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GRAVITATIONAL RADIATION AND THE PROSPECT OF ITS EXPERIMENTAL DISCOVERY

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1. INTRODUCTION

IT has been known for over forty years that the equations of Einstein for a weak gravitational field are similar to the wave equation for the electromagnetic field

$$\square\Phi_{\mu}^{\nu} = -\frac{16\pi k}{c^4}T_{\mu}^{\nu} \quad (1)$$

with the subsidiary condition $\Phi_{\mu,\nu}^{\nu} = 0$. In the set of equations (1) $\Phi_{\mu}^{\nu} = h_{\mu}^{\nu} - \frac{1}{2}\delta_{\mu}^{\nu}h_{\alpha}^{\alpha}$, and h_{μ}^{ν} is a small quantity of the first order describing a distorted metric: $g_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$; T_{μ}^{ν} is the energy-momentum tensor, k the gravitational constant, and c the light velocity[1-3]. These equations, like the Maxwell equations, have solutions in the form of waves which travel with the same velocity as electromagnetic waves. However, the question of the possible observation of such gravitational radiation has been discussed in the literature only in the last few years. The difficulty in observing gravitational waves arises firstly from the small value of the gravitational constant, k, and secondly from the fact that the ratio of the gravitational mass (gravitational charge) to the inertial mass is the same for all bodies.* (In the general theory of relativity this experimental fact underlies the weak principle of equivalence). As a consequence of the latter fact, a non-uniform motion of masses can cause only quadrupole emission of gravitational waves. Thus, for example, the energy loss by gravitational radiation of a system of masses with $v/c \ll 1$ is[2]

$$\frac{d\mathscr{E}}{d\tau} = -\frac{k}{45c^5}(\dddot{D}_{\alpha\beta})^2, \quad (2)$$

where $D_{\alpha\beta}$ is a component of the mass quadrupole tensor

$$D_{\alpha\beta} = \int\limits_V \mu(3x^{\alpha}x^{\beta} - \delta_{\alpha\beta}x_{\gamma}^2)\,dV. \quad (3)$$

Here μ is the density and V the volume. The expression (2) for the power of gravitational radiation agrees, apart from a numerical factor, with the corresponding expression for quadrupole emission in electrodynamics

$$\frac{d\mathscr{E}}{d\tau} = \frac{1}{180c^5}(\dddot{D}_{\alpha\beta})^2, \quad (4)$$

if one replaces $\sqrt{k}\mu$ by the electric charge density ρ. Thus in attempting to design experiments which would observe gravitational radiation one is faced with the same situation as in electrodynamics, but the experimentalist has at his disposal only gravitational charges (gravitational masses) of one sign, all having the same ratio of gravitational charge (gravitational mass) to inertial mass. For this reason the possible emitting and receiving systems can only be of the quadrupole type, and therefore very inefficient. In addition, the specific gravitational charge is extremely weak (for the electron $\sqrt{k}m_{grav}/m_{in} = \sqrt{k}$ is about 10^{21} times smaller than e/m_{in}). Gravitational radiation is a field shed by masses in non-uniform motion, and decreases with increasing distance from the source as R^{-1} if the distance is much greater than the wavelength. In other words, one can define, just as for the electromagnetic case, a wave zone, in which the disturbance of the metric propagates with a velocity equal to the velocity of light, c, and a static zone in which the field can be calculated approximately by Newton's law.

Interest in the design of experiments to find gravitational radiation is growing, and the main reason for this, apart from the continuous improvement in experimental techniques, lies evidently in the relatively recent development of statistical methods for distinguishing weak signals from noise, making optimum use of prior information about the nature of the signal, and using long time intervals. It is worth noting that in this way it turns out to be possible to measure, within a narrow range of frequencies, amplitudes of the vibrations of the centre of mass of a mechanical system below 10^{-13} cm. In this paper we shall give data on the various possible sources of gravitational radiation, discuss the methods proposed for detecting this radiation, and give preliminary results which make it possible to determine an upper limit for the intensity of gravitational radiation of extraterrestrial origin.

2. SOURCES OF GRAVITATIONAL RADIATION

(a) Binary Stars.

The most promising extraterrestrial sources of gravitational radiation are binary stars with short periods of revolution. These may be regarded as practically uniform sources for terrestrial observation, and the attempts to detect their radiation may

*According to the latest measurements[4] the ratio of the inertial to the gravitational mass is the same for different bodies with an accuracy of a × 10^{-11}, where a is of the order of unity.

therefore employ a long-time technique for separating the signal from noise, using a correlation technique which exploits the fact that the signal is exactly synchronous *with the revolution of the components of the* binary star, which can be observed optically. For a system of two stars of masses m_1 and m_2 moving in circular orbits with angular velocity ω, the power radiated by gravitational waves can be worked out from Eq. (2), which for this case takes the form

$$\frac{d\mathscr{E}}{d\tau} = -\frac{32}{5}\frac{k^{7/3}m_1^2m_2^2\omega^{10/3}}{c^5(m_1+m_2)^{2/3}}. \qquad (5)$$

