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PLASMA INSTABILITY AND CONTROLLED THERMONUCLEAR REACTIONS*

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As is well known, plasma instability is the main obstacle in obtaining controlled thermonuclear reactions. It was this instability which dashed the hopes for a quick solution of this problem on the basis of self-constricting high-current discharges. Precisely this instability and the collective processes related to it uncovered the insufficiency of our knowledge for understanding the processes occurring in a plasma and transferred the investigations in the program of controlled reactions to a persistent and painstaking study of plasma physics. One can say without exaggeration that at present one is forced to study one form of instability or another in each plasma installation.

In the theoretical investigations, too, an appreciable portion of the efforts is spent in observing and studying various types of plasma instabilities. Some time ago it appeared, in view of the unending flow of articles in which ever new types of instabilities were observed, that we shall never achieve a full description of the instabilities of a rarefied plasma in a strong magnetic field. Fortunately, the situation has began to improve lately. Despite the fact that the activity in the field of investigations of plasma instabilities has remained at the previous level, it has become clear that the real threat to the containment of high-temperature plasma in a magnetic field is only due to a certain limited number of instabilities.

Why this is so can be understood from the following chain of arguments. The source of plasma instability is some lack of equilibrium in it: spatial inhomogeneity, nonisothermal nature, anisotropy, the presence of streams, etc. From the thermodynamic point of view a nonequilibrium plasma with collisions neglected constitutes a metastable state, while the buildup of oscillations due to the instability corresponds to one of the possible ways of establishing thermodynamic equilibrium. The stabilization of the instability corresponds to forbidding a given form of transition; and just as in the decay of a radioactive nucleus when one of the decay schemes is forbidden another less probable one is realized, so when one of the basic instabilities is suppressed, another less turbulent one appears. Thus, for example, in the experiments of M. S. Ioffe and co-workers [1], in which they studied the behavior of plasma in traps with magnetic mirrors, suppression of the most dangerous flute instability led to the circumstance that the finer cyclotron instability became most prominent. Inasmuch as complete stabilization of all instabilities of a rarefied plasma is apparently impossible, there exists in each specific experiment a limited group of instabilities, the most dangerous among those that have not been stabilized, which will be responsible for the collective processes in the plasma.

Which group of instabilities is most dangerous depends on the specific experimental conditions, i.e., on the configuration of the magnetic field, the temperature and density of the plasma, the method of production of the plasma, etc. Bearing in mind applications to controlled thermonuclear reactions, we shall consider here only toroidal systems which have a much larger "strength margin" compared with adiabatic traps, i.e., are less sensitive to increases of losses above the classical ones.

If we disregard the possibilities of stabilization, then the most dangerous are undoubtedly the magnetohydrodynamic instabilities [2], in which macroscopic portions of the plasma can move with velocities of the order of the thermal velocity of the ions $v_i = (T/m_i)^{1/2}$, where T is the temperature and m_i the mass of the ion. Depending on the energy sources of the instability and the nature of the plasma oscillations, one can differentiate between various particular forms of magnetohydrodynamic instabilities—the screw, flute, and balloon instabilities. The screw instability can develop in toroidal systems in the presence of a longitudinal current; it is the energy of the magnetic field of the current which constitutes the energy reservoir of this instability. However, actually the screw instability should not be considered dangerous, since it is relatively easily stabilized by superposition of a strong longitudinal magnetic field when the Shafranov-Kruskal criteria are fulfilled. The screw instability with finite conductivity (tearing mode) is also not very dangerous. As regards the flute instability and the closely related balloon instability, which develop as a result of the pushing out of the diamagnetic plasma towards the bulge of the lines of force, they can be readily stabilized at a sufficiently low plasma pressure p compared with the magnetic field pressure $H^2/8\pi$ and in the presence of "shear"—crossing of the lines of force. Thus, when account is taken of the possibilities of stabilization the magnetohydrodynamic instabilities need not be considered dangerous. In other words, it is not too dif-

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ficult to impose forbiddenness on the plasma decay by means of magnetohydrodynamic instabilities.

