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Physics of Our Time

THE PRESENT STATE OF THE THEORY OF SUPERCONDUCTIVITY*

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SUPERCONDUCTIVITY, one of the most unusual physical phenomena, is attracting at present particular attention. The construction of powerful permanent magnets of superconducting alloys giving fields up to 100,000 Oe led to a substantial increase in the pace of scientific and technological research on the subject. In this short lecture it is difficult to cover all the various properties of superconductors and the full range of work in this field. I shall therefore confine myself to a brief description of the essential facts and their explanation, and after that I shall say a few words about recent work and about the future prospects.

Superconductivity was discovered by the Dutch physicist Kamerlingh-Onnes in 1911. He measured the electric resistivity of metals at helium temperatures (i.e., temperatures below the boiling point of helium, which is 4.2°K) and found that at a temperature near 4°K the resistance of mercury suddenly drops to zero. Subsequently many more superconducting metals and alloys were discovered with widely differing values of the transition temperature, or so-called "critical temperature." The highest critical temperature, 18°K is that of the alloy Nb_3Sn, and the alloy Bi_2Pt, for example, has a critical temperature of only 0.155°K.

In 1914 Kamerlingh-Onnes discovered that superconductivity is destroyed when the metal is placed in a sufficiently strong magnetic field (critical field). The magnitude of this field depends on temperature. It is greatest at absolute zero (a few hundred or a few thousand Oe for pure metals). It decreases with rising temperature and becomes zero at T_{cr}. If the transition takes place in the presence of a magnetic field it is a normal phase transition, a so-called first-order phase transition, which is accompanied by the release or the absorption of a latent heat. If the transition takes place in the absence of a magnetic field, i.e., at T_{cr}, it is a transition of second order, in which there is no latent heat, and one observes only a jump in the specific heat. Kamerlingh-Onnes showed also that superconductivity is destroyed by the passage of an electric current exceeding a certain magnitude.

Another important property of superconductors was discovered in 1933 by Meissner and Ochsenfeld. It was found that an external magnetic field does not penetrate into the body of a thick superconductor, and that on the transition into the superconducting state the field is, as it were, pushed out of the superconductor. It was shown theoretically by Mrs. de Haas in 1926, and experimentally by Shoenberg in 1939, that this is connected with the appearance of currents in a surface layer of the superconductor of 10^{-5}—10^{-6} cm thickness which screen the interior of the superconductor from the external field. The thickness of this layer was called the penetration depth.

I cannot describe the large number of varied and interesting studies which were carried out subsequently. Many properties of superconductors were discovered and explained thanks to the remarkable experimental work of L. V. Shubnikov, A. I. Shal'nikov, Yu. V. Sharvin, N. V. Zavaritskiĭ, and M. S. Khaĭkin in the U.S.S.R., and K. Mendelssohn, E. T. S. Appleyard, A. B. Pippard abroad, and the theoretical work of A. J. Rutgers, C. H. Gorter, the brothers F. and H. London, R. E. Peierls and the Soviet physicists L. D. Landau and V. L. Ginzburg. However the basic cause of superconductivity remained a mystery until the 1950's.

What made this phenomenon so complicated, and why did it remain for so long unexplained? To answer this question one has to remember a similar phenomenon, the superfluidity of liquid helium. As we know, if liquid helium is cooled below 2.18°K it changes to a peculiar modification which is called superfluid helium or helium II. The ability of helium II to flow through narrow capillaries without friction was discovered by P. L. Kapitza in 1937 and explained by L. D. Landau in 1941. Very roughly, the Landau theory argues as follows. Helium II is a quantum liquid, in which energy and momentum can increase or decrease only by definite amounts or quanta. The friction of a flowing liquid must represent a change in its energy and momentum. It turns out that the helium in motion can absorb such a quantum only if the velocity of flow exceeds a certain critical value. At lower velocities it therefore behaves like a perfect non-viscous liquid.

An attempt to apply this theory mechanically to the electrons in a metal and to assume that superconductivity is the superfluidity of a charged electron fluid ended in failure. It turned out that a fluid consisting of independent electrons could absorb quanta and therefore undergo friction at any velocity of flow. The difference between liquid helium and an electron fluid is connected with the fact that the intrinsic angular momentum (spin) of the helium atom is zero and that

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of an electron is $\frac{1}{2}\hbar$ ($\hbar$ is Planck's constant). However, if two such particles of spin $\frac{1}{2}\hbar$ are bound together we know that the composite particle may have an intrinsic angular momentum of zero or of an integral multiple of $\hbar$. If the electron fluid therefore consisted of such bound pairs it might be superconducting. However, electrons carry like charges. They should repel each other, and there would seem to be no reason for the formation of bound states.

This puzzle remained, as I have mentioned, until the 1950's. In 1950 W. D. Allen* and others, working with mercury, and E. Maxwell, working on tin, discovered an interesting phenomenon: the influence of the isotopic composition of the metal on its critical temperature. In addition to the electrons, the metal contains positively charged ions, which form the crystal lattice. A change in the isotopic composition means a change in the mass of the ions. The latter, however, influences only the vibration frequencies of the lattice. It follows that lattice vibrations are directly connected with superconductivity. Starting from this discovery, H. Fröhlich and J. Bardeen developed a theory of a particular kind of attraction between the electrons. One of the consequences of quantum mechanics is the conclusion that a lattice must always, even at absolute zero, be in a vibrating state (zero-point vibrations). According to the theory of Fröhlich and Bardeen each electron changes the behavior of the zero-point vibrations and this gives rise to a field of force which acts on another electron. In this way there must be an interaction between the electrons which is necessarily attractive and can dominate over the electrostatic repulsion.

On the basis of this fact J. Bardeen, L. N. Cooper, and J. R. Schrieffer in America, and N. N. Bogolyubov in the U.S.S.R., developed a microscopic theory of superconductivity which gave a full explanation of this phenomenon.

