.9) is of the form (3.9). The expression for the hole part of the conductivity tensor can be obtained from (3.9) by replacing the electronic characteristics by hole ones.

Since the off-diagonal elements of the electronic and hole parts differ only in sign, the summary tensor $\sigma_{\alpha\beta}$ remains diagonal in the xyz coordinate frame. We emphasize that the vanishing (or smallness) of all the off-diagonal elements of the conductivity tensor is the consequence of the compensation condition $n_1 = n_2$ and of the isotropy of the carrier dispersion. It will be shown later that in the case (4.9) $\sigma_{\alpha\beta}$ is diagonal also for an arbitrary carrier dispersion law if *the magnetic field* **H** *is parallel to the symmetry axis* of the crystal.

Owing to condition (2.1) and the diagonality of the conductivity tensor, the z component of the electric field in the Alfven wave is negligibly small:

$$\left|\frac{E_z}{E_y}\right| \sim \left|\frac{\sigma_{yy}}{\sigma_{zz}}\right| \ll 1.$$

The spectrum of this wave is therefore determined by (4.4) as before.

*There is a full analogy here with ordinary plasma, in which magnetohydrodynamic waves exist when $\nu \to 0$ (see, e.g., Sec. 14 of [6]).

The element σ_{xx} is real when $\Phi \sim 1$, owing to the spatial dispersion (σ_{xx} is connected with the Landau damping). Therefore the second wave, the electric field in which is polarized along the x axis, is rapidly damped in the case of (4.8) and (4.9). When $\Phi = 0$ there is no Landau damping, and the spectrum of the slow magnetic-sound wave coincides with the spectrum of the Alfven wave:

$$\omega_- = kv_a.$$

To determine whether a wave similar to the fast magnetic-sound wave of magnetohydrodynamics exists in a metal in the case of (4.8) and (4.9), it is necessary to have an expression for the conductivity tensor when

$$|k_z v| \lesssim \omega. \tag{4.12}$$

We note that when $|k_z v| > \omega$ strong Landau damping sets in and hinders the wave propagation.

In the case of (4.12) the dispersion equation for a wave polarized along the x axis is extremely complicated and cannot be solved analytically. It can be analyzed with relative ease in the particular case when the effective masses of the electrons and holes are equal, $m_1 = m_2$ [13]. Then the electron and hole parts of the off-diagonal elements σ_{xy} and σ_{xz} differ only in sign, as a result of which

$$\sigma_{xy} = \sigma_{xz} = 0. \tag{4.13}$$

Therefore the spectrum of the fast magnetic-sound wave is determined only by the element σ_{xx}:

$$k^2c^2 = 4\pi i\omega\sigma_{xx}(\omega, \mathbf{k}, \mathbf{H}). \tag{4.14}$$

An asymptotic expression for σ_{xx} can be obtained easily from the general formula (2.13):

$$\sigma_{xx}(\omega, \mathbf{k}, \mathbf{H}) = \frac{3ne^2}{4\pi m\Omega}\int_0^\pi d\theta \sin^3\theta \int_0^{2\pi} d\tau \times \cos\tau \int_{-\infty}^{\tau} d\tau' \cos\tau' \exp\left\{\frac{1}{\Omega}(\nu - i\omega + ik_z v\cos\theta)(\tau' - \tau) + ik_y R\sin\theta(\cos\tau - \cos\tau')\right\} \tag{4.15}$$

We have used here a polar system of coordinates with polar axis along **H** ($v_x = v\cos\tau\sin\theta$, etc.). Expanding the integrand in (4.15) in powers of the small parameter k_yR, we represent σ_{xx} in the form

$$\sigma_{xx} = \sigma_{xx}^{(0)} + \sigma_{xx}^{(2)}. \tag{4.16}$$

In the limiting case (4.9) under consideration, $\sigma_{xx}^{(0)}$ characterizes the transverse conductivity of the metal in the absence of spatial dispersion:

$$\sigma_{xx}^{(0)} = \frac{ne^2}{m\Omega^2}(\nu - i\omega). \tag{4.17}$$

The term linear in k_yR vanishes, and the main term of the asymptotic expression $\sigma_{xx}^{(2)}$ is given by

$$\sigma_{xx}^{(2)} = \frac{3ne^2}{8m}(k_yR)^2\int_0^\pi \frac{d\theta\sin^5\theta}{\nu - i\omega + ik_z v\cos\theta}. \tag{4.18}$$

Elementary calculation yields for σ_{xx} the expression

$$\sigma_{xx} = \sigma_{xx}^{(0)}\left\{1 - \frac{3}{4}\,\mathrm{tg}^2\Phi\left[\frac{5}{3} - u^2 + \frac{1-u^2}{u}\ln\frac{u+1}{u-1}\right]\right\},$$

$$u \equiv \frac{\omega + i\nu}{k_z v}. \tag{4.19}$$

We can rewrite the dispersion equation (4.14) in the form

$$\frac{\sigma_{xx}(u)}{\sigma_{xx}^{(0)}}\,u^2 = \left(\frac{v_a}{v\cos\Phi}\right)^2. \tag{4.20}$$

Inasmuch as the left-hand side of (4.20) contains functions that depend only on u and Φ, and since $v_a \ll v$, (4.20) reduces to

$$\sigma_{xx}(u, \Phi) = 0. \tag{4.21}$$

Analysis shows that for all values of Φ, including narrow regions near $\Phi = 0$ and $\Phi = \pi/2$, there is one almost-real root $|u| > 1$. In other words, the spectrum and the damping of the fast wave are determined by the formula [13]

$$\omega_+ = A(\Phi)|k_z|v - i\nu, \quad A(\Phi) > 1. \tag{4.22}$$

The possible propagation of an electromagnetic wave with spectrum of the type (4.22) is due to the Fermi statistics of the current carriers and is a specific feature of the metal. In a Boltzmann plasma no such wave could propagate, for in the case (4.12) there would always be some electrons moving in phase with the wave and leading to strong Landau damping.

Thus, a weakly-damped electromagnetic wave with spectrum (4.22) exists in the metal in the case of equal masses and under conditions of strong spatial dispersion (4.12). In the case of unequal masses, $m_1 \neq m_2$, the off-diagonal element of the conductivity σ_{xz} differs from zero. With this, the "renormalization" (2.4) of σ_{xx} becomes important. Each of the elements σ_{xx}, σ_{xz}, or σ_{zz} is now a sum of two terms of a single type, but with different m and ν. Therefore the dispersion equation (4.21) cannot be investigated as readily in this case as when $m_1 = m_2$. It is obvious from physical considerations, however, that there should exist a certain region of values of the angle Φ in which (4.21) has a solution of the type (4.22).

5. Electromagnetic Waves in the Vicinity of Cyclotron Resonance

The collective motion of the electrons in the magnetic field, which is characterized by the cyclotron frequency Ω, leads to the existence of electromagnetic waves in the vicinity of the cyclotron resonances $\omega \approx \Omega_s$. We assume in this section that the magnetic field is sufficiently strong, and the carrier density is relatively small, so that condition (4.7) is satisfied. It is easy to satisfy this condition, which ensures the absence of Landau damping, in metals of the bismuth type.

Following [14], we consider a simplified model, in which the carrier dispersion law is isotropic and the effective mass m_2 of the holes is larger than the electron mass m_1. The conclusions drawn for this model remain qualitatively in force in the general case, too. Many of the results presented in this section have analogs in the theory of a nondegenerate plasma. Nonetheless we deem it expedient to formulate the corresponding deductions for an electron-hole plasma in a metal.

It is quite obvious that at small values of kR the spatial dispersion begins to play a role only in the immediate vicinity of cyclotron resonance, when

$$\frac{|\Omega_s-\omega|}{\omega} \lesssim |k_z R| \ll 1. \tag{5.1}$$

the resultant Landau damping makes wave propagation impossible.

In the isotropic model assumed by us, the nonvanishing elements of the tensor $\sigma_{\alpha\beta}$ are determined by the formulas

$$\left.\begin{aligned} \sigma_{xx}=\sigma_{yy}&=-i\frac{nec}{H}\left(\frac{\beta_1}{1-\beta_1^2}+\frac{\beta_2}{1-\beta_2^2}\right),\\ \sigma_{yx}=-\sigma_{xy}&=\frac{nec}{H}\left(\frac{1}{1-\beta_1^2}-\frac{1}{1-\beta_2^2}\right),\\ \sigma_{zz}&=i\frac{nec}{H}\left(\frac{1}{\beta_1}+\frac{1}{\beta_2}\right), \quad \beta_s=\frac{\omega+i\nu_s}{\Omega_s}. \end{aligned}\right\} \tag{5.2}$$

They differ from the corresponding expressions for $\sigma_{\alpha\beta}$ in the static case in that ν_s is replaced by $\nu_s - i\omega$, and are valid when

$$(kR)^2 \ll 1, \quad k_z R \ll |1-\beta_s|. \tag{5.3}$$

Substituting (5.2) in the dispersion equation (2.3) we can readily represent its solution in the form

$$\left(\frac{kv_a}{\omega}\right)^2=\frac{1-\beta_1\beta_2-\frac{1}{2}\sin^2\Phi \mp \left[(\beta_1-\beta_2)^2\cos^2\Phi+\frac{1}{4}\sin^4\Phi\right]^{1/2}}{\beta_1^2\beta_2^2-\beta_1\beta_2\sin^2\Phi+(1-\beta_1^2-\beta_2^2)\cos^2\Phi}. \tag{5.4}$$

In the limit of a strong magnetic field $|\beta_1| \ll |\beta_2| \ll 1$ formula (5.4) yields the spectrum and damping for the fast magnetic-sound and Alfven waves. The values of the frequency ω, at which the wave vector k becomes infinite (when $\nu_s \to 0$), are the limiting frequencies for the electromagnetic wave of the given type (resonance). If $\cos^2\Phi \gg m_1/m_2$, these frequencies are determined by the formulas

$$\omega_+=\Omega_1\cos\Phi, \quad \omega_-=\Omega_2. \tag{5.5}$$

The frequency ω_+ is the end point of the spectrum of the magnetic-sound wave, and ω_- that of the Alfven wave. As ω approaches Ω_2, the spectrum and the polarization of the Alfven wave change appreciably. In the vicinity of the hole cyclotron resonance the spectrum takes the form

$$k^2=\frac{\Omega_2^2}{v_a^2}\cdot\frac{\omega}{\Omega_2-\omega}\,\frac{1+\cos^2\Phi}{2\cos^2\Phi}, \tag{5.6}$$

and the electric field is elliptically polarized. Formula (5.6) is valid if

$$|k_z|v_2 \ll \Omega_2-\omega \ll \omega. \tag{5.7}$$

Expressing k in terms of ω from (5.6), we can rewrite these inequalities in the form

$$\left(\frac{v_2}{v_a}\right)^2 \ll \left|1-\frac{\omega}{\Omega_2}\right|^3 \ll 1. \tag{5.8}$$

In the region of weaker fields, where

$$\Omega_2 \ll \omega \ll \Omega_1, \tag{5.9}$$

the holes can be regarded as immobile and their contribution to the conductivity neglected (in analogy with the situation in a plasma, where the role of the holes is played by the ions). In this case there is no Alfven wave, and the magnetic-sound wave is transformed into the helicon characteristic of metals with different electron and hole densities. The polarization of this wave is characterized by formulas (3.6), and its spectrum and damping are

$$\omega=\frac{\Omega_1\cos\Phi}{1+\left(\frac{\omega_0}{kc}\right)^2}-i\nu_1. \tag{5.10}$$

Far from the end-point frequency $\Omega_1\cos\Phi$, the value of $(kc/\omega_0)^2$ is much smaller than unity, the spectrum of the wave is quadratic and coincides with (3.4). Near the end point of the spectrum, the wave vector increases rapidly. Figure 1 shows the dependence of the refractive index kc/ω of both waves on the magnetic field.

