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SCATTERING AND TRANSFORMATION OF WAVES IN A MAGNETOACTIVE PLASMA

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INTRODUCTION

A characteristic feature distinguishing a plasma from other media is the strong dependence of its properties on the external fields. The presence of an external magnetic field in the plasma gives rise to anisotropy of its electrodynamic properties; this anisotropy is manifest in dispersion and polarization of the waves propagating in the plasma.

Owing to fluctuations, various waves are always present in the plasma; under thermodynamic equilibrium, the amplitudes of such fluctuation waves are determined by the temperature of the plasma. In the non-equilibrium case the amplitudes of the fluctuation waves can be determined only if the distribution functions of the plasma particles are known.

The interaction between the waves (manifest in the nonlinearity of the equations describing the plasma) makes it possible for the waves to become scattered and for some types of waves to be transformed into others. In a non-equilibrium plasma the intensities of the scattering and transformation processes can become anomalously large if the state of the plasma is near the borderline of the instability region.

A study of the scattering and transformation of the waves can serve as method of plasma diagnostics (determination of the parameters that characterize the state of the plasma).

The study of the foregoing processes is important also in connection with a number of astrophysical and radiophysical problems (sporadic radiation from the sun, radio emission from planets, scattering of radio waves in the ionosphere, etc.).

The investigation of the electrodynamic properties of a plasma in a magnetic field has been the subject of a large number of papers (see [1-8]). This exclusive interest is connected essentially with the possible applications of a magnetoactive plasma for numerous purposes.

The present review is devoted to a theoretical investigation of the electrodynamic properties of a homogeneous magnetoactive plasma. Kinetic theory is used to analyze the propagation of waves and the excitation of waves by external currents in a magnetoactive plasma, to investigate the fluctuations of various physical quantities characterizing the state of a magnetoactive plasma, as well as the scattering of waves and their mutual transformation by fluctuations in the plasma.

1. ELECTRODYNAMIC PROPERTIES OF A MAGNETOACTIVE PLASMA

1. Dielectric Tensor

The behavior of a homogeneous plasma in an external magnetic field, neglecting collisions, will be described by kinetic equations for each species of particles making up the plasma,

$$\frac{\partial F}{\partial t}+\mathbf{v}\frac{\partial F}{\partial \mathbf{r}}+\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial F}{\partial \mathbf{v}}=0 \qquad (1.1)^*$$

and the system of Maxwell's equations

$$\left.\begin{aligned}&\operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},\\ &\operatorname{div}\mathbf{H}=0,\\ &\operatorname{rot}\mathbf{H}=\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}+\frac{4\pi}{c}(\mathbf{j}+\mathbf{j}_0),\\ &\operatorname{div}\mathbf{E}=4\pi(\varrho+\varrho_0),\end{aligned}\right\} \qquad (1.2)^\dagger$$

where $F(\mathbf{v},\mathbf{r},t)$ is the distribution function of the particles of the particular species, e and m the charge and mass of the particles, **E** and **H** the electric and magnetic field intensities in the plasma, and ρ_0 and $\mathbf{j}_0$ the densities of the external charges and currents. The densities of the induced charges and currents are

$$\varrho=\sum e\int F\,d\mathbf{v},\qquad \mathbf{j}=\sum e\int \mathbf{v}F\,d\mathbf{v} \qquad (1.3)$$

(the summation is over the particle species).

Confining ourselves to small-amplitude waves in the plasma, we represent the function F in (1.1) in the form of a sum of the initial distribution function $F_0(\mathbf{v})$ and the deviation $f(\mathbf{v},\mathbf{r},t)$ connected with the propagating wave:

$$F=F_0+f,\qquad f\ll F_0. \qquad (1.4)$$

Expanding the distribution-function and field deviations due to the wave in Fourier integrals in terms of plane waves:

$$f(\mathbf{v},\mathbf{r},t)=\frac{1}{(2\pi)^4}\int f_{\mathbf{k}\omega}(\mathbf{v})\,e^{i\mathbf{kr}-i\omega t}\,d\mathbf{k}\,d\omega \qquad (1.5)$$

etc., we obtain from (1.1) the following linear differential equation for the Fourier component of the distribution function $f_{\mathbf{k}\omega}(\mathbf{v})$:

$$i(\omega-\mathbf{kv})f_{\mathbf{k}\omega}+\frac{e}{m}\left(\mathbf{E}_{\mathbf{k}\omega}+\frac{1}{c}[\mathbf{vH}_{\mathbf{k}\omega}]\right)\frac{\partial F_0}{\partial \mathbf{v}} +\frac{e}{mc}[\mathbf{vH}_0]\frac{\partial f_{\mathbf{k}\omega}}{\partial \mathbf{v}}=0; \qquad (1.6)$$

*$[\mathbf{vH}]\equiv\mathbf{v}\times\mathbf{H}$.

†rot ≡ curl.

H_0 is the intensity of the external constant and homogeneous magnetic field. Integration of (1.6) yields

$$f_{k\omega}(\mathbf{v})=\frac{e}{m\omega_H}\exp\left[\frac{i}{\omega_H}\int_0^{\varphi}(\mathbf{kv}-\omega)\,d\varphi\right]\int^{\varphi}\left(\mathbf{E}_{k\omega}+\frac{1}{\omega}[\mathbf{v}\,[\mathbf{kE}_{k\omega}]]\right)\times\frac{\partial F_0}{\partial\mathbf{v}}\exp\left[-\frac{i}{\omega_H}\int_0^{\varphi}(\mathbf{kv}-\omega)\,d\varphi\right]d\varphi, \tag{1.7}$$

where $\omega_H = eH_0/mc$ is the cyclotron frequency for the particles, and φ is the angle between the planes $(\mathbf{k}, \mathbf{H}_0)$ and $(\mathbf{v}, \mathbf{H}_0)$. In (1.7), the value of the integral is taken at the upper limit, and it is easy to verify that the periodicity condition $f_{k\omega}(\varphi + 2\pi) = f_{k\omega}(\varphi)$ is satisfied.

Substituting (1.7) in (1.3), we obtain the Fourier component of the density of the partial current

$$\mathbf{j}_{k\omega}=-i\omega\hat{\varkappa}E_{k\omega}, \tag{1.8}$$

where $\hat{\kappa} = \kappa_{ij}(\omega, \mathbf{k})$ is the polarizability tensor* component due to the particles of a given species:

$$\varkappa_{ij}(\omega,\mathbf{k})=i\frac{e^2}{m\omega\omega_H}\int v_i\exp\left[\frac{i}{\omega_H}\int_0^{\varphi}(\mathbf{kv}-\omega)\,d\varphi\right]\times\int^{\varphi}\left[\left(1-\frac{\mathbf{kv}}{\omega}\right)\delta_{jl}+\frac{v_jk_l}{\omega}\right]\frac{\partial F_0}{\partial v_l}\times\exp\left[-\frac{i}{\omega_H}\int_0^{\varphi}(\mathbf{kv}-\omega)\,d\varphi\right]d\varphi\,d\mathbf{v}. \tag{1.9}$$

Integrating in (1.9) by parts and using the expansion

$$e^{ia\sin\psi}=\sum_n J_n(a)\,e^{in\psi}$$

($J_n(\alpha)$ —Bessel function), we can represent the partial component of the polarization tensor in the form[10,11]

$$\varkappa_{ij}(\omega,\mathbf{k})=\frac{e^2}{m\omega^2}\left\{\sum_n\int\left(\frac{\omega-k_{\|}v_{\|}}{v_{\perp}}\frac{\partial F_0}{\partial v_{\perp}}+k_{\|}\frac{\partial F_0}{\partial v_{\|}}\right)\times\frac{\Pi_{ij}(n,\mathbf{v})}{\omega-n\omega_H-k_{\|}v_{\|}}d\mathbf{v}-\left(n_0+\int\frac{v_{\|}^2}{v_{\perp}}\frac{\partial F_0}{\partial\mathbf{v}_{\perp}}d\mathbf{v}\right)h_ih_j\right\}, \tag{1.10}$$

where $\Omega^2 = 4\pi n_0e^2/m$ is the square of the Langmuir frequency for particles of a definite species, $v_{\|}$ and $v_{\perp}$ are the components of the particle velocity parallel and perpendicular to the magnetic field H_0, respectively, $\mathbf{h}$ is a unit vector in the direction of the external magnetic field H_0, and the components of the tensor $\Pi_{ij}(n,\mathbf{v})$ are

$$\Pi_{ij}(n,\mathbf{v})=\begin{pmatrix}\frac{n^2\omega_H^2}{k_\perp^2}J_n^2 & iv_\perp\frac{n\omega_H}{k_\perp}J_nJ_n' & v_{\|}\frac{n\omega_H}{k_\perp}J_n^2\\ -iv_\perp\frac{n\omega_H}{k_\perp}J_nJ_n' & v_\perp^2J_n'^2 & -iv_{\|}v_\perp J_nJ_n'\\ v_{\|}\frac{n\omega_H}{k_\perp}J_n^2 & iv_{\|}v_\perp J_nJ_n' & v_{\|}^2J_n^2\end{pmatrix} \tag{1.11}$$

and $\left(J_n=J_n(a),\ J_n'=\frac{\partial J_n(a)}{\partial a}\ \text{и}\ a=\frac{k_\perp v_\perp}{\omega_H}\right)$.

*The dependence of the polarizability on k was first introduced in[9].

The tensor (1.10) is expressed in a coordinate system in which the z axis is directed along the external magnetic field H_0 and the x axis lies in the plane of the vectors $\mathbf{k}$ and H_0.

The path-direction rules for the integrals in the expression for the polarizability tensor (1.10) can be easily established by introducing in the right side of the kinetic equation (1.1) a collision integral in the form $-\nu(F - F_0)$, where ν is the effective collision frequency, and letting ν go to zero in the limit. Formally such a procedure leads to substitution of $\omega + i o$ for ω in (1.10).

The system of Maxwell's equations (1.2) can be transformed with the aid of (1.8) to

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=-\frac{4\pi}{c^2}\frac{\partial\mathbf{j}_0}{\partial t}, \tag{1.12}$$

in which the vectors $\mathbf{D}$ and $\mathbf{E}$ are connected by the relation

$$\mathbf{D}_{k\omega}=\hat{\varepsilon}\mathbf{E}_{k\omega}, \tag{1.13}$$

where $\hat{\epsilon} = \epsilon_{ij}(\omega, \mathbf{k})$ is the dielectric tensor of the plasma:

$$\varepsilon_{ij}(\omega,\mathbf{k})=\delta_{ij}+4\pi\sum\varkappa_{ij}(\omega,\mathbf{k}). \tag{1.14}$$

The dielectric tensor defines completely the electrodynamic properties of the plasma. If we know the explicit dependence of the dielectric tensor on the frequency of the wave vector, then we can use (1.12) and (1.13) to find the dispersion equation, which determines the types of waves propagating in an unbounded magnetoactive plasma and their characteristics.

2. Dielectric Tensor in the Case of a Maxwellian Particle Distribution.

It is easy to calculate the dielectric tensor in explicit form for an equilibrium plasma or for a nonisothermal plasma in which the particles are characterized by Maxwellian distributions with different temperatures. Choosing as the unperturbed distribution functions in (1.10) the Maxwellian functions

$$F_0(v)=n_0\left(\frac{m}{2\pi T}\right)^{\frac{3}{2}}e^{-\frac{mv^2}{2T}} \tag{1.15}$$

with different temperatures for different plasma components, and integrating over the velocities, we obtain ultimately the following expression for the dielectric tensor of a magnetoactive plasma[10]:

$$\varepsilon_{ij}(\omega,\mathbf{k})=\delta_{ij}-\Sigma\frac{\Omega^2}{\omega^2}\left\{e^{-\beta}\sum_n\frac{z_0}{z_n}\pi_{ij}(z_n)\left[\varphi(z_n)-i\sqrt{\pi}z_ne^{-z_n^2}\right]-2z_0^2h_ih_j\right\}, \tag{1.16}$$

where

$$\pi_{ij}(z_n)=\begin{pmatrix}\frac{n^2}{\beta}I_n & \frac{k_{\|}}{|k_{\|}|}\sqrt{\frac{2}{\beta}}nz_nI_n & in(I_n'-I_n)\\ -in(I_n'-I_n) & -i\frac{k_{\|}}{|k_{\|}|}\sqrt{2\beta}z_n(I_n'-I_n) & \left(\frac{n^2}{\beta}+2\beta\right)I_n-2\beta I_n'\\ \frac{k_{\|}}{|k_{\|}|}\sqrt{\frac{2}{\beta}}nz_nI_n & 2z_n^2I_n & i\frac{k_{\|}}{|k_{\|}|}\sqrt{2\beta}z_n(I_n'-I_n)\end{pmatrix}, \tag{1.17}$$

$$\varphi(z) = 2ze^{z^2}\int_0^z e^{x^2}\,dx, \tag{1.18}$$

$I_n = I_n(\beta)$ —modified Bessel function, $I_n' = \partial I_n(\beta)/\partial\beta$, $\sqrt{\beta} = k_\perp s/\sqrt{3}\omega_H$, $s = \sqrt{3T/m}$ —mean-square particle velocity, and

$$z_n = \sqrt{\frac{3}{2}}\,\frac{\omega - n\omega_H}{|k_\parallel|\,s}.$$

3. Dispersion Equation

The different types of plane monochromatic waves propagating in a plasma, $\mathbf{E}(\mathbf{r},t) = \mathbf{E}\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$, differ from one another in the frequency dependence of the phase velocity (dispersion) and in polarization. To find the dispersion and polarization of the waves in a magnetoactive plasma, we shall use Eq. (1.12), in which we put $j_0 = 0$. Introducing the refractive index $\eta = kc/\omega$ and the unit vector $\kappa = \mathbf{k}/k$ in the wave propagation direction, we can write the wave equation in the form

$$\Lambda_{ij}(\omega, \mathbf{k})E_j = 0, \tag{1.19}$$

where

$$\Lambda_{ij}(\omega, \mathbf{k}) = \eta^2(\varkappa_i\varkappa_j - \delta_{ij}) + \varepsilon_{ij}(\omega, \mathbf{k}). \tag{1.20}$$

The vector equation (1.19) has solutions that differ from zero if the determinant made up of the elements of the matrix (1.20) vanishes:

$$\Lambda(\omega, \mathbf{k}) \equiv |\Lambda_{ij}(\omega, \mathbf{k})| = 0. \tag{1.21}$$

The condition (1.21) is the dispersion equation. The roots of this equation determine the dependence of the refractive indices η on the frequency, or else the natural frequencies of the different waves in the plasma.

