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COSMIC MAGNETIC BREMSSTRAHLUNG (SYNCHROTRON RADIATION)*

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1. INTRODUCTION

ELECTROMAGNETIC waves propagating in cosmic space are radiated as the result of various mechanisms. Thus, in the optical part of the spectrum the main role is played by radiation resulting from transitions of electrons between discrete atomic or molecular levels (bound-bound transitions), from recombination (free-bound transitions), and, finally, from transitions in the continuous spectrum (free-free transitions). In the last case when the conditions $\hbar\omega \ll kT$ are satisfied ($\omega = 2\pi\nu$ is the cyclic frequency of the radiation and T is the absolute temperature), the radiation process can be treated classically —we are of course considering bremsstrahlung arising in the acceleration of an electron passing near an atom or an ion. Some components of the cosmic radio emission are generated in similar fashion. Thus, the observed radio emissions of atomic hydrogen (λ = 21 cm) and OH are due to bound-bound transitions, while the thermal radio emission of interstellar and coronal gas is bremsstrahlung.

There are also other important radiation mechanisms in the radio region. Among these there are in particular those incoherent and coherent mechanisms of sporadic solar radio emission whose action is associated with the presence of a quite dense plasma. In other words, we are considering emissions which could not occur from the motion of individual electrons in vacuum. Making the picture rather crude, we may say that these mechanisms are important in the radio region because for the solar atmosphere the plasma frequency lies just in the radio region, $\omega_0 = \sqrt{4\pi e^2 N_e/m} = 5.64 \times 10^4\sqrt{N_e}$ (where N_e is the electron concentration).

There is, however, another emission mechanism which acts even for the motion of electrons in vacuum and plays a tremendous role in radio astronomy. We are speaking of magnetic bremsstrahlung, sometimes called synchrotron radiation.

In the motion of a charged particle in a magnetic field, as soon as its velocity is not directed along the field, the particle experiences an acceleration and consequently must radiate electromagnetic waves. Thus, the appearance of magnetic bremsstrahlung immediately follows simply from the fundamentals of classical electrodynamics. The main features characteristic of the magnetic bremsstrahlung of ultrarelativistic particles have been known for a long time; we may point out that this problem was already considered in quite great detail in the book of Schott,[1] published in 1912. But, as usually occurs in such cases, magnetic bremsstrahlung attracted attention only when its investigation became associated with quite important and specific physical and astrophysical problems. By these, we refer to the magnetic bremsstrahlung in electron accelerators,[2,3] in the terrestrial magnetic field,[4,5] and under cosmic conditions (cf. [16-18] and below).*

The theory of magnetic bremsstrahlung was developed after Schott by many authors (cf. [2-5,14-17]). However, neither the analysis nor the presentation of all these papers are of direct importance for us. The physical aspects of the problem and even the computations, so long as we are interested only in order of magnitude, can be obtained in most cases from elementary arguments (cf. Secs. 2 and 3). As for some of the formulas which are obtained as a result of long computations, we present them without proof, referring to the original paper in which one can find the details. At the same time, we hope that what we have given will be

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*There is a very extensive literature concerning the problems considered in this paper. In this connection, to give a complete reference list would be almost impossible and rather useless. We refer primarily to papers which are of interest for the correct understanding of the history of the problem, to summaries, and finally to individual papers which are directly used in the text. We also mention that at certain points in the present paper we make use of material contained in our book.[34]

sufficient for readers to make use of the theory of magnetic bremsstrahlung and its applications to typical astrophysical problems. In particular, we treat the problem of magnetic bremsstrahlung in the motion of electrons not in a vacuum, but in a plasma (cf. Sec. 4), since this is important in many cases (discrete sources of cosmic radio emission, solar corona, etc.).

The interest in magnetic bremsstrahlung for astrophysics is primarily related to the fact that the non-thermal (non-equilibrium) cosmic radio emission in most cases has precisely the nature of a magnetic bremsstrahlung.[6-9,18-20] This applies to the general galactic radio emission (radiation of the disc and halo), radio emission of the envelopes of supernovae (Cassiopeia A, Taurus A, etc.), and radio emission of normal and radio galaxies (we are referring to the radiation in the continuous spectrum). A partial magnetic bremsstrahlung character is present also in the sporadic radio emission of the sun,[10,21-23] as well as of Jupiter.[23,24] In addition, in particular cases (Crab nebula—Taurus A, radio galaxy M87 ≡ NGC4486 ≡ Virgo A, galaxy M82, and possibly the quasar sources—source 3C273B etc.), one observes optical magnetic bremsstrahlung;[13,25,26] it appears that this is the case also for the optical radiation with a continuous spectrum which sometimes occurs during the time of solar flares.[12] In certain cases, especially in the Crab nebula, one may also expect the appearance of cosmic magnetic bremsstrahlung in the x-ray region.[27-32]

In those cases where the cosmic radio or optical emission has the character of a magnetic bremsstrahlung, the determination of the intensity in the spectrum of this radiation enables one to obtain information about the concentration and the energy spectrum of relativistic electrons in the corresponding source. It is for this reason that the question of magnetic bremsstrahlung in the cosmos is closely connected with the astrophysics of cosmic rays, or, using a more common terminology, with the problem of the origin of cosmic rays,[11,18,19,33,34] and also with gamma and x-ray astronomy.[27-32]

Thus magnetic bremsstrahlung plays an important role in contemporary astrophysics, and one must consider it in analyzing a variety of important problems. It is understood, however, that in the framework of the present paper it is impossible to present in detail all of these problems as well as the results of radioastronomic investigations. We therefore restrict our presentation to the theory of magnetic bremsstrahlung and certain ways of applying it in astrophysics (Sec. 5).

Let us make just a few historical remarks. This seems appropriate to us because in the literature the history of the question is frequently presented incorrectly.

Non-thermal cosmic radio signals were first assumed to be formed in the atmospheres of stars ("radio-star hypothesis").[35,36] At first glance, such a point of view seems natural, considering the existence of the quite intense sporadic radio emission of the sun. However, it is easy to see that to explain the observational data the hypothetical radio stars must be distinguished by very unusual properties. The especially fantastic requirements presupposed in the radio-star hypothesis became clear after the quasi-spherical component of the general galactic radio emission was observed.[37] It then became clear that the sources of the non-thermal galactic radio radiation are mainly in the galactic halo whose existence was predicted only just a little earlier.[38] Nevertheless, even in paper [37] and in some of those which followed,[39,40] the radio-star hypothesis still was not discarded. But if we associate the general galactic radio emission with magnetic bremsstrahlung of relativistic electrons,[7,8] then we immediately arrive at completely tenable and reasonable estimates of the intensities of interstellar fields and the concentration of relativistic electrons. In the case of discrete sources,[6,8] the estimates are also satisfactory. Thus, in the U.S.S.R. the radio-star hypothesis was discarded as early as 1953, and the magnetic-bremsstrahlung character of the main part of the non-thermal cosmic radio emission seemed unquestionable.

For physicists the mechanism of magnetic bremsstrahlung is so simple and lucid that to use this mechanism under cosmic conditions seemed completely natural. But to many of the astronomers the mechanism of magnetic bremsstrahlung at first apparently appeared to be too strange and applicable apparently only to the cosmic radio emission. Because of this the popularity of the magnetic-bremsstrahlung hypothesis rose rapidly after the optical magnetic bremsstrahlung was detected. So far as we know, the question of cosmic optical magnetic bremsstrahlung was first discussed in 1952 by Gordon as applied to solar flares.[12] Later, Shklovskiĭ applied the same picture to explain part of the optical emission of the Crab nebula.[13] Magnetic bremsstrahlung, as is obvious from the most elementary considerations (cf. Secs. 2 and 3), is generally speaking polarized. Thus already in 1953 proposals[41,42,11,12] were made regarding polarization measurements in the optical and radio regions.* Very quickly the polarization of the optical emission of the Crab nebula[25,26] and of the jet in the NGC4486 galaxy (radio galaxy Virgo A) was detected.[44] In the radio region the polarization of the observed magnetic bremsstrahlung generally is much weaker for a variety of reasons and primarily because of the Faraday rotation of the plane of polarization in the sources and in the interstellar media. However, even in this

*It is quite curious that at this first stage the possibility of observing the polarization of the optical radiation of the Crab nebula gave rise to a dispute (cf.[43]; we should like to call attention to the summary[43] as a source of information about the state of the problem at the middle of 1953).

case, the polarization was detected (cf., for example, [45-47]).

The polarization measurements seemed especially convincing to many astronomers apparently because from the point of view of all the known non-magnetic bremsstrahlung mechanisms it was difficult and, in fact, essentially impossible to explain the polarization. However, we feel that there was no reason for doubting the magnetic-bremsstrahlung character of the non-thermal cosmic radio radiation and of the optical radiation in the continuous spectrum in the Crab and the Virgo sources, independent of measurements of the polarization.

In any case, at the Paris symposium on radio astronomy in 1958 (cf. [48]), in contrast to the Manchester symposium of 1955 (cf. [40]), the magnetic-bremsstrahlung theory of the non-thermal cosmic radio emission already was generally accepted.

2. MAGNETIC BREMSSTRAHLUNG OF AN INDIVIDUAL ELECTRON

2.1. Character of Electromagnetic Radiation from the Acceleration of Nonrelativistic and Ultrarelativistic Particles

If a charged particle moves in vacuum, it radiates electromagnetic waves only when it is accelerated (during motion in a medium the picture is changed fundamentally; the influence of the medium will be treated in Sec. 4). In the nonrelativistic case when the velocity of the particle $v \ll c = 3 \times 10^{10}$ cm/sec, the radiation usually has a dipole character. More precisely, the intensity of the quadrupole and higher multipole radiations is proportional to additional factors of the order of $(v/c)^{2n} \sim (a/\lambda)^{2n}$, where a is the size of the radiating system (for example, an oscillator), $\lambda = cT$ is the wave length of the radiation, $T \sim a/v$ is the characteristic period of motion of the particle, $n = 1$ for a quadrupole, $n = 2$ for an octupole, etc. So, for example, the quadrupole radiation is usually important only if the dipole moment of the system is equal to zero or is anomalously small. For a dipole (oscillator) with moment p changing only in magnitude, the electric field in the wave zone varies according to the law $\mathcal{E} \sim \sin\chi$ and the intensity is

$$dJ = \frac{(\ddot{\mathbf{p}})^2}{4\pi c^3}\sin^2\chi\, d\Omega,$$

where χ is the angle of the wave vector of the radiation $\mathbf{k}$ with the axis of the dipole and $d\Omega$ is the element of solid angle (Fig. 1a).

In a magnetic field, a nonrelativistic particle with charge e and mass m moves along a helix where the cyclic frequency of its rotation around the axis of the helix is

$$\omega_H^{(0)} = \frac{eH}{mc} = 1.76\cdot 10^7 H. \tag{2.1}$$

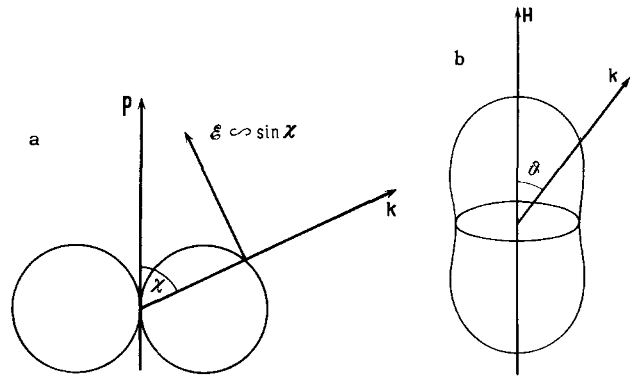

FIG. 1. a) Electric field intensity of a fixed dipole as a function of angle χ between the axis of the dipole p and the wave vector k. b) Intensity of cyclotron radiation as a function of angle ϑ between the magnetic field vector H and the wave vector k.

Here H is the magnetic field strength, and the numerical value is given for an electron (in formula (2.1), and later on, unless specifically stated, we give the absolute value of the charge of the particle).

The radiation from a nonrelativistic electron during its motion in a magnetic field is sometimes called cyclotron radiation. The frequency of the cyclotron radiation is $\omega_H^{(0)}$ and is dipole radiation. In the simplest case of circular motion (orbit radius $r_H = v/\omega_H^{(0)} = mcv/eH$), the particle radiates like two mutually perpendicular linear oscillators shifted in phase by $\pi/2$, or, what is the same thing, like a fixed dipole perpendicular to the magnetic field and rotating with frequency $\omega_H^{(0)}$. The intensity of the cyclotron radiation averaged over a period is

$$dJ = \frac{e^2 r_H^2 (\omega_H^0)^4}{8\pi c^3}(1+\cos^2\vartheta)\, d\Omega,$$

where ϑ is the angle between $\mathbf{k}$ and the field $\mathbf{H}$ (Fig. 1,b). For the helical motion, so long as the component of velocity parallel to the field $v_{\parallel} = \mathbf{v}\cdot\mathbf{H}/H \ll c$, the intensity distribution changes very slightly.

Ultrarelativistic particles radiate completely differently. For them

$$\frac{mc^2}{E} = \sqrt{1-\left(\frac{v}{c}\right)^2} \ll 1. \tag{2.2}$$

When $v \simeq c$ the dipole radiation is by no means predominant in intensity, and the character of the radiation is most simply explained qualitatively as follows: We change to a system of coordinates in which the instantaneous velocity of the particle is zero or is nonrelativistic. Suppose that in this system the radiation has dipole character and occurs at frequency ω_i. We now transform the radiation field by changing to a system in which the velocity of the particle is v. Then the frequency is determined by the well-known formula for the Doppler effect (ψ is the angle between $\mathbf{v}$ and the wave vector $\mathbf{k}$)

$$\omega = \frac{\omega_i \sqrt{1-\left(\frac{v}{c}\right)^2}}{1-\frac{v}{c}\cos\psi} \qquad (2.3)$$

We see that in the ultrarelativistic case (2.2) the frequency $\omega \sim \omega_i/\sqrt{1-(v/c)^2} = \omega_i E/mc^2$ is large compared to ω_i, so long as the angle ψ is sufficiently small, namely, so long as

$$\psi \leqslant \xi = \frac{mc^2}{E}. \qquad (2.4)$$

If, however, $\psi \gg mc^2/E$, the frequency of the radiation drops markedly. Expressions for the field strength and the intensity of the radiation (cf., for example, [17]) also contain in the denominator some power of the factor $(1-(v/c)\cos\psi)$. Thus the radiation is mainly concentrated within a cone of opening angle $\sim mc^2/E$ around the direction of the instantaneous velocity of the particle (Fig. 2). Below we shall always assume that condition (2.2) is satisfied, i.e., that we are dealing with ultrarelativistic particles.

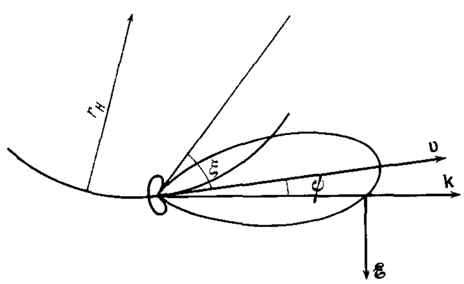

FIG. 2. Projection of the electric field on a plane passing through the axis of the dipole as a function of angle ψ between the translational velocity of the dipole v and the wave vector **k**. The dipole moves perpendicular to its axis. The field distribution is shown for the case v = 2/3 c.

2.2. Magnetic Bremsstrahlung of an Ultrarelativistic Electron (Estimates)

In the motion of an electron with arbitrary total energy E in a magnetic field, the rotation period T $= 2\pi/\omega_H$, where

$$\omega_H = \frac{eH}{mc}\frac{mc^2}{E} = \frac{eH}{mc}\sqrt{1-\left(\frac{v}{c}\right)^2}. \qquad (2.5)$$

The velocity of the electron **v** makes a constant angle θ with the field vector **H** and describes a cone about the field direction (Fig. 3). For $\theta \gg \xi = mc^2/E$ an observer, sitting on the surface of this cone at a large distance from the radiating particle, fixes his attention successively on different radiation pulses following one another at intervals $\tau = 2\pi/\omega_H$. The character of these pulses (Fig. 4) is easily explained if we consider the electric field of a rapidly moving dipole (Fig. 2) which turns relative to the observer as the result of the motion of the particle in the magnetic field (the acceleration vector, which corresponds to the axis of the dipole is always perpendicular to the field **H** and rotates about it with frequency ω_H). The duration of each pulse is

$$\Delta t \sim \frac{r_H^* \xi}{c}\left(\frac{mc^2}{E}\right)^2 = \frac{mc}{eH_\perp}\left(\frac{mc^2}{E}\right)^2, \qquad (2.6)$$

where $r_H^* = E/eH_\perp$ is the radius of curvature of the *space trajectory* of the particle*, $H_\perp = H\sin\theta$ is the magnetic field component perpendicular to the direction of motion (velocity) of the electron and the factor $(mc^2/E)^2$ appears as a consequence of the Doppler effect. In fact, within the limits of angle $\xi = mc^2/E$ the

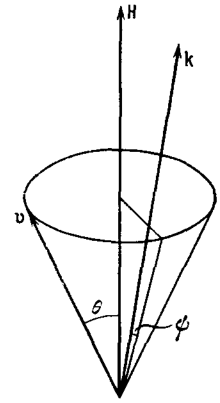

FIG. 3. Velocity cone of electron moving along a helix around a magnetic field **H**. v is the instantaneous velocity of the particle, θ the angle between v and **H**, ψ the angle between **k** and the nearest generator of the velocity cone.

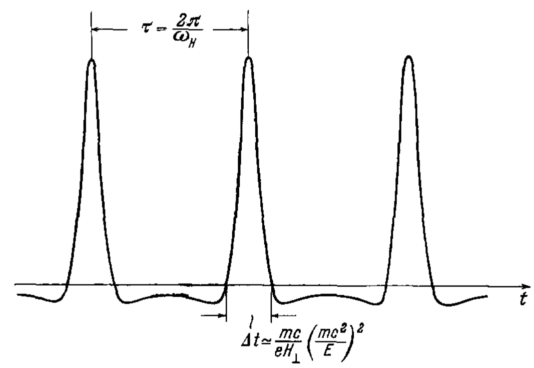

FIG. 4. Electric field in the wave zone as a function of time for a particle rotating in a magnetic field. This picture is obtained if we rotate the field of a rapidly moving dipole, shown in Fig. 2, with angular velocity ω_H.

*We emphasize the difference between the radius of curvature of the space trajectory $r_H^* = \frac{v}{\omega_H \sin\theta} \simeq \frac{E}{eH_\perp}$ and the radius of curvature $r_H = \frac{v\sin\theta}{\omega_H} \simeq \frac{c\sin\theta}{\omega_H} = \frac{E\sin^2\theta}{eH_\perp}$ for the circle which describes the projection of the electron velocity on the plane perpendicular to the field **H** (the radius r_H^* will not appear in what follows, and thus the radii of curvature will usually be designated simply as r_H).

electron moves in the direction of the observer during a time $\Delta t' = r_H^* \xi/c = mc/eH_\perp$. During this time the electron has traversed a path $v\Delta t'$ and the pulse radiated is therefore also contracted by an amount $v\Delta t'$ (this is the Doppler effect). As a result, the observed length of the pulse is of order $(c - v)\Delta t'$ and its duration $\Delta t = \Delta t'(1 - v/c) \simeq 2\Delta t'(mc^2/E)^2$, which is equivalent to (2.6).

