

QUASIPARTICLES IN THE THEORY OF THE NUCLEUS

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Usp. Fiz. Nauk 92, 27-64 (May, 1967).

The nuclei are pure emerald.
But it may be the people are lying.
(A. S. Pushkin, The Legend of Tsar Saltan)

I. INTRODUCTION

SEVERAL years ago methods which had been developed in the theory of elementary particles began to be very intensively applied to the study of systems of strongly interacting particles, such as the electrons in metals, liquid helium, or the atomic nucleus.

If, as in the case of nuclei, there are 100 or 200 particles in a system of strongly interacting particles, it is clear that there is no hope of solving the problem exactly. Even the problem of three interacting particles cannot be solved in general form.

In the case of the nucleus the interaction energy is comparable with the kinetic energy of the particles, and the interaction cannot be regarded as weak—i.e., perturbation theory cannot be applied.

If the system were like a gas, in which the particles interact rarely, then one could develop an approximate method of treatment in which only collisions of pairs of particles are taken into account. In the nucleus, however, the distances between particles are of the same order as the range of the forces, and several particles interact simultaneously.

Different approximate methods must be developed. The idea of these methods is the same as in the dispersion theory of elementary particles, where constants are introduced for the masses of the particles and the characteristics of their interaction. We must introduce constants characterizing the motion of the neutrons and protons in the nucleus, and also constants describing the interactions between the nucleons when they are together in the nucleus. This interaction is very different from that of two nucleons in empty space.

For the infinite system such a program was carried through in Landau's theory of the Fermi liquid.^[1] We shall see below what sort of complications are brought in by the finite size of the nucleus.

After the introduction of suitable constants all nuclear phenomena associated with low energies (energies below 40 MeV) can be calculated. In other words, having found the constants from some of the phenomena, we can quantitatively explain all of the other experimental facts of low-energy nuclear physics. We are of course concerned with the derivation of quantitative relations. The approximate models

ordinarily used in the theory of the nucleus^[2,3] suffice for the qualitative study of almost all nuclear processes. We shall, by the way, present a number of phenomena which cannot be correctly interpreted without the rigorous approach.

A nucleus is a system of two types of Fermi particles—neutrons and protons—strongly interacting with each other. Therefore before proceeding to the examination of various nuclear processes it is useful to elucidate the general physical properties of such systems.

As a rigorous treatment shows, the low-lying excited states of Fermi systems are of a very simple nature, even when the interaction is strong.

First, there are so-called one-particle states, which are analogous to the excitations in an ideal Fermi gas. Excited states of an ideal Fermi gas correspond to the transition of a particle from a state with an energy less than the Fermi limit to a free state above the Fermi limit, or in other words the appearance of a particle and a hole against the background of the Fermi population of the states.

The excitations in a real Fermi system also correspond to the appearance of particles and holes. In particular, such quasiparticles, as they are called, have a mass different from that of free particles. In other words, the one-particle excitations in a real Fermi system coincide with the excitations of an ideal gas composed of quasiparticles with a Fermi distribution in energy.

Physically these results are very natural. A particle moving in a medium sets into motion particles adjacent to it. In weak excitations, when the energy of the particle is close to the Fermi energy, the nature of the distribution of particles brought into motion does not depend very much on the state of the particle under consideration. Therefore in all processes associated with weak excitations the particle and its environment behave as a stable formation, which is called a quasiparticle. Since spin is conserved, the spin of the whole conglomerate forming the quasiparticle is the same as the spin of the particle. Consequently, when quasiparticles move as whole structures they must obey the Pauli principle, like any particle with spin $\frac{1}{2}$. Accordingly in all cases in which a small number of quasiparticles and

quasiholes are involved they behave like excitations in an ideal Fermi gas.

In an infinite system, to define the spectrum of one-particle excitations it suffices to introduce one undetermined constant—the effective mass of the quasiparticles.

To characterize the one-particle excitations in a finite system we must introduce, in addition to the effective mass of the quasiparticles, the parameters of the effective potential well in which the quasiparticles move. For systems with short-range forces of range r_0 these parameters include the depth and width of the well and the width of the layer ($\sim r_0$) in which the density changes from its value inside the system to the value zero.

Besides the one-particle excitations, there also exist in a system of interacting particles so-called collective excitations, which can be interpreted as bound states of a quasiparticle and a quasihole. Sound waves in an infinite system are an example of such excitations. To define the spectrum of collective excitations one must introduce an interaction between the quasiparticles, which, as we shall see, is very different from the interaction between two free particles.

For most physical applications (intensities of transitions, magnetic and quadrupole moments, and so on) it is necessary to know the changes that occur in the system under the influence of an external field. The theory shows that the problem of determining the reaction of a system to an external field reduces to the problem of the behavior in an external field of a gas of interacting quasiparticles placed in a potential well. Here it suffices to include only collisions between pairs of quasiparticles. Multiple collisions of the particles are taken into account exactly in the theory, but lead only to a change of the interaction between the quasiparticles and a change of the “charge” for the interaction of the quasiparticles with the external field. In most cases one can find the “charge” from general arguments (from the conservation laws for charge, energy, momentum, and so on.)

There is a very simple and intuitive explanation of these results. Let the system be acted on by a force which is not very strong, so that the change of the energy of each particle in the field is small in comparison with its kinetic energy. Then the state of the system corresponds to the appearance of a few quasiparticles and a few quasiholes against the background of the Fermi distribution. The number of quasiparticles that arise is a small fraction of the total number of particles in the system. If the average distance between the particles is of the order of the range of the forces, the average distance between quasiparticles will be much larger than the range of the interaction forces, and consequently the quasiparticles form a gas—i.e., we can neglect cases in which three

or more quasiparticles collide simultaneously.

The interaction between quasiparticles is of the same order of magnitude as that between particles, but is very different. As we shall see, in some cases an attraction can be replaced by a repulsion owing to the effect of the other nucleons in the medium, which are a great majority and are present along with the two quasiparticles in question.

As for the “charge” of a quasiparticle in relation to the external field, this “charge” describes the interaction with the field of the conglomerate of particles which forms the quasiparticle. We assume that an electric field, which acts only on the protons, is applied to a nucleus. Since charge is conserved in the interaction of a proton with the other particles of the nucleus, the entire conglomerate forming a protonic quasiparticle has the same charge as a proton. In this case the charge of the quasiparticle is equal to that of the particle. In the case of other external fields, for example for a magnetic field, the interaction of a quasiparticle with the field is different from the corresponding quantity for a particle. A moving neutron in vacuum interacts with a magnetic field only owing to its intrinsic magnetic moment, whereas a neutronic quasiparticle in its motion also brings protons into motion, so that an electric current arises and the interaction with a magnetic field is changed. There is thus an orbital magnetism of neutronic quasiparticles, i.e., a magnetism associated with their orbital motions. In the absence of interactions only the protons have orbital magnetism.

Accordingly, the method of quasiparticles as applied to the theory of the nucleus consists of the following steps. First it is rigorously proved that for weak excitations the nucleus can be regarded as a gas of quasiparticles in a potential well. The interaction between the quasiparticles is characterized by a few universal constants. This interaction is not small and must be taken into account exactly. The only approximation is that for weak excitations, when the number of quasiparticles is small, only their pairwise collisions are taken into account. For the majority of observable nuclear phenomena one can derive formulas which, as the result of the solution of the equations with computing machines, can be expressed in terms of the universal constants of the theory.

The constants which define the interaction of the quasiparticles, like the parameters of the potential well, cannot be calculated without the assumption that the interaction between the particles is small. For the nucleus there are no grounds for such an assumption. Therefore these constants must be found from comparisons of the theory with experiment.

The comparison of the theory with experiment shows that the effective mass of the quasiparticles is close to the mass of the free nucleon, $|(m^* - m)/m| \sim 0.1-0.2$. Therefore the parameters of the potential

well, which can be found in the shell model on the assumption $m^* = m$, will not be changed very much with a more rigorous analysis.

As we shall verify below, the interaction between quasiparticles has a range of the order of r_0 . Therefore for all phenomena associated with distances larger than r_0 , for example with distances of the order of the radius of the nucleus, the interaction can be treated as if it were a δ function. The constants for the δ -function interaction differ little from the analogous constants in infinite nuclear matter, and are therefore the same for all nuclei except the very lightest, to the same accuracy as the density of the nuclei is constant throughout the Mendeleev table, i.e., to the same accuracy as that with which the formula for the nuclear radius R and the atomic weight A ($R = r_0 A^{1/3}$) is satisfied. Accordingly, the interaction between quasiparticles is characterized by universal constants. Furthermore in all formulas that contain matrix elements of the interaction the summation must be taken over all states of the quasiparticles, and not only over the states in the last unfilled shell. We emphasize this fact because in this respect there is an important difference between the consistent method of quasiparticles and an approach frequently used.

In many papers on the theory of the nucleus only the particles in the last unfilled shell are regarded as the object of investigation. We shall see that such a treatment is quite legitimate, but that then the interaction between the quasiparticles can no longer be regarded as universal and of δ -function form.

In fact, in such a treatment one must include in the interaction between the quasiparticles, besides a δ -function term, a term caused by the polarization of the other particles in the nucleus, when one quasiparticle excites the Fermi background and this excitation is then passed on to the second quasiparticle. This mechanism corresponds to an interaction between quasiparticles which is no longer local. The theory allows us to express this nonlocal interaction rigorously in terms of the constants of the universal δ -function interaction. After this effective interaction (consisting of two terms) is introduced, there remains in the sums over states only a summation over the states of the unfilled shell. Instead of this, in papers on the theory of the nucleus one introduces as the effective interaction between the quasiparticles of the unfilled shell either a δ -function interaction, or else (for the study of levels with angular momentum 2) a so-called quadrupole-quadrupole interaction (cf.^[2]) with a constant determined from experiment. With this way of introducing the interaction the constants which define it are not universal, but change from level to level. Moreover, such an interaction leads to inexact results for the intensities of transitions.

The matrix element for a one-particle transition

of a nucleon can be written formally in the form of the product of the effective charge of the nucleon and a matrix element calculated without including the interaction. The effective charge can be expressed in terms of the universal constants of the theory and turns out to depend on the initial and final states of the nucleon. Values of the amplitudes for one-particle electromagnetic transitions are obtained to accuracy 20–30 percent.

For quadrupole transitions the effective charge can also be calculated on the assumption of a quadrupole-quadrupole interaction. It then turns out that the effective charge is independent of the nucleon states and gives the amplitudes for one-particle transitions with an error of the order of 100–200 percent. As compared with such a simplified approach the theory makes especially important changes in the calculation of the intensities and frequencies of collective transitions. It is found that besides the 2^+ levels which are obtained on the assumption of a quadrupole-quadrupole interaction it is also possible to have 2^+ and 0^+ levels which vanish when the interaction is chosen in that form.

A correct treatment of the interaction is also required in problems associated with the change of distribution of the nucleons when a particle is added to the nucleus. One such problem is the change of the electric mean square radius of a nucleus when neutrons are added; this determines the isotope shift of spectral lines in intermediate and heavy elements.

Finally, in many nuclear calculations it is assumed that the interaction between quasiparticles is weak; this is completely impermissible. For example, the magnetic moments of even-odd nuclei sometimes differ by a factor 2.5 from the moment of the added nucleon. This difference is entirely due to the interaction of the odd nucleon with the other nucleons of the medium, and consequently the interaction cannot be regarded as weak. All of these questions have been treated in detail in^[4]. Here we give only intuitive arguments which bring out the physical meaning of the relations.

The most convenient way to derive the formulas is the so-called Feynman diagram method. Therefore we begin with an intuitive explanation of this method.

II. FEYNMAN DIAGRAMS

1. The Graphical Representation of Processes

The method of Feynman diagrams is widely used to derive various relations in the theory of elementary particles. This same method is also used in the approach to the theory of the nucleus which we are to discuss here.

We begin with the fact that the various physical

processes which can happen to particles can be represented with figures:

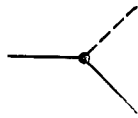
the motion of a light quantum is represented by a dashed line



and that of a particle by a solid line

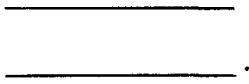


The graph

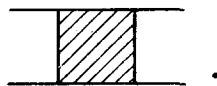


means that a charged particle, say an electron, has emitted a light quantum. The solid line is drawn with a sharp bend to show that the momentum of the electron is different after the emission of the photon.

Suppose there are two noninteracting particles



If they interact, one draws a picture like



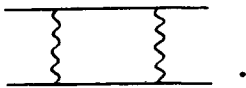
If the interaction is accomplished by means of a light quantum (this means that it is a Coulomb interaction), one joins the lines with a dashed line:



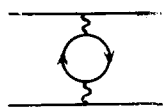
If there are two nucleons and the interaction is accomplished with the exchange of a π meson, a wavy line is drawn between the particle lines:



This graph shows that the two nucleons have interacted with each other once. If they have interacted twice, the drawing is:



The graph



represents a more complicated process: a nucleon has emitted a π meson, which then split up into a nucleon and an antinucleon. These two particles were

then converted into a π meson again, and this was absorbed by the second nucleon.

Still more complicated processes which occur with particles can be represented analogously.

