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ON NONLINEAR WAVE PROCESSES

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WHEN we analyze the scientific work of L. I. Mandel'shtam, we can see what a large place problems in the theory of oscillations occupied in it. I have in mind not only the fact that about a third of his articles are devoted to these problems. L. I. Mandel'shtam is the recognized head and founder of the Soviet school of oscillation theory. Furthermore, he is the founder of the study of oscillations as an independent branch of physics.

Oscillation theory permits us to view phenomena of varying physical natures from a unified standpoint, and find their general "oscillatory" essence. The development of the idea of oscillatory induction facilitates the transition from optical to acoustic phenomena, from problems of radiophysics to those of quantum mechanics, etc.

The oscillatory approach is of great heuristic value. A number of new physical phenomena have been found with this approach, the best-known example being Raman scattering.

The theory of nonlinear oscillations occupies a special place in oscillation theory. In this field, L. I. Mandel'shtam and his closest associates have performed a very great number of studies and got very significant results.

While the theory of oscillations is equally related to all branches of physics, the experimental basis of the theory of nonlinear oscillations has always been radiophysics. This involves the fact that the operation of any modern radio apparatus is unthinkable without nonlinear phenomena.

In fact, the operation of an apparatus designed for radio communication, television, radar, or radio navigation requires that one have a generator of radio waves, transmit these waves through space, and receive them. Correspondingly, the fundamental problems arising in radio physics fall along three lines:

1. Problems of generating and modulating electromagnetic oscillations.
2. Problems involving emission and propagation of radio waves.
3. Problems of radio reception.

Nonlinear effects are the basis of the apparatus for generation and reception of radio waves. This defines their role in radio.

In the first stages of the development of radio technology in the prewar period, the usable frequencies were low, and the length of the radio waves was much longer than the dimensions of the generating and receiving instruments. The nonlinear phenomena that these instruments were based on were in essence oscillatory processes. In distinction, the wave processes involved in the second line of problems were purely linear. Thus, the prewar radio technology required, first of all, the development of a nonlinear theory of oscillations. L. I. Mandel'shtam and his associates played a determining role in this development. However, only the very first steps were taken with regard to nonlinear wave theory.

Beginning in the twenties, after the application of short waves was found possible, radio technology continued to develop in the direction of using shorter wavelengths of radiation. By the time of the second world war, the decimeter range of radio waves was mastered, and the centimeter range began to be taken over. The wavelength became comparable with the dimensions of the transmitting and receiving apparatus. The first amplifiers and generators based on the laws of wave processes began to appear: backward-wave traveling-wave tubes. Materials appeared having nonlinear electromagnetic characteristics in the ultrahigh-frequency range. Wave processes entered as full partners with the oscillatory processes in the first and third lines of development of radio physics.

The subsequent mastery of the millimeter and submillimeter ranges of electromagnetic waves and the recent advance in creating the laser have had the result that wave processes have begun to play the determining role in many problems of radio physics, which we can link under the common term of radio optics. It became necessary to create a nonlinear wave theory, which, in the manner of the nonlinear oscillation theory, was to generalize numerous individual phenomena in the behavior of various devices. We can now consider this task to have been fulfilled to a considerable extent. Some graphic qualitative ideas and concepts have come forth, guiding "wave" conceptions have been developed, and the set of nonlinear wave phenomena has been systematically studied as a whole.

It is interesting to note that in our country the basic role in studying nonlinear wave processes has been played by scientists of the second and third generations of L. I. Mandel'shtam's students. Here, first of all, we should mention the groups at the Institute of Physics of the Academy of Sciences working under the direction of N. G. Basov and A. M. Prokhorov, the group of radio physicists at Gor'kiĭ headed by A. V. Gaponov, and the group of associates in the Department of Radio Physics of Moscow State University working under the direction of S. A. Akhmanov and the present author.

