533.9

# MOTION AND SPREADING OF INHOMOGENEITIES IN A PLASMA

A. V. GUREVICH and E. E. TSEDILINA

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.; Institute of Terrestrial Magnetism, Ionosphere, and Radio Wave Propagation, Academy of Sciences, U.S.S.R.

Usp. Fiz. Nauk **91**, 609–643 (April, 1967)

CONTENTS



## INTRODUCTION

THE development in a plasma of electric and magnetic fields that exert a strong and frequently decisive influence on its motion is a fundamental feature of plasma dynamics, distinguishing it from the dynamics of neutral gases. This feature becomes manifest to the fullest degree in the motion and spreading of macroscopic inhomogeneous formations, i.e., plasma inhomogeneities. Indeed, owing to the difference between the diffusion and drift velocities of the electrons and ions, the electronic and ionic components of the inhomogeneities always tend to separate. This gives rise to an uncompensated electric charge, which produces an internal electric field. The latter retards the fast particles and accelerates the slow ones, thus hindering the charge separation. As a result, the inhomogeneity moves and spreads in such a way that the electron and ion concentrations in it are practically equal. The internal electric field exerts in this case a strong influence on the motion of the inhomogeneity as a whole, giving rise to electric currents that produce an internal magnetic field, which in turn can exert a strong influence. The net result is that it is precisely the internal fields which determine the velocity of the overall motion of the inhomogeneity, and the rate and the character of its spreading.

To describe motions in a plasma, under conditions when the dimension of the inhomogeneities are much larger than the particle mean free paths, it is necessary to consider the system of hydrodynamic equations for all the plasma components (electrons, ions, neutral molecules) jointly with Maxwell's equations for the field. In the general formulation, these equations encompass an exceedingly broad region of plasma dynamics. In the present article we have in mind to explain a much narrower yet clearly delineated group of problems. Namely, the first chapter will consider the diffusion spreading of inhomogeneities in a plasma in a magnetic field. In the absence of an external magnetic field, this process was first considered by Schottky [1] who showed that it reduces to ordinary diffusion, called "ambipolar." The internal electric field influences in this case only the magnitude of the diffusion coefficient, while the plasma situated in the magnetic field is anisotropic. The action of the internal electric field on the motion of the electrons and ions in the anisotropic plasma leads not only to a change in the diffusion coefficient, but also to the occurrence of closed electric currents. As a result, the diffusion spreading of inhomogeneities in a plasma situation in a magnetic field is qualitatively different from ordinary diffusion.

Further, the charged particles in the plasma are frequently made to drift by the inhomogeneity of the magnetic field, by an external electric field, by neutral-particle wind, and by other causes. The analyses of the character of motion and spreading of inhomogeneities under drift conditions is the subject of the second chapter. The internal electric field exerts in this case an even stronger action on the processes in the plasma. It leads, in particular, to a splitting of the inhomogeneities and to the appearance of a special ("dispersion") mechanism of spreading of inhomogeneities; this mechanism is more energetic than diffusion.

We shall consider for the most part the motion and spreading of inhomogeneities in an unbounded plasma. This is important primarily for the physics of ionosphere, of the space next to the earth, and of outer space. Diffusion plays, for example, an important role in the formation and shape of ionospheric of layers [2-9], clouds of artificial ionization produced with the aid of rockets and high-altitude explosions [10-14], the origin and structure of ionospheric inhomogeneities [15-18], especially inhomogeneities in the E layer, connected with hydrodynamic turbulence [19,20], etc. Diffusion processes play a decisive role in the formation of trails of meteors and artificial objects (rockets, satellites) in the ionosphere [21-23], and their scattering of radio waves. [24-28]

Analogous problems arise also in laboratory-plasma investigations of inhomogeneities whose dimensions are much smaller than the dimensions of the entire system. They are important, in particular, in the analysis of stability. [29-32] We shall not stop here to discuss the results of concrete investigations dealing with the distribution and lifetime of plasma in laboratory equipment. They are discussed in the review article by Golant [33] (see also [34-35]). Nor do we touch upon the extensive and important problem of diffusion in an unstable plasma, which was considered in detail in the reviews of Kadomtsev [31], Vedenov [32], Cote [37], and others.*

a) Fundamental equations. The motion and spreading of inhomogeneous formations, whose characteristic dimensions are much larger than the particle mean free path, is described by the following aggregate of macroscopic equations for all the plasma components:†

$$\frac{\partial N_e}{\partial t}+\nabla \mathbf{j}_e=0, \qquad (0.1)$$

$$\frac{\partial N_i}{\partial t}+\nabla \mathbf{j}_i=0, \qquad (0.2)$$

$$\mathbf{j}_i=-\frac{\hat{\sigma}_e}{e}\left(\mathbf{E}+\frac{[\mathbf{V}_m\mathbf{H}]}{c}\right)-\hat{D}_e\nabla N_e+N_e\mathbf{V}_m, \qquad (0.3)‡$$

$$\mathbf{j}_i=\frac{\hat{\sigma}_i}{e}\left(\mathbf{E}+\frac{[\mathbf{V}_m\mathbf{H}]}{c}\right)-\hat{D}_i\nabla N_i+N_i\mathbf{V}_m, \qquad (0.4)$$

$$\frac{\partial N_m}{\partial t}+\nabla(\mathbf{V}_m N_m)=0,$$

$$MN_m\left[\frac{\partial \mathbf{V}_m}{\partial t}+(\mathbf{V}_m\nabla)\mathbf{V}_m\right]=-\nabla(N_mT)-\eta\Delta\mathbf{V}_m$$
$$-\frac{\eta}{3}\nabla(\nabla\mathbf{V}_m)-m\hat{\nu}_{em}(\mathbf{V}_mN_e-\mathbf{j}_e)-M_i\hat{\nu}_{im}(N_i\mathbf{V}_m-\mathbf{j}_i). \qquad (0.5)$$

We assume here for simplicity that the plasma consists of electrons, of one species of singly-charged ions, and of neutral molecules. $N_e$, $N_i$, and $N_m$ are the concentrations of these particles, $\mathbf{E}$ is the electric field, $\mathbf{H}$ the magnetic field, $\mathbf{j}_e$ and $\mathbf{j}_i$ the electron and ion fluxes [36], $\mathbf{V}_m$ the hydrodynamic velocity of the molecules in the same coordinate system, M, $M_i$, and m the masses of molecules, ions, and electrons, e the electron charge, and c the speed of light. Further, $\hat{\sigma}_e$, $\hat{\sigma}_i$, $\hat{D}_e$, and $\hat{D}_i$ are the conductivity and diffusion tensors for the electrons and ions, $\eta$ the viscosity coefficient, and $\hat{\nu}_{em}$ and $\hat{\nu}_{im}$ the tensors of collisions of the electrons and ions with the molecules. All these quantities are determined with the aid of kinetic theory; they will be given in the next section. Here we note only that the terms with the tensors $\hat{\nu}_{em}$ and $\hat{\nu}_{im}$ in Eq. (0.5) describe the neutral-gas friction due to the collisions of the molecules with electrons and ions. The collision tensors $\hat{\nu}_{em}$ and $\hat{\nu}_{im}$ are not independent, and can be expressed with the aid of the conductivity and

*We note that the lifetimes of inhomogeneities in a plasma can also be strongly dependent on recombination, ionization, adhesion, and other microscopic processes, which are not considered in the present paper. Such processes are important, for example, in the lower ionosphere; they can exert a marked influence on diffusion [58,85].

†In order for Eqs. (0.1)–(0.4) to be valid it is sufficient that the dimension of the inhomogeneity in a direction perpendicular to the magnetic field be much larger than the ion Larmor radius.

‡$[\mathbf{V}_m\mathbf{H}] \equiv \mathbf{V}_m \times \mathbf{H}$.

diffusion tensors. Their introduction is useful in a number of cases. Equations (0.1)—(0.5) have been written out under the assumption that the processes in question are isothermal. Otherwise it would be necessary to add to them the equations for the temperatures of the electrons, ions, and molecules.*

The equations of motion of a plasma must be supplemented by Maxwell's equations

$$\nabla \mathbf{E} = 4\pi e (N_i - N_e), \tag{0.6}$$

$$[\nabla \mathbf{E}] = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \tag{0.7}$$

$$\nabla \mathbf{H} = 0, \tag{0.8}$$

$$[\nabla \mathbf{H}] = \frac{4\pi e}{c}(\mathbf{j}_i - \mathbf{j}_e). \tag{0.9}$$

In (0.9) we have neglected the displacement current ($1/4\pi e\ \partial E/\partial t$), bearing in mind that we shall consider below only relatively small quasistationary processes. We point out also the possibility of one more important simplification of the initial equations. Namely, when the inhomogeneities are relatively large and the condition

$$R_D |\nabla N| \ll N \tag{0.10}$$

is satisfied (here N - density of electrons and ions, $R_D = (T/4\pi e^2 N)^{1/2}$ - Debye radius), and the plasma is quasineutral, i.e., the electron and ion densities are approximately equal:

$$N_i \simeq N_e. \tag{0.11}$$

Then the continuity equations (0.1) and (0.2) for the electrons and ions can be replaced by a single equation for the joint density $N = N_e = N_i$:

$$\frac{\partial N}{\partial t} + \nabla \mathbf{j} = 0. \tag{0.12}$$

It is obvious that this is possible only when the additional condition

$$\nabla \mathbf{j}_e = \nabla \mathbf{j}_i = \nabla \mathbf{j} \tag{0.13}$$

is satisfied. The condition (0.13) can always be satisfied by choosing a longitudinal electric field $\mathbf{E}_{\parallel} = \nabla\varphi$. Thus, the additional condition (0.13) should now be regarded as an equation defining the longitudinal electric field and replacing Poisson's equation (0.6). The latter determines, when the field $\mathbf{E}$ is specified, the difference between the electron and ion densities, $N_e - N_i$, required to produce the field. By virtue of condition (0.10), this density difference is only a small fraction of N, on the order of $(R_D |\nabla N| N)^2$. Consequently, the plasma quasineutrality condition (0.11) is satisfied with this degree of accuracy.

Thus, when condition (0.10) is satisfied, we can consider quasineutral motions of an inhomogeneous plasma. The initial equations (0.1), (0.2), and (0.6) are then replaced by equations (0.11)—(0.13).

b) Kinetic coefficients. Weakly ionized plasma. The components of the electronic conductivity tensor in a weakly ionized plasma in a magnetic field are as follows:

$$\begin{aligned}
&\sigma^e_{\parallel} = \sigma^e_{ZZ} = e^2 N_e K_{\sigma e}(0)/m\nu_{em},\\
&\sigma^e_{XZ} = \sigma^e_{ZX} = \sigma^e_{YX} = \sigma^e_{ZY} = 0,\\
&\sigma^e_{\perp} = \sigma^e_{XX} = \sigma^e_{YY} = e^2 N_e \nu_{em} K_{\sigma e}(q_H)/m(\omega_H^2 + \nu_{em}^2),\\
&\sigma^e_{\Lambda} = \sigma^e_{XY} = -\sigma^e_{YX} = -e^2 N_e \omega_H K_{\epsilon e}(q_H)/m(\omega_H^2 + \nu_{em}^2).
\end{aligned} \tag{0.14}$$

The Z axis is directed here along the magnetic field, $q_H = \omega_H/\nu_{em}$, $\omega_H = eH/mc$ is the gyromagnetic frequency, and $\nu_{em}$ is the frequency of collisions between the electrons and the neutral molecules; this frequency plays the major role in a weakly ionized plasma:

$$\nu_{em} = \frac{2\sqrt{2\pi}N_m}{3}\left(\frac{m}{T_e}\right)\int_0^\infty v^2 \exp\left\{-\frac{mv^2}{2T_e}\right\} dv \int_0^\pi \sin\theta\,(1-\cos\theta) \times q_{em}(v,\theta)\,d\theta, \tag{0.15}$$

where $N_m$ is the molecule density, $T_e$ the electron temperature, and $q_{em}(v, \theta)$ the differential effective electron-molecule collision cross section. For example, if $q_{em} = q_0/4\pi v$, then

$$\nu_{em} = N_m q_0. \tag{0.16}$$

If the collision cross section does not depend on the electron velocity $q_{em} = \sigma_{0e}/4\pi$ (collision with elastic sphere), then

$$\nu_{em} = \frac{8\sqrt{2}}{3\sqrt{\pi}} N_m \sigma_{0e} \sqrt{\frac{T_e}{m}}. \tag{0.17}$$

The coefficients $K_{\sigma e}$ and $K_{\epsilon e}$ are close to unity in a weakly ionized plasma. When $\omega_H/\nu_{em} \gg 1$ they are always equal to unity. Their concrete form for an arbitrary value of $\omega_H/\nu_{em}$ is determined by the character of the dependence of the cross section q on the velocity v. For example, in the case of (0.17) $K_\sigma$ and $K_\epsilon$ are strictly equal to unity. For the model of collisions with elastic spheres (0.17), the coefficients $K_{\sigma e}$ and $K_{\epsilon e}$ are given in [38] (see also [39], p. 70). Expressions for the collision frequency $\nu_{em}$ and the coefficients $K_\sigma$ and $K_\epsilon$ for another dependence of the collision cross section q on the velocity v can be obtained in Shkarofsky's paper [40].*

We can present in similar form the ion-conduc-

*The isothermal character of the diffusion processes in an unbounded plasma is apparently well confirmed. The point is that the thermal conductivity in a plasma, which is not connected with the quasineutrality conditions, always proceeds more energetically than diffusion.

*The numerical results given in [40,86] are for functions $h_\sigma$ and $q_\sigma$, which are connected with $K_\sigma$ and $K_\epsilon$ by the relations

$$K_\sigma(q_H) = \frac{(1+q_H^2)\,q_\sigma(q_H)}{q_\sigma^2(q_H) + q_H^2 h_\sigma^2(q_H)}, \quad K_\epsilon(q_H) = \frac{(1+q_H^2)\,h_\sigma(q_H)}{q_\sigma^2(q_H) + q_H^2 h_\sigma^2(q_H)}.$$

Table I

| $\frac{\omega_H}{\nu_{em}}, \frac{\omega_H}{\nu_{ei}}$ | $K_{em\perp}$ | $K_{em\Lambda}$ | $K_{ei\perp}$ | $K_{ei\Lambda}$ | $\frac{\omega_H}{\nu_{em}}, \frac{\omega_H}{\nu_{ei}}$ | $K_{em\perp}$ | $K_{em\Lambda}$ | $K_{ei\perp}$ | $K_{ei\Lambda}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0. | 0,884 | 0 | 0.513 | 0 | 1,0 | 0,927 | 0.0937 | 0.577 | 0.356 |
| 0,01 | 0,884 | 0.0018 | 0.513 | 0.0021 | 2.0 | 0,997 | 0,0342 | 0.644 | 0.279 |
| 0,05 | 0,879 | 0,0085 | 0.513 | 0.0110 | 4.0 | 0.982 | 0.0094 | 0.796 | 0.113 |
| 0,1 | 0,885 | 0,176 | 0,513 | 0,0241 | 6.0 | 0,992 | 0.0032 | 0.854 | 0.0524 |
| 0.2 | 0,895 | 0,0332 | 0,522 | 0.0654 | 10.0 | 1.0 | 0 | 0.946 | 0.0146 |
| 0.5 | 0.915 | 0.538 | 0.544 | 0,236 | | | | | |

tivity tensor $\hat{\sigma}_i$, except that the electron density and mass $N_e$ and m in formulas (0.14) must be replaced by the ion density and mass $N_i$ and $M_i$, and the electron gyrofrequency $\omega_H$ must be replaced by $\Omega_H = eH/M_i c$. The ion-molecule collision frequency (elastic sphere collisions) is determined according to [41] by

$$\nu_{im} = \frac{8\sqrt{2}}{3\sqrt{\pi}} N_m \sigma_{0i} \left(\frac{TM}{M_i(M+M_i)}\right)^{1/2}. \qquad (0.18)$$

Here M is the molecule mass and $\sigma_{0i}$ the total scattering cross section in collisions between ions and and molecules. Expressions for $\nu_{im}$ for other types of interaction can also be found in [41].

The coefficients $K_{\sigma i}(\Omega_H/\nu_{im})$ and $K_{\epsilon i}(\Omega_H/\nu_{im})$ coincide, if $M_i \ll M$, with the corresponding coefficients $K_{\sigma e}(\omega_H/\nu_{em})$ and $K_{\epsilon e}(\omega_H/\nu_{em})$, if the collision cross sections have the same velocity dependence. But if $M_i \gtrsim M$, then the coefficients $K_{\sigma i}$ and $K_{\epsilon i}$ are much closer to unity than $K_{\sigma e}$ and $K_{\epsilon e}$.

The diffusion tensors for the electrons and ions in a weakly ionized plasma are connected with the conductivity tensors by Einstein's relations (see [42] p. 251):

$$\hat{D}_e = \frac{T_e}{e^2 N_e}\hat{\sigma}_e, \qquad \hat{D}_i = \frac{T_i}{e^2 N_i}\hat{\sigma}_i. \qquad (0.19)$$

For collisions between electrons and neutral molecules, the tensor $\hat{\nu}_{em}$ can be represented in the form

$$\hat{\nu}_{em} = \nu_{em}\hat{K}_{em}, \qquad (0.20)$$

where $\nu_{em}$ is the collision frequency considered above, and $\hat{K}_{em}$ is the tensor of the coefficients. The components of the tensor $\hat{K}_{em}$ can be expressed with the aid of the functions $K_\sigma$ and $K_\epsilon$, which were considered above,

$$K_{em\|} = K_{em\perp}(0) = \frac{1}{K_{\sigma e}(0)}, \qquad K_{em\perp} = \frac{K_{\sigma e}(q_H)(1+q_H^2)}{K_{\sigma e}^2(q_H) + q_H^2 K_{\varepsilon e}^2(q_H)},$$

$$K_{em\Lambda} = \frac{q_H[K_{\varepsilon e}(q_H) - K_{\sigma e}^2(q_H)]}{K_{\sigma e}^2(q_H) + q_H^2 K_{\varepsilon e}^2(q_H)}. \qquad (0.21)$$

Here $q_H/\nu_{em}$. For the elastic-sphere collision case (0.17), the coefficients $K_{em\perp}$ and $K_{em\Lambda}$ are listed in Table I. The ion-molecule collision tensor $\hat{\nu}_{im}$ can be represented in perfectly analogous form, except that the frequency $\nu_{em}$ is replaced by $\nu_{im}$, $\omega_H$ by $\Omega_H$, and the functions $K_{\sigma e}$ and $K_{\epsilon e}$ by $K_{\sigma i}$ and $K_{\epsilon i}$ respectively. In addition, the sign of the function $K_{im\Lambda}$ is reversed.