In order that these figures may have not only an illustrative meaning but also a quantitative meaning, we shall take each graph to mean the transition amplitude from one state at the initial time to another state at the final time. The square of the transition amplitude gives the probability of finding the final state at the final time. For example, the graph given above for the emission of a photon denotes the transition amplitude for a charged particle with momentum p to go into a state with a photon of momentum q and a particle of momentum $p - q$.

According to the superposition principle the total transition amplitude, or, as it is usually called, the Green's function, is the sum of all possible physically different transition amplitudes. As an illustration of the graphical method we shall derive the relation connecting the scattering amplitude for two particles with the interaction potential.

According to the superposition principle the scattering amplitude is represented by a sum of graphs:

$$r = \begin{array}{c} \text{[shaded box]} \\ \text{[wavy line]} \end{array} = \begin{array}{c} \text{[wavy line]} \\ \text{[wavy line]} \end{array} + \begin{array}{c} \text{[wavy line]} \\ \text{[wavy line]} \end{array} + \begin{array}{c} \text{[wavy line]} \\ \text{[wavy line]} \end{array} + \dots$$

The first graph represents an interaction between the particles. The second corresponds to a double interaction of the particles. Between the acts of interaction there is the transition amplitude for two noninteracting particles.

We shall assign to the first graph the interaction potential between the particles



and to the lines the Green's function, i.e., the transition amplitude G of a free particle. Then the second graph is written

$$\begin{array}{c} \text{[wavy line]} \\ \text{[wavy line]} \end{array} = UGGU,$$

since the transition amplitude of two free particles is equal to the product of the Green's functions of the separate particles. For the scattering amplitude we get the series

$$\Gamma = U + UGGU + UGGUGGU + \dots$$

The sum of the factors to the right of UGG in the second and subsequent terms is again the series which gives Γ . We get for Γ the equation

$$\Gamma = U + UGG\Gamma.$$

The function G which appears in this equation is

easily found. If the Ψ function of the particle at the initial time is a superposition of different eigenfunctions, the problem of finding G reduces to the problem of the spreading of a wave packet. If, on the other hand, the particle is in a state with a definite energy at the initial time, the transition amplitude can be determined very simply. It is obvious that the expression for Γ is a symbolic way of writing the well known equation of quantum mechanics for the scattering amplitude

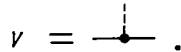
$$\Gamma(\mathbf{p}_1, \mathbf{p}_2) = U(\mathbf{p}_1 - \mathbf{p}_2) + \int U(\mathbf{p}_1 - \mathbf{p}') \frac{\Gamma(\mathbf{p}', \mathbf{p}_2)}{\epsilon_{\mathbf{p}_1} - \epsilon_{\mathbf{p}'} + i\gamma} \frac{d^3\mathbf{p}'}{(2\pi)^3}$$

Comparing the two equations for Γ , we easily establish the exact correspondence between the graphical and analytic expressions.

In an analogous way we can relate the Green's function \tilde{G} of a particle in an external field to the free-particle Green's function G . The Green's function \tilde{G} in the field is represented by a sum of partial transition amplitudes



where the point with the dashed line represents the act of interaction with the external field:



Collecting all of the graphs that appear in \tilde{G} on the right of V , we again get \tilde{G} . Accordingly,

$$\tilde{G} = G + GVG + GVGVG + \dots = G + GV\tilde{G}$$

Comparing the correction to G in first order in V in perturbation theory

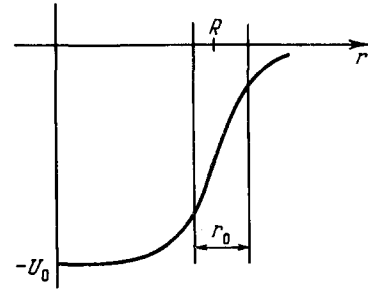
$$G^{(1)} = GVG$$

with the known quantum-mechanical expression, we readily perceive the sense in which the multiplications in the symbolic formula for \tilde{G} are to be understood.

Accordingly, the idea of the graphical method is to use simple examples to establish a correspondence between elements of the graphs and analytic expressions, after which one can learn to decipher arbitrary graphs composed of these elements.

2. The Green's Function of the Quasiparticle

As was mentioned in the Introduction, a system of strongly interacting particles behaves in many cases like a gas of quasiparticles. Therefore the Green's function which describes the behavior of the quasiparticles differs little from the Green's function for the case in which the system consists of a single particle. Let us first consider this case.



Let a system of eigenfunctions be defined by the formula

$$H\varphi_\lambda(r) = \epsilon_\lambda^0 \varphi_\lambda(r), \quad H = \frac{\mathbf{p}^2}{2m} + U(r),$$

where $U(r)$ is a potential well.

As we shall see below, in the case of the nucleus the eigenfunctions and energies for the quasiparticles are defined by these same equations, but with the mass of the free particle replaced by an effective mass m^* and with a potential $U(r)$ which is of the form shown in the figure, where R is the radius of the nucleus and r_0 is the width of the "diffuse edge"—the width of the region in which U changes from its constant value inside to its value outside the nucleus. Besides the potential shown in the figure, U includes corrections associated with the Coulomb and spin-orbit interactions between the particles.

We define the Green's function (the transition amplitude) by the relation

$$\Psi(\mathbf{r}, t + \tau) = \int G_0(\mathbf{r}, \mathbf{r}', \tau) \Psi(\mathbf{r}', t) d\mathbf{r}'$$

The transition amplitude G_0 allows us to express the wave function $\Psi(\mathbf{r}, t + \tau)$ at the time $t + \tau$ in terms of $\Psi(\mathbf{r}, t)$. In the energy representation we get

$$\Psi(\mathbf{r}, t) = \sum_\lambda C_\lambda(t) \varphi_\lambda(\mathbf{r}), \quad C_\lambda(t + \tau) = \sum G_{\lambda\lambda'}^0(\tau) C_{\lambda'}(t)$$

Since φ_λ is an eigenfunction, transitions to other states are impossible and $C_\lambda(t + \tau) = e^{-i\epsilon_\lambda^0 \tau} C_\lambda(t)$; that is,

$$G_{\lambda\lambda'}^0(\tau) = G_\lambda^0(\tau) \delta_{\lambda\lambda'} = e^{-i\epsilon_\lambda^0 \tau} \delta_{\lambda\lambda'}$$

The function $G_{\lambda\lambda'}^0(\tau)$ is defined only for $\tau > 0$. We set $G_{\lambda\lambda'}^0(\tau) = 0$ for $\tau < 0$.

Let us now find the Green's function of a quasiparticle.

The transition amplitude from a state with one quasiparticle λ to a state with one quasiparticle λ' , i.e., the Green's function $G_{\lambda\lambda'}(\tau)$ of a quasiparticle, can be found easily by analogy with the Green's function of a single particle. For a single particle we had in the λ representation

$$G_{\lambda\lambda'}^0(\tau) = \delta_{\lambda\lambda'} \begin{cases} e^{-i\epsilon_\lambda^0 \tau}, & \tau > 0, \\ 0, & \tau < 0. \end{cases}$$

To find the Green's function of a quasiparticle we need only replace the energy ϵ_λ^0 of the free particle

by the energy ϵ_λ of the quasiparticle and take the Pauli principle into account—transitions to occupied states must be excluded. Therefore a factor $(1 - n_\lambda)$ must be included in the Green's function of the quasiparticle, where $n_\lambda = \{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \}$ is the number of quasiparticles in the state λ . Accordingly, we get

$$G_{\lambda\lambda'}^+(\tau) = (1 - n_\lambda) \delta_{\lambda\lambda'} \begin{cases} e^{-i\epsilon_\lambda \tau}, & \tau > 0, \\ 0, & \tau < 0. \end{cases} \quad (2.1)$$

Let us now find the expression for the transition amplitude of a quasihole. Since the number of places for holes in the level λ is proportional to n_λ , in analogy with the case of the quasiparticle we get

$$G_{\lambda\lambda'}^-(\tau) = n_\lambda \delta_{\lambda\lambda'} \begin{cases} e^{-i\epsilon_\lambda^- \tau}, & \tau > 0, \\ 0, & \tau < 0. \end{cases} \quad (2.2)$$

Here ϵ_λ^- is the energy of the hole, or, more exactly, the difference of the energies of the system after and before the appearance of the hole.

As follows from the definition of the Green's function, the quantity $G_\lambda^+(+0)$ must become unity if $n_\lambda = 0$, and the quantity $G_\lambda^-(+0)$ must be unity for $n_\lambda = 1$. The expressions (2.1) and (2.2) satisfy these conditions. Later we shall also verify with examples the correctness of the expressions (2.1) and (2.2) for cases with $n_\lambda \neq 0, 1$ (for example, in the case of pair correlation).

We shall use the following graphs for the quantities $G_\lambda^+(t_1 - t_2)$ and $G_\lambda^-(t_1 - t_2)$:

$$\begin{aligned} G_\lambda^+(t_1 - t_2) &= \lambda t_2 \xrightarrow{\lambda t_1} \\ & \quad t_1 > t_2 \\ -G_\lambda^-(t_1 - t_2) &= \lambda t_2 \xleftarrow{\lambda t_1} \\ & \quad t_1 > t_2 \end{aligned} \quad (2.3)$$

The minus sign in the second formula is adopted in order to simplify the graphical description of the processes of production of a particle and a hole in an external field.

In many cases it is convenient to introduce a Green's function G_λ for the quasiparticle defined in the following way:

$$G_\lambda(\tau) = \begin{cases} G_\lambda^-(\tau), & \tau > 0 \\ -G_\lambda^-(\tau), & \tau < 0 \end{cases} = \frac{\lambda t_2}{\lambda t_1} \lambda t_1, \quad (2.4)$$

where

$$\tau = t_1 - t_2.$$

The change of the Green's function in an external field can be represented more simply in terms of the function G_λ than in terms of the functions G_λ^+ and G_λ^- .

Let us derive the expressions for the functions $G_\lambda^+(\epsilon)$, $G_\lambda^-(\epsilon)$, and $G_\lambda(\epsilon)$, which are the Fourier transforms with respect to τ of (2.1), (2.2), and (2.4). These expressions will be used often in what follows. The Fourier representation $G(\epsilon)$ is connected with the function $G(\tau)$ by the relation

$$G_\lambda(\tau) = \int_{-\infty}^{\infty} G_\lambda(\epsilon) e^{-i\epsilon\tau} \frac{d\epsilon}{2\pi}, \quad G_\lambda(\epsilon) = \int_{-\infty}^{\infty} G_\lambda(\tau) e^{i\epsilon\tau} d\tau.$$

Using the expressions (2.1), (2.2), and (2.4), we get

$$\left. \begin{aligned} G_\lambda^+(\epsilon) &= (1 - n_\lambda) \int_0^{\infty} e^{-i\epsilon_\lambda \tau} e^{i\epsilon\tau} d\tau = i \frac{1 - n_\lambda}{\epsilon - \epsilon_\lambda + i\gamma}, \\ G_\lambda^-(\epsilon) &= n_\lambda \int_0^{\infty} e^{-i\epsilon_\lambda^- \tau} e^{i\epsilon\tau} d\tau = -i \frac{n_\lambda}{\epsilon - \epsilon_\lambda^- + i\gamma}, \\ G_\lambda(\epsilon) &= (1 - n_\lambda) \int_0^{\infty} e^{-i\epsilon_\lambda \tau + i\epsilon\tau} d\tau + n_\lambda \int_{-\infty}^0 e^{i\epsilon_\lambda^- \tau + i\epsilon\tau} d\tau \\ &= i \left[\frac{1 - n_\lambda}{\epsilon - \epsilon_\lambda + i\gamma} + \frac{n_\lambda}{\epsilon + \epsilon_\lambda^- - i\gamma} \right], \end{aligned} \right\} \quad (2.5)$$

where

$$\gamma \rightarrow +0.$$

An important property of the functions $G_\lambda^+(\epsilon)$ and $G_\lambda^-(\epsilon)$ follows from (2.5)—they have a pole at the value of ϵ corresponding to the energy of the quasiparticle or quasihole, as the case may be.

3. Relation of the Green's Function to the Density Matrix. Calculation of Averages

We note one further important property of the Green's function, which we shall use in what follows. Let us introduce the quantity

$$\rho_{\lambda\lambda'} = G_{\lambda\lambda'}^-(0).$$

This quantity is called the density matrix and enables us to calculate the average values of operators which act on the quasiparticles. If there is no external field $G_{\lambda\lambda'}(\tau) = G_\lambda(\tau) \delta_{\lambda\lambda'}$ and $\rho_{\lambda\lambda'} = n_\lambda \delta_{\lambda\lambda'}$. The average value of an operator Q is given by

$$\langle Q \rangle = \sum_\lambda n_\lambda Q_{\lambda\lambda} = \sum_{\lambda\lambda'} \rho_{\lambda\lambda'} Q_{\lambda'\lambda}.$$

This same relation holds also when there are external fields and $G_{\lambda\lambda'} \neq G_\lambda \delta_{\lambda\lambda'}$ —i.e.,

$$\langle Q \rangle = \sum_{\lambda\lambda'} G_{\lambda\lambda'}^-(0) Q_{\lambda'\lambda}.$$

III. PAIR CORRELATION

1. The Influence of Nearby Levels on the Green's Function of a Quasiparticle

If there is an excited state of the system with the same integrals of the motion and with nearly the same energy as a state with one quasiparticle, these states become mixed, and the expression for the Green's function of the quasiparticle is more complicated.