It was shown that the attractive forces lead to the formation of bound pairs (sometimes called Cooper pairs). It is interesting that this is not just a process involving the two electrons which form the pair, but that the whole collective system of electrons in the metal takes part; it turns out in particular that for this reason an arbitrarily weak attraction is sufficient for the formation of pairs. Any transfer of energy to the electron fluid in the superconductor requires the breaking of a pair. But this means that the energy which is transferred cannot be arbitrarily small, but must necessarily exceed the binding energy of a pair. This lower limit to the energy transfer is called the gap in the energy spectrum. As is well known from the theory of semiconductors, such a gap necessarily leads to an exponential temperature dependence of the electronic part of the specific heat and of the thermal conductivity, and this is indeed observed. From the existence of an energy gap it follows that a fluid of electron pairs can move without friction up to a certain velocity. In other words the electric current flows without resistance. As I already stated, the pair formation is a collective effect and is therefore connected with the state of the whole electron system. As the temperature is increased some of the pairs are broken, and this in turn affects the binding energy of the remaining pairs. As a result, the binding energy decreases, and at the critical temperature it becomes zero. No pairs are then left and the metal becomes normal in all respects.

*The author probably means Reynolds et al.—Translator.

I shall now present some equations and diagrams.

(1) The binding energy of a pair is $2\Delta(T)$,

$$\Delta(0) = 2\hbar\omega_D e^{-1/\eta},$$

where ω_D is the Debye frequency; in a metal $\hbar\omega_D$ amounts to 300—400°K. The quantity η is a dimensionless constant which measures the interaction of the electrons with the lattice. In all known cases $\eta < \frac{1}{2}$. The fact that this limit is one of principle has been proved by using rather special assumptions.

(2) The critical temperature is (in energy units)

$$T_{cr} = \frac{\Delta(0)}{1.76}.$$

It is possible that the smallness of the critical temperature is connected with the existence of a limit for η.

(3) The electronic specific heat:

(a) in the normal state

$$C_n = \gamma T \quad \gamma \sim 10^2 - 10^4 \text{ erg/cm}^3\text{deg}^2$$

(b) near $T = 0$

$$\frac{C_s(T)}{C_n(T_{cr})} = 1.35\left(\frac{\Delta(0)}{T}\right)^{3/2} e^{-\Delta(0)/T};$$

(c) near $T = T_{cr}$

$$\frac{C_s(T)}{C_n(T_{cr})} = 2.43 + 3.77\left(\frac{T}{T_{cr}} - 1\right);$$

from which we see that the specific heat has a discontinuity at $T = T_{cr}$.

(4) Temperature dependence of the critical field:

(a) near $T = 0$

$$H_c(T) = H_c(0)\left[1 - 1.06\left(\frac{T}{T_{cr}}\right)^2\right];$$

$H_c(0)$ is proportional to T_{cr} and is in the region of hundreds to thousands of Oersted;

(b) near $T = T_{cr}$

$$H_c(T) = 1.73 H_c(0)\left(1 - \frac{T}{T_{cr}}\right).$$

The comparison of these and other theoretical relations with the experimental data for pure metals shows excellent agreement (Figs. 1—3).

With the new theory one can describe the behavior of superconductors in constant and varying electromagnetic fields. For this the so-called superconducting

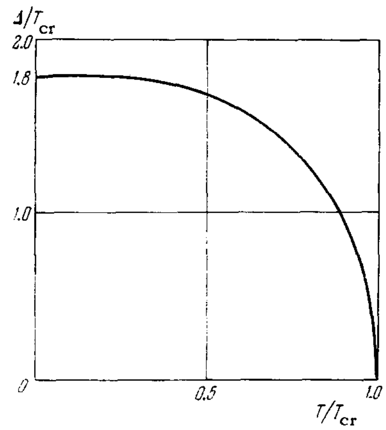

FIG. 1. Dependence of energy gap Δ on temperature.

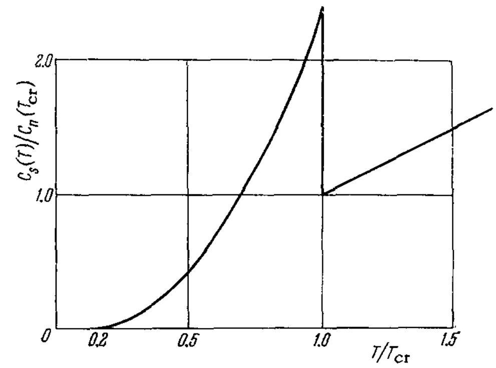

FIG. 2. Temperature dependence of the electronic specific heat.

correlations are essential. The Cooper pairs of electrons are fairly big objects. Their radius is of the order of 10^{-4}—10^{-5} cm, i.e., a few thousand times the interatomic distance. It follows that the motion of electrons at different points of the metal shows correlations over distances of the order of the dimensions of a pair (one uses the term correlation length). It follows that the electric current at a given point in the metal depends on the electromagnetic field not only at that point, but in a region with dimensions of the order of the size of a pair. This leads to an integral, or "non-local," relation between current and field. There are two limiting cases. If the electromagnetic field varies only slightly over distances of the order of the correlation length, the correlation effects are unimportant, and the current depends practically only on the field at the same point. In that case one finds the so-called London equations, which were proposed by the brothers F. and H. London in 1935 on the basis of phenomenological arguments. In the opposite limit one obtains a special kind of integral relation between current and field which was proposed by A. B. Pippard in 1953, also before the appearance of the microscopic theory of superconductivity. The parameter which characterizes the distance over which the field changes appreciably, can be taken to be the penetration depth. Indeed, this is just the distance over which the magnetic field in the superconductor decreases from its external value to zero. Thus we are concerned with the relation between the penetration depth and the correlation length. The experiments show that all pure metals belong either to Pippard's limit or to the intermediate case, i.e., the correlation length is either much greater than the penetration depth (e.g., in aluminium) or of the same order (e.g., in tin).