In the case of strictly transverse propagation ($\cos\Phi \to 0$) the spectrum of the fast magnetic-sound wave terminates at the frequency of the "hybrid" resonance $\Omega_h = (\Omega_1\Omega_2)^{1/2}$. As $\omega \to \Omega_h$, the value of k increases sharply:

$$k^2=\frac{\omega_0^2}{c^2\left[1-(\omega^2/\Omega_h^2)\right]}. \tag{5.11}$$

This formula is valid if

$$\left|1-\frac{\omega^2}{\Omega_h^2}\right| \gg \left(\frac{v_1}{v_a}\right)^2. \tag{5.12}$$

In the immediate vicinity of the "hybrid" resonance it

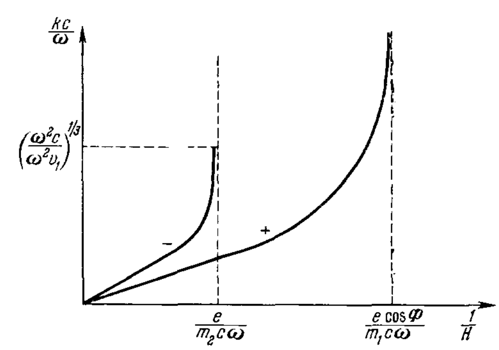

FIG. 1. Refractive index vs. magnetic field for Alfven (−) and magnetic-sound waves (+).

is already necessary to take spatial dispersion into account.

6. Arbitrary Conduction-electron Dispersion. Electric Conductivity Tensor.

We now proceed to study the influence of the anisotropy of the Fermi surface on the properties of electromagnetic waves in metals. We shall show that allowance for anisotropy leads to a number of new distinctive features.

Thus, the damping and polarization of a helicon in a metal with a singly-connected Fermi surface depends on **H** in an entirely different manner than in the case of a multiply connected one. The complicated character of the electron dispersion in metals with $n_1 = n_2$ can leads to a vanishing of the magnetohydrodynamic waves in that region of magnetic fields, where the Alfven velocity is much lower than the Fermi velocity of the carriers. Under certain conditions waves with a quadratic spectrum may propagate in the metal with $n_1 = n_2$ The length of this wave depends on the orientation of the magnetic field **H**, but does not depend on its magnitude. The electric field in this wave is directed parallel to **H**.

In the presence of several groups of carriers (multiply-connected Fermi surface) the summary conductivity tensor is the sum of expressions (2.13) for the different groups. The conduction-electron dispersion is assumed arbitrary, but it is assumed that the electron trajectories in momentum space are closed. It must be emphasized that in the case of open trajectories the elements of the conductivity tensor is of the same order of magnitude as in the presence of a magnetic field. This should lead to the vanishing of the electromagnetic waves (see also [8]). The following asymptotic expression for the Fourier component of the current density was obtained in the case of strong spatial dispersion (3.8) in[15]:

$$\mathbf{j}(\mathbf{k}, \omega, \mathbf{H}) = \frac{Nec}{\mathbf{kH}}[\mathbf{kE}] + \sum C\mathbf{w}(\mathbf{w}^*\mathbf{E}) + s_0\mathbf{H}(\mathbf{EH})H^{-2} + \hat{s}\mathbf{E}. \tag{6.1}$$

Here

$$N = \frac{2}{(2\pi\hbar)^3}(V_1 - V_2) = n_1 - n_2; \tag{6.2}$$

V_1 is the total volume bounded by the Fermi surface, inside of which the states with $\epsilon < \epsilon_F$ are located (electrons, $m > 0$); V_2 is the volume containing the states with $\epsilon > \epsilon_F$ (holes, $m < 0$)[10]. The symbol Σ denotes summation over the different carrier groups.

The quantity C is determined by the formula

$$C = \frac{e^2}{2\pi\hbar^3}\left|\frac{m\mu}{k_z}\right|, \tag{6.3}$$

where $\mu = (\overline{\partial v_z/\partial p_z})^{-1}$ has the dimension and order of magnitude of the effective mass of the electron. The components of the complex "velocity" vector **w** are

$$w_z = \omega/k_z, \quad w_\alpha = \overline{i\mathbf{kv}\varrho_\alpha} \quad (\alpha = x, y), \tag{6.4}$$

$$\varrho_x = \frac{c}{eH}(p_y - \bar{p}_y), \quad \varrho_y = -\frac{c}{eH}(p_x - \bar{p}_x). \tag{6.5}$$

The bar denotes averaging over τ for $\epsilon = \epsilon_F$ and $p_z = \text{const}$. The values of all quantities in (6.3) and (6.4) must be taken at values of p_z satisfying the condition

$$\bar{v}_z(\varepsilon_F, p_z) = \frac{\omega}{k_z}. \tag{6.6}$$

The asterisk in (6.1) denotes complex conjugation, and

$$s_0 = \frac{4\pi e^2}{k_z^2(2\pi\hbar)^3}\sum(\nu - i\omega)\int m\,dp_z = \frac{e^2}{k_z^2}\sum(\nu - i\omega)\left|\frac{\partial n}{\partial \varepsilon_F}\right|. \tag{6.7}$$

The two-dimensional tensor

$$s_{\alpha\beta} = \frac{e^2}{2\pi^2\hbar^3}\sum(\nu - i\omega)\int m\overline{\varrho_\alpha\varrho_\beta}\,dp_z \quad (\alpha, \beta = x, y) \tag{6.8}$$

characterizes the transverse conductivity of the metal in the limit of a homogeneous high-frequency field $(\mathbf{k} \to 0)$.

The first term in (6.1) is the high-frequency Hall current. As in the isotropic case, if the spatial dispersion is strong this current is orthogonal to the vectors **k** and **H**. The second term describes Cerenkov absorption of the wave by the electrons, which move, in the mean, in phase with the wave. However, unlike the isotropic case, this part of the current density is directed not transverse to the vectors **k** and **H**, but along the vector **w**.

If the magnetic field **H** is directed along an axis of symmetry higher than the twofold axis, or if the carrier dispersion is isotropic, then

$$\overline{v_z\varrho_x} = \overline{v_z\varrho_y} = 0, \quad \overline{\varrho_x\varrho_y} = 0, \quad w_y = 0, \tag{6.9}$$

i.e., the tensor $s_{\alpha\beta}$ is diagonal, and the dissipative current connected with the Landau damping is almost perpendicular to the vectors **k** and **H**. Formula (6.1) for **j** corresponds in this case to expression (3.9).

As noted at the end of Sec. 4, the case of metals of the bismuth type actually corresponds to weak spatial dispersion (4.6) and (4.7). The dependence of $\sigma_{\alpha\beta}$ on **k** can then be neglected, and there is no Landau damping, since the phase velocity of the wave exceeds the carrier velocity. The expression for $\sigma_{\alpha\beta}(\omega, \mathbf{H})$ is obtained from the well known asymptotic expression for the electric conductivity tensor[11] by making the simple substitution $\nu \to \nu - i\omega$. When $N = 0$ we have

$$\sigma_{\alpha\beta} = \frac{nec}{H}\sum_s \begin{pmatrix} -i\beta_s a_s^{xx} & -i\beta_s a_s^{xy} & a_s^{xz} \\ -i\beta_s a_s^{yx} & -i\beta_s a_s^{yy} & a_s^{yz} \\ a_s^{zx} & a_s^{zy} & ia_s^{zz}/\beta_s \end{pmatrix}. \tag{6.10}$$

The dimensionless matrices $a_s^{\alpha\beta}$ depend on the form of the Fermi surface and on the orientation of the magnetic field relative to the crystallographic axes.

If the vector **H** is parallel to an axis of symmetry higher than twofold, then all the off-diagonal elements of the matrices $a_{sj}^{\alpha\beta}$ vanish.

7. Helicons (Anisotropic Case)

The anisotropy of the electron dispersion does not exert a strong influence on the characteristics of a helicon in the low-frequency case (3.1). Its spectrum and polarization remain the same as in an isotropic metal. All that changes is the expression for the wave damping. However, the order-of-magnitude estimate $\Gamma \sim \nu/\Omega$ remains valid in this case, too[8].

The situation is different in the region of strong spatial dispersion (3.8). The topology and the shape of the Fermi surface may strongly influence the damping and polarization of the helicon. It is quite evident that in this case we can neglect in (6.1) the small current $\hat{s}\mathbf{E}$. Relation (2.1) allows us to express the longitudinal electric field $\mathbf{kE}/k$ in terms of the transverse part $\mathbf{E}_\perp = \mathbf{E} - \mathbf{k}(\mathbf{kE})/k^2$:

$$\frac{\mathbf{kE}}{k^2} = -\frac{s_0(\mathbf{kh})(\mathbf{hE}_\perp) + \Sigma C(\mathbf{kw})(\mathbf{w}^*\mathbf{E}_\perp)}{s_0(\mathbf{kh})^2 + \Sigma C|\mathbf{kw}|^2}, \tag{7.1}$$

where $\mathbf{h} = \mathbf{H}/H$ is a unit vector in the direction of the constant magnetic field.

Using (7.1) and (1.7), we can easily write down the dispersion equation for the determination of the spectrum and damping of the helicon:

$$1 - \left(\frac{4\pi\omega Ne}{ck\mathbf{kH}}\right)^2 = i\frac{4\pi\omega}{k^2c^2}\left\{\Sigma C|w_x^2| + \frac{s_0\Sigma C|w_y^2| - \sin^2\Phi|\Sigma C w_x w_y|^2}{s_0\cos^2\Phi + \sin^2\Phi\Sigma C|w_y^2|}\right\}. \tag{7.2}$$

In the derivation of (7.2) we have neglected terms of order kR and $\omega/k_z v$ in comparison with those written out. It is seen from this equation that the spectrum of a helicon coincides, apart from small terms, with (3.4). The damping of the wave, on the other hand, which is determined by the right-hand side of (7.2), is in general different. Let us consider several different cases.

a) Let the vector **H** be parallel to a high-order symmetry axis. Then $w_y = 0$, the last term in the right-hand side of (7.2) vanishes, and the relative wave damping is

$$\Gamma = \frac{2\pi\omega}{k^2c^2}\sum C|w_x^2| \approx kR\sin^2\Phi. \tag{7.3}$$

From (7.1) it follows, on the other hand, that $\mathbf{E}\cdot\mathbf{H} \approx 0$, i.e., the vector of the electric field **E** rotates in a plane perpendicular to the constant magnetic field **H**. This case thus corresponds fully to that considered in Sec. 3.

b) Let us consider further the case of a singly-connected but non-isotropic Fermi surface, when the sums of the form $\Sigma C w_\alpha w_\beta^*$ reduce to a single term. It is then convenient to reduce (7.2) to the form

$$1 - \left(\frac{4\pi\omega Ne}{ck\mathbf{kH}}\right)^2 = \frac{\gamma\left[\left(\frac{\omega}{\nu}\right)^2\xi + i\left(1 + \xi + \frac{\omega^2}{\nu^2}\right)\right]}{\left(\frac{\omega}{\nu}\right)^2 + (1+\xi)^2}, \tag{7.4}$$

where

$$\gamma = \frac{2\omega}{(2\pi\hbar)^3 H^2}\left|\frac{m\mu}{k_z}\right|\left[(\overline{v_z p_x})^2 + (\overline{v_y p_y}\sin\Phi - \overline{v_z p_y}\cos\Phi)^2\right]_{\bar{v}_z=0}, \tag{7.5}$$

$$\xi = \left(\frac{k_y k_z}{eH}\right)^2\frac{\pi}{\nu}\int d^3p\,\delta(\varepsilon - \varepsilon_F) \times\delta(k_z\bar{v}_z)(\overline{v_z p_x})^2\left[\int d^3p\,\delta(\varepsilon-\varepsilon_F)\right]^{-1}. \tag{7.6}$$

The value of γ is of the order of kR, and $\xi \sim k_z l(k_y R)^2$. At low frequencies, $\omega \ll \nu$, the change in the spectrum of the wave is much lower than the relative damping, which is determined by the formula

$$\Gamma = \frac{\gamma}{1+\xi} = \frac{akR}{1 + bk_z l(k_y R)^2}, \tag{7.7}$$

where a and b are positive dimensionless quantities of the order of unity, and a depends in a complicated manner on the angle Φ.

Formula (7.7) shows that in the case in question the dependence of the wave damping on the value of H is nonmonotonic. In a relatively weak magnetic field with $\xi \gg 1$, the damping increases with the field: $\Gamma \sim (klkR)^{-1}$. With further increase in H, the value of Γ reaches a maximum, after which it decreases monotonically: $\Gamma \approx akR$. Figure 2 shows the relative damping as a function of H. The dashed line shows the damping of the helicon in the case of isotropic electron dispersion.

The polarization of the wave also exhibits a peculiar variation. The transverse part of the electric field $\mathbf{E}_\perp$ has a circular rotation, as before. The longitudinal component, on the other hand, is determined by the conditions

$$\left.\begin{aligned} \mathbf{Ew} &= 0 && (\xi \gg 1), \\ \mathbf{EH} &= 0 && (\xi \ll 1), \\ (\mathbf{kH})(\mathbf{Ew}) + (\mathbf{kw})(\mathbf{EH}) &= 0 && (\xi = 1), \end{aligned}\right\} \tag{7.8}$$

i.e., the vector **E** rotates in weak magnetic fields in a plane perpendicular to the direction of the dissipative current **w**.