Using (1.20) we can represent the dispersion equation in the form

$$A\eta^4 + B\eta^2 + C = 0, \tag{1.22}$$

where

$$A = \varepsilon_{11}\sin^2\vartheta + 2\varepsilon_{13}\sin\vartheta\cos\vartheta + \varepsilon_{33}\cos^2\vartheta,$$
$$B = -(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2)\sin^2\vartheta + 2(\varepsilon_{12}\varepsilon_{23} - \varepsilon_{22}\varepsilon_{13})\sin\vartheta\cos\vartheta - (\varepsilon_{22}\varepsilon_{33} + \varepsilon_{23}^2)\cos^2\vartheta - \varepsilon_{11}\varepsilon_{33} + \varepsilon_{13}^2,$$
$$C = \varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{23}^2 - \varepsilon_{22}\varepsilon_{13}^2 + \varepsilon_{33}\varepsilon_{12}^2 + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{13},$$

ϑ is the angle between the wave propagation direction $\mathbf{k}$ and the magnetic field $\mathbf{H}_0$.

In the general case the determinant Λ is a complex function of ω and $\mathbf{k}$, and therefore the condition (1.21) reduces to the requirement that the real and imaginary parts of Λ vanish separately:

$$\mathrm{Re}\,\Lambda(\omega, \mathbf{k}) + i\,\mathrm{Im}\,\Lambda(\omega, \mathbf{k}) = 0. \tag{1.23}$$

In the transparency region of the plasma (the region of ω and $\mathbf{k}$ in which the antihermitian part of the dielectric tensor is small compared with the hermitian part), the imaginary part of Λ is small compared with its real part. Therefore, neglecting damping of the waves, the dispersion equation can be approximately written in the form

$$\mathrm{Re}\,\Lambda(\omega, \mathbf{k}) = 0. \tag{1.24}$$

By determining from (1.24) the natural frequency ω of the wave and assuming the damping to be small ($\gamma \ll \omega$) we can easily find the damping coefficient of the wave with the aid of (1.23):

$$\gamma = \frac{\mathrm{Im}\,\Lambda(\omega, \mathbf{k})}{\frac{\partial}{\partial\omega}\mathrm{Re}(\omega, \mathbf{k})}. \tag{1.25}$$

We introduce a matrix λ_{ij}, whose elements are the co-factors of the elements of the matrix Λ_{ij}. By definition,

$$\Lambda_{ij}\lambda_{jk} = \Lambda\delta_{ik}. \tag{1.26}$$

The elements of the matrix λ_{ij} are expressed in terms of the elements of the matrix Λ_{ij} by means of the formula

$$\lambda_{ij} = \frac{1}{2}\varepsilon_{ikl}\varepsilon_{jmn}\Lambda_{mk}\Lambda_{nl}, \tag{1.27}$$

where ϵ_{ikl} is a fully antisymmetrical tensor. It is easy to verify directly that Im Λ can be expressed in the transparency region of the plasma in terms of the hermitian part of the matrix λ_{ij} and the antihermitian part of the dielectric tensor ϵ_{ij}:

$$\mathrm{Im}\,\Lambda = \frac{1}{4i}(\varepsilon_{ij} - \varepsilon_{ji}^*)(\lambda_{ji} + \lambda_{ij}^*). \tag{1.28}$$

4. Wave Polarization

Comparing (1.26) with (1.19), we see that for a wave with dispersion law (1.21) we can choose as the polarization vector

$$e_i = C\lambda_{ij}a_j, \tag{1.29}$$

where a is an arbitrary unit vector and C is a constant determined from the normalization condition $\mathbf{e}\cdot\mathbf{e}^* = 1$.

We shall show that relation (1.29) determines the wave polarization vector accurate to a phase factor. We note first that at all frequencies the matrices λ_{ij} and Λ_{ij} are related by

$$\lambda_{ij}\lambda_{kl} = \lambda_{il}\lambda_{kj} + \Lambda\varepsilon_{ikm}\varepsilon_{jln}\Lambda_{nm}. \tag{1.30}$$

(The correctness of (1.30) can be readily verified by multiplying the left and right sides of the equality $\Lambda\epsilon_{abc} = \epsilon_{mnp}\Lambda_{ma}\Lambda_{nb}\Lambda_{pc}$ by $(1/\Lambda)\lambda_{aj}\lambda_{bl}\epsilon_{kic}$.)

At frequencies satisfying the dispersion law $\Lambda = 0$, relation (1.30) simplifies to

$$\lambda_{ij}\lambda_{kl} = \lambda_{il}\lambda_{kj}. \tag{1.31}$$

Neglecting in the transparency region of the plasma the antihermitian part of λ_{ij}, we can readily derive from (1.31) the equality

$$\frac{\lambda_{il}a_l\lambda_{jk}^*a_k}{\lambda_{mn}a_m a_n} = \frac{\lambda_{il}a_l'\lambda_{jk}^*a_k'}{\lambda_{mn}a_m' a_n'}, \tag{1.32}$$

where **a** and **a′** are arbitrary vectors. We note also that for the arbitrary vectors **a** and **a′** the scalar products $(\mathbf{a}\hat{\lambda}\mathbf{a})$ and $(\mathbf{a}'\hat{\lambda}\mathbf{a}')$ have the same sign:

$$(\mathbf{a}\hat{\lambda}\mathbf{a})(\mathbf{a}'\hat{\lambda}\mathbf{a}') = |(\mathbf{a}\hat{\lambda}\mathbf{a}')|^2. \tag{1.33}$$

(In particular, the signs of the diagonal elements of the hermitian part of the matrix λ_{ij} are identical.)

Using (1.31), we can write for the normalization constant in (1.29)

$$C = \{(\mathbf{a}\hat{\lambda}\mathbf{a})\,\mathrm{Sp}\,\lambda\}^{-\frac{1}{2}}.$$

Consequently the normalized polarization vector for a wave with dispersion (1.21) is

$$\mathbf{e} = \frac{\hat{\lambda}\mathbf{a}}{\sqrt{(\mathbf{a}\hat{\lambda}\mathbf{a})\,\mathrm{Sp}\,\lambda}}. \tag{1.34}$$

According to (1.32), the product $e_i e_j^*$ is invariant to change of the vector **a**, therefore an arbitrary rotation of the vector **a** can lead only to a change of the phase factor in (1.34).

5. Energy Flux Density

Other important characteristics of waves in a plasma are the average energy density W and the average energy flux density in the wave propagation direction S. In a dispersive medium, the energy density is given by [12]

$$W = \frac{1}{16\pi}\left\{\frac{d}{d\omega}(\omega\varepsilon_{ij})E_i^*E_j + |\mathbf{H}|^2\right\}. \tag{1.35}$$

The average energy flux density is connected with W by

$$S = uW, \tag{1.36}$$

where $u = d\omega/dk$ is the group velocity of the wave.

2. WAVES IN A MAGNETOACTIVE PLASMA

1. High-frequency Waves in a Magnetoactive Plasma

Both high-frequency and low-frequency weakly-damped waves can exist in a magnetoactive plasma. To determine the frequency spectrum of weakly damped waves in a plasma it is necessary to use in the general case the expression (1.16) for dielectric tensor.

At high frequencies $\omega \gg ks$, the thermal motion of the particles can be neglected. The dielectric tensor of the plasma takes then the form

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \tag{2.1}$$

where

$$\varepsilon_1 = 1 - \sum \frac{\Omega^2}{\omega^2 - \omega_H^2}, \quad \varepsilon_2 = \sum \frac{\omega_H}{\omega}\frac{\Omega^2}{\omega^2 - \omega_H^2},$$

$$\varepsilon_3 = 1 - \sum \frac{\Omega^2}{\omega^2}. \tag{2.2}$$

We see that neglecting the thermal motion of the plasma particle leads to the absence of spatial dispersion. According to (2.1), a plasma in a magnetic field is an anisotropic and gyrotropic medium even in the absence of spatial dispersion.

Neglecting thermal motion of the particles in the plasma, the dispersion equation is of the form

$$A_0\eta^4 + B_0\eta^2 + C_0 = 0, \tag{2.3}$$

$$\left.\begin{aligned} A_0 &= \varepsilon_1 \sin^2\vartheta + \varepsilon_3\cos^2\vartheta, \\ B_0 &= -[(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\vartheta + \varepsilon_1\varepsilon_3(1+\cos^2\vartheta)], \\ C_0 &= (\varepsilon_1^2 - \varepsilon_2^2)\varepsilon_3; \end{aligned}\right\} \tag{2.4}$$

where the coefficients A_0, B_0, and C_0 are independent of η by virtue of the absence of spatial dispersion. Therefore (2.3) can be directly solved with respect to η, and this yields by the same token the dependence of the refractive indices of the electromagnetic waves in the plasma on the frequency and on the wave propagation direction. Equation (2.3) has two different solutions:

$$\eta_{0,e}^2 = \frac{(\varepsilon_1^2 - \varepsilon_2^2)\sin^2\vartheta + \varepsilon_1\varepsilon_3(1+\cos^2\vartheta) \pm \sqrt{(\varepsilon_1^2 - \varepsilon_2^2 - \varepsilon_1\varepsilon_3)^2\sin^4\vartheta + 4\varepsilon_2^2\varepsilon_3^2\cos^2\vartheta}}{2(\varepsilon_1\sin^2\vartheta + \varepsilon_3\cos^2\vartheta)}, \tag{2.5}$$

which determine the refractive indices of the ordinary (η_0^2) and extraordinary (η_e^2) electromagnetic waves in the plasma.

In the general case, for an arbitrary propagation direction (relative to the magnetic field), the electromagnetic waves in the plasma are not transverse and are characterized by elliptic polarization. For electromagnetic waves with specified frequency, the complex polarization vectors can be chosen in the form

$$\mathbf{e} = \left\{\cos\varphi - i\frac{\varepsilon_2}{\eta^2 - \varepsilon_1}\sin\varphi, \sin\varphi + i\frac{\varepsilon_2}{\eta^2-\varepsilon_1}\cos\varphi, \frac{\eta^2\sin\vartheta\cos\vartheta}{\eta^2\sin^2\vartheta - \varepsilon_3}\right\}, \tag{2.6}$$

(φ is the azimuthal angle of the vector* **k**).

If thermal motion of the particles is neglected, the tensor (2.1) is hermitian and there is no damping of the waves. Taking into account the antihermitian part of the tensor (1.16), we easily obtain with the aid of (1.25) the imaginary part of the refractive index, which determines the damping of the waves in the plasma:

$$\eta'' = \frac{1}{2}\eta \arg(\mathbf{e}^*\hat{\varepsilon}\mathbf{e}). \tag{2.7}$$

In the presence of a magnetic field, the electromagnetic waves in the plasma do not separate, strictly speaking, into longitudinal and transverse. However, if $A \equiv (\kappa\epsilon\kappa) \to 0$, then the longitudinal component of the electric field will be much larger than the trans-

*We shall henceforth find it more convenient to use non-normalized polarization vectors.

verse component. This can be readily verified by multiplying (1.19) by $\boldsymbol{\kappa}$:

$$E_{\|} = -\frac{(\varkappa \varepsilon \mathbf{E}_{\perp})}{A}. \qquad (2.8)$$

We see therefore that for purely longitudinal oscillations in a plasma ($\mathbf{E}_{\perp} = 0$) two conditions should be satisfied: $\Lambda = 0$ and $A = 0$. In the general case these conditions are not satisfied simultaneously. However, in the frequency region for which η^2 is very large, it is necessary to retain in the dispersion equation (1.22) only the term with the highest power of η^2, and then the dispersion equation reduces to the condition $A = 0$.