To derive the Green's function of a quasiparticle as affected by a competing state we write a graphical equation, denoting by thin lines the Green's functions

of quasiparticles as found without any transitions to the competing state. We shall denote the exact Green's function by a thick line. Then we get as the equation for the Green's function G_S

$$G_S = \text{---} = \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots$$

$$= \text{---} + \text{---} \text{---} \text{---} \text{---} \text{---} .$$

Here a block $\text{---} \text{---} \text{---}$ describes the irreducible transition amplitude g from the state with one quasiparticle to the competing state. By "irreducible" amplitude we mean the amplitude containing no parts connected by a straight or a wavy line (the latter represents the Green's function of the competing state). Besides the equation for G_S one must also write the equation for the irreducible amplitude g . The solution of this system of equations gives the final energies and Ψ functions of the states in question.

To write the equations for g one must know the structure of the competing states.

For example, in a nucleus with an odd quasiparticle in the unfilled shell a state with two particles and one hole often lies close to the energy of the state with one quasiparticle. In some cases a competing state may be a state with one quasiparticle and a collective excitation.

We shall follow this effect through in detail for the example of a so-called pair correlation, which we are now going to consider.

2. Pair Correlation in Nuclei

In an infinite Fermi system two particles with opposite spins and momenta will in some cases be in a bound state with an energy close to the Fermi limit, or, as is usually said, these particles form a "Cooper pair". In this case there is a rearrangement in the distribution of the particles near the Fermi limit and an energy gap appears in the spectrum of one-particle excitations. The result is that the system becomes superfluid, or, in the case of charged particles, superconducting. In the language of Green's functions this means that a quasiparticle can go over into a state of a quasihole plus a Cooper pair, or a quasihole into a state of a quasiparticle minus a Cooper pair. In other words, a state with one quasiparticle against a background of N particles gets mixed with a state with one quasihole against a background of $N + 2$ particles, and a state with one quasihole gets mixed with a state with one quasiparticle against a background of $N - 2$ particles.

There is an effect in nuclei reminiscent of the Cooper pair correlation—two quasiparticles (two

(two neutrons or two protons) form a state with total angular momentum equal to zero, so that the Ψ function of these particles is a superposition of one-particle states, of the form

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = \sum_{\nu m} G_{\nu} (-1)^m \varphi_{\nu m}(\mathbf{r}_1) \varphi_{\nu-m}(\mathbf{r}_2),$$

where m is the projection of the total angular momentum and ν is the set of other quantum numbers.

We shall assume $j - j$ coupling, which is known to hold in all nuclei except the very lightest; that is, we assume that the state of a quasiparticle is characterized by the quantum numbers $\lambda = n, l, j, m$.

The existence of pair correlation means that there is a transition amplitude from a state with one quasiparticle to a state with one quasihole and a correlation pair. In other words, there is a block Δ_{λ}^{+} (analogous to the block g of the preceding section)

$$i\Delta_{\lambda}^{+}(\epsilon) = \left| \begin{array}{c} \lambda, \epsilon \\ N \end{array} \right. \text{---} \left. \begin{array}{c} \lambda, \epsilon \\ N+2 \end{array} \right| ,$$

which takes a particle against the background of N particles into a hole against the background of $N + 2$ particles, and there is an analogous block Δ_{λ}^{-} which takes a hole into a particle. We shall later write an equation for $\Delta_{\lambda}(\epsilon)$ from which it can be seen that $\Delta_{\lambda}(\epsilon)$ depends only weakly on ϵ and can be replaced by its value for $\epsilon = \epsilon_F$:

$$\Delta_{\lambda}(\epsilon) \approx \Delta_{\lambda}(\epsilon_F) = \Delta_{\lambda}.$$

To simplify the formulas we shall neglect the distinction between the system-energy differences $E_0(N + 2) - E_0(N)$ for the addition of two particles and $E_0(N) - E_0(N - 2)$ for the subtraction of two particles. Furthermore we shall set $\Delta^{+*}(N) = \Delta^{-}(N + 2) \approx \Delta^{-}(N)$. Then we can write the system of equations for G_S^{+} and G_S^{-} in the form of a single equation for G_S

$$G_{S\lambda}(\epsilon) \equiv \frac{\lambda\epsilon}{N} \frac{\lambda\epsilon}{N} = \frac{\lambda\epsilon}{N} \frac{\lambda\epsilon}{N}$$

$$+ \frac{\lambda\epsilon}{N} \text{---} \text{---} \text{---} \frac{\lambda\epsilon}{N} ,$$

or in analytical form

$$-i \left[(\epsilon - \epsilon_{\lambda}) - \frac{|\Delta_{\lambda}|^2}{\epsilon + \epsilon_{\lambda}} \right] G_{S\lambda} = 1.$$

Here the energy ϵ_{λ} is measured from the Fermi energy limit.

For the extension to the case in which there is an external field it is more convenient to write this equation in the form of two equations

$$-i \{ (\epsilon - \epsilon_{\lambda}) G_{S\lambda} + \Delta_{\lambda} F_{\lambda} \} = 1, \quad (\epsilon + \epsilon_{\lambda}) F_{\lambda} = -\Delta_{\lambda}^* G_{S\lambda}. \quad (3.1)$$

The quantity F which we have introduced denotes the total amplitude for the transition from the state with the hole to that with the quasiparticle. The quantity denoted by F is

$$F \equiv \begin{array}{c} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \begin{array}{c} N \\ N-2 \end{array} = \begin{array}{c} \leftarrow \\ \leftarrow \\ \leftarrow \end{array} \begin{array}{c} \circ \\ \leftarrow \\ \leftarrow \end{array} .$$

As can easily be seen, our equations (together with the equation for Δ which is written below) are invariant with respect to the transformation

$$G' = G, \quad F' = e^{i\varphi}F, \quad \Delta' = e^{-i\varphi}\Delta.$$

Therefore when there are no external fields we can choose the phase φ so that Δ' will be real, i.e., $\Delta_\lambda = \Delta_\lambda^*$.

Using Eq. (3.1), we get

$$\begin{aligned} -iG_\lambda &= \frac{1-n_\lambda}{\varepsilon-E_\lambda+i\gamma} + \frac{n_\lambda}{\varepsilon+E_\lambda-i\gamma}, \\ -iF_\lambda &= \frac{\Delta_\lambda}{2E_\lambda} \left(\frac{1}{\varepsilon-E_\lambda+i\gamma} - \frac{1}{\varepsilon+E_\lambda-i\gamma} \right), \end{aligned} \quad (3.2)$$

where E_λ is the energy of the quasiparticle, $E^2 = \Delta_\lambda^2 + \varepsilon_\lambda^2$, and n_λ is the number of quasiparticles in the state λ ,

$$n_\lambda = \frac{E_\lambda - \varepsilon_\lambda}{2E_\lambda}.$$

We write Eq. (3.1) in the coordinate representation:

$$\begin{aligned} \left(i \frac{\partial}{\partial \tau} - H \right) G_s(\mathbf{r}, \mathbf{r}', \tau) + \Delta(\mathbf{r}) F(\mathbf{r}, \mathbf{r}', \tau) &= i\delta(\mathbf{r}-\mathbf{r}')\delta(\tau), \\ \left(i \frac{\partial}{\partial \tau} + H \right) F(\mathbf{r}, \mathbf{r}', \tau) - \Delta^*(\mathbf{r}) G_s(\mathbf{r}, \mathbf{r}', \tau) &= 0. \end{aligned} \quad (3.3)$$

The change of the relative sign of $i\partial/\partial\tau$ and H in the second equation means that the second equation corresponds to reversed time.^[4] When the equations are written in the form (3.3) it is clear how to introduce an external field. In the case of fields that are unchanged when t is replaced by $-t$ we must add to the Hamiltonian H an effective field which acts on the quasiparticles. In the case of fields that change sign when t is replaced by $-t$ (such as a magnetic field, for example) we must, as before, add to the H in the first equation the effective field V , but in the second equation, since a change of sign of H is associated with time reversal, the quantity added to H is $TV(t) = V(-t)$ (see below). The connection between the effective field and the external field and the equations for G and F in an external field will be considered in what follows. These formulas must be supplemented with an equation for the determination of Δ , which is not hard to derive by the graphical method.^[4] We give the final result:

$$\Delta_{\lambda\lambda'} = \sum (\varphi_\lambda^* \varphi_{\lambda'} \gamma(\mathbf{r}) \varphi_{\lambda_1}^* \varphi_{\lambda_2}) F_{\lambda_1\lambda_2}(\tau) |_{\tau=0}. \quad (3.4)$$

$\gamma(\mathbf{r})$ characterizes the interaction of two quasiparticles with zero angular momentum.

IV. THE INTERACTION BETWEEN QUASIPARTICLES

1. The Mechanism of Interaction of Quasiparticles

The interaction between quasiparticles in a nucleus is decidedly different from the interaction of

two nucleons in vacuum. In fact, the interaction between two nucleons in vacuum is brought about by the exchange of one or more mesons, whereas inside nuclear matter there is another possible mechanism besides this one, namely the exchange of a particle and a hole; in the graphs for the scattering amplitude these two mechanisms are represented in the following way:

$$r = \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} + \dots + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \circ \\ \text{---} \\ \text{---} \end{array} + \dots$$

Wavy lines denote meson Green's functions.

Accordingly, the additional interaction is an interaction owing to polarization of the medium. Besides this, because of the Pauli principle even those graphs of the interaction that are not connected with the polarization are changed owing to the fact that some of the states are occupied by other nucleons and are not accessible to the interacting particles.

Finding the interaction in matter in terms of the interaction of two nucleons in vacuum is a complicated problem, since the influence of the medium very strongly alters the vacuum interaction. This problem is not considered here. The interaction between quasiparticles will be expressed in terms of a few constants, which are not calculated but must be found from a comparison of the theory with experiment. These constants are the same for all nuclei to the same accuracy as the density of the nuclear matter is constant.

We shall show that the range of the interaction forces between quasiparticles is approximately the same as the range r_0 of the interaction potential in vacuum. In fact, the density of the nucleus is determined by the condition that the distance between particles must be of the order of r_0 . Consequently, the momentum at the Fermi limit, which is determined by the density, is connected with r_0 by the relation ($\hbar = m = 1$)

$$p_F r_0 \sim 1.$$

The depth of the effective potential well in which the nuclear particles move is of the order

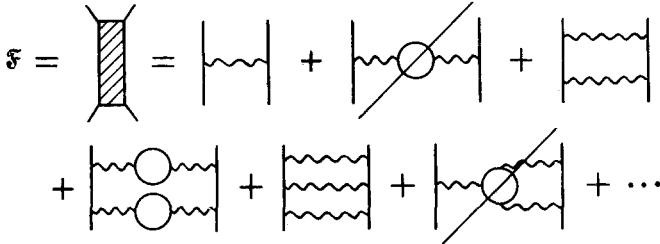
$$U \sim \frac{p_F^2}{2} \sim \frac{1}{r_0^2}.$$

Accordingly, all quantities for nuclear matter, and consequently also the range of the effective interaction forces, are determined by the quantity r_0 , which is the only quantity of the dimensions of length which is characteristic for the vacuum interaction, and also for the additional interaction caused by the polarizability of the nuclear matter.

In many cases it is required to find the scattering amplitude for small four-momentum transfer.

As we shall see, all problems associated with an external field with frequency ω small in comparison with the Fermi limit energy ε_F , and with wave

vectors small in comparison with the momentum p_F at the Fermi limit, reduce to finding scattering amplitudes with small momentum transfers ($k \ll p_F$, $\omega \ll \epsilon_F$). In this case, to obtain a convenient equation we must classify the graphs involved in Γ in the following way. We segregate in a block \mathfrak{F} all graphs which do not contain parts connected by the two lines of a quasiparticle and a quasihole. The graphs contained in \mathfrak{F} are:



The graphs struck out are those which by definition are not included in the block \mathfrak{F} . It will be shown that for small momentum transfers these graphs essentially depend on the states of the particles undergoing scattering.

For small-momentum transfers all of the graphs except those struck out correspond to δ -function contributions to the block \mathfrak{F} . In fact, the first, third, and fifth graphs describe the interaction of free nucleons and therefore are characterized by the range r_0 ; the fourth graph depends only weakly on the momentum transfer, and consequently is also characterized by the single quantity r_0 , the only one of the dimensions of length which appears in the problem. In general all graphs containing more than two lines depend weakly on the momentum transfer, since in the integrations over the four-momenta of internal lines it is large momenta and energies that are important ($p \sim p_F$, $\epsilon \sim \epsilon_F$). When in the momentum representation the region of appreciable variation is $\sim \epsilon_F$, p_F , in the coordinate representation the graph gives a function like a δ function, with a spread $\sim r_0$.

By means of the block \mathfrak{F} all of the graphs occurring in Γ can be classified in the following way: 1) graphs which do not contain two lines in the quasiparticle-quasihole channel (the block \mathfrak{F}); 2) with the quasiparticle-quasihole channel there first appears the block \mathfrak{F} , then the two lines (quasiparticle and quasihole), and afterwards the sum of all graphs which take the quasiparticle and quasihole into the new state (i.e., Γ).