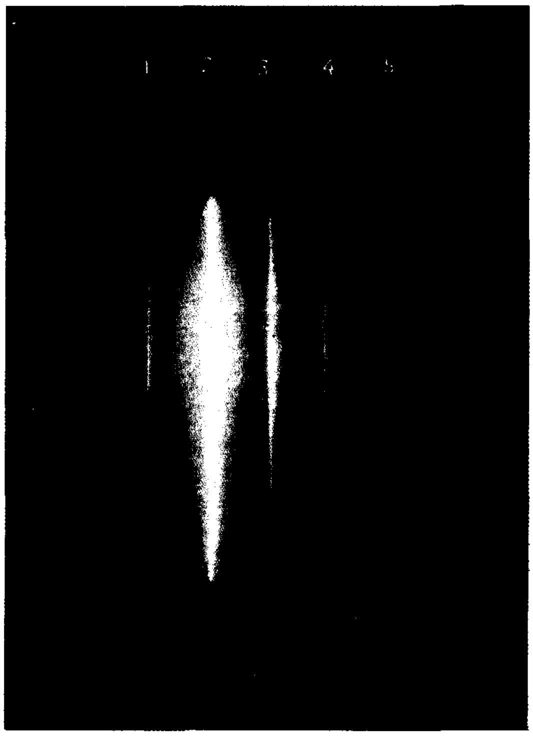

FIG. 1. Stimulated Raman spectrum in benzene excited by a neodymium glass laser via a frequency doubler. 1 – First anti-Stokes component, λ = 0.503 μ; 2 – frequency-doubled radiation from neodymium-glass laser, λ = 0.53 μ; 3 – first Stokes component, shifted by 990 cm⁻¹ (λ = 0.56 μ); 4 – second Stokes component, λ = 0.59 μ; 5 – complex line consisting of the third 909-cm⁻¹ Stokes component of the benzene molecule and the first 3064-cm⁻¹ Stokes component (λ = 0.62 μ). Lines 2 and 3 on the photograph have lost their proper colors owing to strong overexposure.

Wave motion in distributed systems differs from oscillatory motion in systems having lumped parameters in that the state of the system varies continuously not only in time, but also in space. The simplest purely sinusoidal wave in a linear system is characterized not only by a frequency but also by a phase velocity. Depending on the relation between the phase velocity and the frequency, wave processes can completely differ in nature. Therefore, it is useful to distinguish two limiting classes of nonlinear wave systems: highly-dispersive systems and non-dispersive systems.

In the first case, the higher harmonic components of the process arising from nonlinearity propagate with velocities differing from that of the fundamental component. Hence, if the nonlinearity is not very large, the effects of interaction do not accumulate with distance, and the amplitudes of the higher harmonic components do not reach an appreciable value. Only when the form of the dispersion characteristic is specially chosen do one or several of the higher components of the process have phase velocities close to that of the fundamental component. Such components interact strongly with the fundamental component, resulting in energetic beat formation. A typical example of such a system is a degenerate parametric traveling-wave amplifier. In such a system, the dispersion characteristic is chosen (Fig. 2) such that the phase velocity of the fundamental wave of frequency ω_0 and that of the second harmonic are equal, while the phase velocity of the $3\omega_0$ component differs greatly. The relation of the amplitudes A_1 and A_2 of the harmonic components to the space coordinate is shown in Fig. 3. Let a signal of frequency $2\omega_0$ of amplitude A_2 be applied at the input of the amplifier ($z = z_1$), together with a signal of frequency ω_0 with a very small amplitude A_1. Upon propagating through the system, the $2\omega_0$ wave "parametrically excites" the ω_0 wave by transferring energy to it. Gradually the amplitude of the $2\omega_0$ wave declines to zero, while the amplitude of the ω_0 wave increases and reaches a maximum at the point $z = z_2$. Upon propagating further, the system acts as an effective frequency-doubler: the strong ω_0 wave produces through nonlinearity a wave of frequency $2\omega_0$, whose amplitude keeps increasing. Hereby the ω_0 wave decreases in amplitude.

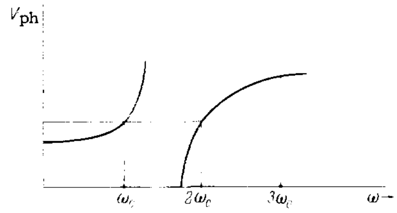

FIG. 2. Phase velocity of the wave vs. the frequency in a traveling-wave parametric amplifier.

Thus, in this case energetic spatial beats occur between the components.

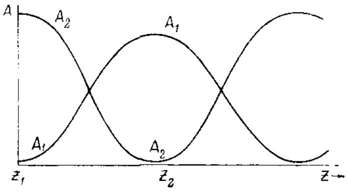

FIG. 3. Amplitudes of the first and second harmonic components of the wave vs. the distance in a traveling-wave parametric amplifier.

In non-dispersive wave systems the processes take place in a completely different way. In these systems, the higher harmonic components arising from nonlinearity are propagated at the same phase velocity as the fundamental. Therefore, all the wave components effectively interact. Consequently, an initially sinusoidal wave is strongly distorted. That is, it is enriched in the higher harmonic components. Under certain conditions, a shock wave can form in these systems, i.e., a wave with abrupt amplitude jumps. A typical example here is an intense acoustic wave, which gradually takes on a sawtooth form as it propagates (Fig. 4).

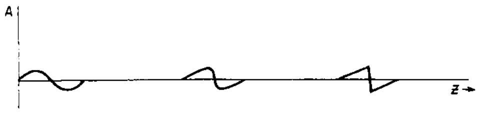

FIG. 4. Waveform vs. distance in non-dispersive nonlinear systems.