Using (1.16) we can write the dispersion equation for longitudinal waves in a magnetoactive plasma in the form

$$A \equiv 1 + \sum \frac{1}{a^2 k^2} \left\{1 - e^{-\beta} \sum_n \frac{z_0}{z_n} I_n(\beta) \left[\varphi(z_n) - i\sqrt{\pi} z_n e^{-z_n^2}\right]\right\} = 0. \qquad (2.9)$$

If $k_{\perp} = 0$, then $\beta = 0$ and the dispersion equation (2.9) has the same form as in the absence of a magnetic field; consequently, the magnetic field does not influence the longitudinal waves propagating in the plasma along the field.

Neglecting the thermal motion of the particles, the dispersion equation for longitudinal waves takes the form

$$A_0 \equiv 1 - \frac{\Omega^2}{\omega^2 - \omega_H^2} \sin^2\vartheta - \frac{\Omega^2}{\omega^2}\cos^2\vartheta = 0. \qquad (2.10)$$

From (2.10) we get the natural frequencies of the longitudinal waves in the plasma:

$$\omega_{\pm}^2 = \frac{1}{2}(\Omega^2 + \omega_H^2) \pm \frac{1}{2}\sqrt{(\Omega^2 + \omega_H^2)^2 - 4\Omega^2\omega_H^2\cos^2\vartheta}. \qquad (2.11)$$

These frequencies correspond to Langmuir oscillations of a plasma in a magnetic field.

The dispersion of the Langmuir oscillations can be found by taking into account the thermal corrections to the dispersion equation. The refractive index of the Langmuir waves is

$$\eta_L^2 = \frac{A_0}{\psi}, \qquad (2.12)$$

where

$$\psi = \frac{s^2}{c^2}\frac{\Omega^2}{\omega^2}\left\{\frac{\omega^4}{(\omega^2 - \omega_H^2)(\omega^2 - 4\omega_H^2)}\sin^4\vartheta + \frac{\omega^2}{3}\frac{6\omega^4 - 3\omega^2\omega_H^2 + \omega_H^4}{(\omega^2 - \omega_H^2)^3}\sin^2\vartheta\cos^2\vartheta + \cos^4\vartheta\right\}. \qquad (2.12')$$

Using (2.9), we can also readily find the damping of the Langmuir waves:

$$\frac{\gamma}{\omega} = \frac{\sqrt{\pi}}{2a^2k^2}\frac{|\omega^2 - \omega_H^2|}{\omega_+^2 - \omega_-^2} z_0 \sum_n I_n(\beta) e^{-z_n^2}. \qquad (2.13)$$

By virtue of the longitudinal character of the Langmuir oscillations, the polarization vector is equal to

$$\mathbf{e} = \boldsymbol{\varkappa}. \qquad (2.14)$$

Let us determine also the average energy flux density in the propagation direction of a high-frequency wave in a plasma. For the ordinary and extraordinary waves, the spatial dispersion is immaterial, therefore S can be determined directly by calculating the projection of the Poynting vector on the wave propagation direction $\boldsymbol{\kappa}$:

$$S = \frac{c}{8\pi}\eta\zeta|\mathbf{E}|^2, \qquad (2.15)$$

where $\zeta = |\mathbf{e}|^2 - |\boldsymbol{\kappa}\cdot\mathbf{e}|^2$.

In the case of Langmuir waves, the energy transport is connected with spatial dispersion, therefore to find S it is necessary to use the general relation (1.36). We note that for Langmuir waves the group velocity and the energy density are equal to

$$u = c \Big/ \frac{d(\omega\eta)}{d\omega}, \quad W = \frac{1}{8\pi}\omega^2\frac{dA_0}{d\omega^2}|\mathbf{E}|^2. \qquad (2.16)$$

We thus obtain for the energy flux density in the wave propagation direction

$$S = \frac{c}{8\pi}\eta\xi|\mathbf{E}|^2, \qquad (2.17)$$

where

$$\xi = \frac{\omega^2 \frac{dA_0}{d\omega^2}}{\eta\frac{d(\omega\eta)}{d\omega}},$$

and η and A_0 are determined by (2.12) and (2.10).

2. Low-frequency Waves in a Magnetoactive Plasma

In a magnetoactive plasma there can exist also weakly damped waves in the low-frequency region of the spectrum [13-16]. To find the natural frequencies of the plasma in the indicated region, we shall use the low-frequency values for the components of the tensor ϵ_{ij} ($\omega \ll \omega_H^i$, $ks_i \ll \omega_H^i$):

$$\left.\begin{aligned}
\varepsilon_{11} &= \varepsilon_0,\\
\varepsilon_{22} &= \varepsilon_0\left\{1 - \frac{2}{3}\frac{k^2s_i^2}{\omega^2}\left[t\varphi(z) + \varphi(\mu z) - i\sqrt{\pi}z\left(te^{-z^2} + \mu e^{-\mu^2z^2}\right)\right]\sin^2\vartheta\right\},\\
\varepsilon_{33} &= 3\varepsilon_0\frac{\omega_H^{i2}}{k^2s_i^2}\left[\frac{1-\varphi(z)}{t} + 1 - \varphi(\mu z) + i\sqrt{\pi}z\left(\frac{e^{-z^2}}{t} + \mu e^{-\mu^2z^2}\right)\right]\cos^{-2}\vartheta,\\
\varepsilon_{12} &= i\varepsilon_0\frac{\omega}{\omega_H^i}\left[1 - \frac{1}{2}\frac{k^2s_i^2}{\omega^2}\left(\sin^2\vartheta - \frac{2}{3}\cos^2\vartheta\right)\right],\\
\varepsilon_{13} &= 0,\\
\varepsilon_{23} &= i\varepsilon_0\frac{\omega_H^i}{\omega}\left[\varphi(z) - \varphi(\mu z) - i\sqrt{\pi}z\left(e^{-z^2} - \mu e^{-\mu^2z^2}\right)\right]\operatorname{tg}\vartheta,
\end{aligned}\right\} \qquad (2.18)^*$$

*tg ≡ tan.

where

$$\varepsilon_0 = \frac{\Omega_i^2}{\omega_H^{i2}}, \quad t = \frac{T_e}{T_i}, \quad \mu = \frac{M}{m} t \ \text{и} \ z = \sqrt{\frac{3}{2}} \frac{\omega}{ks}.$$

We present also expressions for the components of the electronic part of the plasma polarizability tensor in the low-frequency region

$$\left.\begin{aligned} &\varkappa_{11}^e = \frac{\varepsilon_0}{4\pi} \frac{m}{M}, \quad \varkappa_{12}^e = -i \frac{\varepsilon_0}{4\pi} \frac{\omega_H^i}{\omega}, \\ &\varkappa_{22}^e = \frac{\varepsilon_0}{4\pi} \frac{m}{M} \left\{1 - \frac{2}{3} \frac{k^2 s^2}{\omega^2} \left[\varphi(z) - i\sqrt{\pi} z e^{-z^2}\right] \sin^2\vartheta\right\}, \quad \varkappa_{13}^e = 0, \\ &\varkappa_{33}^e = 3 \frac{\varepsilon_0}{4\pi} \frac{\omega_H^{i2}}{k^2 s_i^2} \frac{1}{t} \left[1 - \varphi(z) + i\sqrt{\pi} z e^{-z^2}\right] \cos^{-2}\vartheta, \\ &\varkappa_{23}^e = i \frac{\varepsilon_0}{4\pi} \frac{\omega_H^i}{\omega} \left[\varphi(z) - 1 - i\sqrt{\pi} z e^{-z^2}\right] \operatorname{tg}\vartheta. \end{aligned}\right\} \tag{2.19}$$

Substituting (2.18) in (1.21) we can represent the dispersion equation in the low-frequency region in the form

$$(\eta^2\cos^2\vartheta - \varepsilon_{11})\left[(\eta^2 - \varepsilon_{22})\varepsilon_{33} - \varepsilon_{23}^2\right] + 2\eta^2\varepsilon_{12}\varepsilon_{23}\sin\vartheta\cos\vartheta + \varepsilon_{12}^2\varepsilon_{33} + \ldots = 0. \tag{2.20}$$

It is easy to verify that in (2.20) the second and third terms are smaller than the first by a factor $(\omega/\omega_H^i)^2$ and $(ks_i/\omega_H^i)^2$, respectively. In the case of a sufficiently strong magnetic field these terms can be neglected, and then the dispersion equation breaks up into two independent equations:

$$\begin{gathered} \eta^2\cos^2\vartheta - \varepsilon_{11} = 0, \\ (\eta^2 - \varepsilon_{22})\varepsilon_{33} - \varepsilon_{23}^2 = 0. \end{gathered} \tag{2.21}$$

The first equation of (2.21) determines the refractive index of the Alfven wave:

$$\eta^2 = \frac{\Omega_i^2}{\omega_H^{i2}} \frac{1}{\cos^2\vartheta}. \tag{2.22}$$

Expressing η in terms of k and ω, we can rewrite (2.22) in the form

$$\omega^2 = k^2 v_A^2 \cos^2\vartheta, \quad v_A = \frac{H_0}{\sqrt{4\pi n_0 M}}. \tag{2.23}$$

Taking into account the discarded terms in (2.20), we can find the thermal corrections to the phase velocity and the damping of the Alfven wave:

$$\frac{\gamma}{\omega} = \frac{1}{2}\sqrt{\frac{\pi}{6}} \frac{m}{M} \frac{s}{v_A} \frac{\omega^2}{\omega_H^{i2}} (\operatorname{tg}^2\vartheta + \operatorname{ctg}^2\vartheta), \quad s_i \ll \frac{\omega}{k|\cos\vartheta|} \ll s, \quad \frac{s_i^2}{v_A^2} t \ll 1. \tag{2.24*}$$

The polarization vector of the Alfven wave is

$$\mathbf{e} = \left\{1, \ -i\frac{\omega}{\omega_H^i}\operatorname{ctg}^2\vartheta, \ -\frac{1}{3}\frac{s_i^2}{v_A^2}\frac{\omega^2}{\omega_H^{i2}}\frac{t}{\sin\vartheta\cos\vartheta}\right\}. \tag{2.25}$$

The second equation of (2.21) determines the refractive indices of the magnetic-sound waves. The roots of this equation depend in essential fashion on the relations between the thermal velocities s and s_i of the electrons and ions and the Alfven velocity v_A.

If $s_i^2 \ll v_A^2 \ll s^2$ then, using (2.18), we obtain from (2.21) in first approximation the dispersion of the fast magnetic-sound wave:

$$\eta^2 = \frac{\Omega_i^2}{\omega_H^{i2}}, \quad \omega^2 = k^2 v_A^2. \tag{2.26}$$

The thermal corrections to (2.26) can be obtained by successive approximations:

$$\omega^2 = k^2 v_A^2 + \frac{2}{3} k^2 s_i^2 \left(1 + \frac{t}{2}\right) \sin^2\vartheta. \tag{2.27}$$

The damping of the fast magnetic-sound wave is

$$\frac{\gamma}{\omega} = \frac{1}{2}\sqrt{\frac{\pi}{6}} \frac{m}{M} \frac{s}{v_A} \frac{\sin^2\vartheta}{|\cos\vartheta|}. \tag{2.28}$$

The polarization vector is given by

$$\mathbf{e} = \left\{-i\frac{\omega}{\omega_H^i}\frac{1}{\sin^2\vartheta}, \ 1, \ -i\frac{t}{3}\frac{s_i^2}{v_A^2}\frac{\omega}{\omega_H^i}\sin\vartheta\cos\vartheta\right\}. \tag{2.29}$$

When $T_e \gg T_i$ the second equation of (2.21) has one more solution, corresponding to weakly damped nonisothermal sound waves (slow magnetic-sound wave). Assuming that $s_i \ll \omega/k|\cos\vartheta| \ll s$, we can obtain from (2.21) the following solution:

$$\eta^2 = \frac{c^2}{v_s^2}\frac{1}{\cos^2\vartheta}, \quad v_s^2 = \frac{T_e}{M}, \tag{2.30}$$

or else

$$\omega^2 = k^2 v_s^2 \cos^2\vartheta. \tag{2.31}$$

The damping of the nonisothermal sound is

$$\frac{\gamma}{\omega} = \sqrt{\frac{\pi}{8}\frac{m}{M}}. \tag{2.32}$$

The polarization vector is given by

$$\mathbf{e} = \left\{\sin\vartheta, \ -i\frac{v_s^2}{v_A^2}\frac{\omega_H^i}{\omega}\sin\vartheta\cos^2\vartheta, \ \cos\vartheta\right\}. \tag{2.33}$$

The average energy and energy flux density for low-frequency waves in a plasma can be easily obtained on the basis of the general relations (1.35) and (1.36). In the case of the Alfven and fast magnetic-sound waves, almost all the energy is connected with the electromagnetic field of the wave.

$$W = \frac{1}{8\pi}\frac{c^2}{v_A^2}|\mathbf{E}|^2, \quad S = \frac{1}{8\pi}\frac{c^2}{v_A}\cos\vartheta\,|\mathbf{E}|^2, \tag{2.34}$$

*ctg ≡ cot.

$$W = \frac{1}{8\pi} \frac{c^2}{v_A^2} |\mathbf{E}|^2, \quad S = \frac{1}{8\pi} \frac{c^2}{v_A} |\mathbf{E}|^2. \tag{2.35}$$

In the case of the slow magnetic-sound wave, the kinetic energy of the plasma particles is larger than the field energy by a factor v_A^2/v_S^2. The average energy and energy flux density are determined by the expressions

$$W = \frac{1}{8\pi} \frac{1}{a^2k^2} |\mathbf{E}|^2, \quad S = \frac{1}{8\pi} \frac{v_s \cos\vartheta}{a^2k^2} |\mathbf{E}|^2. \tag{2.36}$$

3. EXCITATION OF WAVES BY EXTERNAL CURRENTS IN A MAGNETOACTIVE PLASMA

1. Excitation Intensity

Electromagnetic waves can be excited in a plasma by external current whose energy is transformed into energy of the electromagnetic field and kinetic energy of the plasma particles. The determination of the intensity of excitation of various waves by external currents may be of practical interest in connection with a study of the feasibility of high-frequency heating of a plasma and the use of a plasma pierced by a beam of charged particles as a source of microwaves.