The finite value of the correlation length made it possible to explain the origin of the surface energy at the border between the normal and superconducting phase. The existence of such an energy follows already from the fact that a solid superconducting cylinder (of pure metal) in a longitudinal field changes discontinuously from the superconducting to the normal state when the field passes its critical value. The point is that we know that the critical field of a thin layer is greater than the critical field of a thick sample. Imagine that the transition occurred by way of the formation of alternate normal and superconducting layers in the cylinder. The layers could then become thinner and thimmer and give a transition at higher and higher fields. This does not happen, just because each boundary requires energy. The magnitude of the surface energy was measured by Yu. V. Sharvin in his very beautiful experiments on the properties of the intermediate state. The hypothesis of the existence of such an

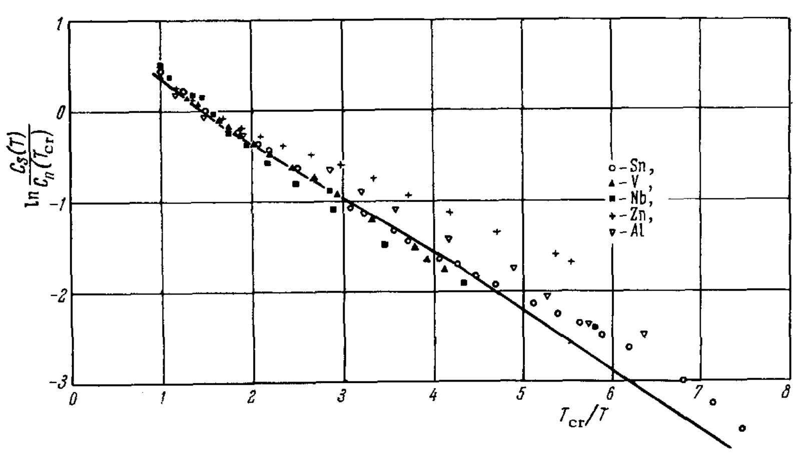

FIG. 3. Comparison between theory and experiment on the electronic specific heat. The systematic deviation at low temperatures (right-hand part of the figures) is due to the anisotropy of the energy gap in real metals, which increases the electronic specific heat compared to the isotropic case assumed in the theory.

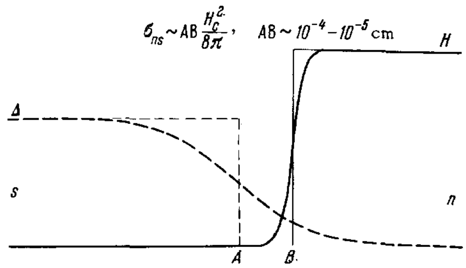

FIG. 4. Origin of the surface energy, schematic. (Explained in the test).

energy was proposed by L. D. Landau in 1937 in a paper on the theory of the intermediate state.

The problem of the surface energy is one of the fundamental ones, and we must therefore consider it in some detail. According to our present ideas the surface energy arises in the following manner. Figure 4 shows the conditions for the formation of a boundary between a normal and a superconducting phase. In the superconducting phase there are pairs with a binding energy of 2Δ. In the normal state $\Delta = 0$. However, the state of the electrons in the metal cannot change over distances less than the correlation length. Therefore Δ varies approximately as shown in the diagram. On the side of the normal phase there is a magnetic field of magnitude H_c (otherwise there could not be equilibrium). The field inside the superconductor must equal zero. This means the field decreases from H_c to zero over a distance of the order of the penetration depth. If we replace the continuous variation by sharp breaks at A and B without changing the area under the curves we are left with a region AB in which, on the one hand, the binding energy of the pairs is zero so that it is like a normal metal, and, on the other hand, the field does not penetrate. Hence this layer AB carries a wasted energy of $(AB)H_c^2/8\pi$ per unit area.

This is the situation when the correlation energy is greater than the penetration depth, but the opposite may also happen. In that case the surface energy is negative. In this way one can classify the superconductors in two types according to the sign of the surface energy: type 1 and type 2 superconductors. The hypothesis that there exist in nature two types of superconductors was proposed by myself in 1952 on the basis of an analysis of the experimental results of N. V. Zavaritskiĭ on the critical fields of thin films. By 1957, before the appearance of a microscopic theory of superconductivity, a general theory of type 2 superconductors was successfully developed. I based this theory on the equations which V. L. Ginzburg and L. D. Landau had obtained in 1950 from semi-phenomenological considerations. After the completion of the microscopic theory, L. P. Gor'kov put it into a suitable form for the description of spatially non-uniform problems. He showed in 1959 that the Ginzburg-Landau equations follow from the exact theory near the critical temperature. As a result it became possible to find the significance of the parameters which appear in these equations.

Why are these problems of interest? After all, every known pure superconductor, with the exception of niobium, belongs to type 1. The point is that any pure superconductor can be converted into a type 2 superconductor by adding impurities or creating defects in the crystal lattice. The electrons in such a superconductor are scattered by the defects, and thereby the superconducting correlations are destroyed. As a result the correlation length can be reduced considerably, and its ratio to the penetration depth can be reversed. As I have pointed out this gives rise to a negative surface energy, i.e., the superconductor becomes one of type 2. This is exactly what happens in most superconducting alloys.

In a type 2 superconductor the character of the transition is radically different from the transition in a type 1 superconductor. Since the concept of a phase boundary here loses its meaning the transition takes place gradually and can extend over a very large range of fields. There are now two critical fields, a lower and an upper one. A field which is less than the lower critical field does not penetrate into a thick superconductor. Then penetration begins gradually until above the upper critical field the superconductor is in the normal state. It is very important that the absence of resistance for a sufficiently weak current persists right up to the upper critical field, which may be very large. It rises in proportion with the concentration of defects, i.e., with the decrease of the correlation length, and may reach the order of several hundred thousand Oersted.