It is not difficult to understand why this forbiddenness can be achieved. The point is that fast displacements of the plasma with velocities of the order of the thermal velocity are possible only at the expense of the appearance of a transverse electrical field **E** in the plasma in which the particles drift with a velocity $\mathbf{v} = c(\mathbf{E} \times \mathbf{H})/H^2$. This relationship can be written in the form $\mathbf{E} = -1/c(\mathbf{v} \times \mathbf{H})$ and from Maxwell's equation we obtain

$$\frac{\partial \mathbf{H}}{\partial t} = -c \,\mathrm{rot}\, \mathbf{E} = \mathrm{rot}\,[\mathbf{vH}]. \qquad (1)^*$$

The latter relationship can be shown to be equivalent to the freezing-in into the plasma of lines of force of the magnetic field. Thus, fast displacement of plasma in the toroidal geometry in the absence of closure of the lines of force leads unavoidably to a deformation of the lines of force, i.e., to a perturbation of the magnetic field. It is precisely the potential barrier connected with the energy increase of the magnetic field during the perturbation which gives rise to the stabilizing effect.

Under conditions when the plasma is magnetohydrodynamically stable, the most dangerous instabilities begin to be the slower instabilities to which above all the drift instabilities belong.[3] Drift instabilities can develop on potential oscillations in which the magnetic field is not perturbed. Their characteristic time of development is on the order of the revolution a/v_d of particles about the plasma column in drift motion with a velocity $v_d \sim v_i\rho_i/a \sim v_e\rho_e/a$ where v_i is the thermal velocity of the ions, $\rho_i = v_i/\Omega_{H_i} = v_i m_i c/eH$ is their average Larmor radius, v_e and ρ_e are the same quantities for electrons, and a is the radius of the plasma column. The time of development of the drift instability with not too small a wavelength is $a/\rho_i \gg 1$ times larger than the characteristic time of magnetohydrodynamic oscillations.

Drift instabilities are not amenable to full stabilization; however, even in this case one can utilize forbiddenness effects. One of these is connected with the fact that the electron component of the plasma has a tendency to be "glued" to the lines of force of the magnetic field. In fact, let us assume that the lines of force of the magnetic field lie on closed toroidal magnetic surfaces placed inside one another whose equation is given by $\psi(x, y, z) = \mathrm{const}$. In view of the very large mobility of the electrons along lines of force and the rather large frequency of electron-electron collisions, a Maxwellian distribution is soon established for the electrons with a temperature T_e which is constant on the magnetic surfaces: $T_e = T_e(\psi)$. Since the distribution function of the electron velocities is close to Maxwellian, one can use for the electrons the hydrodynamic equilibrium equation

$$\nabla n T_e = en\nabla\varphi - \frac{en}{c}[\mathbf{v}_e\mathbf{H}], \qquad (2)$$

where φ is the potential of the electric field.

Projecting the vector equation (2) on the magnetic field direction, we obtain

$$\nabla_{\parallel} n T_e = en\nabla_{\parallel}\varphi. \qquad (3)$$

Hence it follows that along a line of force the Boltzmann distribution is established

$$\varphi = \frac{T_e}{e}\ln n + \varphi_0. \qquad (4)$$

If the lines of force do not close but fill the entire surface $\psi = \mathrm{const}$, then φ_0 depends only on ψ and in this instance in any direction along the surface $\psi = \mathrm{const}$ the gradient of the electron pressure is in equilibrium with the electrical field: $\nabla_\psi n T_e = en\nabla_\psi\varphi$. It follows from (2) that the electron-velocity component v_{en} normal to the surface $\psi = \mathrm{const}$ vanishes. Thus if there is time for a Boltzmann distribution to be established, then even in the presence of slow density oscillations the electrons could not move across magnetic surfaces. In other words, the development of instabilities connected with a density gradient and anomalous transverse diffusion by oscillations occurs only as a result of violation of the Boltzmann distribution.