In analyzing the excitation of waves in a plasma, we start from Eq. (1.12), assuming that the external current $\mathbf{j}_0$ is specified. With the aid of Maxwell's equations it is easy to determine the increase of energy in the entire plasma volume per unit time, i.e., the intensity of the excitation of the waves in the plasma:

$$I \equiv \frac{1}{4\pi} \int \left(\mathbf{E} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \frac{\partial \mathbf{H}}{\partial t} \right) dr = \frac{c}{4\pi} \int [\mathbf{EH}]\, ds - \int \mathbf{j}_0 \mathbf{E}\, d\mathbf{r}. \tag{3.1}$$

In the case of an unbounded plasma, the surface integral in (3.1) vanishes. Using complex notation, we can write the excitation intensity for an unbounded plasma in the form

$$I = -\frac{1}{2} \operatorname{Re} \int \mathbf{j}_0^* \mathbf{E}\, d\mathbf{r}. \tag{3.2}$$

The total energy transferred from the current to the plasma is determined by integrating (3.2) with respect to the time:

$$P = -\frac{1}{2} \operatorname{Re} \int \int \mathbf{j}_0^* \mathbf{E}\, dr\, dt. \tag{3.3}$$

Expanding the external current $\mathbf{j}_0$ and the electric field of the excited waves $\mathbf{E}$ in Fourier integrals in plane waves, we represent the transferred energy in the form

$$P = -\frac{1}{2(2\pi)^4} \operatorname{Re} \int \mathbf{j}_{\mathbf{k}\omega}^* \mathbf{E}_{\mathbf{k}\omega}\, d\mathbf{k}\, d\omega. \tag{3.4}$$

The electric field intensity, is connected according to (1.12) with the exciting current by

$$\mathbf{E}_{\mathbf{k}\omega} = -\frac{4\pi i}{\omega \Lambda(\omega, \mathbf{k})} \hat{\lambda} \mathbf{j}_{\mathbf{k}\omega}. \tag{3.5}$$

Since the external field excites plasma waves for which the following relation holds

$$\lambda_{ij} = e_i e_j^* \operatorname{Sp} \lambda,$$

we get

$$P = \frac{1}{8\pi^3} \operatorname{Re} \left(i \int \frac{\operatorname{Sp} \lambda}{\omega \Lambda} |\mathbf{e} \mathbf{j}_{\mathbf{k}\omega}^*|^2\, dk\, d\omega \right). \tag{3.6}$$

Since $\operatorname{Sp} \lambda$ is real, we can rewrite this formula in the form

$$P = \frac{1}{8\pi^3} \int \frac{\operatorname{Sp} \lambda}{\omega} \frac{\operatorname{Im} \Lambda}{|\Lambda|^2} |\mathbf{e} \mathbf{j}_{\mathbf{k}\omega}^*|^2\, dk\, d\omega. \tag{3.7}$$

The quantity $\operatorname{Im} \Lambda$ can be related to the magnitude of the electric loss in the plasma, defined by the expression[12]

$$Q = -\frac{i\omega}{8\pi} (\varepsilon_{ij} - \varepsilon_{ji}^*) e_i^* e_j |E|^2. \tag{3.8}$$

Indeed, according to (1.28) and (1.34) we have

$$\operatorname{Im} \Lambda = \frac{\operatorname{Sp} \Lambda}{2i} (\varepsilon_{ij} - \varepsilon_{ji}^*) e_i^* e_j. \tag{3.9}$$

Since, by virtue of the increase of entropy, $Q > 0$ for thermodynamically stable systems, it follows from a comparison of (3.9) with (3.8) that

$$\frac{\operatorname{Sp} \lambda}{\omega} \operatorname{Im} \Lambda > 0. \tag{3.10}$$

Taking this condition into account and noting that $|\operatorname{Im} \Lambda| / |\Lambda|^2 \to \pi\delta/A$ in the transparency region of the plasma, we ultimately obtain for the total energy transferred from the current to the plasma

$$P = \frac{1}{8\pi^2} \int \left| \frac{\operatorname{Sp} \lambda}{\omega} \right| |\mathbf{e} \mathbf{j}_{\mathbf{k}\omega}^*|^2 \delta\{\Lambda(\omega, \mathbf{k})\}\, dk\, d\omega. \tag{3.11}$$

The case of greatest interest is when the external current depends harmonically on the time

$$\mathbf{j}_0(\mathbf{r}, t) = \mathbf{j}_0(\mathbf{r}) e^{-i\omega_0 t}.$$

In this case the Fourier component is equal to

$$\mathbf{j}_{\mathbf{k}\omega} = 2\pi\delta(\omega - \omega_0) \mathbf{j}_{\mathbf{k}}. \tag{3.12}$$

Noting that

$$\delta^2(\omega - \omega_0) \longrightarrow \frac{T}{2\pi} \delta(\omega - \omega_0),$$

where T is the interaction time ($T \to \infty$) we obtain from (3.11) for the average energy transferred by the external current to the plasma per unit time:

$$I \equiv \frac{P}{T} = \frac{1}{4\pi} \int \left| \frac{\operatorname{Sp} \lambda}{\omega_0} \right| |\mathbf{e} \mathbf{j}_{\mathbf{k}}^*|^2 \delta\{\Lambda(\omega_0, \mathbf{k})\}\, dk. \tag{3.13}$$

With the aid of this formula we can consider the excitation of both low- and high-frequency waves in a magnetoactive plasma[17,18].

2. Excitation of High-frequency Waves

By way of an example, let us consider excitation of high-frequency waves by an external current in a magnetoactive plasma. For high-frequency waves

$$\delta\{\Lambda\}=\frac{1}{|\mathrm{Sp}\,\lambda|}\left\{\frac{1}{|\mathbf{e}|^2-|\varkappa\mathbf{e}|^2}[\delta(\eta^2-\eta_0^2)+\delta(\eta^2-\eta_e^2)]+\delta(A)\right\}; \quad (3.14)$$

therefore the total excitation intensity (3.13) is the sum of the excitation intensities of the ordinary, extraordinary, and Langmuir waves:

$$I=I_0+I_e+I_L,$$

$$I_{0,e}=\frac{1}{4\pi\omega_0}\int\frac{|\mathbf{e}\mathbf{j}_k^*|^2}{|\mathbf{e}|^2-|\varkappa\mathbf{e}|^2}\delta(\eta^2-\eta_{0,e}^2(\omega_0))\,dk,$$

$$I_L=\frac{1}{4\pi\omega_0}\int|\varkappa\mathbf{j}_k^*|^2\,\delta(A(\omega_0,\mathbf{k}))\,dk. \quad (3.15)$$

The radiation conditions in (3.15) reduce to the requirement that the squares of the refractive indices be positive for the excited waves. We note that in the case of excitation of the Langmuir waves it is essential to take into account the spatial dispersion (the dependence of A on k). Let us consider several particular cases[19].

a) Surface current. If a surface current $\mathbf{j}_0(\mathbf{r}) = \mathbf{j}_0\delta(z)$ flows in a plane perpendicular to the external magnetic field, the waves excited by this current propagate only along the external magnetic field $\mathbf{H}_0$. Then $\varkappa\cdot\mathbf{j}_0 = 0$ and consequently no Langmuir waves are excited. The intensity of excitation of the ordinary and extraordinary waves, per unit surface current, are equal to

$$I_{0,e}=\frac{\pi}{2c\eta_{0,e}}j_0^2,\quad \eta_{0,e}^2=\varepsilon_1\pm\varepsilon_2. \quad (3.16)$$

From the condition $\eta_{0,e}^2 > 0$ it follows that for excitation of the ordinary wave the frequency ω_0 of the external current should be either smaller than ω_H, or larger than $\omega_H/2 + (\omega_H^2/4 + \Omega^2)^{1/2}$, whereas the extraordinary wave is excited by a current whose frequency satisfies the inequality

$$\omega_0>\sqrt{\frac{1}{4}\omega_H^2+\Omega^2}-\frac{\omega_H}{2}.$$

The excitation intensity, defined by (3.16), becomes infinite when $\eta_{0,e}\to 0$. This condition is satisfied at frequencies

$$\omega_0=\sqrt{\frac{1}{4}\omega_H^2+\Omega^2}\pm\frac{\omega_H}{2}. \quad (3.17)$$

Formula (3.17) determines the frequencies at which strong transfer of energy from the external current to the plasma takes place.

b) Linear current. If a linear current $\mathbf{j}_0(\mathbf{r}) = \mathbf{j}_0\delta(x)\delta(z)$ flows in a plane perpendicular to the intensity of the external magnetic field, then the excited waves will propagate in a plane perpendicular to the direction of the external current $\mathbf{j}_0$. No Langmuir wave is excited by the current, and the differential intensities of excitation of the ordinary and extraordinary waves per unit length are equal to

$$di_{0,e}=\frac{\omega_0}{4c^2}\,\frac{\dfrac{\varepsilon_2^2}{(\eta_{0,e}^2-\varepsilon_1)^2}}{\cos^2\vartheta+\dfrac{\varepsilon_2^2}{(\eta_{0,e}^2-\varepsilon_1)^2}+\dfrac{\eta_{0,e}^4\sin^4\vartheta\cos^2\vartheta}{(\eta_{0,e}^2\sin^2\vartheta-\varepsilon_3)^2}}\,j_0^2\,d\vartheta, \quad (3.18)$$

where $\eta_{0,e}^2$ is determined by expression (2.5) with $\omega = \omega_0$. We recall that the frequency of the external current ω_0 should satisfy the condition $\eta_{0,e}^2 > 0$.

The intensity of excitation of waves along the magnetic field ($\vartheta = 0$) is

$$\frac{di_{0,e}}{d\Omega}=\frac{\omega_0}{8c^2}j_0^2. \quad (3.19)$$

If $\vartheta = \pi/2$ and $\epsilon_1^2 - \epsilon_2^2 - \epsilon_1\epsilon_3 > 0$, then the excitation intensity of the ordinary wave is twice as large as the intensity (3.19), and the excitation intensity of the extraordinary wave is equal to zero. In the case when $\epsilon_1^2 - \epsilon_2^2 - \epsilon_1\epsilon_3 < 0$ the situation is reversed.

c) Radial current. Assume that a current whose density is given by the expression

$$\mathbf{j}_0(\mathbf{r})=\begin{cases}0, & r<r_0,\\ \dfrac{J}{2\pi r}\dfrac{\mathbf{r}}{r}\delta(z), & r>r_0,\end{cases}$$

where J is the total current flowing through any circle concentric with a circle of radius r_0, flows in a plane perpendicular to the magnetic field $\mathbf{H}_0$.

The intensity of excitation of the ordinary or extraordinary wave is

$$dI_{0,e}=\frac{\omega_0^3}{8\pi c^3}\eta_{0,e}\left[\cos^2\vartheta+\frac{\varepsilon_2^2}{(\eta_{0,e}^2-\varepsilon_1)^2}+\frac{\eta_{0,e}^4\sin^4\vartheta\cos^2\vartheta}{(\eta_{0,e}^2\sin^2\vartheta-\varepsilon_3)^2}\right]^{-1}\times J_0^2\left(\frac{r_0\omega_0}{c}\eta_{0,e}\sin\vartheta\right)J^2\,do, \quad (3.20)$$

where $J_0(x)$ —Bessel function of zero order.

