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# TRANSITION RADIATION AND OPTICAL PROPERTIES OF MATTER*

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Usp. Fiz. Nauk 87, 189-210 (October, 1965)

## 1. INTRODUCTION

THE Vavilov-Cerenkov radiation was discovered 30 years ago (in 1934).[1] In the intervening period, this radiation has been considered from various points of view. First, the optical properties of the radiation were mainly considered. Usually, the radiation was treated as an example of optics at velocities greater than that of light. The phenomenon was later embraced by nuclear physics. Here, we want to point out that this phenomenon revealed for the first time the direct relationship between nuclear physics and optics. In fact, the characteristic properties of the Vavilov-Cerenkov radiation are governed mainly by three quantities: the charge of the particle, its velocity, and the optical refractive index of the surrounding medium. It has been evident that there are other phenomena in which the optical properties of a substance are important in the case of radiation from a fast-moving particle.[2] Among them is the so-called transition radiation.

Assume that a fast charged particle enters a metal from vacuum. In many metals the radiation of optical frequencies is absorbed over a path which is small or comparable with the wavelength of light. Thus, the optical components of the field of a moving particle disappear almost instantaneously as soon as the particle crosses the boundary. During its subsequent motion in the metal, the particle apparently becomes invisible. Obviously, this process should generate radiation which would be in some respects similar to bremsstrahlung. If the energy of the particle is so high that it is not scattered in the surface layer of the substance, or its velocity is not greatly changed, we may assume that the particle is moving uniformly.† Thus, we have, as in the Vavilov-Cerenkov effect, the emission of radiation, during the uniform motion of a particle, which depends strongly on the optical properties of the medium. The transition from vacuum into a metal is only a special case of this effect. The radiation should appear every time a particle crosses a boundary of two media having different optical properties. The theory of this phenomenon was developed 20 years ago in a paper which V. L. Ginzburg and I wrote[3], and we called the effect transition radiation. The name has now become generally accepted.

It is known that in cathode tubes the anode does indeed emit light under electron bombardment. The emission of light by anodes was already noted in the first investigations of x rays, soon after their discovery by Röntgen. However, in an x-ray tube light is emitted also by the glass and by the residual gases and this interferes with the observations. The first attempt at investigating the phenomenon of light emission by anodes was made by Lilienfeld in 1919,[4] who photographed the emission spectrum of an anode and showed that it was continuous. In spite of several other experimental investigations, the nature of the radiation remained unexplained. The suggestions made about the nature of the radiation included luminescence, and various forms of bremsstrahlung, all which do indeed play some part. Now, it seems surprising that such a natural process as the transition radiation, which follows directly from the equations of electrodynamics, was not proposed earlier. There is no doubt that this was the result of an erroneously held view that a uniformly moving particle does not emit radiation. This view was dropped only after the appearance of the theory of the Vavilov-Cerenkov effect. The calculations based on the theory of transition radiation showed that this type of radiation cannot be ignored in dealing with the observed emission. However, accurate experimental data have only been obtained in the last few years. Since it was found that the theory of transition radiation was applicable to these data, we shall give briefly the main results of this theory.

## 2. THEORY OF TRANSITION RADIATION

The majority of the experiments were carried out using charged particles traversing a solid target in vacuum. The investigations were concerned with that part of the radiation which appeared at the boundary between the target and vacuum. This is the special case of the theory of transition radiation which we shall consider. The formula for the general case is somewhat more complex. In the transition radiation case, as in the Vavilov-Cerenkov radiation, it is unimportant whether the particle is an electron or not. All that is important is the value of the charge on the particle and its velocity of motion. For a nonrelativistic particle, the radiation energy increases as the

*Paper read at the joint session of the General and Applied Physics Division of the U.S.S.R. Academy of Sciences, and the State Committee on the Applications of Atomic Energy, held on December 10, 1964.

†The path over which there should be no scattering depends on the velocity of the particle and the angle of emission of light. For a nonrelativistic particle, this is the distance which the particle traverses in a time much longer than the period of light waves.

square of the velocity. Therefore, the velocity of the particle should be high. The majority of experiments with electrons were carried out using energies from several keV to 100 keV. To produce such electrons, simple apparatus is satisfactory and currents of tens of microamperes are sufficient.

A proton of the same velocity as an electron should, of course, have an energy 1850 times greater. Thus, electrons of energies of, for example, 10 keV correspond to protons of energies of 18.5 MeV. To generate such protons, we need an accelerator, for example, a cyclotron. Therefore, experiments with electrons are easier than those with protons. On the other hand, in the case of electrons, the optical range of frequencies of the bremsstrahlung radiation makes a considerable contribution, whereas with protons, this contribution is unimportant. It will follow from a later treatment that there are some advantages in the use of relativistic electrons, which also require an accelerator such as a linear accelerator or a microtron.

We shall deal first with the case of nonrelativistic energies. We shall assume, for example, that a particle of charge e (for example, an electron) crosses the boundary between vacuum and a material medium, moving uniformly and rectilinearly, and that the square of its velocity obeys $v^2 \ll c^2$ (Fig. 1). We shall consider the field in the wave zone, i.e., far from the boundary. The amplitude of the radiation field, propagated along a direction making an angle $\varphi$ with the normal to the surface, may be represented as the sum of the following terms:

a) The field of the electron moving in vacuum and stopping suddenly at a point A on the boundary of the medium.

b) The field of the electron which begins at the same moment as its motion from the point A into the medium.

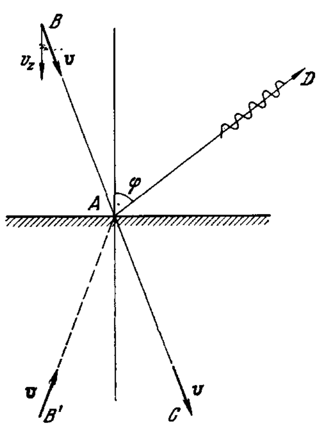


FIG. 1. Schematic explanation of the method of the theoretical consideration of the transition radiation [formula (1)]. BAC is the path of a particle having a charge e and a velocity v moving from vacuum into a medium; B′A is the trajectory of the electrical image of the particle, moving from the interior of the medium to the surface. The density of radiation is calculated for a far point D (the ray making an angle $\varphi$ with the normal).

The first term represents the bremsstrahlung field which appears when the particle is stopped. The second term gives the bremsstrahlung when the particle is suddenly accelerated. Together, they represent the field of a particle in uniform motion. If at a given frequency the medium has the properties $\epsilon = 1$, $\mu = 1$, i.e., it does not differ from vacuum, then the amplitudes of the field are equal in both cases but opposite in sign and their sum is zero. This corresponds to the self-evident observation that a particle moving uniformly in vacuum does not emit light. In other cases, when a medium is present, the compensation of these two terms is incomplete. One must allow also for the presence of a third term:

c) The field of the electrical image of the particle. This is equivalent to the field of some virtual particle moving at a velocity v from the medium toward the same point A and stopping at that point (cf. Fig. 1).