In a rarefied plasma such a violation occurs as a result of the circumstance that the collisions do not have time to reestablish the Maxwellian distribution. With very rare collisions, this leads to the possibility of the developmentof an instability of the enclosed particles: in the inhomogeneous magnetic field of toroidal systems there is always a group of particles closed in between the "mirrors" by regions of enhanced magnetic field and in the absence of collisions a flute-type instability can develop with these particles. If the collisions are not very rare, then there is no instability of the enclosed particles and the Maxwellian distribution is violated either on account of the interaction of resonance electrons with drift waves, on account of the longitudinal inertia of the electrons, or on account of the electron-ion frictional force, i.e., the finite conductivity. The latter effects are small and in the presence of crossing of lines of force the corresponding instabilities are of little effectiveness. Here the drift-temperature instability is most prominent. This instability, as can be seen from its name, belongs to the class of drift instabilities and is related to a temperature gradient. The drift-temperature instability does not lead to strong diffusion of the plasma but produces a large heat flow across the magnetic field. It develops only with a sufficiently large temperature gradient, namely for $d \ln T/d \ln n > 1$. Therefore, by achieving a decrease of the density near the walls of the chamber one can decrease appreciably the value of the anomalous heat

*rot = curl; $[\mathbf{vH}] = \mathbf{v} \times \mathbf{H}$.

conduction due to this instability.

Another forbiddenness effect occurring in drift instabilities consists in the following. All drift instabilities develop via slow oscillations propagating over a small azimuth with a phase velocity of the order of $v_d \sim v_i\rho_i/a$. Drift waves occur only if their phase velocity along the magnetic field exceeds the thermal velocity of the ions v_i, i.e., if the corresponding perturbations are strongly extended along the magnetic field. In the presence of crossing of lines of force—"shear"—this effect leads to strong localization of oscillations in the radial direction. The magnitude of the "shear" can be determined in the following way. Let us consider a quantity c—the angle of twist of lines of force over a small azimuth after circuiting the torus. If the angle c changes with radius, i.e., with distance from the magnetic axis of the system, then this means that the lines of force cross, i.e., change their inclination with respect to the magnetic axis, as the radius r increases. Let Δc be the change of the angle on going over from the axis ($r = 0$) to the boundary of the column ($r = a$). Then one can take $\theta = (a/L_0)\Delta c$ as the value of the "shear" where L_0 is the perimeter of the torus. For small Δc the quantity θ represents an angle between the near-axial and peripheral lines of force.

The width of localization of drift waves is of the order of ρ_i/θ and by increasing θ one can decrease it to a quantity of the order of several ρ_i, where the waves localized at various points along the radius must have a different number of nodes over the azimuth. This leads to the circumstance that different waves turn out to be weakly coupled to each other and the convection of heat or of particles takes on a relay character—the heat transferred by one cell is taken up by the following cell, etc. As a result the transfer process is reminiscent of normal heat conduction or diffusion, and since the localization of the cells can be reduced to several ρ_i, whereas the characteristic transfer rate amounts to a quantity of the order of $v_d \sim \rho_i v_i/a$, the effective coefficient of temperature conduction (and even more that of diffusion) can be reduced in systems with large "shear" to a quantity of the order of $\chi \sim \rho_i v_d \sim \rho_i^2 v_i/a$.

This assertion is, of course, correct only under the condition that closed magnetic surfaces exist in the plasma. If the lines of force do not lie on closed surfaces but go out from the plasma, so that their length within the plasma amounts to a value of the order of L_H, then for a sufficiently long $L_H > a^2/\rho_i$ there is still no disturbance of the equilibrium, since during the time $t \sim a/v_d$ of revolution along a small radius because of the drift motion of the ions will not have time to reach the walls along the lines of force. However, since the mobility of the electrons along the magnetic field is very great, an escape process may develop in which the electrons and ions leave the trap along and across the magnetic field, respectively. In order that there occur a Bohm loss with a lifetime

$$\tau = \frac{\pi a^2 eH}{cT}, \tag{5}$$

it is sufficient that the electrons manage to come out from the trap along the lines of force during a time τ, i.e., $L_H/v_e < \tau \sim a^2/\rho_e v_e$. Thus for

$$a^2/\rho_i < L_H < a^2/\rho_e \tag{6}$$

loss of the order of the Bohm loss should take place.