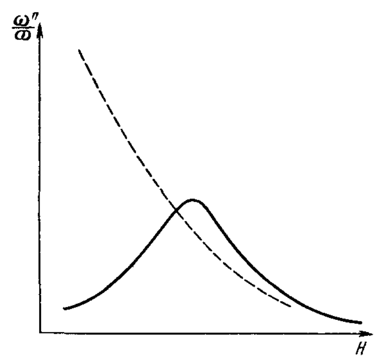

FIG. 2. Schematic plot of the relative damping of a helicon vs. the magnetic field.

A similar nonmonotonic behavior is shown by the damping of a helicon at higher frequencies $\omega \gg \nu$. The maximum of Γ shifts with this towards weaker fields.

In the case of a large number of carrier groups, the damping $\Gamma \sim kR$ decreases monotonically with increasing H. However, the polarization of the helicon varies like (7.8). At relatively low values of H, the electric field **E** rotates in a plane orthogonal to the vector $\Sigma C(\mathbf{k}\cdot\mathbf{w})\mathbf{w}^*$.

The physical nature of the noted singularities of the helicon in the case of a complicated carrier dispersion law consists in the ability of this wave to interact with the longitudinal wave. The dispersion equation for the longitudinal wave is in this case

$$s_0 \cos^2\Phi + \sum C\,|w_y^2|\sin^2\Phi = 0. \qquad (7.9)$$

It has no real solutions, i.e., the longitudinal wave attenuates along its length. Nonetheless, when the lengths of the helical and longitudinal waves become of the same order of magnitude, the damping and polarization of the helical wave change appreciably. Inasmuch as the longitudinal wave attenuates rapidly, the resonance is smeared. These singularities of the helical wave exist only in metals with anisotropic carrier dispersion, when $w_y \neq 0$.

8. Magnetohydrodynamic Waves (Arbitrary Carrier Dispersion)

It was shown in Sec. 4 that in metals with equal electron and hole densities there exist electromagnetic waves of the magnetohydrodynamic type with linear spectrum and plane polarization. In strong magnetic fields, when the Alfven velocity v_a is large compared with the Fermi carrier velocity, the spatial dispersion plays no role. In this case two waves can propagate in the metal, Alfven and fast magnetic-sound. Their phase velocities are of the order of v_a.

In the region of weaker magnetic field, when $v_a \ll v$, a fast wave can propagate in the metal, with a velocity somewhat higher than the Fermi velocity, as well as an Alfven wave with velocity $v_a \cos\Phi$. In this case the spatial dispersion leads to rapid damping of the waves whose phase velocities are small compared with the Fermi velocity. On the other hand, the possibility of propagation of a low-velocity Alfven wave is attributed to the fact that in the isotropic case the dissipative electron and hole currents, which are connected with the Landau damping, are perpendicular to **k** and **H**, i.e., they are parallel, and there is one direction of the electric field in the xy plane for which these currents vanish (y axis). For this direction, the imaginary part of the conductivity is negative, and when $\omega \gg \nu$ it is large compared with the real part; this is what makes possible the propagation of an Alfven wave whose electric field is polarized along the y axis.

In this section we consider the influence of the anisotropy of the carrier dispersion on the character of the electromagnetic waves in metals with $n_1 = n_2$. We consider first the case of not too strong magnetic fields, when there is strong spatial dispersion, and then proceed to investigate the inverse limiting case $v \ll v_a$.

a) Strong spatial dispersion. The necessary condition for the existence of weakly damped electromagnetic waves is, as usual, the smallness of the hermitian (dissipative) part of the conductivity of the metal compared with the anti-hermitian part. In the case of strong spatial dispersion, the asymptotic behavior of the current density is determined by formula (6.1), in which we must put $N = 0$. The dissipative part of the conductivity, which is connected with the Landau damping is much larger than the nondissipative part. Therefore the possibility of propagation of magnetohydrodynamic waves is connected with the vanishing of the Landau damping for some electric-field directions in the xy plane. In other words, in these directions the projections of the velocities w_1 and w_2 should vanish. In the general case, when the magnetic field makes an angle with the symmetry axis, this is impossible and consequently there are no magnetohydrodynamic waves in an anisotropic metal.

If the vector **H** is directed along a symmetry axis of high order, then when $\mathbf{k} \parallel \mathbf{H}$

$$w_{1x} = w_{1y} = w_{2x} = w_{2y} = 0,$$

and two linearly polarized magnetohydrodynamic waves with identical spectra are present in the metal. On the other hand, when the vectors **k** and **H** are not parallel we have $w_{1y} = w_{y2} = 0$ and there exists one (Alfven) wave, whose electric field is polarized along the y axis.

The Alfven wave cannot propagate in the general case because the electron and hole dissipative currents connected with the Landau damping are not parallel to each other. Therefore there are no electric field directions in the xy plane for which there is no Landau damping. However, the presence of large dissipative conductivity in the entire xy plane makes it possible for an entirely new electromagnetic wave to propagate in the metal. In fact, in certain ranges of magnetic-field values, the transverse conductivity $\Sigma C w_\alpha w_\beta^*$ may turn out to be large compared with the longitudinal conductivity s_0 (Eq. (6.7)), and the coupling between them may be weak. Since the imaginary part of s_0 is negative and large compared with the real part when $\omega \gg \nu$, a wave can propagate in the metal, with an electric field polarized along the constant magnetic field ("ordinary" wave). The spectrum of this wave is determined by the dispersion equation

$$k_y^2 c^2 = 4\pi i\omega s_0. \qquad (8.1)$$

Using expression (6.7) for s_0, we can easily obtain the solution of (4.1)[15]

$$\omega(\mathrm{k}) = |k_y k_z| c r_D, \tag{8.2}$$

where

$$r_D = \left| 4\pi e^2 \Sigma \frac{dn}{d\varepsilon_F} \right|^{-1/2} \tag{8.3}$$

is the Debye-Huckel screening radius in the degenerate plasma of the metal. It must be noted that the spectrum of this wave does not depend on the magnitude of the magnetic field H, but depends on its direction. This wave does not appear when $\mathbf{k} \perp \mathbf{H}$ and when $\mathbf{k} \parallel \mathbf{H}$.

For the existence of the wave (8.2) we must have

$$|\sigma_{zz}| \ll |\sigma_{yy}|, \quad |\sigma_{z\alpha}|^2 \ll |\sigma_{zz}\sigma_{\alpha\alpha}| \quad (\alpha = x,\ y). \tag{8.4}$$

These conditions can be rewritten with the aid of (6.3)—(6.7) in the form

$$\left| \frac{\omega}{k_z v} \right| \ll (kR)^2 \ll 1. \tag{8.5}$$

Expressing k in terms of ω (Eq. (8.2)) and substituting in (8.5), we obtain the inequalities

$$\frac{\omega}{\Omega} \frac{v}{v_a} \ll 1 \ll \left(\frac{\omega}{\Omega}\right)^{1/2} \left(\frac{v}{v_a}\right)^{3/2}, \tag{8.6}$$

which determine, together with the condition $\omega \gg \nu$, the region of existence of the weakly damped wave (8.2).

The damping of the wave (8.2) is determined by the scattering of the conduction electrons and its weak coupling with the damped transverse waves. If conditions (1.20) and (8.6) are satisfied, the relative damping of this wave is small compared with unity.

Thus, under conditions of strong spatial dispersion, the anisotropy of the Fermi surface leads to the vanishing of the Alfven wave and to the appearance of an entirely new longitudinal wave with a quadratic spectrum; this wave does not depend on the magnetic field H. The possibility of existence of such a wave is a specific feature of metals.

b) Weak spatial dispersion. We now investigate the influence of the anisotropy of the carrier dispersion on the character of propagation of the electromagnetic waves in the limiting case of a strong magnetic field, (5.7), when the spatial dispersion does not play any role and the asymptotic behavior of $\sigma_{\alpha\beta}$ is characterized by formula (6.10).

The hermitian part of the conductivity tensor (6.10) determines the electromagnetic-wave damping due to the carrier scattering. When $\nu \ll \omega$, this part is small compared with the antihermitian part, which determines the spectrum. Neglecting carrier scattering, which gives a relative wave damping of the order of ν/ω, we write the tensor $\sigma_{\alpha\beta}$ in the form

$$\sigma_{\alpha\beta} = \frac{nec}{H} \begin{pmatrix} -\frac{i\omega}{\Omega} a_1 & -\frac{i\omega}{\Omega} a_{12} & a_{13} \\ -\frac{i\omega}{\Omega} a_{12} & -\frac{i\omega}{\Omega} a_2 & -a_{32} \\ -a_{13} & a_{32} & i\frac{\Omega}{\omega} a_3 \end{pmatrix}, \tag{8.7}$$

where

$$\Omega = \frac{eH}{(m_1 + m_2) c},$$

and the quantities $\{\alpha\}$ are connected in obvious fashion with the matrix elements $a_s^{(\alpha\beta)}$ in (6.10). The elements of the two-dimensional tensor $\tilde{\sigma}_{\alpha\beta}$ (Eq. (3.4)) can be easily expressed in terms of the matrix elements (8.7):

$$\tilde{\sigma}_{\alpha\beta} = -\frac{i\omega}{\Omega} \frac{nec}{H} A_{\alpha\beta}, \tag{8.8}$$

$$A_{\alpha\beta} = \begin{pmatrix} a_1 + \frac{a_{13}^2}{a_3} & \left(a_{12} + \frac{a_{13}a_{32}}{a_3}\right) \sec\Phi \\ \left(a_{12} + \frac{a_{13}a_{32}}{a_3}\right) \sec\Phi & \left(a_2 + \frac{a_{23}^2}{a_3}\right) \sec^2\Phi \end{pmatrix}. \tag{8.9}$$

We have assumed in the calculation that $\cos\Phi \gg \omega/\Omega$.

The solution of the dispersion equation (2.3) yields two waves with a linear spectrum [12]

$$v_{\pm} = \frac{v_a}{(2 \det A_{\alpha\beta})^{1/2}} \left[A_{xx} + A_{\eta\eta} \pm \sqrt{(A_{xx} + A_{\eta\eta})^2 + 4A_{x\eta}^2}\right]^{1/2}. \tag{8.10}$$

The electric field in these waves is linearly polarized. The polarization x′ and η' coincide with the principal axes of the symmetrical real tensor $A_{\alpha\beta}$. If the magnetic field is directed along an axis of high symmetry, then all the off-diagonal elements of the matrix $a_{\alpha\beta}$ are equal to zero. In this case the wave with phase velocity v_- is transformed into the Alfven wave (4.4), and the wave with velocity v_+ into the fast magnetic-sound wave with spectrum (1.21).

In the particular case when $\mathbf{k} \parallel \mathbf{y}$ ($\cos\Phi \ll \omega/\Omega$) the "renormalized" conductivity tensor $\tilde{\sigma}_{\alpha\beta}$ becomes

$$\tilde{\sigma}_{\alpha\beta} = \frac{nec}{H} \begin{pmatrix} -i\frac{\omega}{\Omega}\left(a_1 + \frac{a_{12}^2}{a_2}\right) & a_{13} + \frac{a_{12}a_{23}}{a_2} \\ -a_{13} - \frac{a_{12}a_{23}}{a_2} & i\frac{\Omega}{\omega}\left(a_3 + \frac{a_{23}}{a_2}\right) \end{pmatrix}. \tag{8.11}$$

A wave similar to the Alfven wave is missing in this case, and the velocity of the fast wave v_+ is given by the formula

$$v_+ = v_a \left\{ a_1 + \frac{1}{a_2}\left[a_{12}^2 + \frac{(a_2 a_{13} + a_{12}a_{23})^2}{a_2 a_3 + a_{23}^2}\right]\right\}^{-1/2}. \tag{8.12}$$

The electric field in the wave is polarized along the x axis.

9. Excitation of Electromagnetic Waves in a Metal by an External Field. Surface Impedance.

We have investigated so far the spectrum, damping, and polarization of electromagnetic waves in an unbounded metal. We consider now the excitation of these waves by an external electromagnetic field and their influence on the high-frequency characteristics of the metal. One of the main characteristics of this type is the surface-impedance tensor $Z_{\alpha\beta}$, which relates the components of the total current $\mathbf{J}$ in the metal with the tangential components of the electric field on its surface:

$$\mathscr{E}_\alpha(0) = Z_{\alpha\beta} J_\beta \quad (\alpha,\ \beta = x,\ \eta). \tag{9.1}$$

We shall use in this system a coordinate system $x\eta\zeta$ whose ζ axis coincides with the inward normal to the surface, since the wave inside the metal always propagates perpendicularly to its surface, owing to the large value of the refractive index.