We thus obtain a formula for the spectral density of the radiation per unit solid angle in vacuum along a direction making an angle $\varphi$ with the normal:

$$\frac{dW_\omega}{d\Omega} = \frac{e^2 v_z^2}{4\pi^2 c^3} \sin^2\varphi \left| 1 + r - f\frac{1}{n_2} \right|^2. \tag{1}$$

Here, $v_z$ is the component of the velocity **v** of the particle, directed along the normal to the surface of the medium. We take the square of the modulus of the sum of the field amplitudes of all three terms, and the common factor is taken outside the modulus sign. The unity in the modulus represents the field of a particle terminating at the boundary. The third term in the modulus gives the field of a particle continuing its motion in the medium. The symbol f is Fresnel's coefficient for the reflected wave. The presence of f in this term can be understood because only a fraction of the field, corresponding to the refracted wave, leaves the medium and enters the vacuum. Moreover, the third term in the modulus has the factor $1/n_2$, where $n_2$ is the complex refractive index of the medium. This factor also has a simple physical meaning. The wave observed in vacuum at an angle $\varphi$ moves in the medium at an angle $\varphi'$, and the radiation field is proportional to the sine of the angle. Hence, using Snell's law, we obtain the factor $1/n_2$. Finally, the middle term in the modulus gives the field of the electrical image of the particle. The quantity r is but Fresnel's coefficient for the reflected wave and its meaning is self-evident. The Fresnel coefficients r and f are, interrelated and given by*

$$r = \frac{n_2^2\cos\varphi - \sqrt{n_2^2 - \sin^2\varphi}}{n_2^2\cos\varphi + \sqrt{n_2^2 - \sin^2\varphi}}, \tag{2}$$

*One should not be surprised that the formula includes Fresnel's coefficient f for the wave refracted from vacuum into the medium although, in fact, light proceeds from the medium into vacuum. This is because the source of radiation lies at the boundary and the field is measured at a point far from the boundary. The correctness of the results can be easily checked by using the reversibility principle.[3]

$$f=\frac{2n_2\cos\varphi}{n_2^2\cos\varphi+\sqrt{n_2^2-\sin^2\varphi}}. \tag{3}$$

Therefore, the spectral density of the radiation can be easily written in terms of r or f. In fact,

$$1+r=fn_2. \tag{4}$$

Hence, we find that

$$\frac{dW_\omega}{d\Omega}=\frac{e^2v_z^2}{4\pi^2c^3}\sin^2\varphi\,|1+r|^2\left|1-\frac{1}{n_2^2}\right|^2. \tag{5}$$

Formulas (1) and (5) were obtained initially for the case when the velocity of the particle was directed along the normal to the boundary. However, Pafomov[5] showed that, for a nonrelativistic particle, the same formulas are also applicable in the case of oblique incidence (as shown in Fig. 1) by replacing v with $v_z$. Thus, the radiation energy is proportional to $e^2v_z^2$, and the radiation spectrum is independent of the velocity and governed by the optical properties of the medium. In an optical isotropic medium, the electric vector of the light wave lies in a plane defined by the normal to the surface and the direction of the beam, so that the transition radiation should be completely polarized. This is sometimes used to separate this radiation from other types.

To consider in more detail the characteristics of the transition radiation, we shall replace the quantity r in Eq. (5) with its value given by Eq. (2); we then obtain

$$\frac{dW_\omega}{d\Omega}=\frac{e^2v_z^2}{\pi^2c^3}\sin^2\varphi\left|\frac{(n_2^2-1)\cos\varphi}{n_2^2\cos\varphi+\sqrt{n_2^2-\sin^2\varphi}}\right|^2. \tag{6}$$

It follows from Eq. (6) that for $\epsilon\equiv n_2^2\to\infty$, i.e., for an ideal conductor, the radiation density is

$$\frac{dW_\omega}{d\Omega}=\frac{e^2v_z^2}{\pi^2c^3}\sin^2\varphi. \tag{7}$$

In this case, the radiation is the same as that emitted when two opposite charges e, which are approaching each other at a velocity $v_z$, meet. In the general case, Eq. (6) may be written as follows

$$\frac{dW_\omega}{d\Omega}=\frac{e^2v_z^2}{\pi^2c^3}\sin^2\varphi\,|B|^2, \tag{8}$$

$$|B|^2=\left|\frac{(n_2^2-1)\cos\varphi}{n_2^2\cos\varphi+\sqrt{n_2^2-1+\cos^2\varphi}}\right|^2, \tag{9}$$

where $|B|^2$ is equal to the square of the modulus in Eq. (6) and allows for the real properties of the medium. It can easily be shown that even in metals we cannot simply assume that $|B|^2=1$. In fact, it follows from Eq. (6) that for $\varphi$ close to $\pi/2$ the value of $|B|^2$ decreases as $\varphi$ increases, according to $\cos^2\varphi$, and vanishes at $\varphi=\pi/2$. It can easily be shown that this is because $(1+r)\to 0$ when $\varphi\to\pi/2$. Thus, the field of a particle in vacuum is quenched by the field of its image. The result is the same as if the approaching particles had the same (not opposite) charges.

The exact form of the angular distribution of the radiation obviously depends on $n_2$. However, qualitatively speaking, it is similar for a number of metals and dielectrics and has a maximum lying between 60° and 70° (cf. Figs. 2–4).

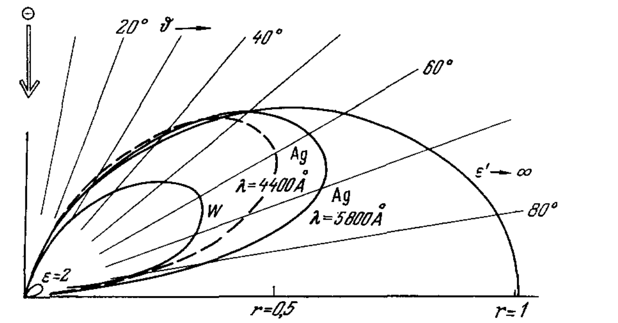


FIG. 2. Polar diagram of the intensity of the transition radiation calculated using formula (8) for an ideal conductor ($\epsilon'\to\infty$), for silver, tungsten, and for a dielectric with $\epsilon=2$. The figure is taken from the work of Boersch et al.[13] Here, $r=\sin^2\varphi|B|^2$. It is evident from the figure that at angles $\varphi<50°$ the intensity of the radiation from silver exceeds that expected for an ideal conductor.

In the general case, the analysis of formula (6) is not very simple since in substances absorbing light the value of $n_2$ is complex:

$$n_2=n+ik. \tag{10}$$

For simplicity, we shall assume that the value of the imaginary component of $n_2^2$ can be neglected compared with the real component. This is true for the majority of dielectrics, for which $k\ll n$.

It is easily shown that for $\epsilon\equiv n_2^2$ slightly greater than unity, the intensity of the transition radiation is considerably less than in an ideal conductor since $|B|^2\ll 1$ for any angle $\varphi$ [cf. Eq. (9)]. In fact, if the angle $\varphi$ is small and $\cos\varphi>(n^2-1)$, then

$$B^2\approx\left(\frac{n_2^2-1}{n_2^2+1}\right)^2<1;$$

at higher values of $\varphi$, for which $\cos\varphi<(n^2-1)$, we have $B^2\sim(n_2^2-1)\cos^2\varphi$, i.e., B decreases to zero as $\varphi\to\pi/2$.

For $\epsilon\equiv n_2^2$ much greater than unity and sufficiently

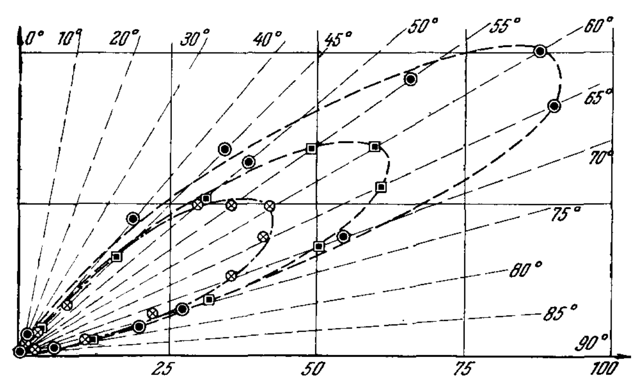


FIG. 3. Comparison of the calculated data on the angular distribution with the experimental results for the polarized component of the visible part of the radiation. The outer curve represents silver, the middle curve aluminum, and the inner curve nickel (dissertation of S. Michalak[14]).