Equation (5) shows that in looking for intense sources of gravitational radiation one is interested in binary stars of large mass and short period, relatively close to the terrestrial observer. Table I lists data on the power of gravitational radiation of six binary stars which lie relatively close to the solar system. The same table lists the period of revolution, the masses m_1 and m_2 in units of the solar mass, the distance L from the earth and the energy flux t of gravitational radiation at the earth. The coefficient A can vary between 0 and a few units according to the orientation of the plane of the orbit of the stars relative to the earth. The first five stars listed in the table are eclipsing (see the catalogue of Kopal[5]) whereas the last, WZ Sagittae, possesses an exceptionally short period of revolution (81 minutes)[6,7]. The integrated energy flux in gravitational radiation from this star may possibly exceed its emission of visible light.

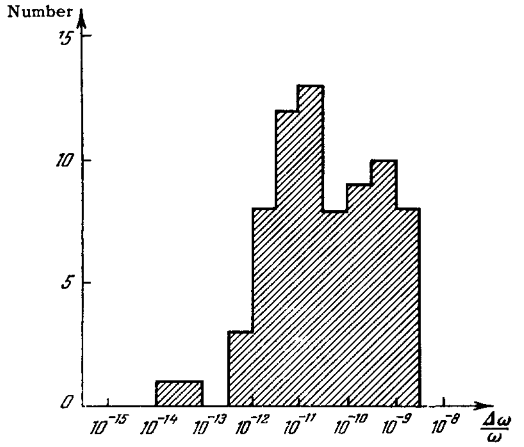
According to estimates by V. N. Mironovskiĭ[8], the binary stars of the type WU Ma should produce the major part of the energy flux of gravitational radiation of extraterrestrial origin; this quantity should be approximately 10^{-9} erg/sec-cm^2. The most probable revolution period of such stars is of the order of 4 hours. Thus a terrestrial receiver for gravitational radiation from the known binary stars would necessarily have to be capable of recording energy fluxes of $10^{-9} - 10^{-10}$ erg/sec-cm^2. A flux of electromagnetic radiation of this magnitude would be easily observed. However, as we shall show below, the quadrupole character of the receiver makes this a very difficult task. The emission of gravitational waves causes an energy loss by the binary star which is known[2] to be at the rate $\epsilon \approx -km_1m_2(2R)^{-1}$ (without relativistic corrections). As a result, the components of the binary star will approach each other[2], and the *angular velocity will increase.* Over a time τ the angular velocity ω should increase by an amount $\Delta\omega$:

$$\frac{\Delta\omega}{\omega} = \frac{96k^{5/3}m_1m_2\omega^{8/3}\tau}{5c^5(m_1+m_2)^{1/3}}. \qquad (6)$$

Table I shows also for the same binary star the increase in angular velocity over a period of $\tau = 3 \times 10^8$ sec (about 10 years). The table shows that, for all the six stars listed, the change in the frequency of revolution which should be produced by gravitational radiation is greater than the relative instability of modern atomic or molecular frequency standards (about 2×10^{-12} for a hydrogen maser). It is worth noting that for most binary stars the revolution frequency is known to 8 figures, and the effect of gravitational radiation appears, as is seen from the table, already in the 9th to 10th figure. Thus a possible crucial experiment, which could confirm the existence of gravitational radiation, would be to observe, over an extensive period, the change in the period of revolution of "suitable" binary stars. The change in the period can be affected also by other phenomena (e.g., the ejection of substantial amounts of matter from one of the components and this makes it difficult to observe the effect by itself. The number of binary stars for which the variation of the revolution frequency due to the emission of gravitational waves can be detected in principle by using modern frequency standards is considerable. The figure shows a histogram of the known eclipsing stars as a function of the expected change $\Delta\omega/\omega$ in the angular velocity during $\tau = 3 \times 10^8$ sec. In compiling this histogram data on m_1, m_2 and τ_{rev} were taken from Kopal's catalogue[4]. The histogram shows that 8 variable stars should change their revolution frequency by more than one part in 10^9, and 48 by more than one part in 10^{11}. One should draw attention to one interesting fact: In compiling Table I one had to select double stars with relatively high m and low τ_{rev}. It turned out that amongst the known double stars there are none for which the mass of the components is of the order of a few solar masses and the period of revolution less than, or equal to, that of WZ Sagittae (81 minutes).