To find $Z_{\alpha\beta}$ it is necessary to solve Maxwell's equations

$$\mathscr{E}''_\alpha(\zeta) = -i4\pi\omega c^{-2} j_\alpha(\zeta) \quad (\alpha = x,\ \eta), \quad j_\zeta(\zeta) = 0. \tag{9.2}$$

The prime denotes differentiation with respect to ζ. From (9.2) follows a relation

$$J_\alpha = \frac{c^2 \mathscr{E}'_\alpha(0)}{4\pi i\omega},$$

which allows us to represent the impedance tensor in the form

$$Z_{\alpha\beta} \equiv R_{\alpha\beta} - iX_{\alpha\beta} = 4\pi i\omega c^{-2} \frac{\partial \mathscr{E}_\alpha(0)}{\partial \mathscr{E}'_\beta(0)}. \tag{9.3}$$

The real part of the tensor $Z_{\alpha\beta}$ determines the absorption of the energy from the external wave

$$Q = \frac{c^2}{8\pi} R_{\alpha\beta} \mathscr{E}_\alpha^{(i)} \mathscr{E}_\beta^{(i)*}, \tag{9.4}$$

where $\mathscr{E}^{(i)}$ is the electric field vector of the incident wave. The diagonal elements of the tensor $X_{\alpha\beta}$ characterize the phase shift upon reflection (change in the frequency of the resonant circuit), and the off-diagonal ones characterize the degree of ellipticity.

In those cases when the electric field is circularly polarized in the $x\eta$ plane (helicon), it is natural to introduce in place of the tensor $Z_{\alpha\beta}$ (9.3) the impedance tensor for circularly polarized waves

$$\mathscr{E}_\pm = \mathscr{E}_x \pm i\mathscr{E}_y. \tag{9.5}$$

The elements of this tensor are defined by the formulas

$$\mathscr{E}_\pm(0) = Z_\pm J_\pm + Z' J_\mp \tag{9.6}$$

and are connected with the elements of $Z_{\alpha\beta}$ by the relations

$$Z_\pm = \frac{1}{2}(Z_{xx} + Z_{\eta\eta}) \pm iZ_{x\eta}, \quad Z' = \frac{1}{2}(Z_{xx} - Z_{\eta\eta}). \tag{9.7}$$

It is convenient to solve Maxwell's equations (9.2) in the Fourier representation. Continuing formally the field $\mathscr{E}_\alpha(\zeta)$ in even fashion to the region $\zeta < 0$ outside the metal, we seek this field in the form

$$\mathscr{E}_\alpha(\zeta) = \frac{1}{\pi} \int_0^\infty dk\, E_\alpha(k) \cos k\zeta. \tag{9.8}$$

The equations for the Fourier components are algebraic;

$$k^2 E_\alpha(\mathbf{k}) + 2\mathscr{E}'_\alpha(0) = \frac{4\pi i\omega}{c^2} \sigma_{\alpha\beta}(\mathbf{k},\ \omega,\ \mathbf{H}) E_\beta(\mathbf{k}) \quad (\alpha = x,\ \eta),$$
$$\sigma_{\zeta\beta}(\mathbf{k},\ \omega,\ \mathbf{H}) E_\beta(\mathbf{k}) = 0, \tag{9.9}$$

the wave vector is parallel to the ζ axis.

We neglect the variation of the conductivity operator due to the collisions of the electrons with the surface of the metal. In the case of small kR, allowance for the surface effects results in small corrections to the real part of the impedance [16]. These corrections are due to the absorption of the energy of the incident wave due to the inelastic (diffuse) scattering of the electrons from the boundary of the metal. If weakly damped waves are excited in the volume of the metal, the surface effects play likewise no noticeable role [12].

After eliminating the longitudinal component E_ζ from (9.9), we get

$$D_{\alpha\beta}(k) E_\beta(k) = -2\mathscr{E}'_\alpha(0) \quad (\alpha,\ \beta = x,\ \eta), \tag{9.10}$$

where

$$D_{\alpha\beta}(k) = k^2 \delta_{\alpha\beta} - 4\pi i\omega c^{-2} \tilde{\sigma}_{\alpha\beta}, \tag{9.11}$$

and $\tilde{\sigma}_{\alpha\beta}$ is determined by formula (2.4).

The solution of (9.10), using (9.8), leads to the following expressions for the field and for the impedance tensor [12]:

$$\mathscr{E}_\alpha(\zeta) = T_{\alpha\beta}(\zeta) \mathscr{E}_\beta(0), \quad Z_{\alpha\beta} = 4\pi i\omega c^{-2} T_{\alpha\beta}(0), \tag{9.12}$$

$$T_{\alpha\beta}(\zeta) = -\frac{2}{\pi} \int_0^\infty dk\, (D^{-1})_{\alpha\beta} \cos k\zeta. \tag{9.13}$$

The elements of the tensor $(D^{-1})_{\alpha\beta}$ are connected with $D_{\alpha\beta}$ by the relations

$$(D^{-1})_{\alpha\beta} = \frac{1}{D} \begin{pmatrix} k^2 - 4\pi i\omega c^{-2} \tilde{\sigma}_{\eta\eta} & -4\pi i\omega c^{-2} \tilde{\sigma}_{\eta x} \\ -4\pi i\omega c^{-2} \tilde{\sigma}_{x\eta} & k^2 - 4\pi i\omega c^{-2} \tilde{\sigma}_{xx} \end{pmatrix}, \tag{9.14}$$

$$D = k^4 - (4\pi i\omega c^{-2})^2 (\tilde{\sigma}_{xx}\tilde{\sigma}_{\eta\eta} - \tilde{\sigma}_{x\eta}\tilde{\sigma}_{\eta x})$$
$$-4\pi i\omega c^{-2} k^2 (\tilde{\sigma}_{xx} + \tilde{\sigma}_{\eta\eta}). \tag{9.15}$$

We now proceed to obtain explicit expressions for the field distribution and the surface impedance when waves of different types are excited by an external electromagnetic field.

a) Excitation of a helicon. Let a circularly polarized external electromagnetic wave be incident on the surface of the metal. At a fixed frequency ω the dispersion equation of the helical wave can be written in the form

$$k^2 = k_0^2 [1 + i\Gamma(k_0,\ H)], \quad k_0^2 = \frac{4\pi\omega Ne}{cH\cos\Phi}, \tag{9.16}$$

and the values of the relative damping of the wave, $\Gamma(k, H)$ are determined for different cases by (3.5), (3.13), and (7.7).

Using formulas (9.5), (9.8), and (9.12)–(9.16), we obtain for the circularly polarized quantities $T_\pm(\zeta)$

$$T_-(\zeta) = -\frac{2}{\pi} \int_0^\infty \frac{dk \cos k\zeta}{k^2 - k_0^2(1 + i\Gamma)}$$

$$= \frac{i}{k_0}\left(1 - \frac{i\Gamma}{2}\right) \exp\left[ik_0\left(1 + \frac{i\Gamma}{2}\right)\zeta\right], \tag{9.17}$$

$$T_+(\zeta) \approx -\frac{2}{\pi}\int_0^\infty \frac{dk \cos k\zeta}{k^2+k_0^2(1+i\Gamma)} = -\frac{1}{k_0}\left(1+\frac{i\Gamma}{2}\right)\exp(-k_0\zeta). \tag{9.18}$$

These functions describe the distribution of the right- and left-rotating field components in the volume of the metal. We see that the function $T_-(\zeta)$, unlike $T_+(\zeta)$, attenuates quite slowly with increasing ζ. The impedance for this wave

$$Z_- = \frac{4\pi\omega}{k_0c^2}\left(1-\frac{i\Gamma}{2}\right) = \frac{4\pi}{c}\left|\frac{\omega H\cos\Phi}{4\pi Nec}\right|^{1/2}\left(1-\frac{i}{2}\Gamma\right) \tag{9.19}$$

is almost real, corresponding to penetration of the wave (9.17) inside the metal. The small imaginary part $X_- \sim R_-\Gamma$ is due to the damping of the wave.

The wave (9.18) with opposite rotation of the polarization vector has an imaginary wave vector ik_0, i.e., it experiences total reflection. Consequently the impedance Z_+ is pure imaginary:

$$Z_+ = -iZ_-^*.$$

The intermingling of the circularly polarized waves with different rotation directions is characterized by the elements Z' and is small when $\Gamma \ll 1$.

b) Excitation of waves of the magnetohydrodynamic type. As shown in Secs. 4, 5, and 8, in metals with identical electron and hole densities ($n_1 = n_2$) there can propagate different electromagnetic waves. Their relative damping is small and its order of magnitude in most cases is ν/ω. When suitable conditions are satisfied, the external field excites these waves in the metal. The electromagnetic field inside the metal represents a weakly damped plane wave of the type $\exp(ik'\zeta - k''\zeta)$, where $k'' \sim k'\nu/\omega \ll k'$. At distances ζ that are smaller than $1/k''$, the wave damping can be neglected.

In the calculation of the surface impedance we neglect the small wave damping and represent the factor $1/D$ in (9.14) in the form

$$\frac{1}{D} \approx \pi i\delta(D') + P\frac{1}{D'}, \tag{9.20}$$

where $D' = \operatorname{Re} D$ and P is the principal-value symbol.

In the case of a strong magnetic field, when the spatial dispersion plays no role (See Sec. 8,b), the impedance tensor is

$$Z_{x'x'} = 4\pi v_+ c^{-2}, \quad Z_{\eta'\eta'} = 4\pi v_- c^{-2}, \tag{9.21}$$

and the axes x' and η' coincide with the principal axes of the real tensor $A_{\alpha\beta}$ (8.9). The electric field in the Alfven and magnetic-sound waves is polarized in these directions. The impedance is independent of the frequency because the phase velocities v_+ and v_- do not depend on ω.

In the case of strong spatial dispersion $v_a \ll v$ and $\Phi \sim 1$, the spectrum of the fast magnetic-sound wave is given by (4.22). The impedance-tensor element corresponding to this wave is

$$Z_{xx} = \frac{4\pi^2 v_a^2}{vc^2}\left(1-i\frac{\nu}{\omega}\right)\int_0^\infty du\,\delta\left\{1-\frac{3}{4}\operatorname{tg}^2\Phi F(u)\right\}, \tag{9.22}$$

where F(u) denotes the function in the square brackets in (4.19). When $\tan\Phi$ is of the order of unity, the integral in (9.22) is likewise of the order of unity.

If the magnetic field H is directed along a high-symmetry axis, then an Alfven wave with spectrum (4.4) can propagate when $v_a \ll v$. The impedance corresponding to its excitation is

$$Z_{\eta\eta} = 4\pi v_a c^{-2}\cos\Phi \tag{9.23}$$

and is $(v/v_a) \gg 1$ times larger than Z_{xx}. This shows that the fast wave (4.22) is excited by an external field with relatively lower amplitude than the Alfven wave.

If the values of ω and H satisfy the conditions (8.6) and (1.20) and if $\Phi \sim 1$, then the external field excited in an anisotropic metal a wave with spectrum (8.2). Then

$$Z_{\eta\eta} = \frac{4\pi i\omega}{c^2}\int_0^\infty \frac{dk}{k^2-\dfrac{4\pi i\omega s_0}{c^2\sin^2\Phi}} = \frac{4\pi}{c}\left(\frac{\omega r_D}{c}\sin\Phi\cos\Phi\right)^{1/2}. \tag{9.24}$$

c) Excitation of electromagnetic waves in a plate. Maxwell's equations in a plate of thickness d $(-d/2 < \zeta < d/2)$ admit of two types of solution: 1) electric field symmetric and magnetic field antisymmetric, and 2) electric field antisymmetric and magnetic field symmetric, Following the paper of Bass, Blank, and Kaganov [8], we denote the solutions of the first type by the index s and those of the second by the index a. If there is no electromagnetic field outside the plate, then the plate acts as a resonator and consequently the wave vector k takes on discrete values. The eigenfunctions and eigenvalues are

$$\left.\begin{aligned} &E_{xn}^{(s)}(\zeta) = E_{xn}^{(s)}(0)\cos k_n^{(s)}\zeta, \quad H_{\eta n}^{(s)}(\zeta) = i\frac{ck_n^{(s)}}{\omega}E_{xn}^{(s)}(0)\sin k_n^{(s)}\zeta,\\ &k_n^{(s)} = \frac{2\pi}{d}\left(n+\frac{1}{2}\right),\end{aligned}\right\} \tag{9.25}$$

$$\left.\begin{aligned} &E_{xn}^{(a)}(\zeta) = \frac{i\omega}{ck_n^{(a)}}H_{\eta n}^{(a)}(0)\sin k_n^{(a)}\zeta,\\ &H_{\eta n}^{(a)}(\zeta) = H_{\eta n}^{(a)}(0)\cos k_n^{(a)}\zeta,\\ &k_n^{(a)} = \frac{2\pi n}{d}.\end{aligned}\right\} \tag{9.26}$$

The connection between the frequency and the wave vector k_n is determined by the dispersion law $\omega = \omega(\mathbf{k})$.