The radial current excites also Langmuir oscillations. The excitation intensity of the Langmuir wave is equal to

$$dI_L=\frac{\omega_0^3}{8\pi c^3}\eta_L\frac{\sin^2\vartheta}{\psi(\omega_0,\vartheta)}J_0^2\left(\frac{r_0\omega_0}{c}\eta_L\sin\vartheta\right)J^2\,do, \quad (3.21)$$

where η_L is the refractive index of the Langmuir wave (2.12) at $\omega = \omega_0$. According to (3.20) and (3.21) the intensity of excitation of the Langmuir wave is c^2/s^2 times larger than that of the ordinary or extraordinary wave.

4. FLUCTUATIONS IN A MAGNETOACTIVE PLASMA

1. Correlation Functions

We proceed now to consider random fluctuations of various physical quantities in a plasma situated in an external magnetic field. We assume that the plasma is homogeneous and stable. We consider first the fluctuations of the current in a magnetoactive plasma. Let us define the partial current density by means of the relation

$$\mathbf{j}(\mathbf{r},t)=\sum e\mathbf{v}(t)\,\delta(\mathbf{r}-\mathbf{r}(t))\}, \quad (4.1)$$

where $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the radius vector and the velocity of some particle at the instant of time t, and the summation is over all the particles in a unit volume. We assume that the mean value of the current is zero in the absence of external influences.

To describe the fluctuations, one usually introduces correlation functions, defined as the mean values of the products of the fluctuation of the quantities at different points of space at different instants of time. If the medium is spatially homogeneous and we consider stationary states of the system, then the quadratic space-time correlation function will depend only on the relative distance and the absolute value of the time interval between the points at which the fluctuations are considered

$$\langle j_i(\mathbf{r}_1, t_1) j_j(\mathbf{r}_2, t_2)\rangle = \langle j_i j_j\rangle_{\mathbf{r}, t}, \qquad (4.2)$$

where $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ and $t = t_2 - t_1$. The brackets $\langle \ldots \rangle$ in the right side of the equation denote the operation of statistical averaging. Formula (4.2) should be regarded as a definition of the space-time correlation function $\langle j_i j_j \rangle_{r,t}$.

The spectral distribution of the space-time correlation function is defined by means of the equation

$$\langle j_i j_j\rangle_{\mathbf{k}\omega} = \int \langle j_i j_j\rangle_{\mathbf{r}, t}\, e^{-i\mathbf{k}\mathbf{r}+i\omega t}\, d\mathbf{r}\, dt. \qquad (4.3)$$

It is obvious that the mean value of the squared product of the Fourier components of the fluctuating quantities is connected with the spectral distribution of the correlation function by the relation

$$\langle j_i^*(\mathbf{k}, \omega) j_j(\mathbf{k}', \omega')\rangle = (2\pi)^4 \delta(\omega - \omega')\, \delta(\mathbf{k} - \mathbf{k}')\, \langle j_i j_j\rangle_{\mathbf{k}\omega}. \quad (4.4)$$

If we neglect the interaction between the electrons and the ions in the plasma, then the correlation functions for the electron and ion currents can be easily obtained on the basis of a direct microscopic calculation[5,20,21]. Indeed, in this case the separate particles move in the plasma along helical lines:

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{R}(t), \quad \mathbf{R}(t) = \left\{ -\frac{V_\perp}{\omega_H}\cos(\omega_H t + \alpha),\ \frac{V_\perp}{\omega_H}\sin(\omega_H t + \alpha),\ V_{\parallel} t \right\}, \qquad (4.5)$$

where $\mathbf{r}_0$ is the radius vector, $V_\perp$ and $V_\parallel$ are the velocity components perpendicular and parallel to the magnetic field, and α is the phase at the initial instant of time $t = 0$.

Substituting (4.1) in (4.2) and noting that the mean value of the current is zero, we get

$$\langle j_i j_j\rangle^0_{\mathbf{r}t} = \langle \sum e^2 v_i(0)\, v_j(t)\, \delta\{\mathbf{r}_0 + \mathbf{R}(0)\}\, \delta\{\mathbf{r} - \mathbf{r}_0 - \mathbf{R}(t)\}\rangle. \quad (4.6)$$

Introducing the single-particle distribution function $F_0(\mathbf{v})$, we can rewrite the correlation function (4.6) in the form

$$\langle j_i j_j\rangle^0_{\mathbf{r}t} = e^2 \int v_i(0)\, v_j(t)\, \delta\{\mathbf{r} - \mathbf{R}(t) + \mathbf{R}(0)\}\, F_0(\mathbf{v})\, d\mathbf{v}. \quad (4.7)$$

The spectral density of the correlation function, according to (4.3), is

$$\langle j_i j_j\rangle^0_{\mathbf{k}\omega} = e^2 \int\int_{-\infty}^{\infty} v_i(0)\, v_j(t)\, e^{-i\mathbf{k}\{\mathbf{R}(t) - \mathbf{R}(0)\} + i\omega t}\, F_0(\mathbf{v})\, dt\, d\mathbf{v}. \quad (4.8)$$

Using (4.5) and integrating with respect to t, we get

$$\langle j_i j_j\rangle^0_{\mathbf{k}\omega} = 2\pi e^2 \sum_n \int \Pi_{ji}(n, \mathbf{v})\, \delta(\omega - n\omega_H - k_\parallel v_\parallel)\, F_0(\mathbf{v})\, d\mathbf{v}, \quad (4.9)$$

where the tensor Π_{ij} is defined by (1.11).

To find the correlation functions with allowance for the self-consistent interaction between the charged particles in the plasma, we introduce extraneous electron and ion currents into the material equations (1.8):

$$\mathbf{j}^\alpha = -i\omega \varkappa^\alpha \mathbf{E} + \mathbf{j}^\alpha_{\text{extr}}. \qquad (4.10)$$

Since the extraneous currents should not depend on the self-consistent interaction, the correlation functions will be the same for them as for the currents of the non-interacting particles (4.9). Eliminating the self-consistent field $\mathbf{E}$ from (4.10) with the aid of Maxwell's equations, we get[5,22]

$$\langle j_i^\alpha j_j^\beta\rangle_{\mathbf{k}\omega} = \sum_\gamma (\delta_{\alpha\gamma}\delta_{im} - 4\pi\varkappa^\alpha_{ik}\Lambda^{-1}_{km})^* (\delta_{\beta\gamma}\delta_{jn} - 4\pi\varkappa^\beta_{jl}\Lambda^{-1}_{ln})\, \langle j_m^\gamma j_n^\gamma\rangle^0_{\mathbf{k}\omega}, \qquad (4.11)$$

where $\Lambda^{-1}_{ij} = \lambda_{ij}/\Lambda$.

Formula (4.11) establishes a general connection between the correlation function for the current fluctuations with allowance for the self-consistent interaction between the charged particles in the plasma, and the correlation function for the fluctuations of the current of the independent particles.

The spectral distributions of the fluctuations of the total current and of the electric field in a magnetoactive plasma are determined by the formulas[5,23]

$$\langle j_i j_j\rangle_{\mathbf{k}\omega} = (\delta_{im} - 4\pi\varkappa_{ik}\Lambda^{-1}_{km})^* (\delta_{jn} - 4\pi\varkappa_{jl}\Lambda^{-1}_{ln})\, \langle j_m j_n\rangle^0_{\mathbf{k}\omega}, \quad (4.12)$$

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = \frac{16\pi^2}{\omega^2}\Lambda^{-1*}_{ik}\Lambda^{-1}_{jl}\langle j_k j_l\rangle^0_{\mathbf{k}\omega}. \qquad (4.13)$$

It is easy to obtain in similar fashion the correlation functions for the fluctuations of all other quantities in the plasma.

In the equilibrium case, when the plasma particle distribution functions are Maxwellian, the correlation function for the fluctuations of the total current of the non-interacting particles is expressed in terms of the dielectric tensor of the plasma:

$$\langle j_i j_j\rangle^0_{\mathbf{k}\omega} = i\frac{\omega}{4\pi} T(\varepsilon^*_{ij} - \varepsilon_{ji}). \qquad (4.14)$$

In the case of a Maxwellian nonisothermal plasma, the correlation functions for the fluctuations of the electron and ion extraneous currents can be expressed in terms of the electronic and ionic polarizabilities of the plasma:

$$\langle j_i^\alpha j_j^\beta\rangle^0_{\mathbf{k}\omega} = i\delta_{\alpha\beta}T_\alpha(\varkappa^{\alpha*}_{ij} - \varkappa^\alpha_{ji}). \qquad (4.15)$$

In the general case, in the absence of thermodynamic equilibrium, the correlation functions $\langle j_i^\alpha j_j^\beta\rangle^0_{k\omega}$ are not expressed in terms of the ionic and electronic polarizabilities of the plasma κ^α_{ij}, and therefore in a nonequilibrium plasma mere specification of κ^α_{ij} is not sufficient for a complete description of the fluctuations, unlike the case of an equilibrium or quasiequilibrium nonisothermal plasma.

2. Collective Coherent Fluctuations. Effective Temperature

The spectral distributions of the correlation functions in the transparency region of the plasma have sharp delta-like maxima at frequencies ω and vectors $\mathbf{k}$ satisfying the dispersion equation $\mathrm{Re}\,\Lambda(\omega, \mathbf{k}) = 0$. It is easy to establish the form of the spectral distributions of the correlation functions near such maxima. Since $\mathrm{Im}\,\Lambda \ll \mathrm{Re}\,\Lambda$ in the plasma transparency region, we have

$$\Lambda_{ik}^{-1*}\Lambda_{jl}^{-1} = \frac{\pi\lambda_{ik}^{*}\lambda_{jl}}{|\mathrm{Im}\,\Lambda|}\,\delta(\Lambda).$$

Noting further that the relation $\lambda_{ij} = e_i e_j^* \,\mathrm{Sp}\,\lambda$ is valid for ω and $\mathbf{k}$ satisfying the dispersion relation, we can represent the spectral distribution for the electric-field fluctuations in the form

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 8\pi^2 e_i^* e_j \tilde{T}\left|\frac{\mathrm{Sp}\,\lambda}{\omega}\right|\delta\{\Lambda(\omega, \mathbf{k})\}, \tag{4.16}$$

where $\tilde{T}$ is defined by

$$\tilde{T} = \frac{2\pi}{\omega}\frac{\mathrm{Sp}\,\lambda}{\mathrm{Im}\,\Lambda}\langle j_i j_j\rangle^0_{\mathbf{k}\omega}\, e_i e_j^*. \tag{4.17}$$

In the case of an equilibrium plasma, using (4.14) and (3.9), we can easily verify that $\tilde{T}$ is equal to the plasma temperature T. In the general case $\tilde{T}$ can be regarded as an effective temperature characterizing the rms amplitude of the fluctuation oscillations of the electric field in the plasma. In a nonequilibrium plasma, the effective temperature can take on large values. If the state of the plasma approaches the borderline of the kinetic instability region, then $\mathrm{Im}\,\Lambda \to 0$, and the effective temperature increases without limit[24,25]. Using (3.9), we can represent the effective temperature (4.17) in the form

$$\tilde{T} = \frac{4\pi}{i\omega}\frac{\langle j_i j_j\rangle^0_{\mathbf{k}\omega} e_i e_j^*}{(\varepsilon_{kl}^* - \varepsilon_{lk})\, e_k e_l^*}. \tag{4.18}$$

In particular, for a nonisothermal plasma the effective temperature is

$$\tilde{T} = \frac{\sum\limits_{\alpha} T_\alpha\,(\varkappa_{ij}^{\alpha *} - \varkappa_{ji}^{\alpha})\, e_i e_j^*}{\sum\limits_{\alpha}(\varkappa_{kl}^{\alpha *} - \varkappa_{lk}^{\alpha})\, e_k e_l^*}. \tag{4.19}$$

The spectral distribution of the correlation function for the partial currents in the plasma transparency region is expressed directly in terms of the correlation function for the electric field:

$$\langle j_i^\alpha j_j^\beta\rangle_{\mathbf{k}\omega} = \omega^2 \varkappa_{ik}^{\alpha}\varkappa_{jl}^{\beta}\langle E_k E_l\rangle_{\mathbf{k}\omega}. \tag{4.20}$$

This connection between the correlation functions can be obtained for the partial currents and the electric field on the basis of (4.10) from which we leave out the extraneous current. This indicates that the fluctuation oscillations in the region of the maxima are characterized not only by dispersion and polarization, but also by connections between the different quantities, the same as for free waves in the plasma. For example, the magnetic field, the partial current, and the partial charge are connected with the electric field of the fluctuation wave by the relations

$$\mathbf{H} = \eta\,[\varkappa\mathbf{E}],\quad \mathbf{j}^\alpha = -i\omega\hat{\varkappa}^\alpha\mathbf{E},\quad \varrho^\alpha = -i\mathbf{k}\hat{\varkappa}^\alpha\mathbf{E}. \tag{4.21}$$

Therefore the correlation functions of all the quantities in the region of the collective fluctuations are expressed in terms of the correlation function of the electric field.