Table I

Star	τ_{rev}, days	m_1	m_2	L, cm	$d\mathscr{E}/d\tau$ erg/sec	At, erg/cm^{2}sec	$\frac{\Delta\omega}{\omega}\Big\vert_{\tau=3\cdot10^8 sec}$
UV Leo	0.60	1.36	1.25	$2.1\cdot10^{20}$	$1.8\cdot10^{31}$	$3.5\cdot10^{-12}$	$3.2\cdot10^{-10}$
V Pup	1.45	1.66	9.8	$1.2\cdot10^{21}$	$4\cdot10^{31}$	$2.3\cdot10^{-12}$	$1.4\cdot10^{-10}$
i Boo	0.268	1.35	0.68	$3.8\cdot10^{19}$	$1.9\cdot10^{30}$	$1.1\cdot10^{-10}$	$1.9\cdot10^{-9}$
YY Eri	0.321	0.76	0.50	$1.3\cdot10^{20}$	$2.6\cdot10^{29}$	$1.3\cdot10^{-12}$	$5.1\cdot10^{-10}$
SW Lac	0.321	0.97	0.83	$2.3\cdot10^{20}$	$1.1\cdot10^{30}$	$1.7\cdot10^{-12}$	$9.5\cdot10^{-10}$
WZ Sge	81 min	0.6	0.03	$3\cdot10^{20}$	$3.5\cdot10^{29}$	$3\cdot10^{-13}$	$4\cdot10^{-9}$



If such double stars existed, their life as double stars would be relatively short because of the energy loss by gravitational radiation. For the "exceptional" double star WZ Sagittae the hypothetical life time would be of the order of 100 million years[6]. Thus the absence of known double stars with large values of $\Delta\omega/\omega$ may be regarded as a somewhat indirect confirmation of the existence of gravitational radiation.

(b) Hypothetical Sources of Gravitational Radiation.

Phenomena associated with the asymmetrical collapse of stars may produce very powerful gravitational radiation. It was shown by Ya. B. Zel'dovich and I. D. Novikov[9] that when a body of mass m falls on a spherically symmetrical contracting star of mass M with a radius close to the gravitational radius $r_g = 2kM/c^2$, a few percent of the energy mc^2 will be converted into a pulse of gravitational radiation, provided $m \sim M$. The gravitational wave emitted when mass m moves in the direction of the radius of M consists of a single pulse of duration $\Delta\tau \sim r_g/c$, whereas a continuing motion in an orbit with a radius comparable to r_g gives rise to a succession of such pulses. Each such pulse contains an energy of the order of $\alpha m^2c^2/M$, where $\alpha \sim 0.01 - 0.1$. It is interesting that the total amount of energy emitted in the form of gravitational waves in finite motion of a mass m in the neighborhood of a star M does not depend on the mass ratio m/M. If such a source is placed at a distance L = 500 megaparsec from the solar system and if $m_\odot$ and $M = 10^3\, m_\odot$, then the expected energy flux at the earth is

$$t \approx \beta \frac{mc^3}{4\pi L^2 r_g} = 0.7 \text{ erg/cm}^2\text{sec}, \qquad (7)$$

if we assume $\beta = 10^{-2}$. In this case the dominant part of the spectrum of the radiation should lie near the frequency $f \approx c/r_g = 10^3$ cps. However, we do not know how often such processes take place, and therefore, if one constructs a receiver with the object of detecting the radiation from such sources, one would have to carry out observations over a long period of time. In asymmetrical collapse of a star, intense gravitational radiation can be produced by a different mechanism (rotation or vibration of the star). According to the estimates of I. S. Shklovskiĭ and N. S. Kardashev[10], based on certain model assumptions about the asymmetrical collapse of a star of a mass $M = 10^{41}$, the power of gravitational radiation should reach $\sim 10^{54} - 10^{58}$ erg/sec. If such a source is located at a distance of 500 megaparsec one may expect at the earth a flux of gravitational radiation of $t \approx 10^{-1} - 10^3$ erg/cm^2 sec at a frequency around $f \approx 10^4$ cps.

If there exist the so-called neutron stars[11], which would have relatively large masses ($m \sim 0.5 m_\odot$) and small dimensions ($R \sim 16$ km), then double neutron stars would also be powerful sources of gravitational radiation. Dyson[11] estimates that a double neutron star should emit, two seconds prior to the fusion of its components, a flux of gravitational radiation of about 10^{52} erg/cm^2 sec with a frequency of about 10^3 cps. If there was such a source at a distance of 300 kiloparsec from the earth, the expected energy flux would be about 10^3 erg/cm^2 sec. For the sources listed above, the gravitational radiation is a mechanism for dissipating energy, and not for the transfer of energy from one part of the interacting system to another.[13] We note that these hypothetical sources of gravitational radiation are transient; although they should produce, during a short time interval, a considerably more intense gravitational radiation than the known double stars, an estimate of the frequency of occurrence of such sources is required before setting up an experiment based on this idea. Such estimates have not yet been worked out.

(c) High-frequency Gravitational Radiation of Extraterrestrial Origin.

A possible source of gravitational radiation of high frequency is the thermal motion of matter. According to an estimate by V. I. Mironovskiĭ[8] the dominant part of the gravitational radiation of the sun is due to (gravitational) bremsstrahlung in the Coulomb scattering of electrons, and this is of the order of magnitude of 10^{15} erg/sec. A flux of this magnitude represents a few gravitons (with frequencies in about the optical region) incident on one square meter of the surface of the earth in each second.