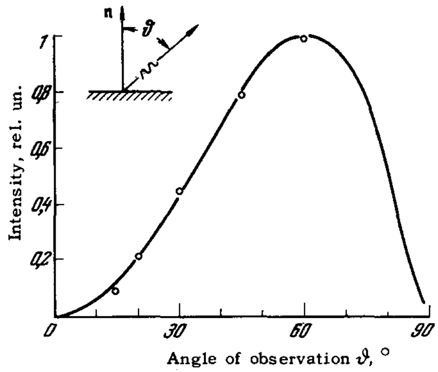


FIG. 4. Comparison of the observed angular distribution for tungsten with that expected from the theory (data of Boersch et al.[13]).

low values of $\varphi$, the quantity B is close to unity, i.e., the density of the radiation approaches the value given by Eq. (7).

The case of metals, for which usually $k \gg n$, is of interest. It is found that even for metals the approximate value of $|B|^2$ can be obtained by assuming, in some cases, that $n_2^2$ is real.[6] In fact,

$$\varepsilon = n_2^2 = (n^2 - k^2) + 2ink, \tag{11}$$

and at sufficiently high values of k compared with n we have $|n^2 - k^2| \gg |2nk|$. The characteristic feature of this case, typical of metals, is that the real part of $\epsilon$ is negative. It is easy to show that in this case, at not too high values of $\varphi$, we have $|B|^2 > 1$, i.e., the density of the radiation is even greater for an ideally conducting metal. In the approximate treatment, we shall assume that $n_2^2 = n^2 - k^2 < 0$. Thus, the quantity in the numerator of the expression for $|B|^2$ is

$$|(n_2^2 - 1)\cos\varphi|^2 = [(n^2 - k^2 - 1)\cos\varphi]^2$$
$$= [(k^2 - n^2 + 1)^2 \cos^2\varphi] > |n_2^2|^2 \cos^2\varphi,$$

and the following expression[6] is obtained for $|B|^2$*

$$|B|^2 = \frac{(k^2 - n^2 + 1)^2 \cos^2\varphi}{(k^2 - n^2)^2 \cos^2\varphi + (k^2 - n^2) + \sin^2\varphi}. \tag{12}$$

Hence, at low values of $\varphi$, we find that

$$|B|^2 \approx 1 + \frac{1}{k^2 - n^2}. \tag{13}$$

The possibility of $|B|^2 > 1$ is obviously due to the fact that if $n_2^2$ is negative, the presence of the term $-1$ in the factor $|n_2^2 - 1|^2$ does not decrease but increases $|B|^2$. On the other hand, comparing Eqs. (1), (5), and (6), we easily find that the term $-1$ is due to that part of the field which appears during the motion of a particle in a medium. Thus, even in metals with high values of the absorption coefficient, we cannot always neglect this part of the field. This feature, which distinguishes the transition radiation in a real metal from the ideal case given by formula (7), was not predicted; it was established by comparison of the experimental data with the theory.[6] (The results of the calculation can be seen also in Fig. 2.) Another feature, which had been overlooked until it was found experimentally, was that the spectral density of the radiation could become high when $|n_2|^2$ was small. In the particular case of silver, this occurs in the ultraviolet region. For silver, k = 1 and n = 0.16 (cf. Fig. 11) at $\lambda$ = 3500 Å. Thus, the real component of $\epsilon$ is equal to $n^2 - k^2 = -1$ and the imaginary component is much less than $|2nk| = 0.32$. Consequently, we are not far wrong if we use Eq. (13) at $\lambda$ = 3500 Å. Then we find that $|B|^2 = 2$. As $\lambda$ decreases, the imaginary component of $n_2$ decreases (the maximum of the transparency of silver occurs at $\lambda$ = 3250 Å), and the real component increases; at $\lambda$ = 3300 Å, we have n = k = 0.4, i.e., $n^2 - k^2 = 0$. However, the value of $|B|^2$ is not equal to infinity as might follow from Eq. (13). This formula is now inapplicable since the imaginary component of $n_2^2$, equal to $|2nk| = 0.32$, can no longer be neglected. If we now use formula (9) and make an allowance for the real and imaginary components of $n_2^2$, we find that the maximum of the quantity $|B|^2$ lies in the wavelength range between $\lambda$ = 3300 Å and $\lambda$ = 3500 Å (cf. Fig. 12).

All the formulas just given refer to the case when the thickness of the target in which the particle is moving is considerably greater than the thickness of the layer from which light may emerge. If the target is so thin that the particle and light are capable of being transmitted through it, we must allow for the fact that the transition radiation appears at both surfaces of the target. If, moreover, the particle crosses the target without undergoing scattering and without a marked reduction in its velocity, the radiation from both surfaces is coherent. The strength of the radiation depends on the angle of emission, the plate thickness, and the particle velocity.[7-9] The presence of such coherence distinguishes the transition radiation from bremsstrahlung and from fluorescence. This feature was, in fact, observed experimentally. In the general case, particularly when a particle is obliquely incident on a surface, the formulas are quite cumbersome and will not be given here.

Before considering the experimental data on the transition radiation, we shall deal briefly with the case of the radiation of a relativistic particle. For the latter formula (1) has the form

$$\frac{dW_\omega}{d\omega} = \frac{e^2v^2}{4\pi^2c^3}\sin^2\varphi\left|\frac{1}{1-\beta\cos\varphi} + \frac{r}{1+\beta\cos\varphi} - f\frac{1}{n^2}\frac{1}{1-\beta n_2\cos\varphi'}\right|^2. \tag{14}$$

Here, $\varphi'$ is the angle in the medium (which, in general, is complex), which becomes $\varphi$ after refraction. The values of $\varphi'$ and $\varphi$ are obviously related by

$$n_2\cos\varphi' = \sqrt{n_2^2 - \sin^2\varphi}. \tag{15}$$

*To calculate the square of the modulus in the denominator of Eq. (9), we must bear in mind that

$$\sqrt{n_2^2 - 1 + \cos^2\varphi} = \sqrt{n_2^2 - \sin^2\varphi} = i\sqrt{k^2 - n^2 + \sin^2\varphi}.$$

Formula (14) applies to the case when the particle moves along the normal to the surface of separation in the direction from the medium into vacuum and the radiation is observed in vacuum at an angle $\varphi$ to the normal. For the opposite direction of the velocity, i.e., when the particle moves from vacuum into the medium, the sign in front of $\beta$ in all terms in the denominator should be reversed. Thus, in contrast to a nonrelativistic particle, the radiation in the present case depends on the sign of the particle velocity with respect to the medium, i.e., a characteristic directivity appears in the phenomenon. When the particle is not moving along the normal to the surface of separation, we cannot simply replace v with $v_z$. The formula for the relativistic particle then becomes more complicated.[5] When the refractive index is higher than unity, the formula also includes the Vavilov-Cerenkov radiation. If the value of $n_2$ is real and greater than unity, the denominator in the third term vanishes when the well-known condition is satisfied:

$$\cos\varphi' = 1/\beta n_2, \tag{16}$$

i.e., the intensity of the radiation becomes infinite. For a complex refractive index, i.e., in the presence of absorption, the spectral density remains finite for any angle.

Formula (14) differs from formula (1) by the presence of factors of the type $1/(1 \pm \beta n \cos\varphi)$, in each of the terms; these factors are characteristic of the radiation of a relativistic particle. Consequently, the transition radiation of a relativistic particle has a number of features which have recently attracted considerable attention and have stimulated theoretical investigations.[8-11] These problems are outside the scope of my lecture and I shall therefore deal with them very briefly.