The spectrum of the radiation, consisting of pulses following one another at intervals $\tau = 2\pi/\omega_H$, will obviously consist of overtones of the frequency ω_H. In fact, since $\tau \gg \Delta t$, in the region of high harmonics the spectrum can be considered to be continuous, where the maximum in the spectrum is at the frequency

$$\omega_m \sim \frac{1}{\Delta t} \sim \frac{eH_\perp}{mc}\left(\frac{E}{mc^2}\right)^2. \tag{2.7}$$

An important point is that the field of the radiation changes sign (cf. Fig. 4). This is why the spectrum has a maximum. The effective width of the spectrum of the radiation is also of order ω_m, and thus the mean spectral density of power of the magnetic bremsstrahlung can be estimated by dividing the total power of this radiation (cf. formula (2.10) below) by ω_m. As a result

$$\bar{p} \sim \frac{P(E)}{\omega_m} \sim \frac{e^3H_\perp}{mc^2}. \tag{2.8}$$

One of the characteristic features of magnetic bremsstrahlung is its polarization. In the coordinate system fixed on the electron the preferential direction for the electric vector in the radiated waves lies in the same plane as the direction of acceleration (cf. Fig. 1,a). Since during the motion of a particle in a magnetic field the direction of the acceleration is continually changing, the waves will generally be elliptically polarized. If the oscillator is moving in the direction of the observer, the polarization of the radiation moving along the direction of translation does not change. It is thus clear that magnetic bremsstrahlung of a single electron in general is polarized elliptically with the electric field $\mathscr{E}$ in the wave a maximum in a plane passing through the acceleration direction. This means that the preferential direction of the field $\mathscr{E}$ in the wave is perpendicular to the projection of the magnetic field and the plane of the diagram. (As usual, by this plane we mean the plane perpendicular to the line of sight.)

Before proceeding to the results of the quantitative theory, we should emphasize that the magnetic bremsstrahlung of electromagnetic radiation treated in the present paper is by no means the only possible type of magnetic bremsstrahlung. In fact, a charged particle moving in a magnetic field will radiate all those fields with which it interacts. Thus particles of all types will radiate gravitational waves, while, for example, protons should also radiate π^+, π^0 mesons (processes $p \to n + \pi^+$, $p \to p + \pi^0$), positrons and neutrinos (β^+ decay of the proton in a magnetic field, i.e., the process $p \to n + e^+ + \nu$). However, the intensity of the magnetic bremsstrahlung of nonelectromagnetic radiation is negligibly small and plays no particular role in astrophysics.[49]

2.3. Magnetic Bremsstrahlung of an Electron (Formulas)

During the motion in a uniform magnetic field $\mathbf{H}$ the frequency of rotation of the electron is given by formula (2.5), and the radius of the projection of the orbit on the plane perpendicular to $\mathbf{H}$ is

$$r_H = \frac{v\sin\theta}{\omega_H} = \frac{mcv\sin\theta}{eH\sqrt{1-\left(\frac{v}{c}\right)^2}}.$$

The total power in the magnetic bremsstrahlung is easily calculated from the general formulas (cf. [17], Secs. 73, 74) without carrying out a spectral resolution. This power is

$$P(E) = \frac{2e^4H_\perp^2v^2}{3m^2c^5\left(1-\frac{v^2}{c^2}\right)} = \frac{2e^4H_\perp^2}{3m^2c^3}\left[\left(\frac{E}{mc^2}\right)^2 - 1\right]. \tag{2.9}$$

For the ultrarelativistic case

$$P(E) = \frac{2e^4H_\perp^2}{3m^2c^3}\left(\frac{E}{mc^2}\right)^2 = 1.57\cdot10^{-15}H_\perp^2\left(\frac{E}{mc^2}\right)^2\frac{\text{erg}}{\text{sec}}$$
$$= 0.98\cdot10^{-3}H_\perp^2\left(\frac{E}{mc^2}\right)^2\frac{\text{eV}}{\text{sec}}, \tag{2.10}$$

where the numerical estimates refer to electrons (and positrons; $mc^2 = 0.51 \times 10^6$ eV); for a nucleus with charge eZ and mass M

$$P(E) = 0.98\cdot10^{-3}H_\perp^2\left(\frac{Z^2m}{M}\right)^2\left(\frac{E}{Mc^2}\right)^2\frac{\text{eV}}{\text{sec}}. \tag{2.11}$$

Expression (2.10) obviously determines the rate of loss of energy by an ultrarelativistic electron moving in a constant magnetic field. We note that in a field of electromagnetic radiation with characteristic frequency $\omega \ll (mc^2)^2/\hbar E$ the so-called Compton losses of energy differ from the expression (2.10) by the replacement of $H_\perp^2$ by $(16\pi/3)w_p$ where w_p is the density of energy in the radiation (for more details, cf. [34], Sec. 8).

The calculation of the electromagnetic field for each of the harmonics of the magnetic bremsstrahlung is quite involved (detailed calculations are given in [15]; for circular motion of the electron, i.e., for $\sin\theta = 1$, the appropriate expressions are not difficult to obtain by using the potentials given in [17]). If the electric field of the radiation of an ultrarelativistic particle is expressed in Fourier series

$$\mathscr{E} = \operatorname{Re}\left(\sum_{n=1}^{\infty}\mathscr{E}_n e^{-in\omega_H t}\right),$$

where Re is the real part of the expression, at distance r from the particle the amplitude of the n-th harmonic of the radiation in the direction $\mathbf{k}$ is

$$\mathscr{E}_n=\frac{2e^*\omega_H}{\sqrt{3}\,\pi cr}\exp\left(in\omega_H\frac{r}{c}\right)\frac{n}{\sin\theta}\{(\xi^2+\psi^2)K_{2/3}(g_n)\,\mathbf{l}_1$$
$$+i\psi(\xi^2+\psi^2)^{1/2}K_{1/3}(g_n)\,\mathbf{l}_2\}. \qquad (2.12)$$

Here e^* is the charge of the radiating particle (for an electron $e^* = -e$), ψ is the angle between the wave vector $\mathbf{k}$ and the nearest generator of the velocity cone, $\mathbf{l}_1$ and $\mathbf{l}_2$ are two mutually perpendicular unit vectors in the plane of the diagram, where $\mathbf{l}_2$ is parallel to the projection of H on this plane, and $\mathbf{l}_1 = \mathbf{k}\times\mathbf{l}_2/k$. The functions $K_{1/3}(g_n)$ and $K_{2/3}(g_n)$ are Bessel functions of imaginary argument of the second kind, while

$$g_n=\frac{n}{3\sin\theta}(\xi^2+\psi^2)^{3/2}. \qquad (2.13)$$

In the expression (2.12), as throughout in symbolic formulas, we use only the absolute (Gaussian) system of units.

As one sees from (2.12), the electric vector $\mathrm{Re}\{\mathscr{E}_n e^{-in\omega_H t}\}$ for a given harmonic describes an ellipse in the course of time. One of the axes of this ellipse (the minor axis) is along the projection of H on the figure plane, and the second (major) axis is perpendicular to this projection, and their ratio, which we denote by $\tan\beta$, by virtue of (2.12) is equal to

$$\mathrm{tg}\,\beta=\frac{\psi K_{1/3}(g_n)}{(\xi^2+\psi^2)^{1/2}K_{2/3}(g_n)}. \qquad (2.14)^*$$

When $\psi > 0$ the direction of rotation is right-handed (clockwise relative to the observer), and when $\psi < 0$ it is left-handed. The angle ψ is taken to be positive if the direction of the radiation and the magnetic field vector lie on the same side of the velocity cone (Fig. 5).

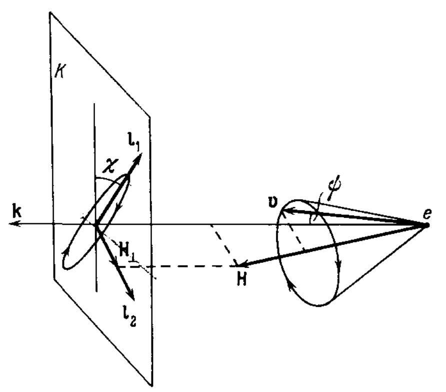

FIG. 5. Ellipse of vibration of the electric vector in a wave radiated by a particle moving in a magnetic field. The charge is assumed to be positive. For a negatively charged particle (an electron) the direction of rotation is opposite to that shown. K is the figure plane (the plane perpendicular to the direction of radiation, or, what is the same thing, to the direction of the observer); $\mathbf{l}_1$ and $\mathbf{l}_2$ are two mutually orthogonal unit vectors in the figure plane, of which $\mathbf{l}_2$ is directed along the projection of the magnetic field $\mathbf{H}$ on the figure plane.

*tg ≡ tan.

The polarization degenerates to linear only when $\psi = 0$, i.e., if the wave vector lies precisely on the surface of the velocity cone. For large ψ (i.e., when $\psi \gg \xi$) the polarization tends toward circular polarization, since for large values of the argument $K_{2/3}(x) \simeq K_{1/3}(x) \simeq (\pi/2x)^{1/2}e^{-x}$; however, the intensity of the radiation then becomes negligibly small (cf. below, Fig. 6).

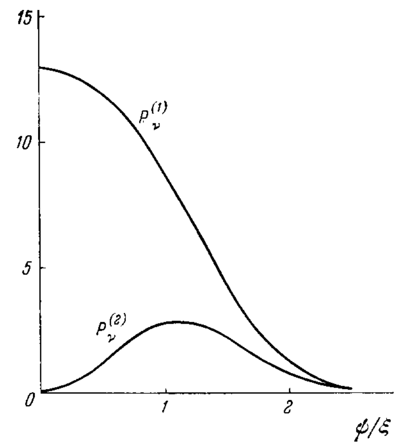

FIG. 6. Magnetic bremsstrahlung of a single electron. The angular dependence of the radiation fluxes for the two principal polarization directions: perpendicular to the magnetic field projection on the figure plane ($p_\nu^{(1)}$) and along this projection ($p_\nu^{(2)}$) for $\nu/\nu_c = 0.29$. The unit for the vertical scale is the coefficient $3e^3H/4\pi^2\xi\cdot r^2mc^2(\nu_c/\nu)^2$ in expressions (2.17) and (2.18). The angle $\psi = 0$ corresponds to the direction of the instantaneous velocity of the electron.

The density of flux of energy of the radiation averaged over a period, for the energy contained in the n-th harmonic, is

$$p_n=\frac{c}{8\pi}|\mathscr{E}_n|^2. \qquad (2.15)$$

Since when $\xi = mc^2/E \ll 1$ the radiated energy is almost entirely concentrated in the region of very high harmonics where the spectrum is practically continuous, it is convenient to change from harmonic number n to frequency

$$\nu=n\frac{\omega_H}{2\pi}=\frac{2}{3}\frac{n\xi^3}{\sin\theta}\nu_c,$$

where we have introduced the notation

$$\nu_c=\frac{3eH\sin\theta}{4\pi mc\xi^2}=\frac{3eH_\perp}{4\pi mc}\left(\frac{E}{mc^2}\right)^2. \qquad (2.16)$$

Then, by virtue of (2.15), (2.12), and (2.16), the spectral densities of flux of radiation with the two principal polarization directions are

$$p_\nu^{(1)}=\frac{3}{4\pi^2r^2}\frac{e^3H}{mc^2\xi}\left(\frac{\nu}{\nu_c}\right)^2\left(1+\frac{\psi^2}{\xi^2}\right)^2K_{2/3}^2(g_\nu), \qquad (2.17)$$

$$p_\nu^{(2)}=\frac{3}{4\pi^2r^2}\frac{e^3H}{mc^2\xi}\left(\frac{\nu}{\nu_c}\right)^2\frac{\psi^2}{\xi^2}\left(1+\frac{\psi^2}{\xi^2}\right)K_{1/3}^2(g_\nu), \qquad (2.18)$$

where

$$g_\nu = \frac{\nu}{2\nu_c}\left(1+\frac{\psi^2}{\xi^2}\right)^{3/2}, \qquad (2.19)$$

and we write $p_\nu = p_n\, dn/d\nu = 2\pi p_n/\omega_H$.

The angular distributions of the radiation fluxes $p_\nu^{(1)}$ and $p_\nu^{(2)}$ are shown in Fig. 6. For the unit on the vertical scale we have taken the coefficient $(3e^3H/4\pi^2r^2mc^2\xi)(\nu/\nu_c)^2$ in expressions (2.17) and (2.18). The curves are drawn for $\nu/\nu_c = 0.29$ which corresponds, as we shall see later, to the maximum in the frequency spectrum of the total radiation (over all directions) of an electron. Figure 6 shows that in the region of small angles ψ the main contribution to the radiation comes from $p_\nu^{(1)}$, i.e., from vibrations for which the electric field direction is perpendicular to the projection of **H** on the figure plane.

We now find the spectral distribution of the total radiation (over all directions) of a single ultrarelativistic electron. To do this we must integrate expressions (2.17) and (2.18) over all solid angles. Here we can use the fact that the quantities $p_\nu^{(1)}$ and $p_\nu^{(2)}$ as functions of angle ψ tend rapidly to zero outside an interval $\Delta\psi \sim mc^2/E$ and thus, in integrating over solid angle, the only important contribution comes from the narrow ring sector $\Delta\Omega = 2\pi \sin\vartheta\, \Delta\psi$ around the velocity cone, where $\vartheta = \theta - \psi \simeq \theta$ is the angle between the direction of observation **k** and the field **H**. Thus we must find the quantity

$$r^2 \int p_\nu^{(1,2)}\, d\Omega = 2\pi r^2 \sin\theta \int_{-\infty}^{+\infty} p_\nu^{(1,2)}\, d\psi,$$

where in the last expression the limits of integration are replaced by $\pm\infty$. As the computation shows (cf., for example, [15])

$$\int_{-\infty}^{+\infty} p_\nu^{(1)}\, d\psi = \frac{\sqrt{3}e^3H}{2\pi mc^2r^2}\frac{\nu}{2\nu_c}\left[\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta + K_{2/3}\left(\frac{\nu}{\nu_c}\right)\right],$$

$$\int_{-\infty}^{+\infty} p_\nu^{(2)}\, d\psi = \frac{\sqrt{3}e^3H}{2\pi mc^2r^2}\frac{\nu}{2\nu_c}\left[\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta - K_{2/3}\left(\frac{\nu}{\nu_c}\right)\right]. \qquad (2.20)$$

The spectral distribution of the power in the total radiation from a single electron is

$$p(\nu) = 2\pi r^2 \sin\theta \int (p_\nu^{(1)} + p_\nu^{(2)})\, d\psi$$

$$= \frac{\sqrt{3}e^3H_\perp}{mc^2}\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta. \qquad (2.21)$$

The graph of the function $F(x) = x\int_x^\infty K_{5/3}(\eta)\, d\eta$, showing the spectral distribution of the power in the total radiation (2.21) is given in Fig. 7. The polarization of the total radiation (for more details see section 3.3) is

$$\Pi = \frac{\int (p_\nu^{(1)} - p_\nu^{(2)})\, d\Omega}{\int (p_\nu^{(1)} + p_\nu^{(2)})\, d\Omega} = \frac{K_{2/3}(\nu/\nu_c)}{\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta} = \frac{F_p(\nu/\nu_c)}{F(\nu/\nu_c)}. \qquad (2.22)$$

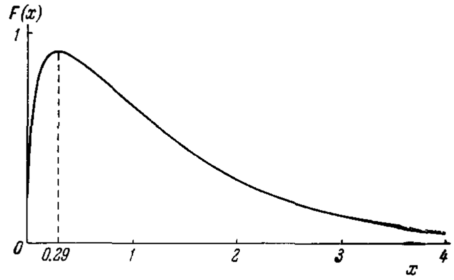

FIG. 7. Spectral distribution of the power of the total radiation (over all directions) from a charged particle moving in a magnetic field.

Values of the function $F(x)$ and $F_p(x) = xK_{2/3}(x)$ and their approximate expressions are given in Table I.

The maximum in the spectrum of the radiation from a single electron is at the frequency

$$\nu_m \simeq 0.29\nu_c = 0.07\frac{eH_\perp}{mc}\left(\frac{E}{mc^2}\right)^2 = 1.2\cdot 10^6 H_\perp\left(\frac{E}{mc^2}\right)^2$$

$$= 1.8\cdot 10^{18} H_\perp (E_{\rm erg})^2 = 4.6\cdot 10^{-6} H_\perp (E_{\rm eV})^2. \qquad (2.23)$$

Here the frequency ν_m is expressed in hertz (cycles per second) and the component of the field $H_\perp$ perpendicular to the line of sight is measured in oersted.

At the maximum frequency (2.23) the spectral density of the power in the total radiation (2.21) is

$$p(\nu_m = 0.29\nu_c) \equiv p_m = 1.6\frac{e^3H_\perp}{mc^2} = 2.16\cdot 10^{-22} H_\perp \frac{\rm erg}{\rm sec\text{-}Hz} \qquad (2.24)$$

and, of course, is in agreement with the estimate (2.8).

Expressions for the intensity of the magnetic bremsstrahlung for the case of an aggregate of electrons, with which one actually deals in astrophysical cases, will be obtained and discussed in the following Section 3. However, it is already useful here to consider the simplest case when there are monoenergetic electrons with a distribution of velocities which is isotropic over all directions. We denote the concentration of such electrons with energy E at point **r** by $N(\mathbf{r})$ and assume that the total power $p(\nu)$ is radiated strictly in the direction of the motion. Then the spectral density of the flux of radiation from electrons in volume dV $= r^2\, dr\, d\Omega$ at distance r from the observer and moving in the solid angle $d\Omega'$ is

$$d\Phi_\nu = \frac{1}{4\pi} p(\nu) N(\mathbf{r})\, d\Omega'\, dV.$$

The intensity J_ν is taken per unit solid angle $d\Omega$ and is the flux through unit area, i.e., in this case $d\Omega' = dS/r^2 = 1/r^2$.

Thus

$$J_\nu = \frac{d\Phi_\nu}{d\Omega} = \frac{p(\nu)}{4\pi}\int N(\mathbf{r})\, dr, \qquad (2.25)$$

where the integration is taken along the line of sight and the magnetic field on which $p(\nu)$ depends is assumed to be uniform along the whole path.

Actually we do not have to regard the electrons as being distributed isotropically—the important point is

Table I. Values of the functions $F(x) = x\int_x^\infty K_{5/3}(\eta)d\eta$ and $F_p(x) = xK_{2/3}(x)$

x	$F(x)$	$F_p(x)$	x	$F(x)$	$F_p(x)$	x	$F(x)$	$F_п(x)$
0	0	0	0.30	0.918	0.596	2.0	0.301	0.250
0.001	0.213	0.107	0.40	0.901	0.607	2.5	0.200	0.168
0.005	0.358	0.184	0.50	0.872	0.603	3.0	0.130	0.111
0.01	0.445	0.231	0.60	0.832	0.590	3.5	0.0845	0.0726
0.025	0.583	0.312	0.70	0.788	0.570	4.0	0.0541	0.0470
0.050	0.702	0.388	0.80	0.742	0.547	4.5	0.0339	0.0298
0.075	0.772	0.438	0.90	0.694	0.521	5.0	0.0214	0.0192
0.10	0.818	0.475	1.0	0.655	0.494	6.0	0.0085	0.0077
0.15	0.874	0.527	1.2	0.566	0.439	7.0	0.0033	0.0031
0.20	0.904	0.560	1.4	0.486	0.386	8.0	0.0013	0.0012
0.25	0.917	0.582	1.6	0.414	0.336	9.0	0.00050	0.00047
0.29	0.918	0.592	1.8	0.354	0.290	10.0	0.00019	0.00018

Approximate expressions for $x \ll 1$

$$F(x) = \frac{4\pi}{\sqrt{3}\,\Gamma\left(\frac{1}{3}\right)}\left(\frac{x}{2}\right)^{1/3} \times\left\{1-\frac{\Gamma\left(\frac{1}{3}\right)}{2}\left(\frac{x}{2}\right)^{2/3}+\frac{3}{4}\left(\frac{x}{2}\right)^2-\frac{9}{40}\frac{\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{5}{3}\right)}\left(\frac{x}{2}\right)^{10/3}+\dots\right\},$$

$$F_p(x) = \frac{2\pi}{\sqrt{3}\,\Gamma\left(\frac{1}{3}\right)}\left(\frac{x}{2}\right)^{1/3} \times\left\{1-\frac{\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{5}{3}\right)}\left(\frac{x}{2}\right)^{4/3}+3\left(\frac{x}{2}\right)^2-\frac{3}{5}\frac{\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{5}{3}\right)}\left(\frac{x}{2}\right)^{10/3}+\dots\right\};$$

for $x \gg 1$

$$F(x) = \sqrt{\frac{\pi}{2}}\, e^{-x} x^{1/2}\left\{1+\frac{55}{72}x^{-1}-\frac{10151}{10368}x^{-2}+\dots\right\},$$

$$F_p(x) = \sqrt{\frac{\pi}{2}}\, e^{-x} x^{1/2}\left\{1+\frac{7}{72}x^{-1}-\frac{455}{10368}x^{-2}+\dots\right\}.$$

that within the range of angles $\psi \sim \xi = mc^2/E$ along the line of sight their distribution should not change a great deal, while their concentration per unit solid angle is equal to $N(\mathbf{r})/4\pi$. Thus we can carry out an average over the angle ψ, and the simplifying assumptions made above about the radiation being strictly forward are unimportant. A more consistent derivation of formula (2.25) is given in Sec. 3.3.