We note that the collision tensors $\hat{\nu}_{em}$ and $\hat{\nu}_{im}$ enter not only in the equations of motion (0.5) of the neutral molecules, but also in the equations for the velocity of the macroscopic motion of the electrons and ions:

$$mN_e\hat{\nu}_{em}\mathbf{V}_e = -eN_e\mathbf{E} - \frac{e}{c}[\mathbf{V}_e\mathbf{H}]N_e - T_e\nabla N_e, \qquad (0.22)$$

$$M_iN_i\hat{\nu}_{im}\mathbf{V}_i = eN_i\mathbf{E} + \frac{e}{c}N_i[\mathbf{V}_i\mathbf{H}] - T_i\nabla N_i. \qquad (0.23)$$

Using these equations, we can readily establish the connection expressed by formulas (0.21) between the collision tensors and the conductivity tensors.*

It is important that for collisions between electrons and neutral molecules the tensor of the coefficients $\hat{K}_{em} = \hat{\nu}_{em}/\nu_{em}$ is very close to a unit tensor. This pertains to an even greater degree to the tensor $\hat{K}_{im}$. In approximate calculations it is therefore possible to replace, with sufficient accuracy, the collision tensors $\hat{\nu}_{em}$ and $\hat{\nu}_{im}$ by the scalar collision frequencies $\nu_{em}$ and $\nu_{im}$. Such an approximation is called in [39] the "elementary theory." In the "elementary theory" approximation, the expressions for the conductivity and diffusion tensors are given as before by formulas (0.14), with $K_\sigma = K_\epsilon \equiv 1$. In the absence of a magnetic field (as $H \to 0$) the conductivity, diffusion, and collision tensors go over naturally into scalar quantities equal to the longitudinal components of the corresponding tensors.

### Arbitrary Degree of Ionization

We have considered above only a weakly ionized plasma, when the principal role is played by collisions with neutral molecules. At higher degrees of ionization, collisions between electrons and ions also become important. To take these into account in the calculation of the conductivity and diffusion tensors,

*In operator form, the connection between the collision tensor and the conduction tensor in a weakly ionized plasma has the simple form

$$\hat{\sigma}_{em} = \frac{e^2N_e}{m}(\hat{\nu}_{em} - \hat{\omega}_H)^{-1}, \quad \hat{\sigma}_{im} = \frac{e^2N_i}{M_i}(\hat{\nu}_{im} + \hat{\Omega}_H)^{-1},$$

where

$$\hat{\omega}_H\mathbf{x} = \frac{e}{mc}[\mathbf{H}, \mathbf{x}], \qquad \hat{\Omega}_H\mathbf{x} = \frac{e}{M_ic}[\mathbf{H}, \mathbf{x}].$$

it is necessary to add in the right sides of (0.22) and (0.23) the terms $mN_e\hat{\nu}_{ei}(V_e - V_i)$ and $mN_e\hat{\nu}_{ei}(V_i - V_e)$ respectively. Here $\hat{\nu}_{ei}$ is the electron-ion collision tensor:

$$\hat{\nu}_{ei} = \nu_{ei}\hat{K}_{ei}, \tag{0.24}$$

where $\nu_{ei}$ is the collision frequency:

$$\nu_{ei} = \frac{4\sqrt{2\pi}}{3}\frac{e^4 N_i}{m^{1/2}T_e^{3/2}}\ln\Lambda, \tag{0.25}$$

$N_i$ is the ion density and $\ln\Lambda = \ln(T_e R_D/e^2)$ is the Coulomb logarithm ($R_D$ is the Debye radius).

The tensor $\hat{K}_{ei}$ is given by (0.21), i.e., it is described as before by the two functions $K_{ei\perp}(q_H)$ and $K_{ei\Lambda}(q_H)$ ($K_{ei\parallel} = K_{ei\perp}(0)$). For a fully singly-ionized plasma, these functions are listed in Table I as functions of $\omega_H/\nu_{ei}$. It is seen from the table that the deviation of $\hat{K}_{ei}$ from a unit tensor is in general more appreciable than in the case of collisions with molecules.* In a partly ionized plasma, where collisions of electrons with both neutral molecules and ions are important, the form of the functions $K_{ei\perp}$ and $K_{ei\Lambda}$ changes, depending on the ratio $\nu_{ei}/\nu_{em}$. The same pertains to the functions $K_{em\perp}$, $K_{em\Lambda}$, $K_{im\perp}$, and $K_{im\Lambda}$. In the general case, one can apparently state that the functions $K_\perp$ and $K_\Lambda$ assume values between those listed in Table I for the corresponding functions in weakly-ionized and fully-ionized plasma.† It is also important to emphasize that under real conditions the gyrofrequency is usually comparable with the collision frequency only in a weakly ionized plasma. At high degrees of plasma ionization, when $\nu_{ei} \gtrsim \nu_{em}$, the gyrofrequency is usually much higher than the collision frequency. For example, in the ionosphere $\omega_H \sim \nu_{em}$ at altitudes $h \sim 80$–$90$ km, and $\Omega_H \sim \nu_{im}$ at $h \sim$ 1( 100–120 km. At these altitudes, the ionosphere plasma is weakly ionized. On the other hand, at altitudes on the order of 200 km and higher, where the plasma is strongly ionized, the gyrofrequency is larger by 2–3 orders of magnitude than the collision frequency. The coefficient tensors $\hat{K}$ are in this case close to the unit tensor. Under these conditions we can confine ourselves to the "elementary theory" approximation, setting the tensors $\hat{K}$ equal to the unit tensor. In this approximation, accurate to small terms of order $m\nu_{em}/M_i\nu_{im} \sim \sqrt{m/M} \ll 1$ the components of the conductivity and diffusion tensors, for arbitrary degree of ionization, are:

$$\sigma_{e\parallel} = \frac{e^2N_e}{m(\nu_{em}+\nu_{ei})}, \quad \sigma_{e\perp} = \frac{e^2N_i}{mA}\left[\nu_{ei} + \nu_{em}\left(1+\frac{\Omega_H^2}{\nu_{im}^2}\right)\right],$$

$$\sigma_{e\Lambda} = -\frac{e^2N_e}{mA}\omega_H[1 + m\nu_{ei}/M_i\nu_{im} + \Omega_H^2/\nu_{im}^2],$$

$$\sigma_{i\parallel} = \frac{e^2N_i}{M_i\nu_{im}}\frac{\nu_{em}}{(\nu_{ei}+\nu_{em})}, \quad \sigma_{i\perp} = \frac{e^2N_i}{M_i\nu_{im}A}(\nu_{em}^2 + \nu_{em}\nu_{ei} + \omega_H^2),$$

$$\sigma_{i\Lambda} = \frac{e^2N_i\Omega_H}{M_i\nu_{im}^2A}(\nu_{em}^2 + M_i\nu_{ei}\nu_{im}/m + \omega_H^2),$$

$$D_{e\parallel} = T\left(1+2\frac{m\nu_{ei}}{M_i\nu_{im}}\right)\Big/ m(\nu_{em}+\nu_{im}), \quad D_{i\parallel}$$

$$= T(\nu_{em}+2\nu_{ei})/M_i\nu_{im}(\nu_{em}+\nu_{ei})$$

$$D_{e\perp} = \frac{T}{mA}\left[(\nu_{em}+\nu_{ei})\left(1+2\frac{m\nu_{ei}}{M_i\nu_{im}}\right) + \frac{\Omega_H^2}{\nu_{im}^2}(\nu_{em}+2\nu_{ei})\right],$$

$$D_{e\Lambda} = -\frac{T\omega_H}{mA}\left\{1 + 3\frac{m\nu_{ei}}{M_i\nu_{im}} + \frac{\Omega_H^2}{\nu_{im}^2}\right\},$$

$$D_{i\perp} = \frac{T}{M_i\nu_{im}A}\left[(\nu_{em}+\nu_{ei})(\nu_{em}+2\nu_{ei}) + \omega_H^2\left(1+2\frac{m\nu_{ei}}{M\nu_{im}}\right)\right],$$

$$D_{i\Lambda} = \frac{T\Omega_H}{M_i\nu_{im}^2A}\left\{\nu_{em}^2 + \omega_H^2 - \frac{M_i}{m}\nu_{ei}\nu_{im}\right\},$$

$$A = (\nu_{em}+\nu_{ei})^2 + \omega_H^2\left(1+2\frac{m\nu_{ei}}{M_i\nu_{im}} + \frac{\Omega_H^2}{\nu_{im}^2}\right). \tag{0.26}$$

We see from the foregoing formulas that relations (0.19) are valid only in a weakly ionized plasma.*

The coefficient of kinetic viscosity in a gas of neutral molecules, for different laws of interactions between them, is given in [41]. For example, for the elastic-sphere collision model

$$\eta = 0.563\frac{\sqrt{MT}}{\sigma_0}, \tag{0.27}$$

where $\sigma_0$ *is the total scattering cross section in the* collision.

## 1. DIFFUSION SPREADING OF INHOMOGENEITIES

### 1.1. Equation of Ambipolar Diffusion in a Plasma Situated in a Magnetic Field

Let us consider the spreading of a quasineutral inhomogeneity in a plasma.† We neglect the influence of the solenoidal electric field on the molecule mo-

*We note that for a fully ionized plasma with ionization multiplicity Z the difference between $K_{ei}$ and the unit tensor is even greater. This follows from [42,44,45].

†The kinetic coefficients for the electrons in a plasma of arbitrary degree of ionization were calculated in [40]. No account was taken there, however, of the ion motion, a procedure valid apparently only in the case of an alternating electric field of sufficiently high frequency $\omega \gg \Omega_H$. In general form, transport phenomena in a three-component plasma were considered in [46]. The electron and ion conductivity and diffusion tensors for an arbitrary degree of ionization are expressed with the aid of the collision tensors in [43].

*We note that the components of the complete plasma electric conductivity tensor $\hat{\sigma} = \hat{\sigma}_e + \hat{\sigma}_i$ have the following form in the same approximation ($N_e = N_i = N$):

$$\sigma_\parallel = \frac{e^2N}{m(\nu_{em}+\nu_{ei})}, \quad \sigma_\perp = \frac{e^2N}{mA}\left(\nu_{em} + \nu_{ei} + \frac{\Omega_H}{\nu_{im}}\omega_H\right),$$

$$\sigma_\Lambda = -\frac{e^2N\omega_H}{mA}.$$

†The question of establishment of the quasineutral state is considered in Sec. 1.4c, where it is shown that if condition (0.10) or the more exact condition (1.48) is satisfied, an arbitrary initial charge diverges rapidly and a quasineutral state is established and spreads out by diffusion.

tion. The roles of these two factors will be discussed in Sec. 1.4, where we shall show that their influence is indeed immaterial under a wide range of conditions. The general equation describing the diffusion spreading of the inhomogeneities follows in this case directly from (0.12) and (0.13). Substituting in them the expressions (0.3) and (0.4) for the electron and ion currents and recognizing that $E = \nabla\varphi$, where $\varphi$ is the potential of the electric field, we obtain

$$\frac{\partial N}{\partial t} = \frac{1}{2}(\nabla\hat{D}_i\nabla + \nabla\hat{D}_e\nabla)N + \frac{1}{2e}(\nabla\hat{\sigma}_e\nabla - \nabla\hat{\sigma}_i\nabla)\varphi,$$
$$\frac{1}{e}(\nabla\hat{\sigma}_e\nabla + \nabla\hat{\sigma}_i\nabla)\varphi = (\nabla\hat{D}_i\nabla - \nabla\hat{D}_e\nabla)N. \tag{1.1}$$

Here $\hat{\sigma}_e$, $\hat{D}_e$ and $\hat{\sigma}_i$, $\hat{D}_i$ are the electron and ion conductivity and diffusion tensors. Under stationary conditions in a weakly ionized plasma, equations of this type were considered by Johnson and Hulbert [3].

In a weakly ionized plasma, when the electron-ion collisions are immaterial and the tensors $\hat{\sigma}_e$ and $\hat{\sigma}_i$ are proportional to N, the following condition is satisfied:

$$(\nabla\hat{\sigma}_e\nabla)(\nabla\hat{\sigma}_i\nabla) = (\nabla\hat{\sigma}_i\nabla)(\nabla\hat{\sigma}_e\nabla). \tag{1.2}$$

It is satisfied also for an arbitrary degree of ionization in the linear approximation, when $N = N_0 + \delta N$ and the tensors $\hat{\sigma}_e$ and $\hat{\sigma}_i$ depend only on the homogeneous concentration $N_0$. In these cases, multiplying the first equation of (1.1) by $(\nabla\sigma_e\nabla + \nabla\sigma_i\nabla)$ and the second by the difference of the same quantities, we can eliminate the electric-field potential. The ambipolar diffusion equation then takes the form [47,48]

$$(\nabla\hat{\sigma}_e\nabla + \nabla\hat{\sigma}_i\nabla)\frac{\partial N}{\partial t} = [(\nabla\hat{\sigma}_e\nabla)(\nabla\hat{D}_i\nabla) + (\nabla\hat{\sigma}_i\nabla)(\nabla\hat{D}_e\nabla)]N. \tag{1.3}$$

In the absence of a magnetic field, the conductivity and the diffusion coefficient are scalars. The equations in (1.1) then take the form

$$\frac{\partial N}{\partial t} = \frac{D_e\sigma_i + D_i\sigma_e}{\sigma_e + \sigma_i}\Delta N,$$
$$(\nabla\sigma_e + \nabla\sigma_i)\nabla\varphi = e(\nabla D_i - \nabla D_e)\nabla N. \tag{1.4}$$

We have taken into account here the relations $\sigma_i\nabla\sigma_e = \sigma_e\nabla\sigma_i$ and $\sigma_e\nabla D_i = -\sigma_i\nabla D_e$, which hold for an arbitrary degree of plasma ionization accurate to small terms of order $m\nu_{em}/M_i\nu_{im} \sim \sqrt{m/M_i}$. The first of these equations is the equation of ambipolar diffusion obtained by Schottky [1] (see also [33,34,42,49,50]). Equation (1.1) for strictly longitudinal or strictly transverse diffusion also reduces to this form in the presence of a magnetic field. In the latter case, naturally, $\sigma_e = \sigma_{e\perp}$, $D_e = D_{e\perp}$, etc.

Equations (1.1) and (1.3) are of fourth order. They differ significantly from the usual equation of diffusion in an anisotropic medium:

$$\frac{\partial N}{\partial t} = D_\parallel\frac{\partial^2 N}{\partial z^2} + D_\perp\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right). \tag{1.5}$$

It should be noted that the authors of many papers devoted to diffusion in a plasma in a magnetic field obtained the ambipolar-diffusion equation in the form (1.5). They did it by replacing div $j_e$ = div $j_i$ the derivation of (0.13) by the stronger condition $j_e = j_i$, i.e., they imposed an additional requirement that prohibits solenoidal currents in the plasma (see, for example, [21,33,36,51,52]). Such a requirement greatly limits the class of possible solutions. It does not follow from the initial equations and is in general unjustified (see [53]).

In an unbounded plasma, which will be considered henceforth, we have at infinity the natural conditions $N \to N_0$ and $\varphi \to 0$. We note that if the boundary and initial conditions of the problem contain no characteristic dimensions, then Eqs. (1.1), (1.3), and (1.4) admit of a self-similar solution of the type

$$N(\mathbf{r}, t) = t^{-\alpha}N(\xi), \tag{1.6}$$

where $\xi = \mathbf{r}/\sqrt{t}$, and $\alpha$ is a constant. The transformation (1.6) decreases the number of variables in the equations under consideration. The latter retain the same form, except that $\partial N/\partial t$ is replaced by $-(\alpha + \frac{\xi}{2}\nabla)N$, the operator $\nabla$ denoting now differentiation with respect to $\xi$. The transformation (1.6) can be used, in particular, in the calculation of the Green's function in an unbounded plasma. In this case $\alpha = 3/2$ (for the three-dimensional problem).

The boundary conditions for Eqs. (1.1) and (1.3), which are specified on the surfaces bounding the plasma, relate the concentration with the particle flux on the surface. On each boundary, there are in this case two conditions defining the electron and ion currents. This is as it should be, since Eqs. (1.1) and (1.3) are of fourth order (unlike the ordinary diffusion equation). In the general case the boundary conditions are

$$\left.\begin{aligned} -\frac{1}{e}\mathbf{n}\hat{\sigma}_e\nabla\varphi - \mathbf{n}\hat{D}_e\nabla N = \lambda_e N, \\ \frac{1}{e}\mathbf{n}\hat{\sigma}_i\nabla\varphi - \mathbf{n}\hat{D}_i\nabla N = \lambda_i N. \end{aligned}\right\} \tag{1.7}$$

Here $\mathbf{n}$ is the normal to the boundary at the point under consideration, and $\lambda_e$ and $\lambda_i$ are factors that depend on the character of the interaction of the electrons and the ions with the surface bounding the plasma and on the plasma potential.* In particular, in the case of total absorption (neutralization) of the electrons and the ions on the boundary surface, conditions (1.7) take the form†

*The calculation of the factors $\lambda_i$ and $\lambda_e$ is a task of kinetic theory. No such calculations have been made for the general case as yet. For a weakly ionized plasma in the absence of a magnetic field, the problem was considered in [87].

