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L. I. MANDEL'SHTAM'S WORK ON THE THEORY OF THE OPTICAL IMAGE AND MODERN QUASIOPTICS

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I wish to demonstrate, using as an example L. I. Mandel'shtam's work on the theory of the optical image, the depth and broad scope of his scientific method, which has enabled him to see, in an already-thoroughly studied topic, perfectly new and unexpected aspects, whose significance has come to be appreciated only recently, after the lapse of many decades.

In connection with the present great expansion of research in the theory and techniques of coherent light, and also in connection with problems involving generation, amplification, and transmission of oscillation in the longer-wavelength sections of the electromagnetic spectrum (the millimeter and submillimeter bands), many workers have become greatly interested in quasioptics, a branch of electrodynamics dealing with the asymptotic laws of diffraction by bodies whose dimensions are large compared with the wavelength.

To understand the laws of wave diffraction by large bodies, dealt with in quasioptics, it is necessary to make use of both the geometric-optics and wave properties of the electromagnetic field. Moreover, it is precisely the diffraction phenomena that determine the specific characteristics of the wave processes in quasioptical systems.

Recent advances in quasioptics include, first, the development of a theory of free oscillation of open resonators used in lasers [1-4], the theory of propagation of coherent wave beams in lens and mirror waveguides [5], the theory of excitation of open resonators and waveguides [6], and the solution of several problems in the theory of wave propagation in smooth waveguides of large cross section [7].

Even long before these topics were intensively tackled in quasioptics, its methods were successfully applied to the theory of optical instruments, to explain the diffractive aberrations of images. Mandel'shtam paid much attention to questions of the theory of the optical image. One of the approaches developed by him in this theory is especially interesting in light of problems now being solved by modern quasioptics.

In 1912, while at the Physics Institute of the Strasbourg University, Mandel'shtam wrote a paper "On the Use of Integral Equations in the Theory of the Optical Image." [8] This paper was a regular development of his earlier research on the theory of the microscopic image [9]. As is well known, Mandel'shtam analyzed critically Abbe's theory and demonstrated the feasibility of a unified treatment of the structure of the diffraction image of both self-luminous (i.e., incoherent) and nonluminous (i.e., coherent) objects.

The paper "On the Use of Integral Equations in the Theory of the Optical Image" differed from earlier work on this topic primarily by a fundamentally new statement of the problem. Here is what Mandel'shtam writes:

"Principal attention has been paid hitherto to a study of how the image of a given object is produced through a given diaphragm; for example, to the minimum width that a diaphragm must have in order for the image still to exhibit a certain structure. This problem reduces, as is well known, to a determination of the resolving power of optical instruments.

The question which I want to discuss briefly consists in the following: What structures result in images similar to themselves if a specified diaphragm is used?" (emphasis is by Mandel'shtam [8], p. 230).

An important factor in such a formulation of the question of the structure of the optical image is the fact, especially noted by Mandel'shtam, that it leads directly to a homogeneous integral equation of the type

$$pf(x)=\int_{-a}^{a} K(x, x')\, f(x')\, dx' \qquad (1)$$

for the sought field structure f(x). The kernel of this equation is the diffraction image of a luminous point on the object in image space; the integration limits are determined by the boundaries of the object and its image.

We thus see here precisely the formulation of the problem of a periodic field structure, which is the basis of contemporary work by Goubau and Schwering [5]. Fox and Li [1], Boyd and Gordon [2], Boyd and Kogelnik [3], Vaĭnshteĭn [4], and many others [7] working on the diffraction theory of open resonators and beam waveguides.

The investigation of integral equations of type (1), with kernels K(x, x′) possessing the properties of the diffraction image of a luminous point, is presently one of the fundamental problems of quasioptical theory. Extensive use is made in its solution of modern computational techniques. The difficulty in investigating these equations lies in the fact that in the arbitrary case the kernels K(x, x′) obtained are complex and symmetrical, but not hermitian, and a general theory of integral equations with such kernels has not been fully developed as yet. Only very recent papers deal

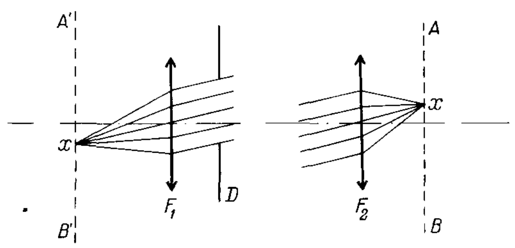

FIG. 1

with the existence of solutions of these equations under the same limitation on the kernels K(x, x') as are imposed by the physical conditions of quasioptics [10].