The problem of the presence or absence of magnetic surfaces is therefore of first-order importance for the entire problem of magnetic thermal insulation. More specifically one can formulate this as follows: what requirements must be made of a magnetic configuration so that even in the presence of some initial perturbations the magnetic field should in the presence of a plasma tend in time towards a state with magnetic surfaces located within one another? This question has so far not been fully analyzed. One can apparently only assert that in axially symmetric systems (of the Tokamak, levitron and other types) there exists, in the presence of a minimum average magnetic field an effect of re-establishment of magnetic surfaces. As regards systems with a weak longitudinal field (of the Zeta type), when the lines of force are convex towards the outside, or complex magnetic configurations of the stellarator type, they require further investigation.

Thus if there are magnetic surfaces, then one can in a system with a sufficiently large "shear" decrease appreciably the anomalous loss of a sufficiently dense plasma, by a factor of the order of ρ_i/a compared with the Bohm loss.[4] However, the corresponding losses are still much larger than the classical ones. It is of interest to consider the significance of such losses from the point of view of achieving a self-sustaining thermonuclear reaction, and to what requirements they lead.

In order to achieve a self-sustaining reaction in an equal mixture of deuterium and tritium for $\beta = 8\pi p/H^2 < 1$, one must ensure a containment time[5] of $\tau > 6 \times 10^7/H^2\beta$. If we take the Bohm time (6) as the scale, taking into account the possibility of increasing it by a factor $\alpha^{-1} > 1$, then the condition for a self-sustaining reaction can be written in the form

$$a^2H^3 > 2\cdot 10^7 \frac{\alpha}{\beta}\frac{cT}{e}. \tag{7}$$

It is seen hence that it is essential for a self-sustaining reactor to use the maximum possible magnetic field. At present it is in principle possible to produce with the aid of a super-conducting winding a magnetic field $H = 10^5$ Oe. Substituting this value in (7) and taking $T = 10$ keV, we obtain

$$a > 1.4\cdot 10^2 \sqrt{\frac{\alpha}{\beta}}. \tag{8}$$

Hence we see that for $\beta = 10^{-2}$ and Bohm losses

($\alpha = 1$) the small radius of the torus turns out to be 14 meters. For losses smaller by two orders of magnitude ($\alpha = 10^{-2}$) the radius a takes on the more acceptable value of 1.4 m. With this we have $\rho_i/a \sim 10^{-3}$, i.e., the possibility of reaching $\alpha = 10^{-2}$ and even smaller values of α appears quite realistic. For $\beta \sim 10^{-2}$, which corresponds to a density $n \sim 10^{15}$ cm^{-3} at a temperature T = 10 keV one should expect the plasma to be stable and escape should only be due to drift instabilities. As regards instabilities on the enclosed particles, then for the indicated density it is also not dangerous. Thus the quantities $a \approx 10^2$ cm, $H \approx 10^5$ Oe can be considered to be tentative values for the dimension and magnetic field of a thermonuclear reactor. The reactor itself can be imagined in the form of a sharp torus of the Tokamak type in which the magnetic surfaces are produced by means of a longitudinal current.