At the maximum frequency, which is related to the electron energy E by formula (2.23), we find, according to (2.24) and (2.25)

$$J_{\nu,m} = 0{,}13\frac{e^3H_\perp}{mc^2}\int N(\mathbf{r})\,dr$$
$$= 1.7\cdot 10^{-23}H_\perp\int N(\mathbf{r})\,dr \quad \frac{\text{erg}}{\text{cm}^2\ \text{sec-Hz-sr}}$$
$$= 1.7\cdot 10^{-26}H_\perp\int N(\mathbf{r})\,dr \quad \frac{\text{W}}{\text{m}^2\ \text{Hz-sr}}. \qquad (2.26)$$

To the monoenergetic electron spectrum, there obviously corresponds a distribution $N(\mathbf{r}, E) = N(\mathbf{r})\,\delta(E - E')$ where $\delta(E)$ is the delta-function. A similar result is obtained when the spectrum of the electrons is arbitrary, but the energy of the predominant majority of particles lies in an interval $\Delta E \ll E$.

3. MAGNETIC BREMSSTRAHLUNG OF AN AGGREGATE OF ELECTRONS

3.1. The Stokes Parameters

Before proceeding to give the basic formulas characterizing magnetic bremsstrahlung of an aggregate of electrons, we recall the definition of the Stokes parameters.[50]

An arbitrary flux of radiation, in addition to its frequency dependence, is characterized in general by four independent parameters; for example, the position of the principal axis of the polarization ellipse, the intensities along the two principal axes, and the direction of rotation of the electric vector. The choice of these parameters is, of course, not unique. In many cases it is convenient to make use of the Stokes parameters, which are defined as follows:

Let us choose, at the point of observation, in a plane perpendicular to the direction of arrival of the electromagnetic wave (that is, in the so-called figure plane) two mutually perpendicular directions $\mathbf{s}_1$ and $\mathbf{s}_2$ (Fig. 8, the wave vector of the radiation is directed toward the reader). Then the intensity of any harmonic of the electric field produced at the point of observation by an individual radiating particle (with label i) has the projections

$$\begin{aligned}\xi_1^i(t) &= \xi_1^i \cos(\omega t + \varphi_i),\\ \xi_2^i(t) &= \xi_2^i \cos(\omega t + \varphi_i - \psi_i),\end{aligned} \tag{3.1}$$

where ξ_1^i and φ_i are the amplitude and phase of the oscillation along axis $\mathbf{s}_1$, while ξ_2^i and $(\varphi_i - \psi_i)$ are the similar quantities for direction $\mathbf{s}_2$. The field of the radiation from an aggregate of particles is equal to the sum of the respective components for all particles:

$$\xi_1(t) = \sum_i \xi_1^i(t), \quad \xi_2(t) = \sum_i \xi_2^i(t). \tag{3.2}$$

The quantity measured experimentally is the time average of the flux of energy in the radiation (or the intensity of the radiation, when we are talking about the flux per unit solid angle) $J = (c/4\pi)\overline{E^2}$. One can get complete information about the flux of radiation by introducing some additional phase difference for one of the projections of the electric field and measuring, as a function of the position of the analyzer, the intensity of the radiation with a given vibration direction as selected by the analyzer.

Suppose that, for the projection of the electric vector of the vibrations along direction $\mathbf{s}_2$, we introduce an additional phase difference ϵ relative to the vibrations along $\mathbf{s}_1$ (cf. Fig. 8).

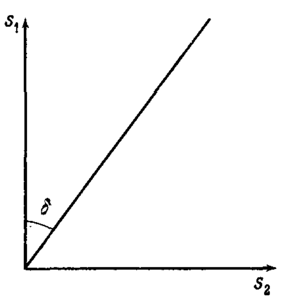

FIG. 8. Definition of the Stokes parameters. In the direction s_2 we introduce an additional retarding phase ϵ relative to the vibrations in the perpendicular direction s_1. Angle δ determines the position of the plane of the analyzer. The measured flux of radiation is directed toward the reader.

Then the wave (3.2) becomes

$$\xi_1(t) = \sum_i \xi_1^i \cos(\omega t + \varphi_i), \qquad \xi_2(t) = \sum_i \xi_2^i \cos(\omega t + \varphi_i - \psi_i - \varepsilon). \tag{3.3}$$

If the plane of vibration of the electric vector selected by the analyzer makes an angle δ with the direction $\mathbf{s}_1$ (cf. Fig. 8), then at the output of the analyzer the electric field is equal to

$$\xi(t) = \xi_1(t)\cos\delta + \xi_2(t)\sin\delta, \tag{3.4}$$

while the time averaged energy flux of the radiation (intensity) is

$$J(\delta, \varepsilon) = \frac{c}{4\pi}\overline{[\xi(t)]^2} = \frac{c}{4\pi}\{\overline{[\xi_1(t)]^2}\cos^2\delta + \overline{[\xi_2(t)]^2}\sin^2\delta + \overline{\xi_1(t)\xi_2(t)}\sin 2\delta\}. \tag{3.5}$$

If the radiations of individual particles are incoherent (phases for different particles independent and distributed randomly), then from expression (3.3), after averaging over time and phase, we easily find

$$\begin{gathered}\overline{[\xi_1(t)]^2} = \frac{1}{2}\sum_i (\xi_1^i)^2, \quad \overline{[\xi_2(t)]^2} = \frac{1}{2}\sum_i (\xi_2^i)^2,\\ \overline{\xi_1(t)\xi_2(t)} = \frac{1}{2}\sum_i \xi_1^i\xi_2^i \cos\psi_i \cos\varepsilon - \frac{1}{2}\sum_i \xi_1^i\xi_2^i \sin\psi_i \sin\varepsilon.\end{gathered} \tag{3.6}$$

Thus, if we introduce the notation

$$\left.\begin{aligned} J &= \sum_i (J_1^i + J_2^i) = \frac{c}{8\pi}\sum_i [(\xi_1^i)^2 + (\xi_2^i)^2],\\ Q &= \sum_i (J_1^i - J_2^i) = \frac{c}{8\pi}\sum_i [(\xi_1^i)^2 - (\xi_2^i)^2],\\ U &= \frac{c}{4\pi}\sum_i \xi_1^i\xi_2^i \cos\psi_i,\\ V &= \frac{c}{4\pi}\sum_i \xi_1^i\xi_2^i \sin\psi_i,\end{aligned}\right\} \tag{3.7}$$

the intensity (3.5) as a function of position of the analyzer (angle δ) and the additional phase difference ϵ becomes

$$J(\delta, \varepsilon) = \frac{1}{2}[J + Q\cos 2\delta + (U\cos\varepsilon - V\sin\varepsilon)\sin 2\delta]. \tag{3.8}$$

The quantities J, Q, U, and V are called the Stokes parameters and completely characterize the flux of radiation. By changing the phase difference ϵ and the analyzer position δ, we can, as is clear from (3.8) experimentally determine all these parameters. For independent (incoherent) fluxes of radiation the Stokes parameters are additive, as is immediately seen from their definition (3.7).

For the radiation of an individual particle, the Stoke parameters J_e, Q_e, U_e, and V_e are expressed in terms of the densities of flux of radiation with the two principal directions of vibration $p_\nu^{(1)}$ and $p_\nu^{(2)}$, the ratio of the minor and major axes of the ellipse of the vibration of the electric vector, which we denote by $\tan\beta$, and by the angle χ between a fixed direction (direction $\mathbf{s}_1$) and the major axis of the ellipse of vibration (the angle χ is measured clockwise and obviously is defined in the interval $0 \le \chi < \pi$). Let us find these expressions.

The vector of the electric vibrations of the radiation from a single particle can be written in the form (cf. Eq. (2.12))

$$\mathcal{E} = A(\cos\beta\cos\omega t\,\mathbf{l}_1 + \sin\beta\sin\omega t\,\mathbf{l}_2). \tag{3.9}$$

If the principal axes of the vibration ellipse $\mathbf{l}_1$ and $\mathbf{l}_2$ are turned through an angle χ relative to the axes $\mathbf{s}_1$ and $\mathbf{s}_2$ respectively, then the vibrations along axes $\mathbf{s}_1$ and $\mathbf{s}_2$ are expressed as follows:

$$\left.\begin{aligned} \xi_1(t) &= A(\cos\chi\cos\beta\cos\omega t - \sin\chi\sin\beta\sin\omega t) \equiv \xi_1\cos(\omega t + \psi_1), \\ \xi_2(t) &= A(\sin\chi\cos\beta\cos\omega t + \cos\chi\sin\beta\sin\omega t) \equiv \xi_2\cos(\omega t - \psi_2). \end{aligned}\right\} \tag{3.10}$$

The phase difference of these vibrations $\psi = \psi_1 + \psi_2$ and their amplitudes ξ_1 and ξ_2 (cf. (3.1)), as one can easily verify from (3.10), are given by the relations

$$\left.\begin{aligned} &\xi_1^2 + \xi_2^2 = A^2, \quad \xi_1^2 - \xi_2^2 = A^2\cos 2\beta\cos 2\chi, \\ &2\xi_1\xi_2\cos\psi = A^2\cos 2\beta\sin 2\chi, \\ &2\xi_1\xi_2\sin\psi = A^2\sin 2\beta. \end{aligned}\right\} \tag{3.11}$$

For a single particle the densities of radiation flux with the two principal directions of polarization are, as a consequence of (3.9), equal to

$$p^{(1)} = \frac{c}{8\pi}A^2\cos^2\beta, \quad p^{(2)} = \frac{c}{8\pi}A^2\sin^2\beta. \tag{3.12}$$

Thus, according to (3.7), (3.11), and (3.12), relative to the axes $\mathbf{s}_1$ and $\mathbf{s}_2$ the Stokes parameters of the radiation of a single particle are equal to

$$\left.\begin{aligned} J_e &= p^{(1)} + p^{(2)}, \\ Q_e &= (p^{(1)} - p^{(2)})\cos 2\chi, \\ U_e &= (p^{(1)} - p^{(2)})\sin 2\chi, \\ V_e &= (p^{(1)} - p^{(2)})\,\mathrm{tg}\,2\beta. \end{aligned}\right\} \tag{3.13}$$

The first Stokes parameter J obviously determines the total density of energy flux (or intensity) of the radiation. The degree of polarization of the radiation is given by

$$\mathrm{P} = \frac{\sqrt{Q^2 + U^2 + V^2}}{J}, \tag{3.14}$$

while the angle χ, characterizing the position of the principal axis of the polarization ellipse, is, according to (3.13),

$$\mathrm{tg}\,2\chi = \frac{U}{Q}. \tag{3.15}$$

Of the two values of the angle χ $(0 \le \chi < \pi)$, given by (3.15), we select the one which lies in the first quadrant if $U > 0$, and in the second quadrant if $U < 0$.

The degree of ellipticity (ratio of principal axes of the vibration ellipse) is characterized by the angle β defined by the relation

$$\sin 2\beta = \frac{V}{J}. \tag{3.16}$$

The angle β is defined within the interval $-\pi/2 \le \beta \le \pi/2$; for $\beta > 0$ the direction of rotation of the electric vector is right-handed (clockwise relative to the observer), while for $\beta < 0$ it is left-handed (cf. equation (3.9)).

In the absence of elliptical (and circular) polarization $V = 0$ and the degree of polarization is

$$\mathrm{P} = \frac{J_{\max} - J_{\min}}{J_{\max} + J_{\min}}, \tag{3.17}$$

where $J_{\max}$ and $J_{\min}$ are the maximum and minimum values of the observed intensity (3.8) as a function of the analyzer angle δ (without introducing retardation, i.e., for $\epsilon = 0$).

3.2. Radiation from an Aggregate of Particles

Let us now consider the radiation of a system of particles. Let $N(E, \mathbf{r}, \boldsymbol{\tau})dE\,dV\,d\Omega_\tau$ be the number of particles in the volume element $dV = r^2\,dr\,d\Omega$, whose energies are contained in the interval E to $E + dE$, and with velocity within the solid angle $d\Omega_\tau$ in the neighborhood of the direction $\boldsymbol{\tau}$. Since the radiation of the individual electrons is incoherent and the Stokes parameters are therefore additive, the intensity of the radiation of such a system along the direction $\mathbf{k}$ is

$$J_\nu \equiv J(\nu, \mathbf{k}) = \int J_e(\nu, E, \mathbf{r}, \theta, \psi)N(E, \mathbf{r}, \boldsymbol{\tau})\,dE\,d\Omega_\tau r^2\,dr. \tag{3.18}$$

Here $J_e(\nu, E, \mathbf{r}, \theta, \psi)$ is determined by the first of the expressions (3.13), and consequently for the magnetic bremsstrahlung of a single electron is equal to $J_e(\nu, E, \mathbf{r}, \theta, \psi) = p_\nu^{(1)} + p_\nu^{(2)}$ (cf. (2.17) and (2.18)); the integration over r is carried out along the line of sight in the direction $-\mathbf{k}$. The other Stokes parameters are expressed similarly.

We emphasize that, unlike the Stokes parameters for the radiation of a single electron (3.13), with the dimensions of spectral density of flux of radiation energy, expression (3.18) determines the intensity of the radiation, i.e., the flux of energy per unit area perpendicular to the direction of the observer, taken per unit solid angle and per unit frequency interval. The usual unit for the measurement of intensity of radiation in radioastronomy is $\mathrm{W/m^2\,Hz\text{-}sr} = 10^3\ \mathrm{erg/cm^2\text{-}sec\text{-}Hz\text{-}sr}$.

If the source (the radiating system of electrons) has small angular size, then the quantity measured experimentally is (as in the case of an individual particle) the spectral density of flux of radiation

$$\Phi_e = \int J_\nu\,d\Omega = \int J_e(\nu, E, \mathbf{r}, \theta, \psi)N(E, \mathbf{r}, \boldsymbol{\tau})\,dE\,d\Omega_\tau\,dV, \tag{3.19}$$

where $dV = r^2\,dr\,d\Omega$ and the integration is taken over the whole volume of the source.

In the expressions (3.18) and (3.19) and the analogous expressions for the other Stokes parameters, the integration over $d\Omega_\tau$ can be carried out in general for an arbitrary distribution of particles $N(E, \mathbf{r}, \boldsymbol{\tau})$. In fact, as we have seen in deriving expressions (2.20) and (2.21), the functions $p_\nu^{(1)}$ and $p_\nu^{(2)}$ differ from zero only within the small solid angle $\Delta\Omega_\tau = 2\pi\sin\theta\,\Delta\psi$

where $\Delta\psi \lesssim mc^2/E$. Thus, the contribution to the radiation comes only from particles moving within this angle. If the distribution of particles over the angle θ between the velocity and the field is sufficiently smooth, then considering that $\vartheta = \theta - \psi \approx \theta$ we may set $N(E, \mathbf{r}, \tau) = N(E, \mathbf{r}, \mathbf{k})$, and from now on not make any distinction between the angles ϑ and θ. Then the integration over $d\Omega_\tau$ reduces to an integration over $d\psi$. As a result, using expressions (2.20) and (2.21) we get

$$J_\nu = J(\nu, \mathbf{k}) = \int N(E, \mathbf{r}, \mathbf{k})\, p(\nu)\, dE\, dr$$
$$= \frac{\sqrt{3}\, e^3}{mc^2} \int \left\{ N(E, \mathbf{r}, \mathbf{k}) H \sin\theta \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta \right\} dE\, dr. \qquad (3.20)$$

Here in the general case the field intensity $\mathbf{H}$, the angle θ between $\mathbf{k}$ and $\mathbf{H}$, and the density of particles $N(E, \mathbf{r}, \mathbf{k})$ depend on the distance $\mathbf{r}$.

We can similarly express the other Stokes parameters, for example

$$Q(\nu, \mathbf{k}) = \frac{\sqrt{3}\, e^3}{mc^2} \int \left\{ N(E, \mathbf{r}, \mathbf{k}) H \sin\theta \cos 2\chi \frac{\nu}{\nu_c} \times K_{2/3}\left(\frac{\nu}{\nu_c}\right) \right\} dE\, dr. \qquad (3.21)$$

The parameter $U(\nu, \mathbf{k})$ differs from $Q(\nu, \mathbf{k})$ only in the replacement of $\cos 2\chi$ in the integrand of (3.21) by $\sin 2\chi$. As for the parameter $V(\nu, \mathbf{k})$, which characterizes the presence of elliptically polarized radiation, in the ultrarelativistic approximation considered here it is equal to zero.[16] This result is valid up to terms of order mc^2/E and is easily understood if we recall that the sign of ψ determines the direction of rotation of the electric vector in the wave radiated by an individual electron. Since the power of the radiation (cf. (2.17) and (2.18)) is independent of the sign of ψ, while the distribution of particles over directions of motion within the limits of very small angles $\psi \lesssim mc^2/E$ is practically constant by assumption, the contributions to the radiation in a given direction from particles with positive and particles with negative ψ are the same, and the polarization will be linear. A significant elliptical polarization in the ultrarelativistic case could occur only for a markedly anisotropic distribution of velocities of the electrons. For this it would be necessary that the distribution vary markedly within the very small angle $\psi \sim mc^2/E$, i.e., essentially there would have to be a discontinuity in the angular distribution of the electrons just along the direction toward the observer. If, in addition, we consider the possible fluctuations in direction of the magnetic field, the realization of this sort of possibility is extremely improbable.

3.3. Intensity and Polarization of the Radiation in the Case of Monoenergetic and Power-law Spectra of the Electrons.

We now present the expressions for the intensity and polarization of the radiation in various specific cases.

If all the electrons have the same energy (monoenergetic spectrum) and the magnetic field is uniform, the intensity of the radiation, according to equation (3.20), is

$$J(\nu, \mathbf{k}) = \frac{\sqrt{3}\, e^3}{mc^2} \tilde{N}(\mathbf{k}) H \sin\theta \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta = \tilde{N}(\mathbf{k})\, p(\nu), \qquad (3.22)$$

where $\tilde{N}(\mathbf{k}) = \int N(\mathbf{r}, \mathbf{k})\, dr$ is the number of electrons per unit solid angle along the line of sight, whose velocities are directed toward the observer. Formula (3.22) obviously coincides with (2.25) obtained in less rigorous fashion. The degree of polarization in this case, according to (3.14), (3.20), and (3.21), is

$$P = \frac{F_p\left(\frac{\nu}{\nu_c}\right)}{F\left(\frac{\nu}{\nu_c}\right)} = \begin{cases} \frac{1}{2}\left\{1 + \frac{\Gamma\left(\frac{1}{3}\right)}{2}\left(\frac{\nu}{2\nu_c}\right)^{2/3}\right\} & \text{for } \nu \ll \nu_c, \\ 1 - \frac{2}{3}\frac{\nu_c}{\nu} & \text{for } \nu \gg \nu_c \end{cases} \qquad (3.23)$$

and coincides with the degree of polarization of the total radiation (over all directions) of a single electron (2.22). Values of the functions F_p and F are given in Table I.

The energy spectrum of the electrons along the line of sight can be approximated within a limited energy interval $E_1 \le E \le E_2$ by a power function of the form

$$\tilde{N}(E, \mathbf{k})\, dE = \tilde{K}(\mathbf{k}) E^{-\gamma} dE. \qquad (3.24)$$

Here $\tilde{N}(E, \mathbf{k})$ is the number of electrons along the line of sight moving in the direction of the observer and taken per unit solid angle and per unit energy interval.