The graphical equation for Γ is then

$$\Gamma = \text{[Diagram: square with hatched interior and four external lines]} = \text{[Diagram: hatched rectangle with two external lines]} + \text{[Diagram: hatched rectangle with two external lines and a hatched square with two external lines]} \quad (4.1)$$

and the analytic form is

$$\Gamma = \mathfrak{F} + \mathfrak{F}GG\Gamma. \quad (4.2)$$

Since the block \mathfrak{F} does not contain the two lines, it is of δ -function form with respect to the difference of the times. According to (4.2) this is also true of Γ . Therefore both Green's functions are taken for equal times. Let the 4th component transferred in Γ be $\omega [\Gamma(t) \rightarrow \Gamma e^{-i\omega t}]$. Then the expression GG in (4.2) is given by

$$\int G_{\lambda_1}(t_1-t) G_{\lambda_2}(t-t_1) e^{-i\omega t_1} dt_1 = e^{-i\omega t} \frac{n_{\lambda_1} - n_{\lambda_2}}{\epsilon_{\lambda_1} - \epsilon_{\lambda_2} + \omega}.$$

The equation obtained for Γ is

$$(\lambda_1 \lambda_2 | \Gamma | \lambda_3 \lambda_4) = (\lambda_1 \lambda_2 | \mathfrak{F} | \lambda_3 \lambda_4) + \sum_{\lambda \lambda'} (\lambda_1 \lambda_2 | \mathfrak{F} | \lambda' \lambda) \frac{n_{\lambda'} - n_{\lambda}}{\epsilon_{\lambda'} - \epsilon_{\lambda} + \omega} (\lambda' \lambda | \Gamma | \lambda_3 \lambda_4). \quad (4.3)$$

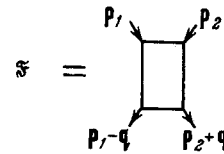
Since in the coordinate representation the block \mathfrak{F} is like a δ -function and is determined by a region of radius r_0 near the point in question, we shall call the quantity \mathfrak{F} the amplitude of the local interaction, or simply the local interaction.

In the case in which pair correlation is important, or in which there are levels near the Fermi surface which compete with the one-particle states, the expression for G_{λ} is more complicated and the equation for Γ is not of the simple form (4.3).

2. Local Interaction of Quasiparticles

As has already been stated, the effective local interaction between quasiparticles will be characterized by a few numbers.

Let us first consider uniform nuclear matter, and then introduce corrections brought about by the finiteness of the size of the nucleus. In the momentum representation the amplitude of the local interaction depends on the two momenta p_1 , p_2 and on the momentum transfer q :



Since the block \mathfrak{F} depends weakly on the momenta (changes appreciably for changes of the momenta by amounts $\delta p \sim p_F$, $\delta \epsilon \sim \epsilon_F$), for small momentum transfer we can set $q = 0$ (to accuracy $\sim k/p_F$, ω/ϵ_F). Moreover, for the study of the amplitude Γ near the Fermi surface it is sufficient to know \mathfrak{F} for $|p_1| = |p_2| = p_F$ and $\epsilon_1 = \epsilon_2 = \epsilon_F$. Therefore \mathfrak{F} depends only on the angle between the initial momenta p_1 and p_2 . Besides this the interaction between the quasiparticles depends on the spins of the quasiparticles and on the isotopic spin. Assuming isotopic invariance, we get

$$\mathfrak{F} = C \{f + f' \tau_1 \tau_2 + (g + g' \tau_1 \tau_2) \sigma_1 \sigma_2\}, \quad (4.4)$$

where f, f', g, g' are functions of the angle between \mathbf{p}_1 and \mathbf{p}_2 , and τ, σ are the isotopic and spin matrices.

We take the normalization factor C to be

$$C = \frac{\pi^2}{m^* p_F}.$$

Then f, f', g, g' are dimensionless quantities of the order 1.

We have not included in (4.4) terms of the form $(\mathbf{p}_1 \sigma_1)(\mathbf{p}_2 \sigma_2)$, which arise as a relativistic correction and go to zero for small speeds of the particles.

By the way, since the speeds at the Fermi limit are not very small in comparison with the speed of light ($v/c \sim 1/4$), these terms can be important and their size must be found from experiment. To begin with, we shall try to make the comparison with experiment on the assumption that such terms are small.

The so-called tensor forces are proportional to k^2 and therefore are not included in (4.4), although of course they have affected the numerical values of the terms that have been included.

Let us expand \mathfrak{F} in a series of Legendre polynomials depending on the cosine of the angle between \mathbf{p}_1 and \mathbf{p}_2 :

$$x = \frac{\mathbf{p}_1 \mathbf{p}_2}{p_F^2}, \quad \mathfrak{F} = \sum_l \mathfrak{F}_l P_l(x). \quad (4.5)$$

The numbers f_l, f'_l, g_l, g'_l must be found from a comparison between theory and experiment. We note that this expansion has nothing in common with the usual expansion of the scattering amplitude in partial waves, in which the expansion uses functions P_l of the angle of deflection, whereas in \mathfrak{F} the angle of deflection has been set equal to zero ($k = 0$).

The comparison with experiment shows that in nuclei the zeroth harmonics are most important in the expansion (4.5); that is, the local interaction of quasiparticles does not depend much on their velocities.

Let us now consider what complications are introduced by the finite size of the nucleus. Immediately outside the nucleus the interaction between nucleons changes sharply—only the direct interaction mechanism remains. Still, however, the interaction is very different from that of free nucleons, since the Ψ functions of the nucleons are distorted by reflection from the surface of the nucleus. To take into account the transition from the interaction inside the nucleus to that outside it one must assign some sort of dependence $\mathfrak{F}(r)$. Since this transition occurs in a thin layer of the order of r_0 , and the interaction is always averaged over a range of values of r which is larger than r_0 , the results do not depend very strongly on the form of the function $\mathfrak{F}(r)$. We can take, for example,

$$\mathfrak{F}(r) = \frac{n(r)}{n(0)} \mathfrak{F}_{in} + \left[1 - \frac{n(r)}{n(0)}\right] \mathfrak{F}_{ex},$$

where \mathfrak{F}_{ex} (\mathfrak{F}_{in}) is the value outside (inside) the nucleus. The interaction \mathfrak{F}_{ex} can be expressed in terms of the interaction of free particles.

The equation for the effective field in an infinite system involves \mathfrak{F} for a momentum transfer \mathbf{q} equal to the four-vector of the external field. Therefore in sufficiently homogeneous fields we can set $\mathbf{q} = 0$ in \mathfrak{F} (to accuracy $\sim k/p_F, \omega/\epsilon_F$).

In a finite system, even when the external field V^0 is uniform the effective field V is not uniform, but changes considerably in distances of the order of R . Therefore we must have an expression for \mathfrak{F} for $k \sim 1/R$; along with this we can set $\omega = 0$ if $\omega \ll \epsilon_F$. Since

$$\frac{k}{p_F} \sim \frac{1}{p_F R} \sim \frac{1}{A^{1/3}},$$

it is sufficient to take into account in F only terms linear in k and neglect terms $\sim k^2$, and consequently to neglect the tensor forces, which contain k^2 . The most general expression linear in k , symmetric in the indices of the particles, and unchanged when t is replaced by $-t$ is the following:

$$\mathfrak{F}^{sl} = \hat{\kappa} (\sigma_1 + \sigma_2) [(\mathbf{p}_1 - \mathbf{p}_2), \mathbf{k}].$$

Under the interchange $1 \leftrightarrow 2$ the quantity $\mathbf{k} = \mathbf{p}'_1 - \mathbf{p}'_2$ goes over into $-\mathbf{k}$. The quantity $\hat{\kappa}$ is different for like and unlike nucleons. It can be shown that $\kappa = 1/2(\kappa^{nn} + \kappa^{np})$ determines the spin-orbit term in the Hamiltonian of a single quasiparticle,

$$\delta U = -\kappa \frac{1}{r} \frac{dn}{dr} \sigma l.$$

The interaction \mathfrak{F}^{sl} leads to a spin-orbit correction to the magnetic moments of nuclei.

In some cases the finite size of the system has a still stronger effect on the magnitude of the essential momentum transfer. Namely, in some cases there is an important contribution from terms describing the reflection of quasiparticles from the boundaries of the system.^[5] Since on reflection there is a change of the order p_F in the momentum of the particle, the momentum transfer will be of this same order. In such cases one further constant must be introduced into the theory (cf. ^[5]).

3. Interaction through the "Core"

As is well known, in many nuclear calculations an effective interaction is introduced with different constants for different cases. The form used most often is the quadrupole-quadrupole interaction, which in the λ representation is written in the following way:

$$\mathfrak{F}^{QQ} = C Q_{\lambda_1 \lambda_2} Q_{\lambda_3 \lambda_4},$$

where $Q_{\lambda_1 \lambda_2}$ is the matrix element of the quadrupole moment

$$Q_{\lambda_1\lambda_2} = \int \varphi_{\lambda_1}^* r^2 P_2(\cos \theta) \varphi_{\lambda_2} d^3r.$$

Octupole-octupole and other interactions are written analogously. All of these interactions have played an important part in the qualitative systematization of the experimental data.

We shall see that all of these interactions are approximate ways of writing analogous interactions which can be derived rigorously from the expression for the scattering amplitude and can be expressed in terms of the constants of the local interaction \mathfrak{F} .

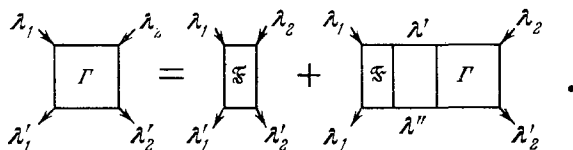
For simplicity we neglect the pair correlation, and find the effective interaction between particles belonging to a single shell.

Let us write the equation for the scattering amplitude in the form

$$\Gamma(v_1, v_2) = \mathfrak{F}(v_1, v_2) + \sum_{\nu'} \mathfrak{F}(v_1, \nu') A(\nu') \Gamma(\nu', v_2).$$

Here ν_1 denotes the set of indices (λ_1, λ'_1) of the particle before and after the collision, and $\nu_2 = (\lambda_2, \lambda'_2)$ denotes the same for the second particle; ν' denotes the indices for the intermediate state.

Graphically we have



The summation in this equation goes over all states λ' and λ'' such that one of them is above the Fermi limit and the other below it, in accordance with the value

$$A(\nu') = \frac{n_{\lambda'} - n_{\lambda''}}{\epsilon_{\lambda'} - \epsilon_{\lambda''} + \omega}.$$

We can make a renormalization of the equation for Γ so that the summation will go only over the states of the shell to which the particles undergoing scattering belong, for example over only the states of the last unfilled shell. In the renormalized equation a nonlocal interaction \mathfrak{F}' appears instead of the local interaction \mathfrak{F} ; \mathfrak{F}' includes the part of the interaction caused by the excitation of other shells (excitation of the "core"). In fact, let us introduce the quantity \mathfrak{F}' according to the formula

$$\mathfrak{F}' = \mathfrak{F} + \mathfrak{F}' A_2 \mathfrak{F} = (1 + \mathfrak{F}' A_2) \mathfrak{F}. \tag{4.6}$$

The index 2 on the A shows that the summation goes over all shells except the one to which the particles undergoing scattering belong. Let us multiply the equation

$$\Gamma = \mathfrak{F} + \mathfrak{F} A \Gamma$$

by the factor $1 + \mathfrak{F}' A_2$ on the left. Then

$$\Gamma + \mathfrak{F}' A_2 \Gamma = \mathfrak{F}' + \mathfrak{F}' A \Gamma, \quad \Gamma = \mathfrak{F}' + \mathfrak{F}' A_1 \Gamma. \tag{4.7}$$

Here the index 1 means that the summation goes over only the states of the shell we have singled out.

Since the usual way of including the interaction in nuclear calculations is to sum over only the states of the last shell, the formulas must involve the complicated quantity \mathfrak{F}' instead of the simple local interaction \mathfrak{F} . This is the reason it is necessary to introduce different constants for different processes. Meanwhile, by Eq. (4.6), \mathfrak{F}' can be expressed in terms of the quantity \mathfrak{F} , which is the same for all nuclei (except the very lightest) and for all types of transitions.

The quantity \mathfrak{F}' , like \mathfrak{F} , cannot be put in the form of a product of two factors:

$$(\lambda_1\lambda_2 | \mathfrak{F}' | \lambda_3\lambda_4) \neq C Q_{\lambda_1\lambda_2} Q_{\lambda_3\lambda_4}.$$

For example, the simplest form of \mathfrak{F} is that with neglect of velocity-dependent terms:

$$(\lambda_1\lambda_2 | \mathfrak{F} | \lambda_3\lambda_4) = \mathfrak{F}_0 \int \varphi_{\lambda_1}^* \varphi_{\lambda_2} \varphi_{\lambda_3}^* \varphi_{\lambda_4} d^3r,$$

and this does not break up into factors.

In studying quadrupole excitations, for example, it is necessary to form from the ingoing indices of \mathfrak{F} and \mathfrak{F}' a superposition which has angular momentum 2, and solve the corresponding equation for the radial matrix elements.

The representation of \mathfrak{F}' in the form of a product distorts the radial dependence of this quantity and can lead to considerable errors. The solution of Eqs. (4.6) and (4.7) (after separating the angular variables) with a computing machine is such a simple problem that it is pointless to introduce assumptions to simplify the interaction \mathfrak{F} .