If the plate is situated in an external field of frequency ω, then waves (9.25) and (9.26) will be excited in it. If the plate is at an antinode of the electric field in the resonator, then the distribution of the electric field in it is symmetric. The antisymmetric case is realized in an antinode of the magnetic field. When a weakly damped wave with given polarization is excited we have

$$Z_s = i\frac{4\pi\omega}{kc^2}\operatorname{ctg}\left(\frac{1}{2}kd\right), \quad Z_a = -i\frac{4\pi\omega}{kc^2}\operatorname{tg}\left(\frac{1}{2}kd\right). \tag{9.27}$$*

*ctg ≡ cot.

The relation

$$k = k'(\omega) + ik''(\omega) \qquad (9.28)$$

in (9.27) is determined from the inverse of the dispersion equation. It follows from (9.27) that in the limit as $k'' \to 0$ the impedance of the plate becomes infinite when $kd = 2\pi n$ for case s, and when $kd = 2\pi(n + \frac{1}{2})$ for case a.

In the general case, both symmetrical and antisymmetrical electric fields are excited in the plate. Near resonance, a sharp change takes place in the real and imaginary parts of the impedance:

$$Z_n = \frac{4\pi\omega}{k'c^2 d} \frac{k'' - i\left(k' - \frac{\pi n}{d}\right)}{(k'')^2 + \left(k' - \frac{\pi n}{d}\right)^2}. \qquad (9.29)$$

In order for the resonance curve to be sufficiently "sharp," the plate thickness must be small compared with attenuation length of the wave

$$d \ll \frac{1}{k''}. \qquad (9.30)$$

The maximum values of the real and imaginary parts of the impedance are of the order of $\omega d/c^2\Gamma n^2$ and decrease with the number like $1/n^2$. The excitation of helicons in the plate under conditions of weak spatial dispersion is considered in [8], and a similar analysis for magnetohydrodynamic waves is given in [17].

10. Experimental Observation of Electromagnetic Waves in Metals

Several recent experimental papers are devoted to the observation of weakly damped electromagnetic waves in metals in a strong magnetic field. The helicons (3.4)—(3.5) were detected by Bowers and coworkers[18] by observing standing waves in single-crystal sodium plates. Low-frequency helicons were subsequently observed by a number of workers[19-22] in many other metals (Ag, Au, Cu, Pb, Al, In, etc). In all these investigations, the helicons were revealed by resonances in the impedance of a plate (9.29) or a rectangular bar. In the latter case the sample was a cavity resonator, in which all three components of the wave vector are "quantized":

$$k_\alpha = \frac{\pi n_\alpha}{d_\alpha}. \qquad (10.1)$$

When (10.1) is substituted in the spectrum (3.4), the latter yields a set of frequencies $\omega(n_1, n_2, n_3)$, which are observed on curves similar to those shown in Fig. 3. Observation of helicons in K, Na, In, Al, and Si at T = 4.2°K was reported in [7]. In [23] there were observed high-frequency helicons with $\omega \sim 10^{11}\ \text{sec}^{-1}$ in degenerate InSb at nitrogen and room temperatures. In all these experiments, the spatial and temporal dispersions were negligible (very low frequencies in the case of the metals and low electron density in the case of InSb).

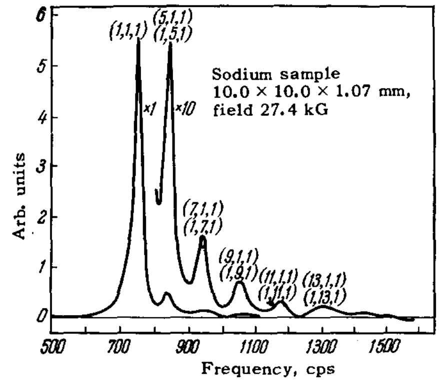

FIG. 3. Impedance vs. frequency in sodium (H = 27.4 kOe; the figure is taken from [21]).

Magnetohydrodynamic waves were observed for the first time under conditions of weak spatial dispersion, $kv \ll \omega$, by Aubrey and Chambers[24]. They observed a sharp linear rise in the real part of the surface impedance of bismuth in fields H > 3 kOe at $\omega \sim 6 \times 10^{10}\ \text{sec}^{-1}$. The magnetic field was parallel to the sample surface. The impedance increase was due to excitation of a fast magnetic-sound wave (8.12).

Buchsbaum and Galt have shown[9] that Alfven waves, with phase velocities $v_{1,2} = v_a$, were observed in the earlier investigation of Galt et al.[25], devoted to cyclotron resonance in bismuth in a magnetic field perpendicular to the surface of the metal ($\mathbf{k} \parallel \mathbf{H}$). In bismuth samples with unequal electron and hole densities ($n_1 \neq n_2$), they observed helicons. This apparently was the first observation of helicons in metals.

Khaĭkin, Édel'man, and Mina[26-28] investigated in detail the Alfven and fast magnetic-sound waves (8.10) and (8.12) by observing standing waves in a bismuth plate. Figure 4 shows one of the recorded plate-impedance oscillations as a function of the reciprocal magnetic field. In the case of a linear dependence of the phase velocity $v_{ph} = \omega/k$ on H, the oscillations are periodic in 1/H. The oscillation period is determined from the condition

$$\frac{\omega}{v_{ph}} = \frac{\pi n}{2d}.$$

The curve shows two systems of resonances, due to two different waves. It was shown in [26-28] that one of the waves (Alfven) vanishes when $\Phi \to \pi/2$, that the wave damping is due to carrier scattering; that their phase velocities depend on the frequency, and that v_{ph} is proportional to the magnetic field H. Kirsch[29] observed Alfven waves in bismuth in a magnetic field perpendicular to the sample surface. Williams[30] studied the fast magnetic-sound wave (8.12) in a field parallel to the surface ($\mathbf{H} \perp \mathbf{k}$). Smith et al.[31] reported observation of the same wave up to the "hybrid resonance" (5.11). The experiments of [26,31] revealed a change in the period of the plate-impedance oscillations, due to the change in the wave spectrum on approaching cyclotron resonance (5.6) and (5.10).

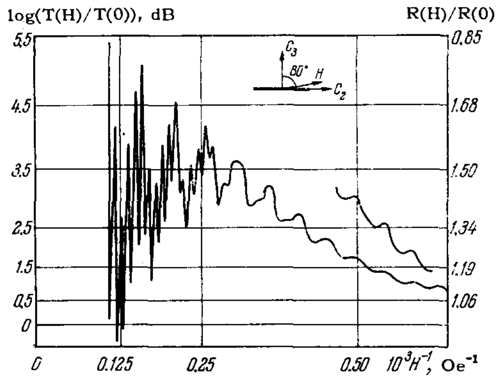

FIG. 4. Plot of the surface impedance of bismuth, demonstrating simultaneous excitation of Alfven and fast magnetic-sound waves. Smaller period–Alfven wave, larger period-magnetic-sound wave. On the top is indicated the arrangement of the vectors of the field H and of the axes C_3 and C_2 in a plane perpendicular to the sample surface, which is represented by the heavy line. On the right is shown part of the curve with larger magnification. The figure is borrowed from [28].

The results of all these experiments are in good agreement with the theory, and also with the model of the Fermi surface in bismuth [28]. It is reported in [32] that transparency of a bismuth plate thicker than 1 mm was observed in experiments on the passage of electromagnetic waves.

The reason for so large a number of papers devoted to undamped waves in bismuth is that the case of weak spatial dispersion $v_a \gg v$ is easy to realize in bismuth, owing to the low carrier density ($n_1 = n_2 \sim 10^{17}$ cm^{-3}). In this field region, the length v_a/ω of the magnetohydrodynamic waves is large enough, and the length v_a/ν within which the waves are damped is much larger than the carrier mean free path $l = v/\nu$.

The fields required to realize the case of weak spatial dispersion in typical metals with carrier densities of the order of one per atom are very strong, $H > 10^6$ Oe. In weaker fields, an important role is assumed by Landau damping. To observe waves of the type (4.2) and (8.2), whose spectrum is independent of H, it is necessary to vary the frequency ω or the inclination of the magnetic field relative to the sample. There are so far no published reports of such waves in metals.

Table I contains a summary of all weakly-damped waves whose length is large compared with the Larmor radius.

II. SHORT WAVES (kR ≫ 1)

11. Qualitative Considerations

We have considered so far electromagnetic waves of great length ($kR \ll 1$). The possibility of propagation of excitations with a wavelength small compared with the electron orbits is much more complicated. The reason is that in this wavelength region a particularly

Table I

Name of wave	Existence conditions	Spectrum	End point of spectrum ($m_1 \ll m_2$)	Relative damping	Polarization	Remark
1. Helicon	$n_1 \neq n_2$ $\nu,\ \omega,\ kv \ll \Omega$ $\Phi \neq \frac{\pi}{2}$	$\omega(k) = \frac{k^2 cH \cos\Phi}{4\pi e \lvert n_1 - n_2 \rvert}$		$\frac{\nu}{\Omega}\sec\Phi + \frac{3\pi}{16} kR\sin^2\Phi$	$E_x = -iE_y\cos\Phi$ $E_z \ll E_x$	In the case of a singly-connected anisotropic Fermi surface, the damping and polarization of the wave change strongly with change in H (see Sec. 7)
2. Magnetohydrodynamic wave	$n_1 = n_2$ $\nu \ll \omega \ll \Omega$					
	a) Weak spatial dispersion					
3. Alfven wave	$\Phi \neq \frac{\pi}{2}$	$\omega = kv_a\cos\Phi$	$\Omega_1\cos\Phi + (\Omega_1\Omega_2)^{1/2}$	$\nu/2\omega$	$E \parallel [[kH]H]$	In the case of an anisotropic Fermi surface, the spectrum and polarization of the waves are given in Sec. 8.
4. Fast magnetic-sound wave		$\omega = kv_a$	Ω_2	$\nu/2\omega$	$E \parallel [kH]$	
	b) Strong spatial dispersion					
5. Alfven wave	$\Phi \neq \frac{\pi}{2}$	$\omega = kv_a\cos\Phi$		$\nu/2\omega$	$E \parallel [[kH]H]$	Exists if the direction of the vector H coincides with a symmetry axis higher than twofold
6. Fast wave	$\omega > kv\cos\Phi$ $\Phi \neq 0,\ \pi/2$	$\omega = kvA(\Phi)\cos\Phi$ $A(\Phi) > 1$		ν/ω	$E \parallel [kH]$	
7. Slow wave with quadratic spectrum	$\frac{\omega}{\Omega}\frac{v}{v_a} \ll 1 \ll \left(\frac{\omega}{\Omega}\right)^{1/2}\left(\frac{v}{v_a}\right)^{3/2}$	$\omega = k^2 c r_D \sin\Phi\cos\Phi$			$E \parallel \bar{H}$	Exists only if the direction of the vector H does not coincide with a symmetry axis

important role is played by Cerenkov absorption of the wave by electrons and the main part of the conductivity of the metal is dissipative. Therefore weakly-damped waves can exist only in those special cases when the Cerenkov absorption vanishes for certain directions of the vectors **k** and **E**. We shall consider these special cases below.

In a magnetic field, the Cerenkov absorption is due to electrons that move, in the mean, in phase with the wave and for which the following relations are satisfied:

$$k_z v_z = \omega + N\Omega, \quad N = 0, \ \pm 1, \ \pm 2, \ \ldots \tag{11.1}$$

If the wave vector **k** is exactly perpendicular to the magnetic field H ($k_z = 0$) and $\omega \neq N\Omega$, then condition (11.1) is not satisfied for any of the electrons, and there is no Landau damping. With this, waves much shorter than the characteristic Larmor radius can exist near cyclotron resonance. The propagation of these waves will be considered in Sec. 16.