Let us consider in somewhat greater detail the general scheme of the optical instrument discussed by Mandel'shtam. It is shown in Fig. 1.

Here F_1 and F_2 are two lens systems. The object is in the focal plane A'B' of the lens system F_1. The image of a certain point x' of this object is the point x in the focal plane AB of the second lens system. In the second focal plane of the lens system F_1 is a diaphragm D. In addition, diaphragms in planes A'B' and AB limit the dimensions of the object and of its image. It is assumed for simplicity that the distance from the points x and x' to the optical axis of the system is the same, i.e., the magnification of the system is equal to unity. The question of finding periodic distributions is formulated for this very system. Mandel'shtam raises this question both for the field, in the case of coherent objects, and for the intensity in the case of completely incoherent (self-luminous) objects. We shall be interested in what follows only in the first case. The limiting diaphragm considered by Mandel'shtam is a rectangular aperture, and the kernel of integral equation (1) is chosen to be the function

$$K(x, x') = \mathrm{const} \cdot \frac{\sin C(x - x')}{x - x'}, \qquad (2)$$

which, as is well known, is the Fraunhofer diffraction pattern of a rectangular aperture. The constant C in (2) depends in a known fashion on the diaphragm dimension and on the wavelength λ.

It is interesting to find out which of the modern quasioptical problems in the theory of open waveguides and resonators corresponds to this concrete formulation. It is easy to show that the optical system corresponding to the kernel (2) in question can be reduced to the form shown in Fig. 2. To be sure, in the latter the magnification is equal to – 1 and not to 1, as

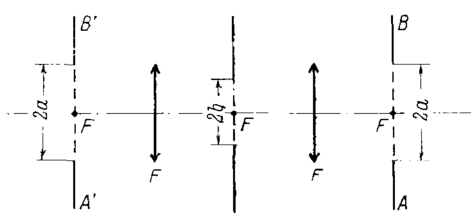

FIG. 2

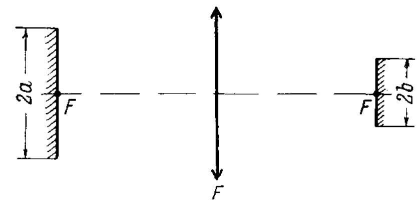

FIG. 3

is assumed by Mandel'shtam for simplicity in writing the integral equation, but this has no fundamental significance and is manifest only in the sign of the eigenvalues p. It is perfectly obvious that if we find the solution of integral equation (1) for the indicated optical system, the same solutions will describe the periodic field distribution in the lens-type diaphragmed waveguide obtained by periodically continuing the unit optical cell under consideration. The eigenvalues **p** will determine in this case the damping and the phase of the wave beams in such a waveguide. It is also easy to construct the scheme of an open resonator corresponding to the optical system shown in Fig. 2. To this end we can consider in lieu of the beam passing through the diaphragm D a beam reflected from a plane conducting metal screen replacing the diaphragm aperture. A similar procedure can be used for diaphragm A'B'. As a result we obtain the open resonator shown in Fig. 3.

Equation (1) describes, obviously, the field distribution on the left-hand mirror of this resonator. From the system of Fig. 3 we can go over, by replacing the single lens F with two equivalent ones (in the sense of image transmission), to the resonator shown in Fig. 4.

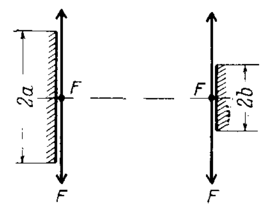

FIG. 4

In this resonator lenses with focal lengths F are located near mirrors separated by a distance F.

This system is in turn equivalent, with respect to the field distribution on the mirrors, to a resonator with two confocal spherical mirrors of focal length F/2 (Fig. 5). The latter is the presently well-known

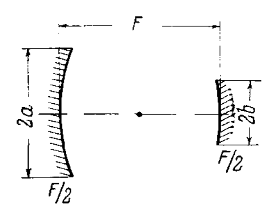

FIG. 5

and well-studied model of the confocal resonator widely used in lasers [1-4].