It is interesting that in a surface layer of a thickness of the order of the correlation length the superconductivity persists to even bigger fields (1.7 times in a longitudinal field). But this phenomenon can be observed only in very carefully prepared samples.

The penetration of a magnetic field into a type 2 superconductor makes a very characteristic picture. At fields close to the lower critical value the penetration begins with the formation of separate filaments of magnetic flux. In the center of such a filament the binding energy of the pairs has dropped to zero. It reaches the normal value at a distance from the center of the order of the correlation length. The magnetic field is greatest in the center of the filament and decreases to zero over a distance of the order of the penetration depth. Each filament is a tiny solenoid which carries a well-determined amount of magnetic flux. This flux equals the so-called flux quantum, and its magnitude is 2.06×10^{-7} Oe-cm^2. Near the first critical field the distance between these filaments (they are called quantum vortices, or filaments of magnetic flux) is infinite. As the external field increases they approach each other. In ideal circumstances they form a periodic structure, and in cross-

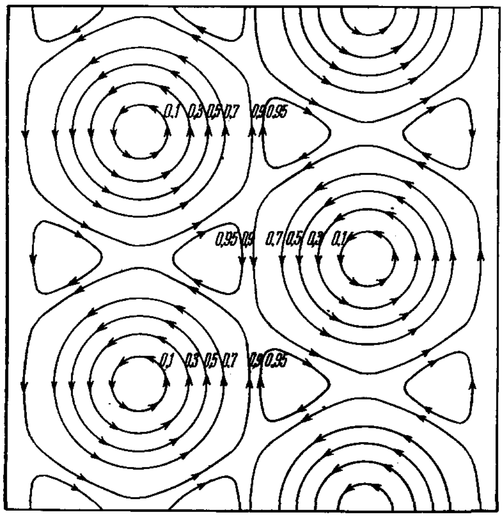

FIG. 5. Distribution of currents in the mixed state near H_{c2}. The streamlines are also lines of constant field and constant energy gap. The numbers indicate the ratio $(H_c - H)/H_c$.

section give a triangular lattice. The accumulation of filaments continues to the point when the distance between their centers is of the order of the correlation length. After this the field between the centers increases with increasing external field until it equals the external field when the upper critical field is reached. This describes the nature of this peculiar state, which is called the mixed state. We note that according to this description there is no Meissner effect in a type 2 superconductor above the first critical field, i.e., there is superconductivity, but the magnetic field partly penetrates.

At this point we should add some remarks about thin superconducting films. They are produced by depositing metal from vapor. If the thickness of the film is less than the correlation length of the substance of which it is made, then this thickness will play the part of the correlation length. If the film is also thinner than the penetration depth then the film becomes a superconductor of type 2 with a correlation length of the order of the film thickness. The critical field of the film will therefore increase as the thickness is decreased.

I shall now again present some equations and diagrams (Figs. 5—16) descriptive of type 2 superconductors.

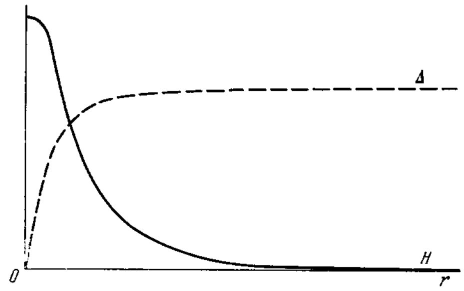

FIG. 6. Binding energy of the pairs and field in a magnetic flux filament.

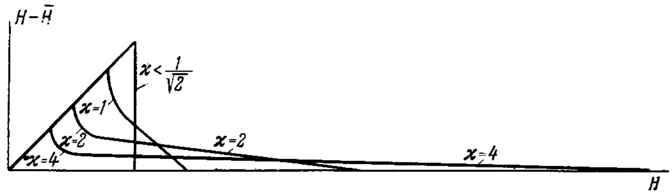

FIG. 7. Theoretical variation of the magnetization M as a function of the external field H for different values of κ. The straight piece, rising at 45° to the horizontal, indicates that the field does not penetrate into the superconductor. The curve starts deviating from it at H_{c1}. M(H) goes to zero at $H = H_{c2}$.

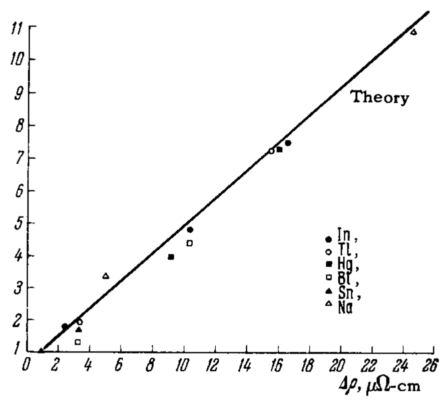

FIG. 9. Comparison of theory and experiment for the variation of $H_{c1}/H_{cm} = \kappa\sqrt{2}$ with the residual resistance $\Delta\rho$ for alloys of lead with different metals.

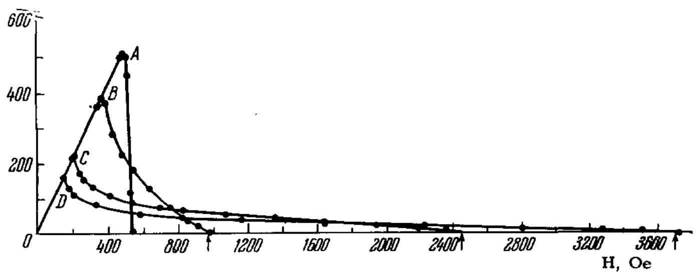

FIG. 8. Experimental results for M(H) in Pb-In alloys at different concentrations (the ordinate scale is twice that of the abscissae). The ordinates are $H - \bar{H}$.