On the other hand, if the angle between the vectors **k** and **H** differs from $\pi/2$ and if $\omega \ll \Omega$, then relation (11.1) is satisfied for many groups of electrons:

$$N \sim k_z R \gg 1. \tag{11.2}$$

In this case the Cerenkov absorption is strong, and the electromagnetic waves cannot propagate in general. However, under certain conditions the Landau damping can experience sharp oscillations, depending on the relation between the wavelength and the diameter of the electron orbit. The physical nature of these oscillations is essentially the same as in the case of "geometric resonance" in the absorption of ultrasound [33]. The occurrence of these oscillations is due to the fact that the electron interacts most effectively with the electromagnetic field on those sections of its orbit, where it moves along the equal-phase planes of the wave. In the case of a convex electron orbit, the latter has two such sections (in the vicinity of the points A and B, see Fig. 5). On the other hand, when the electron moves between these sections, it is acted upon by rapidly oscillating field of the wave, which does not change its average energy. Therefore the energy absorbed by the electrons depends on the phase difference of the wave field at the points A and B, where they move in opposite directions. If the orbit diameter subtends an odd number half-waves, then the action of the wave on the electron is the same in sections A and B. The absorption is then maximal. On the other hand,

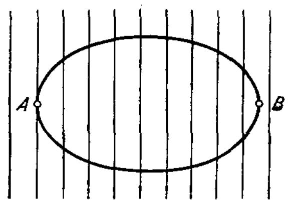

FIG. 5

if the number of half-waves subtended by the diameter is even, then the wave field acts in these sections in opposite directions, and there is no absorption.

When $k_z R \gg 1$, there is always a group of electrons whose contribution to the absorption is large. This results in a relatively small amplitude of the oscillations when kR is varied. On the other hand, if $k_z R$ is small, then the condition (11.1) is satisfied only when N = 0. In this case the "geometric resonance" oscillations are large [34], and the Cerenkov absorption vanishes at the minima. It was shown in [12,35] that for definite values of the vector k the dissipative conductivity becomes so small, and that the main role is assumed by the Hall conductivity. It then becomes possible for electromagnetic waves to propagate with discrete wave-vector and frequency spectra. We shall now consider them in greater detail.

12. Asymptotic Behavior of the Conductivity Tensor

To investigate the properties of waves with a discrete spectrum in the case of large kR, it is necessary to find the asymptotic form of the tensor $\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H})$. We assume that the angle Φ between the vectors **k** and **H** is close to $\pi/2$:

$$\varphi = \frac{\pi}{2} - \Phi \ll 1. \tag{12.1}$$

At the same time we assume that the spatial inhomogeneity of the field wave along **H** is strong. In other words, we are interested in the asymptotic behavior of the tensor (2.13) when the following conditions are satisfied:

$$|\nu - i\omega| \ll k_z v \ll \Omega \ll kv. \tag{12.2}$$

An asymptotic expression for the conductivity tensor was obtained in [35] for the case of an isotropic quadratic dispersion of the conduction electrons. In the coordinate system x, η, ζ it is of the form

$$\sigma_{\alpha\beta}(\mathbf{k}, \omega, \mathbf{H}) = \frac{3ne^2}{2mk_z vkR} \begin{pmatrix} 1-F & \dfrac{\cos\left(2kR-\frac{\pi}{4}\right)}{(\pi kR)^{1/2}} - \dfrac{\nu-i\omega}{k_z v}G, & -\dfrac{\nu-i\omega}{k_z v}G \\ -\dfrac{\cos\left(2kR-\frac{\pi}{4}\right)}{(\pi kR)^{1/2}} + \dfrac{\nu-i\omega}{k_z v}G & \dfrac{\nu-i\omega}{k_z v} & \dfrac{\nu-i\omega}{kv} \\ \dfrac{\nu-i\omega}{k_z v}G & \dfrac{\nu-i\omega}{kv} & 2k_z R\,\dfrac{\nu-i\omega}{kv} \end{pmatrix}, \tag{12.3}$$

where

$$F = \frac{2}{\sqrt{\pi}} \int_0^\infty dx \sin\left(2kR - \frac{\pi}{4} + x^2\right) \exp\left(-2x\sqrt{kR}\,\frac{\nu-i\omega}{k_z v}\right), \tag{12.4}$$

$$G=\frac{2}{\sqrt{\pi}}\int_0^\infty dx\cos\left(2kR-\frac{\pi}{4}+x^2\right)\exp\left(-2x\sqrt{kR}\,\frac{\nu-i\omega}{k_zv}\right). \quad (12.5)$$

It is possible to show with the aid of these formulas that, owing to the smallness of the angle φ, the distinction between the axes η and z, and also ζ and $-y$, can be neglected. In addition, it follows from (12.3) that the longitudinal part of the field E_ζ is much larger than the transverse one:

$$E_\zeta\approx -E_y\approx-\frac{1}{2k_zR}\,(GE_x+E_z). \quad (12.6)$$

However, in spite of this, the "renormalization" (2.4) does not play an important role and

$$\tilde{\sigma}_{\alpha\beta}\cong\sigma_{\alpha\beta}\quad(\alpha,\ \beta=x,\ \eta). \quad (12.7)$$

It follows from (12.4) and (12.5) that the asymptotic behavior of the elements $\sigma_{\alpha\beta}$ depends on the magnitude of the parameter

$$w=(kR)^{1/2}\,\frac{\nu-i\omega}{k_zv}. \quad (12.8)$$

If

$$|w|\ll 1 \quad (12.9)$$

we have[35]

$$\sigma_{xx}=\frac{3ne^2}{2mk_zvkR}\Big[1-\sin(2kR)+\frac{2}{\sqrt{\pi}}\,w\cos\left(2kR-\frac{\pi}{4}\right)\Big], \quad (12.10)$$

$$\sigma_{xz}=-\sigma_{zx}=\frac{3\pi ne^2}{2mk_zv\,(\pi kR)^{3/2}}\Big[\cos\left(2kR-\frac{\pi}{4}\right)-w\sqrt{\pi}\cos(2kR)\Big]; \quad (12.11)$$

Re σ_{xx} is responsible for the dissipation of the wave energy. It is a sharply oscillating function of kR, taking on minimum values at

$$kR=\alpha_n\equiv\left(n+\frac{1}{4}\right)\pi,\qquad n=0,\,1,\,2,\,\ldots \quad (12.12)$$

These oscillations of Re σ_{xx} are due to the fact that the main role in the absorption is played by the electrons that move, in the mean, in phase with the wave. The velocity of these electrons in the direction of the magnetic field is

$$v_z\sim\frac{|\omega+i\nu|}{k_z}, \quad (12.13)$$

and the scatter of their orbit diameters is

$$\delta D=2R\left(1-\sqrt{1-\frac{v_z^2}{v^2}}\right)\cong R\,\frac{v_z^2}{v^2}\cong R\left|\frac{\nu-i\omega}{k_zv}\right|^2. \quad (12.14)$$

Thus, condition (12.9) is sufficient to make the scatter of the diameters of these effective electrons small compared with the length of the electromagnetic wave. Therefore in the case (12.9) all the effective electrons are under identical conditions relative to the wave, and σ_{xx} oscillates sharply when kR changes.

In the inverse limiting case

$$|w|\gg 1 \quad (12.15)$$

there is a large scatter of the orbit diameters of the effective electrons over the wavelength. Owing to the averaging of their contributions, the amplitude of the oscillations σ_{xx} and σ_{xz} is strongly decreased:

$$\sigma_{xx}=\frac{3ne^2}{2mk_zvkR}\left[1-\frac{1}{w\sqrt{\pi}}\sin\left(2kR-\frac{\pi}{4}\right)\right], \quad (12.16)$$

$$\sigma_{xz}=\frac{3\pi ne^2}{2mk_zv\,(\pi kR)^{3/2}}\,\frac{\sin\left(2kR-\frac{\pi}{4}\right)}{2w^2}. \quad (12.17)$$

13. Low-frequency Waves with Discrete Spectrum

We consider first the case of low frequencies $\omega\ll\nu$, and assume that condition (12.9) is satisfied. We compare the minimum value of σ_{xx} with σ_{xz}. When kR $=a_n$ we have

$$\frac{\sigma_{xx}^{\min}}{|\sigma_{xz}|}=\frac{2\nu}{\varphi\Omega}. \quad (13.1)$$

If this ratio is small, i.e., if the angle φ satisfies the conditions

$$\frac{\nu}{\Omega}\ll\varphi\ll\frac{1}{\alpha_n}, \quad (13.2)$$

then the *nondissipative* conductivity σ_{xz} is much larger than the dissipative terms $\sigma_{xx}^{\min}$ and σ_{zz}. In other words, at wave-vector values

$$k_n=\frac{\alpha_n}{R} \quad (13.3)$$

the effective dielectric constant of the electron gas becomes real. Therefore waves with a discrete spectrum can propagate in the metal. The permissible values of k, for which the Landau damping vanishes, are determined by (13.3). The corresponding frequencies ω_n are obtained from the dispersion equation (2.3). Taking (12.7) into account and putting $k=k_n$, we obtain

$$\omega_n=\frac{k_n^2c^2}{4\pi\,|\sigma_{xz}(k_n)|}\equiv\left(\frac{8\pi}{9}\right)^{1/2}\left(\frac{v_a}{v}\right)^2\alpha_n^{5/2}\varphi\Omega. \quad (13.4)$$

The relative damping of the wave

$$\Gamma_n(k)=-\frac{\operatorname{Im}\omega}{\omega_n}=\frac{\sigma_{xx}(k)}{2\,|\sigma_{xz}(k_n)|}=\frac{\nu}{\varphi\Omega}+(2\pi\alpha_n)^{1/2}\sin^2(kR-\alpha_n) \quad (13.5)$$

is minimal when $kR=\alpha_n$ and increases sharply when $|kR=\alpha_n|$ increases. The width in that region of k-space in which a weakly-damped wave exists is given by

$$\Gamma_n(k)\ll 1. \quad (13.6)$$

If the inequality $\nu\ll\varphi\Omega$ is violated, even the minimum value of the relative damping Γ_n becomes larger than or of the order of unity, and the wave attenuates rapidly.

Substituting the wave spectrum (13.4) into Maxwell's equations (1.7), we can easily show that the transverse part of the electric field of the wave is circularly polarized: $E_x\approx iE_z$. On the other hand, the magnitude of the longitudinal field E_y should be determined from (12.6).

The region of existence of the wave is determined by the conditions (13.2) and the inequality $\omega_n \ll \nu$. For typical metals ($n \sim 10^{22}$ cm^{-3}, $m \sim 10^{-27}$ g) they are satisfied in fields $H \gtrsim 10^3$ Oe at angles φ on the order of one degree and mean free paths v/ν on the order of several millimeters.