†It is necessary also that the mean free path and the Larmor radii (for strictly transverse diffusion) be much larger than the Debye radius.

$$N=0,\quad \mathbf{n}(\hat{\sigma}_e\nabla\varphi+e\hat{D}_e\nabla N)=\frac{\lambda_e}{\lambda_i}\mathbf{n}(e\hat{D}_i\nabla N-\hat{\sigma}_i\nabla\varphi). \tag{1.8}$$

The first of these conditions is analogous to the usual condition at an absorbing wall in the case of diffusion (see [54]). On a nonconducting surface, the field potential is in addition established in such a way that the currents of electrons and ions which become absorbed (neutralized) at a given point of the surface, be equal, i.e.,

$$\lambda_e=\lambda_i. \tag{1.9}$$

Then the boundary conditions (1.8) do not depend on the factors $\lambda_e$ and $\lambda_i$. The potential of an isolated conducting surface is established in such a way that the integral electron and ion fluxes are equal at the surface. Relation (1.9) is satisfied then only in special symmetrical cases, namely a spherical or cylindrical surface with an axis parallel to the magnetic field[55,56].

### 1.2. Solution of Equation of Ambipolar Diffusion. Diffusion Coefficient.

We assume that the initial inhomogeneity is a perturbation of the main density of the homogeneous plasma: $N(\mathbf{r}, 0) = N_0 + \delta N(\mathbf{r}, 0)$, $\delta N \ll N$. The spreading of such inhomogeneities is described by Eq. (1.3). It can be readily solved for an unbounded plasma by expanding the unknown functions in Fourier integrals with respect to the coordinates

$$\delta N(\mathbf{r}, t)=\frac{1}{(2\pi)^3}\int \delta N_{\mathbf{k}}(t)\, e^{i\mathbf{kr}}\, d^3k. \tag{1.10}$$

Substituting the expansion (1.10) in (1.3) we get

$$\delta N_{\mathbf{k}}(t)=\delta N_{\mathbf{k}}(0)\exp\{-D_a(\beta)k^2t\}. \tag{1.11}$$

Here $\delta N_k(0)$ are the Fourier components of the initial perturbation of the density, $\delta N_k(0) = \delta N(\mathbf{r}, 0)\exp(-i\mathbf{k}\cdot\mathbf{r})\,d^3r$, and $D_a(\beta)$ is the coefficient of ambipolar diffusion

$$D_a(\beta)=\frac{(\sigma_{e\parallel}\cos^2\beta+\sigma_{e\perp}\sin^2\beta)(D_{i\parallel}\cos^2\beta+D_{i\perp}\sin^2\beta)+(\sigma_{i\parallel}\cos^2\beta+\sigma_{i\perp}\sin^2\beta)(D_{e\parallel}\cos^2\beta+D_{e\perp}\sin^2\beta)}{(\sigma_{e\parallel}+\sigma_{i\parallel})\cos^2\beta+(\sigma_{e\perp}+\sigma_{i\perp})\sin^2\beta}. \tag{1.12}$$

Here $\sigma_{\parallel}$ and $\sigma_{\perp}$, $D_{\parallel}$ and $D_{\perp}$ are the longitudinal and transverse components of the conductivity and diffusion tensors of the electrons and ions, and $\beta$ is the angle between the direction of the vector k and the direction of the magnetic field $H_0$. In the derivation of (1.12) we took account of the fact that $\sigma_{ji}k_jk_i = (\sigma_{\parallel}\cos^2\beta + \sigma_{\perp}\sin^2\beta)k^2$. We now analyze the concrete form of the ambipolar diffusion coefficient. For the components of the tensors $\sigma$ and D we shall use in this case the equations of the elementary theory.

a) Weakly ionized plasma. Substituting (0.14) and (0.19) in (1.12) we get[47]:

$$D_a(\beta)=(T_e+T_i)\{M_i\nu_{im}[1+(\Omega_H/\nu_{im})^2]/[1+(\Omega_H/\nu_{im})^2\cos^2\beta] + m\nu_{em}[1+(\omega_H/\nu_{em})^2]/[1+(\omega_H/\nu_{em})^2\cos^2\beta]\}^{-1}. \tag{1.13}$$

The coefficient of ambipolar diffusion (1.13) depends in essential fashion on the angle $\beta$. In longitudinal diffusion ($\beta = 0$) the diffusion coefficient (1.13) is equal to

$$D_{a\parallel}=\frac{T_e+T_i}{m\nu_{em}+M_i\nu_{im}}. \tag{1.14}$$

It coincides with the diffusion coefficient (1.4) in an isotropic plasma. Consequently, the magnetic field does not influence the diffusion along the force lines. In the case of transverse diffusion ($\beta = \pi/2$) we have [21]

$$D_{a\perp}=\frac{T_e+T_i}{M_i\nu_{im}+m\omega_H^2/\nu_{em}}. \tag{1.15}$$

It follows therefore that in a sufficiently strong magnetic field, $\Omega_H\omega_H \gg \nu_{im}\nu_{em}$, the coefficient of ambipolar diffusion transverse to the field coincides with the coefficient of transverse diffusion of the electrons, multiplied by a factor $(1 + T_i/T_e)$ due to the ion influence.[60-62]

The solution of Eq. (1.5), which describes ordinary diffusion in an isotropic medium, can also be represented in the form (1.11). In this case the diffusion coefficient for arbitrary angle $\beta$ is determined by the transverse and longitudinal diffusion coefficients:

$$D_0(\beta)=D_{\parallel}\cos^2\beta+D_{\perp}\sin^2\beta. \tag{1.16}$$

In our case the diffusion coefficient (1.13) has a more complicated dependence on the angle $\beta$. It differs quite strongly from $D_0(\beta)$. This is seen from Fig. 1, which shows a plot of $D_a$ against $\cos\beta$ for $m\nu_{em}/M_i\nu_{im} = 0.01$ and for the different values of $\Omega_H/\nu_{im}$ indicated in the figure. The dashed curves in the figure show for comparison, at the same values of the parameters, the coefficient $D_0(\beta)$. We see that the difference between the coefficients $D_a(\beta)$ and $D_0(\beta)$ is large. It increases with increasing ratio $\Omega_H/\nu_{im}$. It is seen from (1.13) that when $\omega_H\Omega_H/\nu_{em}\nu_{im} \ll 1$, i.e., $H \ll (c/e)(M_i m\nu_{em}\nu_{im})^{1/2}$, the influence of the magnetic field is negligible and

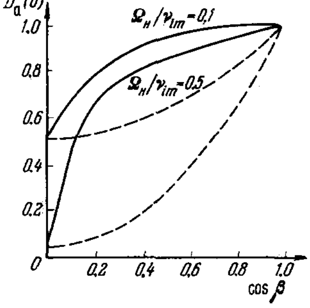


FIG. 1.

the diffusion is isotropic. Under the opposite conditions, considerable anisotropy sets in. It is precisely this case which is represented by the curve $\Omega_H/\nu_{im} = 0.5$ in Fig. 1. In a strong magnetic field when $\Omega_H \gg \nu_{im}$, i.e., $H \gg M_i\nu_{im}(c/e)$, the anisotropy is very strong. In this case, at values of $\beta$ not too close to $\pi/2$, the approximate value of the coefficient $D_a(\beta)$ is

$$D_a(\beta) \simeq \left(1+\frac{T_e}{T_i}\right)(D_{i\parallel}\cos^2\beta + D_{i\perp}\sin^2\beta), \tag{1.17}$$

where $D_{i\parallel}$ and $D_{i\perp}$ are coefficients of the longitudinal and transverse diffusion of the ions. Under these conditions the diffusion proceeds at the ion-diffusion rate increased by a factor $(1 + T_e/T_i)$. However at angles very close to $\pi/2$ (when $\pi/2 - \beta \lesssim (\nu_{im}/\Omega_H)\sqrt{m\nu_{em}/M_i\nu_{im}}$) the coefficient $D_a(\beta)$ decreases sharply to a value $D_{\perp i}(1 + T_e/T_i) \times \sqrt{m\nu_{em}/M_i\nu_{im}}$. This is seen from Fig. 2, which shows in logarithmic scale a plot of $D_a$ against $\cos\beta$ for $\Omega_H/\nu_{im} = 10$. The dashed curve represents $D_0(\beta)$, and the dash-dot curve the coefficient $D_a(\beta)$ calculated with formula (1.17).

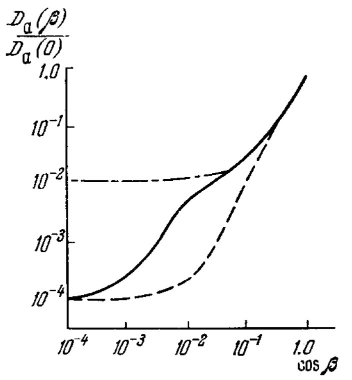


FIG. 2.

The indicated singularity of the function $D_a(\beta)$ at angles $\beta$ close to $\pi/2$ greatly influences the character of the spreading of the inhomogeneities in the strong magnetic field. In fact, we recognize that if the inhomogeneity is strongly elongated in the H direction, then its Fourier transform has a sharp maximum at angles $\beta$ close to $\pi/2$. The width of this maximum is $\Delta\beta \sim R_\perp/R_\parallel$, where $R_\parallel$ and $R_\perp$ are the characteristic dimensions of the inhomogeneity along and across the magnetic field. It is clear therefore that the character of the spreading of the inhomogeneities differs greatly with the shape of the inhomogeneity, or more accurately with the ratio of $R_\parallel$ to $R_\perp$. Indeed, if

$$R_\perp/R \gg \frac{\nu_{im}}{\Omega_H}\sqrt{\frac{m\nu_{em}}{M_i\nu_{im}}}, \tag{1.18}$$

then the presence of a sharp maximum in the diffusion coefficient would have little effect on the spreading of the inhomogeneity. In this case one can use the approximate formula (1.17) for $D_a(\beta)$. The inhomogeneity transverse to the magnetic field moves then at the rate of the transverse ion diffusion, increased by a factor $(1 + T_e/T_i)$. But in the case of inhomogeneities that are very strongly elongated along H, when the condition inverse to (1.18) is satisfied, the rate of spreading of the inhomogeneities transversely to the field is much smaller, of the same order as the rate of transverse diffusion of the electrons. Thus, the rate of spreading of the inhomogeneity in a plasma, transversely to the magnetic field, depends strongly on the shape of the inhomogeneity.

b) Arbitrary degree of ionization. For arbitrary degree of plasma ionization, both collisions with neutral molecules and collisions between electrons and ions are important. Substituting in this case the general formulas (0.26) for $\sigma$ and D in (1.12), we get

$$D_a(\beta) = \frac{2T\left[\left(1+\frac{\nu_{ei}}{\nu_{em}}\right)^2 + \frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\left(1+2\frac{m\nu_{ei}}{M_i\nu_{im}}\right) + \frac{\omega_H^2\Omega_H^2}{\nu_{em}^2\nu_{im}^2}\cos^4\beta\right]}{M_i\nu_{im}\left[\left(1+\frac{\nu_{ei}}{\nu_{em}}\right)^2 + \frac{\Omega_H\omega_H}{\nu_{im}\nu_{em}}\left(1+\frac{\nu_{ei}}{\nu_{em}}\right) + \frac{\omega_H^2}{\nu_{em}^2}\left(1+\frac{m\nu_{ei}}{M_i\nu_{im}}+\frac{\Omega_H^2}{\nu_{im}^2}\right)\cos^2\beta\right]}. \tag{1.19}$$

We have neglected small terms of the order of $m\nu_{em}/M_i\nu_{im} \sim \sqrt{m/M_i}$ compared with unity. In addition, we have assumed for simplicity that $T_e = T_i = T$. Formula (1.19) was derived by Grigor'ev [63].

When $\nu_{ei} = 0$, expression (1.19) coincides with formula (1.13) which was considered in the preceding section. When $H = 0$, the diffusion coefficient (1.19) goes over into (1.14); the electron-ion collisions consequently do not influence the diffusion when there is no magnetic field. The same result is obtained also for longitudinal diffusion in a magnetic field ($\beta = 0$). For transverse diffusion ($\beta = \pi/2$), the electron-ion collisions, to the contrary, are very important. In this case, as shown by Golant [64,33]

$$D_\perp = \frac{2T(\nu_{em}+\nu_{ei})}{M_i\nu_{im}(\nu_{ei}+\nu_{em})+m\omega_H^2}. \tag{1.20}$$

We see therefore that the collisions of the electrons with the ions exert a decisive influence on the transverse diffusion at high degrees of plasma ionization. When $\nu_{em} \to 0$ formula (1.20) leads to the well known expression which determines the transverse diffusion in a strongly ionized plasma [65]

$$D_\perp = \frac{2T}{m}\frac{\nu_{ei}}{\omega_H^2}. \tag{1.21}$$

The dependence of the diffusion coefficient $D_a$ on the angle $\beta$ is in general analogous to the case of a weakly ionized plasma. This is seen from Fig. 3,

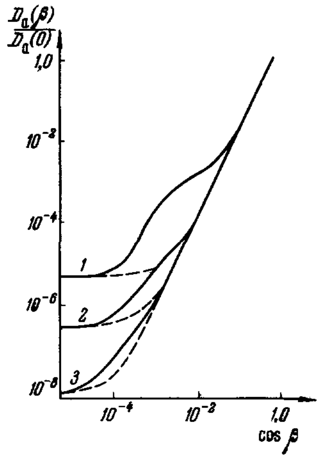


FIG. 3.

which shows a plot of $D_a(\beta)/D_a(0)$ for $m\nu_{em}/M_i\nu_{im} = 10^{-3}$ at different values of the parameters $\Omega_H/\nu_{im}$ and $\nu_{ei}/\nu_{em}$: curve 1 – for $\Omega_H/\nu_{im} = 29.5$; $\nu_{ei}/\nu_{em} = 3.7$; curve 2 – for $\Omega_H/\nu_{im} = 3.5 \times 10^2$ and $\nu_{ei}/\nu_{em} = 40$; curve 3 – for $\Omega_H/\nu_{im} = 2.4 \times 10^4$ and $\nu_{ei}/\nu_{em} = 4 \times 10^3$.* The dashed curves in the figure show plots of (1.16), where $D_{\|}$ and $D_{\perp}$ are given by (1.14) and (1.20). We see from the figure that the dashed curves differ greatly, as before, from the continuous ones. However, with increasing electron-ion collision frequency (more accurately, with increasing ratio $m\nu_{ei}/M_i\nu_{im}$), the difference between them decreases and becomes small when $m\nu_{ei}/M_i\nu_{im} \gg 1$. Consequently, when $\nu_{ei} \gg M_i\nu_{im}/m$, the diffusion spreading of the inhomogeneities in a plasma situated in a magnetic field is close to ordinary diffusion in an anisotropic medium with diffusion coefficients $D_{\|}$ and $D_{\perp}$ given (1.14) and (1.20). This can be readily seen also from formula (1.19). Indeed, when $\nu_{ei} \gg \nu_{im}M_i/m$, neglecting small terms, we get

$$D_a(\beta) = \frac{2T}{M_i\nu_{im}}\cos^2\beta + \frac{2T\nu_{ei}}{M_i\nu_{im}\nu_{ei} + m\omega_H^2}\sin^2\beta. \qquad (1.22)$$

Thus, at large values of the ratio $m\nu_{ei}/M_i\nu_{im}$ the ambipolar diffusion in the plasma assumes a character close to ordinary diffusion in an anisotropic medium. It is easy to understand the cause of this phenomenon. The point is that the singularities indicated above in the diffusion of a plasma transversely to a magnetic field are connected with the large differences in the rates of the transverse diffusion of the electrons and ions. The ions diffuse transversely to the magnetic field, in general, much more rapidly than the electrons. As a result, the electronic and ionic components of the inhomogeneity separate and an electric charge appears and hinders the motion of the ions transversely to the field. The same charge leads to an intensification of the motion of the electrons transversely to the field, owing to the formation of conduction currents. This permits the inhomogeneity spread transversely to the field at a rate much larger than the rate of transverse electron diffusion. However, with increasing degree of plasma ionization, the rate of transverse electron diffusion increases and approaches, when $m\nu_{ei} > M_i\nu_{im}$, the rate of transverse diffusion of the ions. When $m\nu_{ei} \gg M_i\nu_{im}$, the electrons and ions diffuse transversely to the field at equal rates, with a diffusion coefficient (1.21); consequently the effects connected with the difference in the rate of transverse diffusion of the electrons and ions become weaker, as we have seen before. It must be emphasized at the same time that in those cases when these effects are, generally speaking, small, they again become quite appreciable, and even decisive, at large distances $r \gg \sqrt{Dt}$ (see Sec. 1.3b).

c) Polarization of inhomogeneity. Perturbations of magnetic field. The potential of the electric field in the inhomogeneity is determined by the second equation of (1.1). The Fourier components of the potential $\varphi_k$ are

$$\varphi_{\mathbf{k}} = \frac{e\,[(D_{i\|} - D_{e\|})\cos^2\beta + (D_{i\perp} - D_{e\perp})\sin^2\beta]}{(\sigma_{e\|} + \sigma_{i\|})\cos^2\beta + (\sigma_{e\perp} + \sigma_{i\perp})\sin^2\beta}\,\delta N_{\mathbf{k}}, \qquad (1.23)$$

where $\beta$ is again the angle between k and H, and $\delta N_k$ are the Fourier components of the density perturbations (1.11). Recognizing that $D_{e\|} \gg D_{i\|}$ and $D_{i\perp} > D_{e\perp}$ in a strong magnetic field, we see that the sign of $\varphi_k$ changes with the angle $\beta$. Consequently, the sign of the electric-field potential in the inhomogeneity in a strong magnetic field depends on the shape of the inhomogeneity: it becomes positive for inhomogeneities which are very strongly elongated along the field. A similar singularity takes place, naturally, also in the difference of the electron and ion densities in the homogeneity:

$$\delta N_{i\mathbf{k}} - \delta N_{e\mathbf{k}} = \frac{(D_{e\|} - D_{i\|})\cos^2\beta + (D_{e\perp} - D_{i\perp})\sin^2\beta}{(\sigma_{e\|} + \sigma_{i\|})\cos^2\beta + (\sigma_{i\perp} + \sigma_{e\perp})\sin^2\beta}\,\frac{k^2}{4\pi}\,\delta N_{\mathbf{k}}. \qquad (1.24)$$