As we shall see later, for the electrons responsible for cosmic radio emission, such an approximation is applicable over a quite broad interval of energy. Here the limits E_1 and E_2 of the spectrum (3.24) frequently can be taken so that, within the range of frequencies of radiation of interest to us, the radiation from electrons with energies $E < E_1$ and $E > E_2$ will be negligible. On this assumption, in the integrals (3.20) and (3.21) we can take the spectrum (3.24) over the whole energy interval and make use of the relations

$$\int_0^\infty dE\, E^{-\gamma} \frac{\nu}{\nu_c} K_{2/3}\left(\frac{\nu}{\nu_c}\right) = \frac{1}{4}\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+7}{12}\right)\left[\frac{3eH\sin\theta}{2\pi m^3 c^5 \nu}\right]^{\frac{\gamma-1}{2}},$$

$$\int_0^\infty dE\, E^{-\gamma} \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta = \frac{1}{4}\frac{\gamma + \frac{7}{3}}{\gamma+1}\Gamma\left(\frac{3\gamma-1}{12}\right) \times \Gamma\left(\frac{3\gamma+7}{12}\right)\left[\frac{3eH\sin\theta}{2\pi m^3 c^5 \nu}\right]^{\frac{\gamma-1}{2}}, \qquad (3.25)$$

where $\Gamma(x)$ is the Euler gamma function and we assume that the condition $\gamma > 1/3$ is satisfied. Then (3.20) reduces to the following expression for the intensity of the radiation of a system of electrons with energy spectrum (3.24) in a homogeneous magnetic field **H**:

$$J(\nu, \mathbf{k}) = \frac{\sqrt{3}}{\gamma+1}\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right)\frac{e^3}{mc^2}\left(\frac{3e}{2\pi m^3c^5}\right)^{\frac{\gamma-1}{2}} \times \tilde{K}(\mathbf{k})\,[H\sin\theta]^{\frac{\gamma+1}{2}}\nu^{-\frac{\gamma-1}{2}}. \tag{3.26}$$

Here $\tilde{K}(\mathbf{k})$ is the coefficient in the spectrum (3.24).

Let us assume that the distribution of electrons can be regarded as homogeneous and isotropic, i.e., $N(E, \mathbf{r}, \mathbf{k}) = (1/4\pi)N(E)$, where

$$N(E)\,dE = KE^{-\gamma}\,dE \tag{3.27}$$

is the number of electrons per unit volume with arbitrary directions of motion and with energies within the interval E to $E+dE$.

Then

$$\tilde{K}(\mathbf{k}) = \frac{L}{4\pi}K,$$

where K is the coefficient in the energy spectrum (3.27) and L is the extent of the radiating region along the line of sight. Of course, in the general case, $\tilde{K}(\mathbf{k})$ may depend on the angle θ between the direction of the magnetic field and the line of sight.

In the case of a homogeneous field the degree of polarization of the radiation depends only on the exponent γ in the energy spectrum (3.24) and, as can be seen by using (3.14) and (3.25), is equal to

$$P = \frac{\gamma+1}{\gamma+\frac{7}{3}}, \tag{3.28}$$

which amounts to 75% when $\gamma = 3$ and 69% when $\gamma = 2$.

It is not appropriate to apply formulas (3.26) and (3.28) to magnetic bremsstrahlung of cosmic electrons since the observed radiation is collected from a large region of space, over different portions of which the magnetic field is oriented differently. One should rather assume that along the line of sight the direction of the magnetic field varies chaotically. In this case there is no polarization of the radiation, and its intensity is easily found by averaging (3.26) over all magnetic field directions. By using the relations

$$\frac{1}{2}\int_0^\pi (\sin\theta)^{\frac{\gamma+1}{2}}\sin\theta\,d\theta = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{\gamma+5}{4}\right)\Big/\Gamma\left(\frac{\gamma+7}{4}\right) \tag{3.29}$$

this gives the following expression for the intensity of the radiation from a homogeneous and isotropic distribution of electrons with energy spectrum (3.27) in a random magnetic field:

$$J_\nu = a(\gamma)\frac{e^3}{mc^2}\left(\frac{3e}{4\pi m^3c^5}\right)^{\frac{\gamma-1}{2}}H^{\frac{\gamma+1}{2}}KL\nu^{-\frac{\gamma-1}{2}} = 1.35\cdot 10^{-22}a(\gamma)LKH^{\frac{\gamma+1}{2}}\left(\frac{6.26\cdot 10^{18}}{\nu}\right)^{\frac{\gamma-1}{2}}\frac{\text{erg}}{\text{cm}^2\ \text{sec-sr-Hz}} \tag{3.30}$$

Here K is the coefficient in the spectrum (3.27) per unit volume, and by $H^{(\gamma+1)/2}$ we mean some average value of this quantity in the radiating region, while $a(\gamma)$ is a coefficient depending on the exponent of the energy spectrum γ:

$$a(\gamma) = 2^{\frac{\gamma-1}{2}}\sqrt{3}\,\Gamma\left(\frac{3\gamma-1}{12}\right)\Gamma\left(\frac{3\gamma+19}{12}\right) \times\Gamma\left(\frac{\gamma+5}{4}\right)\Big/8\sqrt{\pi}(\gamma+1)\Gamma\left(\frac{\gamma+7}{4}\right). \tag{3.31}$$

Values of the coefficient $a(\gamma)$ are given in Table II.

As we see from expressions (3.26) and (3.30), for a power-law energy spectrum of the radiating particles with exponent γ, the corresponding exponent in the frequency spectrum of the radiation is

$$J_\nu \propto \nu^{-\alpha}, \quad \alpha = \frac{\gamma-1}{2}. \tag{3.32}$$

We have assumed above that the energy spectrum of the electrons is a power-law spectrum (cf. (3.24) and (3.27) within some sufficiently wide range of energies. Now we present quantitative estimates of this interval. The error caused by the replacement in (3.20) and (3.21) of the finite integration limits by 0 and ∞ for a given frequency ν does not exceed 10% for each of the limits if the conditions

$$E_1(\nu) \leqslant mc^2[4\pi mc\nu/3eHy_1(\gamma)]^{1/2} \approx 2{,}5\cdot 10^2[\nu/y_1(\gamma)H]^{1/2}\ \text{eV},$$
$$E_2(\nu) \geqslant mc^2[4\pi mc\nu/3eHy_2(\gamma)]^{1/2} \approx 2{,}5\cdot 10^2[\nu/y_2(\gamma)H]^{1/2}\ \text{eV} \tag{3.33}$$

are satisfied.

The values of the numerical factors $y_1(\gamma)$ and $y_2(\gamma)$ for different values of γ are given in Table II. As we see, in the case of a power-law spectrum the energy interval giving the main contribution to the radiation at a given frequency is strongly dependent on the exponent γ. For $\gamma \geq 1.5$ ($\alpha \geq 0.25$) more than 80% of the radiation at a given frequency comes from electrons with energies different by no more than a factor of ten. For $\gamma < 1.5$ this energy interval increases

Table II

γ	1	1,5	2	2,5	3	4	5
$a(\gamma)$	0.283	0.147	0.103	0.0852	0.0742	0.0725	0.0922
$\tilde{a}(\gamma)$	0.34	0.22	0.15	0.11	0.074	0.036	0.018
$y_1(\gamma)$	0.80	1.3	1.8	2.2	2.7	3.4	4.0
$y_2(\gamma)$	0.00045	0.011	0.032	0.10	0.18	0.38	0.65

rapidly, and when $\gamma \to 1/3$ ($\alpha \to -1/3$) it becomes infinite. The point is that within the range of frequencies $\nu < \nu_m$, the intensity of the radiation from an individual particle $p_\nu \equiv p(\nu, E) \sim (\nu/\nu_c)^{1/3} \sim \nu^{1/3} E^{-2/3}$, and for the spectrum (3.17) the total intensity

$$J_\nu \infty \int p(\nu, E) N(E) dE \infty \int \frac{dE}{E^{\gamma+\frac{2}{3}}}$$

is unbounded if the energy spectrum of the particles is taken with index $\gamma \leq 1/3$ to arbitrarily large energies.

The value $\alpha = -1/3$ is minimal for magnetic bremsstrahlung in vacuum since even the spectrum of the radiation from an individual particle does not contain regions with a more rapid rise in intensity with frequency.

In applications of the theory one frequently comes across the problem of evaluating the interval of energies of electrons (E_1, E_2) giving radiation with a power spectrum (3.32) within the frequency interval ν_1 to ν_2. If the frequency interval is sufficiently large ($\nu_2/\nu_1 \gtrsim y_1(\gamma)/y_2(\gamma)$), then from the results given we conclude that the electrons should have a power-law energy spectrum within the energy interval $E_1 < E < E_2$, where

$$E_1 = mc^2 [4\pi mc\nu_1/3eHy_1(\gamma)]^{1/2} \simeq 2.5\cdot 10^2 [\nu_1/y_1(\gamma) H]^{1/2} \ \mathrm{eV},$$
$$E_2 = mc^2 [4\pi mc\nu_2/3eHy_2(\gamma)]^{1/2} \simeq 2.5\cdot 10^2 [\nu_2/y_2(\gamma) H]^{1/2} \ \mathrm{eV}. \tag{3.34}$$

If, however, the frequency interval is small, or α is small (in practice $\alpha < 0.25$, i.e., $\gamma < 1.5$), then we can only make a rough estimate of the interval of energy of the electrons by assuming that all the radiation of an electron with energy E occurs at frequency $\nu_m = 0.29\,\nu_c$. Then, in expressions (3.24) we must set $y_1(\gamma) = y_2(\gamma) = 0.24$.

If we make such a simplifying assumption, i.e., if we set (cf. (2.10) and (2.23)) $p_\nu \equiv p(\nu, E) = P(E)\delta(\nu - \nu_m)$, then for isotropically moving electrons with the spectrum (3.27) in a random magnetic field ($H_\perp^2 = 2/3\, H^2$) we easily find

$$J_\nu = \frac{L}{4\pi} \int P(E)\,\delta(\nu - \nu_m)\, KE^{-\gamma}\, dE$$
$$= \widetilde{a}(\gamma) \frac{e^3}{mc^2} \left(\frac{3e}{4\pi m^3 c^5}\right)^{\frac{\gamma-1}{2}} H^{\frac{\gamma+1}{2}} KL\nu^{-\frac{\gamma-1}{2}}, \tag{3.35}$$

where $\widetilde{a}(\gamma) = 0.31(0.24)^{(\gamma-1)/2}$. The values of the coefficient $\widetilde{a}(\gamma)$ are given in Table II. As we see, within the interval of values of γ considered, the exact formula (3.30) and the elementary approximate formula (3.35) differ only by an unimportant numerical coefficient.

3.4. Radiation in a Non-uniform Field

If the magnetic field over the extent of the line of sight L is inhomogeneous or cannot be regarded as completely random, the assumptions made in deriving equations (3.26) and (3.30) are not valid, and we must use expression (3.20). Then, in the general case, we must take account of the dependence of the magnetic field intensity H and the distribution function for the radiating particles $N(E, \mathbf{r}, \tau)$ on the coordinates. In this general form the problem was solved, for example, for a dipole magnetic field in papers [51,52], in order to determine the characteristics of the magnetic bremsstrahlung of the radiation belts of the earth and Jupiter.

It is sometimes of interest to consider a somewhat different formulation of the problem where, without giving a specific dependence of magnetic field on coordinates, we can restrict ourselves simply to some of its average characteristics. For example, in the problem of the polarization of magnetic bremsstrahlung, an important role is played by a certain effective anisotropy of the field, if the field cannot be regarded as homogeneous or completely random.

The calculation of the degree of polarization in such an "intermediate" case was carried out in [16] for two models of the magnetic field.

The first of these assumes that there is superposed on a homogeneous field some random (isotropic on the average over the radiating region) field H_c whose absolute value is constant. We may imagine that such a situation exists approximately in the neighborhood of the galactic plane and in particular in the spiral arms of the galaxy. If $H_\perp$ is the projection of the intensity of the homogeneous magnetic field on the figure plane and $\beta = H_\perp/H_c$, then in the two limiting cases of weak and strong homogeneous field the degree of polarization turns out to be

$$P = \frac{(\gamma+3)(\gamma+5)(\gamma+1)}{32\left(\gamma+\frac{7}{3}\right)} \beta^2 \qquad (\beta \ll 1), \tag{3.36}$$

$$P = \left(1 - \frac{2}{3\beta^2}\right) \frac{\gamma+1}{\gamma+\frac{7}{3}} \qquad (\beta \gg 1). \tag{3.37}$$

The second model corresponds to the situation where there is no homogeneous field, but because of the more or less regular character of the field (for example, a mixture of dipole or toroidal fields) certain directions occur more frequently than others. This case may be realized in the discrete sources of cosmic radio emission. If the distribution of magnetic fields over direction differs little from isotropic, while the intensity of the field H can be assumed to be approximately constant in absolute value, then the degree of polarization is

$$P = \frac{15(\gamma+1)(\gamma+5)}{8\left(\gamma+\frac{7}{3}\right)(\gamma+7)} \frac{\overline{\Delta H^2}}{H^2}, \tag{3.38}$$

where $\overline{\Delta H^2} = \overline{H_1^2} - \overline{H_2^2}$ is the difference between the averages over the volume of the source of the squares of the components of the magnetic field along two perpendicular directions in the figure plane; these directions are chosen so that this difference is a maximum. Thus, for both models the degree of polarization serves as a measure of the anisotropy of the magnetic field in the source of radiation.

4. INFLUENCE OF COSMIC PLASMA ON THE PROPAGATION AND RADIATION OF ELECTROMAGNETIC WAVES

In most of the cases one meets it may be assumed that the magnetic bremsstrahlung develops and is propagated in vacuum, as we have assumed above. This does not mean at all, however, that the effect of the medium, and in particular the cosmic plasma, can always be neglected.

On the contrary, the medium sometimes has a radical effect on the character of the electromagnetic radiation. Consider, for example, an oscillator (dipole) vibrating in an isotropic, non-absorbing plasma, for which the square of the index of refraction has the form

$$n^2(\omega) = 1 - \frac{4\pi e^2 N_e}{m\omega^2} = 1 - 3.18\cdot 10^9 \frac{N_e}{\omega^2} = 1 - 8.06\cdot 10^7 \frac{N_e}{\nu^2}, \tag{4.1}$$

where N_e is the concentration of electrons in the plasma. If the frequency of vibration of the oscillator ω_i is significantly higher than the plasma frequency

$$\omega_0 = 2\pi\nu_0 = \sqrt{\frac{4\pi e^2 N_e}{m}} = 5.64\cdot 10^4 \sqrt{N_e}, \tag{4.2}$$

the oscillator in the plasma radiates in approximately the same way as in vacuum. But when $\omega_i \lesssim \omega_0$ the influence of the plasma becomes decisive, since for $\omega_i < \omega_0$ the radiation is absent in general. This last point is already clear from the fact that when $\omega_i < \omega_0$ the index $n(\omega_i)$ becomes imaginary, and the field far from the oscillator is damped so that*

$$\mathscr{E} \sim \exp\left\{-\frac{\omega_i}{c}\sqrt{|n(\omega_i)|}\,r\right\}.$$

Another characteristic example is the radiation from a uniformly moving electron: In vacuum this radiation is absent, whereas in a medium it can occur—we are speaking of Cerenkov radiation which appears when the velocity of motion v exceeds the phase velocity of waves in the medium $c_p = c/n(\omega)$. Formally we may say that it is just the replacement of c by $c_p = c/n(\omega)$ that distinguishes the theory of radiation in a medium from the case of a vacuum. In order to understand the situation more specifically we shall show that formula (2.3) for the Doppler effect in the motion of a radiator in a medium becomes (cf. [53])

$$\omega = \frac{\omega_i\sqrt{1-\frac{v^2}{c^2}}}{\left|1-\frac{v}{c}n(\omega)\cos\psi\right|} \tag{4.3}$$

In the expressions in the denominator for the intensity we must naturally also replace the factor $1-(v/c)\cos\psi$ by $|1-(v/c)n\cos\psi|$. This replacement is extremely important because when $(v/c)n(\omega)\cos\psi_0/c = 1$, the frequency (4.3) and the intensity go to infinity. Of course, for observed quantities there is no divergence if we take account of dispersion and some other factors (for example, absorption) in actual cases. But the possibility of fulfilling this condition, which we write in the form

$$\cos\psi_0 = \frac{c}{n(\omega)v}, \tag{4.4}$$

already shows various points. For example, whereas in vacuum the frequency $\omega \to \infty$ only for $v/c \to 1$, and only in the direction of the velocity of the radiator (i.e., for angle $\psi = 0$; cf. (2.3)) in a medium $\omega \to \infty$ on the cone $\psi = \psi_0$ (cf. (4.3), (4.4)). Condition (4.4) determines the direction of the Cerenkov radiation, i.e., the cone $\psi = \psi_0$ is the Cerenkov cone. This cone separates the whole space of wave vector directions into two parts. Outside the cone (region of angles $\psi > \psi_0$) the Doppler effect is normal. Here formula (4.3) differs from (2.3) only by the replacement of c by c/n, while the physical processes in the radiation occur qualitatively just as in vacuum (for example, the radiating atom goes from a higher energy level to a lower). Within the cone (when $\psi < \psi_0$) the Doppler effect is said to be anomalous or "superluminal." Of course, the anomalous Doppler effect exists only for motions with "superluminal" velocity, i.e., when $v > c/n(\omega)$. For the anomalous effect we must take the absolute value of the denominator as we have written in (4.3) and as is automatically obtained in a quantum or classical calculation. The physical feature of the radiation in the region of the anomalous Doppler effect consists in the fact that the radiation is accompanied by the transition, for example, of an atom from a lower energy state to a higher; for a classical oscillator this corresponds to a jump of the vibration when the radiation occurs, whereas for the normal Doppler effect the vibration of the oscillator is damped as a result of radiation. (The increase of amplitude of oscillation during radiation or, in quantum language, the transition of the system to a higher energy state, is accompanied by a reduction of the kinetic energy of translational motion of the radiator, which guarantees the satisfying of the laws of conservation of energy and momentum; cf. [53].)

The influence of the medium on the radiation is essentially different depending on whether $n(\omega) > 1$ or $n(\omega) < 1$. If $n(\omega) < 1$, as is the case in an isotropic plasma (cf. (4.1)), then $vn/c < 1$ always, and the anomalous Doppler effect cannot occur. In this case, even at the very highest energies, when $v \to c$, the denominator in (4.3) does not tend to zero, and the radiation does not have the features typical for the radiation of ultrarelativistic particles in vacuum (cf. Sec. 2.1). Thus, even when $E/mc^2 \to \infty$, the radiation is concentrated not within a cone with opening angle $\sim mc^2/E$ (cf. (2.4)), but within the range of angles $\psi \lesssim \sqrt{1-n(\omega)}$ (for simplicity we are assuming that $1-n \ll 1$; cf.

*Here, obviously, the field is assumed to be sufficiently small so that we need not consider non-linear effects.

(4.3)). From similar arguments one can easily see that the effect of the medium is unimportant if

$$1-n^2(\omega) \ll \left(\frac{mc^2}{E}\right)^2.$$

This condition is also obtained, of course, from a direct calculation of the intensity of radiation in a medium (cf. (4.24)). If, however, $n > 1$, then the radiation is very similar in its properties to the radiation of ultrarelativistic particles in vacuum even when $v < c$, namely in the vicinity of the Cerenkov cone. Specifically, this means that the very highest frequencies and the main fraction of the energy will be radiated not along the direction of the instantaneous velocity of the moving radiator, but in the neighborhood of the Cerenkov cone. Here we must say that when dispersion is taken into account (dependence of n on ω), the situation becomes much more complicated since the Cerenkov angle ψ_0 itself depends on ω. All we wished to do, however, was to emphasize the features which occur for a radiator moving in a medium. This applies in particular to the magnetic bremsstrahlung in a medium, which will be considered in Sec. 4.3.