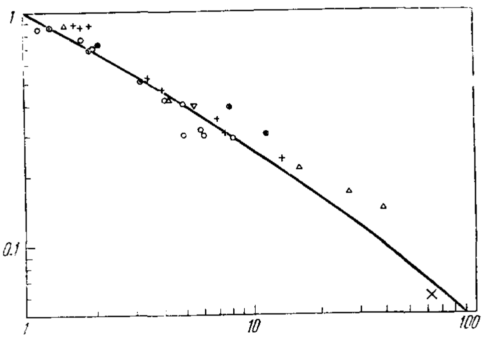

FIG. 10. Comparison of theory and experiment for the variation of H_{c1}/H_{cm} for various alloys of tantalum, niobium, vanadium, and alloys of lead with tellurium, bismuth with indium, and the compound V_3Ga.

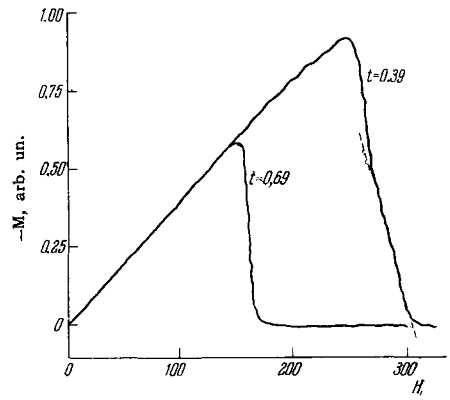

FIG. 11. Change of the alloy In + 1.5% Bi with decreasing temperature from type 1 to type 2. H is in Oe. Since κ increases by 25% as the temperature is lowered from T_{cr} to 0, it is possible to select an alloy for which $\kappa < 1\sqrt{2}$ (type 1) when $T = T_{cr}$ ($t = T/T_{cr}$) and $\kappa > 1\sqrt{2}$ (type 2) near $T = 0$. This condition is satisfied for In + 1.5% Bi. At the higher temperature there is a sharp transition, whereas at the lower temperature there appears a region with the mixed state.

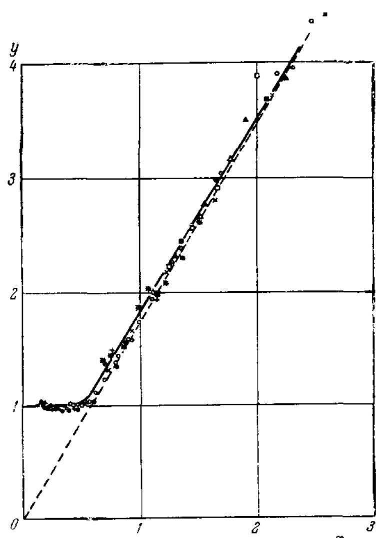

FIG. 12. Comparison of theory and experiment for the critical field of thin films. Mercury films were formed by deposition at low temperatures. The mean free path in such films is $l \sim 10^{-7}$ cm, so that they belong at all thicknesses to type 2.

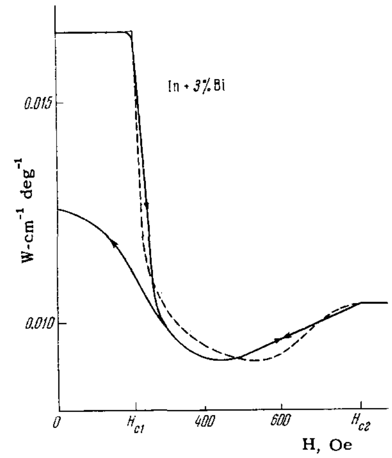

FIG. 13. Comparison of theory and experiment for the thermal conductivity of In + 3% Bi in a magnetic field. The rising-field curve corresponds to the equilibrium structure.

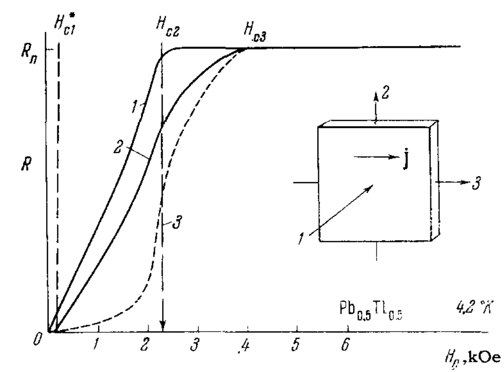

FIG. 14. The real part of the high-frequency surface resistance of the alloy $Pb_{0.5}Tl_{0.5}$ in a magnetic field. The arrows show the direction of the ac current (j) and the dc magnetic field (1, 2, or 3). If the applied field is parallel to the surface the surface layer remains superconducting up the field $H_{c3} = 1.7\ H_{c2}$.

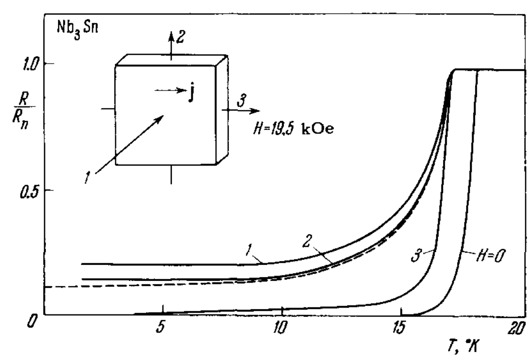

FIG. 15. Temperature dependence of the real part of the surface resistance of Nb_3Sn in a magnetic field for different directions of the dc magnetic field relative to the ac current. As $T \rightarrow 0$, R does not tend to zero if the current is perpendicular to the vortex filaments.

