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Letters to the Editor*

# *MORE ABOUT THE ENERGY-TIME UNCERTAINTY RELATION (REPLY TO AHARONOV AND BOHM)*

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## 1 GNOSIOLOGICAL STATEMENT OF THE PROBLEM OF MEASUREMENT IN QUANTUM MECHANICS

For a correct discussion of the quantum mechanical uncertainty relations, it is absolutely essential to have a clear conception of the gnosiological statement of the problem of measurement This question was clarified a number of times by Bohr, who indicated that all properties (including purely quantum properties) of an object are obtained by means of a classically described instrument We are, thus, dealing in the act of measurement with the interaction of two systems a quantum-mechanically described micro-object and a classically described instrument This is a feature peculiar to the measurement act In the measurement process, the object cannot be separated from the instrument a physical and logical relation exists between them [1]

The probabilities expressed by the wave function are the probabilities of some result of the interaction of the micro-object and the instrument (of some reading on the instrument) The wave function itself can be interpreted as the reflection of the potential possibilities of such an interaction of the micro-object (prepared in a definite way) with various types of instruments

A quantum mechanical description of an object by means of a wave function corresponds to the relativity requirement with respect to the means of observation This extends the concept of relativity with respect to the reference system familiar in classical physics

The dependence of the wave function on time (according to the Schrodinger equation) describes the temporal variation of the predictions referring to the measurements This is precisely what is understood in quantum mechanics as the change of state of a system with time The act of measurement itself cannot be described by the wave function As a result of the measurement act, the wave function serving for prediction becomes unreal (is canceled). A measurement need not necessarily serve as a basis for new predictions; moreover, the object of the measurement (for example, the photon) may as a result of the measurement even cease to exist (be absorbed). However, the case is also not excluded where the measurement is set up in such a way that it also constitutes a "preparation" of the object, i e , that it leads to new predictions It can then result in a new wave function, but this wave function is written anew and is not obtained from the old one by means of the Schrodinger equation This is evident already from the fact that the new predictions are not simple developments of the old ones, but are set up anew on the basis of data characterizing the new preparation of the object

The fact that the act of measurement represents a whole and cannot be itemized by introducing into it a series of other measurements is related to these features Bohr pointed out this indivisibility many times

## 2 REPLY TO AHARONOV AND BOHM

In an article published in 1964 entitled "Reply to Fock Concerning the Time Energy Indeterminacy Relation" [2] Aharonov and Bohm repeat their attempt of 1961 [3] to disprove the Heisenberg-Bohr uncertainty relation As can be seen from the title itself, the authors argue mainly with us (our papers of 1947 [4] and 1962 [5]), although we are not the only defenders of quantum mechanics; this fact prompted us this time to reply Aharonov and Bohm

The authors ignore in their article entirely the gnosiological statement of the problem of measurement in quantum mechanics presented in the previous section They subdivide the measurement into stages and find it possible to speak about the results of each stage of measurement separately At the same time they, apparently, assume that during the course of the measurement the wave function also varies according to the Schrodinger equation All this represents such a clear lack of understanding of the fundamentals of quantum mechanics that it would be sufficient merely to point it out in order to refute the reasoning of these authors However, we will consider their reasoning in somewhat greater detail

In the Heisenberg-Bohr uncertainty relation

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†This does not mean that the interaction between the object and the body which serves as the instrument cannot be described quantum mechanically, but such a description implies that the body is included in the quantum mechanical system. In such a case the interaction between the body and the initial object can no longer be considered a measurement to measure the more complex system including both the body and the object, a new instrument is required whose action must again be described classically

$\Delta(E' - E)\Delta t > h$, the subject of the discussion, we are dealing with an energy exchange between the object and the instrument and with the instant at which this exchange occurred. The premise for a discussion of the change in the energy of the object is the application of the law of conservation of energy to the system consisting of the object and instrument. One cannot therefore, as the authors do, renounce the application of the law of conservation of energy in this problem. Also, one must not introduce separately the time of the object and the time of the instrument and consider them different dynamical variables (the authors do this in order to avoid the fact that the energy and the time are canonically conjugate variables). The time is common to the object and instrument and is reckoned by means of the laboratory clock.