It follows from the above that the methods of analyzing wave processes must differ for dispersive and non-dispersive (or weakly-dispersive) nonlinear systems. For strongly dispersive systems, it is adequate to represent the wave process as a sum of several harmonic components having amplitudes and phases slowly varying in space and time. The behavior of the amplitudes and phases is described by equations analogous to the abbreviated equations in oscillation theory, but essentially differing in that they are partial differential equations. A large number of theoretical studies of the behavior of various wave systems have been carried out by using these equations.

For the other limiting case of weakly dispersive systems, it is adequate to use a representation not involving resolution into harmonic components. Here the wave form gradually varies when the nonlinearity is small. This fact of gradual variation in space and time can be used to derive approximate equations describing the propagation process. A number of phenomena occurring in weakly-dispersive nonlinear wave systems have been analyzed using these equations.

In addition to the distinction between dispersive and non-dispersive systems, it is useful to classify wave systems in terms of the type of nonlinearity. The nonlinear parameters can be the reactive parameters of the medium (e.g., the dielectric constant in electrodynamics) affecting the velocity of propagation of the wave, or else the dissipative parameters determining the attenuation of the wave. Thus, nonlinear wave systems can be divided into the following four limiting classes:

1. Dispersive wave systems having a dissipative type of nonlinearity.
2. Dispersive systems with reactive nonlinearity.
3. Non-dispersive wave systems with reactive nonlinearity.
4. Non-dispersive systems with dissipative nonlinearity.

We must note that a number of practically-important systems are of mixed type, and the wave processes in them are somewhat more complex than in any of these classes. Thus, for example, in traveling-wave electron tubes acting as amplifiers in the ultrahigh frequency range, two of the three interacting waves do not show dispersion, while the other does. Hence, traveling-wave electron tubes are systems having reactive nonlinearity combining the characteristics of the second and third classes. Nevertheless, many very important wave systems can be referred to one of the cited four classes, and hence this classification is convenient.

All laser instruments belong to the first class of dispersive wave systems with dissipative nonlinearity. These systems are clearly dispersive, since dispersion is always strong in optics, and the nonlinear parameters in them are those determining the attenuation. At low light intensities, the attenuation is negative, and this gives rise to the self-excitation of lasers, while at large intensities the attenuation becomes positive because of the saturation effect.

Nonlinear optical filters belong to the same class. They are systems having varying transmission at different transmitted light intensities. These filters are used in so-called Q-spoiling systems of lasers capable of producing light pulses of record-breaking power.

All nonlinear optically transparent systems belong to the second class of dispersive systems with reactive nonlinearity. In these, the polarization of the medium is the nonlinear parameter. These systems permit one to modulate transmitted light waves in amplitude and phase, and most important, they make possible an efficient frequency transformation of powerful radiation. It is hard to overestimate the importance of the latter problem. At present there are only two powerful lasers, the neodymium glass laser of wavelength $\lambda = 1.06\ \mu$, and the ruby laser with $\lambda = 0.7\ \mu$. There are no other powerful lasers. At the same time, we need sources of other wavelengths to solve a number of important physical and technical problems. Nonlinear optical instruments permit one efficiently to transform light from one wavelength to another. The simplest transformers are frequency doublers, which make use of the nonlinearity of certain crystals. In order to transform the light frequency one can also use such phenomena as stimulated Raman scattering. This phenomenon permits one to shift the laser frequency by an amount corresponding to the vibration frequency of the molecules. The efficiency of this transformation amounts to several dozen per cent. Figure 1 shows a photograph of a series of coherent-radiation lines in the visible. These lines were obtained by successive use of the frequency-doubling effect on the laser frequency and the stimulated Raman-scattering phenomenon. In the same way, one can obtain powerful coherent radiation in almost any region of the spectrum.

The third class of wave systems includes, first, all of nonlinear acoustics, and second, the systems showing electromagnetic shock waves recently proposed by the Gor'kiĭ radio physicists and developed by them. The simplest example of these systems is a ferrite-filled coaxial line.

The characteristic nonlinear phenomena that appear when waves propagate in these systems are used in radio technology to obtain field pulses of record-breaking brevity.

Under certain conditions, abrupt field jumps appear in electromagnetic shock-wave devices, based on a mechanism of nonlinear dissipation of the energy of the wave. In these cases, they must be assigned to the fourth class of nonlinear wave systems.

The cited approaches do not exhaust, of course, all the fields of physics and technology in which nonlinear wave processes play the determining role. Gasdynamics at supersonic velocities and chemical physics are examples of such fields.

For lack of space, there is no opportunity to describe even briefly the mechanisms of typical phenomena of each of the four classes of nonlinear wave systems.

The aim of presenting this paper is only to throw some light on one of the major trends in the development of the theory of nonlinear oscillations founded by L. I. Mandel'shtam. The importance and interest of the enumerated problems again illustrates the significance of his scientific work in modern physics.

Translated by M. V. King