If one follows the hypothesis of the mutual conversion of energy between ordinary matter and gravitation (D. D. Ivanenko and A. A. Sokolov[14]) one can estimate the reaction cross section for the gravitational transmutation of fermions[15-21]. However, the cross sections for such reactions are of an extremely small magnitude: according to an estimate by G. M. Gandel'man and V. S. Pinaev[48], the gravitational radiation in the Coulomb scattering of electrons is by ten orders of magnitude less than the neutrino emission. Naturally, therefore, no real experiments based on this kind of transmutation have so far been discussed.

We should mention one more mechanism which could lead to gravitational radiation of high frequency. In the propagation of electromagnetic waves through a constant electric or magnetic field, the components of the energy momentum tensor will vary with the frequency of the electromagnetic wave, and should, according to Eq. (1), give rise to gravitational radiation of the same frequency.[22] Moreover, since the velocity of propagation of the two waves is the same, there should be a resonance between the electromagnetic and gravitational waves. In the absence of the constant field no gravitational waves should be emitted. The conversion efficiency can be characterized by the ratio of the amplitude a(x) of the gravitational wave to that of the electromagnetic wave b(x). If one assumes plane waves traveling in the x direction and interacting during a time τ, one finds[22] that

$$\left|\frac{a(x)}{b(0)}\right|^2 \approx \frac{k}{\pi c^2}(F^0)^2\tau^2, \quad (8)$$

where F^0 is the intensity of the constant field. Assuming $F^0 = 10^{-5}$ Oe and $\tau = 3 \times 10^8$ sec, and taking the total transit time of the electromagnetic radiation from its distant cosmic source to be 10^7 years, we find

$$\left|\frac{a(x)}{b(0)}\right|^2 \approx 10^{-17}.$$

It is essential that this does not represent a "red shift" of all photons in the electromagnetic wave, but a conversion of photons into gravitons.[15]

(d) Possible Terrestrial Sources of Gravitational Radiation.

In terrestrial conditions it seems very difficult to make sources of gravitational radiation which could give fluxes comparable with those expected from the relatively low-frequency sources described above (transient processes in the collapse of stars and radiation from double stars). For example, if we rotate a 10^4-gram mass with a speed at which the centrifugal tension approaches the short-term strength of high-quality steel, the greatest power of gravitational radiation obtainable at optimum rotor shape is about 10^{-30} erg/sec (about 10 gravitons a year).

Mechanical vibrations in solid bodies also produce gravitational radiation. The sustained vibration of a specimen in its fundamental mode produces gravitational radiation of a power which can be calculated from the expression

$$\frac{d\mathscr{E}}{d\tau} = -\frac{16k\mu^2S^2\xi^2v^6}{15c^5}, \quad (9)$$

which can be obtained from (2) by elementary transformations (cf.[23]). In (9) μ is the density of matter in the specimen, S its cross section, ξ the amplitude of the relative elongation, and v the velocity of the longitudinal waves in the specimen. According to estimates by Weber[23] one could, with the optimum choice of μ, S and ξ, expect gravitational radiation of a power of 10^{-13} erg/sec, but the excitation of such vibrations would require a power of $\sim 10^8$ watt.

In explosions one may also expect a pulse of gravitational radiation. Schücking (cf. the table in Wheeler's book[24]) estimated that in the explosion of a uranium bomb (17 kilotons) the radiated power amounts to $\sim 10^{-4}$ erg/sec over $\tau \sim 10^{-8}$ sec.

The conclusion from these estimates for various kinds of sources of gravitational radiation would seem to be that one should give preference to experiments with extra-terrestrial sources. However, as will be shown below, there is no obstacle of principle against experiments with terrestrial sources.

3. DETECTORS FOR GRAVITATIONAL RADIATION

(a) Conditions for the Design of a Detector.

As was pointed out earlier, detectors for gravitational radiation, like the sources, must be of the quadrupole type. In other words, the design of an experiment to detect gravitational radiation requires at least two test masses. Since they have identical specific gravitational charges (identical ratios of gravitational to inertial mass) a relative motion of the masses will result only from the gradient of the field. The difference of the forces acting on the two masses in the field of a gravitational wave is[23,46]

$$F^{\mu}_{\text{grav}} \approx -mc^2R^{\mu}_{0\alpha0}l^{\alpha}, \quad (10)$$

if the distance l^{α} between the masses is small compared to the wavelength and their velocities are not too high ($v \ll c$). In Eq. (10) m is the mass of each test body, and $R^{\mu}_{0\alpha0}$ a component of the Riemann curvature tensor. In electrodynamics the difference between the forces acting on two equal electric charges q, placed at a distance l from each other, is

$$F_{\text{el}} \approx q\frac{\partial E}{\partial l}\,l. \quad (11)$$

The expressions (10) and (11) are similar; the quantity $R^{\mu}_{0\alpha0}$ is the equivalent of the gradient of the field intensity.