V. NUCLEI IN EXTERNAL FIELDS

Many properties of nuclei (static moments, transition probabilities, the energies of the first levels, and so on) can be determined easily if one knows the change of the density matrix of the quasiparticles in an external field and its change when particles are added to the system. As we have seen, the density matrix is simply related to the Green's function (page 290). The procedure for finding it is as follows. One first determines the change of the Green's function in the effective field which arises in the system under the influence of the external field. The effective field can in turn be expressed in terms of the change of the density matrix. The result is that one has a system of equations for the determination of the effective field. Knowing the effective field, we can easily find the change of G, and consequently the change of the density matrix.

1. The Effective Field

Let us determine the change of the Green's function of a quasiparticle in an external field, at first neglecting the pair correlation. For simplicity we confine ourselves to the first approximation in the field treated as a perturbation. We shall take the

interaction between the particles into account exactly. We write down some graphs which appear in the Green's function G of the quasiparticle in the field:

$$\tilde{G} = \text{---} + \text{---} \circ \text{---} + \text{---} \triangle \text{---} + \text{---} \triangle \text{---} + \dots = \text{---} + \text{---} \triangle \text{---} \quad (5.1)$$

$$= G + GV\tilde{G}.$$

Here a circle denotes direct interaction of the quasiparticle with the external field V^0

$$\text{---} \circ \text{---} = e_q V^0,$$

e_q being the "charge" of the quasiparticle. As we shall see, for some types of field $e_q \neq 1$, which means that the external field acting on the quasiparticles is different from the external field applied to the particles. For noninteracting particles we have

$$\tilde{G}_0 = \text{---} + \text{---} \bullet \text{---} = G_0 + G_0 V^0 G_0.$$

Accordingly, the shaded triangle in (5.1) replaces the point in this diagram and represents the effective field V acting on a quasiparticle.

Let us derive the equation for the field V . Among the graphs that appear in V there is one graph which does not contain the interaction between the quasiparticles ($e_q V^0$). All the other graphs have the following structure. If we move in the direction from the base toward the vertex of the triangle, all graphs begin with an interaction, after which there are two lines of free motion, and then a set of graphs representing the effective field. We introduce the block \mathfrak{F} , which does not contain any parts connected by two lines. Then the effective field is determined by the equation

$$V = e_q V^0 + \mathfrak{F} G G V. \quad (5.2)$$

Or graphically

$$V = \text{---} \triangle \text{---} = \text{---} \circ \text{---} + \text{---} \triangle \text{---} \mathfrak{F} \text{---}.$$

The first term in V describes the direct action of the external field on the quasiparticle. The second term gives the additional field which arises owing to polarization of the medium, i.e., is brought about by

the action of the redistributed nucleons of the nucleus.

In the λ representation we get

$$V_{\lambda_1 \lambda_2} = e_q V_{\lambda_1 \lambda_2}^0 + \sum (\lambda_1 \lambda_2 | \mathfrak{F} | \lambda \lambda') A_{\lambda \lambda'} V_{\lambda' \lambda_2}, \quad (5.3)$$

where

$$A_{\lambda \lambda'} = \int G_{\lambda}(t) G_{\lambda'}(-t) e^{-i\omega t} dt. \quad (5.4)$$

As we have seen (page 293)

$$A_{\lambda \lambda'} = \frac{n_{\lambda} - n_{\lambda'}}{\varepsilon_{\lambda} - \varepsilon_{\lambda'} + \omega}. \quad (5.4')$$

2. The Effective Charge of Quasiparticles in an Unfilled Shell

For processes associated with transitions inside the last unfilled shell, it is convenient to carry out a renormalization of Eq. (5.3) in such a way that the summation will go over only the states of the last shell.

We break A up into two terms

$$A = A_1 + A_2.$$

$(A_2)_{\lambda \lambda'}$ is different from zero only when both states lie outside the last shell, and $(A_1)_{\lambda \lambda'}$ is different from zero when one or both states lie in the last shell. Since in A_2 both states are sufficiently far from the Fermi surface, for them one can always neglect the pair correlation and in general the influence of any states close to the Fermi surface. Therefore A_2 is given by the simple expression (5.4').

We write the equation for V in the symbolic form

$$V = e_q V^0 + \mathfrak{F} A_1 V + \mathfrak{F} A_2 V.$$

In the third term we replace V by the right member of this equation. Repeating this operation, we get

$$V = e_q \{1 + \mathfrak{F} A_2 + \mathfrak{F} A_2 \mathfrak{F} A_2 + \dots\} V^0 + \{\mathfrak{F} + \mathfrak{F} A_2 \mathfrak{F} + \mathfrak{F} A_2 \mathfrak{F} A_2 \mathfrak{F} + \dots\} A_1 V.$$

We denote the coefficient of $A_1 V$ in curly brackets by \mathfrak{F}' :

$$\mathfrak{F}' = \mathfrak{F} + \mathfrak{F} A_2 \mathfrak{F} + \mathfrak{F} A_2 \mathfrak{F} A_2 \mathfrak{F} + \dots = \mathfrak{F} + \mathfrak{F} A_2 \mathfrak{F}'. \quad (5.5)$$

In the λ representation we have

$$(\lambda_1 \lambda_2 | \mathfrak{F}' | \lambda_3 \lambda_4) = (\lambda_1 \lambda_2 | \mathfrak{F} | \lambda_3 \lambda_4) + \sum'' (\lambda_1 \lambda_2 | \mathfrak{F} | \lambda' \lambda) \frac{n_{\lambda} - n_{\lambda'}}{\varepsilon_{\lambda} - \varepsilon_{\lambda'} + \omega} (\lambda \lambda' | \mathfrak{F}' | \lambda_3 \lambda_4). \quad (5.6)$$

The sign "'' on the sum means that neither λ nor λ' lies in the last shell.

This is the equation for the effective interaction of quasiparticles in the last shell.

We now denote by V' the quantity

$$V' = e_q \{1 + \mathfrak{F} A_2 + \mathfrak{F} A_2 \mathfrak{F} A_2 + \dots\} V^0 = e_q V^0 + \mathfrak{F} A_2 \{e_q V^0 + \mathfrak{F} A_2 V^0 + \dots\} = e_q V^0 + \mathfrak{F} A_2 V' = e_q (1 + \mathfrak{F}' A_2) V^0. \quad (5.6')$$

In these notations the equation for V takes the form

$$V = V' + \mathfrak{F}' A_1 V. \quad (5.7)$$

In Eq. (5.7) the summation goes over states, one or both of which are in the last shell. The quantity A_1 in (5.7) must be determined with effects included of the distortion of the Green's functions caused by the influence of nearby levels. The quantity V' can be written formally as a product

$$V'_{\lambda\lambda'} = (e_{\text{eff}})_{\lambda\lambda'} V_{\lambda\lambda'}^0.$$

The effective charge $(e_{\text{eff}})_{\lambda\lambda'}$ depends on the states λ and λ' .

3. The Effective Field in the Case of Pair Correlation

We first write the equation for V in such a form that it is valid independent of the form of the Green's functions of the quasiparticles. The second term in the equation for V can always be represented in the form

$$V - e_q V^0 = \mathfrak{F} \delta G, \quad (5.8)$$

where δG is the change of the Green's function in the field. In fact if we move from the direction of the free ends of the lines, the graphs of $V - e_q V^0$ begin with the interaction, and if we separate off the block \mathfrak{F} , which does not contain two lines, after this there are graphs beginning and ending with a quasiparticle and containing the external field—i.e., the change of the Green's function in the field. In the special case in which

$$\delta G = G V \tilde{G},$$

we get the former expression for V . In the case of pair correlation there is a more complicated expression for the determination of δG .

We note that the expression (5.8) has a very intuitive meaning. Since \mathfrak{F} is of δ -function form in the time, the factor $\delta G(t_1, t_2)$ in (5.8) is taken for equal times. Let us agree to take $\delta G(t_1, t_2)$ at $t_2 = t_1 + 0$. Then, as we have seen in III. 3, the Green's function $G(t_1, t_1 + 0)$ is equal to the density matrix

$$\rho_{\lambda\lambda'}(t) = G_{\lambda\lambda'}(t, t + 0),$$

and therefore we can write the right member of (5.8) in the form

$$(V - e_q V^0)_{\lambda_1 \lambda_2} = (\mathfrak{F} \delta G)_{\lambda_1 \lambda_2} = \sum (\lambda_1 \lambda_2 | \mathfrak{F} | \lambda \lambda') \delta \rho_{\lambda \lambda'}. \quad (5.8')$$

In this form the expression (5.8) acquires a simple physical meaning—the additional field which arises owing to the polarization of the medium is the matrix product of the local interaction of the quasiparticles and the change of the density matrix. In particular, if \mathfrak{F} does not depend on the velocities and spins, i.e., if its form in the coordinate representation is

$$\mathfrak{F}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \mathfrak{F}_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(\mathbf{r}_1 - \mathbf{r}_3) \delta(\mathbf{r}_3 - \mathbf{r}_4),$$

we get

$$(\mathfrak{F} \delta \rho)_{\lambda_1 \lambda_2} = \mathfrak{F}_0 \sum_{\lambda \lambda'} \int \Phi_{\lambda_1}^* \Phi_{\lambda_2} \Phi_{\lambda'}^* \Phi_{\lambda'} d^3 r (\delta \rho)_{\lambda \lambda'}.$$

But the quantity

$$\delta n(\mathbf{r}, t) = \sum (\delta \rho)_{\lambda \lambda'} \Phi_{\lambda'}^*(\mathbf{r}) \Phi_{\lambda}(\mathbf{r})$$

is the change of the density of quasiparticles, and the additional field in the coordinate representation is

$$V - e_q V^0 = \mathfrak{F}_0 \delta n(\mathbf{r}, t),$$

as must be the case for a δ -function interaction.

Accordingly, our problem is to find the change $\delta \rho$ of the density matrix in an external field, for the case of pair correlation. To do so we find the change of G in the external field. We have already stated in III. 2 how to introduce the effective field in the equations for the functions G and F . We have only to make the procedure somewhat more precise. The equations (3.3) for G and F in the field now take the form

$$\begin{aligned} \left(i \frac{\partial}{\partial t} - H - V \right) \tilde{G} &= i \tilde{\Delta} \tilde{F} + i \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \\ \left(i \frac{\partial}{\partial t} + H - V^T \right) \tilde{F} &= i \tilde{\Delta}^* \tilde{G}. \end{aligned} \quad (5.9)$$

The field V^T in the equation for \tilde{F} is connected with the field $V = V(\omega) e^{-i\omega t}$ by the relation^[4]

$$V^T(t) = \pm V(-\omega) e^{-i\omega t}.$$

The plus sign corresponds to fields which are even under the replacement of t by $-t$, and the minus sign to fields which are odd.

We have here inserted the quantity $\tilde{\Delta}$ instead of Δ , since the change of Δ in the field must also be taken into account.

Accordingly, Eq. (5.9) must be supplemented with an equation for the Δ in the field. In sufficiently uniform fields the relation between Δ and F is the same as before:

$$\tilde{\Delta}_{\lambda \lambda'}(t) = \sum (\Phi_{\lambda'}^* \Phi_{\lambda} V(r) \Phi_{\lambda_1}^* \Phi_{\lambda_2}) \tilde{F}_{\lambda_1 \lambda_2}(t, t). \quad (5.10)$$

Equations (5.9) and (5.10) allow us to find \tilde{G} , \tilde{F} , and $\tilde{\Delta}$ in fields which, while they are weak in comparison with the energy at the Fermi limit, are comparable with Δ and with the distance between the levels in the nucleus:

$$\Delta \lesssim V \ll \epsilon_F.$$

For fields $V \sim \epsilon_F$ all of the characteristics of the quasiparticles are changed, and the theory becomes too cumbersome to be worth studying.

In what follows we confine ourselves to the case of weak fields, for which $V \ll \Delta$. Then Eqs. (5.9) and (5.10) can be expanded in powers of the field V . Confining ourselves to the first term of the expansion with respect to V , after simple algebraic steps we get

$$\delta G = GVG - F^*V^TF - Gd^{(1)}F - F^*d^{(2)}G,$$

$$\delta F = -Fd^{(1)}F + G^Td^{(2)}G + FVG + G^TV^TF. \quad (5.11)$$

Here

$$d^{(1)} = \delta\Delta = \tilde{\Delta} - \Delta, \quad d^{(2)} = \delta\Delta^* = \tilde{\Delta}^* - \Delta^*,$$

where $d^{(1)}$ and $d^{(2)}$ can be expressed in terms of δF and δF^* by means of (5.10).

Equations (5.9) and (5.10), together with (5.8'), which expresses the effective field in terms of the change of the density matrix, are a complete system of equations for the determination of V and $\delta\rho$.

Knowing V and $\delta\rho$, one can, as we shall see, calculate the probabilities and frequencies of transitions.