The process of spreading of the inhomogeneity in the magnetic field is accompanied by the occurrence of a closed electric current **j**:

$$\mathbf{j} = e(\mathbf{j}_i - \mathbf{j}_e), \qquad \nabla\mathbf{j} = 0,$$

where $j_i$ and $j_e$ are the ion and electron fluxes (0.3) and (0.4). This leads to the appearance of magnetic perturbations, too. The Fourier components of the magnetic-field perturbations are

$$\delta\mathbf{H}_{\mathbf{k}} = i\,\frac{4\pi}{ck^2}[\mathbf{k}\mathbf{j}_{\mathbf{k}}] = \frac{4\pi e}{ck^2}\,\delta N_{\mathbf{k}}\left\{[\mathbf{k}\,(\hat{D}_i - \hat{D}_e)\,\mathbf{k}] + [\mathbf{k},\,(\hat{\sigma}_e + \hat{\sigma}_i)\,\mathbf{k}] \times \frac{(D_{e\|} - D_{i\|})\cos^2\beta + (D_{e\perp} - D_{i\perp})\sin^2\beta}{(\sigma_{e\|} + \sigma_{i\|})\cos^2\beta + (\sigma_{e\perp} + \sigma_{i\perp})\sin^2\beta}\right\}. \qquad (1.25)$$

In particular, in a weakly ionized plasma, substituting the expressions for the components of the diffusion and conductivity tensors, we get hence [47]

*These values of the parameters correspond to altitudes 150, 200, and 400 km in the ionosphere.

$$\delta \mathbf{H_k} = -\frac{4\pi e}{c}\,\delta N_{\mathbf{k}}\;\frac{(T_e+T_i)\left\{\left(\frac{\omega_H}{\nu_{em}}\right)^2\cos\beta\left[\frac{\mathbf{k}}{k}\,\frac{\mathbf{H}_0}{H_0}\right]+\frac{\omega_H}{\nu_{em}}\left(1+\frac{\omega_H\Omega_H}{\nu_{em}\nu_{im}}\cos^2\beta\right)\left(\frac{\mathbf{H}_0}{H_0}-\frac{\mathbf{k}}{k}\cos\beta\right)\right\}}{M_i\nu_{im}\left[1+\frac{\omega_H\Omega_H}{\nu_{em}\nu_{im}}+\frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\,(1+\Omega_H^2/\nu_{im}^2)\right]}.$$

When $H = 0$, and also for the case of strong longitudinal diffusion, there are no magnetic-field perturbations, as should be the case.

### 1.3. Spreading of Small Perturbations. Green's Function

We now consider the spreading of inhomogeneities which had at the initial instant of time very small (pointlike) dimensions: $\delta N(\mathbf{r}, 0) = n_0\delta(\mathbf{r})$. The Fourier transform of such an initial inhomogeneity is a constant: $\delta N_k(0) = n_0$. It follows then from (1.10) and (1.11) that at any instant of time the particle-density perturbations are described by the expression

$$\delta N(\mathbf{r}, t) = n_0 \int e^{i\mathbf{kr}-D_a(\beta)k^2t}\, d^3k = n_0 G(\mathbf{r}, t). \tag{1.26}$$

The function $G(\mathbf{r}, t)$ is the source function, or the Green's function, for Eq. (1.3).*

We now integrate in (1.26) with respect to $d^3k$. To this end, we introduce in the space of the vectors $\mathbf{k}$ the spherical coordinates $k$, $\mu = \cos\beta = \mathbf{k}\cdot\mathbf{H}/kH$, and $\varphi$ - the angle between the planes $\mathbf{rH}$ and $\mathbf{kH}$. Integrating then with respect to $dk$, we obtain [66]:

$$G(\mathbf{r}, t) = G(r, \alpha, t) = \frac{1}{8\pi^{5/2}t^{3/2}}\int_0^1 \frac{d\mu}{D_a^{3/2}(\mu^2)}\int_0^\pi \exp\left\{-\frac{r^2B^2}{4D_a(\mu^2)t}\right\} \times \left\{1-\frac{r^2B^2}{4D_a(\mu^2)t}\right\}d\varphi,$$

$$B = \mu\cos\alpha + \sqrt{1-\mu^2}\sin\alpha\cos\varphi. \tag{1.27}$$

Here $\alpha$ is the angle between $\mathbf{r}$ and $\mathbf{H}$.

a) Small distances. When $r \ll \sqrt{Dt}$ we get from (1.27):

$$G(0, t) = \frac{1}{8\pi^{3/2}t^{3/2}}\int_0^1 \frac{d\mu}{D_a^{3/2}(\mu^2)}. \tag{1.28}$$

From this we see that the perturbations of the concentration in the center of the inhomogeneity, which are proportional to $G(0, t)$, decrease with time like $t^{-3/2}$, i.e., in the same manner as in ordinary diffusion. The value of $G(0, t)$ (more accurately, the value of the integral in formula (1.28)) characterizes the rate of spreading of the inhomogeneity. It depends essentially on the rate of diffusion of both the electrons and the ions. In particular, in a weakly ionized plasma with $\omega_H \gtrsim \nu_{em}$ we have

$$I = \int_0^1 \frac{d\mu}{D_a^{3/2}(\mu^2)} \simeq D_{i\parallel}^{-3/2}\left(1+\frac{T_e}{T_i}\right)^{-3/2}\left\{1+\left(\frac{\Omega_H}{\nu_{im}}\right)^2 + \left(\frac{\omega_H}{\nu_{em}}\right)^{1/2}\left(\frac{\Omega_H}{\nu_{im}}\right)^{3/2}\right\}. \tag{1.29}$$

In a strongly ionized plasma ($\nu_{ei} \gg M_i\nu_{im}/m$) with $T_e = T_i$ we get from (1.22)

$$I \simeq 2^{-3/2}D_{i\parallel}^{-3/2}\left\{1+\frac{m\omega_H^2}{M\nu_{im}\nu_{ei}}\right\}. \tag{1.30}$$

It is seen from (1.29) and (1.30) that the values of the integral I increase in a strong magnetic field in proportion to $H^2$. The rate of spreading of the inhomogeneity decreases accordingly. It is interesting that in a strong magnetic field ($\omega_H \gg \nu_{ei}$, $\Omega_H \gg \nu_{im}$) expressions (1.29) and (1.30) can be represented when $T_e = T_i$ in the form

$$I = \frac{1}{\sqrt{(2D_{i\parallel})(2D_{i\perp})(2D_{e\perp})}}, \tag{1.31}$$

where $D_{i\parallel}$, $D_{i\perp}$, and $D_{e\perp}$ are respectively the coefficients of longitudinal and transverse diffusion of the electrons and ions. Let us compare this expression with the value $I = 1/\sqrt{D_\parallel D_\perp^2}$, which is obtained in the case of ordinary diffusion in an anisotropic medium. It is natural to assume that the coefficient of ambipolar diffusion along the magnetic field is $D_{a\parallel} = 2D_{i\parallel}$. It then follows from (1.31) that the effective coefficient of plasma diffusion transversely to the magnetic field is the mean square of the doubled coefficients of transverse electron and ion diffusion.

b) Asymptotic behavior. We now determine the character of the perturbations at large distances from the main inhomogeneity, $r \gg 2\sqrt{Dt}$. We consider first a simple case, when $\mathbf{r} \parallel \mathbf{H}$, i.e., $\cos\alpha = 1$. In this case

$$G(r, 0, t) = G_\parallel(r, t) = \frac{1}{8\pi^{3/2}t^{3/2}}\int_0^1 \frac{d\mu}{D_a^{3/2}(\mu^2)}\left(1-\frac{r^2\mu^2}{2D_a(\mu^2)t}\right)\exp\left\{-\frac{r^2\mu^2}{4D_a(\mu^2)t}\right\}. \tag{1.32}$$

Replacing $\mu$ by a new variable $x = r\mu/2\sqrt{D_a(\mu^2)t}$, we rewrite (1.32) in the form

$$G_\parallel(r, t) = \frac{1}{4\pi^{3/2}tr}\int_0^{r/2\sqrt{D_a(1)t}} \frac{e^{-x^2}[1-2x^2]\,dx}{D_a-\mu^2D_a'}. \tag{1.33}$$

*The general solution of the linearized equation (1.3) can be expressed in terms of the Green's function for arbitrary initial perturbation $\delta N(\mathbf{r}', 0)$ and in the presence of a source $I(\mathbf{r}', t')$:

$$\delta N(\mathbf{r}, t) = \int \delta N(\mathbf{r}', 0)\, G(\mathbf{r}-\mathbf{r}', t)\, d^3r' + \int_0^t \int G(\mathbf{r}-\mathbf{r}', t-t')\, I(\mathbf{r}'t')\, d^3r'\, dt'.$$

Here and below $D'_a = dD_a/d\mu^2$, $D''_a = d^2D_a/(d\mu^2)^2$, etc. In the integral (1.33), the significant values are $x \sim 1$. Recognizing that $r/2\sqrt{Dt} \gg 1$, we replace the upper limit of integration by infinity. Expanding the denominator in a series in the vicinity of $\mu = 0$ and expressing $\mu$ in terms of x, we find that

$$G_{\parallel}(r, t) = -\frac{3D''_a(0)\,t}{\pi r^5}. \tag{1.34}$$

In particular, for a weakly ionized plasma, when $D_a(\mu^2)$ is given by expression (1.13), we have

$$G_{\parallel}(r, t) = \frac{12D_{i0}t\,[1+\Omega_H^2/\nu_{im}^2]\,[1+\omega_H^2/\nu_{em}^2]\,\frac{\omega_H^3\Omega_H}{\nu_{em}^3\nu_{im}}}{\pi r^5\{1+\omega_H\Omega_H/\nu_{em}\nu_{im}\}^3}. \tag{1.35}$$

We see therefore that the perturbations of the density $\delta N$ decrease in proportion to $1/r^5$ with increasing distance. They also increase with increasing magnetic field. In particular, $\delta N$ increases in proportion to $H^2$ in a strongly magnetized plasma ($\Omega_H \gg \nu_{im}$). As $H \to 0$, the coefficient $D''_a(0)$ approaches zero in proportion to $H^4$. When $H = 0$ the asymptotic expression for $G_{\parallel}$ degenerates and the Green's function decreases exponentially with distance, as in the case of ordinary diffusion.

The change in the asymptotic behavior of the Green's function in the case of diffusion in a plasma in a magnetic field is the consequence of violation of the analyticity of its Fourier components $G_k$ in the vicinity of the point $k = 0$. Indeed, according to (1.26)

$$G_{\mathbf{k}} = \exp\{-D_a(\mu^2)\,k^2t\}.$$

We see therefore that if $D_a(\mu^2) = C_0 + C_1\mu^2$, where $\mu = k_z/k$, then $D_ak^2$ is an analytic function of k and the function $G_k$ is also an analytic function of k. But if the diffusion coefficient has a different dependence on $\mu^2$ (given, for example, by formula (1.12)), then a singularity arises at the point $k = 0$. This singularity is indeed the cause of the change in the asymptotic properties of the Green's function. The physical cause of the phenomenon lies in the fact that the character of the perturbations at large distances from the main inhomogeneity is determined by the influence of the electric charge and not by diffusion.

We have considered above only the case $\alpha = 0$. Similar calculations show that for an arbitrary angle $\alpha$ between r and H the density perturbations at large distances are again proportional to $t/r^5$.[66]

c) Shape of inhomogeneity. The Green's function G depends on three variables: distance r, angle $\alpha$, and time t. From (1.27) it follows, however, that the product $t^{3/2}G$ is the function of only two variables, the angle $\alpha$ and $x = r/2\sqrt{2D_{i\parallel}t}$. This is as it should be, since the Green's function in an unbounded plasma is determined by the self-similar equation (1.6). It is therefore convenient to consider the dimensionless function $G(x, \alpha) = G(r, t, \alpha)/G(0, t)$, where $G(0, t) \sim 1/t^{3/2}$ is the value of the Green's function at $r = 0$, determined by formula (1.28).

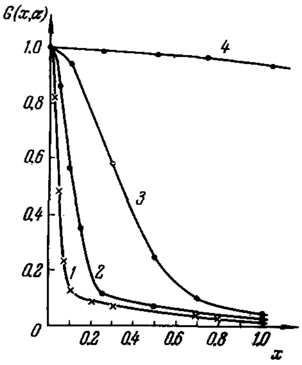


FIG. 4.

The result of the numerical calculation of the function $G(x, \alpha)$ is shown in Fig. 4 (weakly ionized plasma, $\Omega_H/\nu_{im} = 1$, $m\nu_{em}/M_i\nu_{im} = 3 \times 10^{-3}$). The dependence of G on x is shown here for different values of the angle $\alpha$: curves 1, 2, 3, and 4 are constructed for $\cos\alpha = 0$, 0.9, 0.99, and 1 respectively. We see from the figure that for small values of x the function G decreases rapidly, exponentially. When $x \sim 1$ the rate of decrease of G is much slower. At large values of x, according to the asymptotic expressions given above, G decreases in proportion to $1/x^5$. At small x, the function G decreases especially rapidly in a direction perpendicular to the magnetic field ($\cos\alpha = 0$), as expected. The anisotropy of the function $G(x, \alpha)$ at $x \sim 0.1$ is quite strong; when $s \sim 1$ it is much less pronounced.

The curves $G(x, \alpha) = \text{const} = G_0$ characterize the shape acquired by the inhomogeneity during the process of its spreading in the plasma. They are shown in Fig. 5a in a logarithmic scale for the different values of $G_0$ indicated in the figure. In Fig. 5b, the

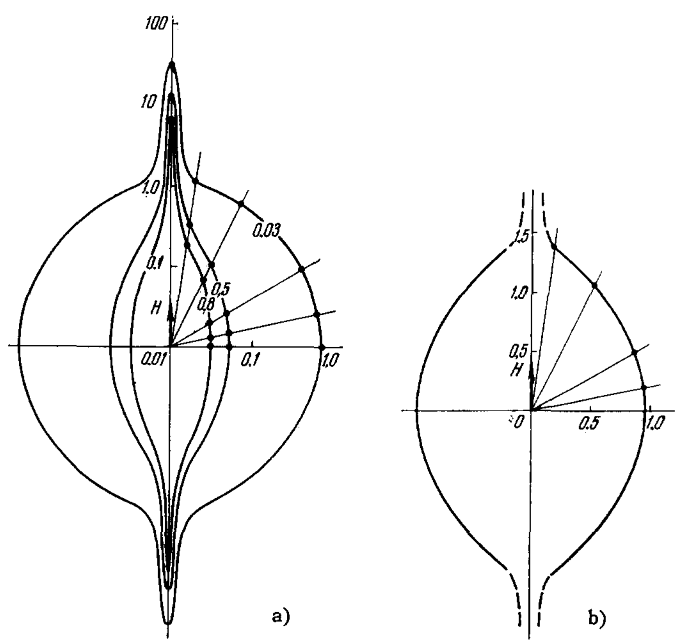


FIG. 5.

curve $G_0 = 0.03$ is drawn in a linear scale. The abscissas and ordinates indicate the values of x, and the angle $\alpha$ is measured from the ordinate axis, which is directed along the magnetic field. We see from the figure that the inhomogeneity acquires, as a result of the spreading, a shape which differs noticeably from the ellipsoids which are produced in ordinary diffusion in an anisotropic medium.

d) Lifetime of inhomogeneity. The lifetime of an inhomogeneity is naturally defined as the time during which the perturbation $\delta N(0, t)$ of the particle density at the maximum of the inhomogeneity, decreases by a specified factor p (say, 10 or 100). Assume that at the initial instant of time the perturbation of the density at the maximum of the inhomogeneity was $\delta N(0, 0)$. The lifetime t of the inhomogeneity is then determined from the relation $\delta N(0, 0) = p\delta N(0, t)$. Using for $\delta N(0, t)$ expressions (1.26) and (1.28), we get

$$\frac{t}{t_0} = \left\{ \int_0^1 \left[ \frac{D_{i\parallel}(1 + T_e/T_i)}{D_a(\mu^2)} \right]^{3/2} d\mu \right\}^{2/3}, \tag{1.36}$$

$$t_0 = \left[ \frac{n_0 p}{\delta N(0, 0)} \right]^{2/3} \Big/ 4\pi D_{i\parallel}(1 + T_e/T_i). \tag{1.37}$$

Here $t_0$ is the lifetime of the inhomogeneity in the plasma in the absence of a magnetic field, $n_0 = \int \delta N(\mathbf{r}, 0)\, d^3r$ is the total number of particles in the initial inhomogeneity.* The ratio $n_0/\delta N(0, 0)$ is of the order of $R_0^3$, where $R_0$ is the characteristic dimension of the initial inhomogeneity. It follows from (1.37) that the lifetime of the inhomogeneity is proportional to $R_0^2/D_{i\parallel}$.