Let us now consider the problem of the possibility of attaining thermonuclear temperatures ($T > 5$ keV) by means of Joule heating. Joule-heat release per unit volume is

$$j^2/\sigma \approx \frac{c^2 H_\vartheta^2}{4\pi^2 \sigma a^2}, \qquad (9)$$

where σ is the conductivity, j—the current density and H_ϑ—the value of the azimuthal magnetic field on the boundary of the discharge. To heat the plasma, the Joule heating must be larger than the energy losses $\alpha(cT/eH\pi a^2)3nT$. Taking into account that $2nT = \beta H^2/8\pi$, we obtain

$$H > \alpha\beta \frac{H^2}{H_\vartheta^2} \frac{\sigma T}{ec}. \qquad (10)$$

Substituting here T = 5 keV (with $\sigma = 4 \times 10^{18}$), we find

$$H > 10^9 \alpha\beta \frac{H^2}{H_\vartheta^2}. \qquad (11)$$

It is hence seen that for $\alpha = \beta = 10^{-2}$ and $H = 10^5$ Oe, which seems entirely sensible, Joule heating would be quite sufficient if it were possible to raise H_ϑ to the value H. The experiments carried out to date on Tokamak installations indicate that a column with current is macroscopically stable for $q > q_0 = 2$—4 where $q = aH/RH_\vartheta$ is the "margin coefficient," i.e., for $H^2/H_\vartheta^2 > R^2 q_0^2/a^2$. However, it is not seen from the theoretical considerations why one could not obtain lower values of q by choosing a suitable radial current distribution. It would therefore be desirable to carry out a more detailed investigation of the screw instability on the Tokamak instruments. In addition, one should explain what maximum toroidality (i.e., maximum ratio of the smaller radius of the torus a to the larger one R) can in practice be produced.

If H^2/H_ϑ^2 cannot be reduced to a value of the order of 10, then one can still make use of the possibility of decreasing α and β, at least during the time of the Joule heating. As we see from (11), for $\alpha = \beta = 10^{-3}$ and $H = 10^5$ Oe Joule heating occurs even for $H_\vartheta/H \sim 10^{-1}$ (for $\alpha \gtrsim 10^{-3}$ the synchrotron radiation is still insignificant). It follows from theoretical considerations that the possibility of a temporary decrease of α exists. To do this one can make use of the effect of the smallness of the diffusion coefficient compared with the coefficient of thermal conduction and produce during the heating a "detachment" of the plasma from the walls either by increasing the magnetic field or by displacement of the diaphragm. In addition one can add from outside the column impurities in order to decrease the conductivity and increase the Joule heat release in this region. One must therefore not exclude entirely Joule heating as one of the possible methods of producing a "thermonuclear plasma" although its efficiency lies on the border of its essential value.

Summing up, one can say that although so far no hopes remain for the total stabilization of a plasma, it appears theoretically possible to decrease considerably the effect due to the instabilities by increasing the dimensions of the system, the magnitude of the magnetic field, and the crossing of the lines of force. At the same time one must fully stabilize the fast magnetohydrodynamic instabilities of an ideal plasma, and the next most-dangerous drift instabilities are strongly suppressed. To achieve controlled thermonuclear reactions in this manner one must overcome the enormous technical difficulties connected with the production of a magnetic field of the order of 10^5 Oe within a volume of the order of many cubic meters. These conclusions are tentative. In order to be certain that they are correct one must carry out extensive physical investigations with the general aim of obtaining a full picture of collective processes in plasmas.

[1] Yu. V. Gott, M. S. Ioffe, and E. E. Yushmanov, in Plasma Physics and Controlled Nuclear Fusion Research, vol. 1, Vienna, 1966, p. 35.

[2] B. B. Kadomtsev, coll. Voprosy teorii plazmy (Problems of Plasma Theory), vol. 1, Gosatomizdat, 1963.

[3] A. B. Mikhaĭlovskiĭ, ibid. vol. 3, Gosatomizdat, 1963.

[4] B. B. Kadomtsev and O. P. Pogutse, in Plasma Physics and Controlled Nuclear Fusion Research, vol. 1, Vienna, 1966, p. 365.

[5] L. A. Artsimovich, Upravlyaemye termoyadernye reaktsii (Controlled Thermonuclear Reactions), Moscow, Fizmatgiz, 1961, p. 19.

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