14. High-frequency Waves with Discrete and Continuous Spectrum

In the region of high frequencies $\omega \gg \nu$, the element σ_{xx} has (12.10) at the minima of the oscillations an appreciable imaginary part proportional to ω. It was shown in [35] that the expressions for ω_n and Γ_n differ in this case from (13.4) and (13.5) in that they contain an additional factor $1/\rho_n$, where

$$\varrho_n^2 = 1 + \left(\frac{32\pi}{9}\right)^{1/2}\left(\frac{v_a}{v}\right)^2 \alpha_n^{9/2}. \qquad (14.1)$$

The polarization of the transverse part of the electric field of the wave is elliptical:

$$E_x = i\varrho_n E_z. \qquad (14.2)$$

Let us consider the variation of the properties of this wave when the magnetic field H changes.

a) In the region of weak fields, when the second term in the right-hand side of (14.1) is small compared with unity, we have $\rho_n \to 1$. Then the spectrum, the damping, and the polarization of the high-frequency wave remain the same as for the low-frequency wave (see Sec. 13).

b) In the region of stronger fields, when

$$\varrho_n^2 \gg 1, \qquad (14.3)$$

we have [35]

$$\omega_n = \left(\frac{2\pi}{9}\right)^{1/4} \frac{v_a}{v}\, \alpha_n^{9/4}\varphi\Omega, \qquad (14.4)$$

$$\Gamma_n(k) = \frac{1}{2\omega_n}\left[\nu + \varphi\Omega\,(2\pi\alpha_n)^{1/2}\sin^2(kR - \alpha_n)\right]. \qquad (14.5)$$

The transverse part of the electric field in the wave is polarized along the x axis. Its spectrum and damping are determined by the value of σ_{xx}^{min}. The existence of such a wave is due to the fact that when $kR = \alpha_n$ we have

$$\mathrm{Re}\,\sigma_{xx}^{min} \ll -\mathrm{Im}\,\sigma_{xx}. \qquad (14.6)$$

c) In the case $\omega \gg \nu$, the real part of the element σ_{zz} (see (12.3)) is small compared with its imaginary part, which is negative. This should lead to the existence of a weakly-dampled wave. However, owing to the noticeable magnitude of the off-diagonal element σ_{xz} there is, in general, a rather strong coupling with the second wave, which attenuates rapidly. Since the element σ_{xz} (12.11) is a rapidly oscillating function of kR when $|w| \ll 1$, it follows that when

$$kR = \beta_N = \frac{\pi}{2}\left(N + \frac{3}{4}\right), \quad N = 0, 1, 2, \ldots, \qquad (14.7)$$

the coupling of the wave weakens, and the term with σ_{xz} in the dispersion equation can be neglected. With this, the dispersion equation splits into two, one of which determines the damped wave (when $k = \beta_N/R$ the element σ_{xx} is real). The second wave,

$$k_N^2 c^2 = 4\pi i\omega\sigma_{zz}(k_N, \omega), \qquad (14.8)$$

gives a weakly damped wave with a discrete spectrum and linear polarization [35]:

$$\omega_N = \left(\frac{2}{3}\right)^{1/2} \frac{v_a}{v}\, \beta_N^{5/2}\varphi\Omega, \quad k_N = \frac{\beta_N}{R}, \qquad (14.9)$$

$$E_x = 0, \quad E_z = 2\varphi\beta_N E_y. \qquad (14.10)$$

In addition to the conditions (12.2) and $\omega_N \gg \nu$, the existence of this wave calls for satisfaction of the inequality

$$\frac{|\sigma_{xz}^{max}|^2}{\sigma_{xx}|\sigma_{zz}|} \gg 1, \qquad (14.11)$$

which ensures strong coupling of the waves when $k \neq k_N$. The most stringent of these conditions can be represented in the form

$$\frac{\nu}{\varphi\Omega} \ll \frac{v_a}{v}\beta_N^{5/2} \ll 1, \quad \varphi \ll \frac{1}{\beta_N} \ll 1. \qquad (14.12)$$

d) Finally, if in addition to the conditions (12.2) and $\omega_N \gg \nu$ the inequality

$$\varphi \ll \frac{\omega}{\Omega} \qquad (14.13)$$

is satisfied, then the degree of wave coupling is weak (inequality (14.11) is reversed). Equation (14.8) becomes valid also for $kR \neq \beta_N$, and the spectrum of the wave turns from discrete to continuous [12]:

$$\omega(k) = \left(\frac{2}{3}\right)^{1/2} \frac{v_a}{v}(kR)^{5/2}\varphi\Omega. \qquad (14.14)$$

For a wave with spectrum (14.14) to exist, it is necessary to satisfy the conditions (1.20) and (14.13) and the inequalities

$$1 \ll \left(\frac{v}{v_a}\right)^{2/5}\left(\frac{\varphi\Omega}{\omega}\right)^{3/5} \ll \frac{\Omega}{\omega}, \qquad (14.15)$$

which are obtained from (12.2) by expressing k in terms of ω with the aid of (14.14).

The foregoing analysis of the possibilities of propagation of weakly damped waves is based on the use of expressions (12.3) for the elements of the conductivity tensor. These asymptotic formulas were obtained for a singly-connected spherical Fermi surface. It is quite obvious that the character of the asymptotic behavior of the tensor $\sigma_{\alpha\beta}$, and in particular the deduced presence of sharp oscillations of the elements σ_{xx} and σ_{xz}, remains in force also in the case of a nonspherical but convex Fermi surface. The difference lies in the fact that in (12.10) and (12.11) the arguments of the oscillating functions will contain not 2kR but kD, where D is the diameter of the central section of the Fermi surface along the $\mathbf{k} \times \mathbf{H}$ direction. In addition, the dissipative current connected with the Landau damping will be directed along the electron velocity $\partial\epsilon(\mathbf{p})/\partial\mathbf{p}$ at the point $\mathbf{p} \cdot \mathbf{H} = 0$ on the Fermi surface $\epsilon(\mathbf{p}) = \epsilon_F$.

From the considerations presented in Sec. 11 it follows that in the case of a multiply connected or non-convex Fermi surface the amplitude of the conductivity oscillations will be small. In this case, indeed, there is not one but several groups of "effective" electrons, moving on the average in phase with the wave and satisfying the condition $k_z v_z = \omega$. Therefore σ_{xx} will be the sum of expressions of the type (12.10) for many groups:

$$\sigma_{xx} \sim \sum_j a_j (1 - \sin kD_j). \qquad (14.16)$$

This quantity cannot vanish for any real value of the wave vector.* Therefore there are no waves with discrete spectrum in the case of a multiply connected or non-convex Fermi surface. Only the wave (14.14) with *continuous spectrum can propagate in the metal, if all* the dissipative currents from different groups of "effective" electrons are collinear. Thus, the predicted waves with discrete spectrum can be observed only in metals with a convex Fermi surface. This requirement is satisfied by alkaline metals.

15. Excitation of Waves with Discrete Spectrum. New Resonance Effect

An external wave incident on the surface of a metal gives rise to the skin effect. The highly inhomogeneous distribution of the electromagnetic field at the surface of the metal can be represented as a superposition of plane waves with different values of k. Owing to the small depth of the skin layer, there are always some waves satisfying the condition $kR = \alpha_n$. Resonant excitation of the natural oscillations will occur whenever the frequency ω of the external wave coincides with one of the natural frequencies ω_n. If the frequency ω is fixed, then waves with discrete spectrum are excited at magnetic-field values $H = H_n$ for which $\omega_n(H) = \omega$.

By way of an example, we can consider the excitation of a wave with spectrum (14.4), in which the transverse part of the electric field is polarized along the x axis. For this wave, the resonant values of the magnetic field are

$$H_n = \frac{mc}{e}\left(\frac{\omega\omega_0 v}{\varphi c}\right)^{1/2}\left(\frac{9}{2\pi}\right)^{1/8}\alpha_n^{-9/8}. \qquad (15.1)$$

Let us find first the field $\mathcal{E}_x(\zeta)$ at large distances from the metal surface $\zeta = 0$. This field is characterized by the function

$$T_{xx}(\zeta) = -\frac{2}{\pi}\int_0^\infty \frac{dk\, k^{5/2}\cos k\zeta}{k^{9/2} - \left(\frac{H_n}{H}\right)^4 k_n^{9/2}[1 + 2i\Gamma(k)]}, \qquad (15.2)$$

where $k_n = \alpha_n/R$, and the expression for $\Gamma(k)$ is determined by formula (14.5), in which ω_n must be replaced by ω. We introduce the "detuning" from resonance

$$\Delta = \frac{8}{9}\frac{H - H_n}{H_n} \quad (|\Delta| \ll 1) \qquad (15.3)$$

and a new integration variable $\tau = (k/k\ )^{1/2}$. In the region of resonance ($\Delta \to 0$) the main contribution to $T_{xx}(\zeta)$ is made by a small vicinity of the pole of the integrand near the point $\tau = 1$ on the complex τ plane. Therefore we can represent $T_{xx}(\zeta)$ approximately in the form

$$T_{xx}(\zeta) \approx -\frac{2}{\pi k_n}\int_0^\infty \frac{d\tau\, \tau^6[\exp(ik_n\zeta\tau^2) + \exp(-ik_n\zeta\tau^2)]}{\tau^9 - \left[1 - \frac{9}{2}\Delta + \frac{9}{4}i\Gamma(\Delta)\right]}, \qquad (15.4)$$

where

$$\Gamma(\Delta) = \frac{1}{2\omega}[\nu + (2\pi\alpha_n)^{1/2}\varphi\Omega\Delta^2]. \qquad (15.5)$$

In the first term of (15.4) we swing the contour of integration towards the line $\arg\tau = \pi/4$, and in the second to the line $\arg\tau = -\pi/4$. As a result we obtain the sum of the residues and the integrals along the lines $\arg\tau = \pm\pi/4$. For large ζ, the main contribution to $T_{xx}(\zeta)$ is made by the first residue at the point

$$\tau_0 = 1 - \frac{\Delta}{2} + \frac{i}{4}\Gamma(\Delta). \qquad (15.6)$$

In the vicinity of the resonance, where $\Gamma(\Delta) \ll 1$, we obtain

$$T_{xx}(\zeta) \approx -\frac{4i}{9}\frac{R}{\alpha_n\left[1 - \Delta + \frac{i}{2}\Gamma(\Delta)\right]} \times \exp\left\{\frac{i\alpha_n\zeta}{R}\left[1 - \Delta + \frac{i}{2}\Gamma(\Delta)\right]\right\}. \qquad (15.7)$$

This means that at large distances from the surface the field is a plane monochromatic wave of length

$$\lambda_n = \frac{2\pi R}{\alpha_n} \approx \frac{2\pi R_n}{\alpha_n}, \quad R_n = R\frac{H}{H_n}, \qquad (15.8)$$

having a small damping proportional to $\Gamma(\Delta)$. Further, from (15.2) with $\zeta = 0$ and $\Delta \to 0$ we obtain the resonant values of the surface impedance:

$$Z_{xx}^{(n)} = \frac{16\pi}{9}\frac{\omega R_n}{\alpha_n c^2}\left(1 - i\,\mathrm{ctg}\,\frac{2\pi}{9}\right). \qquad (15.9)$$

They increase slowly with increasing n:

$$Z_{xx}^{(n)} \sim \alpha_n^{1/8}. \qquad (15.10)$$

This increase in the resonant values of Z with decreasing magnetic field is due to the increase in the wavelength (15.8) of the natural electromagnetic os-

*An exception is the case when the diameters of the central sections of different carrier groups have a rational ratio. Then the spectrum of the weakly-damped waves is more "rarefied."

cillations. The ratio of the width of the resonant maximum to the distance between neighboring maxima has the same order of magnitude as the relative width of the region in k-space in which the wave attenuates weakly.

Far from resonance, when $\Gamma(\Delta) \gg 1$, the expression for $T_{xx}(0)$ can be written in the form

$$T_{xx}(0) = -\frac{2}{\pi}\int_0^\infty \frac{dk \cdot k^2}{k^4 - i\delta_{eff}^{-4}[1-\sin(2kR)]}, \quad (15.11)$$

where

$$\delta_{eff} = \left(\frac{2}{3}\frac{v^2c^2\varphi}{\omega_0^2\omega\Omega}\right)^{1/4} \quad (15.12)$$

is the effective depth of penetration of the electromagnetic field into the metal. The value of $T_{xx}(0)$ was calculated for this case in [36]. When $\delta_{eff} \ll R$ we have

$$Z_{xx}^{(nonres)} = \left(\frac{2}{\pi}\right)^{1/2}\gamma\left(\frac{1}{4}\right)\frac{\omega\delta_{eff}}{c^2}\exp\left(\frac{3\pi i}{8}\right) \sim H^{-1/4}\omega^{3/4}, \quad (15.13)$$

where $\gamma(x)$ is the Euler gamma function.

Thus, narrow maxima, corresponding to resonant excitation of natural oscillations with spectrum (14.4), are superimposed on the smooth dependence of the impedance on the magnetic field (15.13). The ratio of the resonant value of the impedance $Z_{xx}^{(n)}$ to the nonresonant value (15.13) is

$$\frac{Z_{xx}^{(n)}}{Z_{xx}^{(nonres)}} \approx \frac{R}{\alpha_n\delta_{eff}} \approx \left(\frac{\varphi\Omega_n}{\omega}\alpha_n^{1/2}\right)^{1/4} \gg 1. \quad (15.14)$$

Figure 6 shows schematically the dependence of the impedance on the reciprocal of the magnetic field.

The dependence of $Z_{\alpha\beta}$ on H has a similar character also when other weakly-damped waves with discrete spectrum are excited.