It is evident from formula (14) that at low values of $\varphi$, for which $\cos\varphi$ is close to 1, the denominator in the first term in the square of the modulus decreases as the particle energy is increased, i.e., $\beta$ increases to 1. The third term can be neglected compared with the first term only if the angle of observation differs considerably from the angle characteristic of the Vavilov-Cerenkov radiation and if $n_2 \neq 1$. In this case, only the first term in the transition radiation formula is important at very small angles. This first term is identical with the radiation from a relativistic particle that stops suddenly in vacuum. Simultaneously with an increase in the radiation at low angles, the cone of the radiation in which this increase is important becomes smaller. Consequently, the total density of the radiation per unit frequency interval, integrated over all angles, increases only logarithmically with the particle energy. It is important to note that, in contrast to a nonrelativistic particle, the transition radiation at low values of $\varphi$ is not weakened by the use of a transparent dielectric with a refractive index differing slightly from 1. This makes it possible to sum the radiation from many surfaces of separation.[10] This case is of considerable interest because the number of the transition-radiation photons in the optical region of the spectrum generated when one boundary is crossed is only of the order of 0.01 for a relativistic particle. Therefore, to increase the efficiency of detection of a particle using its transition radiation, it is advisable to sum the radiation from many surfaces.

We shall deal later with other characteristic properties of the transition radiation of a relativistic particle. The most interesting is the consequence of a feature considered here: the spectrum of the transition radiation of an ultrarelativistic particle extends to the x-ray and $\gamma$-ray regions and this extension increases with the particle energy. In fact, at very high frequencies, the refractive index is close to but less than unity and this deviation from unity decreases as the square of the frequency. On the other hand, the higher the particle energy, i.e., the smaller the value of $(1 - \beta \cos\varphi)$ at low values of $\varphi$, the smaller is that difference between the refractive index and unity which is sufficient to make the third term in Eq. (14) small compared with the first term. Consequently, on stepping up the particle energy, the threshold frequency of the transition radiation increases approximately proportionally to the energy.

The fact that this property was not observed at first was undoubtedly due to the accepted idea that the transition radiation is produced only in the optical range of the spectrum. When Garibyan[8] showed that the total energy of the transition radiation of a relativistic particle increased as its energy increased, the result seemed at first to be paradoxical. In fact, formula (14), as mentioned before, gives only a logarithmic rise for increasing particle energy. However, it soon became understood that it was not permissible to compare these particular quantities. Formula (14) gives the spectral density of the radiation while Garibyan's result refers to the total energy of the radiation, which increases due to the enrichment of the spectrum with new frequencies.

Several investigators predicted other interesting properties for the radiation of an ultrarelativistic particle. In particular, it was found that, in some cases, the bremsstrahlung and transition radiation cannot be considered to be independent.[11] It seems to me that these problems should be the subject of a separate discussion.

The object of my discussion is the optical part of the spectrum. I have mentioned the radiation of a relativistic particle because it is rational to use relativistic particles to obtain the optical part of the spectrum. I shall speak about this in the last part of my lecture. Here, I shall only give some estimates of the transition radiation intensity. To determine the number of photons emitted per unit solid angle in the frequency range $\Delta\omega$, it is necessary to divide the quantity given in Eq. (8) by $\hbar\omega$; we then obtain

$$\frac{dN_\omega}{d\Omega}\Delta\omega=\frac{\alpha}{\pi^2}\beta^2\sin^2\varphi\,|B_\omega|^2\frac{\Delta\omega}{\omega}, \tag{17}$$

where $\alpha$ is the fine structure constant ($\alpha = 1/137$), and $|B_\omega|^2$ for a nonrelativistic particle is given by Eq. (9) but for a relativistic particle is equal to one quarter of the quantity under the squared-modulus sign in Eq. (14). Formula (17) gives the radiation energy per single particle. If we have a current of singly-charged particles, equal to i milliamperes, the value given by Eq. (17) must obviously be multiplied by $6 \times 10^{15}$i. The behavior of the quantity $|B|^2$ in a real system and the difference from the behavior in an ideally conducting medium ($|B|^2 = 1$) is shown in Fig. 2.

## 3. EXPERIMENTAL INVESTIGATIONS OF THE TRANSITION RADIATION

A large number of papers have already been published which report quite comprehensive investigations of the radiation from various targets under the action of charged particles. With the exception of Goldsmith and Jelley,[12] who used 1—5 MeV protons, electrons were used in all these investigations. I shall not present them in chronological order nor shall I give a complete description of each investigation.* I shall attempt only to summarize the main results obtained.

Up to the present, investigations have been made of the radiation generated by the electron bombardment of the following substances: aluminum, nickel, silver, vanadium, tantalum, molybdenum, titanium, cesium, copper, tin, antimony, germanium,[13-16] as well as some dielectrics:[17] NiO, CoO, MnO. Thus, the radiation is seen to be universal. In practically every paper, the polarization of the radiation was investigated as well as the dependence of the output on the particle energy and the absolute value of the radiation energy. In all cases, the light was found to be partly polarized, and the sign of the polarization was that predicted by the theory of transition radiation (the electric vector was in a plane passing through the normal to the surface and the direction of observation). In the case of metals, the polarization increased with the electron energy and became practically 100% in the 50—100 keV range of energies. This was in accordance with expectations.

In metals, luminescence is possible in the surface layer. We may expect the brightness of such luminescence not to increase but to decrease as the electron energy is increased, since the losses in the particle energy due to ionization in the surface layer decrease as the particle velocity rises.

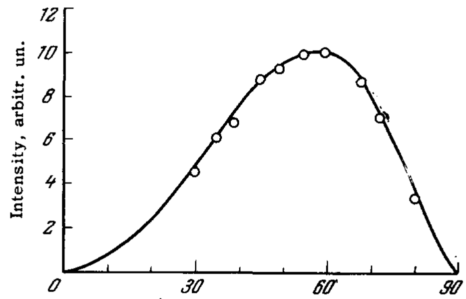


FIG. 5. Angular distribution obtained for the polarized component of the radiation produced when a dielectric ($\epsilon = 5.4$) was bombarded with electrons. The continuous curve represents the predictions of the theory of transition radiation (the results of Tanaka and Katayama[17]).

The same is true of the optical part of the bremsstrahlung. Bremsstrahlung is associated with the scattering of electrons and the probability of scattering in a thin layer decreases as the electron energy is increased. Since the light from a metal may emerge only from a thin layer, the bremsstrahlung yield should be inversely proportional to the electron energy.

On the other hand, the transition radiation yield increases, as mentioned earlier, as the square of the electron velocity. Therefore, as the electron energy is increased the transition radiation should become increasingly dominant compared with bremsstrahlung and luminescence. In the case of the dielectrics,[17] NiO, CoO, MnO, the polarization was found to be less than that in metals. The value of the polarization was about 50% and it depended weakly on the electron energy (the range up to 30 keV was investigated). This was explained in a natural way by the fact that in dielectrics luminescence appears not only on the surface but also in the interior. Since the coefficient

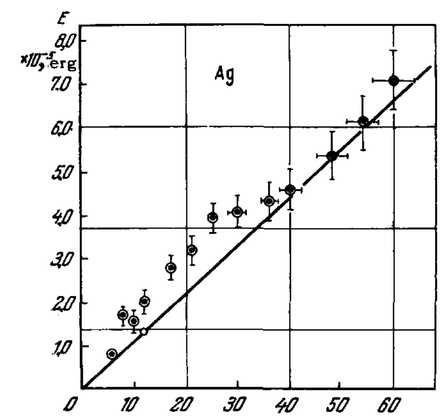


FIG. 6. Dependence of the yield E = f(U) of the transition radiation from silver (at an angle $\varphi = 60°$) on the electron energy (solid angle $\Omega = 7.7 \times 10^{-2}$, wavelength interval 3900–6600 Å, current I in $\mu$A, electron energy in keV) and comparison with the theory (represented by the straight line) (data in the dissertation of S. Michalak[14]).

*For the same reason, the literature cited is not intended to be complete. In particular, I am citing only some of the papers of the extensive literature on the problems associated with the theory of transition radiation.

of absorption of light is not very high, the light is summed over all parts of the electron path. Therefore, the luminescence yield, like the transition radiation yield, should, in this case, increase approximately proportionally to the electron energy. If the fraction of the unpolarized light is subtracted from the total intensity, the remainder has an angular distribution with a maximum at an angle of about 60° (Fig. 5), which is typical of the transition radiation. Good quantitative agreement of the angular distribution with the theory of transition radiation was also obtained for the emission of metals (Figs. 3 and 4).