Let us write the change of the density matrix symbolically in the form

$$\delta\rho = AV.$$

On the other hand, we had for V :

$$V = e_q V^0 + \mathfrak{F}\delta\rho.$$

Substituting V in the expression for $\delta\rho$, we get an equation for the change of the density matrix of the quasiparticles in the field

$$A^{-1}\delta\rho = e_q V^0 + \mathfrak{F}\delta\rho.$$

In the λ representation, for the case in which pair correlation is unimportant, we can use the expression (5.4') for A . Then

$$(\omega + \varepsilon_\lambda - \varepsilon_{\lambda'}) \delta\rho_{\lambda\lambda'} = (n_\lambda - n_{\lambda'}) \{ e_q V_{\lambda\lambda'}^0 + \sum (\lambda\lambda' | \mathfrak{F} | \lambda_1\lambda_2) \delta\rho_{\lambda_1\lambda_2} \}.$$

This equation coincides exactly with the kinetic equation for the density matrix of a gas of particles with the interaction \mathfrak{F} with the charge e_q , which equation can be derived very simply if we consider a gas of noninteracting quasiparticles in the field V .^[4]

Accordingly in the case in which there is no pair correlation the equation for the effective field can be derived even without the Green's functions. In the case in which pair correlation is important, no simple equation for ρ exists, and the derivation of equations for the effective field without the Green's functions is an extremely cumbersome task.

4. The Change of the Density Matrix when the Number of Particles is Changed

Let us now consider the change of the density matrix which arises from a change of the number of particles in the system. Suppose that along with this there have been changes δn_λ in the occupation numbers of the quasiparticles. We shall show later how the δn_λ are determined.

The expression for the change of the nondiagonal part of the density matrix

$$(\delta\rho)_{\lambda_2\lambda_1} = A_{\lambda_1\lambda_2} V_{\lambda_1\lambda_2}$$

remains valid also in the absence of an external field,

if we take V to mean the effective field which has arisen from the rearrangement of the occupation numbers,

$$V_{\lambda_1\lambda_2} = \sum_{\lambda\lambda'} (\lambda_1\lambda_2 | \mathfrak{F} | \lambda\lambda') (\delta\rho_{\lambda\lambda'}) (1 - \delta_{\lambda\lambda'}) + \sum_{\lambda} (\lambda_1\lambda_2 | \mathfrak{F} | \lambda\lambda) \delta n_\lambda$$

$$= \sum_{\lambda\lambda'} (\lambda_1\lambda_2 | \mathfrak{F} | \lambda\lambda') A_{\lambda\lambda'} V_{\lambda\lambda'} + \sum_{\lambda} (\lambda_1\lambda_2 | \mathfrak{F} | \lambda\lambda) \delta n_\lambda. \quad (5.12)$$

Accordingly, if the δn_λ are known, we can find V and the nondiagonal part of the density matrix. For the change of the density matrix we have the inhomogeneous equation

$$(\delta\rho)_{\lambda_1\lambda_2} = \delta_{\lambda_1\lambda_2} \delta n_{\lambda_1} + (1 - \delta_{\lambda_1\lambda_2}) A_{\lambda_1\lambda_2} \sum (\lambda_1\lambda_2 | \mathfrak{F} | \lambda\lambda') (\delta\rho)_{\lambda\lambda'}. \quad (5.13)$$

By means of the equation for the amplitude Γ we can put Eqs. (5.12) and (5.13) in a different form.

We write (5.13) in the symbolic form

$$V = \mathfrak{F}\delta_0\rho + \mathfrak{F}AV,$$

where

$$(\delta_0\rho)_{\lambda_1\lambda_2} = \delta_{\lambda_1\lambda_2} \delta n_{\lambda_1}.$$

Let us compare this equation with the equation for $\Gamma\delta_0\rho$

$$\Gamma\delta_0\rho = \mathfrak{F}\delta_0\rho + \mathfrak{F}A\Gamma\delta_0\rho.$$

We see that

$$V = \Gamma\delta_0\rho,$$

or in the λ representation

$$V_{\lambda_1\lambda_2} = \sum (\lambda_1\lambda_2 | \Gamma | \lambda\lambda) \delta n_\lambda. \quad (5.14)$$

By means of Γ we can write the equation for V in the form

$$V = e_q V^0 + \mathfrak{F}AV = (1 + A\Gamma) e_q V^0. \quad (5.15)$$

Similarly, Eq. (5.13) can be written in the form

$$\delta\rho = \delta_0\rho + A\mathfrak{F}\delta\rho = \delta_0\rho + A\mathfrak{F}\delta_0\rho + A\mathfrak{F}A\mathfrak{F}\delta_0\rho + \dots = (1 + A\Gamma) \delta_0\rho. \quad (5.16)$$

We shall now see how to determine $\delta_0\rho$, i.e., the diagonal part of the change of the density matrix of the quasiparticles. The effective field changes the energy of the quasiparticles. In first order in V we have

$$\tilde{\varepsilon}_\lambda = \varepsilon_\lambda + V_{\lambda\lambda}.$$

The new occupation numbers \tilde{n}_λ are found from the condition that the energy of the system be a minimum. If the distance between levels near the Fermi surface is large in comparison with the interaction energy, the requirement that the energy be a minimum reduces to the condition

$$\tilde{n}_\lambda(\tilde{\varepsilon}_\lambda) = \begin{cases} 1, & \tilde{\varepsilon}_\lambda < \tilde{\mu}, \\ 0, & \tilde{\varepsilon}_\lambda > \tilde{\mu}, \end{cases} \quad (5.17)$$

where $\tilde{\mu}$ is the new chemical potential and is determined by the change of the total number of particles

in the system

$$\delta N = \sum_{\lambda} (\tilde{n}_{\lambda}(\tilde{\epsilon}_{\lambda}) - n_{\lambda}(\epsilon_{\lambda})).$$

If the change of the energy ϵ_{λ} , i.e., $V_{\lambda\lambda}$, is smaller than the distance to the nearest level, the change of the n_{λ} consists of the added particles going into the nearest free places. If, on the other hand, the effective field V is so large that the levels intersect, then there is a more complicated rearrangement of the occupation numbers in accordance with the condition (5.17). It is easy to see that when an odd particle is added to a filled shell the field which arises is of the order $V \sim \epsilon_F/A$, which is smaller than the distance to the nearest level. Therefore in this case the added particle goes into the nearest level λ_0 , and the change in the occupation numbers is

$$\delta n_{\lambda} = \delta_{\lambda\lambda_0}.$$

Accordingly, as long as there is no intersection of the levels, δn is determined very simply. If, however, the levels intersect, then δn_{λ} is determined in the following way. Having assigned a definite rearrangement of the particles among the levels, we must determine $\delta\rho$. Then from $\delta\rho$ we determine V , and by means of V we determine the new energies $\tilde{\epsilon}_{\lambda}$. Then the energy of the system of quasiparticles is calculated, with the interaction \mathfrak{F} taken into account. This sort of calculation must be made for several test distributions \tilde{n}_{λ} . The distribution with the lowest energy gives the new state of the system.

Accordingly, in fields which cause intersection of levels the determination of the δn_{λ} with the interaction between quasiparticles included is a complicated problem, which must be solved specially for each concrete case.

In the limiting case of very strong fields the problem becomes simpler again, since in this case many levels are involved and the change of the density matrix can be found by using the quasiclassical approximation.

5. The Calculation of Averages

Suppose it is required to calculate the change of the average value of a quantity Q , for example the quadrupole moment or the magnetic moment, when the number of particles is changed or the system is excited.

The quantity $\delta \langle Q \rangle$ is given by

$$\delta \langle Q \rangle = e_q Q \delta\rho = e_q \sum_{\lambda\lambda'} Q_{\lambda\lambda'} (\delta\rho)_{\lambda\lambda'}.$$

Here we have allowed for the fact that the charge of a quasiparticle with respect to a field Q may be different from the charge of the particle ($e_q \neq 1$). Using the formulas (5.15) and (5.16) given in the preceding section, we can write this expression in a more convenient form

$$\begin{aligned} \delta Q &= e_q Q \delta\rho = e_q Q (1 + A\Gamma) \delta_0\rho = V [Q] \delta_0\rho, \\ V [Q] &= e_q Q + e_q Q A\Gamma = e_q Q + \mathfrak{F}AV [Q]. \end{aligned} \quad (5.18)$$

Here we have denoted by $V [Q]$ the effective field caused by the external field Q . Accordingly, the change of the average value is given by the following simple formula:

$$\delta Q = e_q \sum_{\lambda} V_{\lambda\lambda} [Q] \delta n_{\lambda}. \quad (5.19)$$

Let one particle be added to a magic nucleus. Since in this case there is no intersection of terms, one quasiparticle is added to the nucleus in a state λ_0 above the filled levels of the magic nucleus. Therefore δn_{λ} is given by

$$\delta n_{\lambda} = \delta_{\lambda\lambda_0},$$

and in this case the change of the average value is given by the diagonal matrix element of the effective field for the state λ_0

$$\delta Q = e_q V_{\lambda_0\lambda_0} [Q].$$

6. The Frequencies and Probabilities of Transitions

Let us write the equation for V , as before, in symbolic form

$$V = e_q V^0 + \mathfrak{F}AV.$$

Here $\delta\rho = AV$ is the change of the density matrix of the quasiparticles in the field. The eigenfrequency ω_s of any state s is determined by the condition

$$V_{\lambda\lambda'} \underset{\omega \rightarrow \omega_s}{=} \frac{\chi_{\lambda\lambda'}^{(s)}}{\omega - \omega_s} + V_{\lambda\lambda'}^r.$$

The residue of V at the pole satisfies the equation

$$\chi^{(s)} = \mathfrak{F}A_s \chi^{(s)}. \quad (5.20)$$

The solution of this equation determines the eigenfrequencies ω_s and eigenfunctions $\chi_{\lambda\lambda'}^{(s)}$; we shall see that the transition probabilities can be expressed in terms of these quantities. Since (5.20) determines χ up to a constant factor, we still have to find the normalization of χ . Substituting the expression for V near the pole in the equation for V , we get

$$V^r (\omega - \omega_s) = e_q V^0 (\omega - \omega_s) + \mathfrak{F} \frac{dA}{d\omega} \chi (\omega - \omega_s) + \mathfrak{F}A_s V^r (\omega - \omega_s),$$

or

$$V^r = e_q V^0 + \mathfrak{F} \frac{dA}{d\omega} \chi + \mathfrak{F}A_s V^r.$$

Multiplying by χA and using (5.20), we get

$$\chi A V^r = e_q \chi A V^0 + \chi \frac{dA}{d\omega} \chi + \chi A V^r.$$

Accordingly, χ is normalized in the following way:

$$\sum_{\nu} \chi(\nu) \frac{dA(\nu)}{d\omega} \chi(\nu) = -e_q \sum \chi(\nu) A(\nu) V^0(\nu), \quad (5.21)$$

where ν denotes the set of indices $\lambda_1 \lambda_2$.

We have still to express the transition probability

in terms of the quantities χ and A . The number of transitions per unit time into the state s is given by the well known formula

$$W_{0s} = 2\pi |H'_{0s}|^2 \delta(\omega - \omega_s), \quad \omega_s = E_s - E_0, \quad (5.22)$$

where the perturbation operator H' is given by

$$H' = \sum_n V^0(r_n).$$

The transition probability can be expressed simply in terms of the average value of the perturbation over the excited ground state

$$\langle H' \rangle = \langle \Phi' H' \Phi' \rangle = - \sum_s \left[\frac{|H'_{0s}|^2}{E_s - E_0 - \omega + i\gamma} + \frac{|H'_{0s}|^2}{E_s - E_0 + \omega - i\gamma} \right].$$

The transition probability is

$$W_{0s} = 2 \operatorname{Im} \langle H' \rangle_s, \quad (5.23)$$

where $\langle H' \rangle_s$ is the term of $\langle H' \rangle$ which corresponds to the transition to the state s . But $\langle H' \rangle$ can be expressed in terms of the change $\delta\rho$ of the density matrix of the quasiparticles in the field,

$$\langle H' \rangle = e_q V^0 \delta\rho = \sum e_q V_{\lambda\lambda'}^0 (\delta\rho)_{\lambda\lambda'} = e_q V^0 A V.$$

Introducing V near the pole, we find

$$\langle H' \rangle = e_q V^0 A V = e_q V^0 A \frac{\chi}{\omega - \omega_s + i\gamma},$$

and then by (5.23)

$$W_{0s} = 2\pi e_q \sum_v V^0(v) A(v) \chi(v) \delta(\omega - \omega_s). \quad (5.24)$$

Let us denote by ρ' the residue at the pole of the density matrix in the field

$$\delta\rho = A V = \frac{\rho'}{\omega - \omega_s}, \quad \rho' = A \chi.$$

Then from the normalization of χ we get the normalization for ρ'

$$e_q V^0 \rho' = \rho' \frac{dA^{-1}}{d\omega} \rho', \quad (5.25)$$

and W_{0s} can also be written in the form

$$W_{0s} = 2\pi \sum_v \rho'(v) \frac{dA^{-1}}{d\omega} \rho'(v). \quad (5.26)$$

7. The Conservation Laws and the Charges of Quasiparticles for Various Fields

The conservation laws impose strong restrictions on the charge e_q of quasiparticles. The derivation of the results given below is contained in^[4]. We confine ourselves to suggestive physical arguments.

First let us consider the physical consequence which arise from the requirement of gauge invariance.

The physical meaning of this requirement is as follows. Let vector fields $\partial f_n / \partial x_i$ and $\partial f_p / \partial x_i$ be applied to neutrons and protons. As is well known, such fields do not produce any physical changes in the system. In particular, there is no polarization of the medium, and the effective field acting on the

quasiparticles is the same as the external field.