We see thus that the concrete example considered by Mandel'shtam is from the modern point of view equivalent to the problem of the modes in a diaphragmed confocal beam waveguide (Fig. 2), or the equivalent confocal resonator (Fig. 5).

Mandel'shtam solved the integral equation (1) with kernel (2) for $a = \infty$, i.e., for the distribution of a field which is not diaphragmed in the object and image planes. By the same token he actually found the field distribution on the non-diaphragmed (left-hand) mirror of the resonator shown in Fig. 3, or the resonators equivalent to it (Figs. 4 and 5).

Were Mandel'shtam to consider the solution of his Eq. (1) for finite a, then by the same token he would have investigated the presently known modes of a diaphragmed confocal beam waveguide, or a confocal resonator with finite mirrors. At that time, however, integral equations with a kernel of type (2) were not yet thoroughly investigated. The main results for such equations were obtained several decades later, in connection with problems arising in information theory, antenna theory, and finally, the theory of open waveguides and resonators [11-14].

Mandel'shtam therefore did not have at his disposal a well developed mathematical formalism for a detailed investigation of the properties of the solution of the equations derived by him. This makes all the more interesting those opinions concerning the character of the solutions, and hence the character of the repeating field distributions, which were expressed by Mandel'shtam on the basis of the general theory of integral equations, and which apply in their entirety, by virtue of the foregoing, to modern quasioptical waveguides and resonators. Since the kernel (2) in the example considered by Mandel'shtam is real and symmetrical, he indicates that there exists an infinite set of eigenvalues p and corresponding solutions of the integral equation (1) or, as we now say, when speaking of open resonators and waveguides, an infinite set of natural modes.

Especially modern-sounding is the following remark by Mandel'shtam: "Of practical importance is the following situation: ... from among the infinite number of structures which yield (for different values of p) similar images, only a finite number corresponds to a noticeably luminosity of the image ... Structures corresponding to all other values p will have images with only negligible brightness" ([8], p. 236.) Thus, Mandel'shtam called attention even then to the most important property of diaphragmed optical systems, that of transmitting selectively, with different illuminations, periodic field distributions that are similar in structure. In modern quasioptical waveguides and resonators this property is the basis of selection of modes and resolution of their spectrum. It is connected with the possibility of obtaining single-mode operation in multimode systems with dimensions much larger than the wavelength. The selection of modes in open resonators and beam waveguides is at present one of the most important problems of modern quasioptics. On its solution depend, in particular, the possibilities of successfully using lasers for communication, navigation, radar, microscopic operations, i.e., wherever it is required that the laser beam have a high degree of coherence, monochromaticity, and large directivity.

From the mathematical point of view the problem of optimal selection consists in finding a wave-beam-transformation operator compatible with the electromagnetic-field equations, such that its largest eigenvalue exceeds all others appreciably in absolute value, but at the same time remains sufficiently close to unity. Yet no one has considered this problem in this formulation. An analysis of the hitherto investigated concrete lens and mirror systems shows that the best selectivity in this sense is possessed by confocal resonators and waveguides with diaphragms having constant transparency over the entire aperture, i.e., just the systems analyzed by Mandel'shtam in his 1912 paper. Additional possibilities of suppressing undesirable oscillations in open systems can be connected with the use of special selective elements.

In connection with the problem of beam transmission of electromagnetic energy, interest is attached to one more aspect of Mandel'shtam's problem. We refer to a search for an optimal wave-beam configuration realizing the transmission of energy between two specified apertures with minimum loss.

In the optical scheme considered by Mandel'shtam (Fig. 2), the role of such apertures is played by the openings in screens A'B' and AB in object and image space, respectively. It can be shown that in the case of rectangular apertures the solution of this variational problem again leads to integral equation (1) with a kernel of the type (2) [15]. It follows therefore that both the main field distribution in Mandel'shtam's optical system (Fig. 2) and the fundamental mode in a confocal resonator or waveguide have the lowest radiation loss compared with any other possible field distribution among the specified apertures in these systems. In this sense the field distributions that can be obtained from the integral equation formulated by Mandel'shtam are optimal.

Thus, the approach developed by L. I. Mandel'shtam in the theory of the optical image more than 50 years ago discloses almost all the main elements that are characteristic of modern quasioptical theory of open resonators in a waveguide.

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