As expected, the brightness of the radiation was proportional to the electron energy. The value of the absolute yield was found to be in agreement, within the limits of experimental error, with the value calculated from the theory of transition radiation (Fig. 6).

Several experiments have been carried out in which some of the possible radiation mechanisms, other than transition radiation, were eliminated. Thus, for example, it was carefully checked that the energy of the radiation was proportional to the electron current (Fig. 7). This proved that there were no radiation mechanisms of the cascade kind. In particular, electron bombardment could liberate a gas and then excite it. Such an effect would be proportional to the square of the electron current. It was also established that the brightness of the radiation was independent of the presence of a gas.[13] In high vacuum (pressure at least $10^{-5}$ torr), the brightness did not change when the vacuum was improved by 4 orders of magnitude (to $10^{-9}$ torr).

It seems to me that the experiments carried out at various target temperatures are very interesting.[13] Such experiments give the most direct method of finding whether the observed effects are associated with luminescence. In the case of tungsten, it was found that the brightness of the radiation remained constant

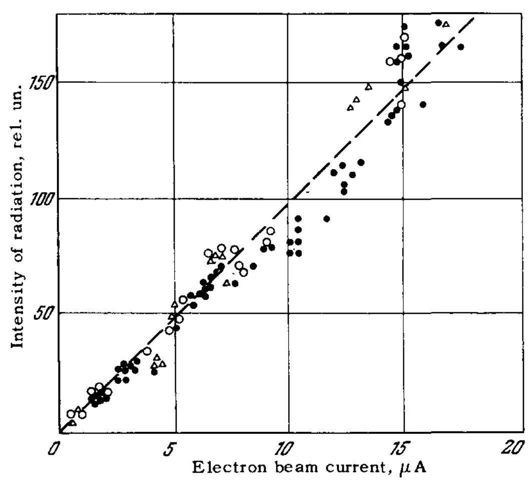


FIG. 7. Dependence of the brightness of the transition radiation (in relative units) on the current (for 12 keV energy) (dissertation of S. Michalak[14]).

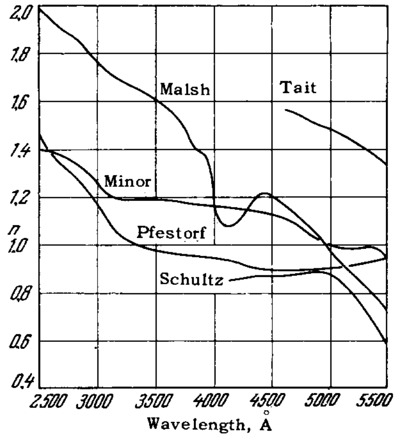


FIG. 8. Dependence of the real component of the refractive index of copper on the wavelength, according to different authors (the curves are taken from the work of Emerson et al.[15]).

when the target was heated to 2000°K. This experiment was not simple because the brightness of the thermal radiation was strong and the transition radiation was very weak. The measurement was successful because of the use of light filters, which transmitted only the ultraviolet part of the spectrum, and the use of a modulated electron beam. By amplifying only the alternating component of the photomultiplier current, it was possible to separate the component of light excited by electrons from the continuous background of thermal radiation. Experiments were carried out also on other metals, varying the temperature within the limits which were permissible for a given substance.[13] If there was a film on the metal surface, the brightness did indeed change on heating. Thus, in the case of platinum, a radiation maximum was observed at 400°C and the polarization of the radiation decreased. In the case of aluminum covered with an oxide layer, the brightness of the radiation increased approximately by an order of magnitude and the polarization disappeared almost completely. We must mention that 30 years ago, P. A. Cerenkov carried out, on the advice of S. I. Vavilov, analogous experiments on the influence of temperature on the brightness in order to prove that the polarized radiation discovered by Cerenkov was not luminescence.

Thus, we may assert that the polarized component of the radiation in these experiments was not luminescence. The universal nature of the radiation, its polarization, yield, dependence of the yield on the particle energy, and its angular distribution were in good agreement with the theory of transition radiation.

The experiments on the spectra of the transition radiation are also interesting, and we must consider this problem in some detail. In order to compare the observed spectrum of the transition radiation with the theoretical predictions, it is necessary to know the values of the real and imaginary components of the refractive index, considered as functions of the frequency of light. This is not always simple. For ex-

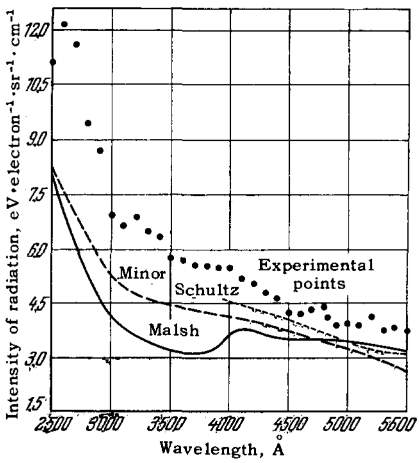


FIG. 9. Comparison of the dependence of the experimental photon yield of copper on the wavelength (points) with the theoretically expected dependence (curves). Electron energy 100 keV, copper foil thickness 1270 Å, angle of observation $\varphi = 30°$ (Emerson et al.[15]).

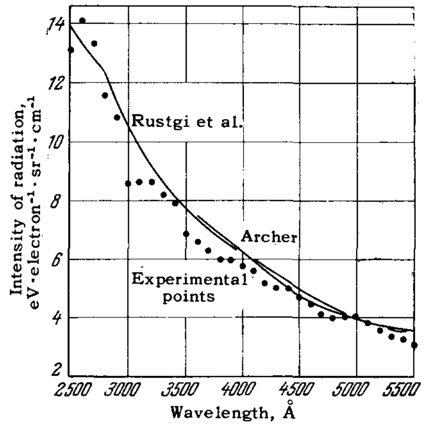


FIG. 10. Experimental dependence of the photon yield of germanium on the wavelength (points) and the theoretically expected dependence (curve). Electron energy 100 keV, foil thickness 690 Å, angle of observation $\varphi = 30°$ (Emerson et al.[15]).

ample, let us consider the dependence of the real component of the refractive index of copper on the wavelength, as measured by various authors (Fig. 8). It is evident from the curves in Fig. 8 that there is practically no agreement between the data of different authors. To a considerable extent, these differences are real and are due to the quality of the metal being investigated. The agreement of the data for the imaginary component of the refractive index is somewhat better but far from satisfactory. Figure 9 shows the experimental data for the transition radiation from copper at an angle $\varphi = 30°$ and an electron energy of 100 keV, as well as the curves calculated using the optical data just referred to. If we allow for all the possible errors, we still find that there is no basic contradiction of the theory. One should compare not only the shape of the curves but the absolute values of the ordinates and this makes agreement even more surprising. Obviously, the experiment in which the same targets were to be used for optical measurements and for measurements of the transition radiation would be decisive. However, so far, the optical and transition radiation problems have attracted the attention of different kinds of physicists. This will obviously continue until optics specialists become interested in the transition radiation.

Although the applicability of the theory of transition radiation to the explanation of the form of the radiation spectrum is not fully proved in the case of copper, better agreement is obtained in other cases. The next figure (Fig. 10) shows the experimental data for the spectrum of the transition radiation of germanium and the theoretical curve calculated on the basis of available optical data.[15] It can be seen in Fig. 10 that agreement is quite good.