1. Criteria for type 1 and 2 superconductors. One introduces a parameter κ defined by

$$\varkappa(T) = \varkappa(T_{cr}) A(T),$$

where A(T) varies from 1 at T_{cr} to 1.25 at T = 0. For pure superconductors

$$\varkappa_0(T_{cr}) = \frac{\delta L(0)}{\xi_0},$$

where $\delta_L(0) = (mc^2/4\pi Ne^2)^{1/2}$ is the penetration depth at T = 0, $\xi_0 = 0.18\hbar v/T_{cr}$ the correlation length, v the mean electron velocity, m the electron mass, e the electron charge, and N the number of electrons per cm^3. For superconductors with defects or, what amounts to the same thing, for alloys

$$\varkappa(T_{cr}) = \varkappa_0(T_{cr}) + 7.5\cdot 10^3 \gamma^{1/2}\varrho;$$

γ is the coefficient of the temperature in the linear law for the electronic specific heat in the normal state in $erg/cm^3\,deg^2$, and ρ is the resistivity in the normal state (its constant part or "residual resistivity") in ohm cm.

We have then the criterion

$$\varkappa < \frac{1}{\sqrt{2}} \text{ — for a type 1 superconductor}$$

$$\varkappa > \frac{1}{\sqrt{2}} \text{ — for a type 2 superconductor.}$$

2. Simple order-of-magnitude estimate of the upper critical field H_{c2}.

In a magnetic field H a Cooper pair moves in a spiral with the radius

$$r_H = \frac{cp_\perp}{eH},$$

where $p_\perp$ is the momentum component of the pair at right angles to H. The radius r_H should exceed the dimensions of the pair, i.e., $r_H > \xi$. On the other hand the momentum of the pair cannot exceed $\hbar/\xi$ where $\hbar$ is Planck's constant. This condition arises from the uncertainty principle. Hence $p_\perp < \hbar/\xi$. It follows that $H < H_{c2} \sim c\hbar/e\xi^2$. In alloys $\xi \sim \sqrt{\xi_0 l}$, where l is the mean free path of the electrons in the normal state. Consequently

$$H_{c2} \sim \frac{ch}{e\xi_0 l}.$$

3. Critical field (theoretical results).

$$H_{c1} = \frac{H_{cm}}{\sqrt{2}\varkappa}[\ln(\varkappa + 1.8) + 0.08],$$

$$H_{c2} = H_{cm}\sqrt{2}\varkappa,$$

where H_{cm} is determined from the area under the magnetization curve as a function of the field:

$$\frac{H_{cm}^2}{8\pi} = -\int_0^{H_{c2}} M(H)\,dH.$$

4. Field dependence of the magnetization. At $H = H_{c1}$ the gradient dM/dH becomes infinite, and at $H \to H_{c2}$

$$-4\pi M = \frac{H_{c2} - H}{1.16(2\varkappa^2 - 1)}.$$

5. A typical quantitative comparison of theory with experiment. In the alloy In + 2.5% Bi, κ was determined from the four different relations quoted above (H_{c1}, H_{c2}, M and the expression for κ in terms of the resistance in the normal state); with the results

$$\varkappa^{(1)} = 1.76, \quad \varkappa^{(2)} = 1.80,$$
$$\varkappa^{(3)} = 1.78, \quad \varkappa^{(4)} = 1.77.$$

6. The only pure metal for which $\kappa > 1/\sqrt{2}$ is pure niobium ($\kappa(0) = 1.2$). However, in alloys one can reach $\kappa \sim 100$; for example $\kappa(0) = 95$ in the alloy Ti + 25% V.

Recently much theoretical and experimental work has been done on the mixed state. We should mention the work of the French physicist Jacrot and his collaborators, who have studied the mixed state by means of the diffraction of neutrons by the vortex lattice. They found the diffraction peaks and determined the lattice spacing which agreed with the theoretical predictions. Interesting work was also done by the French group of theoreticians working with de Gennes. Apart from predicting the phenomenon already mentioned of the retention of superconductivity in a surface layer, they showed that, from the point of view of thermal properties, the quantized vortices behave like grains of normal metal of a diameter of the order of the correlation length. The low-temperature specific heat of type 2 superconductors contains a term proportional to the temperature and to the number of vortex filaments.

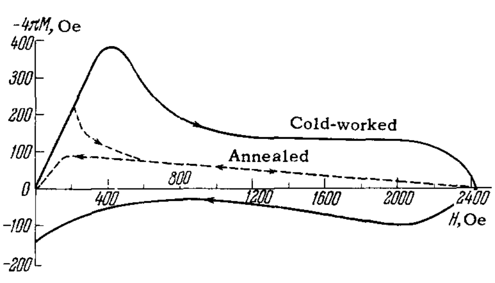

FIG. 16. Magnetization curve for an alloy with macroscopic inhomogeneities. This curve does not correspond to an equilibrium state, as is evident from the substantial difference between the curves for rising and falling field. After long annealing at a temperature close to the melting point the alloy has become more homogeneous and the magnetization curve approaches its equilibrium form. Some difference between the up and down curves is accounted for by the formation of a magnetic "potential barrier" at the surface, which impedes the ejection of magnetic flux lines from the superconductor. In that case the equilibrium curve is the one taken with rising field. We should draw attention to the fact that the annealing has practically no effect on the upper critical field H_{c2}.

These results are confirmed by experiment. Finally I should mention the work of M. P. Kemoklidze, I. M. Khalatnikov, and myself, which determined theoretically the frequencies of normal vibrations of the vortex filaments in the superconductor. There are as yet no experiments on this question.