If we could carry out a measurement of an electromagnetic wave $E = E_0 \sin(\omega_0\tau - kx)$ by means of two similar electric charges (with the same ratio q/m_{in}), using the difference in the forces F_{el} acting on the two charges, then from the knowledge of the frequency ω_0, the charge q, and the distance l, we could easily compute the flux of electromagnetic radiation to which these charges are exposed. For this purpose we must insert the value for F_e from (11) in the expression for the Poynting vector

$\mathbf{S} = c(4\pi)^{-1}\mathbf{E}\times\mathbf{H}$, using the fact that $E = E_0 \sin(\omega_0\tau - kx)$. We then find for the energy flux in the sinusoidal electromagnetic wave

$$S \approx \frac{c^3F_{el}^2}{4\pi q^2\omega^2 l^2}, \tag{12}$$

which applies if $l \ll \lambda$ and the charges are moving with small velocity.

By extending the same argument to the case of a gravitational wave of energy flux t, we arrive at the analogous expression

$$t \approx \frac{c^3(F^{\mu}_{grav})^2}{8\pi k m^2 l^2\omega^2}. \tag{13}$$

Equation (13), like (12), was derived for a sinusoidal wave. By comparing (12) and (13) we see that (apart from a numerical factor) they coincide if we replace $k^{1/2}m$ by q, as in the case of emission.

Thus any pair of test masses can "receive" gravitational radiation if there is some device for detecting the small difference between the forces acting on these masses when they are exposed to the field of a gravitational wave. Such a pair may consist of the earth and a satellite, the earth and a star, two planets, two test masses in the laboratory, and an extended rigid body in which the gravitational wave excites mechanical vibrations. Such a receiver, like an electric quadrupole, has a directivity pattern (cf. [25]). The design of an experiment to detect gravitational radiation must evidently be discussed from two points of view. Firstly one must discuss the arrangement of the test masses for an experiment in laboratory conditions. Secondly one has to find out by what method to measure the small relative displacement of the mass probes caused by F^{μ}_{grav} ("instrumental" limitations).

Consider the first question. If we have at our disposal an oscillator consisting of two equal point masses m, connected with each other by an element of stiffness K and an element with damping constant H, the equation of motion for the relative displacement of the masses under the influence of gravitational radiation has the form

$$m\frac{d^2\xi^{\mu}}{d\tau^2} + H\frac{d\xi^{\mu}}{d\tau} + K\xi^{\mu} = F^{\mu}_{grav} + F^{\mu}_{fl} = -mc^2R^{\mu}_{0\alpha 0}l^{\alpha} + F^{\mu}_{fl}, \tag{14}$$

where F^{μ}_{fl} is the resultant of all fluctuating forces acting on the test masses. The condition for F^{μ}_{grav} to be observable against the background of F^{μ}_{fl} is known in the case of equilibrium thermal fluctuations: it is necessary that

$$F^{\mu}_{grav} \geqslant B\sqrt{\overline{F^2_{fl}}} = B\sqrt{4\varkappa TH\Delta f}. \tag{15}$$

In Eq. (15) B is a numerical factor of the order of a few units, which depends on the chosen confidence level for the detection, κ is Boltzmann's constant, T the temperature, and Δf the frequency interval to which one can restrict the observation of the relative motion of the masses. For this reason the availability of advance information about the frequency spectrum plays an important part. For example, in carrying out the coherent detection of a "pure" sinusoidal wave $\Delta f = 1/\hat{\tau}$, where $\hat{\tau}$ is the duration of the measurement; in this case $\hat{\tau}$ may amount to several days, (cf. e.g. the conditions of the experiment in [26]) so that Δf may be as small as $10^{-5} - 10^{-6}$ cps. In cases in which $\Delta f \ll f_{av}$, the experiment may be continued for a considerable time $\hat{\tau}$, and then Δf in (15) has to be replaced by $(\Delta f/\hat{\tau})^{1/2}$. In this case one also has to change the factor B slightly (for details cf. [27]).

Thus, prior information about the spectrum of the gravitational radiation which is to be detected plays an important part in determining the magnitude of the lower limits on F^{μ}_{grav} and t, but evidently the value of Δf in (15) (of the order of $10^{-5} - 10^{-6}$ cps) is a limiting factor for a real experiment. In a terrestrial laboratory experiment it would seem unrealistic to think of an experiment with relatively large macroscopic masses at temperatures below 4°K. Therefore any substantial gain in the theoretically attainable sensitivity of a quadrupole detector of gravitational radiation can come only from a reduction in H, the damping in the connection between the two trial masses. We note an important point: reducing H means reducing the fluctuating forces of other than gravitational origin; therefore the limiting sensitivity of possible detectors depends on the standard of experimental skill which, for the present problem, amounts to the extent to which one succeeds in reducing the quantity H in the conditions of the experiment.