Still the formulas for the magnetic bremsstrahlung in vacuum in many cases are retained completely. This is explained by the low concentration of the cosmic plasma. Thus, in intergalactic space the influence of the medium is unimportant over the whole radio range (cf. the criteria (4.26)). In the interstellar medium ($N_e \lesssim 1$) the formulas obtained for the vacuum also can be used in a large part of the radio range, and the situation is changed only for waves with wave length $\lambda \gtrsim 30-100$ meters. The inclusion of the effect of the medium is more important for the long wave part of the spectrum of various discrete sources of radiation and also in the solar corona and, in general, in stellar atmospheres.

The process of radiation of electromagnetic waves is directly affected by the medium which is in the vicinity of the radiator, in a region with dimensions of the order of the wavelength in the medium, $\lambda = 2\pi c/n(\omega)\omega$. But at distances $r \gg \lambda$ from the radiator the wave field already is formed and "stripped off" from the source. Thus the influence of the medium on the radiation when $r \gg \lambda$ can be treated without making any connections with the character and nature of the radiation. Questions of this sort are usually called problems of propagation of electromagnetic waves. Here we must first of all explain how the amplitude (intensity) and the state of polarization of a plane wave of the type

$$\mathcal{E} = \mathcal{E}_0 \exp\left\{-\frac{\omega}{c}\varkappa z + i\left(\frac{\omega}{c}nz - \omega t\right)\right\}$$

changes as it traverses a path L through some medium.

The index of refraction n and the absorption index κ (absorption coefficient $\mu = 2\omega\kappa/c$) depend on the properties of the medium and the frequency ω of the radiation. Here we have to deal with the most varied conditions, and there are no universal formulae for all media and all frequencies. Thus γ rays with energy $E \gtrsim 10^{11}$ eV may be absorbed in the cosmos as a result of the process $\gamma + \gamma' \to e^+ + e^-$, i.e., creation of pairs from thermal photons (γ'), which are present in space as a result of radiation from stars. When $10^8 < E < 11^{11}$ eV, γ rays are absorbed mainly because of production of pairs $e^+ + e^-$ on nuclei and electrons, while for $E < 10^8$ eV we must also consider Compton scattering. X rays and soft γ rays are absorbed primarily as the result of photoeffects in atoms. In the optical region the important effect is the absorption in atomic transitions as well as in interstellar dust. Finally in the radio region the absorption in the cosmos occurs at the lines of neutral hydrogen ($\lambda = 21$ cm) and, in principle, in some other lines, but in the rest of the spectrum is associated with the collision of electrons with protons in the cosmic plasma. Later we shall restrict our treatment of the propagation of radio waves in plasma and leave out of account the effects of neutral atoms (cf. Sec. 4.1), since this case is the most interesting one in radioastronomy. Furthermore it is especially important to consider the reabsorption of the magnetic bremsstrahlung by the radiating relativistic electrons themselves (Sec. 4.2).

4.1. Propagation of Radio Waves in the Cosmic Plasma

The presence of magnetic fields in interstellar space and generally under cosmic conditions makes the plasma magnetoactive. The propagation of waves in such a plasma in general is strongly dependent on the intensity of the constant magnetic field, the angle between this field and the wave vector, etc. (cf., for example, [54]). However, if we do not consider stellar atmospheres and, specifically, the solar corona, under cosmic conditions the influence of magnetic fields shows itself only in a rotation of the plane of polarization of the radio waves. The point is that the gyrofrequency $\omega_H^{(0)} = 1.76 \times 10^7 H$ (cf. (2.1)), even in a field $H \sim 10^{-2}$ Oe, amounts to $\omega_H^{(0)} \sim 10^5$, from which $\lambda_H^{(0)} = 2\pi c/\omega_H^{(0)} = 1.07 \times 10^4\ H \sim 10^6$ cm. Usually, however, the field H in the cosmos is weaker, and consequently the frequency $\omega_H^{(0)}$ is even smaller (for $H \sim 10^{-5}$ we already have $\lambda_H^{(0)} \sim 10^9$ cm). Thus, even in the radio region (especially in the region of meter waves, which are most widely used in radioastronomy) the frequency of the radiation

$$\omega \gg \omega_H^{(0)}. \tag{4.5}$$

If this inequality is satisfied, the plasma can be regarded as practically isotropic (with index of refraction (4.1)), except when we are computing the phase difference between the ordinary and extraordinary waves. This difference $(\omega/c)(n_2 - n_1)L$ is proportional not only to the difference of the indices of refraction $n_2 - n_1$ of the waves of the two types, but also to the path length L. Therefore, obviously, even for negligible values of $|n_2 - n_1|$ the phase difference may become quite large. From the general formulas for a magnetoactive plasma one can easily show that for practically all angles θ between the constant magnetic field **H** and the wave vector **k**, the propagation of waves in the cosmic plasma can be regarded as "quasi-longitudinal."* As a result, for the angle Ψ through which the plane of polarization of the radiation rotates in traversing the path L, we may use the formula

$$\Psi = \frac{1}{2}\frac{\omega}{c}(n_2 - n_1)L = \frac{2\pi e^3 N_e H \cos\theta}{m^2c^2\omega^2}L$$
$$= 0.93\cdot 10^6 \frac{N_e LH\cos\theta}{\omega^2} = 2.36\cdot 10^4 \frac{N_e LH\cos\theta}{\nu^2}. \tag{4.6}$$

In the case where the quantities H, N_e, and θ vary along the line of sight but this change is small over a wave length, we must replace the product $N_e LH \cos\theta$ in (4.6) by an integral $\int_0^L N_e H \cos\theta\, dr$, which is taken along the line of sight. It is also of interest to consider the rotation of the polarization and the depolarization of the radiation when there are various inhomogeneities along the line of sight (gas clouds, local inhomogeneity of the magnetic field, etc.), but we shall not consider this here (cf. [55]).

The inequality (4.5) may be violated in stellar atmospheres, and it frequently cannot be used in the analysis of the propagation of radio waves in the solar corona. In such cases, one must use the very familiar general formulas for a magnetoactive plasma.[53,23] Under cosmic conditions the absorption of radio waves usually is comparatively small. Thus under the condition (4.5) in first approximation one can, for the average value of n, use the formula (4.1), and for Ψ (or $n_2 - n_1$), use formula (4.6), in which absorption is not included. At the same time total absorption along the line of sight may be important, and therefore one must know the coefficient of absorption of radiowaves μ during their propagation through the cosmic plasma.

The expression for μ depends on the ratio ω/ω_0, i.e., the ratio of the frequency of the radiation to the plasma frequency (4.2). If we exclude the case of stellar atmospheres, the electron concentration in the cosmic plasma $N_e < 10^4$ cm^{-3} and consequently $\omega_0 < 5\times 10^6$ ($\lambda_0 = 2\pi c/\omega_0 > 3\times 10^4$ cm = 300 m). Usually, however, $N_e \lesssim 10$ cm^{-3}, $\omega_0 \lesssim 10^5$ and $\lambda_0 \gtrsim 10$ km. At the same time, in radioastronomy usually one uses waves shorter than 30 meters, and only with satellites can one systematically carry out measurements at longer wavelengths.[56,57] We shall therefore limit ourselves to the case where

$$\frac{\omega}{\omega_0} = 1.77\cdot 10^{-5}\frac{\omega}{\sqrt{N_e}} \ll 1. \tag{4.7}$$

Under the condition (4.7) the absorption coefficient is given by the expression (the derivation for this formula can be found in [54], Sec. 37)

$$\mu = \frac{2\omega}{c}\varkappa = \frac{32\pi^2 e^6 N_e^2}{3\sqrt{2\pi}(kT_e m)^{3/2}c\omega^2}\ln\frac{(2kT_e)^{3/2}}{2.115e^2m^{1/2}\omega}$$
$$\approx \frac{10^{-2}N_e^2}{T_e^{3/2}\nu^2}\left(17.7 + \ln\frac{T_e^{3/2}}{\nu}\right), \tag{4.8}$$

where the electron temperature T_e is measured in °K and ν in Hz. (formula (4.8) is identical with formula (35) given in paper [22]; we shall throughout drop the subscript on T_e).

The absorption of radio waves, which we are considering occurs in the process of collisions of electrons with ions, i.e., in a process which is inverse to bremsstrahlung. Formula (4.8) is a purely classical formula (the quantum constant $\hbar$ does not appear in it), since it applies to the frequency region satisfying the condition

$$\hbar\omega \ll kT. \tag{4.9}$$

Furthermore, in using classical theory to describe the collisions of an electron with an ion, it is assumed that $e^2/\hbar v \ll 1$, i.e., $T \ll 3\times 10^5$ degrees. If $T \gtrsim me^4/k\hbar^2 = 3\times 10^5$ °K, then in (4.8) the logarithmic term has approximately the form $\ln(\sqrt{3\times 10^5}\,T/\nu)$. For $T \gg 3\times 10^5$ the factor $\ln[(2kT)^{3/2}/2.115\,e^2m^{1/2}\omega]$ in (4.8) is replaced by

$$\ln\left(\frac{4kT}{1.781\hbar\omega}\right) \approx 24.6 + \ln\left(\frac{T}{\nu}\right) \approx \left[17.7 + \ln\left(\frac{10^3 T}{\nu}\right)\right].$$

What we have stated, of course, does not mean that formula (4.8) could not be obtained by quantum methods or by using various quantum pictures, for example, the Einstein relations between the probabilities of spontaneous emission and absorption (cf., for example, [54]). But for the major part of the problems of radioastronomy one can limit oneself to the classical formula (4.8) if its conditions of validity are satisfied.

Knowing the absorption coefficient μ one can compute the optical thickness of the gas in the direction considered

$$\tau = \int \mu\, dr. \tag{4.10}$$

If $\tau \gg 1$, then an ionized gas with temperature T (or, more precisely, with an electron temperature T_e) radiates like a black body, i.e., the intensity of the radiation under the condition (4.9) is

$$J_\nu = \frac{2k\nu^2}{c^2}T. \tag{4.11}$$

In this case the spectral index α, i.e., the exponent in the relation $J_\nu = \text{const}\cdot\nu^{-\alpha}$ is $\alpha = -2$. For an arbitrary optical thickness

*The condition for "quasi-longitudinal" waves in this case has the form (cf.[54], Sec. 37) $u\sin^4\theta/4\cos^2\theta \ll 1$, $u\sin^2\theta \ll 1$, $\sqrt{u} = \omega_H^{(0)}/\omega$. When $\lambda = 2\pi c/\omega = 10^4$ cm and $H \sim 10^{-5}$, the parameter $u \sim 10^{-12}$.

$$J_\nu = \frac{2k\nu^2}{c^2} T_{\text{eff}} = 3.07\cdot 10^{-37}\nu^2 T_{\text{eff}} \ \frac{\text{erg}}{\text{cm}^2\ \text{sec-Hz-sr}}$$
$$= \frac{2.76\cdot 10^{-17}}{\lambda^2\ (\text{in meters})} T_{\text{eff}} \ \frac{\text{W}}{\text{m}^2\ \text{mHz-sr}}, \quad (4.12)$$

where

$$T_{\text{eff}} = T(1 - e^{-\tau}). \quad (4.13)$$

When $\tau \ll 1$ (optically thin layer), according to (4.8) and (4.10)

$$T_{\text{eff}} \approx T\tau \sim \nu^{-2},\ J_\nu = \frac{2k\nu^2}{c^2} T\tau = \text{const}, \quad \alpha = 0 \quad (4.14)$$

(the weak logarithmic dependence of (4.8) on frequency ν can usually be neglected). Thus the spectral index of the thermal radiation varies within the range $-2 \le \alpha \le 0$, while the effective temperature $T_{\text{eff}} \le T$. These two facts permit one in principle to separate out the thermal radiation of the medium from the non-equilibrium radiation and in particular from the radiation of the magnetic bremsstrahlung type.

4.2. Reabsorption of Magnetic Bremsstrahlung by Relativistic Electrons

If the dimension of the region which is filled with relativistic electrons is sufficiently large, one begins to feel the effect of absorption of the magnetic bremsstrahlung by the relativistic electrons themselves. This process of reabsorption leads to a redistribution of the energy over the spectrum of the magnetic bremsstrahlung of the system.

Let us determine the coefficient for absorption (self-absorption) in an ultrarelativistic electron gas which is in a magnetic field. Let $N(\mathbf{p})$ be the distribution function of the electrons in momentum space and J_ν the intensity of the radiation in a given direction. The reduction in the number of quanta in the radiation flux with intensity J_ν associated with true absorption caused by transitions of electrons from state 1 with energy $E - h\nu$ to state 2 with energy E is $B_{12}N(\mathbf{p} - \hbar\mathbf{k})J_\nu$, where $\hbar\mathbf{k}$ is the momentum of a photon with frequency $\nu = kc/2\pi$ and B_{12} is the Einstein absorption coefficient. On the other hand, the number of quanta in the flux increases as a result of stimulated emission (transitions from state 2 to state 1) by an amount $B_{12}N(\mathbf{p})J_\nu$. Thus the net change in number of quanta per unit volume per unit time is

$$B_{21}N(\mathbf{p})J_\nu - B_{12}N(\mathbf{p} - \hbar\mathbf{k})J_\nu,$$

while the reabsorption coefficient, taking account of all possible transitions, is

$$\mu_r = -\frac{1}{J_\nu}\frac{dJ_\nu}{dx}$$
$$= \int \left\{B_{12}N(\mathbf{p} - \hbar\mathbf{k}) - B_{21}N(\mathbf{p})\right\} \hbar\nu p^2\, dp\, d\Omega. \quad (4.15)$$

We now make use of the Einstein relation

$$B_{21} = B_{12} = A_{21}\frac{c^2}{2h\nu^3},$$

where the spontaneous radiation probability A_{21} is equal to the number of quanta radiated by an electron into unit solid angle per unit time in the absence of external radiation. We assume the radiation to be occurring in vacuum, i.e., we set the index of refraction equal to unity.

Since, for ultrarelativistic electrons, the radiation is concentrated in the direction of motion and its power $p(\nu) = p(\nu, E)$ is given by (2.21), on substitution in (4.15) we should set $A_{21}\, d\Omega = p(\nu, E)/h\nu$. Then, taking account of the fact that $\mathbf{p}$ and $\mathbf{k}$ are parallel,

$$N(\mathbf{p} - \hbar\mathbf{k}) = N(p - \hbar k) = N\left(p - \frac{h\nu}{c}\right).$$

Here for simplicity we assume the distribution of electrons to be isotropic; but we may take $N(p - (h\nu/c))$ to be the distribution function per unit solid angle for the direction $\mathbf{k}$. If we consider that the only transitions having significant intensity are those with $h\nu \ll pc$, we may set

$$N(p) - N\left(p - \frac{h\nu}{c}\right) = \frac{h\nu}{c}\frac{\partial N}{\partial p}.$$

As a result, expression (4.15) takes the form

$$\mu_r = \frac{c}{2\nu^2}\int \frac{dN(p)}{dp} p(\nu, E)\, p^2\, dp. \quad (4.16)$$

We now go over from the spectrum in momentum space to the energy spectrum of the electrons using the equations $E = cp$ and $N(p)4\pi p^2 dp = N(E)\,dE$, where the electrons are assumed to be distributed isotropically. Expression (4.16) now becomes

$$\mu_r = \frac{c^2}{8\pi\nu^2}\int E^2 \frac{d}{dE}\left(\frac{N(E)}{E^2}\right) p(\nu, E)\, dE. \quad (4.17)$$

Then for the power spectrum $N(E) = KE^{-\gamma}$ we get (cf. (2.21) and (3.25))

$$\mu_r = g(\gamma)\frac{e^3}{2\pi m}\left(\frac{3e}{2\pi m^3 c^5}\right)^{\frac{\gamma}{2}} K H_\perp^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}}, \quad (4.18)$$

where the coefficient $g(\gamma)$ which depends on the index of the energy spectrum is equal to

$$g(\gamma) = \frac{\sqrt{3}}{4}\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+22}{12}\right). \quad (4.19)$$

Values for the coefficient $g(\gamma)$ are given in Table III.

Table III

γ	1	2	3	4	5
$g(\gamma)$	0.96	0.70	0.65	0.69	0.83

Substituting numerical values in (4.18) we have

$$\mu_r = g(\gamma)\, 0.019\, (3.5\cdot 10^9)^\gamma K H_\perp^{\frac{\gamma+2}{2}} \nu^{-\frac{\gamma+4}{2}}. \quad (4.20)$$

Taking account of reabsorption, for example, for a homogeneous radiating layer of thickness L,

$$J_\nu \propto \int_0^L p(\nu) e^{-\mu_r r}\, dr = \frac{p(\nu)}{\mu_r}(1 - e^{-\mu_r L}).$$

If the depth of the radiating region $L \gg 1/\mu_r$, the spectral dependence of the emerging radiation will have the form

$$J_\nu \propto p_\nu/\mu_r \propto \nu^{-\frac{\gamma-1}{2}} \nu^{\frac{\gamma+4}{2}} = \nu^{5/2}. \tag{4.21}$$

As we see, this dependence is different from the frequency dependence of the equilibrium thermal radiation of an optically thick layer, which is proportional to the square of the frequency. This difference is related to the fact that the assumed power spectrum of the electrons is not an equilibrium spectrum.

4.3. Magnetic Bremsstrahlung in a Medium (Plasma)

In Sec. 4.1 we have already discussed qualitatively the effect of the index of refraction of the medium on the process of radiation of electromagnetic waves. We now must dwell on the quantitative inclusion of the effect of the medium on the intensity of the magnetic bremsstrahlung. In the cosmos, under the condition (4.5), we may assume the plasma to be isotropic and use formula (4.1) for n, with $n \leq 1$. This is just the case which we will consider. More general calculations[23,58] are needed, for example, for the solar atmosphere where the frequencies ω and $\omega_H^{(0)} = eH/mc$ may be comparable to one another.

The computation of the power radiated by an electron moving in a magnetic field in a medium with $n < 1$ is analogous to the computation mentioned above for motion in vacuum, and under the condition $1 - n \ll 1$ gives the following expression for the spectral density of the power of the radiation

$$p(\nu) = \sqrt{3}\, \frac{e^3 H_\perp}{mc^2} \left[1 + (1-n^2)\left(\frac{E}{mc^2}\right)^2 \right]^{-\frac{1}{2}} \times \frac{\nu}{\nu_c'} \int_{\nu/\nu_c'}^{\infty} K_{5/3}(\eta)\, d\eta, \tag{4.22}$$

where

$$\nu_c' = \nu_c \left[1 + (1-n^2)\left(\frac{E}{mc^2}\right)^2 \right]^{-\frac{3}{2}}. \tag{4.23}$$

We see from (4.22) and (4.23) that the medium has a strong influence on the radiation only under the condition

$$(1-n^2)\left(\frac{E}{mc^2}\right)^2 \gtrsim 1.$$

If, however,

$$1 - n^2 \ll \left(\frac{mc^2}{E}\right)^2, \tag{4.24}$$

the influence of the medium can be neglected (this same criterion was obtained in section (4.1)). Using expression (4.1) for the index of refraction in a plasma, the inequality (4.24) can be written in the form of a condition on the frequency interval for which the influence of the medium is not appreciable:

$$\nu^2 \gg \frac{e^2}{\pi m} N_e \left(\frac{E}{mc^2}\right)^2 = \frac{4}{3} \frac{ecN_e}{H_\perp} \nu_c, \tag{4.25}$$

where the characteristic frequency ν_c is determined by the expression (2.16). In order for the effect of the medium to be negligible over the main interval of frequencies of the magnetic bremsstrahlung $\nu \sim \nu_c$, it is necessary that the criterion

$$\nu \gg \nu_n \equiv \frac{4ecN_e}{3H_\perp} \approx 20 \frac{N_e}{H_\perp} \tag{4.26}$$

be satisfied. Based on this condition one can easily verify the validity of the statement made at the beginning of Sec. 4, that it is possible in most cases occurring in the cosmos to neglect the effect of the medium on the intensity of the magnetic bremsstrahlung.