In their new article, as in the article of 1961, the authors consider a certain artificially chosen Hamiltonian operator containing a discontinuous function $g(t)$ treated as the external field; this function is assumed to differ from zero (and be constant) only during a short time interval.

In our critical article of 1962 we indicated that the introduction of discontinuous functions of the time into the Hamiltonian operator constitutes in itself a violation of the Heisenberg relations. The validity of these relations is thus rejected a priori; this constitutes an error in logic known as petitio principii (begging the question).

Admitting to some extent the validity of our objection, the authors attempt to deprive it of its convincing nature by means of the following considerations. They agree that instantaneous switching on and off of the interaction introduces an uncertainty (actually an infinite one) into the energy of the field; the authors speak of field quanta of infinite energy. On the other hand, they deny for some reason that these quanta can be transferred to a particle. This denial, however, signifies a denial of the applicability of the law of conservation of energy to the object-instrument system (the authors speak of this explicitly), whereas the law of conservation of energy offers, as we have already mentioned, the only means for discussing the energy of the object.

In their further discussions which refer already to the case of a smooth (during a time on the order of $\Delta t$) switching on and off of the field, the authors in fact renounce their enunciated refusal to admit energy transfer between the field and the particle. They admit that after switching on the field the kinetic energy of the particle becomes uncertain to the order $\Delta E > h/\Delta t$. However, they then make an assumption even stranger than all the previous assumptions: they assume that after switching on of the field this uncertainty disappears. As if the field quanta emitted by the instrument when the field is switched on could be re-absorbed by the instrument after the field is switched off. This strange conclusion is apparently connected with the above-mentioned completely unacceptable conception of the authors that the course of the measurement can be followed as a function of time in all its details, and that parallel to the course of the measurement a wave function continues to exist which changes according to the Schrödinger equation.

At the end of their article, the authors make one more attempt to avoid conclusions unfavorable to their point of view, an attempt based on substitution of the measurement of the z coordinate of an auxiliary heavy particle for measurement of the time t. Here, however, the authors do not take into account the fact that the uncertainty relations also hold for the heavy particle with the relations for the energy and time readily obtainable in this case from the relations for the momentum and the coordinate admitted by the authors themselves. In fact, if we measure the time by the path traversed by the heavy particle, then $\Delta t = (1/v_z)\Delta z$ and, on the other hand, $\Delta E = v_z \Delta p_z$, where $v_z$ and $p_z$ are the velocity and momentum of the heavy particle. Hence, from $\Delta p_z \Delta z > h$, one obtains $\Delta E \Delta t > h$. Since the system is conservative, the latter relation, derived for the heavy (auxiliary) particle is also valid for the light (investigated) particle.

Of course, if one rejects the uncertainty relation for the auxiliary particle, then it is not difficult to conclude from this that it is also not fulfilled for the investigated particle. But such a method of reasoning, starting from a rejection of the uncertainty relations for the means of observation, means a repetition of the same error of logic—petitio principii—which, as we have indicated, was made by the authors in their first article (of 1961).

One is thus forced to state that the initial assumptions of Aharonov and Bohm contradict the fundamentals of quantum mechanics, and that their discussions and conclusions are erroneous. The Heisenberg-Bohr uncertainty relations are an organic component of quantum mechanics, and their rejection is equivalent to a denial of all quantum mechanics as a whole.


[1] V. A. Fock, A Discussion with Niels Bohr, Voprosy filozofii, No. 8, 49 (1964).

[2] Y. Aharonov and D. Bohm, Phys. Rev. **B134**, 1417 (1964).

[3] Y. Aharonov and D. Bohm, Phys. Rev. **122**, 1649 (1961).

[4] N. S. Krilov and V. A. Fock, JETP **17**, 93 (1947); V. Fock and N. Krilov, J. Phys. USSR **11**, 112 (1947).

[5] V. A. Fock, JETP **42**, 1135 (1962), Soviet Phys. JETP 15, 784 (1962).


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