In order to give an idea what fluxes of gravitational radiation can in principle be detected in terms of specific experiments, we list a few values of t on the assumption that the fluctuating forces acting on the test masses are caused only by exposure to hydrogen vapor at helium temperatures ($p = 10^{-10}$ mm, T = 4°K)*.

According to Eq. (13), the smallest detectable t depends on the mass m of the test bodies, the distance l between them, and the frequency of the gravitational radiation which is to be detected. We have therefore listed in Table II a number of values of t for different m, l, ω, and Δf. In calculating these values, it was assumed that $F^{\mu}_{grav} = (\overline{F^2_{fl}})^{1/2}$ (for details see [29,30]).

We see from the table that for a frequency $\omega = 2.5 \times 10^{-3}$ radians/sec (corresponding approximately to the frequency of emission from WZ Sagittae), under the conditions indicated in the table, the flux t which could in principle be observed is less than the predicted value of t for a star of type WU Ma ($t \sim 10^{-9}$ erg/cm² sec, see above). For a frequency ω of $2\pi \times 10^2$ radians/sec the estimate for the ideal lower limit of detection is $t = 2 \times 10^{-15}$ erg/cm² sec, fairly close to the power level ($N \sim 10^{-13}$ erg/sec) of the emitter proposed by Weber[23]. Thus, at least

*Data on hydrogen at such temperatures and pressures may be found in[28].

in principle, the proposed combination of experimental "transmitter" and "receiver" is feasible.

Conditions in which the fluctuating forces which act on the two test masses would be due only to the pressure of a dilute gas could be most easily achieved in free fall. Such an experiment could be designed even in terrestrial conditions by using so-called magnetic suspensions,[31,32] which have already been used to "suspend" masses of 25 kg in vacuo without mechanical contact. For a rotary motion of the suspended masses such a system introduces only an extremely weak damping.[32]

The estimates of the quantity t were based on the assumption that the only forces acting on the test masses were mechanical fluctuating forces. It is easy to estimate fluctuating interactions of other kinds, in particular fluctuations of the electromagnetic field and forces due to fluctuations of the gravitational potential. The latter determine a lower bound for the values of F^{μ}_{grav} and t which are in principle observable. Estimates show, however, that these two kinds of fluctuations produce forces which are substantially less than those due to fluctuations in gas pressure, even for $p = 10^{-10}$ mm and $T = 4°K$ (For details see [29,30]). In the terminology used for electromagnetic detectors, the noise figure of a gravitational receiver will be much greater than unity, even in the conditions we have described. We note one interesting point. If one decreases the damping constant H, the time constant $\tau^* = m/H$ of the oscillator becomes larger* than the time $\hat{\tau}$ that can reasonably be used for the experiment ($\hat{\tau} \sim 10^5 - 10^6$ sec). This situation requires a somewhat unusual statistical method for separating a weak signal from the noise[33], since usually $\hat{\tau} \gg \tau^*$. *It then turns out that if the test* masses are subject to a very weak damping force H, then the less H (or the longer τ^*), the smaller the minimum power $\hat{N}_1$ which can be extracted from, or added to, the energy of the probes[33]:

$$N_1 = \frac{C\varkappa T}{\sqrt{\hat{\tau}\tau^*}}, \quad \text{if} \quad \tau^* \gg \hat{\tau}. \tag{16}$$

The factor C in Eq. (16) has the same meaning as B in (15). This expression for N_1 differs from the well-known expression for the smallest observable power in the coherent reception of a sinusoidal signal against a background of thermal noise in equilibrium,

$$N_2 = c\frac{\varkappa T}{\hat{\tau}}, \tag{17}$$

which applies if $\hat{\tau} \gg \tau^*$. If the damping of the system is so small that $\tau^* \gg \hat{\tau}$, one can clearly, for given $\hat{\tau}$, detect with certainty a much weaker signal [according to Eq. (16)] than is usually believed. The expression (16) ceases to apply when $\hbar\omega \approx N_1\hat{\tau} = C\kappa T(\hat{\tau}/\tau^*)$.

*In the magnetic suspension of Beams[32] a rotor of 13 kg mass lost only 10 rpm per day, at an angular velocity of ~ 25,000 rpm; this corresponds to $\tau^* \sim 10^9$ sec.

Table II

t, erg/cm²sec	$2\cdot10^{-8}$	$1.5\cdot10^{-10}$	$2\cdot10^{-15}$
m, grams	10^4	$2\cdot10^5$	10^1
ω, radians/sec	$2\pi\cdot10^{+2}$	$2.5\cdot10^{-3}$	$2\pi\cdot10^2$
l, cm	10^2	10^4	10^3
Δf, cps	1	10^{-6}	10^{-5}
F^{μ}_{grav}, dynes	$2\cdot10^{-14}$	$1.4\cdot10^{-17}$	$6\cdot10^{-17}$
Δl, cm	$5\cdot10^{-24}$	$1.2\cdot10^{-17}$	$1.5\cdot10^{-26}$

These considerations about the weakest forces and powers that can, in principle, be observed in experiments with test masses, may, in our point of view, also be of interest for non-gravitational experiments with test bodies.