With increasing magnetic field, the lifetime of the inhomogeneity increases. For example, in a weakly ionized plasma, as follows from (1.29) and (1.36), we have

$$t/t_0 = \left\{ 1 + \left(\frac{\Omega_H}{\nu_{im}}\right)^2 + \left(\frac{\omega_H}{\nu_{em}}\right)^{1/2} \left(\frac{\Omega_H}{\nu_{im}}\right)^{3/2} \right\}^{2/3}$$

We see therefore that in a strong magnetic field the lifetime of the inhomogeneity is proportional to $H^{4/3}$. At an arbitrary degree of ionization of a plasma in a strong magnetic field ($\Omega_H \gg \nu_{im}$, $\omega_H \gg \nu_{ei}$) we have

$$t/t_0 = D_{i\parallel}^{2/3}/D_{i\perp}^{1/3} D_{e\perp}^{1/3}.$$

## 1.4. Influence of Motion of Neutral Molecules and of Solenoidal Electric Field

a) Molecule motion. We have neglected in the foregoing the motion of the neutral molecules ($V_m = 0$). Actually, when the inhomogeneities of an electron-ion component of the plasma spread out, the collisions with the molecules give rise to motion of the neutral gas, too. It is natural to expect that when $N \ll N_m$ it will affect only slightly the diffusion spreading of the inhomogeneities.

We consider first a weakly ionized plasma in the absence of a magnetic field. In this case the dispersion equation for the system of linearized equations (0.12), (0.13), (0.3)—(0.5), describing the quasineutral motion of the electrons, ions, and neutral molecules with $T_e = T_i = T$ is written in the form

$$(i\omega + D_\eta k^2)^2 \left\{ i\omega^3 + \omega^2 k^2 \left( D_a + \frac{4}{3} D_\eta \right) - i\omega k^2 \left( D_a \frac{N}{N_m} \nu_{im} + S^2 \right) - k^4 S^2 D_a \right\} = 0. \tag{1.38}$$

Here $D_\eta = \eta/M_i N_m$ is the viscosity coefficient, S the speed of sound in the neutral gas, and $D_a$ the coefficient of ambipolar diffusion (1.14). Equation (1.38) has four roots. The root $\omega = iD_\eta k^2$ describes the damping of the hydrodynamic motions of the incompressible gas of neutral particles ($V \perp k$), and the roots $\omega = i(2/3) D_\eta \pm Sk\sqrt{1 + 2N/N_m}$ describe the sound waves.* Finally, the root

$$\omega = ik^2 \frac{T_e + T_i}{M_i \nu_{im} + M_i \nu_{im} (T_e + T_i) N/TN_m} \tag{1.39}$$

describes the diffusion spreading of inhomogeneities of the electron and ion density - ambipolar diffusion. It is seen from (1.39) that at low degrees of ionization, the change in the diffusion coefficient, connected with the particle motion, is small. At high degrees of ionization it becomes appreciable.

It is interesting that the coefficient of ambipolar diffusion (1.39 retains its meaning also when $N_m \to 0$: in this case $\omega = ik^2 TN_m/M_i N\nu_{im}$. At the same time it is known that no diffusion spreading of the inhomogeneity takes place in a fully ionized isothermal plasma in the absence of a magnetic field. The point is that a state with constant pressure must be produced for diffusion spreading of the inhomogeneity. In particular, the process of ambipolar diffusion in an isothermal plasma in the absence of a magnetic field is always a process in which mutual diffusion takes place of the electron-ion gas and of the neutral-molecule gas. It is easy to see that in this case the following relation is always satisfied:

$$\delta N_m(\mathbf{r}, t) = -\frac{T_e + T_i}{T} \delta N(\mathbf{r}, t). \tag{1.40}$$

Because of this, the pressure perturbation is $\Delta p = T\delta N_m + T_e \delta N_e + T_i \delta N_i \equiv 0$, so that the total pressure in the plasma is constant. If $N/N_m \ll 1$, then the pressure of the neutrals does not affect the mutual diffusion, as is usually the case for diffusion in gas mixtures[41]. When $N/N_m \gtrsim 1$, on the other hand, the diffusion is quite appreciable. However, when

*Formula (1.36) is valid under the assumption that the lifetime of the inhomogeneity is sufficiently large: $t > R_{0\perp}^2/D_{e\perp}^{1/2} D_{i\perp}^{1/2}$, where $R_{0\perp}$ is the characteristic dimension of the initial inhomogeneity in a plane perpendicular to $\mathbf{H}$. At large values of the number p, as is clear from (1.36), the foregoing condition is always satisfied.

*The longitudinal waves in a partially ionized gas were considered, for example, in [88,89].

$N \gg N_m$, as is clear from (1.40), only very weak perturbations of the charged-particle concentration can spread out: $\delta N \sim \delta N_m \ll N_m$. In a fully-ionized plasma (when $N_m = 0$) such motions are generally impossible.

In the presence of a magnetic field the dispersion equation with allowance for the motion of the neutrals has in general a very complicated form.[43] In longitudinal diffusion $k \parallel H$ the root describing the ambipolar diffusion is determined, as before by (1.39). For transverse diffusion, $k \perp H$,

$$\omega = ik^2 D_{a\perp} \Big/ \left[1 + \frac{N(T_e+T_i)}{N_m T(1+\omega_H \Omega_H/\nu_{im}\nu_{em})}\right],$$

where $D_{a\perp}$ is the coefficient of transverse ambipolar diffusion (1.15). We see therefore that the magnetic field only weakens the influence of the neutral-molecule motion on the diffusion. Apparently in the general case, when the condition

$$N(T_e+T_i) \ll N_m T \tag{1.41}$$

is satisfied, the motion of the neutral molecules likewise does not affect the ambipolar diffusion.*

b) Solenoidal electric field. So far we have assumed throughout that the electric field E is longitudinal. We shall now take into account the influence of a solenoidal electric field on the motion of the ions and electrons. In other words, we consider in lieu of Eq. (0.6) the complete system of quasistationary Maxwell equations (0.6)—(0.9). The general dispersion equation describing the spreading of quasineutral inhomogeneities is then written in the form[48]

$$i\omega^3 + A(D_M k^2)\omega^2 - iB(D_M k^2)^2\omega - C(D_M k^2)^3 = 0, \tag{1.42}$$

where $D_M = c^2/4\pi\sigma_e$ is the magnetic viscosity. The coefficients A, B, and C in a weakly ionized plasma take the form:

$$A = 2 + p + \frac{\omega_H \Omega_H}{\nu_{em}\nu_{im}}(1+\cos^2\beta),$$

$$B = 1 + 2p + \frac{\omega_H \Omega_H}{\nu_{em}\nu_{im}}(1+2p\cos^2\beta) + \frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\left(1+\frac{\Omega_H^2}{\nu_{im}^2}\right),$$

$$C = p\left[1 + \frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\left(1+\frac{\Omega_H^2}{\nu_{im}^2}\cos^2\beta\right)\right].$$

Here $\beta$ is, again the angle between the vectors **k** and H, and p is a dimensionless parameter:

$$p = \frac{D_{a\parallel}}{D_M} = \frac{4\pi}{c^2}(\sigma_{e\parallel}D_{i\parallel} + \sigma_{i\parallel}D_{e\parallel}) = \frac{(T_e+T_i)}{mc^2}\frac{4\pi e^2 N}{M_i \nu_{im}\nu_{em}}. \tag{1.43}$$

The dispersion equation (1.42) is of third order. All its roots $\omega$ are proportional to $k^2$, and consequently the spreading of the quasineutral inhomogeneities in the plasma is described in general, when a solenoidal field is taken into account, by three independent diffusion processes.

For example, in the case of transverse diffusion ($\cos\beta = 0$) the roots of the dispersion equation (1.42) are

$$\omega_{1,2} = ik^2 D_M \left\{p + 1 + \frac{\omega_H \Omega_H}{\nu_{em}\nu_{im}} \mp \sqrt{(p-1)^2 + \frac{\Omega_H^2\omega_H^2}{\nu_{im}^2\nu_{em}^2} + \frac{2\omega_H\Omega_H}{\nu_{em}\nu_{im}}(p+1)}\right\}, \qquad \omega_3 = ik^2 D_M.$$

At small values of the parameter, $p \ll 1 + \omega_H \Omega_H/\nu_{em}\nu_{im}$, the root $\omega_1$ is equal to $ik^2 D_{a\perp}$, where $D_{a\perp}$ is the coefficient of transverse ambipolar diffusion (1.15). It is precisely this root which describes the spreading of the particle-density perturbations. The two other roots are much larger under the same conditions - on the order of $D_M$. They describe the spreading of the magnetic perturbations.

For longitudinal diffusion ($\cos\beta = 1$):

$$\omega_1 = ik^2 D_{a\parallel},$$

$$\omega_{2,3} = ik^2 D_M\left[1 + \frac{\omega_H \Omega_H}{\nu_{em}\nu_{im}} \pm i\frac{\omega_H}{\nu_{em}}\right].$$

The first of these roots describes longitudinal ambipolar diffusion, and the two others the spreading of the magnetic disturbances. The latter process has not only diffusion but also wave properties. If $\Omega_H \ll \nu_{im}$ and $\omega_H \gg \nu_{em}$, respectively, the corresponding waves are weakly damped. Their dispersion is quadratic ($\omega \sim k^2$). These waves are observed in the earth's magnetosphere[90-92] ("whistlers") and in solids[67-70] ("helicons").

For an arbitrary diffusion diffusion, if the condition

$$p = \frac{4\pi e^2 N(T_e+T_i)}{mMc^2\nu_{im}\nu_{em}} \ll 1 + \frac{\omega_H\Omega_H}{\nu_{em}\nu_{im}}, \tag{1.44}$$

is satisfied, one of the diffusion roots, $\omega_1$, is much smaller than the other two. It is this minimal root which describes the spreading of the particle-density perturbations. It coincides with the coefficient of ambipolar diffusion (1.13).

Thus, in the general case, the spreading of quasineutral inhomogeneities in a plasma is a composite of three independent diffusion processes, which describe the time variation of the perturbations of the magnetic field and of the electron and ion densities. When condition (1.44) is satisfied, these processes separate: two of them become much faster than the third, and the latter is connected just with the spreading of the plasma-density perturbations. A solenoidal electric field has no great influence on the latter process, which coincides with the already considered ambipolar diffusion. The condition (1.44) limits the pressure of the electron-ion plasma component. In particular, in a sufficiently strong magnetic field $\omega_H \Omega_H > \nu_{em}\nu_{im}$, condition (1.44) has a simple physical meaning: the pressure of the electron-ion gas

*It must be emphasized that in this section we consider only the molecule motion brought about by the spreading of an electron-ion inhomogeneity. The gas of neutral molecules, as a whole, is assumed to be as rest. The role of the motion of the entire neutral-particle gas is discussed in the next section.

$N(T_e + T_i)$ should be smaller than the magnetic-field pressure $H^2/4\pi$. Under real conditions, in outer-space and in laboratory plasma, this requirement is usually satisfactorily met.

c) Establishment of quasineutral state. So far we have considered only quasineutral inhomogeneities, i.e., we have assumed that the uncompensated electric charge has already time to diffuse. This is true, of course, only in the case of sufficiently slow processes.

Let us consider now the spreading of an arbitrary initial inhomogeneity in a plasma and let us show how the quasineutral state, from which the ambipolar diffusion starts out, is produced.[47]

In a weakly ionized plasma, the equations (0.1)—(0.6) take the form

$$\left.\begin{aligned} &\frac{\partial N_e}{\partial t}-\nabla\left(\frac{\hat\sigma_e}{e}\mathbf{E}\right)-\nabla(\hat D_e\nabla N_e)=0,\\ &\frac{\partial N_i}{\partial t}+\nabla\left(\frac{\hat\sigma_i}{e}\mathbf{E}\right)-\nabla(\hat D_i\nabla N_i)=0,\\ &\nabla\mathbf{E}=-4\pi e(N_i-N_e).\end{aligned}\right\}\quad(1.45)$$

In the derivation of (1.45) we have neglected, as usual, the motion of the neutral molecules and the influence of the solenoidal electric field. Let us assume, as before, that the inhomogeneity of the homogeneous plasma is $\delta N_{e,i} \ll N_0$. Expanding the unknown functions $\delta N_e$, $\delta N_i$, and $\mathbf{E}$ in a Fourier integral in terms of the coordinates, we get from (1.45) the following system of equations for the functions $\delta N_{ek}(t)$ and $\delta N_{ik}(t)$:

$$\frac{\partial N_{e\mathbf{k}}}{\partial t}+(D_{\|e}\cos^2\beta+D_{\perp e}\sin^2\beta)k^2\delta N_{e\mathbf{k}}+4\pi(\sigma_{e\|}\cos^2\beta+\sigma_{e\perp}\sin^2\beta)(\delta N_{e\mathbf{k}}-\delta N_{i\mathbf{k}})=0,\quad(1.46a)$$

$$\frac{\partial N_{i\mathbf{k}}}{\partial t}+(D_{\|i}\cos^2\beta+D_{\perp i}\sin^2\beta)k^2\delta N_{i\mathbf{k}}+4\pi(\sigma_{\|i}\cos^2\beta+\sigma_{\perp i}\sin^2\beta)(\delta N_{i\mathbf{k}}-\delta N_{e\mathbf{k}})=0.\quad(1.46b)$$

The solution of the linear equations (1.46) can be written in the form:

$$\begin{aligned}\delta N_{\mathbf{k}e}(t)&=\delta N^{(1)}_{e\mathbf{k}}e^{i\omega_1 t}+\delta N^{(2)}_{e\mathbf{k}}e^{i\omega_2 t},\\ \delta N_{\mathbf{k}i}(t)&=\delta N^{(1)}_{i\mathbf{k}}e^{i\omega_1 t}+\delta N^{(2)}_{i\mathbf{k}}e^{i\omega_2 t},\end{aligned}\quad(1.47)$$

where $\omega_1$ and $\omega_2$ are the roots of the characteristic equation. If the characteristic dimensions of the disturbed region are large compared with the Debye radius $R_D$ (condition (0.10), then

$$\sigma_e \gg D_e k^2,\qquad \sigma_i \gg D_i k^2.\quad(1.48)$$

In this case the roots $\omega_1$ and $\omega_2$ have the simple form:

$$\omega_1=4\pi i\,[(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta],\quad(1.49a)$$

$$\omega_2=ik^2\,\frac{(\sigma_{\|e}\cos^2\beta+\sigma_{\perp e}\sin^2\beta)(D_{\|i}\cos^2\beta+D_{\perp i}\sin^2\beta)+}{(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta}\times\frac{+(\sigma_{\|i}\cos^2\beta+\sigma_{\perp i}\sin^2\beta)(D_{\|e}\cos^2\beta+D_{\perp e}\sin^2\beta)}{(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta}.\quad(1.49b)$$

For $\delta N^{(1)}_{ke}$, $\delta N^{(2)}_{ke}$, $\delta N^{(1)}_{ki}$, and $\delta N^{(2)}_{ki}$ we obtain in this case

$$\delta N^{(1)}_{\mathbf{k}e}=\frac{\sigma_{\|e}\cos^2\beta+\sigma_{\perp e}\sin^2\beta}{(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta}[\delta N_{\mathbf{k}e}(0)-\delta N_{\mathbf{k}i}(0)],\quad(1.50)$$

$$\delta N^{(1)}_{\mathbf{k}i}=\frac{\sigma_{\|i}\cos^2\beta+\sigma_{\perp i}\sin^2\beta}{(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta}[\delta N_{\mathbf{k}i}(0)-\delta N_{\mathbf{k}e}(0)],$$

$$\delta N^{(2)}_{\mathbf{k}e}=\delta N^{(2)}_{\mathbf{k}i}=\frac{(\sigma_{\|e}\cos^2\beta+\sigma_{\perp e}\sin^2\beta)\,\delta N_{\mathbf{k}i}(0)+(\sigma_{\|i}\cos^2\beta+\sigma_{\perp i}\sin^2\beta)\,\delta N_{\mathbf{k}e}(0)}{(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta}.\quad(1.51)$$

Here $\delta N_{ke}(0)$ and $\delta N_{ki}(0)$ are the Fourier components of the initial perturbation of the electron and ion density.

From (1.47)—(1.51) we see that the root $\omega_1$ describes the spreading of the initially uncompensated charge in the plasma, which is proportional to $[\delta N_{ki}(0) - \delta N_{ke}(0)]$. The charge vanishes very rapidly, and after a time

$$\Delta t \gg 1/4\pi\,[(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta+(\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta]\quad(1.52)$$

the inhomogeneity becomes quasineutral, $\delta N_{ke}(t) = \delta N_{ki}(t)$.* The spreading of the quasineutral inhomogeneities is described by the second root $\omega_2$. This process coincides with the ambipolar diffusion which was considered above. It is much slower than the leakage of the initial charge, since $\omega_2/\omega_1 \approx (kR_D)^2 \ll 1$.

Thus, the quasineutral state of the plasma becomes established after a time (1.52) or (1.52a). When the conditions (0.10) or (1.48) are satisfied, this time is small compared with the characteristic time of ambipolar diffusion.

## 2. MOVING INHOMOGENEITIES

We have considered above the diffusion spreading of inhomogeneities in a plasma, and assumed that the electron and ion gas in the homogeneous (unperturbed) plasma is at rest. Under real conditions, however, the electrons and ions frequently execute also a common motion or drift. The drift may be due to various causes: external electric field, motion of neutral gas (wind), inhomogeneity of the magnetic field, gravitational field, etc.

*In choosing the condition (1.52), we used the macroscopic equations. Therefore satisfaction of this condition is essential only in a sufficiently dense plasma, when the electron collision frequency $\nu$ exceeds the plasma frequency $\omega_0$. If $\nu \lesssim \nu_0$, then the establishment of the quasineutral state proceeds via emission of plasma waves. The time of establishment is of the order of $1/\omega_0$. The condition (1.52) is then replaced by the condition

$$\Delta t \gg 1/\omega_0.$$

When drift is present in the plasma, the inhomogeneity moves. If the electron and ion drift velocities $V_e$ and $V_i$ are equal, then the entire inhomogeneity will move at the same velocity, of course. Usually, however, the electrons and ions drift with different velocities. Then the drift tends to separate the electronic and ionic components of the inhomogeneity. This gives rise to an electric field that hinders the separation. The action of the field causes the inhomogeneity to move at some mean velocity intermediate between $V_e$ and $V_i$, and the electron and ion densities in the inhomogeneity are always equal to each other. However, the same electric field causes the inhomogeneity to spread. This mechanism of spreading of moving inhomogeneities, called here "dispersion mechanism," is radically different from diffusion. It leads to a "splitting" of the moving inhomogeneity. The present section is devoted to an analysis of the motion and spreading of inhomogeneities in a plasma under drift conditions.