16. Electromagnetic Waves in the Vicinity of Cyclotron Resonances

With the exception of Sec. 5, we have considered waves with frequencies much lower than the carrier cyclotron frequency Ω. It was noted in Sec. 5 that near cyclotron resonance an important role is played by spatial dispersion and the Cerenkov absorption associated with it. The results of Sec. 5 pertain to metals with low carrier density (such as bismuth). These metals are characterized by a small value of the parameter $\omega_0 R/c = v/v_a$, which determines the role played by the spatial dispersion. For typical metals, the actual situation is reversed,

$$\frac{\omega_0 R}{c} \gg 1, \quad (16.1)$$

where the length of the electromagnetic wave turns out to be much smaller than the diameter of the electron

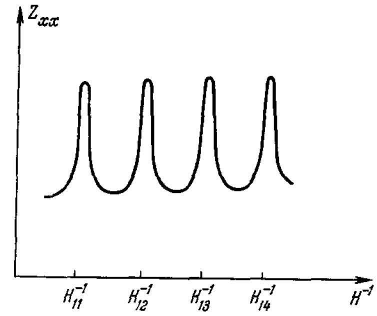

FIG. 6. Impedance vs. reciprocal magnetic field upon excitation of a wave with discrete spectrum (14.4).

orbits ($kR \gg 1$). Therefore cyclotron resonance in such metals is possible only in a magnetic field that is strictly parallel to the surface [37]. With this, the wave vector **k** is orthogonal to the vector **H**. It was noted in Sec. 11 that in this case, when $\omega \neq N\Omega$, there are no electrons that move, in the mean, in phase with the wave. Consequently there is no Cerenkov absorption. Mathematically this is manifest by the fact that the spatial dispersion does not influence the form of the factors

$$(\omega - \Omega N + i\nu)^{-1}, \quad (16.2)$$

which describe the effect of the resonant interaction of the electrons with the electromagnetic field.

The wave-energy dissipation in the case of strictly transverse propagation $\mathbf{k} \perp \mathbf{H}$ is the due only to the carrier scattering, which is characterized by the collision frequency ν. Therefore in the case when

$$|\omega - N\Omega| \gg \nu \quad (16.3)$$

we can neglect ν on (16.2), and the dielectric constant ϵ of the conductor turns out to be real. Its sign is determined by the sign of the difference $\omega - N\Omega$. Thus, on one side of the resonance we have $\epsilon < 0$, and the wave experiences total reflection. On the other side of the resonance the dielectric constant ϵ is positive, and in accordance with the general point of view expressed in the introduction, weakly-damped waves can propagate in the metal [14]. This is indeed the cause of the sharp asymmetry of the resonance surface-impedance curves at cyclotron resonance [37].

We confine ourselves for simplicity to examination of an isotropic electron dispersion law. It follows from the general expression (2.13) that in the case under consideration ($\mathbf{k} \parallel y$, $\mathbf{H} \parallel z$) the elements σ_{zy}, σ_{yz}, σ_{xz}, and σ_{zx} vanish identically. Expanding

$$\exp[ikR(\cos\tau - \cos\tau')]$$

in a double Fourier series in the variables τ and in τ' and integrating with respect to them, we obtain [14]

$$\sigma_{\alpha\beta} = \frac{3ne^2}{2m}\sum_{N=-\infty}^{\infty}\frac{1}{\nu - i(\omega - N\Omega)}\int_0^\pi d\theta\sin\theta\begin{pmatrix}[J'_N(x)]^2\sin^2\theta & -J_N(x)J'_N(x)\frac{\nu - i\omega}{ikv}\sin\theta & 0\\ J_N(x)J'_N(x)\frac{\nu - i\omega}{kv}\sin\theta & -[J_N(x)]^2\frac{N\Omega}{(kv)^2}(\nu - i\omega) & 0\\ 0 & 0 & [J_N(x)]^2\cos^2\theta\end{pmatrix}, \quad (16.4)$$

where $x = kR \sin\theta$ and the prime denotes the derivative of the Bessel function $J_N(x)$ with respect to x.

The arguments of the oscillating functions in the asymptotic expression, $J_N(x)$ and $J'_N(x)$, differ by $\pi/2$ when $x \gg 1$. Therefore σ_{xy}^2 is smaller by a factor kR than the product $\sigma_{xx}\sigma_{yy}$ and the tensor $\sigma_{\alpha\beta}$ is practically diagonal. Its elements are given by the formulas [14]

$$\sigma_{xx} = \sigma_{zz} = \frac{3\pi i}{4}\frac{ne^2}{mkv}\operatorname{ctg}\left(\pi\frac{\omega+i\nu}{\Omega}\right), \tag{16.5}$$

$$\sigma_{yy} = \frac{3ne^2(\nu - i\omega)}{m(kv)^2}\left[1 - \frac{\pi}{2}\frac{\omega+i\nu}{kv}\operatorname{ctg}\left(\pi\frac{\omega+i\nu}{\Omega}\right)\right]. \tag{16.6}$$

It follows from them that the dispersion equations for both transverse waves are identical and are not connected with each other

$$k^2c^2 = 4\pi i\omega\sigma_{\substack{xx\\zz}}. \tag{16.7}$$

The wave whose electric vector is polarized along the field H is called in magnetoactive-plasma theory [6] the ordinary wave. The second transverse wave, in which $\mathbf{E} \parallel x$, is called extraordinary. We use the same terminology.

Near the cyclotron resonances, when

$$\nu \ll N\Omega - \omega \ll \Omega, \tag{16.8}$$

both waves are weakly damped. Their spectrum and damping are

$$\omega_N(k) = N\Omega \times \left[1 - \frac{3}{4}\left(\frac{\omega_0 R}{c}\right)^2\frac{1}{(kR)^3}\right], \tag{16.9}$$

$$-\operatorname{Im}\omega = \nu. \tag{16.10}$$

Similar results were obtained by Demidov [38] for a nondegenerate plasma. Since the frequency ω is almost fixed by the condition (16.8) near resonance, it is more convenient to solve the dispersion equation (16.9) with respect to k:

$$(kR)^3 = \frac{3}{4}\left(\frac{\omega_0 R}{c}\right)^2\left(1 - \frac{\omega}{N\Omega}\right)^{-1}. \tag{16.11}$$

It follows from (16.11) that the frequencies $N\Omega$ are limiting (resonant) frequencies. The condition $kR \gg 1$ is automatically satisfied near resonance in the case (16.1)

Inequalities (16.8) ensure smallness of the dispersion and damping of the ordinary and extraordinary waves. Figure 7 shows schematically the dependence of Re ω_N on k (solid lines).

Besides the transverse waves, a third, longitudinal wave should exist near the cyclotron resonances [14]. The electric field in it is potential and polarized along the wave vector **k**. Unlike the case of small kR, when the spectrum of the longitudinal wave has a large "gap" ω_0, this spectrum shifts towards much frequencies $N\Omega$ much smaller than ω_0 when kR is large. Similar longitudinal waves can exist also in a dense high-tempera-

Table II

Waves	Existence condition	Wave spectrum	Relative damping	Polarization	Remark
1. Low-frequency wave with discrete spectrum	$\omega \ll \nu \ll \varphi\Omega \ll \Omega/\alpha_n$, $1 \ll \alpha_n$	$\omega_N = \left(\frac{8\pi}{9}\right)^{1/2}\left(\frac{v_a}{v}\right)^2 \times \varphi\Omega\alpha_N^{9/2}$, $k_N = \alpha_N/R$	$\frac{\nu}{\varphi\Omega} + (2\pi\alpha_N)^{1/2} \times \sin^2(kR - \alpha_n)$	$E_x = iE_z$, $E_y = E_z/2\varphi\alpha_N$	Exists in a metal with with singly-connected and convex Fermi surface
High-frequency waves with discrete spectrum	$\nu \ll \omega_N$, $\varphi \ll \alpha_N^{-1} \ll 1$				
2. Wave (13.4)	$\nu \ll \varphi\Omega\varrho_N$, $\left(\frac{v_a}{v}\right)^2\alpha_N^4 \ll \varrho_N$	$\omega_N = \left(\frac{8\pi}{9}\right)^{1/2} \times \left(\frac{v_a}{v}\right)^2\frac{\varphi\Omega\alpha_N^{9/2}}{\varrho_N}$, $k_N = \alpha_N/R$	$\frac{1}{\varrho_N}\left[\frac{\nu}{\varphi\Omega} + (2\pi\alpha_N)^{1/2} \times \sin^2(kR - \alpha_n)\right]$	$E_x = i\varrho_N E_z$, $E_y = \frac{1 + \frac{4}{3}\alpha_N^4\left(\frac{v_a}{v}\right)^2}{2\varphi\alpha_N}E_z$	Ditto
3. Wave (14.4)	$\omega_N \ll \varphi\Omega$	$\omega_N = \left(\frac{2}{3}\right)^{1/2}\frac{v_a}{v}\varphi\Omega\beta_N^{5/2}$, $k_N = \beta_N/R$	$\frac{1}{2\omega_N}\left[\nu + \frac{4\varphi\Omega\sin^2(kR - \beta_N)}{\pi(1 - \sin 2\beta_N)}\right]$	$E_x = 0$, $E_y = E_z/2k_zR$	Ditto
4. High-frequency wave with continuous spectrum	$\nu \ll \omega$, $\left(\frac{\omega}{\Omega}\right)^{2/5}\left(\frac{v_a\varphi}{v}\right)^{3/5} \ll \varphi \ll \frac{\omega}{\Omega} + \left(\frac{\Omega}{\omega}\frac{v_a}{v}\varphi\right)^{2/5}$	$\omega = \left(\frac{2}{3}\right)^{1/2} \times \left(\frac{v_a}{v}\right)\varphi\Omega(kR)^{5/2}$	$\nu/2\omega$		
Waves near cyclotron resonances	$\nu \ll \lvert N\Omega - \omega\rvert \ll \Omega$, $\frac{\omega_0 R}{c} \gg 1$, $\varphi \ll \frac{\lvert N\Omega - \omega\rvert}{kR}$				
5. Ordinary wave	$\omega < N\Omega$	$\omega_N = N\Omega \times \left[1 - \frac{3}{4(kR)^3}\left(\frac{\omega_0 R}{c}\right)^2\right]$	ν/ω_N	$E_x = E_y = 0$, $\mathbf{E} \parallel \mathbf{H}$	
6. Extraordinary wave	$\omega < N\Omega$	$\omega_N = N\Omega \times \left[1 - \frac{3}{4(kR)^3}\left(\frac{\omega_0 R}{c}\right)^2\right]$	ν/ω_N	$E_z = E_y = 0$, $\mathbf{E} \parallel [\mathbf{kH}]$	
7. Longitudinal wave	$\omega > N\Omega$	$\omega_N = N\Omega\left(1 + \frac{1}{2kR}\right)$	ν/ω_N	$E_x = E_z = 0$, $\mathbf{E} \parallel \mathbf{k}$	

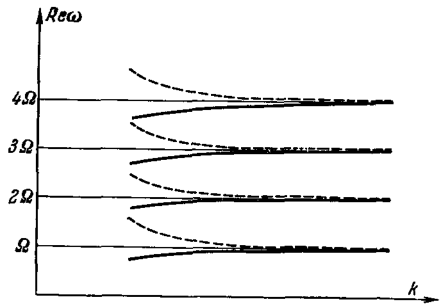

FIG. 7. Dispersion of weakly damped ordinary and extraordinary waves (solid curves) and of the longitudinal wave (dashed curves) in the vicinity of cyclotron resonances.

ture plasma when inequalities (16.8) and (16.1) are satisfied (v must be replaced by $(2T/m)^{1/2}$).

The T longitudinal waves can propagate because of their weak coupling to the transverse ones:

$$|\sigma_{xy}^2| \ll |\sigma_{xx}\sigma_{yy}|. \quad (16.12)$$

Their spectrum and damping are determined by the dispersion equation

$$\varepsilon_{yy} \equiv \frac{4\pi i \sigma_{yy}(k, H, \omega)}{\omega} = 0. \quad (16.13)$$

Upon satisfaction of the condition

$$\nu \ll (\omega - N\Omega) \ll \Omega \quad (16.14)$$

the dispersion equation yields

$$\omega_N(k) = N\Omega\left(1 + \frac{1}{2kR}\right), \quad \text{Im}\,\omega = -\nu. \quad (16.15)$$

Unlike the transverse waves, the dispersion of the longitudinal waves is anomalous, i.e., their frequency decreases with increasing k. The spectrum of the longitudinal wave is shown by the dashed lines of Fig. 7.

An essential condition for the existence of all three waves is strict perpendicularity of the vectors **k** and **H**:

$$|k_z v| < |\omega - N\Omega|. \quad (16.16)$$

If inequality (16.16) is violated, large Cerenkov absorption appears. The cyclotron resonance is then "smeared out" and the weakly damped waves vanish.

To conclude the section, we present Table II, listing the characteristics of weakly damped waves whose length is much smaller than the Larmor radius.

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