(e) "Instrumental" Limitations.

In the preceding section we formulated the conditions in which we have to arrange two test masses if we want to detect gravitational radiation by means of their relative motion. However, the implementation of these conditions requires apparatus for the measurement of extremely small mechanical displacements. Table II lists the amplitude of linear displacement of the test masses for three cases of detectors*. At present there exist radio-frequency[26] and optical[34] methods for measuring the amplitude of mechanical vibrations in a narrow frequency band to $\Delta l(\tau) \sim 10^{-11} - 10^{-12}$ cm. However, this estimate should *not be regarded as the attainable limit.*

It is easy to show that the limiting sensitivity of the usual capacitance gauges depends on the natural line width of the radio-frequency generator used.

A capacitance gauge for measuring mechanical displacements consists of an electric circuit in which a radio-frequency generator excites electric oscillations. A change in the spacing d between the plates of a capacitor which forms part of the circuit causes a change in the resonance frequency of the circuit, and consequently in the voltage amplitude of the circuit, which can be measured with the aid of a precision amplifier. In order for the change $\Delta l(\tau)$ in the spacing of the plates to be measurable, it is necessary that the resulting frequency shift δf of the circuit be greater than the effect of fluctuations on the frequency of the generator:

$$\delta f = f_0\frac{\Delta l(\tau)}{2d} > \sqrt{W(f)\Delta f}. \tag{18}$$

In this expression W(f) is the spectral density of the frequency deviations of the generator,[35,36], and Δf the width of the frequency band for $\Delta l(\tau)$. Assuming

*The values of $\Delta l(\tau) \sim 10^{-17}$ cm or less quoted in Table II represent the Fourier components of the center of mass of the test bodies, over a narrow frequency interval, caused by thermal fluctuations in the cases listed in the table ($F^{\mu}_{grav} = (\overline{F^2_{f1}})^{1/2}$).

that the line width of the generator is determined purely by the shot effect, and using the well-known analytic expression for $W(f)$[36], one can estimate the smallest detectable $\Delta l(\tau)$:

$$\Delta l(\tau) = 2\beta d \sqrt{\frac{eI_0 r\Delta f}{2N}}. \qquad (19)$$

In Eq. (19) e is the electron charge, I_0 the mean current in the generator tube, r the effective resistance in the generator resonant circuit, N the power, and β a compensation coefficient, which may be of the order of $10^{-3} - 10^{-4}$. If we take in (19) $I_0 = 10^{-4}$ A, $r = 10^{-3}\ \Omega$ (achievable in superconducting circuits at helium temperatures), and $N = 10^{-4}$ W, then $\Delta l(\tau) = (10^{-16} - 10^{-17})(\Delta f)^{1/2}$ cm. Apparatus capable of detecting such mechanical displacements has not yet been made. The estimate of the observable $\Delta l(\tau)$ which we have obtained shows that there still exists a large "reserve" of sensitivity for the radio-frequency measurements of small mechanical vibrations. Obviously it would not be right to express in advance preference for capacitance gauges for observing the small relative displacement of the test masses. It may well be that an improvement in interferometric measuring techniques for small displacements could give comparable results.[38]

As noted in the preceding section, we could use as test masses the earth and a satellite, two satellites, two planets, or the earth and a star. This would probably be particularly appropriate for detecting gravitational radiation of extra-terrestrial origin and of extremely low frequency (one may then select an $l \sim \lambda$ whereas on the earth $l \ll \lambda$). Such "test" bodies are in a very suitable state for the reception of gravitational waves (from the point of view of the considerations set out above). However, the radio-frequency and optical techniques available at present for measuring displacements are in this case much worse than those available in terrestrial laboratory conditions. Equation (13), which gives the energy flux t in a gravitational wave resulting from the difference between the forces $F^{\mu}_{\rm grav}$ acting on two test masses, gives for two "free" masses the result

$$t \approx \frac{c^3\omega^2(\Delta l)^2}{8\pi k l^2}, \qquad (20)$$

where Δl is the amplitude the variation in the distance between the masses caused by the forces $F^{\mu}_{\rm grav}$. The estimates given in this section show that in terrestrial conditions we can at present count on reaching $\Delta l/l \sim 10^{-15} - 10^{-16}$ (assuming $l = 10^3$ cm) and one can consider experiments based on $\Delta l/l \approx (10^{-19} - 10^{-20})(\Delta f)^{1/2}$. By means of optical measurements of the relative motion of stars we might, on the other hand, obtain $\Delta l/l \geq 10^{-8}$ [39]. Radar methods for measuring the distances of planets would give about the same relative accuracy.[37] The use of radio interference techniques for this purpose has not yet been discussed. For the present, therefore, one should probably give preference to terrestrial detectors for gravitational radiation.