To justify the theory, we are obviously interested in the case when the behavior of the refractive index has those singularities which would appear characteristically in the spectrum of the transition radiation. From this point of view, the experiments on silver, carried out by a number of authors, are undoubtedly important. The behavior of the real component n and the imaginary component k of the refractive index of silver is shown in Fig. 11. For any radiation emerging from the interior of a silver sample, the wavelengths at which silver is most transparent ($\lambda$ = 3250 Å) are the most favorable. As far as the transition radiation is concerned, its maximum, as mentioned earlier, should occur at somewhat longer wavelengths, where the real component of the refractive index is small and the imaginary component is still not large. Figure 12 shows the calculated radiation spectra for silver. The graphs given in Fig. 12 were taken from the work of German physicists.[13] Similar graphs were calculated independently in the U.S.S.R. by Pariĭskaya. The agreement, as expected, is good.

Figure 12 shows that the transition radiation spec-

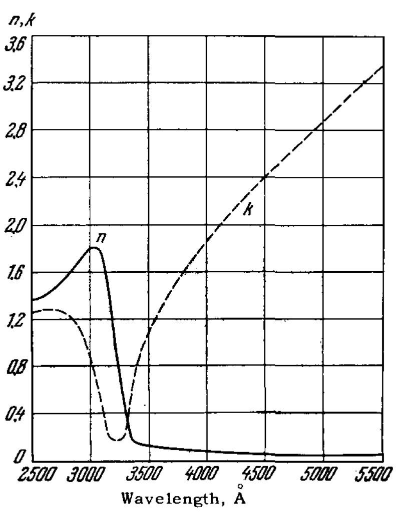


FIG. 11. Dependence of the real n and imaginary k components of the refractive index of silver on the wavelength (figure taken from[15]).

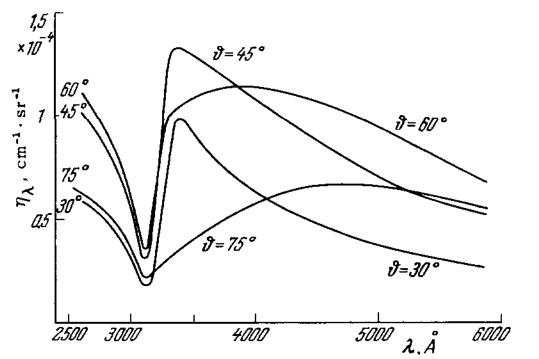

FIG. 12. Transition radiation spectrum of silver for various angles of observation expected on the basis of the refractive index data (Boersch et al.[13]).

trum should, in this case, have a very characteristic form. Having reached a maximum with the reduction of the wavelength to λ = 3450 Å, it should drop suddenly in the region in which silver is transparent. For low angles of emission, for example, $\varphi$ = 30°, the maximum is quite sharp. For an angle of emission equal to 60°, the intensity increases (this, as already mentioned, is typical of the transition radiation), but the maximum becomes flatter. At still higher angles, $\varphi$ = 75°, the intensity of the radiation decreases and the maximum disappears. These features have, in fact, been detected experimentally. The German physicists[13] proceeded as follows. Having measured the spectral density of the reduction in radiation as a function of the energy of the electrons bombarding a target, these physicists represented the data obtained in the form of a sum of two terms: one which was proportional to the energy, and another which was inversely proportional to the electron energy. The result of such an approach at an electron energy of 30 keV is shown in Fig. 13. It is evident from this figure that the spectrum of that part of the radiation which increases in proportion to the electron energy does indeed satisfy the predictions of the theory of transition radiation. The part which decreases as the electron energy increases has the form of a narrow peak lying at the wavelength of 3250 Å, i.e., coinciding with the transparency maximum of silver. The German physicists suggest that this radiation is probably the optical part of the spectrum of the bremsstrahlung radiation, generated in the interior of the silver sample. The maximum peak of this radiation at 30 keV is approximately equal to the intensity of the transition radiation at its maximum. At 100 keV, the peak should be an order of magnitude smaller and can be neglected.

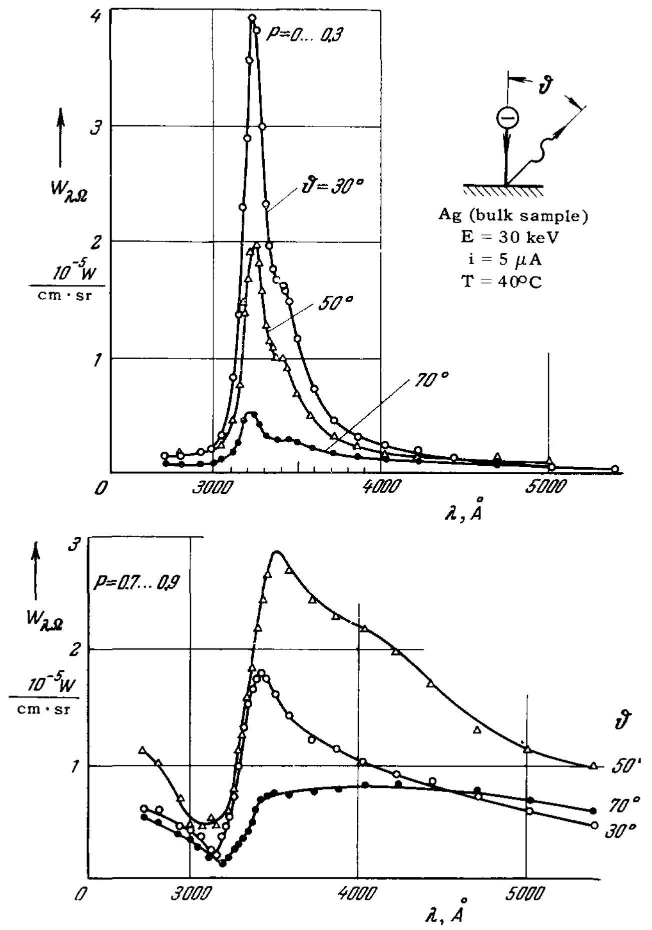


FIG. 13. The separation of the spectrum of silver, observed under electron bombardment, into two components. The separation is made on the assumption that the intensity at each wavelength is the sum of two terms: one which is directly proportional and the other which is inversely proportional to the electron energy E (taken from[13]). The top part of the figure represents the component proportional to 1/E, while the bottom part shows the other component. The analysis was carried out for different angles of observation and E = 30 keV.

The layer from which light may emerge from silver is thin. Even at the transparency maximum, light is attenuated in silver by a factor e in a thickness of the order of 3500 Å. However, in a thin foil we can observe the radiation which appears due to the interference of the transition radiation at the two surfaces of the foil, i.e., where the electrons enter the foil and where they leave it. Consequently, at certain angles and foil thicknesses, the radiation intensity should pass through maxima and minima. This indeed is observed.[18] The next figure (Fig. 14) shows the transition radiation spectrum of silver at an electron energy of 100 keV. The emission angle is 30° and the foil thickness 710 Å. This represents approximately the maximum for the addition of amplitudes from the two surfaces. It is evident from Fig. 14 that the expected radiation maximum (continuous curve) is indeed observed. The experimental points are in reasonable agreement with the calculations although the experimentally observed peak is somewhat flatter than that theoretically expected. In short, we can say that the theory of transition radiation is in satisfactory agreement with experiment by providing an explanation of the radiation spectrum; the data obtained on silver are most convincing in this respect.

Nevertheless, the data for silver have been the sub-

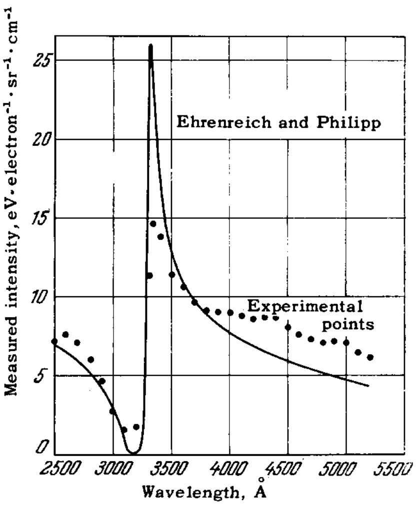


FIG. 14. Theoretically expected and experimentally measured photon intensities from a silver foil 710 Å thick when observed at an angle of 30° for an electron energy of 100 keV (Emerson et al.[15]).

ject of some discussion. The reason for this is that the physicists[18] who investigated the emission of thin silver layers did not know of the existence of the theory of transition radiation. They used the treatment put forward by Ferrel in 1958, who showed that the radiation could be used to detect electron plasma oscillations in a metal, which is excited by fast electrons traversing it.[20,21] Therefore, the observed radiation peak, which depended on the plate thickness, was assumed to confirm the theory. Physicists seem to be obsessed by "plasma." If one had simply suggested an investigation of the transition radiation and not of plasma oscillations, the investigation might not have been carried out.