5. SOME APPLICATIONS OF THE THEORY OF MAGNETIC BREMSSTRAHLUNG TO ASTROPHYSICS

5.1. General Remarks

The theory of magnetic bremsstrahlung has a very extensive and rapidly increasing region of application in astrophysics. This fact is explained by two circumstances which have become clear during the last ten to fifteen years.

First of all, relativistic particles, and in particular relativistic electrons, occur under cosmic conditions not as an exception, but as a rule. Their appearance is caused by the fact that in a moving or turbulent plasma there are practically always various instabilities and various accelerating mechanisms.

Second, as a rule there are magnetic fields in the cosmos. Their occurrence is also related to instabilities, in this case to the instability of the motion of a conducting medium (cosmic plasma) in the absence of magnetic fields. In other words, the blowups of various oscillations and turbulences (in the broad sense of this concept) leads on the one hand to the appearance of "superthermal" particles and generally insures the injection of fast particles. On the other hand, the production of a turbulent plasma, especially in the absence of collisions, means precisely that in it there are blowups, and different electromagnetic "normal" waves are propagated, including the low frequency waves which are called magnetohydrodynamic waves. The appearance in a plasma of different motions leads to the "twisting" of the force lines, i.e., an increase in intensity of the magnetic field.

The question of what level the energy density of relativistic particles (cosmic rays) reaches and how high the magnetic field rises does not become completely clear and in general under nonequilibrium conditions we cannot give an entirely general answer. But apparently, in the cosmos, one frequently has conditions which are close to quasi-equilibrium, when

$$w_{\text{c.r.}} \sim \frac{H^2}{8\pi} \sim \frac{\varrho u^2}{2}. \tag{5.1}$$

Here $w_{\text{c.r.}}$ is the density of energy of cosmic rays, $H^2/8\pi$ is the energy density of the field, and $\rho u^2/2$ is the density of kinetic energy of chaotic (turbulent) motion of the gas.

The presence of relativistic electrons and magnetic fields—these are necessary and also practically sufficient conditions for the appearance of magnetic bremsstrahlung. As we have already pointed out in the Introduction, and as is very well known, the magnetic bremsstrahlung process accounts for the radiation and, especially, the radio emission of a very large number of cosmic objects.

The *primary use of the theory of magnetic bremsstrahlung* consists in drawing conclusions about relativistic electrons and magnetic fields in the sources of radiation on the basis of measurements of the intensity, spectrum, and polarization of cosmic radiation. Another application is associated with the analysis of the changes in intensity of the radiation of the source, reabsorption, depolarization, and rotation of the plane of polarization of magnetic bremsstrahlung, for the purpose of determining various parameters, (for example, electron concentration), characterizing both the source of radiation itself as well as the medium on the path from the source to the earth.

It would be impossible and useless to discuss in detail, within the framework of the present paper, the various methods and possibilities for using the theory of magnetic bremsstrahlung. Our problem is much more prosaic, to point out some of the most important relations and formulas which allow one to make typical calculations.

5.2. Electronic Component of Cosmic Rays in Extended and Discrete Sources of Radio Emission

Quite frequently one deals with a situation where the spectrum of the radiation in a given region of frequencies can be regarded to sufficient accuracy as a power law, i.e., $J_\nu \sim \nu^{-\alpha}$. We furthermore assume that from some arguments (presence of polarization, very high effective temperature, or considering that for the thermal radiation $\alpha \leq 0$) we are sure that the radiation is of magnetic bremsstrahlung origin. Then, as we see from (3.32) one immediately determines the exponent γ in the differential energy spectrum of electrons $N(E) = KE^{-\gamma}$. Namely

$$\gamma = 2\alpha + 1. \tag{5.2}$$

If the magnetic field in the radiating region is assumed on the average over the line of sight to be random in direction and equal to H, then from formula (3.30) we have

$$K = \frac{7.4 \cdot 10^{21} J_\nu}{a(\gamma) LH} \left(\frac{\nu}{6.26 \cdot 10^{18} H}\right)^{\frac{\gamma-1}{2}} = \frac{8.9 \cdot 10^{22} H T_{\text{eff}}}{a(\gamma) L} \left(\frac{\nu}{6.26 \cdot 10^{18} H}\right)^{\frac{\gamma+3}{2}} \tag{5.3}$$

Here

$$T_{\text{eff}} = \frac{c^2}{2k\nu^2} J_\nu, \tag{5.3'}$$

the path length L is measured in cm, H in oersted, ν in Hz, T_{eff} in degrees, and K in $\text{erg}^{-\gamma} \cdot \text{cm}^{-3}$. We recall furthermore that $N(E)dE = KE^{-\gamma}dE$ is the number of electrons per unit volume (cm^3) in the energy interval E to E+dE. Furthermore, the distribution of electrons along the line of sight (path L) is assumed to be isotropic and homogeneous. For an *inhomogeneous distribution, other things being unchanged,* KL in (3.30) and (5.3) is replaced by $\int_0^L K\,dr$. As for the assumption that the electrons are isotropic, it is used in the derivation of formulas (3.30) and (5.3) since it is assumed that the distribution of electrons over direction does not depend on the direction of the vector **H** at a given point in space. If, however, the field in the radiating region can be assumed to be uniform (in particular, on the basis of polarization measurements), then one should use formula (3.26). In deriving this formula the assumption of isotropy was, in fact, not made. The only thing necessary was that the distribution over direction vary slightly within the limits of the cone with opening angle $\sim mc^2\gamma E$ along the line of sight.

If the spectrum is a power spectrum with $\nu_1 \leq \nu \leq \nu_2$, then according to formulas (3.34) we can determine the values of the energy E_1 and E_2 between which the electron spectrum also can be taken to be a power law. For a rough estimate it is convenient to use the simple relation (2.23) between E and $\nu = \nu_m$ for monoenergetic electrons.

We remark that information on the quantity $\gamma = 2\alpha + 1$ can also be obtained from polarization measurements (cf. (3.28)) so long as depolarizing factors can be assumed to be absent, as is the case for sufficiently high frequencies for radiation in a quasi-uniform field. Unfortunately, this latter condition occurs only as an exception in the cosmos.

If the spectral index α is unknown or if one wants to obtain a lower limit for the total number of relativistic electrons, one should use formula (3.22) for monoenergetic electrons, according to which (cf. also (2.25) and (2.26))

$$\widetilde{N}(\mathbf{k}) = J(\nu, \mathbf{k})/p(\nu) = \frac{J_\nu(\mathbf{k})}{1.6 e^3 H_\perp / mc^2}. \tag{5.4}$$

Here it is assumed that for all electrons the maximum in the radiation spectrum occurs at the observed frequency ν, i.e., their energy is determined by expression (2.23), while the spectral density of the power of the radiation is given by expression (2.24). In the isotropic case when $H_\perp^2 = \frac{2}{3}H^2$, while $\tilde{N}(\mathbf{k}) = NL/4\pi$, the concentration of relativistic electrons according to (5.4), is equal to

$$N = \frac{4\pi mc^2}{1.6e^3 LH_\perp} J_\nu(\mathbf{k}) = 7.2\cdot 10^{22}\frac{J_\nu(\mathbf{k})}{LH}. \qquad (5.5)$$

When one considers discrete sources, then usually the measured quantity is not the intensity J_ν, but the spectral density of the flux of radiation

$$\Phi_\nu = \int J_\nu \, d\Omega, \qquad (5.6)$$

where the integration is taken over all solid angle occupied by the source. If the linear size of the source L is small compared to the distance R to it, while the absolute value of the magnetic field intensity and the concentration of relativistic electrons can be assumed to be approximately constant over the volume of the source, we have from (5.6) and (3.30)

$$\Phi_\nu = 1.35\cdot 10^{-22} a(\gamma)\frac{KVH^{\frac{\gamma+1}{2}}}{R^2}\left(\frac{6.26\cdot 10^{18}}{\nu}\right)^{\frac{\gamma-1}{2}}, \qquad (5.7)$$

where V is the volume of the source (for a spherical source obviously $V = \pi L^3/6$).

Expressing K in terms of the spectral density of the flux of radiation Φ_ν observed at some frequency ν, we get

$$K = \frac{7.4\cdot 10^{21} R^2 \Phi_\nu}{a(\gamma) HV}\left(\frac{\nu}{6.26\cdot 10^{18} H}\right)^{\frac{\gamma-1}{2}}. \qquad (5.8)$$

From this one can determine the total number of relativistic electrons in the energy interval (E_1, E_2):

$$N_t = V\int_{E_1}^{E_2} KE^{-\gamma}\,dE = \frac{7.4\cdot 10^{21}}{(\gamma-1)\,a(\gamma)}\frac{R^2\Phi_\nu}{H}\left[\frac{y_1(\gamma)\,\nu}{\nu_1}\right]^{\frac{\gamma-1}{2}} \times\left\{1-\left(\frac{y_2(\gamma)\,\nu_1}{y_1(\gamma)\,\nu_2}\right)^{\frac{\gamma-1}{2}}\right\} \qquad (5.9)$$

Here E_1 and E_2 are the limits of the energy interval in which the electron spectrum has the form $KE^{-\gamma}$. The frequencies ν_1 and ν_2 are related to E_1 and E_2 according to (3.34); in the frequency interval (ν_1, ν_2) the spectrum of the radiation will be a power law with index $\alpha = (\gamma-1)/2$ (cf. Sec. 3.3). Since usually $\nu_1 \ll \nu_2$ and $y_2(\gamma) < y_1(\gamma)$, for $\gamma > 1$ the number of electrons is determined practically only by the lower limit of the frequency interval and is equal to

$$N_t(>E_1) = \frac{7.4\cdot 10^{21}}{(\gamma-1)\,a(\gamma)}\frac{R^2\Phi_\nu}{H}\left[\frac{y_1(\gamma)\,\nu}{\nu_1}\right]^{\frac{\gamma-1}{2}}. \qquad (5.10)$$

The values of the factors $a(\gamma)$ and $y_1(\gamma)$ are given in Table II.

Similarly we can represent the total energy of the electrons in the source responsible for radiation in the observed interval of frequencies $\nu_1 \le \nu \le \nu_2$ as

$$W_e = V\int_{E_1}^{E_2} KE^{-\gamma+1}\,dE = A(\gamma,\nu)\frac{R^2\Phi_\nu}{H^{3/2}}, \qquad (5.11)$$

where

$$A(\gamma,\nu) = \begin{cases} \dfrac{2.96\cdot 10^{12}}{(\gamma-2)\,a(\gamma)}\nu^{1/2}\left[\dfrac{y_1(\gamma)\,\nu}{\nu_1}\right]^{\frac{\gamma-2}{2}}\left\{1-\left[\dfrac{y_2(\gamma)\,\nu_1}{y_1(\gamma)\,\nu_2}\right]^{\frac{\gamma-2}{2}}\right\} & \text{for } \gamma > 2, \\ 1.44\cdot 10^{13}\nu^{1/2}\ln\left\{\dfrac{y_1(\gamma)\,\nu_2}{y_2(\gamma)\,\nu_1}\right\} & \text{for } \gamma = 2, \\ \dfrac{2.96\cdot 10^{12}}{(2-\gamma)\,a(\gamma)}\nu^{1/2}\left[\dfrac{y_2(\gamma)\,\nu}{\nu_2}\right]^{\frac{\gamma-2}{2}}\left\{1-\left[\dfrac{y_2(\gamma)\,\nu_1}{y_1(\gamma)\,\nu_2}\right]^{\frac{2-\gamma}{2}}\right\} & \text{for } \frac{1}{3} < \gamma < 2. \end{cases} \qquad (5.12)$$

5.3. Cosmic Rays and Magnetic Fields in Discrete Sources of Magnetic Bremsstrahlung

The formulas (5.3), (5.8), and (5.11) given above permit one to determine the electron concentration along the line of sight in an extended source (for example, in the galactic halo) or to determine the total energy of relativistic electrons in a discrete source from the known Φ_ν and R only if we know the field H. Unfortunately, there are still no reliable, independent methods for estimating the strength of the magnetic field in sources, and therefore in calculating W_e one must make additional assumptions.

As the basic assumption of this sort, one usually assumes that the energy of the magnetic field in the source, $W_H = (H^2/8\pi)V$, and the energy of relativistic particles (cosmic rays, including relativistic electrons) $W_{c.r.}$, in first approximation are equal to one another. Actually, this assumption corresponds to a minimum total energy of the system of field and particles for a given power in the magnetic bremsstrahlung. More precisely, the minimum of the total energy of the relativistic electrons (5.11) and the magnetic field in the source, i.e., the minimum of the quantity $W = W_e + W_H = C_1 H^{-3/2} + C_2 H^2$, where C_1 and C_2 are coefficients independent of H, occurs when $W_H = \frac{3}{4}W_e$. (A similar result, $W_H = \frac{3}{4}W_{c.r.}$, is also obtained using the formula (5.14) below, so long as κ_r is independent of H.) We note furthermore that a magnetic field with an energy density significantly less than the energy density of the relativistic particles could not retain the relativistic particles within the limited volume of the source. As a result, the outflow of particles from the system itself would probably lead to a state of energetic quasi-equilibrium between the magnetic field and the relativistic particles. Thus, it seems quite reasonable to assume that in the source

$$W_H = \varkappa_H W_{c.r.}, \qquad (5.13)$$

where κ_H is a numerical coefficient of order one.

Since the data on radio observations permit one to judge only the number and energy of electrons in the source, to determine the total energy of all relativistic particles one must also establish a relation between this quantity and the energy of relativistic electrons W_e. Any sort of reliable method for estimating the fraction of relativistic electrons in the total energy of relativistic particles does not exist at present, and so one must introduce some proportionality coefficient between the energies of all cosmic rays in the source and the energy of the relativistic electrons:

$$W_{\text{c.r.}} = \varkappa_r W_e. \tag{5.14}$$

Usually one assumes that the proportionality coefficient is of order $\kappa_r = 100$. The choice of this value is to a large extent arbitrary, but a basis for it may be the relation between cosmic rays and electrons in the galaxy and in some radio nebulae (for example, in Cassiopeia A, cf. [59]).

Under these assumptions, from the observed flux of radio emission one can directly determine both the magnetic field intensity and the total energy of cosmic rays and electrons in the source, if one knows the spectrum, the angular diameter, and the distance of the source. In fact, from (5.11), (5.13), and (5.14) it follows that

$$W_H \equiv V\frac{H^2}{8\pi} = \varkappa_H \varkappa_r A(\gamma,\ \nu)\frac{R^2\Phi_\nu}{H^{3/2}}. \tag{5.15}$$

Then

$$H = \left[48\varkappa_H\varkappa_r A(\gamma,\ \nu)\frac{\Phi_\nu}{R\varphi^3}\right]^{2/7}, \tag{5.16}$$

where $A(\gamma, \nu)$ is determined by expression (5.12), $V = (\pi/6)L^3$ and $\varphi = L/R$ is the angular diameter of the source. Then the total energy of cosmic rays in the source is equal to

$$W_{\text{c.r.}} = \varkappa_r W_e = 0.19\varkappa_H^{-3/7}[\varkappa_r A(\gamma,\ \nu)\Phi_\nu R^2]^{4/7}(R\varphi)^{9/7}. \tag{5.17}$$

5.4. Emission Spectrum and Characteristics of Discrete Sources

We assumed above (in Secs. 5.2 and 5.3) that the spectrum of the radiation and the corresponding spectrum of the electrons followed power laws. Of course, such an assumption has only a limited validity, and actually the spectra of all sources somewhere deviate (kink or pile up). The study of the reasons for the change in spectral index is a matter of overwhelming interest since it discloses possibilities for determining various parameters of the sources.

The inclusion of all possible factors makes the picture very difficult to see through. It is, therefore, natural to restrict oneself to two of the more frequent formulations of the problem.

Within the framework of one of these, we assume that the electrons radiate in vacuum, and that their radiation is propagated without distortion, that the spectrum of electrons no longer is assumed to be a power law, and in general is not assigned beforehand. In this case, the problem consists in (based on some specific pictures) determining the nature of the energy spectrum of the electrons and the corresponding frequency spectrum of the magnetic bremsstrahlung.

In the second formulation of the problem, the electron spectrum will be assumed to be given (a power law in the simplest case), but the effect of the medium in the process of radiation and propagation of electromagnetic waves will not be neglected. Here the problem reduces to explaining the nature of the changes in the frequency spectrum, polarization, and intensity of the radiation caused by the influence of the medium.

Let us look at the first formulation of the problem, i.e., let us consider the factors determining the character of the energy spectrum of ultrarelativistic electrons. If for simplicity we assume the distribution of electrons to be isotropic, for which there is a justification in some cases, then this distribution is completely characterized by the function $N(e, \mathbf{r}, t)$ giving the number of electrons per unit volume and unit energy interval at the time t. When we include spatial diffusion, energy losses and the contributions from sources of electrons, the function $N = N(E, \mathbf{r}, t)$ satisfies the equation

$$\frac{\partial N}{\partial t} - D\Delta N + \frac{\partial}{\partial E}[b(E)N] + \frac{N}{T} = Q(E,\ \mathbf{r},\ t). \tag{5.18}$$

Here D is the diffusion coefficient for electrons, $b(E) = dE/dt$ is the rate of change of the energy of the electrons as a result of continuous loss due to radiation and collision (this term by necessity includes the systematic acceleration of the particles in the variable magnetic field), T is the lifetime of the electrons with respect to catastrophic losses, for example, radiation with large energy transfer in one collision, $Q(E, \mathbf{r}, t)$ is the strength of the sources (number of electrons emerging per unit time) taken per unit volume and unit energy interval.

In applications to the general, non-thermal emission of the Galaxy, in first approximation it is natural to limit oneself to the stationary picture, setting $N = N(E, \mathbf{r})$ and $Q = Q(E, \mathbf{r})$ in (5.18). Here it is assumed that over the last $(1-3) \times 10^8$ years (the lifetime of cosmic rays in the galaxy), the galaxy has changed very little. In particular, if during this time there have occurred, as is assumed in paper [60], explosions of the galactic nucleus, we assume that they led to no significant change in intensity of relativistic particles (cosmic rays and electrons) in the galaxy. For such a stationary model, the spectrum and distribution of electrons in the galaxy was determined by means of Eq. (5.18) in [61,62]. In [61] the spectrum of the sources was assumed to be a power law with index $\gamma = 2$, while in [62] the source of relativistic electrons was taken to be the process of generation of electrons in the collisions of cosmic rays with the nuclei of the interstellar gas, i.e., the electrons were assumed to be secondary with respect to the proton and nuclear components of cosmic rays. In papers [61,62] spatial diffusion and continuous loss of energy for the electrons were taken into account.

The spectrum of secondary electrons in the galaxy was also calculated in [63] for a spatially homogeneous stationary model where $N = N(E)$ (diffusion was not considered, but to estimate the outflow of particles, in (5.18) T was taken to be the time T_e for diffuse emergence of particles from the galaxy).

If we include all these various factors (non-power spectrum of electron sources, energy losses, and diffusion), the spectrum of electrons no longer, of course, can be a power law if we are not considering individual small regions. Thus, to calculate the spectral intensity of the magnetic bremsstrahlung, one must use the general expression (3.20), which, for an isotropic distribution of electrons in a field **H** which is random in direction, takes the form

$$J_\nu = \frac{\sqrt{3}\, e^3}{4\pi mc^2} \int_0^L dr \int_{mc^2}^{\infty} dE N(E, \mathbf{r})\, H(\mathbf{r})\, \Phi(\nu/\nu_0). \quad (5.19)$$

Here

$$\nu_0 = \frac{\nu_c}{\sin\theta} = \frac{3eH}{4\pi mc}\left(\frac{E}{mc^2}\right)^2, \quad (5.20)$$

while the function (cf. [34,62])

$$\Phi(\zeta) = \zeta^3 \int_\zeta^\infty \frac{d\xi}{\xi^2\sqrt{\xi^2-\zeta^2}} \int_\xi^\infty K_{5/3}(\eta)\, d\eta = \zeta^3 \int_\zeta^\infty \frac{d\xi}{\xi^3\sqrt{\xi^2-\zeta^2}} F(\xi) \quad (5.21)$$

gives the spectral distribution of the radiation from an electron, averaged over all angles θ, between its velocity and the field **H**. Thus, for example, the spectral density of the power of radiation from electrons with energy E (monoenergetic spectrum) and an isotropic distribution in the chaotic field is equal to

$$\bar{p}_\nu = \frac{1}{2} \int_0^\pi p_\nu \sin\theta\, d\theta = \frac{\sqrt{3}\, e^3 H}{mc^2} \Phi\left(\frac{\nu}{\nu_0}\right), \quad (5.22)$$

where p_ν is the spectral density (2.21) of radiation of an electron moving at an angle θ to the magnetic field.