3. Fluctuations in an Equilibrium and Nonisothermal Plasma

Let us determine the explicit form of the correlation function of the electric field for various fluctuation waves in the plasma transparency region. Since the damping due to the ion motion is negligible compared with the damping due to electron motion for both high- and low-frequency waves (when $\omega \ll \omega_H^i$), the effective temperature will be equal to T_e. Therefore the spectral distribution for the electric-field fluctuations (4.16) will assume the simpler form

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} \equiv e_i^* e_j \langle E^2\rangle_{\mathbf{k}\omega} = 8\pi^2 e_i^* e_j T_e\left|\frac{\mathrm{Sp}\,\lambda}{\omega}\right|\delta\{\Lambda(\omega, \mathbf{k})\}. \tag{4.22}$$

Using in the high-frequency region the relation (3.14), we obtain the spectral distributions for the fluctuations connected with the ordinary and extraordinary waves in the form

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 8\pi^2\frac{e_i^* e_j}{|\mathbf{e}|^2 - |\varkappa\mathbf{e}|^2}\frac{T_e}{|\omega|}\,\delta(\eta^2 - \eta^2_{0,\,e}), \tag{4.23}$$

where the polarization vectors are determined by the expression (2.6). The spectral distribution for Langmuir fluctuations are determined by the expression

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 4\pi^2\varkappa_i\varkappa_j T_e\frac{|\omega^2 - \omega_H^2|}{\omega_+^2 - \omega_-^2}\{\delta(\omega - \omega_+) + \delta(\omega + \omega_+) + \delta(\omega - \omega_-) + \delta(\omega + \omega_-)\} \tag{4.24}$$

or else

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 8\pi^2\varkappa_i\varkappa_j\frac{T_e}{|\omega|}\frac{\eta_L^2}{A_0}\,\delta(\eta^2 - \eta_L^2), \tag{4.25}$$

if account is taken of the dispersion of the Langmuir waves.

It is easy to obtain similarly the correlation functions for the electric field in the region of low-frequency fluctuations. Thus, the spectral distribution for Alfven fluctuations is of the form

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 4\pi^2 e_i^* e_j T_e\frac{v_A^2}{c^2}\{\delta(\omega - kv_A\cos\vartheta) + \delta(\omega + kv_A\cos\vartheta)\}, \tag{4.26}$$

where the vector $\mathbf{e}_A$ is determined by (2.25). The magnetic-sound fluctuations of the electric field are characterized by the spectral distributions

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 4\pi^2 e_i^* e_j T_e\frac{v_A^2}{c^2}\{\delta(\omega - kv_A) + \delta(\omega + kv_A)\}, \tag{4.27}$$

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 4\pi^2 e_i^* e_j T_e a^2 k^2 \{\delta(\omega - kv_s\cos\vartheta) + \delta(\omega + kv_s\cos\vartheta)\}, \tag{4.28}$$

where the vectors $\mathbf{e}_M$ and $\mathbf{e}_S$ are respectively equal to (2.29) and (2.33). We note that the spectral distributions for the low-frequency fluctuations are proportional to the square of the ratio of the phase velocity of the corresponding wave to the velocity of light in vacuum.

4. Fluctuations in a Nonequilibrium Plasma

By way of an example of a nonequilibrium system, let us consider a plasma through which passes a compensated beam of charged particles, with a velocity u directed along the magnetic field. If the particle distributions in the stationary plasma and in the beam are Maxwellian with temperatures T and T′, then the components of the dielectric tensor are

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \delta_{ij} - \sum \frac{\Omega^2}{\omega^2} \times \left\{ e^{-\beta} \sum_n \frac{z_0}{z_n} \pi_{ij}(z_n)\left[\varphi(z_n) - i\sqrt{\pi} z_n e^{-z_n^2}\right] - 2z_0^2 h_i h_j \right\} - \sum \frac{\Omega'^2}{\omega^2} \left\{ e^{-\beta'} \sum_n \frac{y_0}{y_n} \pi_{ij}(z_n') \times \left[\varphi(y_n) - i\sqrt{\pi} y_n e^{-y_n^2}\right] - 2z_0'^2 h_i h_j \right\}, \tag{4.29}$$

where

$$y_n = \sqrt{\frac{3}{2}} \frac{\omega - n\omega_H - k_\parallel u}{|k_\parallel| s'}.$$

The primes denote quantities pertaining to the beam.

The spectral distribution of the fluctuations of the free-particle current is determined by the expression

$$\langle j_i j_j\rangle^0_{\mathbf{k}\omega} = \sqrt{\frac{2\pi}{3}} \frac{e^2}{|k_\parallel|} \left\{ \sum n_0 s e^{-\beta} \sum_n \pi_{ji}(z_n) e^{-z_n^2} + \sum n_0' s' e^{-\beta'} \sum_n \pi_{ji}(z_n') e^{-y_n^2} \right\}. \tag{4.30}$$

Using (4.29) and (4.30) we can find, in accord with (4.16), the spectral distribution of the field fluctuations in the plasma+beam system. We confine ourselves to a beam of low density ($n_0' \ll n_0$). In this case the influence of the beam on the dispersion of the waves in the plasma can be neglected. The beam will, however, greatly affect the effective temperature of the fluctuation oscillations.

In the high-frequency region, when considering wave dispersion, we can neglect also the thermal motion of the particles. The spectral distribution of the field fluctuations is determined in this case by the formula

$$\langle E_i E_j\rangle_{\mathbf{k}\omega} = 8\pi^2 \frac{\tilde{T}}{|\omega|} \left\{ \frac{e_i^* e_j}{|\mathbf{e}|^2 - |\varkappa\mathbf{e}|^2} \left[\delta(\eta^2 - \eta_0^2) + \delta(\eta^2 - \eta_e^2)\right] + \varkappa_i \varkappa_j \delta(A) \right\}. \tag{4.31}$$

Since there is no wave damping in a cold plasma it is necessary, in calculating the effective temperature that enters in (4.31), to take into account both the thermal motion of the electrons in the plasma and the presence of the beam. (The thermal motion of the ions in either the beam or in the stationary plasma can be neglected.)

Using (4.29) and (4.30), we can represent the effective temperature for the high-frequency fluctuation oscillations in the form [26]

$$\tilde{T} = T \frac{R(\omega, \mathbf{k})}{\left|1 - \frac{u}{\tilde{u}}\cos\vartheta\right|}, \tag{4.32}$$

where

$$R(\omega, \mathbf{k}) = \frac{1 + \frac{n_0'}{n_0}\left(\frac{T}{T'}\right)^{\frac{1}{2}} K}{1 + \frac{n_0'}{n_0}\left(\frac{T}{T'}\right)^{\frac{3}{2}} K}, \quad \tilde{u} = \frac{\omega}{k}\left[1 + \frac{n_0}{n_0'}\left(\frac{T'}{T}\right)^{\frac{3}{2}} K^{-1}\right]. \tag{4.33}$$

In the case of Langmuir fluctuation oscillations the value of K is given by

$$K = e^{\beta - \beta'} \frac{\sum_n I_n(\beta') e^{-y_n^2}}{\sum_n I_n(\beta) e^{-z_n^2}}. \tag{4.34}$$

For ordinary and extraordinary electromagnetic fluctuation waves we have

$$K = e^{\beta-\beta'} \times \frac{\sum_n \left\{ \left[\varrho'^2 + (n^2 + \beta'^2)\frac{\varepsilon_2^2}{(\eta^2 - \varepsilon_1)^2} \right] I_n(\beta') - 2\beta'\varrho' \frac{\varepsilon_2}{\eta^2 - \varepsilon_1} I_n'(\beta') \right\} e^{-y_n^2}}{\sum_n \left\{ \left[\varrho^2 + (n^2 + \beta^2)\frac{\varepsilon_2^2}{(\eta^2 - \varepsilon_1)^2} \right] I_n(\beta) - 2\beta\varrho \frac{\varepsilon_2}{\eta^2 - \varepsilon_1} I_n'(\beta) \right\} e^{-z_n^2}},$$

$$\varrho = n + \beta\frac{\varepsilon_2}{\eta^2 - \varepsilon_1} + \sqrt{2\beta}\frac{k_\parallel}{|k_\parallel|}\frac{\eta^2\sin\vartheta\cos\vartheta}{\eta^2\sin^2\vartheta - \varepsilon_3} z_n. \tag{4.35}$$

The quantity $\tilde{u}$ plays the role of a critical velocity, which when reached causes the fluctuations in the plasma to increase without limit, so that the plasma becomes unstable. In the limiting case when $\eta^2 \gg 1$ expression (4.35) coincides with (4.34). We note also that when $\eta^2 \gg 1$ the spectral distribution for the fluctuations of the ordinary and extraordinary waves coincide with the distribution for the Langmuir fluctuation oscillations.

The effective temperature for low-frequency fluctuation oscillations in a magnetoactive plasma pierced by a beam of charged particles is determined by the formula

$$\tilde{T} = \frac{T}{\left|1 - \frac{u}{\tilde{u}}\cos\vartheta\right|}, \tag{4.36}$$

in which the critical velocities for the Alfven and fast and slow magnetic-sound waves are respectively

$$\left.\begin{aligned} \tilde{u}_A &= \frac{\omega}{k}\left\{1+\frac{n_0}{n_0'}\sqrt{\frac{T}{T'}}\frac{\sin^4\vartheta+\cos^4\vartheta}{\cos^4\vartheta+\left(\frac{T}{T'}-\cos^2\vartheta\right)^2}e^{v_0^2}\right\},\\ \tilde{u}_M &= \frac{\omega}{k}\left\{1+\frac{n_0}{n_0'}\sqrt{\frac{T}{T'}}\frac{e^{v_0^2}}{1+\left(1-\frac{T}{T'}\right)^2}\right\},\\ \tilde{u}_s &= \frac{\omega}{k}\left\{1+\frac{n_0}{n_0'}\left(\frac{T'}{T}\right)^{3/2}e^{v_0^2}\right\}. \end{aligned}\right\} \quad (4.37)$$

The spectral distributions of the low-frequency fluctuations in a nonequilibrium plasma will be determined by formulas (4.26)—(4.28), in which T_e must be replaced by the effective temperature (4.36).

5. SCATTERING AND TRANSFORMATION OF ELECTROMAGNETIC WAVES IN A MAGNETOACTIVE PLASMA

1. Field of Scattered Waves. Scattering and Transformation Cross Sections.

Electromagnetic waves propagating in a plasma may become scattered by thermal fluctuations. Since the spectrum of the fluctuations is characterized, besides the main maximum at low frequencies, also by maxima at frequencies corresponding to the natural oscillations of the plasma, combination (Raman) scattering, accompanied by a change in the scattered-wave frequency by an amount equal to the natural frequency of the plasma oscillations, is possible in addition to the incoherent scattering of the electromagnetic waves in which the frequency change is small[27,22]. The interaction of the propagating waves with the fluctuation oscillations can lead also to mutual transformation of the waves. The intensities of the Raman scattering and of the wave scattering are determined by the magnitude of the fluctuations. Under nonequilibrium conditions, these intensities can increase anomalously if the plasma is situated near the region of kinetic instability.

The electromagnetic field in the plasma is determined by the system of equations (1.1)—(1.2). If we neglect the nonlinear terms in the kinetic equation and assume that there are no external currents ($j_0 = 0$), then the system (1.1)—(1.2) reduces to the following linear equation for the electric field:

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{\hat{\varepsilon}}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=0. \quad (5.1)$$

The solutions of (5.1) satisfy the superposition principle and describe different oscillations that propagate independently in the plasma.

Actually, owing to the nonlinearity of the system (1.1)—(1.2), different oscillations do not propagate in the plasma independently, but interact with one another. The interaction between the oscillations leads to different processes of scattering and transformation of the waves in the plasma[28-33].

Let us denote the field of the incident wave by $\mathbf{E}_0(\mathbf{r}, t)$. We assume that this field satisfies Eq. (5.1). Owing to the interaction between the incident wave and the fluctuation field, scattered waves are produced. The summary electric field is represented in the form

$$\mathbf{E}_{\text{sum}}=\mathbf{E}_0+\delta\mathbf{E}+\mathbf{E},$$

where $\delta\mathbf{E}$ is the fluctuation field and $\mathbf{E}$ is the field of the scattered waves. Taking into account the nonlinear terms in the kinetic equation (1.1), we obtain for the field of the scattered waves the equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{\hat{\varepsilon}}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=-\frac{4\pi}{c^2}\frac{\partial\mathbf{J}}{\partial t}, \quad (5.2)$$

where $\mathbf{J}$ is the current due to the field of the incident wave and the fluctuations in the plasma. For an incident plane monochromatic wave

$$\mathbf{E}_0(\mathbf{r}, t)=\mathbf{E}^0 e^{i\mathbf{k}_0\mathbf{r}-i\omega_0 t},$$

the Fourier components of the current $\mathbf{J}$ are

$$\mathbf{J}_{\mathbf{k}\omega}=-\sum\frac{e^2}{m\omega_H}\int\left\{\left[\left(\mathbf{E}^0+\frac{1}{c}[\mathbf{v}\mathbf{H}^0]\right)\frac{\partial\delta f_{\mathbf{q}\Delta\omega}}{\partial\mathbf{v}}\right.\right.$$
$$\left.+\left(\delta\mathbf{E}_{\mathbf{q}\Delta\omega}+\frac{1}{c}[\mathbf{v}\delta\mathbf{H}_{\mathbf{q}\Delta\omega}]\right)\frac{\partial f^0}{\partial\mathbf{v}}\right]$$
$$\left.\times e^{-\frac{i}{\omega_H}\int_0^{\varphi}(\mathbf{k}\mathbf{v}-\omega)\,d\varphi}\int^{\varphi}\mathbf{v}e^{\frac{i}{\omega_H}\int_0^{\varphi}(\mathbf{k}\mathbf{v}-\omega)d\varphi}\,d\varphi\right\}d\mathbf{v},$$

$$\mathbf{q}=\mathbf{k}-\mathbf{k}_0,\ \Delta\omega=\omega-\omega_0. \quad (5.3)$$

Taking into account the large difference between the masses of the electrons and the ions, we can confine ourselves in (5.3) to allowance for the electronic component only.