Silin and Fetisov[22] were the first to show that the results obtained in the U.S.A. fitted well the theory of transition radiation if one used Pafomov's formula[7] for a thin plate. At that time, the data on the transition radiation of silver at longer wavelengths were already available; they had been obtained by Michalak. It was evident that the transition radiation could also be emitted at shorter wavelengths. Therefore, the agreement with theory for shorter wavelengths, and for a thin plate was important.

It would be incorrect, however, to assume that a choice has to be made between the transition radiation and Ferrel's mechanism. The theory of transition radiation is macroscopic, while Ferrel considered the microscopic process. The latter may, to some extent, govern the value of those optical characteristics of a metal which occur in the macroscopic theory. Thus, Ferrel's mechanism is allowed for completely by the theory of transition radiation.

When speaking of that part of the radiation which is coherent with the field of the particle, in a lecture delivered in 1961 in memory of S. I. Vavilov, I formulated the problem as follows:[23] "We can now say that the occurrence of interference indicates that the radiation is governed by the electromagnetic field associated with a moving particle. This field should be described by Maxwell's equations for a medium if they are written correctly for the given case. Since the theory of transition radiation is nothing but a simple consequence of Maxwell's equations, it follows that if the experimental conditions are allowed for correctly, it should give a complete description of the observed radiation. It should include also various types of effect based on special assumptions (with the exception of luminescence). If a more detailed comparison of the theory with experiment should show some discrepancies, this obviously will mean that the equations for the propagation of light in silver require correction. In other words, such investigations represent nothing but a refinement of the data on the optical properties of thin silver layers."

I would like to draw attention to the last sentence, where I spoke of the possibility of measuring optical constants. Unfortunately, my suggestions have met with objections from our American colleagues. In publishing a translation of my lecture, they added (which is a quite unusual practice in journals) the remarks of the translator. These present the formulation of the problem given by Stern.[21] He points out that in the region of the radiation peak of silver, agreement with experiment may be obtained both on the basis of the theory of transition radiation and from the plasma oscillations representation, i.e., "... there are two different ways of considering the same phenomenon. Since the transition radiation includes all types of radiation of a film, it is more general. Ferrel's method makes it possible to calculate correctly the region of the peak and gives the physical mechanism which is responsible for the peak." Thus, this remark confirms rather than rejects my point of view and, in any case, gives no grounds for polemics. If I understand the remark correctly, it contains only the additional proposition that in the region of the peak the main contribution to the behavior of the refractive index is made by plasma oscillations and that the microscopic theory is so far incapable of determining its dependence on the frequency at neighboring wavelengths.

The behavior of the refractive index is governed by the presence of the natural oscillation frequencies of a substance. The mechanism of such oscillations is, in many cases, different from that of plasma oscillations. Partly in an effort to explain this, experiments have been carried out on dielectrics.[17] To me the main problems seems to be to determine to what extent the transition radiation may be used to obtain information about microscopic processes in a substance.

The theory of transition radiation uses only the data on the complex refractive index, which has been determined so far from optical measurements. In con-

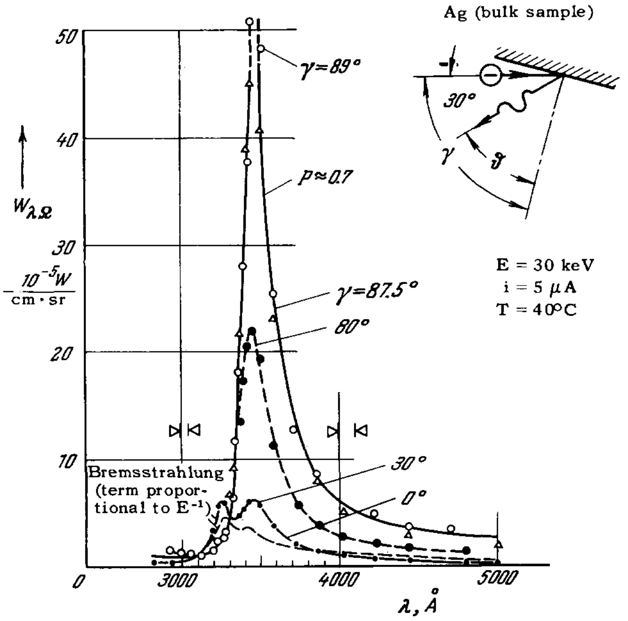


FIG. 15. Radiation of silver for the oblique incidence of electrons on its surface. For angles of incidence of 87.5° and 89° with respect to the normal and an electron energy of 30 keV, the amplitude of the peak is an order of magnitude greater than the intensity of the bremsstrahlung and transition radiations obtained for normal incidence (data given in[19]).

nection with this, the question arises as to whether the information obtained from the observations of the transition radiation is identical with the data on the refractive index obtained by optical methods. The results of investigations of the transition radiation spectrum and intensity, summarized in this lecture, show that at least in the first approximation these data are indeed equivalent. Further experiments will show how exact this agreement is.

The second problem which also requires elucidation is whether it is possible to use the transition radiation to determine not the average properties of a medium but the singularities of a very thin surface layer on the medium. In this connection, I would like to draw attention to the interesting result obtained recently by German physicists.[19] They investigated the radiation of silver in the region of the maximum, to which I have referred earlier, using electrons incident at a very low angle (down to 1°) on the metal surface. (The electron energy was up to 30 keV.) It was found that in this case the radiation peak increased approximately by one order of magnitude and did not coincide with the transparency maximum of silver (Fig. 15). They concluded that the radiation was due to surface plasma waves excited by electrons. It seems to me that this conclusion requires further discussion.

If over the whole of its path an electron remains in the surface layer, the conditions for the observation of the optical part of bremsstrahlung should be very favorable since the light output is then summed over the whole electron path. The intensity of bremsstrahlung should increase with the electron energy and could easily be an order of magnitude greater than in the case of normal incidence.

However, bremsstrahlung is generated when electrons are scattered and the electrons may either penetrate deeper into the layer or they may come back through the target surface. Obviously, this case is not simple to deal with and it seems to me that the applicability of the theory of transition radiation to this case should be carefully analyzed.

## 4. CHARACTERISTICS OF THE OPTICAL PART OF THE TRANSITION RADIATION SPECTRUM OF RELATIVISTIC PARTICLES

We have already seen that the experimental data on the transition radiation of nonrelativistic particles are in agreement with the theory (with the possible exception of the case of grazing incidence of particles on the surface, which is still not clear). So far, the spectrum and intensity of transition radiation have been calculated from the known optical properties of a substance. Obviously, the opposite procedure is also possible, i.e., the optical parameters of a substance can be determined from the transition radiation. We shall not consider whether this has any advantages compared with the usual optical methods. However, this new possibility, like any independent method, may be found useful. In fact, experiments involving the observation of the transition radiation are very simple and, in the case of nonrelativistic electrons, can be carried out in any optical laboratory. Actual experiments are always richer in information than theory and we cannot yet say whether optical measurements are completely identical with measurements using the transition radiation, i.e., whether they might yield data which in some way are supplementary.