## 2.1. Equation of Motion of the Inhomogeneities

We assume that condition (0.10) is satisfied, i.e., we shall consider only the motion of a quasineutral plasma. We neglect, as before, the influence of the internal vortical electric field and the perturbations in the motion of the neutral gas. We then get directly from (0.12) and (0.13) an equation describing the motion and spreading of an inhomogeneous plasma, an equation which is essentially the generalization of the ambipolar diffusion equation (1.1) to include the case of a plasma with drift. Indeed, substituting into (0.12) and (0.13) the expressions (0.3) and (0.4) for the electron and ion fluxes and taking into account their drift motion, we get:

$$\frac{\partial N}{\partial t}+\frac{1}{2}\nabla(N\mathbf{V}_{e0}+N\mathbf{V}_{i0})-\frac{1}{2}(\nabla\hat{D}_i\nabla+\nabla D_e\nabla)N$$
$$-\frac{1}{2e}(\nabla\hat{\sigma}_e\nabla-\nabla\hat{\sigma}_i\nabla)\varphi=0,$$
$$\frac{1}{e}(\nabla\hat{\sigma}_e\nabla+\nabla\hat{\sigma}_i\nabla)\varphi=\nabla(N\mathbf{V}_{e0}-N\mathbf{V}_{i0})+\nabla(\hat{D}_i\nabla-\nabla\hat{D}_e\nabla)N. \quad (2.1)$$

Here $\varphi$ is the potential of the internal electric field, and $\mathbf{V}_{e0}$ and $\mathbf{V}_{i0}$ are the drift velocities of the electrons and ions:

$$\mathbf{V}_{e0}=\mathbf{U}_m-\frac{\hat{\sigma}_e}{eN}\left\{\mathbf{E}_0+\frac{1}{c}[\mathbf{U}_m\mathbf{H}]-\frac{mg}{e}\right\}$$
$$-\frac{\hat{\sigma}_{He}T_e}{2e^2N}\left(\frac{[\mathbf{H}\nabla\mathbf{H}]}{H^2}+\frac{[\mathbf{H},(\mathbf{H}\nabla)\mathbf{H}]}{H^3}\right),$$
$$\mathbf{V}_{i0}=\mathbf{U}_m+\frac{\hat{\sigma}_i}{eN}\left\{\mathbf{E}_0+\frac{1}{c}[\mathbf{U}_m\mathbf{H}]+\frac{Mg}{e}\right\}+\frac{\hat{\sigma}_{He}T_i}{2e^2N}\left(\frac{[\mathbf{H}\nabla,\mathbf{H}]}{H^2}\right.$$
$$\left.+\frac{[\mathbf{H},(\mathbf{H}\nabla)\mathbf{H}]}{H^3}\right). \quad (2.2)$$

Here $\mathbf{E}_0$ is the external electric field, $\mathbf{U}_m$ the velocity of the neutrals, and $\mathbf{g}$ the acceleration due to gravity. Expressions (2.2) can be derived from (0.3), (0.4) by substituting for $\mathbf{E}$ the effective force acting on the electrons and ions in the gravitational field and an inhomogeneous magnetic field (see [57,65,71]). The tensors $\hat{\sigma}_{He}$ and $\hat{\sigma}_{Hi}$ coincide in the elementary-theory approximation with $\hat{\sigma}_e$ and $\hat{\sigma}_i$.* Therefore, in a coordinate frame moving together with the neutral gas the drift velocities $\mathbf{V}_{e0}$ and $\mathbf{V}_{i0}$ coincide with the velocities produced by the effective external electric field $\mathbf{E}_{eff}$. The latter, however, is different for the electrons and the ions:

$$\mathbf{E}_{\mathrm{eff},e}=\mathbf{E}_0+\frac{1}{c}[\mathbf{U}_m\mathbf{H}]-\frac{mg}{e}-\frac{T_e}{2e^2N}\left(\frac{[\mathbf{H},\nabla\mathbf{H}]}{H^2}+\frac{[\mathbf{H},(\mathbf{H}\nabla)\mathbf{H}]}{H^3}\right),$$
$$\mathbf{E}_{\mathrm{eff},i}=\mathbf{E}_0+\frac{1}{c}[\mathbf{U}_m\mathbf{H}]+\frac{Mg}{e}+\frac{T_i}{2e^2N}\left(\frac{[\mathbf{H},\nabla\mathbf{H}]}{H^2}+\frac{[\mathbf{H},(\mathbf{H}\nabla)\mathbf{H}]}{H^3}\right). \quad (2.3)$$

The boundary conditions of (2.1) have the same form (1.7) as before.

If the inhomogeneity is only a perturbation of the main density of the homogeneous plasma $N_0$, i.e., if $N = N_0 + \delta N$, where $\delta N \ll N_0$, then Eq. (2.1) can be linearized and $\varphi$ eliminated. The linearized equation takes the form

$$(\nabla\hat{\sigma}_e\nabla+\nabla\hat{\sigma}_i\nabla)\frac{\partial\delta N}{dt}+\Big[(\nabla\hat{\sigma}_e\nabla)\frac{\partial}{\partial N_0}(N_0\mathbf{V}_{i0}\nabla)$$
$$+(\nabla\hat{\sigma}_i\nabla)\frac{\partial}{\partial N_0}(N_0\mathbf{V}_{e0}\nabla)\Big]\delta N-[(\nabla\hat{\sigma}_e\nabla)(\nabla\hat{D}_i\nabla)$$
$$+(\nabla\hat{\sigma}_i\nabla)(\nabla\hat{D}_e\nabla)]\delta N=0. \quad (2.4)$$

The tensors $\hat{\sigma}_e$, $\hat{\sigma}_i$, $\hat{D}_e$, and $\hat{D}_i$ and the velocities $\mathbf{V}_{e0}$ and $\mathbf{V}_{i0}$ depend here only on the density $N_0$ of the homogeneous plasma.

The solutions of the linearized equation (2.4) will be considered later on.† It is important to emphasize the limitations that follow from the very formulation of the linearized problem in a homogeneous plasma with a homogeneous magnetic field. Indeed, the drift velocities of the electrons and ions are not equal to each other. Consequently, a current of constant density flows in the plasma. This current produces its own proper magnetic field, which is inhomogeneous. It upsets the homogeneity of the plasma. It is therefore meaningful to consider only inhomogeneities whose dimensions are bounded by the condition

$$L<\frac{c}{V}R_D. \quad (2.5)$$

Here $R_D$ is the Debye radius, $V = |\mathbf{V}| = |\mathbf{V}_{i0} - \mathbf{V}_{e0}|$,

*The difference between the tensors $\hat{\sigma}_{He}$ or $\hat{\sigma}_{Hi}$ and $\hat{\sigma}_e$ or $\hat{\sigma}_i$ is connected with the fact that the effective force acting on a particle moving in an inhomogeneous magnetic field depends on its velocity ($F \sim V^2$). This affects only the form of the collision tensor or the tensor of the coefficients R.

†The motion and spreading of cylindrical inhomogeneities with strong density perturbation $\delta N \sim N_0$ (meteor trails in the ionosphere) were considered in a number of papers [72-77]. In these papers, however, the problem was not solved rigorously. Assumptions which were not always sufficiently corroborated were made, and the accuracy of the results is therefore not certain.

and L is the dimension of the inhomogeneity in the direction perpendicular to $\mathbf{V}$: $L = V\delta N/(\mathbf{V}\cdot\nabla\delta N)$. When the condition (2.5) is satisfied, the pressure of the proper magnetic field is smaller than the plasma pressure, so that the field cannot greatly upset the plasma inhomogeneity. We note that if the problem is considered in a homogeneous magnetic field $H_0$, then it is necessary to satisfy besides (2.5) also the condition

$$L < \frac{c}{V}\frac{H_0}{eN_0}, \qquad (2.6)$$

which follows from the fact that the magnitude of the inhomogeneous magnetic field of the current is limited by the requirement that this field be small compared with $H_0$. If the pressure of the external magnetic field $H_0^2/4\pi$ exceeds the plasma pressure $2N_0T$, only the condition (2.6) is important.

## 2.2. Solution of Equation of Motion

a) Velocity of ambipolar drift. The solution of the linear equation (2.4) in an unbounded plasma is obtained by expanding the sought function in a Fourier integral in terms of the coordinates. This yields:

$$\delta N(\mathbf{r}, t) = \frac{1}{(2\pi)^3}\int \delta N_{\mathbf{k}}(0)\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{V}_0(\beta)t) - D_a(\beta)k^2 t}\, d^3k, \qquad (2.7)$$

where $\delta N_k(0)$ are the Fourier components of the initial concentration perturbation, $D_a(\beta)$ is the coefficient of ambipolar diffusion (1.12), and $\mathbf{V}_a(\beta)$ is the velocity of the ambipolar drift:

$$\mathbf{V}_a = \frac{(\sigma_{\|e}\cos^2\beta + \sigma_{\perp e}\sin^2\beta)\frac{\partial}{\partial N_0}(N_0\mathbf{V}_{i0}) + (\sigma_{\|i}\cos^2\beta + \sigma_{\perp i}\sin^2\beta)\frac{\partial}{\partial N_0}(N_0\mathbf{V}_{e0})}{(\sigma_{\|e}+\sigma_{\|i})\cos^2\beta + (\sigma_{\perp e}+\sigma_{\perp i})\sin^2\beta}. \qquad (2.8)$$

This formula expresses the velocity of the quasineutral inhomogeneity (velocity of ambipolar drift) in terms of the drift velocities of the electrons and ions in the inhomogeneous plasma $\mathbf{V}_{e0}$ and $\mathbf{V}_{i0}$. It is very important that the drift velocity $\mathbf{V}_a$ turns out to depend on the angle $\beta$ between the wave vector $\mathbf{k}$ and the direction of the magnetic field $\mathbf{H}_0$. Consequently, the different Fourier-components of the inhomogeneity have different velocities. This means that when we consider an arbitrary inhomogeneity we can speak only of some mean value of its velocity, which depends on the Fourier spectrum of the inhomogeneity, i.e., on the shape of the inhomogeneity. In addition, the dispersion of the ambipolar drift velocity $\mathbf{V}_a(\beta)$ leads to a spreading of the inhomogeneities, in analogy with the spreading of wave packets in a dispersive medium.

Taking account of expression (2.2) for the drift velocities of the electrons and ions in a homogeneous plasma, we can rewrite (2.8) in the form

$$\mathbf{V}_a = \mathbf{U}_m + \frac{1}{e}\,\frac{(\sigma_{\|e}\cos^2\beta + \sigma_{\perp e}\sin^2\beta)\frac{\partial\hat{\sigma}_i}{\partial N}\mathbf{E}_{\text{eff},i} - (\sigma_{\|i}\cos^2\beta + \sigma_{\perp e}\sin^2\beta)\frac{\partial\hat{\sigma}_e}{\partial N_0}\mathbf{E}_{\text{eff},e}}{(\sigma_{e\|}+\sigma_{i\|})\cos^2\beta + (\sigma_{e\perp}+\sigma_{i\perp})\sin^2\beta}. \qquad (2.9)$$

Here $\mathbf{E}_{\text{eff},i}$ and $\mathbf{E}_{\text{eff},e}$ are the effective electric fields for the electrons and ions, determined by formulas (2.3). In a weakly ionized plasma the conductivity is proportional to $N_0$, and consequently $\partial\hat{\sigma}/\partial N_0 = \hat{\sigma}/N_0$. At an arbitrary degree of ionization, the components of the tensors $\partial\hat{\sigma}_e/\partial N_0$ and $\partial\hat{\sigma}_i/\partial N_0$ have the following form in the elementary-theory approximation:

$$\frac{\partial\sigma_{\|i}}{\partial N_0} = \frac{e^2\nu_{em}^2}{M_i\nu_{im}(\nu_{em}+\nu_{ei})^2}, \quad \frac{\partial\sigma_{\|e}}{\partial N_0} = \frac{e^2\nu_{em}}{m(\nu_{em}+\nu_{ei})^2},$$

$$\frac{\partial\sigma_{\perp i}}{\partial N_0} = \frac{e^2\nu_{em}^2}{M_i\nu_{im}A^2}\left\{(\nu_{em}+\nu_{ei})^2 + \omega_H^2\left(2 + 2\frac{\nu_{ei}}{\nu_{em}} - \frac{\nu_{ei}^2}{\nu_{em}^2}\right) + \frac{\omega_H^4}{\nu_{em}^2}\left(1 + \frac{\Omega_H^2}{\nu_{im}^2} + 2\frac{\nu_{ei}\nu_{em}}{\nu_{im}^2}\left(\frac{m}{M_i}\right)^2\right)\right\},$$

$$\frac{\partial\sigma_{\perp e}}{\partial N_0} = \frac{e^2\nu_{em}}{mA^2}\left\{(\nu_{em}+\nu_{ei})^2 + \omega_H^2\left(1 + 2\frac{\nu_{ei}}{\nu_{em}} + 2\frac{m\nu_{ei}^2}{M_i\nu_{em}\nu_{im}}\right) + \omega_H^2\frac{\Omega_H^2}{\nu_{im}^2}\left(2 + 2\frac{\nu_{ei}}{\nu_{em}} + \frac{\Omega_H^2}{\nu_{im}^2}\right)\right\},$$

$$\frac{\partial\sigma_{Ai}}{\partial N_0} = \frac{e^2\omega_H\nu_{em}}{M_i\nu_{im}A^2}\left\{(\nu_{em}+\nu_{ei})\left(2\nu_{ei} + \frac{m\nu_{em}^2}{M_i\nu_{im}}\right) + \omega_H\nu_{em}\frac{\Omega_H}{\nu_{im}}\left[2 + \left(\frac{\nu_{ei}}{\nu_{em}}\right)^2 + \frac{2\nu_{ei}M_i\nu_{im}}{m\nu_{em}^2}\right] + \frac{\omega_H^3}{\nu_{em}}\frac{\Omega_H}{\nu_{im}}\left[1 + \left(\frac{\Omega_H}{\nu_{im}}\right)^2 + 2\frac{m\nu_{ei}}{M_i\nu_{im}}\right]\right\},$$

$$\frac{\partial\sigma_{Ae}}{\partial N_0} = -\frac{e^2\omega_H}{mA^2}\left\{\nu_{em}^2 - \nu_{ei}^2 + \omega_H^2\left[1 + \frac{m\nu_{ei}}{M_i\nu_{im}}\left(2 + \frac{m\nu_{ei}}{M_i\nu_{im}}\right)\right] + \omega_H^2\frac{\Omega_H^2}{\nu_{im}^2}\left(2 + 2\frac{m\nu_{ei}}{M_i\nu_{im}} + \frac{\Omega_H^2}{\nu_{im}^2}\right)\right\},$$

$$A = (\nu_{em}+\nu_{ei})^2 + \omega_H^2\left(1 + 2\frac{m\nu_{ei}}{M_i\nu_{im}} + \frac{\Omega_H^2}{\nu_{im}^2}\right). \qquad (2.10)$$

Expressions (2.9), (2.10), and (2.3) solve the problem of the velocity of a quasineutral inhomogeneity in a plasma, i.e., of the velocity of ambipolar drift.

We note here that in the absence of a magnetic field the drift in the plasma can give rise to an external electric field and to motion of the neutrals. From (2.9) it follows that in this case we have

$$\mathbf{V}_a = \mathbf{U}_m + \frac{\left(\sigma_e\frac{\partial\sigma_i}{\partial N_0} - \sigma_i\frac{\partial\sigma_e}{\partial N_0}\right)}{e(\sigma_e+\sigma_i)}\mathbf{E}_0.$$

Taking the relation $\sigma_e(\partial\sigma_i/\partial N_0) = \sigma_i(\partial\sigma_e/\partial N_0)$ into account, we find that the velocity of the ambipolar drift in the absence of a magnetic field is equal to the velocity of the neutral gas. It does not depend on the external electric field. This is understandable: a quasineutral inhomogeneity with $\delta N_e = \delta N_i$ becomes polarized in a plasma situated in an external electric field E, but does not move, since the force

$\mathbf{F} = e\mathbf{E}(\delta N_e - \delta N_i)$ acting on it is equal to zero (accurate to small terms of order $(kR_D)^2$).

b) Plasma in an electric field. For the ambipolar-drift velocity in a weakly ionized plasma situated in an external field $E_0$ in a magnetic field $H_0$, we get from (2.9)[78]

$$\mathbf{V}_a = \frac{e}{M_i \nu_{im} A}\left\{\frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\mathbf{E}_0 + \frac{\omega_H}{\nu_{em}}\left(1 + \frac{\omega_H}{\nu_{em}}\frac{\Omega_H}{\nu_{im}}\cos^2\beta\right)\right.$$

$$\left.\times\frac{[\mathbf{E}_0\mathbf{H}]}{H} - \frac{\omega_H^2}{\nu_{em}^2}\frac{\mathbf{H}(\mathbf{E}_0\mathbf{H})}{H^2}\right\},$$

$$A = 1 + \frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\left(1 + \frac{\Omega_H^2}{\nu_{im}^2}\right) + \frac{\omega_H \Omega_H}{\nu_{em}\nu_{im}}. \qquad (2.11)$$

In the absence of the magnetic field, the velocity $\mathbf{V}_a$ vanishes. The dependence of the drift velocity on $\cos\beta$ for different values of $\omega_H$ is shown in Fig. 6. The ordinates represent the following components of the drift velocity $\mathbf{V}_a$ in units of $(eE\cos\alpha)/(M_i\nu_{im})$: $V_1$ - the component in the magnetic-field direction, $V_2$ - the Hall component (in the $\mathbf{E}\times\mathbf{H}$ direction), $V_3$ - the component in the $\mathbf{H}\times[\mathbf{E}\times\mathbf{H}]$ direction; $\cos\alpha = (\mathbf{H}\cdot\mathbf{E})/HE = 1/\sqrt{2}$, $m\nu_{em}/M_i\nu_{im} = 0.01$ (all the subsequent figures are plotted in terms of the same units and at the same values of $\cos\alpha$ and $m\nu_{em}/M_i\nu_{im}$). With increasing H and when $\omega_H < \nu_{em}$, the first to grow is the Hall component of the velocity (proportional to $\mathbf{E}\times\mathbf{H}$). This is seen from Fig. 6a, which shows a plot of the velocity $\mathbf{V}_a$ for $\omega_H = 0.5\nu_{em}$. When $\omega_H \sim \nu_{em}$ the region of angles $\beta$ close to $\pi/2$ (i.e., $\mathbf{k}\perp\mathbf{H}$) is distinct. The component of the velocity $\mathbf{V}_a$ parallel to H increases rapidly when $\cos\beta \to 0$, and reaches the value of the electron drift velocity $eH/m\nu_{em}$ when $\Omega_H \sim \nu_{im}$. This is seen from Fig. 6c ($\omega_H = 50\nu_{em}$). In a strong magnetic field the velocity $\mathbf{V}_a$ decreases with increasing ratio $\Omega_H/\nu_{im}$ (Fig. 6d) ($\omega_H = 100\nu_{em}$). However, as before, the component of $\mathbf{V}_a$ parallel to H increases rapidly as $\cos\beta \to 0$.