I shall mention a little later the practical applications of type 2 superconductors, which arise from their high critical fields. For such applications it is important to have not only a high critical field but also a high critical current. What is the position in this respect? In bulk superconductors of type 1 the critical current is determined by the so-called Silsbee condition. This gives a current of such strength that the field produced on the surface of the superconductor reaches the critical value. If this is exceeded, a finite resistance appears. In bulk superconductors of type 2 the situation is different. In the case of homogeneous superconductors the Silsbee condition is also valid, and the field must then not exceed the lower critical value. This is connected with the fact that in a type 2 superconductor in the mixed state no resistanceless current can flow at right angles to the filaments. This is illustrated in Fig. 15. However, the field H_{c1} decreases with increasing κ, so that the product $H_{c1}H_{c2}$ stays approximately constant. This results in very unfavorable conditions for the current. However, in reality the position is not so serious. The point is that in the inhomogeneous superconductors one uses in practice the magnitude of the critical current depends strongly on their structure. This is connected with the fact that the currents one observes in practice are not necessarily the true critical currents for equilibrium conditions. In practice the current flows with an extremely small, but finite, resistance, which is connected with the dissipative processes in the non-uniform superconductor. These processes are so slow that one can in practice reach quite high values of the current with practically no resistance. Besides this it is possible to make "sponges" with a higher critical parameter, allowing the whole current to flow. One interesting feature of the currents in nonuniform superconductors is the fact that they flow not only on the surface, as is the case in uniform superconductors, but to a considerable extent also in the body of the specimen. In such non-uniform specimens current densities up to 10^5 A/cm^2 have been reached with practically zero resistance.

A particularly interesting question concerns currents in thin films. If the film thickness is less than the penetration depth, and also less than the correlation length, current densities of hundreds of millions of amperes per square centimeter can be reached. These are theoretical figures, the practical results are about ten times lower, but that is already very high.

As many of you undoubtedly know, superconducting alloys are now used to make extremely strong permanent magnets. In principle this is very simple. If one makes a closed superconducting ring and induces a current in it, then, because of the absence of resistance, the current goes on circulating in the ring for ever without decreasing. In place of a ring one can also use a short closed coil. Such a coil carrying a current differs from an ordinary electromagnet by not requiring an external power source. In an electromagnet the energy of the generator is used up to overcome the metallic resistance and is ultimately converted into heat. This is an undesirable effect. The heat has to be conducted away. A superconducting coil, once it is "charged up" with field, does not require any further power supply. Naturally such coils also have their weak point; this is the very low temperature. But in return for this one can obtain extremely high fields. At present there exist magnets using the alloy Nb_3Sn with fields up to 100 kOe. This is not yet the limit. It is known that the critical field of this alloy, Nb_3Sn, is at low temperatures approximately 200 kOe, and in V_3Ga it exceeds 300 kOe. The credit for the discovery of such alloys and the construction of superconducting magnets belongs chiefly to the American group of Kunzler. The first work on such magnets dates from 1961. For me, as a theoretician, it is difficult to judge the technological difficulties arising in the manufacture of the coils. However, it seems to me that the future lies not in the present type of magnet containing windings of wire or ribbon, but in superconducting films of a thickness below 10^{-5}–10^{-6} cm.

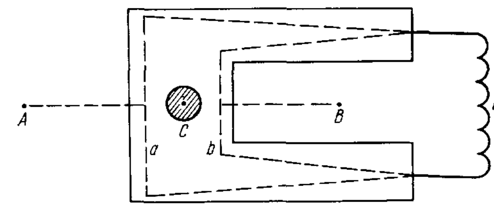

FIG. 17. Schematic diagram of "pumping" field into a coil (explanation in the text).

In connection with the problem of coils it is interesting to remember that the charging up of such coils can in principle be carried out by means of a much weaker field using the method of "magnetic flux pumping." One way of doing this is illustrated in Fig. 17. The coil is connected with a superconducting plate. A certain field is then applied at right angles to the plate, which destroys the superconductivity in a certain region. This region is then moved to the interior of the circuit and then the direction of the field is reversed. This induces in the circuit a current which will compensate the change of magnetic flux. Now the normal region is again moved along the plate outside the circuit; the magnetic field is again reversed, and the cycle repeated. This method has not yet been used technologically (fields of only 15 kOe have been reached) but it is possible that in the future the coils are going to be charged in this manner.

Apart from these coils superconductors have one other important application. The disruption of superconductivity by a magnetic field forms the basis for the so-called cryotrons, which may be used as memory units in electronic computers. Cryotrons have now been developed which consist of superconducting films of a few microns thickness and a few square millimeters in area. The operating time of such devices is less than 10^{-8} seconds. They will be of value for the miniaturization of computers.

Let us now turn to other aspects of research on superconductors. A number of interesting studies have been carried out recently. One finds very unusual properties in superconductors with an admixture of magnetic atoms, i.e., atoms with incomplete inner shells, which have magnetic moments. The theory of such superconductors was worked out in 1960 by myself and Gor'kov. It was found that magnetic impurities had a strong effect on the critical temperature. This decreases rapidly with increasing concentration of the impurity, and a concentration of the order of one per cent is already sufficient to destroy superconductivity completely. It should be noted here that ordinary nonmagnetic impurities change the behavior of a superconductor in a magnetic field, but in low concentrations have very little effect on the transition temperature.

It was also found that in the presence of magnetic impurities the gap in the energy spectrum disappears even before the metal loses its superconductivity. This occurs for an impurity concentration which is about 90 percent of the value which would completely destroy superconductivity, and in this situation the superconductor can receive energy in arbitrary amounts. In that case the electronic specific heat does not show the exponential temperature law but a linear law like the normal metal although with a different coefficient. The absence of a gap in the spectrum means that the pair binding energy is variable and there exist, in particular, pairs for which the binding energy is arbitrarily small. This does not rule out superconductivity since one can apply here a reasoning similar to that of the Landau theory for helium. Subsequently Maki, in Japan, found that the gap in the spectrum disappears also for thin superconducting films in a magnetic field which is 95 per cent of critical.

The experimental study of superconductors with magnetic impurities is in its early stages, but the findings so far confirm the theory.