In the problems considered in [62,63] the reason for the non-power nature of the spectrum of the radiation was primarily the non-power spectrum of the sources of electrons. However, even in the case of a power spectrum of the sources, [61] the character of the spectrum of the electrons (and consequently also the spectrum of the radiation) may be changed drastically as a result of energy losses.

This can be easily seen on the example of a stationary homogeneous problem where $N = N(E)$. In this case, the solution of (5.18) has the form (cf. [34,61]; it is assumed that $b(E) < 0$, i.e., there is no acceleration of particles, or it is less important than the losses)

$$N(E) = \frac{1}{|b(E)|} \int_E^\infty \exp\left\{-\frac{1}{T}\int_{E_0}^{E} \frac{dE'}{b(E')}\right\} Q(E_0)\, dE_0. \quad (5.23)$$

If there are no catastrophic losses ($T = \infty$), and the sources have a power spectrum of the form $Q(E_0) = Q_0 E_0^{-\gamma_0}$, then from (5.23) we get

$$N(E) = \frac{Q_0 E^{-(\gamma_0-1)}}{(\gamma_0-1)\,|b(E)|}. \quad (5.24)$$

For losses of energy due to magnetic bremsstrahlung and Compton effect, $b(E) \backsim E^2$ (cf. (2.10)); therefore the spectrum (5.24) has the form $N(E) = KE^{-\gamma}$, where the index is

$$\gamma = \gamma_0 + 1. \quad (5.25)$$

In the case of radiation losses, if we assume approximately that these losses are continuous, then $b(E) \backsim E$ and, as is clear from (5.24)

$$\gamma = \gamma_0. \quad (5.26)$$

For ionization losses (loss in collision with particles of the medium) $b(E)$ for ultrarelativistic electrons depends only logarithmically on energy. This dependence can be neglected in first approximation and then

$$\gamma = \gamma_0 - 1. \quad (5.27)$$

In various parts of the energy spectrum usually losses of different types predominate (ionization losses at low energies, magnetic bremsstrahlung and Compton losses at high energies). Therefore even for a power law over the whole range of the energy spectrum of the sources, the spectrum of the electrons will not be of this type.

Now let us consider the non-stationary case which apparently applies to such discrete sources of radio and optical magnetic bremsstrahlung as the radio galaxies, exploding nuclei of galaxies, and supernovae. In the non-stationary case, the spectrum of the electrons is determined by the expression (cf. [34,61])

$$N(E,\ t) = \frac{1}{|b(E)|} \int_E^\infty \exp\left\{-\frac{1}{T}\int_{E_0}^{E} \frac{dE'}{b(E')}\right\} \times Q\left(E_0,\ t - \int_{E_0}^{E} \frac{dE'}{b(E')}\right) dE_0. \quad (5.28)$$

A detailed analysis of this expression for the case of magnetic bremsstrahlung losses was given in [64]. However, the main consequences can already be obtained from the expression for the magnetic bremsstrahlung loss (2.10). In this case $b(E) = -P(E)$ and integrating the equation

$$\frac{dE}{dt} = -P(E) \equiv -\beta E^2,$$

where

$$\beta = \frac{2e^4}{3m^4c^7} H_\perp^2 = 1.95 \cdot 10^{-9} \frac{H_\perp^2}{mc^2}, \quad (5.29)$$

we get

$$E = \frac{E_0}{1 + \beta E_0 t}, \quad (5.30)$$

where E_0 is the energy of the electron at time $t = 0$. It then follows that the energy of the electron decreases by a factor of two during the time

$$T_M = \frac{1}{\beta E_0} = \frac{5.1 \cdot 10^8}{H_\perp^2} \frac{mc^2}{E} \text{ sec.} \quad (5.31)$$

Furthermore, as is clear from expression (5.30), for any initial energy of the electron, at time t its energy does not exceed the value

$$E_m(t) = \frac{1}{\beta t} = \frac{5.1\cdot 10^8}{H_\perp^2 t} mc^2 = \frac{2.6\cdot 10^{14}}{H_\perp^2 t}\,\mathrm{eV}. \quad (5.32)$$

Therefore, if the generation of electrons in the source ceases at $t = 0$, then after time t one will observe in the electron spectrum a sharp drop at the energy, determined by the expression (5.32). This drop will correspond in the frequency spectrum of the magnetic bremsstrahlung to a sharp edge at frequencies (cf. (1.23))

$$\nu > \nu_m(E_m(t)) = \frac{3.1\cdot 10^{23}}{H_\perp^3 t^2}\,\mathrm{Hz}. \quad (5.33)$$

If, however, the source is "switched on" at time $t = 0$ and is stationary starting from that moment, then in the energy range $E > 1/\beta t$ one can have a stationary spectrum developed, which, in the case of a power law for the sources, will have the exponent (5.25). For $E < 1/\beta t$ the spectrum will differ very little from the spectrum of the sources, since the losses in this energy region are small, according to (5.31). Thus, the quantities $E_m(t)$ and $\nu_m[E_m(t)]$ (cf. (5.32) and (5.33)) in this case determine the position of the kink in the energy spectrum of the electrons and in the partial spectrum of their radiation, respectively.

In a certain sense an analogous situation also holds in the stationary case, when we include diffusion emergence of particles from the radiating region. In this case, the role of the time t is played by the effective time of diffuse emergence $T_e = L^2/2D$ where L is the size of the region and D the diffusion coefficient. For particles with energies $E \gg E_c$, where

$$E_c = \frac{1}{\beta T_e} = \frac{2D}{\beta L^2}, \quad (5.34)$$

the spectrum will be distorted by the effect of losses and, for example, for sources with a power spectrum, will have an exponent (5.25), whereas for $E < E_c$ the spectrum will differ little from the spectrum of the sources. Accordingly, in the frequency spectrum of the radiation, in the region of frequencies

$$\nu \sim \nu_m(E_c) = \frac{3.1\cdot 10^{23}}{H_\perp^3 T_e^2}\,\mathrm{Hz} \quad (5.35)$$

one will observe a kink. Thus the analysis of singularities in the frequency spectrum can give valuable information about the age of sources, diffusion coefficient, etc.

For a given form of the energy spectrum of the electrons, a change in intensity of the magnetic bremsstrahlung may be caused both by the outflow of electrons from the radiating region, as well as by the change in dimensions of the source of radiation, for example due to the expansion of a radiating nebula.[65] Let us consider in more detail the change in intensity of radiation when the dimensions of the region occupied by relativistic electrons and magnetic field changes. Here we shall assume that there is no injection or "pumping in" of energy to the relativistic particles.

Since, under cosmic conditions, to high accuracy, we satisfy the condition for freezing in of force lines, i.e., conservation of magnetic flux through a material contour, for a uniform expansion

$$H \propto L^{-2}, \quad (5.36)$$

where L is the size of the source of radiation. A reduction of the magnetic field leads to an adiabatic "cooling" of the particles, namely the energy of a relativistic particle in the isotropic case changes according to the law

$$E \propto H^{1/2} \propto L^{-1}. \quad (5.37)$$

This relation is easy to obtain by taking account of the conservation of the adiabatic invariant $p_\perp^2/H$ in a slowly varying magnetic field. Here $p_\perp$ is the component of the momentum of a particle perpendicular to **H** where $p = E/c$; if in the process of expansion the isotropic distribution of the particles is maintained, then $p_\perp^2 = \frac{2}{3}p^2$.

Furthermore, in the expansion the number of particles does not change, but they get shifted to some other energy interval. Thus

$$VN(E)\,dE = VKE^{-\gamma}\,dE = \mathrm{const},$$

from which with $V \propto L^3$ and $E \propto L^{-1}$, we get

$$K \propto L^{-(\gamma+2)}. \quad (5.38)$$

Consequently, the flux of radiation from a discrete source, according to (5.7) and (5.36)—(5.38), changes with the size of the source according to the law

$$\Phi_\nu \propto L^{-2\gamma}. \quad (5.39)$$

From this, for example, for a nebula expanding with a radial velocity v (and $dL/dt = 2v$), the percentage change in flux is

$$\frac{1}{\Phi_\nu}\frac{d\Phi_\nu}{dt} = -\frac{2\gamma}{L}\frac{dL}{dt} = -\frac{4\gamma v}{L}. \quad (5.40)$$

Such an effect was detected in [66] for the radio source Cassiopeia A (rate of expansion of this nebula—the envelope of a supernova of the second type—reaches 7,000 km/sec).

We point out here that when in such a source there is a kink or some other singularity in the spectrum, the frequency ν_1 corresponding to this singularity changes as the source expands according to the law

$$\nu_1 \propto L^{-4} \quad (5.41)$$

(cf. (2.23) and (5.36), (5.37)). In addition to the assumptions made above about the conservation of isotropy in the distribution of electrons over direction during expansion, we also assume that other processes which could lead to a change in the position of the kink (primarily loss of energy of the electrons) are slow compared to the expansion. For example, if the kink is due to magnetic bremsstrahlung losses and, consequently, its position changes with time according to the law $dE/dt = -\beta E^2$, then one must satisfy the condition $L^{-1}dL/dt \gg \beta E$.

Now let us briefly consider those changes in the characteristics of magnetic bremsstrahlung which are caused by effects of the medium in the process of generation and propagation of magnetic bremsstrahlung.

The polarization and intensity of magnetic bremsstrahlung can change because of the effect of the medium not only in the source, but also along the path from it to the earth. In the latter case, however, the nature of the radiation is not specific (if we do not consider the fact that in the cosmos polarization is primarily a characteristic of the magnetic bremsstrahlung). We shall therefore not spend time on discussion of the "method of de-excitation" consisting in finding information about inter-galactic, interstellar, or solar plasmas on the basis of a study of the polarization characteristics of the cosmic radio emission (cf. Sec. 4.1 and [55]). As for the usual absorption along the path from the source, as a result of such absorption the intensity will change according to the law $J = J_0 e^{-\tau}$ where J_0 is the intensity near the source and τ is the thickness of material traversed (cf. Sec. 4.1). Therefore in the low frequency region where τ becomes large, one should observe a rapid fall-off in intensity in the spectrum of the radiation. For the galactic radio emission such a fall-off is observed at frequencies $\nu \lesssim 3$ MHz.

Absorption in the source itself also should lead to a change in the spectrum of the radiation. Thus if $g(\nu)$ is the intensity of the radiation from unit volume of the source (radiative power), then the total intensity is

$$J(\nu) = \int_0^L g(\nu) e^{-\mu l} dl = g(\nu) \frac{1 - e^{-\mu L}}{\mu}. \qquad (5.42)$$

Since the absorption coefficient μ depends on frequency (for radio waves in plasma, according to (4.8), $\mu \propto \nu^{-2}$), obviously the spectrum of the total radiation of the source $J(\nu)$ differs from the spectrum of the radiation of the particles $g(\nu)$ if the optical thickness $\tau = \mu L$ is sufficiently large. In application to the radio-galactic radio emission, the case of interest is that where the radiating region is interspersed with absorbing regions (clouds of ionized hydrogen). Such a case was treated in [67].

Reabsorption of radiation in an ultrarelativistic electron gas leads in a qualitative way to a result analogous to ordinary absorption, since the absorption coefficient in the case of reabsorption (4.18) also increases rapidly with decreasing frequency. As a result of reabsorption in the low frequency region, where the optical thickness $\mu_r L$ for reabsorption is large, the spectrum of the radiation changes markedly in form, namely, the intensity drops with decreasing frequency like $\nu^{5/2}$ (cf. (4.21)), in contrast to the case for the high frequency region, for which $J_\nu \propto \nu^{-\alpha}$, $\alpha > 0$. Since, in the case of reabsorption, the absorption coefficient (cf. (4.18)) depends on the concentration of relativistic electrons and the intensity of the magnetic field, the possibility arises of obtaining information about the values of these quantities in the source starting from the observed positions of kinks in the spectrum of the radiation due to reabsorption.

Finally, if condition (4.26) is not satisfied in the source, one must take into account the effect of the index of refraction on the process of radiation itself, i.e., one must use expression (4.22) for the intensity of the radiation of a plasma with $n < 1$. The analysis of this expression shows that in the frequency region $\nu < \nu_n$ (cf. (4.26)) the intensity of the radiation falls off markedly (cf., for example, [69]).

From the remarks in the present section it is clear that the study of magnetic-bremsstrahlung spectra makes it possible to obtain a variety of important information about relativistic electrons, magnetic fields, and gas in sources, and also about the time scales and ages of sources.

5.5. Optical and X-ray Magnetic Bremsstrahlung

Optical and x-ray magnetic radiation does not differ from the magnetic bremsstrahlung in the radio region, and here again the qualitative picture given in Sec. 2 remains, and the quantitative results presented in Secs. 2 and 3 also. At the same time, in a given magnetic field, for radiation of optical and even more so for x-ray frequencies, the electrons should have considerably higher energy than in the case of radiation of radio frequencies. If, however, the energy of the electron is unchanged, then we must increase the magnetic field even more. Specifically, for an estimate we make use of formula (2.23), by virtue of which

$$\frac{\nu_2}{\nu_1} = \frac{H_{\perp,2}}{H_{\perp,1}} \frac{E_2^2}{E_1^2}. \qquad (5.43)$$

Suppose, for example, $\nu_1 = 3 \times 10^8$ ($\lambda = c/\nu_1 = 1$ m) in a typical field $H_{\perp,1} = 3 \times 10^{-6}$ for the galaxy. Then, according to (2.23), the energy of the radiating electrons $E_1 \sim 5 \times 10^9$ eV. In the same field $H_{\perp,2} = H_{\perp,1}$, the optical frequencies $\nu_2 = 10^{14}$–10^{15} ($\lambda = 0.3$–$3\,\mu$) can be radiated only by electrons with energies $E_2 \sim 5 \times 10^{12}$ eV. For x-rays, $\nu_2 \sim 10^8$ and, consequently, for the same magnetic field the electrons must now have energy $E_2 \sim 3 \times 10^{14}$ eV.

One must keep in mind that magnetic bremsstrahlung losses are proportional to $H_\perp^2 E^2$ (cf. (2.10)) and therefore particles with very high energy or in a very strong field are slowed down very quickly. An estimate of the energy and "lifetime" in a magnetic field can be made conveniently using formulas (5.30) and (5.31). Then in formula (5.31) one can express the energy of the electron in terms of the characteristic frequency of its radiation (2.23) and thus obtain a direct connection between the observed frequency and the characteristic lifetime (the time during which the energy decreases by a factor of two) of the radiating electrons:

$$T_M = \frac{5\cdot 10^8}{H_\perp^2}\frac{mc^2}{E}\sec \simeq \frac{5{,}5\cdot 10^{11}}{H_\perp^{3/2}\nu^{1/2}}\sec = \frac{1{,}8\cdot 10^4}{H_\perp^{3/2}\nu^{1/2}}\ \text{yr}. \quad (5.44)$$

Here $H_\perp$ is measured in Oe and ν in Hz. The time T_M expressed in terms of frequency has, of course, a somewhat conventional character since we have used for ν the frequency corresponding to the maximum in the spectrum of the radiation of monoenergetic electrons.

In a field $H_\perp = 3 \times 10^{-6}$, the times T_M for electrons with energy 5×10^9, 5×10^{12}, and 3×10^{14} eV are respectively 2×10^8, 2×10^5, and 3×10^3 years. For our galaxy, and generally for normal galaxies for which the value $H_\perp = 3 \times 10^{-6}$ can be regarded as typical, a characteristic time T_M of the order of 10^5 years, let alone 10^3 years, seems to be extremely small, and it is therefore natural that the optical and x-ray magnetic bremsstrahlung will be weak. More precisely, the situation can be changed only when there is an intense injection of electrons at high energy in interstellar space from some of the sources, for example from the envelopes of supernova. A discussion of the question of interstellar x-ray magnetic bremsstrahlung can be found in [29]. In the case of a field $H_\perp \sim 3 \times 10^{-4}$ which is typical for the envelopes of supernovae, the electrons responsible for optical and x-ray radiation are those with energies 5×10^{11} and 3×10^{13} eV, for which the lifetimes T_M are of order 10^2 years and 1 year respectively. Thus, for example, for the Crab nebula, whose age is about 900 years, to assume that the electrons responsible for the optical radiation were formed in the explosion can only be possible by stretching things (this is possible in a field $H_\perp \sim 10^{-4}$, which from other considerations is already too weak). But if the x-ray radiation of the Crab nebula is magnetic bremsstrahlung, as now appears most probable, the existence in the Crab of "pumpings"—injections of electrons of high energies right at the present time—appears completely definite.

As stated, the optical and x-ray magnetic bremsstrahlung is described completely by the formulas given earlier. There even arises a simplification in that at high energies one can neglect the effects of the index of refraction $n(\omega) \neq 1$ in the radiating region, reabsorption, and rotation of the plane of polarization in the cosmic plasma. Thus, one need only include absorption of the radiation along the path from the source to the earth or in the source itself, for example, when it contains dust (as for the galaxy M82).

For convenience, we give some expressions which are useful for calculation. In the x-ray region and sometimes also in the optical region, one uses not the energy flux, but the flux or intensity of the number of particles (photons), which we denote respectively by F_ν and I_ν. The change obviously is made by dividing the energies by the quantum energy $h\nu$. Thus, according to (3.30) the intensity of the number of quanta is

$$I(\nu) = \frac{J\nu}{h\nu} = 3.26\cdot 10^{-15} a(\gamma) LKH^{\frac{\gamma+1}{2}}\left(\frac{6.26\cdot 10^{18}}{\nu}\right)^{\frac{\gamma+1}{2}} \frac{\text{photons}}{\text{cm}^2\ \text{sec-sr-Hz}} \quad (5.45)$$

or, if we go over from frequency ν to photon energy $\epsilon = h\nu$, expressed in eV,

$$I(\varepsilon) = I(\nu)\frac{d\nu}{d\varepsilon} = 0.79 a(\gamma) LKH^{\frac{\gamma+1}{2}}\left(\frac{2.59\cdot 10^4}{\varepsilon}\right)^{\frac{\gamma+1}{2}} \frac{\text{photons}}{\text{cm}^2\ \text{sec-sr-eV}}. \quad (5.46)$$

Here L is measured in cm, K in $(\text{ergs})^{\gamma-1}\cdot\text{cm}^{-3}$, H in Oe and ϵ in eV. Similarly the flux of photons from a discrete source (cf. (5.7)) is

$$F(\nu) = \frac{\Phi(\nu)}{h\nu} = 3.26\cdot 10^{-15} a(\gamma)\frac{VKH^{\frac{\gamma+1}{2}}}{R^2}\left(\frac{6.26\cdot 10^{18}}{\nu}\right)^{\frac{\gamma+1}{2}} \frac{\text{photons}}{\text{cm}^2\ \text{sec-Hz}} \quad (5.47)$$

or, taken relative to the photon energy $\epsilon = h\nu = 4.14 \times 10^{-15}\,\nu$ eV,

$$F(\varepsilon) = 0.79 a(\gamma)\frac{VKH^{\frac{\gamma+1}{2}}}{R^2}\left(\frac{2.59\cdot 10^4}{\varepsilon}\right)^{\frac{\gamma+1}{2}} \frac{\text{photons}}{\text{cm}^2\ \text{sec-eV}}. \quad (5.48)$$

Furthermore when the electron spectrum can be assumed to be the same over the whole volume of the source, it is convenient to use the following expression for the ratio of the fluxes of radiation at different frequencies ν_1 and ν_2 (cf. (5.7))

$$\frac{\Phi_2(\nu_2)}{\Phi_1(\nu_1)} = \frac{V_2}{V_1}\left(\frac{H_2}{H_1}\right)^{\frac{\gamma+1}{2}}\left(\frac{\nu_1}{\nu_2}\right)^{\frac{\gamma-1}{2}}. \quad (5.49)$$

Here it is assumed that the radiation at frequency ν_1 arises in a region of the source with volume V_1, while in this region the magnetic field intensity is H_1, whereas the radiation at frequency ν_2 comes from a volume V_2 with field H_2. If we are considering the radiation of electrons with the same energy $E_2 = E_1$, then the frequencies ν_2 and ν_1 are related by (5.43) and the ratio of fluxes is

$$\frac{\Phi_2(\nu_2)}{\Phi_1(\nu_1)} = \frac{V_2 H_2}{V_1 H_1}. \quad (5.50)$$

Formulas (5.49) and (5.50) are important when, within a small volume V_2 of the source with total volume V_1, the field $H_2 \gg H_1$, and in the electron spectrum there is a cutoff on the high energy side so that the electrons from the volume V_1 do not radiate at frequencies $\nu_2 \gg \nu_1$ while the radiation from volume V_2 at frequencies ν_1 is small because of the smallness of volume V_2. Then the observed ratio of fluxes at frequencies ν_2 and ν_1 from the whole source will be determined by the ratio of the fluxes from the volumes V_2 and V_1 Such a situation can exist, for example, in the case of a nebula, having in its central region a collapsed star with a very strong magnetic field.