Equation (5.2) describes all the processes of scattering and transformation of waves in a magnetoactive plasma. Using the results of Sec. 3, we can easily find the average increase in the energy of the scattered-wave field per unit time. Noting that

$$\langle\mathbf{J}^*_{\mathbf{k}\omega}\mathbf{J}_{\mathbf{k}\omega}\rangle=TV\langle\mathbf{J}\mathbf{J}\rangle_{\mathbf{k}\omega}, \quad (5.4)$$

where the angle brackets denote statistical averaging, V the volume of the plasma, and T the interaction time, we can represent the average radiation intensity, in accord with (3.11), in the form

$$I=\frac{V}{8\pi^2}\int\left|\frac{\operatorname{Sp}\lambda}{\omega}\right|\langle|\mathbf{e}\mathbf{J}^*|^2\rangle_{\mathbf{k}\omega}\,\delta\{\Lambda(\omega,\mathbf{k})\}\,d\omega\,d\mathbf{k}. \quad (5.5)$$

Dividing the radiation intensity I by the energy flux density in the propagation direction of the incident wave S_0 and by the value of the scattering volume V, we can obtain the cross section (coefficient) for the scattering or transformation of the waves:

$$\sum=\frac{I}{VS_0}. \quad (5.6)$$

We note that the cross section can be defined differently, taking S_0 in (5.6) to mean the total energy flux. The results will differ only by a normalization factor.

2. Excitation of High-frequency Waves

If high-frequency waves are produced as a result of the scattering or the transformation, then the radiation intensity is equal to

$$I=\frac{V}{8\pi^2}\int\frac{1}{|\omega|}\langle|\mathbf{e}\mathbf{J}^*|^2\rangle_{\mathbf{k}\omega}\left\{\frac{1}{|\mathbf{e}|^2-|\varkappa\mathbf{e}|^2}[\delta(\eta^2-\eta_0^2)+\delta(\eta^2-\eta_e^2)]+\delta(A)\right\}d\omega\,dk. \qquad (5.7)$$

The first term in the curly brackets of (5.7) describes the increase in the energy of the high-frequency electromagnetic waves in the plasma, and the second the increase in the energy of the scattered Langmuir waves. From the energy and momentum conservation laws

$$\omega=\omega_0+\Delta\omega,\quad \mathbf{k}=\mathbf{k}_0+\mathbf{q} \qquad (5.8)$$

it follows that the high-frequency waves can be excited by either high-frequency or low-frequency incident waves. On the other hand, if the incident wave is of low frequency, it can be transformed into a high-frequency one only by interacting with high-frequency fluctuations.

Since the phase velocities in high-frequency waves in a plasma are much higher than the thermal velocity of the electrons, the expression for the current $\mathbf{J}_{\mathbf{k}\omega}$ can be simplified by expanding the integrand in (5.3) in powers of $\mathbf{k}\cdot\mathbf{v}/\omega$. As a result we get

$$J_i(\mathbf{k},\omega)=i\omega\varkappa_{ij}\left\{\delta_{jk}\frac{\delta n(\mathbf{q},\Delta\omega)}{n_0}+\frac{1}{\omega_0}\left[k_j^0\delta_{kl}-\delta_{jk}k_l^0-4\pi\frac{\omega_0^2}{\Omega^2}(\varkappa_{jk}^0k_l+\delta_{jl}k_m\varkappa_{mk}^0)\right]\delta v_l(\mathbf{q},\Delta\omega)-\frac{i}{en_0}\left[k_l^0\varkappa_{lk}^0\delta E_j(\mathbf{q},\Delta\omega)+\frac{\omega_0}{c}\varepsilon_{jlm}\varkappa_{lk}^0\delta H_m(\mathbf{q},\Delta\omega)\right]\right\}E_k^{01}, \qquad (5.9)$$

where δn and $\delta\mathbf{v}$ are the fluctuations of the density and of the macroscopic velocity of the electrons, and $\delta\mathbf{E}$ and $\delta\mathbf{H}$ are the fluctuations of the electric and magnetic fields in the plasma. Expression (5.9) also follows directly from hydrodynamic considerations.

It is possible to investigate with the aid of (5.9) the following processes of scattering and transformation of high-frequency waves in a magnetoactive plasma: scattering of electromagnetic (ordinary and extraordinary) waves, transformation of electromagnetic waves into Langmuir waves, scattering of Langmuir waves, and transformation of Langmuir waves into electromagnetic waves. In addition, it is possible to investigate on the basis of (5.9) the transformation of low-frequency waves into high-frequency ones.

3. Scattering and Transformation of Electromagnetic Waves by Incoherent Fluctuations.

The scattering of electromagnetic waves with small change of frequency ($\Delta\omega \ll \omega_0$) in a plasma in the presence of a magnetic field, just as without the field, is produced primarily by fluctuations of the electron density. Neglecting scattering by the fluctuations of the velocity and of the electric and magnetic fields, we can write for the differential scattering cross section

$$d\Sigma_{E\to E}=\frac{1}{2\pi}\left(\frac{e^2}{mc^2}\right)^2\frac{\omega_0^2\omega^2}{\Omega^4}N\langle\delta n^2\rangle_{\mathbf{q}\Delta\omega}\,d\omega\,do, \qquad (5.10)$$

where

$$N=\eta^3|\mathbf{e}^*(\varepsilon_0-1)\mathbf{e}_0|^2/\eta_0(|\mathbf{e}_0|^2-|\varkappa_0\mathbf{e}_0|^2)(\mathbf{e}^*\hat{\varepsilon}\mathbf{e}),$$

$\mathbf{e}_0$ and $\mathbf{e}$ are the polarization vectors of the incident and scattered waves. The factor N depends on the directions of propagation of the incident and scattered waves relative to the magnetic field (angles ϑ_0 and ϑ), and also on the difference of the azimuthal angles of the wave vectors $\mathbf{k}_0$ and $\mathbf{k}$ (angle φ).

It is easy to trace the transformation of formula (5.10) into the corresponding expression for the differential cross section for scattering in an isotropic plasma[22]. Indeed, when $H_0 = 0$ we have

$$\varepsilon_{ik}^0-\delta_{ik}=-\frac{\Omega^2}{\omega_0^2}\delta_{ik},\quad \varkappa_0\mathbf{e}_0=\varkappa\mathbf{e}=0,\quad \eta_0=\sqrt{\varepsilon(\omega_0)}.$$

Consequently $N = (\Omega^4/\omega_0^4)e_{\perp 0}^2$ ($e_{0\perp}$ is the component of the polarization vector $\mathbf{e}_0$ perpendicular to k) and expression (5.10) goes over, after averaging over the different orientations of the vector $\mathbf{e}_0$, into formula (10.20) of [5].

The spectral distribution for the electron density fluctuations in the general case of a non-isothermal magnetoactive plasma is determined by the expression

$$e^2\langle\delta n^2\rangle_{\mathbf{q}\Delta\omega}=\frac{2}{\Delta\omega}\operatorname{Im}\{T_e(q_m-4\pi q_l\varkappa_{ik}^e\Lambda_{km}^{-1})^*\times(q_n-4\pi q_j\varkappa_{jl}^e\Lambda_{ln}^{-1})\varkappa_{mn}^{e*}+16\pi^2T_i(q_i\varkappa_{ik}^e\Lambda_{km}^{-1})^*q_j\varkappa_{jl}^e\Lambda_{ln}^{-1}\varkappa_{mn}^{i*}\}. \qquad (5.11)$$

If $\Delta\omega^2 \ll q^2c^2$, then the correlation function (5.11) greatly simplifies and takes the same form as for a free plasma

$$e^2\langle\delta n^2\rangle_{\mathbf{q}\Delta\omega}=\frac{2q^2}{\Delta\omega|\varepsilon|^2}\{T_e|1+4\pi\varkappa^i|^2\operatorname{Im}\varkappa^e+16\pi^2T_i|\varkappa^e|^2\operatorname{Im}\varkappa^i\}, \qquad (5.12)$$

where κ and ϵ must be taken to mean the longitudinal components of the corresponding tensors.

If the change of the wave vector q is parallel to $\mathbf{H}_0$ during scattering, then the spectrum of the scattered radiation has the same characteristic frequencies as in an isotropic plasma. The scattering intensity depends essentially on the magnitude of the magnetic field. If the direction of $\mathbf{q}$ does not coincide with that of H_0, then the magnetic field influences also the spectrum of the scattered radiation.

In the case of an isothermal plasma, the spectrum of the scattered radiation, for angles other than $\pi/2$ between $\mathbf{q}$ and $\mathbf{H}_0$, is characterized by a sharp minimum at $\Delta\omega = 0$, just as in the absence of a magnetic field. This maximum is due to the interaction between the incident wave and the incoherent fluctuations of the electron density in the plasma. The interaction between the incident wave and the fluctuations of the electron velocity and of the electric and magnetic fields can be neglected at small frequency deviations. Although the scattering is by the electron-density fluctuations, the Doppler broadening of the main maximum is determined by the thermal velocity of the ions, since the electrons and ions interact via the self-consistent field.

When $T_e = T_i$ and $\omega_0 \gg \omega_\pm$ ($\omega_\pm$ are the frequencies of the Langmuir oscillations of the plasma in the magnetic field) we can find the integral scattering coefficient of the electromagnetic waves in the plasma by using the dispersion relation for (5.11). The integral scattering coefficient turns out to be

$$d\Sigma = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^4}{\Omega^4} N_{\omega=\omega_0} \frac{1+a^2q^2}{2+a^2q^2}\, do. \tag{5.13}$$

When $a^2q^2 \gg 1$, formula (5.13) can be regarded as a generalization of the well-known Rayleigh formula to include the case of a magnetoactive medium.

In a nonisothermal plasma, the maximum in the spectrum of the scattered radiation, due to interaction with incoherent fluctuations, is greatly reduced. In a strongly nonisothermal plasma ($T_e \gg T_i$) the height of the maximum is $(M/m)^{1/2}$ times smaller than in the isothermal case.

Owing to the interaction between the electromagnetic waves and the density fluctuations in the plasma, these waves can also be transformed into Langmuir waves. According to (5.7) the differential cross section for the transformation of the ordinary or extraordinary wave into a Langmuir wave is, when $\Delta\omega \ll \omega_0$,

$$d\Sigma_{E\to L} = \frac{1}{2\pi}\left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^2\omega^2}{\Omega^4} \frac{\eta_L^3\,|\varkappa(\hat{\varepsilon}_0 - 1)\,\mathbf{e}_0|^2}{\eta_0\,(|\mathbf{e}_0|^2 - |\varkappa_0\mathbf{e}_0|^2)\,(\varkappa\hat{\varepsilon}\varkappa)} \langle\delta n^2\rangle_{\mathbf{q}\Delta\omega}\, d\omega\, do. \tag{5.14}$$

Just as in scattering, the principal role in the transformation of electromagnetic waves with a small change in frequency is played by the interaction with the incoherent fluctuations. As $H_0 \to 0$ formula (5.14) goes over into expression (10.40) of [5].

The ratio of the transformation coefficient (5.14) to the scattering coefficient (5.10) has an order of magnitude c^3/s^3. Therefore in the region of frequencies ω_0 close to $\omega_+(\vartheta_0)$ and $\omega_-(\vartheta_0)$ the absorption connected with the transformation of electromagnetic waves into Langmuir waves is more important than the scattering of the electromagnetic waves.

We note that formula (5.14) can be derived directly from (5.10) by taking the scattered wave in the latter to mean a Langmuir wave with $\eta = \eta_L$ and $\mathbf{e} = \boldsymbol{\kappa}$.

4. Scattering and Transformation of Waves by Coherent Fluctuations.

In addition to the main maximum at $\Delta\omega = 0$, the scattered-radiation spectrum contains also maxima connected with the scattering and transformation of electromagnetic waves by coherent (collective) fluctuations in the plasma.