On the question of type 2 superconductors and superconductors with magnetic impurities one of the originators of the theory of superconductivity, John Bardeen, said that after the discovery of superconductivity one found first the Meissner effect, and then, after a long time, the gap in the energy spectrum. And then the Russians first abolished the Meissner effect and then closed the gap.

Continuing the study of superconductors with magnetic impurities, Gor'kov and Rusinov showed in 1963 the possibility of a phase which would at the same time be superconducting and ferromagnetic, i.e., the magnetic moments of the impurity atoms would in such a phase be oriented. This had already been noticed in experiments, but the result seemed to give rise to objections in principle and was ascribed to subsidiary effects.

A number of important papers are connected with the discovery of the so-called tunnel diode by Giaever in America in 1960. This consists of a system of two superconductors, or a superconductor and a normal metal, separated by a very thin layer of dielectric of a thickness of the order of only a few atomic distances. Such a layer is usually provided by the oxide film on the surface of the metal. Electrons and even pairs can pass through the dielectric layer by means of the so-called tunnel effect. This simple device turned out to be extremely valuable for studying the nature of superconductivity. From the variation of the current with the applied potential difference one can deduce the binding energy of the pairs, its temperature dependence and its anisotropy in a single crystal. It was found further that one can determine the singularities in the spectrum of lattice vibrations, i.e., the variation of frequency with wavelength. Quite recently (1962) a whole group of phenomena were predicted theoretically and later found experimentally, which are known by the name of the Josephson effect. It was found in particular that the application of a constant potential difference across such a tunnel diode can give rise to an alternating current with a frequency of $2eV/\hbar$, where V is the potential difference (for $V \sim 10^{-4}$ volts, $\omega \sim 10^{11}$ sec^{-1}). The amplitude of these oscillations is very small (for $V \sim 10^{-4}$ volts, $W_{osc} \sim 10^{-10}$ watt), but they have already been discovered experimentally, although only indirectly.*

Next we should mention the discovery, in 1964, of two superconducting semiconductors; germanium telluride and strontium titanate. Admittedly we are here concerned with such a high concentration of local levels that we obtain not only a so-called impurity band, but that it merges with the conduction band (or the valence band, according to the sign of the carriers). This is therefore not really a case of true semiconductors but of semimetals. The critical temperature of these materials depends on a concentration of carriers. In practice one reaches temperatures up to 0.3°K.

Time does not permit me to go into details of other investigations, and in conclusion I want to add a few words about a question which concerns most of us

*Since this paper was read a group of physicists in Khar'kov, I. K. Yanson, C. M. Svistunov and I. M. Dmitrenko have succeeded in producing Josephson oscillations with a wavelength of 3 cm; the radiated power was about 10^{-14} watt.

particularly: is it not possible to increase the critical temperature so as to find superconductivity not merely at helium temperatures, but at liquid air temperatures, or perhaps even at room temperature? As mentioned already, superconductivity depends in all metals except perhaps in one case, ruthenium, on the lattice vibrations. The evidence for this lies in the dependence of the critical temperature on the mass of the lattice ions. In ruthenium this effect does not exist. I have already pointed out that with a very simplified model one finds a limitation on the critical temperature which corresponds to about 40°K. In practice the highest temperature reached is 18°K. We seem to find here therefore good agreement.

In view of this the question arises whether other mechanisms of superconductivity might exist. In principle they do exist. If one considers the mutual attraction of the electrons from a more general point of view then any mechanism to give attraction consists of the following. One requires a system which can be excited, i.e., which can change to a state of higher energy. One electron interacting with such a system can transfer it into the excited state and change its momentum. Another electron interacting with the same system transfers it back to the ground state and also changes its momentum. The result of this two-step process can be regarded as a mutual scattering of the two electrons since the rest of the system remains finally in the initial state. However, this represents an interaction between the electrons, and one can prove that it is necessarily attractive. The usual mechanism involving lattice vibrations is one example of this type of interaction. The intemediary system here consists of the ions, which can vibrate.

The magnitude of the binding energy of a pair and the critical temperature, which is proportional to this, depends on two factors: the distance between the energy levels of the intermediary system and the strength of the interaction between the electrons and this system. It is desirable that both be as large as possible. Various ideas of this kind have been suggested but unfortunately there has not yet been any real progress. For example in a magnetic material the system of magnetic moments could serve as the intermediary system, but it can be shown that it would give only about as large an effect as the phonon mechanism. One might have hoped that the electrons inside the ions might also make transitions to excited states but it turns out that this mechanism also is very ineffective. It is possible that one of these mechanisms is in operation in ruthenium (critical temperature 0.47°K).

Recently, Little in America has put forward the idea that superconductivity might be found in certain linear polymers, which consist of a main chain with side links. These links could be excited and would then serve as intermediary system for the electrons in the main chain. Simple estimates show that such a molecule would have a critical temperature of the order of a thousand degrees. However, a more careful approach raises serious doubts about the properties of such one-dimensional systems.* Besides one cannot attach terminals to such a molecule and measure its resistance. Presumably if it is superconducting this can best be found out by using high frequency electromagnetic fields. That is how this particular question stands at present.

I have tried to report on the most interesting facts relating to the phenomenon of superconductivity. Some physicists thought that after the appearance of the theory of superconductivity in 1957 the interest in this phenomenon would die out; in fact the opposite has happened. Work on superconductivity has proceeded extremely actively throughout the world. In fact it has become a large and distinct part of solid state physics. I believe that we have not yet heard the last word about superconductivity. There will still be much research and perhaps discoveries. I also believe that the existing superconducting magnets, cryotrons, and other devices, are only the very first and perhaps not even the most important, practical applications of this remarkable phenomenon.

Translated by R. E. Peierls

*Quite recently Yu. P. Bychkov, L. P. Gor'kov, and I. E. Dzyaloshinskii have shown that superconductivity in one dimension is possible in principle.