It follows from this that in the case of a scalar field acting on the protons the charges of the proton and neutron quasiparticles are:

$$e_q^{pp} = 1, \quad e_q^{np} = 0.$$

For a vector field there is a more complicated expression, which we shall give below.

We can obtain further information on the charges of quasiparticles by using the fact that in some fields which are not merely fictitious there is no redistribution of the particles, and consequently the effective field is equal to the external field. For example, in a uniform field which acts equally on the two types of particles, the system vibrates as a whole without any internal changes. From this condition we easily find that

$$V_{\lambda\lambda'}^p [p_\alpha^p + p_\alpha^n] = p_\alpha = (e_q^{pp} + e_q^{np}) p_\alpha,$$

i.e.,

$$e_q^{pp} + e_q^{np} = e_q^{nn} + e_q^{pn} = 1.$$

Similarly we conclude that the sum $e_q^{pp} + e_q^{np} = 1$ for any perturbation which commutes with the Hamiltonian and has only diagonal matrix elements in the λ representation, i.e., for perturbations of the form

$$H' = \sum a_{\lambda\lambda'}^{\pm} a_{\lambda\lambda'} Q_{\lambda\lambda},$$

if the operator H' commutes with H .

In fact, it is not hard to see that such a perturbation produces no polarization of the medium. We shall call this kind of perturbation a diagonal perturbation. Let us derive the expression for the effective charge in the case of a perturbation of the form $\sigma \mathcal{H}$. Since the charge e_q is determined by the local interaction of the particles, its value in the nucleus differs little from the corresponding quantity in unbounded nuclear matter of the same density. Since the spin-orbit interaction in nuclear matter is small, the operator for the total spin of the system commutes with the Hamiltonian. Moreover, in a sufficiently large system the spin-orbit correction to the Hamiltonian of the quasiparticles is unimportant and can be omitted. Then the functions are eigenfunctions of the operator H' . Accordingly, the perturbation is diagonal and

$$e_q^{pp} + e_q^{np} = 1.$$

Let us write this condition in the form

$$e_q^{pp} = 1 - \zeta_s, \quad e_q^{np} = \zeta_s.$$

The quantity ζ_s cannot be calculated and must be found from experiment. This same quantity occurs in the renormalization of the axial-vector β -decay constant in the nucleus. For allowed transitions the interaction with the electron-neutrino field gives a perturbation in the Hamiltonian of the nucleons which is proportional to $(\tau_x \pm i\tau_y) \sigma_z$ (Gamow-Teller transi-

tions). Let us find the local charge of the quasiparticles for this sort of external field.

We first consider the field $\tau_z \sigma_z$:

$$\tau_z \sigma_z = \frac{1+\tau_z}{2} \sigma_z - \frac{1-\tau_z}{2} \sigma_z = \sigma_z^p - \sigma_z^n.$$

The inhomogeneous term (containing the external field) in the equation for V is

$$e_q V^0 \{ \tau_z \sigma_z \} = e_q [\tau_z \sigma_z] \tau_z \sigma_z = e_q [\sigma_z^p] \sigma_z - e_q [\sigma_z^n] \sigma_z \\ = \begin{Bmatrix} e_q^{pp} - e_q^{pn} \\ e_q^{np} - e_q^{nn} \end{Bmatrix} \sigma_z = \begin{Bmatrix} 1 - 2\zeta_s \\ -(1 - 2\zeta_s) \end{Bmatrix} \sigma_z = (1 - 2\zeta_s) \tau_z \sigma_z.$$

Accordingly,

$$e_q [\tau_z \sigma_z] = 1 - 2\zeta_s.$$

Owing to isotopic invariance the charge will be the same for the field $(\tau_x \pm i\tau_y) \sigma_z$. For the field $\tau_x \pm i\tau_y$ (Fermi transitions) we get (by first considering the field τ_z) the value $e_q = 1$.

We give a table of the values of the charges e_q for various fields.

Type of field	Charges
Scalar, $f(z)$	$e_q^{pp} = 1, \quad e_q^{pn} = 0$
Vector, p_α	$e_q^{pp} = 1 - \frac{1}{3} f_1^{np} \frac{1}{m^*},$ $e_q^{pn} = \frac{1}{3} f_1^{np} \frac{1}{m^*}$
Spinor, σ_α	$e_q^{pp} = 1 - \zeta_s, \quad e_q^{pn} = \zeta_s$
Orbital, $\mathbf{r} \times \mathbf{p}$	$e_q^{pp} = 1 - \frac{1}{3} f_1^{np} \frac{1}{m^*},$ $e_q^{pn} = \frac{1}{3} f_1^{np} \frac{1}{m^*}$
β -decay, $\begin{cases} \tau_x \pm i\tau_y, \\ (\tau_x \pm i\tau_y) \sigma_z \end{cases}$	$e_q = 1$ $e_q = 1 - 2\zeta_s$

Since the charges e_q are determined by the local properties of the medium, these same expressions are also valid when the fields in question are multiplied by smooth functions of the coordinates. For the same reason the charge for the orbital field ($\mathbf{r} \times \mathbf{p}$) is equal to that for the vector field (p_α).

VI. NUCLEAR MOMENTS

1. The Scheme for Calculating Nuclear Moments

The change of the energy of the nucleus in a static external field, for example in the electric or magnetic field of the atomic electrons, is determined by various moments. The change of energy in a uniform magnetic field is determined by the dipole magnetic moment of the nucleus, which is called simply the magnetic moment. In cases in which nonuniformity of a magnetic field is important, one must introduce octupole magnetic moments. The interaction with the

electric field of the atomic electrons is in practice determined by two moments—the mean square electric radius of the nucleus, which occurs in the formula for the isotopic shift of atomic spectral lines, and the quadrupole moment, which is found from hyperfine structure. All of these quantities are expressed as averages of appropriate operators over the ground state of the nucleus. The moment Q is given by

$$Q = (\Phi_0 \sum_n Q_n \Phi_0).$$

As we have seen, the changes of averages when the number of particles in the nucleus is changed can be calculated in terms of the change $\delta\rho$ of the density matrix of the quasiparticles, or, still more simply, by finding the effective field produced by the external field Q (i.e., the addition of a term $H' = \sum_n Q_n$ to the Hamiltonian) and determining the change in the occupation numbers of the quasiparticles which occurs in the change from one nucleus to the other. The change of the moment Q is given by

$$\delta Q = \text{Sp } e_q \delta\rho \hat{Q} = \sum_\lambda V_{\lambda\lambda} [Q] \delta n_\lambda, \tag{6.1}$$

where e_q is the charge of the quasiparticles in relation to the field Q . Since even-even nuclei have no magnetic moments, to calculate the magnetic moment of an even-odd or odd-even nucleus it suffices to find the change of the density matrix when one particle is added to an even-even nucleus. In the domain of even-even nuclei the quadrupole moments are zero, and therefore also for the calculation of the quadrupole moment of an adjacent nucleus it suffices to know the change of the density matrix when one particle is added.

Accordingly, the scheme for calculating static moments is as follows.

One finds the effective field $V[Q]$ corresponding to the field Q . The field Q is the quantity $r^2 P_2(\cos \theta)$ in the case of quadrupole moments, the quantity r^2 in the case of the isotope shift, or, finally, the operator for the magnetic moment of one particle, when one is finding the magnetic moment of a nucleus. One next determines the change δn_λ of the number of quasiparticles in the level λ when one particle is added. After this Q is calculated by means of Eq. (6.1).

In the simplest case of the addition of one particle to a doubly magic nucleus we have

$$\delta n_\lambda = \delta_{\lambda\lambda_0},$$

where λ_0 is the state in which the odd quasiparticle appears. In this case

$$Q = V_{\lambda_0\lambda_0} [Q].$$

Let us display the isotopic indices in these relations. Suppose that, as in the case of quadrupole

moments and isotope shifts, the operator Q acts only on protons. Then

$$\delta Q = \delta Q^p = Q^p \delta \rho^p = \sum V_{\lambda\lambda}^p [Q^p] \delta n_\lambda^p + V_{\lambda\lambda}^n [Q^p] \delta n_\lambda^n.$$

There is an analogous calculation for magnetic moments, with contributions from the operators for both the proton and the neutron moments.

2. Magnetic Moments

The operator for the dipole magnetic moment of a single nucleon can be written as the sum of two terms

$$\mu = \mu^j + \mu^s, \\ \mu^j = \frac{1+\tau_z}{2} \mathbf{j}, \quad \mu^s = \left[\frac{1+\tau_z}{2} \left(\gamma_p - \frac{1}{2} \right) + \frac{1-\tau_z}{2} \gamma_n \right] \boldsymbol{\sigma},$$

where $\frac{1}{2}(1+\tau_z)$ and $\frac{1}{2}(1-\tau_z)$ are matrices corresponding to the proton and neutron states and the total angular momentum is $\mathbf{j} = \mathbf{l} + \frac{1}{2}\boldsymbol{\sigma}$, where \mathbf{l} is the orbital angular momentum; γ_p and γ_n are the proton and neutron gyromagnetic ratios.

It is necessary to find the effective field $V[\mu_0]$ corresponding to the operator μ , and to calculate the magnetic moment from the formula

$$\langle \mu \rangle = \sum_{\lambda} V_{\lambda\lambda}[\mu] \delta n_{\lambda}.$$

Magnetic moments of higher multipole orders can be calculated by an analogous procedure.

It turns out that the change of the orbital part of the magnetic moment under the influence of the interaction between quasiparticles is small. The spin part, however, changes by a large amount.

The paramagnetic susceptibility tensor of a nucleus is determined by the constant g in the spin-orbit interaction between the quasiparticles. In some cases the susceptibility differs from 1 by a sizable factor, and it takes its largest values at the periphery of the nucleus. Moreover, the susceptibility depends on the angle between the direction of the magnetic field and the radius vector \mathbf{r} . This fact gives a quantitative explanation of the so-called l -forbidden transitions (see below). For a table of experimental and calculated values of the magnetic moments and octupole magnetic moments see [4,6,7]. There is satisfactory agreement with experiment.

3. Quadrupole Moments and the Isotope Shift

In order to calculate quadrupole moments it is necessary to find the effective field produced by an external scalar field equal to $V^0 = r^2 P_2(\cos \theta)$. The isotope shift is determined by a field $V^0 = r^2$.

The change of the quadrupole moments and the quantity $\langle r^2 \rangle$ have been calculated in [8]. The equation for the effective field was solved with computing machines.

A comparison of the theory with experimental data showed that in this case (unlike that of the calculation

of magnetic moments) it is necessary to take into account the dependence of the interaction on the radius near the surface of the nucleus. The comparison with experiment makes it possible to determine both internal and external values of the constants f_0 and f'_0 , and then to calculate the values of the quadrupole moments and of the quantity $\delta \langle r^2 \rangle$ for a large number of elements. Although, as must be the case, the values found for f_0 and f'_0 outside the nucleus are not equal to the values for free nucleons (see below), they are not very different from these values.

A table comparing the theoretical and experimental values of $\delta \langle r^2 P_2 \rangle$ and $\delta \langle r^2 \rangle$ is given in [4].

In all cases there is agreement with the experimental values to within 30 to 40 percent (except for light elements).

VII. ELECTROMAGNETIC AND β -DECAY TRANSITIONS

1. Dipole Excitations

We shall consider dipole transitions under the action of γ -rays of not very high frequency, $\omega \ll \epsilon_F$. For this case we can neglect the variation of the field over the radius of the nucleus ($k^2 R^2 \ll 1$). To study the excitation of the nucleus, and not its motion as a whole, it is convenient to go over to the center-of-mass system. There is then an inertial field in addition to the electric field acting on the protons, and the perturbation of the Hamiltonian of the system is of the form ($e = 1$)

$$H' = \mathbf{E} \left\{ \frac{N}{A} \sum_p \mathbf{r}_i - \frac{Z}{A} \sum_n \mathbf{r}_i \right\}.$$

The effective field which arises from the perturbation H' satisfies the equations

$$V^p = \frac{N}{A} E x + \mathfrak{F}^{pp} A^p V^p + \mathfrak{F}^{pn} A^n V^n, \\ V^n = -\frac{Z}{A} E x + \mathfrak{F}^{nn} A^n V^n + \mathfrak{F}^{np} A^p V^p. \quad (7.1)$$

For the position of the maximum of the giant-resonance curve we get

$$\omega_s^2 = \omega_0^2 (1 + 2f'_0) = \omega_0^2 \frac{3\beta}{\epsilon_F}.$$

An analogous relation can be derived for the width of the maximum [9,10]

$$\Gamma^2 = \Gamma_0^2 \frac{3\beta}{\epsilon_F},$$

where Γ_0 is the width and ω_0 is the position of the maximum as found for a system of noninteracting particles in a well with a diffuse edge. These results are in satisfactory agreement with experiment.

Inclusion of the velocity dependence of the forces between the quasiparticles changes the sum rule for dipole transitions. The result is

$$\int \sigma(\omega) d\omega = 2\pi^2 \frac{NZ}{A} \left[\frac{1+2/3f'_1}{1+2/3f_1} \right],$$

where f_1 and f'_1 are determined by the first harmonic of the expansion (4.5). For $f_1 = f'_1$, which means that there are no exchange forces, the sum rule is of the usual form.