At the present state of development of technology, experiments with relativistic particles, particularly electrons, are completely practicable. In fact in some respects, the use of relativistic particles is more convenient. If the transition radiation is observed at an angle $\vartheta$, which is sufficiently large compared with the maximum which is characteristic of a relativistic particle, i.e., $\vartheta \gg E/mc^2$, we can assume that $\beta = 1$ in Eq. (14). In this range of angles, the angular distribution and the spectral density of the radiation are independent of the particle energy. At the same time, the experimental conditions for the observation of the radiation become in some respects simpler, Above all, the intensity of the radiation increases by a factor of $\beta^2$. Compared with 20 keV electrons ($\beta = 0.2$), this gives an increase in intensity by a factor of 25. Moreover, the scattering and energy losses in the surface layer of a substance become less important. This makes it possible to investigate more conveniently the transition radiation, including the radiation generated when a particle emerges from a target, since a thicker

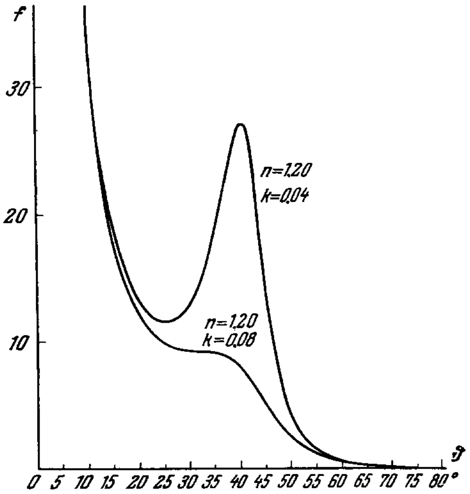


FIG. 16. Angular distribution of the transition radiation of a relativistic particle for angles $\vartheta < 90°$ and two values of the refractive index: n = 1.20, k = 0.04; n = 1.20, k = 0.08.

$$f = \frac{|1-\varepsilon|^2\,|\varepsilon - \sin^2\vartheta|\cot^2\vartheta}{|(\sqrt{\varepsilon-\sin^2\vartheta}+\varepsilon\cos\vartheta)\,(1-\sqrt{\varepsilon-\sin^2\vartheta})|^2},$$
$$\varepsilon = (n+ik)^2.$$

target can be used than that for particles of lower energies.

In this connection, I want to quote some results of calculations that Pafomov and I carried out recently.[25] If a substance is relatively transparent, the measurement of its refractive index and absorption coefficient presents no great difficulties. Therefore, we shall speak only of substances in which light is absorbed in thicknesses equal to a small number of wavelengths. If the absorption is not too strong, the information about the optical properties of a substance may be obtained from the Vavilov-Cerenkov effect. As already mentioned, formula (14) allows automatically for this effect.

Figure 16 shows the angular distribution of the transition radiation for angles which are acute with respect to the direction of motion of the particle. The upper curve in Fig. 16 has a clear maximum, characteristic of the Vavilov-Cerenkov radiation. This curve was calculated for a real component n of the refractive index equal to 1.20, and for an imaginary component k equal to 0.04. For this value of k, the light is reduced in intensity by a factor of e over a path which is approximately equal to 4λ, where λ is the wavelength of light in vacuum, i.e., for visible light, the radiation is absorbed in a thickness approximately equal to 2μ. Thus, the absorption of light is quite strong. The lower curve in Fig. 16 was calculated for a value of the imaginary component of the refractive index which was twice as large: k = 0.08. It is evident from this curve that the maximum is now completely absent. Hence, it follows that even from the amplitude of the maximum we can determine qualitatively the order of magnitude of the absorption coefficient if it is not too high. The amplitude of the maximum and its width obviously depend on the absorption coefficient of a medium, and the position of the maximum depends mainly on the real component of the refractive index. It should be mentioned, however, that at high values of n the Vavilov-Cerenkov radiation will not emerge from the medium even in the case of weak absorption because of the total internal reflection. This can be easily avoided by making the particle cross the boundary of separation not along the normal but at some angle. Formula (14) is then replaced by another, more com-

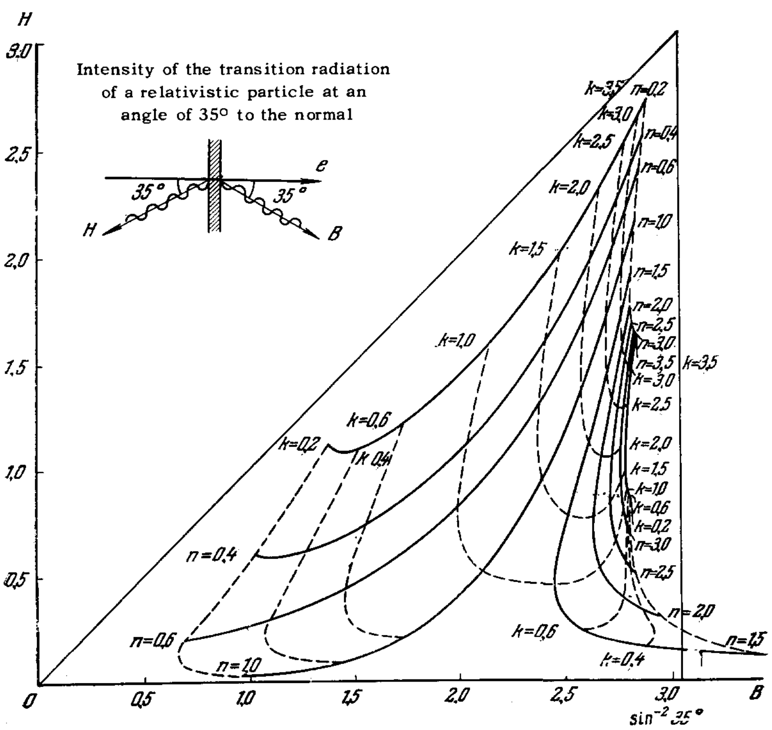


FIG. 17. The abscissa gives the expected intensity f of the transition radiation at an angle of 35° (with respect to the line of motion of a relativistic particle), and the ordinate gives the same for an angle $\pi - 35°$. The continuous curves represent n = const and varying k, the dashed curves k = const and varying n.

plex, formula, but if a computer is used to find the angular distribution curve, this point is not very important.

The question now arises whether there might be simple ways of measuring the optical properties of strongly absorbing media, such as metals. Various possible experimental arrangements may be suggested. I would like to point out one of them, which attracted the attention of Pafomov and myself in the analysis of the angular distribution curves of the transition radiation. It is found that the intensity of the radiation observed at acute and obtuse angles $\varphi$, measured from the line of motion of the particle (for particles incident normal on the surface), not only depends on the real and imaginary components of the refractive index but that these dependences are also very different. This can be seen in Fig. 17. The letter B represents the intensity of radiation emitted at an acute angle $\varphi = 35°$, while the letter H represents the radiation observed at an angle $\pi - 35°$, as shown at the top of the figure. The abscissa represents the value of B and the ordinate represents H. The continuous curves represent a fixed value of n but a varying value of k, while the dashed curves represent a fixed value of k but a varying value of n. It is evident from Fig. 17 that two series of curves are obtained and that the curves in one series do not intersect. Therefore, each of the points on the graph represents a unique value of the magnitudes of the real and imaginary components of the refractive index. This means that having measured the transition radiation at acute and obtuse angles, for a given electron current, using any wavelength and comparing the intensities with some standard, we can determine directly the values of the optical properties of a substance. It is at present difficult to say whether such a method would be convenient or practical. However, since the transition radiation of relativistic particles has hardly been investigated,* we can at least hope that such graphs would be useful in making a comparison of the experimental data with the theory.

I have attempted to describe the present state of the knowledge of the transition radiation. It is very difficult to forecast what applications any particular effect might have. It seems, however, that the investigations of the transition radiation have reached a stage when applications might be considered.

*Some data are given in[24].


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Translated by A. Tybulewicz