The normal component of the drift velocity, $V_n = (\mathbf{k}\cdot\mathbf{V}_0)/k$ was determined in a paper by Clemmow and Johnson[79]. It depends only on the angle $\beta$ between the direction of the magnetic field and the wave vector k, but also on another angle, which characterizes the direction of the vector k. The velocity $V_n$ as a function of $\cos\beta$ is shown for $\mathbf{k}\parallel[\mathbf{E}\times\mathbf{H}]$ in Fig. 7a and for $\mathbf{k}\perp[\mathbf{E}\times\mathbf{H}]$ in Fig. 7b. Curves 1, 2, and 3 are for $\omega_H = \nu_{em}$, $50\nu_{em}$, and $100\nu_{em}$, respectively. It is seen from Fig. 7 that the normal component of the drift velocity also changes strongly with changing angle $\beta$. In a strong magnetic field the normal component of the drift velocity increases sharply at angles $\beta$ close to $\pi/2$.

In the case of crossed fields, $\mathbf{E}\perp\mathbf{H}$, the component of $\mathbf{V}_a$ parallel to H which is the most strongly dependent on the angle $\beta$, vanishes. This case is particularly interesting, since in other drift mechanisms the effective electric field, as is clear from (2.3) is usually perpendicular to the magnetic field. The velocity $\mathbf{V}_a$ then lies in a plane perpendicular to

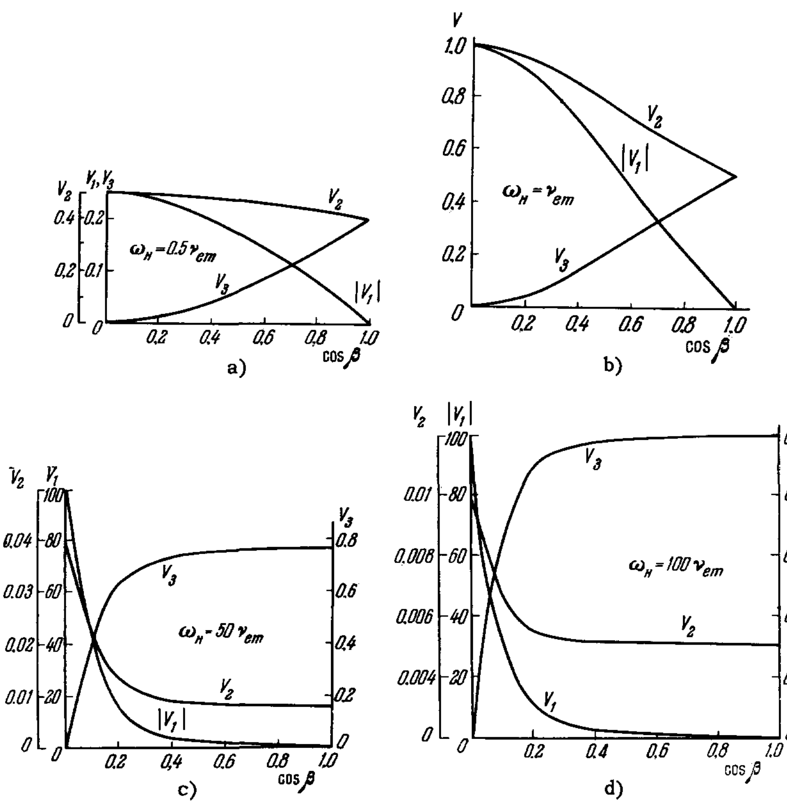


FIG. 6.

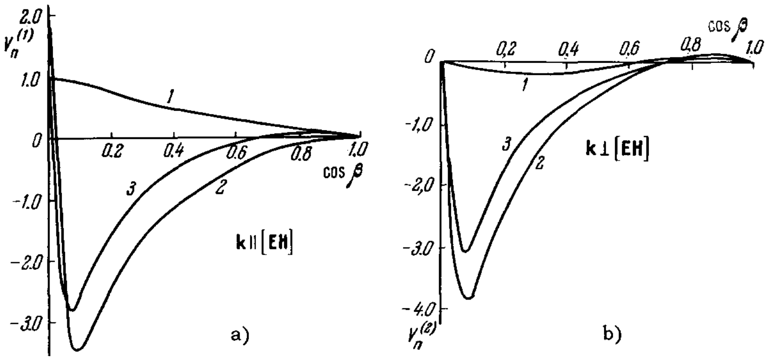


FIG. 7.

H. The hodograph of the velocity $V_a$ is shown in Fig. 8 as a function of the angle $\beta$ in the case when $E \perp H$ ($\omega_H = \nu_{em}$). The drift velocity is seen to have appreciable dispersion.

The dispersion of the ambipolar-drift velocity leads, first, to an influence of the shape of the inhomogeneity on its mean velocity. Indeed, if the inhomogeneity is strongly elongated in the direction of the magnetic field, then the main contribution to its Fourier spectrum is made by the region of angles $\beta$ close to $\pi/2$. In this case the inhomogeneity can move at the velocity of the electron drift. Inhomogeneities that are not strongly elongated in the direction of the magnetic field have velocities that are only of the same order as the ion drift. The dispersion of the drift velocity leads also to dispersion spreading (attenuation) of the inhomogeneities. It will be considered in the next section.

In a strongly ionized plasma in crossed fields $E \perp H$, under conditions $\Omega_H \gg \nu_{im}$ and $\nu_{ei} \gg \nu_{em}$ (these conditions obtain in the ionosphere at altitudes exceeding 200—300 km), the dispersion is small and the inhomogeneity moves at a velocity [80-82]

$$\mathbf{V}_a = c\,[\mathbf{EH}]/H^2. \tag{2.12}$$

This is connected with the fact that under the indicated conditions the electron and ion drift velocities are approximately equal. The components of the drift velocity in the directions of H and of $H \times [E \times H]$ are in this case small; for these components, however, a considerable dispersion exists as before.

### 2.3. Spreading and Shape of Moving Inhomogeneities

a) Green's function. Dispersion mechanism of spreading. The spreading of inhomogeneities, which had at the initial instant of time very small dimensions (points) is described in the presence of drift by formulas (1.26) and (1.27), where

$$B = \mu\gamma + \sqrt{(1-\mu^2)(1-\gamma^2)}\cos\varphi,\ \ \gamma = \boldsymbol{\rho}\mathbf{H}/\rho H,\ \ \boldsymbol{\rho} = \mathbf{r} - \mathbf{V}_a(\mu^2)\,t. \tag{2.13}$$

Here $\mu = \cos\beta = \mathbf{k}\cdot\mathbf{H}/kH$ and $\varphi$ is the angle between the **rH** and **kH** planes.

The maximum of the function G is attained at $\rho = 0$. In diffusion without drift, this condition determines one maximal point $\mathbf{r} = 0$; in the case of drift with only one velocity $\mathbf{V}_0$ the condition defines only one point $\mathbf{r} = \mathbf{V}_0 t$. In our case, owing to the dispersion of the drift velocity, $V_a = V_a(\mu^2)$, the condition $\rho = 0$ at each instant of time t is satisfied not for one point, but for a one-dimensional set of points - a curve defined by the relation

$$\mathbf{r} = \mathbf{V}_a(\mu_0^2)\,t, \tag{2.14}$$

where $\mu_0$ runs through all values from 0 to 1. With increasing time, the locus of the maxima (2.14) moves in space, becomes elongated, and conserves similarity. This is seen from Fig. 9, where the loci

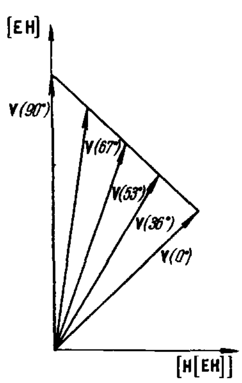


FIG. 8.

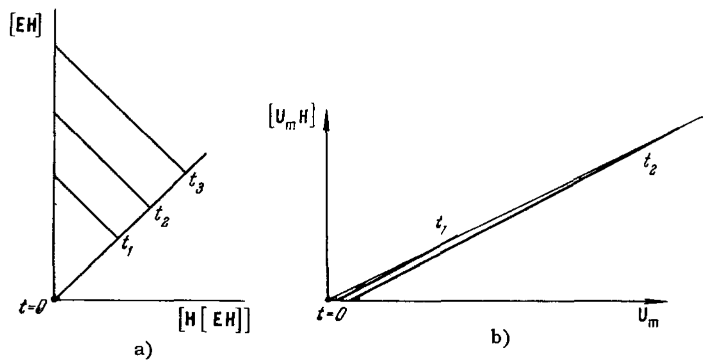


FIG. 9.

of the maxima (2.14) were plotted in a plane perpendicular to H for instants of time $t_1 < t_2 < t_3$.*

The length of the locus of the maxima increases linearly with time. The maximum value of the Green's function should decrease accordingly. Consequently, the dispersion of the drift velocity leads to a spreading of the inhomogeneities. This process of dispersion spreading is faster than the diffusion spreading, but proceeds only along the locus of the maxima. In directions perpendicular to this locus, the inhomogeneity spreads out as a result of diffusion. Simultaneous action of the dispersion and diffusion mechanisms should cause the particle density in the moving inhomogeneities to decrease in time more rapidly than in stationary inhomogeneities, when only diffusion is important.

The asymptotic values of the function $G(r, t)$ at points lying on the locus of the maxima (2.14), i.e., for r and t satisfying the relation $\mathbf{r} = \mathbf{V}_a(\mu_0^2)t$, can be calculated by the saddle-point method [66]. It turns out here that the principal maximum of the function G is determined by the condition $\mathbf{r} = \mathbf{V}_a(0)t$. The value of the Green's function at this maximum is

$$G(\mathbf{r}=\mathbf{V}_a(0)\,t) = 0.641\pi^{-3}D_a^{-5/4}(0)\,[1-\gamma^2(0)]^{-1/4} \times\left[\frac{d}{d\mu^2}\left|\mathbf{V}_a(\mu^2)-\mathbf{V}_a(0)\right|\right]_{\mu=0}^{-1/2} t^{-7/4}. \quad (2.15)$$

Thus, in the principal maximum the density decreases with time like $1/t^{7/4}$. This maximum moves with a velocity $\mathbf{V}_a(0)$. In other words, the velocity of the principal maximum of the inhomogeneity is determined by formulas (2.8) or (2.9) with $\cos^2\beta = 0$.

The second maximum moves with a velocity $V(\mu^2)$, where $\mu_{01}^2$ satisfies the condition $\mu_{01}^2 + \gamma(\mu_{01}^2) = 1$. The Green's function at this maximum decreases like $t^{-15/8}$. When $\gamma = 0$ the second maximum moves with a velocity $\mathbf{V}_a(1)$. The Green's function decreases in this case like $t^{-11/6}$.

In all other directions of the locus of the maxima (2.14), the Green's function decreases in proportion to $1/t^2$. Thus, the concentration of the particles in moving inhomogeneities decreases with time more rapidly than in stationary ones when $G_{max} \sim 1/t^{3/2}$. It is this increase in the rate of spreading which manifests the dispersion mechanism of inhomogeneity spreading.

b) Shape and lifetime of the inhomogeneities. As indicated above, the asymptotic expression for the Green's function has two maxima if drift is present in the plasma. Accordingly, during the course of its motion the inhomogeneity should "split" it were into two parts moving with essentially different velocities $\mathbf{V}_a(0)$ and $\mathbf{V}_a(\mu_{01})$. In addition, the inhomogeneity becomes elongated in the direction of the locus of the maxima (2.14), which joins both indicated maxima. The direction in which the inhomogeneity becomes elongated does not coincide in general with the direction of the magnetic field.

Figures 10a and 10b show equal-concentration curves for two instants of time in a moving inhomogeneity [83]. This pertains to a weakly ionized plasma, and the drift is due to motion of the neutral gas with velocity $U_m$ in a direction perpendicular to H. In this case the velocity of the ambipolar drift is

$$\mathbf{V}_a = \frac{\left(1+\frac{\omega_H^2}{\nu_{em}^2}\cos^2\beta\right)\mathbf{U}_m + \frac{\Omega_H\omega_H^2}{\nu_{im}\nu_{em}^2}\cos^2\beta\left[\mathbf{U}_m\frac{\mathbf{H}}{H}\right]}{1+\frac{\Omega_H}{\nu_{im}}\frac{\omega_H}{\nu_{em}}+\frac{\omega_H^2}{\nu_{em}^2}\left(1+\frac{\Omega_H^2}{\nu_{im}^2}\right)\cos^2\beta}. \quad (2.16)$$

It was assumed in the numerical calculation pre-

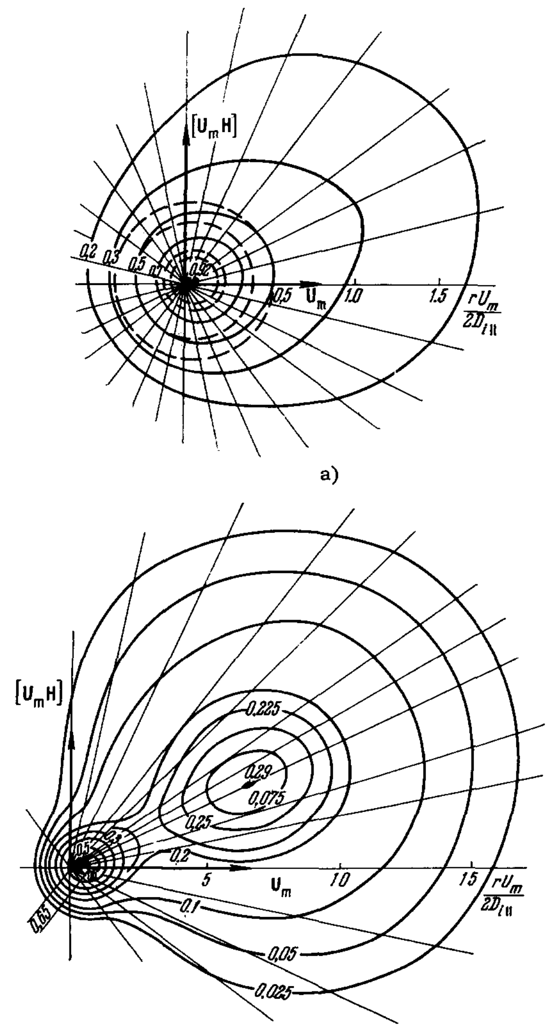


FIG. 10.

*The curves in Fig. 9a were plotted for the case when the drift in the plasma is produced by an external electric field perpendicular to H (the conditions are the same as for Fig. 8.). In Fig. 9b, the drift is produced by the wind of neutral particles moving with a velocity $U_m$ perpendicular to H ($\omega_H = 50\,\nu_{em}$). In both cases, the locus of the maxima is nearly straight and lies in a plane perpendicular to H. In the general case this is a three-dimensional curve.

sented in Fig. 10 that $\omega_H/\nu_{em} = 50$ and $\omega_H/\nu_{im} = 0.5$. Figure 10 shows curves of constant values of the ratio $G(r, t)/G_g(0, t)$ in a plane perpendicular to H ($G(r, t)$ = Green's function for the moving inhomogeneity, $G_g(0, t)$ = Green's function at the maximum of the inhomogeneity if the latter were at rest and its spreading were determined only by the diffusion as per (1.28)). The curves of Fig. 10a are plotted for the instant $t = 2D_{i\parallel}/U_m^2$, and those in Fig. 10b for $t = 20D_{i\parallel}/U_m^2$. When $t < 2D_{i\parallel}/U_m^2$ the shape of the inhomogeneity is determined by diffusion spreading. It is seen from Fig. 10a that when $t = 2D_{i\parallel}/U_m^2$ the influence of the drift on the shape of the inhomogeneity is already quite pronounced (the curves determined by diffusion alone are shown dashed in the figure). When $t \ll 2D_{i\parallel}/U_m^2$ the shape of the inhomogeneity is determined completely by the drift. The inhomogeneity splits in two - two maxima appear. This is seen from Fig. 10b. The principal maximum of the inhomogeneity moves in this case, according to (2.15) and (2.16), in the direction of $U_m$ at a low velocity

$$V_a(0) = U_m \Big/ \left[1 + \frac{\Omega_H \omega_H}{\nu_{im}\nu_{em}}\right] = 0.0385 U_m. \qquad (2.17)$$

The value of the ratio $G(r, t)/G_g(0, t)$ in the principal maximum, at $t = 20D_{i\parallel}/U_m^2$, is already much smaller than unity; this decrease is the result of the dispersion mechanism of spreading. The second maximum, according to the asymptotic formulas, moves in a direction which does not coincide with the direction of motion of the principal maximum:

$$V_a(1) = \frac{1 + \omega_H^2/\nu_{em}^2}{1 + \frac{\omega_H^2}{\nu_{em}^2}\left(1 + \frac{\Omega_H^2}{\nu_{im}^2}\right) + \frac{\omega_H \Omega_H}{\nu_{em}\nu_{im}}} U_m + \frac{\Omega_H \omega_H^2/\nu_{im}\nu_{em}^2}{1 + \frac{\omega_H^2}{\nu_{em}^2}\left(1 + \frac{\omega_H^2}{\nu_{im}^2}\right) + \frac{\Omega_H \omega_H}{\nu_{im}\nu_{em}}} \left[U_m \frac{H}{H}\right]$$

$$= 0.794 U_m + 0.397 \left[U_m \frac{H}{H}\right]. \qquad (2.18)$$

The velocity of the second maximum is larger by more than 20 times than the velocity of the principal maximum.