5.6. Summary of Important Formulas

We may assume that the present paper will be used not only to familiarize the reader with the physical aspects and results of the theory of magnetic bremsstrahlung, but simply in order to find quickly formulas that are needed. We therefore, in conclusion, give a summary of the most important formulas which we have already met and discussed in the text.*

The total power of the magnetic bremsstrahlung of an ultrarelativistic electron (magnetobremsstrahlung loss) is

$$P(E)=\int_0^\infty p(\nu)\,d\nu=0.98\cdot 10^{-3}H_\perp^2\left(\frac{E}{mc^2}\right)^2\text{eV/sec}. \quad (2.10)$$

The spectral density of the power of the total radiation from a single electron is

$$p(\nu)=\frac{\sqrt{3}\,e^3H_\perp}{mc^2}F\left(\frac{\nu}{\nu_c}\right)$$
$$=2.37\cdot 10^{-22}H_\perp F\left(\frac{\nu}{\nu_c}\right)\ \frac{\text{erg}}{\text{sec}\cdot\text{Hz}}; \quad (2.21)$$

a plot of F(x) is given in Fig. 7, and its values and approximate expressions in Table I. The spectral density $p(\nu)$ has a maximum

$$p(\nu_m)=1.6\,\frac{e^3H_\perp}{mc^2}=2.16\cdot 10^{-22}H_\perp\ \frac{\text{erg}}{\text{sec}\cdot\text{Hz}} \quad (2.24)$$

at the frequency

$$\nu_m\simeq 0.29\nu_c=1{,}2\cdot 10^6H_\perp\left(\frac{E}{mc^2}\right)^2$$
$$=4.6\cdot 10^{-6}H_\perp\,[E\,(\text{eV})]^2\ \text{Hz}. \quad (2.23)$$

The electron energy for which the maximum of the radiation occurs at frequency $\nu=\nu_m$ is

$$E=7.5\cdot 10^{-10}\left(\frac{\nu}{H_\perp}\right)^{1/2}\text{erg}=4.7\cdot 10^2\left(\frac{\nu}{H_\perp}\right)^{1/2}\text{eV}. \quad (2.23)$$

The intensity of the radiation in a homogeneous field, for electrons distributed isotropically along the line of sight (length L), with one and the same energy (monoenergetic spectrum) and concentration $N(\mathbf{r})$, is

$$J_\nu=\frac{p(\nu)}{4\pi}\int_0^L N(\mathbf{r})\,dr; \quad (2.25)$$

its value at the maximum is

$$J_{\nu,m}=1.7\cdot 10^{-23}H_\perp\int_0^L N(\mathbf{r})\,dr\ \frac{\text{erg}}{\text{cm}^2\ \text{sec-sr-Hz}}. \quad (2.26)$$

The average concentration of electrons (assuming $H^2=\frac{3}{2}H_\perp^2$) is

$$N=\frac{1}{L}\int_0^L N(\mathbf{r})\,dr=7.2\cdot 10^{22}\frac{J_\nu}{LH}. \quad (5.5)$$

The intensity of the radiation in a random magnetic field, for electrons with a homogeneous and isotropic distribution along the path L and energy spectrum $N(E)=KE^{-\gamma}$ (radiative power $g_\nu=J_\nu/L$) is

$$J_\nu=1.35\cdot 10^{-22}a(\gamma)\,LKH^{\frac{\gamma+1}{2}}\left(\frac{6.26\cdot 10^{18}}{\nu}\right)^{\frac{\gamma-1}{2}}\frac{\text{erg}}{\text{cm}^2\ \text{sec-sr-Hz}}. \quad (3.30)$$

The intensity of number of photons under these same conditions (photon energy $\epsilon=h\nu=4.14\times 10^{-15}\nu$ measured in eV) is

$$I(\varepsilon)=0.79a(\gamma)\,LKH^{\frac{\gamma+1}{2}}\left(\frac{2.59\cdot 10^4}{\varepsilon}\right)^{\frac{\gamma+1}{2}}\frac{\text{photons}}{\text{cm}^2\ \text{sec-sr-Hz}}. \quad (5.46)$$

The values of the coefficient $a(\gamma)$ are given in Table II on page 685.

The limits of the energy interval for a power spectrum of electrons given a power spectrum of radiation in the interval $\nu_1\le\nu\le\nu_2$ are

$$E_1=2.5\cdot 10^2\,[\nu_1/y_1(\gamma)\,H]^{1/2}\ \text{eV},$$
$$E_2=2.5\cdot 10^2\,[\nu_2/y_2(\gamma)\,H]^{1/2}\ \text{eV}; \quad (3.34)$$

The coefficients $y_1(\gamma)$ and $y_2(\gamma)$ are given in Table II. For rough estimates, and also when $\gamma<1.5$, we may set $y_1(\gamma)=y_2(\gamma)=0.24$.

The intensity of radiation, expressed in terms of effective temperature, is

$$J_\nu=3.07\cdot 10^{-37}\nu^2T_{\text{eff}}\frac{\text{erg}}{\text{cm}^2\ \text{sec-sr-Hz}}. \quad (4.12)$$

The coefficient in the electron spectrum expressed in terms of the intensity or effective temperature of the radiation at frequency ν is

$$K=\frac{7.4\cdot 10^{21}J_\nu}{a(\gamma)\,LH}\left(\frac{\nu}{6.26\cdot 10^{18}H}\right)^{\frac{\gamma-1}{2}}$$
$$=\frac{8.9\cdot 10^{22}HT_{\text{eff}}}{a(\gamma)\,L}\left(\frac{\nu}{6.26\cdot 10^{18}H}\right)^{\frac{\gamma+3}{2}}\text{erg}^{\gamma-1}\cdot\text{cm}^{-3}. \quad (5.3)$$

The flux of radiation Φ_ν from a discrete source of volume V, located at distance R, is

$$\Phi_\nu=1.35\cdot 10^{-22}a(\gamma)\frac{KVH^{\frac{\gamma+1}{2}}}{R^2}\left(\frac{6.26\cdot 10^{18}}{\nu}\right)^{\frac{\gamma-1}{2}}\frac{\text{erg}}{\text{cm}^2\ \text{sec-Hz}}; \quad (5.7)$$

In this case

$$K=\frac{7.4\cdot 10^{21}R^2\Phi_\nu}{a(\gamma)\,HV}\left(\frac{\nu}{6.26\cdot 10^{18}H}\right)^{\frac{\gamma-1}{2}}\text{erg}^{\gamma-1}\cdot\text{cm}^{-3}. \quad (5.8)$$

The total energy of relativistic electrons in a source is

$$W_e=A(\gamma,\ \nu)\frac{R^2\Phi_\nu}{H^{3/2}}. \quad (5.11)$$

Expressions for the coefficient $A(\gamma,\nu)$ are given in formula (5.12).

The magnetic field energy W_H and the cosmic ray energy $W_{c.r.}$ in the source are

*In all cases the magnetic fields H and $H_\perp$ are measured in oersted, the path L in cm, the time in sec, the frequency ν in hertz (cps), electron concentrations N and N_e in cm^{-3}, the coefficient K in the energy spectrum of electrons $N(E)dE=KE^{-\gamma}dE$ in $(\text{erg})^{\gamma-1}\ \text{cm}^{-3}$.

$$W_H = \varkappa_H W_{c.r.} = \varkappa_H \varkappa_r W_e$$
$$= 0.19\,[\varkappa_H \varkappa_r A(\gamma,\ \nu)\,\Phi_\nu R^2]^{4/7}(R\varphi)^{9/7}. \quad (5.17)$$

The magnetic field intensity is

$$H = [48\varkappa_H \varkappa_r A(\gamma,\ \nu)\,\Phi_\nu/R\varphi^3]^{2/7}. \quad (5.16)$$

The characteristic time for magnetic bremsstrahlung losses (the time during which the energy of the electron decreases by a factor two) is

$$T_M = \frac{5.1\cdot 10^8}{H_\perp^2}\frac{mc^2}{E}\ \mathrm{sec} \quad (5.31)$$

or if ν is the frequency at which the maximum in the spectrum of radiation of the electron occurs, then

$$T_M \simeq 5.5\cdot 10^{11}/H_\perp^{3/2}\nu^{1/2}\ \mathrm{sec}. \quad (5.44)$$

The maximum energy of electrons at time t after their injection into the magnetic field is

$$E_m(t) = \frac{2.6\cdot 10^{14}}{H_\perp^2 t}\ \mathrm{eV}. \quad (5.32)$$

The frequency in the spectrum of radiation of these electrons corresponding to the dropoff of the spectrum is

$$\nu_m(E_m(t)) = \frac{3.1\cdot 10^{23}}{H_\perp^3 t^2}\ \mathrm{Hz}. \quad (5.33)$$

The change in flux of radiation due to expansion of the source with dimension L in the absence of "pumping in" of energy is

$$\Phi_\nu(t) \propto [L(t)]^{-2\gamma}. \quad (5.39)$$

The coefficient of absorption of radio waves in a plasma is

$$\mu = \frac{10^{-2}N_e^2}{T^{3/2}\nu^2}\left[17.7 + \ln\frac{T_e^{3/2}}{\nu}\right]\mathrm{cm}^{-1}. \quad (4.8)$$

The coefficient for reabsorption in a gas of ultrarelativistic electrons is

$$\mu_r = g(\gamma)\,0.019\,(3.5\cdot 10^9)^\gamma K H_\perp^{\frac{\gamma+2}{2}}\,\nu^{-\frac{\gamma+4}{2}}\ \mathrm{cm}^{-1}. \quad (4.20)$$

The values for the coefficient $g(\gamma)$ were given in Table III (coefficient $g(\gamma) \sim 1$). The characteristic frequency above which the deviation of the index of refraction of the plasma from unity has no effect on the magnetic bremsstrahlung is

$$\nu_n \simeq 20\frac{N_e}{H_\perp}\ \mathrm{Hz}, \quad (4.26)$$

where N_e is the concentration of electrons in the plasma. The angle of rotation of the plane of polarization of the radiation in passing through a path L at an angle θ to the field **H** is

$$\Psi = 2.36\cdot 10^4\frac{N_e HL\cos\theta}{\nu^2}\ \mathrm{rad}. \quad (4.6)$$

[1] G. A. Schott, Electromagnetic Radiation, Cambridge University Press, Cambridge, 1912.

[2] L. A. Artsimovich and I. Ya. Pomeranchuk, JETP **16**, 379 (1946).
[3] L. A. Schiff, Rev. Sci. Instr. **17**, 6 (1946).
[4] I. Ya. Pomeranchuk, JETP **9**, 919 (1939).
[5] V. V. Vladimirskiĭ, JETP **18**, 392 (1948).
[6] H. Alfvén and N. Herlofson, Phys. Rev. **78**, 616 (1950).
[7] K. O. Kiepenheuer, Phys. Rev. **79**, 738 (1950).
[8] V. L. Ginzburg, DAN SSSR **76**, 377 (1951).
[9] G. G. Getmantsev, DAN SSSR **83**, 557 (1952).
[10] G. G. Getmantsev and V. L. Ginzburg, DAN SSSR **87**, 187 (1952).
[11] V. L. Ginzburg, UFN **51**, 343 (1953), Fortschr. Phys. **1**, 659 (1954).
[12] I. M. Gordon, DAN SSSR **94**, 413 (1954).
[13] I. S. Shklovskiĭ, DAN SSSR **90**, 983 (1953).
[14] J. Schwinger, Phys. Rev. **75**, 1912 (1949).
[15] K. C. Westfold, Astrophys. J. **130**, 241 (1959).
[16] A. A. Korchak and S. I. Syrovat-skiĭ, Astron. Zh. **38**, 885 (1961), Soviet Astron. AJ **5**, 678 (1962).
[17] L. D. Landau and E. M. Lifshitz, Teoriya Polya, Moscow, Fizmatgiz, 1962; translation "Classical Theory of Fields," Pergamon, 1963.
[18] V. L. Ginzburg, UFN **62**, 37 (1957); also in Progr. Elem. Particle and Cos. Ray Phys. (Amsterdam) **4**, 337 (1958).
[19] I. S. Shklovskiĭ, Kosmicheskie radioizluchenie, Moscow, Gostekhizdat, 1956; translation, "Cosmic Radio Waves," Cambridge, Mass., Harvard, 1960.
[20] J. L. Pawsey and E. R. Hill, Repts. on Progr. Phys. **24**, 69 (1961).
[21] A. Boischot and J. E. Denisse, Compt. rend. **245**, 2149 (1957).
[22] Wild, Smerd, and Weiss, Ann. Rev. Astron. and Astrophys. **1**, 291 (1963).
[23] V. V. Zheleznyakov, Radioizluchenie Solntsa i planet (Radio Emission of the Sun and Planets), Moscow, Nauka, 1964.
[24] J. A. Roberts, Planet. and Space Sci. **11**, 221 (1963).
[25] M. A. Vashakidze, Astr. tsirkulyar #147, 11 (1954); V. A. Dombrovskiĭ, DAN SSSR **94**, 1021 (1954).
[26] J. H. Oort and Th. Walraven, BAN **12**, 285 (1956).
[27] G. W. Clark, Nuovo Cimento **30**, 727 (1963).
[28] V. L. Ginzburg and S. I. Syrovat-skiĭ, JETP **45**, 353 (1963), **46**, 1865 (1964), Soviet Phys. JETP **18**, 243 (1964), **19**, 1255 (1964).
[29] V. L. Ginzburg and S. I. Syrovat-skiĭ, UFN **84**, 201 (1964), also Space Sci. Rev. **4**, 267 (1965) and Soviet Phys. Uspekhi **7**, 681 (1965).
[30] V. L. Ginzburg, DAN SSSR **156**, 43 (1964), Soviet Phys. Doklady **9**, 329 (1964).
[31] V. L. Ginzburg and S. I. Syrovat-skiĭ, DAN SSSR **158**, 808 (1964), Soviet Phys. Doklady **9**, 831 (1965).
[32] L. Woltjer, Astrophys. J. **140**, 1309 (1964).
[33] P. Morrisons, Hand. Phys. **46/1**, 1 (1961).
[34] V. L. Ginzburg and S. I. Syrovat-skiĭ, Proiskhozhdenie kosmicheskikh luchei, Moscow, AN SSSR, 1963;

enlarged edition "Origin of Cosmic Rays," Pergamon, 1964.

[35] M. Ryle, Proc. Phys. Soc. (London) **A62**, 491 (1949).

[36] A. Unsold, Z. Astrophys. **26**, 176 (1949), Phys. Rev. **82**, 857 (1951).

[37] I. S. Shklovskiĭ, Astron. Zh. **29**, 418 (1952).

[38] S. B. Pikel'ner, DAN SSSR **88**, 229 (1953).

[39] A. Unsold, Z. Phys. **141**, 70 (1955).

[40] Radio Astronomy, Manchester Symposium, Cambridge, England, 1957.

[41] I. M. Gordon, Reports of the Third Conference on Problems of Cosmogony, AN SSSR, 1954, pp. 253, 267.

[42] V. L. Ginzburg, ibid., p. 260; DAN SSSR **92**, 1133 (1953).

[43] Reports of the Third Conference on Problems of Cosmogony, Moscow, Press of the Academy of Sciences USSR, 1954.

[44] W. Baade, Astrophys. J. **123**, 550 (1956).

[45] V. A. Razin, Astron. Zh. **35**, 241 (1958), Soviet Astron. AJ **2**, 216 (1958).

[46] Westerhout, Seeger, Brouw, and Tinbergen, BAN **16**, 187, 213 (1962).

[47] Mayer, McCullough, and Sloanaker, Astrophys. J. **139**, 248 (1964).

[48] Paris Symposium on Radioastronomy, Stanford University Press, Stanford, 1959.

[49] V. L. Ginzburg and G. F. Zharkov, JETP **47**, 2279 (1964), Soviet Phys. JETP **20**, 1525 (1965).

[50] S. Chandrasekhar, Radiative Transfer, Oxford, Clarendon Press, 1950.

[51] A. A. Korchak, Astron. Zh. **40**, 994 (1963), Soviet Astron. AJ **7**, 764 (1964).

[52] K. S. Thorne, Astrophys. J. Suppl. **8**, N73 (1963).

[53] V. L. Ginzburg, UFN **69**, 537 (1959), Soviet Phys. Uspekhi **2**, 874 (1960).

[54] V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme, Moscow, Fizmatgiz, 1960; translation "The Propagation of Electromagnetic Waves in Plasmas," Pergamon, 1964.

[55] V. L. Ginzburg and V. V. Pisareva, Izv. Vuz. (Radiofiz) **6**, 877 (1963).

[56] Benediktov, Getmantsev, and Ginzburg, Iskusstv. Sputn. Zemli #7, 3 (1961); Planet. and Space Sci. **9**, 109 (1962).

[57] Walsh, Haddock, and Schult, Space Res. Amsterdam **4**, 935 (1964).

[58] V. Ya. Éĭdman, JETP **34**, 131 (1958) and **36**, 1335 (1959), Soviet Phys. JETP **7**, 91 (1958) and **9**, 947 (1959).

[59] S. B. Pikel'ner, Astron. Zh. **38**, 21 (1961), Soviet Astron. AJ **5**, 14 (1961).

[60] G. R. Burbidge and F. Hoyle, Astrophys. J. **138**, 57 (1963).

[61] S. I. Syrovat-skiĭ, Astron. Zh. **36**, 17 (1959), Soviet Astron. AJ **3**, 22 (1959).

[62] V. L. Ginzburg and S. I. Syrovat-skiĭ, Astron. Zh. **41**, 430 (1964), Soviet Astron. AJ **8**, 342 (1964).

[63] S. Hayakawa and H. Okuda, Progr. Theoret. Phys. Kyoto) **28**, 517 (1962).

[64] Kardashev, Kuz'min, and Syrovat-skiĭ, Astron. Zh. **39**, 216 (1962), Soviet Astron. AJ **6**, 167 (1962).

[65] I. S. Shklovskiĭ, Astron. Zh. **37**, 256 (1960), Soviet Astron. AJ **4**, 243 (1960).

[66] Högbom and Shakeshaft, Nature **189**, 561 (1961); D. S. Heeschen and B. L. Meredith, Nature **190**, 705 (1961).

[67] S. Ya. Braude and V. V. Vaĭsberg, Izv. Vuz. (Radiofiz) **7**, 193, 1032 (1964).

[68] V. I. Slysh, Nature **199**, 682 (1963).

[69] V. A. Razin, Izv. Vuz. (Radiofiz) **3**, 584 (1960).

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