2. Quadrupole Transitions

There are many well studied quadrupole transitions with small frequencies ($\omega \sim \Delta$). For these transitions one must write expressions which take pair correlations into account.

For the study of quadrupole transitions we must find the effective field caused by an external field of frequency ω of the form

$$V^0 = r^2 P_2(\cos \theta) \equiv Q.$$

The equation for the effective field is of the form

$$V^n(\mathbf{r}) = V^0(\mathbf{r}) + [f_0^{pp} \delta n^p(\mathbf{r}) + f_0^{nn} \delta n^n(\mathbf{r})] \left(\frac{dn}{d\varepsilon_F} \right)^{-1},$$

where $\delta n^p(\mathbf{r})$ and $\delta n^n(\mathbf{r})$ are the changes of the densities of protons and neutrons in the field.

For the study of quadrupole collective vibrations it is common practice to use a so-called quadrupole-quadrupole interaction of the form

$$(\lambda_1 \lambda_2 | \mathfrak{F}^Q | \lambda_3 \lambda_4) = -\kappa Q_{\lambda_1 \lambda_2} Q_{\lambda_3 \lambda_4}.$$

The only reason for this choice of the interaction is that with it Eq. (7.1) reduces to an algebraic equation.

The interaction \mathfrak{F}^Q leads to an effective field of the form

$$V = C(\omega) V^0,$$

i.e., it is equivalent to the assumption that the change of density in the field is

$$\delta n(r) = a(r) P_2(\cos \theta) = C_1 r^2 P_2(\cos \theta).$$

This assumption distorts the radial dependence of $\delta n(\mathbf{r})$, and consequently also distorts the quantity $V - V^0$. The error is particularly large in cases in which states with large angular momenta are important. The main contribution to $V - V^0$ is then from values of r near the surface of the nucleus, where $f_0(r)$ and $\delta n(r)$ vary rapidly with r .

For a number of cases in which pair correlation is unimportant the intensities of one-particle quadrupole transitions have been calculated in^[11]. There is satisfactory agreement with the experimental data. The value for $f_0(\mathbf{r})$ used in the solution is the same as in the calculation of quadrupole moments and the isotope shift.

3. Magnetic Transitions

The intensities of one-particle of collective magnetic transitions can be calculated by the same scheme as is used for electric transitions.^[12]

The only difference between the equations here and those used for calculating magnetic moments is that the frequency ω appears in the denominator

$A_{\lambda\lambda}'$. For small frequencies the probability of the transition from state λ_0 to state λ_1 is determined by the matrix element $V_{\lambda_0\lambda_1}$ of the static effective field, which replaces the quantity $V_{\lambda_0\lambda_1}^0$ of the one-particle model in the formulas for the transition.

For transitions with small frequencies there is usually a strong effect of interaction between configurations, which leads to additional restrictions on the transitions.^[13]

The configuration interaction is to be calculated by the same scheme as used with the shell model, except that we must take as the interaction the effective interaction \mathfrak{F}' in the last shell, and it must be found from the equation

$$\mathfrak{F}' = \mathfrak{F} + \mathfrak{F} A' \mathfrak{F}',$$

where A' means that the sum is taken over all shells except the last.

4. β Decay

As is well known, β transitions in nuclei (β decay, K capture) are caused by the following perturbation:

$$H' = G_V \int \bar{\Psi} \tau_+ \gamma_\mu (1 + C_1 \gamma_5) \Psi \bar{\varphi}_e \gamma_\mu (1 + \gamma_5) \varphi_\nu d^3r,$$

where $\Psi = \Psi^+ \gamma_4$ and $\bar{\Psi}^+$, Ψ are the operators for creation and annihilation of a nucleon, while φ_e and φ_ν are the neutrino and electron functions. The factor C_1 is the ratio of the axial-vector and vector interaction constants:

$$C_1 = \frac{G_A}{G_V} = -1.2.$$

τ_+ is the isotopic matrix that takes a proton into a neutron:

$$\tau_+ = \frac{1}{2} (\tau_x + i\tau_y) = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}.$$

Neglecting relativistic corrections for the nucleons, we get

$$\bar{\Psi} \tau_+ \gamma_\mu (1 + C_1 \gamma_5) \Psi = \delta_{\mu 0} \Psi^+ \tau_+ \Psi + C_1 \delta_{\mu \alpha} \Psi^+ \tau_+ \sigma_\alpha \Psi,$$

where $\alpha = 1, 2, 3$.

In the λ representation we get

$$H' = G_V \sum_{\lambda\lambda'} a_\lambda^\dagger a_{\lambda'} [(\tau_+ j_0)_{\lambda\lambda'} + C_1 (\tau_+ \sigma_\alpha j_\alpha)_{\lambda\lambda'}], \quad (7.2)$$

where $j_\mu = (j_0, j_\alpha)$ is the current density of the light particles.

To obtain H' in the case of allowed transitions we regard $j_\mu(\mathbf{r})$ as independent of r (the Coulomb function of the electron is replaced by its value at the surface of the nucleus). In the case of Fermi transitions the difference between the effective and external fields is due only to the Coulomb corrections, since when the Coulomb field is neglected the perturbation

$$H' = \sum a_\lambda^\dagger a_{\lambda'} (\tau_\pm)_{\lambda\lambda'},$$

produces no polarization of the medium, owing to

isotopic invariance, just as no polarization is produced by the perturbation

$$H' = \sum a_{\lambda\lambda'}^{\dagger} (T_Z)_{\lambda\lambda'} = T_Z.$$

The equation for $V[\tau_+]$ shows that corrections appear only in second order in the ratio V_Q/ϵ_F , where V_Q is the Coulomb field. Therefore the corrections to the Fermi matrix element, as calculated on the assumption of strict isotopic invariance, are negligibly small (~ 0.3 percent).

In the case of Gamow-Teller transitions the equations for the effective field are very similar to those for the effective field corresponding to the spin part of the magnetic moment.

The charge of the quasiparticles for the field $\tau_+ \sigma_+$ is given by

$$e_q = 1 - 2\zeta_s.$$

In the sums over λ and λ' one state refers to the neutrons, the other to the protons. As in the case of the spin part of the magnetic moment, the main terms in the sums are those for which λ and λ' differ only in the sign of the projection of the spin along the direction of the angular momentum j .

In the case of mirror nuclei the Gamow-Teller matrix element can be expressed rigorously in terms of the magnetic moment of the daughter or mother nucleus in the ground state. Substitution of the observed magnetic moments gives amplitudes for the β transitions which agree with the observed values within experimental error.

A calculation of the probabilities of allowed β transitions with pair correlation taken into account approximately is given in^[14]. The same paper gives a comparison of the theoretical and experimental values of the probabilities for allowed transitions.

In all cases in which there is no configuration perturbation one gets satisfactory agreement of the absolute transition probabilities with the experimental values. The values taken for the spin-spin interaction constant in this calculation are the same as in the case of magnetic moments.

5. l -forbidden Transitions

Among magnetic transitions and among allowed β transitions there are some in which there is a change of the orbital angular momentum by two units.

Such l -forbidden transitions are impossible in the one-particle model, since the matrix element of σ or $\sigma\tau_+$ is equal to zero for transitions in which there is a change of the orbital angular momentum.

The explanation of l -forbidden transitions is as follows.

The transition probability depends on the matrix element of the effective field, not the external field. In the cases of either an external field σ or an external field $\sigma\tau_+$ the effective field is of the form

$$V_\alpha = V_1(r^2) \sigma_\alpha + V_2(r^2) \frac{r_\alpha r_\beta \sigma_\beta}{r^2}.$$

The second term in this expression has matrix elements for states with values of the orbital angular momentum differing by two units. The intensities calculated in this way agree well with the experimental probabilities of l -forbidden transitions.^[12]

6. μ Capture

In μ capture, unlike β decay and K capture, the momentum carried away by neutrinos is large, and the energy imparted to the nucleus is of the order of 10 to 15 MeV. In the perturbing Hamiltonian the light-particle current cannot be regarded as independent of r . Moreover, one must supplement the perturbation with the induced pseudoscalar interaction and so-called weak magnetism. These terms make a negligibly small contribution in the case of β decay, but give a considerable correction for μ capture. In other respects the calculations of the probability of μ capture is made according to the same scheme as the calculation of dipole or quadrupole transitions with large excitation energies. The summation over states in the equation for the effective field is carried out in the quasiclassical approximation in^[15]. The formula obtained for the dependence of the μ -capture time on A and Z is in good agreement with experiment. More exact numerical results are obtained in^[16] by solving the equation for the effective field with a computer. A comparison of the theoretical and experimental values of μ -capture times is given in the latter paper.

CONCLUSION

The purpose of the approach to nuclear calculations which has been described is to formulate all of the problems which have been semiquantitatively solved with models in rigorous language and express them in terms of universal constants of the theory.

A satisfactory correlation of the various phenomena of nuclear physics has been obtained. As we have seen, for this it is sufficient to introduce an interaction between quasiparticles which is characterized by constants f_0, f'_0, g_0, g'_0 inside the nucleus and analogous constants outside the nucleus.

To get more reliable results one must make the interaction between quasiparticles more exact. A more precise comparison of the theory with experiment will also make it possible to find further harmonics in the expansion of \mathfrak{F} in terms of the angle between the initial momenta of the particles.

We shall list some problems which have not yet been solved.

Let us begin with nuclear reactions.

For the treatment of reactions in which an intermediate nucleus is formed it is not hard to reformulate existing calculations in the language of interact-

ing quasiparticles, using the gas approximation for the quasiparticles, as before.

More interesting results can be obtained from the consideration of direct nuclear reactions. Direct nuclear reactions have been studied in detail in^[17] by the method of separating off the singular parts of the appropriate graphs. As in the approach given here, one then has to introduce constants characterizing the nonsingular parts of the graphs.

In the simplest cases these constants can be expressed in terms of the constants we have introduced for the interaction between quasiparticles.

First, the elastic and inelastic scattering of nucleons is described by the graphs for the scattering amplitude which we have studied above,

$$r = \begin{array}{c} \lambda_1 \swarrow \quad \nearrow \lambda \\ \circ \\ \lambda_2 \searrow \quad \swarrow \lambda' \end{array} .$$

Here λ_1, λ_2 are the initial and final states of the nucleon undergoing scattering, and λ and λ' correspond to a one-particle excitation.

Accordingly, this reaction can be expressed in terms of the interaction constants which appear in \mathfrak{F} .

Furthermore the block

$$g = \begin{array}{c} \lambda_1 \swarrow \quad \nearrow \\ \circ \\ \lambda_2 \searrow \quad \swarrow \end{array} \text{---} \text{wavy line} ,$$

which corresponds to scattering with production of a collective excitation, is the residue at the corresponding pole of the scattering amplitude, and consequently can be expressed in terms of \mathfrak{F} .

The reactions (γn) and (γp) also do not require the introduction of any new constants.

For processes occurring near the Fermi surface, reactions with production of deuterons or α particles can be described with one additional constant.

In the case of α particles this same constant appears in an improved theory of α decay. In fact the irreducible block which characterizes the formation of an α particle from quasiparticles lying near the Fermi surface is of δ -function type in the coordinate representation and is characterized by a single constant.

There are interesting effects connected with abrupt changes of nuclear radius and shape on the addition of particles.

As a small number of particles are added to a magic nucleus, as long as the field of the added particles does not cause intersection of terms, a smooth redistribution of the density occurs without change of the radius of the nucleus. The density at the center of the nucleus is larger than the average value corresponding to the formula $R = r_0 A^{1/3}$. When the number

of added particles becomes large enough, the radius or shape of the nucleus changes abruptly, and the average value of the density is reestablished. This effect could be observed in experiments like those done by Hofstadter.

It is of great interest to make calculations with the effective interaction \mathfrak{F}' for all cases of mixing of configurations, which leads to decided changes of transition probabilities or static moments.

Useful information will also be given by calculations of dipole absorption of radiation, on the assumption that the initial reaction is a one-particle excitation of the nucleus, which then gets redistributed over more complicated excitations. In this way this example can be used to follow the mechanism of initial reactions through to the end.

It is of great interest to calculate the constants which appear in the so-called generalized model of the nucleus.

Quite a number of problems are connected with the interaction between a nucleus and a μ meson which is in the K shell, because in this case there is strong electric and magnetic polarization of the nucleus. The only problem of this sort that has been solved is that of the isotopic shift in a muonic atom.^[18]

It is particularly interesting to calculate effects associated with the transition from the spherical shape to a deformed shape. Among the excited states of a spherical nucleus near the transition point there is a deformed state, and a deformed nucleus in this region has a spherical state among its excited states. Moreover, the ground state is a superposition of spherical and deformed states (fifty percent of each at the transition point).

In deformed nuclei there is another mechanism of interaction of the quasiparticles besides the local interaction, namely the exchange of rotational excitations of the nucleus. Inclusion of this mechanism will also be an important test of the theory.

The calculation of interaction constants and constants characterizing the potential well in terms of the interaction of free nucleons presents very great difficulties, and is not a problem of the theory discussed here.

Present approximate methods for solving this problem start in one way or another from the assumption that the interaction is small or that the gas approximation can be used, and there is no theoretical basis for this.

If, however, the result of such calculations should be to get correct results for those interaction constants that are already known, then after such a check one could place confidence in the calculations.

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Translated by W. H. Furry