(c) Radiation Detectors with Extended Masses.

In the preceding two sections we have discussed the conditions for the detection of weak fluxes of gravitational radiation by means of localized test masses, and the difficulties in the way of measuring small displacements of such masses. These difficulties are greatly reduced if one uses extended masses.

As we remarked in Sec. (a), we can use as a detector for gravitational radiation an extended solid body, in which a gravitational wave excites mechanical vibrations according to Eq. (10). We can then use the piezoelectric effect to detect $F^{\mu}_{\rm grav}$. Weber[23,40] estimates that such observations of the mechanical stresses near the lowest mechanical mode (about 10^3 cps) of a duraluminum rod 1.5 m long and with a mass of 1 ton, at a temperature of 4°K and $\hat{\tau} = 1$ sec, should permit the detection of gravitational radiation at a flux of $t \sim 10^2 - 10^3$ erg/cm^{2}sec. With coherent reception and an excitation time $\hat{\tau} = 10^7$ sec the corresponding limit is $t = 10^{-3} - 10^{-4}$ erg/cm^{2}sec. A full description of an experiment of this kind, which is at present being set up in the University of Maryland, can be found in [25]. Such receiving equipment will be capable of detecting gravitational radiation of extra-terrestrial origin from hypothetical sources of several of the types listed in Ch. 1 above. In the event of a negative result one can set a limit to the rate of occurrence of powerful pulses of gravitational radiation (processes occurring in the collapse of stars, or radiation from nearby neutron stars). The limiting sensitivity in an experiment using this principle is determined by the large value of the internal friction H of the material of the rod; an increased sensitivity can apparently be achieved only through an increase in the scale of the experiment. If it is intended to detect high-frequency gravitational radiation by means of extended test bodies, it would seem appropriate to make use of phonon counters[41-43]. It is important that for bodies of small dimensions one can reach temperatures below 4°K. According to estimates[42] of the performance of such receivers one can expect to measure $\Delta l/l \sim 10^{-25}$. We should note that no estimate has yet been made of the intensity of gravitational waves which could be detected with the aid of phonon counters.

4. UPPER BOUND TO THE DENSITY OF GRAVITATIONAL RADIATION OF EXTRA-TERRESTRIAL ORIGIN

One can obtain an upper bound on the density of gravitational radiation of extra-terrestrial origin from the known experimental data on the rate of expansion of the universe. This quantity may be[44,45] of the order of $10^3 - 10^2$ erg/cm^2 sec.

Dirac (cf.[23]) has suggested that certain astronomical anomalies may be caused by gravitational radiation. Weber[23] estimates that the known anomalies in the rotation of the earth could be caused by a flux of gravitational radiation of $t \sim 5 \times 10^{-8}$ erg/cm^2 sec. If one follows this suggestion one can deduce from the known data on the relative velocities of stars (allowing for the known accuracy in the astronomical observations of the radial and tangential velocities of stars) an upper bound on the energy flux of gravitational radiation of very low frequency. Such estimates, which are derived easily from Eq. (20), give for $\omega = 2 \times 10^{-7}$ and 2×10^{-8} radians/sec, 8×10^7 and 8×10^5 erg/cm^2 sec, respectively. The earth may be regarded as an extended body and as such can also serve as a receiver for gravitational radiation. If the low-frequency seismic oscillations in a "quiescent" state of the earth are due to gravitational radiation[47] this would require a spectral density of the energy flux of gravitational radiation near the resonance frequencies of about 10^8 erg/cm^2 sec/radians $\cdot$ sec^{-1}.

Since the lowest-frequency modes of the earth which could be excited by gravitational radiation have periods of about 54 minutes, and the q value for these modes is about 400, the flux of gravitational radiation which could excite the observed vibrations of the earth with period of about 54 minutes should amount to about $t \sim 10^2$ erg/cm^2 sec.

CONCLUSION

In summarizing the considerations presented in this article about the possible sources of gravitational radiation and the requirements for making detectors, we have to remark that the art of experimentation in this field has not yet reached a sufficiently high level. In the region of low frequencies there seems to be no fundamental limitation which would prevent the detection of gravitational radiation from known sources. In terrestrial conditions it is much more difficult to construct a system of transmitter and receiver, than to detect gravitational radiation of extra-terrestrial origin.

The relations and estimates used in this review follow from the theory of gravitation which is found in most monographs and textbooks and which in essence is generally accepted, at least for low-frequency ("classical") processes. There exists, however, a point of view according to which gravitational waves could not carry energy. The problem of quantization of the gravitational field and the related question of the possibility of observing gravitational quanta, or gravitons, are often regarded as controversial. Therefore it would be highly desirable to see such experiments performed successfully both for low and for high frequencies. We note that the discovery of the existence of gravitational radiation would constitute yet another confirmation of Einstein's general theory of relativity. The discovery and study of gravitational radiation of extra-terrestrial origin would probably provide a new channel for astrophysical information.

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