The values of the ratio $G(r, t)/G_g(0, t)$ for the principal maximum ($G_1$), the second maximum ($G_2$), and the saddle point ($G_3$) are shown in Table II for different instants of time t. It is seen from the table that the concentration of the particles in the second maximum decreases for some time more slowly than in the first maximum. The relative height of the second maximum therefore increases and approaches that of the first maximum. An apparent transfer of particles from the first to the second maximum takes place. The value of the function G decreases much more rapidly at the saddle point between the maxima than at the two maxima. This shows that the splitting of the moving inhomogeneity into two parts becomes more intense in the course of time. The first of these two inhomogeneities is strongly elongated along the magnetic field. It moves at a velocity $V_a(0)$. The second, which is elongated in a plane perpendicular to H, moves with velocity $V_a(1)$.

Table II

| $tU_m^2/D_{i\parallel}$ | $G_1$ | $G_2$ | $G_3$ | $tU_m^2/D_{i\parallel}$ | $G_1$ | $G_2$ | $G_3$ |
|---|---|---|---|---|---|---|---|
| 0 | 1 | — | — | 50 | 0.54 | 0.27 | 0.098 |
| 2 | 0.92 | — | — | 200 | 0.44 | 0.25 | 0.045 |
| 20 | 0.65 | 0.29 | 0.22 | 800 | 0.34 | 0.23 | 0.020 |

It is natural to define the lifetime t of the moving inhomogeneity, as before, as the time during which the particle density at the maximum of the inhomogeneity decreases by a specified number of times (p times). For the case considered here this is shown in Fig. 11. The ordinates represent the ratio $t/t_0$, where $t_0$ is the lifetime (1.37) of the inhomogeneity in the plasma in the absence of a magnetic field; the abscissas are the dimensionless quantities

$$x = \frac{U_m}{D_{i\parallel}}\left[\frac{n_0 p}{\delta N(0,0)}\right]^{1/3}.$$

The quantity x is proportional to $U_m R_0/D_{i\parallel}$, where $R_0 \sim [n_0/\delta N(0, 0)]^{1/3}$ is the characteristic dimension of the initial inhomogeneity. It is seen from the figure that when $x \lesssim 1$ the motion has practically no effect on the lifetime of the inhomogeneity; conversely, when $x \gg 1$ the lifetime decreases markedly.

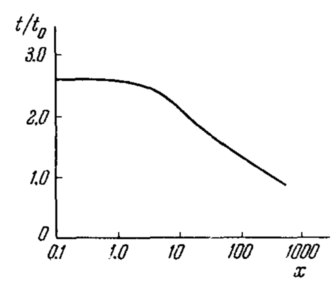


FIG. 11.

It is easy to verify that in the general case, if the condition

$$\frac{p^{1/3} R_0 \,|\, V_a(1) - V_a(0)\,|}{D_{i\parallel}} \ll 1, \qquad (2.19)$$

is satisfied, then the principal role is played by the diffusion spreading, and the lifetime of the inhomogeneity is determined by formula (1.36) and (1.37). If a condition inverse to (2.19) is satisfied, the drift spreading exerts a strong influence. It decreases noticeably the lifetime of the inhomogeneity.

Let us dwell in conclusion on the conditions limiting the applicability of Eq. (2.1), which describes ambipolar motion and spreading of electron-density and ion-density inhomogeneities in a plasma. Equation (2.1) was derived neglecting the influence of the solenoidal electric field and the perturbation of the molecule motion. In addition, in the presence of drift, corrections must be introduced in the expressions used in the derivation of (2.1), those for the electron and ion fluxes (0.3) and (0.4). These corrections are connected with the influence of the inertial ($\partial V/\partial t$) and nonlinear ($(V \cdot \nabla)V$) terms in the equations for the macroscopic velocities of the ions and electrons. The influence of the latter factors is explained in [43,84], where it is shown that their role is negligible if

$$V_a^2 \ll V_T^2, \tag{2.20}$$

where $V_T$ is the thermal velocity of the ions. It can be assumed that the role of the perturbations of the molecule motion is also negligible when the condition (2.20) is satisfied (here $V_T$ is the thermal velocity of the molecules), supplemented by the condition (1.41).

As already indicated above, in the presence of drift a solenoidal electric field upsets the general homogeneity of the plasma, thus limiting the dimensions of the inhomogeneities (2.5) and (2.6) considered here. In addition, when account is taken of the solenoidal electric field, as indicated above (Sec. 1.4b), three types of diffusion processes come into play. However, when condition (1.44) is satisfied, the associated corrections to the ambipolar diffusion and to the drift are apparently small.

Thus, to be able to use Eq. (2.1) to describe the motion and spreading of inhomogeneities it is necessary to satisfy the conditions (1.41), (1.44), (2.5), (2.6), and (2.20). In addition, it must be emphasized once more that the entire analysis has been carried out here only under the assumption that the plasma as a whole is stable.

## CONCLUDING REMARKS

For further investigations of the motion and spreading of macroscopic inhomogeneous formations in a plasma, it is expedient to note the following. The question of the behavior of inhomogeneous perturbations in a laminar unbounded plasma can be regarded as generally clarified. The purpose of further work would be essentially to solve the inherently nonlinear problems (when the initial particle density in the inhomogeneity exceeds by many times the density of the homogeneous plasma), and also to take into considerations the limits and, of course, to solve analogous problems for an unstable (turbulent) plasma. It is desirable to investigate in detail the influence of various microprocesses in the plasma on the structure and character of spreading of the inhomogeneities.

Further progress in the study of the phenomena under consideration is hindered, however, by the lack of clear and sufficiently complete experimental data. It is necessary to investigate experimentally, first, the influence of the form of the inhomogeneity on its velocity (2.2a) on the speed of spreading (Sec. 1.2a). It would be of interest to observe the effect of the splitting of the moving inhomogeneities (sec. 2.3b), and to separate their dispersion damping (Sec. 2.3a). The dependence of the velocity of ambipolar drift on the angle between k and H (sec. 2.2a) can apparently be investigated experimentally with the aid of radio waves directed at different angles to the magnetic field and scattered by the inhomogeneity. The form of the moving inhomogeneities can probably be studied by adding luminous or absorbing ionized impurities.* An extensive experimental investigation of different singularities of motion and spreading of inhomogeneities in a plasma is essential.

*It is possible that the complex shapes of the artificial inhomogeneities observed in the ionosphere [12,13] are connected with the effects considered here. The shape of the inhomogeneities observed by Biermann [93] agrees with the theory (see Sec. 1.3c).


[1] W. Schottky, Phys. Zs. 25, 635 (1924).

[2] V. C. A. Ferraro, Terr. Magn. and Atm. Elect. 50, 213 (1945); 51, 427 (1946).

[3] M. H. Johnson, E. O. Hulbert, Phys. Rev. 79, 802 (1950).

[4] E. R. Schmerling, Nature 188, 133 (1960).

[5] H. Rishbeth, Nature 193, 56 (1961).

[6] J. A. Lyon, Nature 193, 55 (1961).

[7] P. C. Kendall, J. Atm. Terr. Phys. 24, 85 (1962).

[8] S. Chandra, J. Atm. Terr. Phys. 26, 113 (1964).

[9] I. E. C. Gliddon, Planet. Space Sci. 13, 959 (1965).

[10] F. F. Marmo, I. Pressman, Planet. Space Sci. 2, 174 (1960).

[11] I. Y. Rees, Planet. Space Sci. 3, 35 (1961).

[12] N. W. Rosenberg, J. Geoph. Res. 68, 3057 (1963).

[13] S. P. Zimmerman, K. S. Champion. J. of Geoph. Res. 68, 3049 (1963).

[14] N. P. Mar'in, Geomagnetizm i aêronomiya, 4, 647 (1963).

[15] B. N. Gershman, ibid. 3, 878 (1963).

[16] B. N. Gershman and G. I. Grigor'ev, ibid. 5, 843 (1965).

[17] S. Kato, Space Sci. Rev. 4, 223 (1965).

[18] M. Dagg, J. Atm. and Terr. Phys. 10, 194 (1957); 11, 139 (1957).

[19] F. Villars, H. Feshbach, J. Geoph. Res. 68, 1303 (1963).

[20] E. A. Novikov, Izv. AN SSSR ser. geofiz. 11, 1624 (1960).

[21] V. P. Dokuchaev, Radiofizika (Izv. vuzov) 3, 50 (1960).

[22] Ya. L. Al'pert, A. V. Gurevich, and L. P. Pitaevskiĭ, Iskusstvennye sputniki v razrezhennoĭ plazme (Artificial Satellites in a Rarefied Plasma) Nauka, 1964.

[23] C. D. Wotkins, Nature **206**, 1027 (1965).
[24] T. R. Kaiser, Phylos. Mag. Suppl. 2, 499 (1953).
[25] J. S. Guenhaw, E. L. Neufeld, J. Atm. Terr. Phys. **6**, 133 (1955); **3**, 780 (1963).
[26] A. V. Gurevich and L. P. Pitaevskiĭ, Geomagnetizm i aéronomiya **3**, 823 (1963); **6**, 842 (1966).
[27] Yu. K. Kalinin, ibid. **4**, 124 (1964).
[28] E. I. Fialko, ibid. **5**, 97 (1965).
[29] B. B. Kadomtsev, ZhTF **31**, 1273 (1961), Soviet Phys. Tech. Phys. **6**, 927 (1962).
[30] A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, UFN **73**, 701 (1961), Soviet Phys. Uspekhi **4**, 332 (1961).
[31] A. A. Vedenov, in: Voprosy teorii plazmy (Problems in Plasma Theory), No. 4, 188 (1964), Gosatomizdat.
[32] A. A. Vedenov, Teoriya turbulentnoĭ plazmy (Theory of Turbulent Plasma), VINITI, 1965.
[33] V. E. Golant, UFN **79**, 377 (1963), Soviet Phys. Uspekhi **6**, 161 (1963).
[34] A. Simon, An Introduction to Thermonuclear Research, London, Pergamon Press, 1959.
[35] F. C. Hoh, Rev. Mod. Phys. **34**, 267 (1962).
[36] W. P. Allis, Handbuch d. Physik, Springer-Verlag, Berlin, v. 21, 1956.
[37] O. Cote, Turbulent diffusion of Sodium Vapor Tails in the Upper Atmosfere, NASA Report CR-301, 1965.
[38] V. L. Ginzburg and A. V. Gurevich, UFN **70**, 201 (1960), Soviet Phys. Uspekhi **3**, 115 (1960).
[39] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Fizmatgiz, 1960).
[40] J. P. Shkarofsky, Canad. J. Phys. **39**, 1619 (1961).
[41] S. Chapman and T. G. Cowling, The Mathematical Theory of Nonuniform Gases, Cambridge, 1939.
[42] S. I. Braginskiĭ, in: op. cit.[31], No. 1, 183 (1963).
[43] E. E. Tsedilina, Candidate's dissertation, Moscow, 1966.
[44] R. Landshoff, Phys. Rev. **76**, 904 (1949); **82**, 442 (1951).
[45] A. R. Hochstim, Convergent transport coefficients in plasma (preprint), 1965.
[46] M. V. Samokhin, ZhTF **33**, 667 and 675 (1963), Soviet Phys. Tech. Phys. 8, 498 and 504 (1963).
[47] A. V. Gurevich, JETP **44**, 1302 (1963), Soviet Phys. JETP **17**, 878 (1963).
[48] A. V. Gurevich and E. E. Tsedilina, Geomagnetizm i aéronomiya **5**, 251 (1965).
[49] V. L. Granovskiĭ, Elektricheskiĭ tok v gaze (Electric Current in Gas), Gostekhizdat, 1952, p. 310.
[50] N. A. Kaptsov, Élektricheskie yavleniya v gazakh i v vakuume (Electric Phenomena in Gases and in Vacuum), Gostekhizdat, 1950.
[51] B. N. Gershman, Radiotekhnika i élektronika 1, 720 (1956).
[52] R. W. Friedrichs, F. Mastrup, Phys. Fluids **6**, 36 (1963).
[53] L. H. Holway, Phys. Fluids 8, 1207 (1965).
[54] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Mechanics of Continuous Media), Gostekhizdat, 1953.
[55] V. E. Golant, O. B. Danilov, and A. P. Zhilinskiĭ, ZhTF **33**, 1043 (1963), Soviet Phys. Tech. Phys. **8**, 728 (1963).
[56] A. S. Syrgii and V. L. Granovskiĭ, Radiotekhnika i élektronika **5**, 1522 (1960).
[57] A. I. Morozov and A. S. Solov'ev, in op. cit[31], No. 2, 177 (1963).
[58] E. I. Ginzburg, Radiofizika (Izv. vuzov) **8**, 626 (1965).
[59] L. H. Holway, J. of Geophys. Res. **70**, 3635 (1965).
[60] M. A. Biondi, S. C. Brown, Phys. Rev. **75**, 1700 (1949).
[61] A. Simon, Phys. Rev. **98**, 317 (1955).
[62] I. S. Townsend, Philos. Mag. **25**, 459 (1938).
[63] G. I. Grigor'ev, Geomagnetizm i aéronomiya **4**, 183 (1964).
[64] V. E. Golant, ZhTF 30, 881 (1960), Soviet Phys. Tech. Phys. 5, 831 (1961).
[65] L. Spitzer, Physics of Fully Ionized Gases, Interscience, 1956.
[66] A. V. Gurevich and E. E. Tsedilina, Geomagnetizm i aéronomiya **6**, 255 (1966).
[67] P. Aigrain, Proc. Int. Conf. on Semiconductor Physics, Prague, 1960, p. 224.
[68] R. Bowers, G. Legendy, F. Rose, Phys. Rev. Letts **7**, 339 (1961).
[69] J. A. Lehane, P. C. Thonemann, Proc. Phys. Soc. 85, 301 (1965).
[70] I. R. Klozenberg, B. McNamara, P. C. Thonemann, J. of Fluid Mech. **21**, 543 (1965).
[71] D. V. Sivukhin, in op cit.[31], no. 1, 7 (1963).
[72] D. F. Martyn, Phil. Trans. Roy. Soc. **A246**, 306 (1953).
[73] K. Weeks, J. Atm. Terr. Phys., Special Suppl. Proc. Polar, Atmospher. Simposium II, p. 12, 1957.
[74] S. Kato, Planet. Space Sci. **11**, 823, 1297 (1963); **12**, 1 (1964).
[75] P. C. Clemmow, M. A. Johnson, K. Weeks, Rep. Conf. on Phys. of Ionosphere, London, Phys. Soc. **136**, 1955.
[76] W. G. Baker, D. F. Martyn, Phil. Trans. Roy. Soc. **A246**, 281 (1953).
[77] V. P. Dokuchaev, Radiofizika (Izv. vuzov) 1, 34 (1958).
[78] E. E. Tsedilina, Geomagnetizm i aéronomiya **5**, 679 (1965).
[79] P. C. Clemmow, M. A. Johnson, J. Atm. Terr. Phys. **16**, 21 (1959).
[80] B. N. Gershman and V. L. Ginzburg, Radiofizika (Izv. vuzov) 2, 8 (1959).
[81] I. P. Dougherty, J. Geophys. Res. **64**, 2215 (1959).
[82] D. F. Farley, J. Geophys. Res. **68**, 6083 (1963).
[83] A. V. Gurevich and E. E. Tsedilina, Geomagnetizm i aéronomiya **7** (1967).

[84] V. Fiala, ibid. 6, 597 (1966).
[85] V. L. Granovskiĭ, Radiotekhnika i élektronika 11, 371 (1966).
[86] J. P. Shkarofsky, T. W. Johnson, M. P. Bachynski, Particle Kinetics in Plasma, Eddison Wesey, Reeding-Mass. 1966.
[87] E. Wasserstrom, C. H. Su, R. F. Probstein, Phys. Fluids 8, 56 (1965).
[88] G. M. Sessler, Phys. Fluids 7, 90 (1964).
[89] U. Ingard, K. W. Gentle, Phys. Fluids 8, 1396 (1965).
[90] L. R. Storey, Phil. Trans. Roy. Soc. A246, 113 (1953).
[91] B. N. Gershman and V. A. Ugarov, UFN 72, 235 (1960), Soviet Phys. Uspekhi 3, 743 (1961).
[92] B. N. Gershman, UFN 89, 201 (1966), Soviet Phys. Uspekhi 9, 414 (1966).
[93] L. Biermann, Paper at XV URSI Assembly, Munich, 1966.

Translated by J. G. Adashko