The current that induces the scattered waves (5.9) can be expressed in the case of coherent fluctuations by using (4.21), in terms of the fluctuations of the electric field only. Thus, the current correlator, which enters in the general formula for the intensity (5.7) of the radiated waves, can be expressed in the form

$$\langle|\mathbf{eJ}^*|^2\rangle_{\mathbf{k}\omega} = \frac{\omega^2\Delta\omega^2}{e^2n_0^2c^2}|\mathbf{B}|^2\langle|\delta\mathbf{E}|^2\rangle_{\mathbf{q}\Delta\omega}|\mathbf{E}_0|^2, \tag{5.15}$$

where

$$\begin{aligned} B = e_i^*\varkappa_{ij}\Big\{&\tilde{\eta}e_j^0\tilde{\varkappa}_k\tilde{\varkappa}_{kl}\tilde{e}_l + \eta_0(\varkappa_j^0e_k^0 - e_j^0\varkappa_k^0)\tilde{\varkappa}_{kl}\tilde{e}_l \\ &+ \frac{\omega_0}{\Delta\omega}[\eta^0\tilde{e}_j\varkappa_k^0\varkappa_{kl}^0e_l^0 + \tilde{\eta}(\tilde{\varkappa}_j\tilde{e}_k - \tilde{e}_j\tilde{\varkappa}_k)\varkappa_{kl}^0e_l^0] \\ &- 4\pi\eta\frac{\omega_0\omega}{\Omega^2}[\varkappa_{jk}^0e_k^0\varkappa_l\tilde{\varkappa}_{lm}\tilde{e}_m + \tilde{\varkappa}_{jk}\tilde{e}_k\varkappa_l\varkappa_{lm}^0e_m^0]\Big\}. \end{aligned} \tag{5.16}$$

Here $\mathbf{e}^0$, $\boldsymbol{\kappa}^0$, and η^0 are the polarization vector, the unit vector in the direction of propagation, and the refractive index of the incident wave; $\tilde{\mathbf{e}}$, $\tilde{\boldsymbol{\kappa}}$, $\tilde{\eta}$ and $\mathbf{e}$, $\boldsymbol{\kappa}$, η are the corresponding quantities for the fluctuation and scattered waves. We recall that the frequencies and the wave vectors of the incident, scattered, and fluctuation waves are related by

$$\omega = \omega_0 + \Delta\omega, \quad \mathbf{k} = \mathbf{k}_0 + \mathbf{q}, \tag{5.8}$$

which represent the energy and momentum conservation laws. Knowing the spectral distributions of the coherent fluctuations, and also the properties of the incident and excited waves, it is easy to separate in (5.15) the main terms responsible for transitions of a definite type.

5. Scattering and Transformation of Electromagnetic Waves by Langmuir Fluctuations

Let us consider first scattering and transformation of electromagnetic waves by high-frequency Langmuir fluctuations. We assume that the refractive index of the fluctuation Langmuir oscillations is $\tilde{\eta} \gg 1$, whereas the refractive indices η_0 and η of the incident and scattered waves are of the order of unity. From the conservation laws (5.8) it follows that $\omega \cong \omega_0 \gg \Delta\omega$. It can be readily verified here that the main role is played by the interaction of the incident wave with the density fluctuations of the electrons (first term in (5.16)). Therefore the electromagnetic-wave scattering cross section will be determined by formula (5.10). The spectral distribution (5.12) in the case $a^2q^2 \ll 1$ has delta-like maxima at frequencies $\omega_+(\tilde{\vartheta})$ and $\omega_-(\tilde{\vartheta})$, where $\tilde{\vartheta}$ is the angle between $\mathbf{q}$ and $\mathbf{H}_0$ and is connected with the angles ϑ_0, ϑ, and φ by the relation

$$\operatorname{tg}^2\tilde{\vartheta} = \frac{k_0^2 \sin^2\vartheta_0 + k^2\sin^2\vartheta + 2k_0 k \sin\vartheta_0 \sin\vartheta \cos\varphi}{(k_0\cos\vartheta_0 - k\cos\vartheta)^2}. \qquad (5.17)$$

The cross section for the scattering of the electromagnetic waves, at frequency deviations close to the frequencies $\omega_\pm(\tilde{\vartheta})$ of the Langmuir oscillations, is

$$d\Sigma_{E\to\tilde{L}\to E} = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^2\omega^2}{\Omega^4} a^2q^2 N \frac{|\Delta\omega^2 - \omega_H^2|}{\tilde{\omega}_+^2 - \tilde{\omega}_-^2}$$
$$\times \{\delta(\Delta\omega - \tilde{\omega}_+) + \delta(\Delta\omega + \tilde{\omega}_+) + \delta(\Delta\omega - \tilde{\omega}_-)$$
$$+ \delta(\Delta\omega + \tilde{\omega}_-)\}\, d\omega\, do. \qquad (5.18)$$

The cross section for the transformation of electromagnetic waves by Langmuir fluctuations into Langmuir waves is

$$d\Sigma_{E\to\tilde{L}\to L} = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^2\omega^2}{\Omega^4} a^2q^2 \frac{\eta_L^3 |\varkappa\hat{Q}^L \mathbf{e}_0|^2}{\eta_0 (|\mathbf{e}_0|^2 - |\varkappa\mathbf{e}_0|^2)(\varkappa\hat{\varepsilon}\varkappa)}$$
$$\times \frac{|\Delta\omega^2 - \omega_H^2|}{\tilde{\omega}_+^2 - \tilde{\omega}_-^2} \{\delta(\Delta\omega - \tilde{\omega}_+) + \delta(\Delta\omega + \tilde{\omega}_+) +$$
$$+ \delta(\Delta\omega - \tilde{\omega}_-) + \delta(\Delta\omega + \tilde{\omega}_-)\}\, d\omega\, do, \qquad (5.19)$$

where

$$Q_{ij}^L = (\varepsilon_{ik} - \delta_{ik}) \Big\{ \delta_{kj} + \frac{\omega_0\Delta\omega}{\Omega^2} [(\varepsilon_{kj}^0 - \delta_{kj}) \varkappa_l (\tilde{\varepsilon}_{lm} - \delta_{lm}) \tilde{\varkappa}_m$$
$$+ (\tilde{\varepsilon}_{kl} - \delta_{kl}) \tilde{\varkappa}_l \varkappa_m (\varepsilon_{mj}^0 - \delta_{mj})] \Big\}.$$

In the case of a nonequilibrium plasma (for example, a plasma pierced by a beam of charged particles) it is necessary to take into account in the cross sections (5.18) and (5.19) and additional factor $R(1 - (u/\tilde{u}) \cos \tilde{\vartheta})^{-1}$, due to the replacement of the temperature T by the effective temperature (4.32).

The relative contribution of Raman scattering by Langmuir fluctuations (5.18) to the integral scattering cross section (5.13) in an equilibrium plasma is a quantity of the order of a^2q^2. However, under nonequilibrium conditions the cross section for the transformation of electromagnetic waves into Langmuir waves may increase anomalously in the case when the plasma is near the region of kinetic instability [34-37] (see also [38-41]).

6. Scattering and Transformation of Electromagnetic Waves by Low-frequency Fluctuations.

In a magnetoactive plasma, Raman scattering of electromagnetic waves can also be produced by low-frequency magnetic-sound and Alfven fluctuations. Using the general expression (5.7) for the intensity and formulas (4.26)–(4.28) for the spectral distributions of the low-frequency fluctuations, we can easily investigate different concrete cases of scattering and transformation.

In a nonisothermal plasma, the most important are scattering and transformation of electromagnetic waves by slow magnetic-sound fluctuations. In this case the principal role is played in (5.9) by density fluctuations. The cross sections for the scattering and transformation of electromagnetic waves by slow magnetic-sound fluctuations are determined by the expression

$$d\Sigma_{E\to\tilde{S} \nearrow E \atop \searrow L} = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^4}{\Omega^4} \frac{\eta^3 |\mathbf{e}^*(\hat{\varepsilon} - 1)\mathbf{e}_0|^2}{\eta_0 (|\mathbf{e}_0|^2 - |\varkappa_0\mathbf{e}_0|^2)(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$
$$\times \{\delta(\Delta\omega - qv_s\cos\tilde{\vartheta}) + \delta(\Delta\omega + qv_s\cos\vartheta)\}\, d\omega\, do, \qquad (5.20)$$

where η and $\mathbf{e}$ stand for the refractive index and polarization for the ordinary, extraordinary, or Langmuir waves respectively. The differential scattering cross section (5.20) differs from the corresponding cross section in an isotropic plasma (see formula (10.32) of [5]) only in the form of the dispersion for nonisothermal fluctuations.

The ratio of the cross section for the scattering of electromagnetic waves in a strongly nonisothermal plasma by slow magnetic-sound fluctuations, integrated over the frequencies, to (5.13) is a quantity of the order of unity. Therefore in a strongly nonisothermal plasma the main line in the scattered-radiation spectrum splits into two lines, connected with scattering by slow magnetic-sound fluctuations.

In the case of scattering and transformation of electromagnetic waves by fast magnetic-sound fluctuations, it is necessary to take into account, besides the density fluctuations, the fluctuations of the magnetic field. The corresponding cross sections for the scattering and transformation of the electromagnetic waves are

$$d\Sigma_{E\to\tilde{M} \nearrow E \atop \searrow L} = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^4}{\Omega^4} \frac{v_s^2}{v_A^2} \frac{\eta^3 |\mathbf{e}^*\hat{Q}^M\mathbf{e}_0|^2}{\eta_0 (|\mathbf{e}_0|^2 - |\varkappa_0\mathbf{e}_0|^2)(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$
$$\times \{\delta(\Delta\omega - qv_A) + \delta(\Delta\omega + qv_A)\}\, d\omega\, do, \qquad (5.21)$$

where

$$Q_{ij}^M = (\varepsilon_{ik} - \delta_{ik}) \Big\{ -i\delta_{kj}\sin\tilde{\vartheta} + \frac{\omega_0\omega_H}{\Omega^2} (\tilde{\varkappa}_k\tilde{e}_l - \tilde{\varkappa}_l e_k)(\varepsilon_{lj}^0 - \delta_{lj}) \Big\}.$$

The ratio of the scattering cross section (5.21), integrated over the frequencies, to (5.13) is of the order of v_S^2/v_A^2.

In the case of scattering and transformation of electromagnetic waves by Alfven fluctuations, the principal role is played by fluctuations of the magnetic field, since Alfven oscillations are not accompanied by a change in density. The cross sections for the scattering and transformation of electromagnetic waves by Alfven fluctuations are

$$d\Sigma_{E\to\tilde{A}\genfrac{}{}{0pt}{}{\nearrow E}{\searrow L}} = \frac{1}{6} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^6}{\Omega^6} \frac{s^2}{c^2\cos^2\tilde{\vartheta}}$$

$$\times \frac{\eta^3 |\mathbf{e}^*\hat{Q}^A\mathbf{e}_0|^2}{\eta_0 (|\mathbf{e}_0|^2 - |\varkappa_0\mathbf{e}_0|^2)(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})} \{\delta(\Delta\omega - qv_A\cos\tilde{\vartheta})$$

$$+ \delta(\Delta\omega + qv_A\cos\tilde{\vartheta})\} \, d\omega \, do, \tag{5.22}$$

where

$$Q_{ij}^A = (\varepsilon_{ik} - \delta_{ik})(\tilde{\varkappa}_k\tilde{e}_l - \tilde{\varkappa}_l\tilde{e}_k)(\varepsilon_{lj}^0 - \delta_{lj}).$$

Formula (5.22) is valid for both the isothermal and the nonisothermal case.

The ratio of the cross section for the scattering of electromagnetic waves by Alfven fluctuations, integrated over the frequencies, to (5.13) is of the order of s^2/c^2.

The cross sections for the scattering and transformation of electromagnetic waves by low-frequency fluctuations, just as in the case of Langmuir fluctuations, can increase strongly in a nonequilibrium plasma that is near the region of kinetic instability.

6. SCATTERING AND TRANSFORMATION OF LANGMUIR WAVES IN A MAGNETOACTIVE PLASMA

1. Scattering and Transformation by Incoherent Fluctuations

The general formulas (5.7) and (5.9) make it also possible to investigate the scattering of Langmuir waves and the transformation of Langmuir waves into high-frequency electromagnetic waves. Choosing as the incident wave a Langmuir wave and using for the density of the incident energy flux expression (2.17), we can easily obtain with the aid of (5.7) concrete expressions for the cross sections of the various scattering and transformation processes.

In the case of scattering and transformation of Langmuir waves by incoherent fluctuations in an isothermal plasma, just is in the case of interaction of an incident wave with density fluctuations, the principal role is played by the interaction of the incident wave with the density fluctuations. The cross sections for the scattering and transformation of Langmuir waves by incoherent fluctuations are

$$d\Sigma_{L\genfrac{}{}{0pt}{}{\nearrow L}{\searrow E}} = \frac{1}{2\pi} \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^4}{\Omega^4} \frac{\eta^3 |\mathbf{e}^*(\hat{\varepsilon}-1)\varkappa_0|^2}{\eta_0\xi_0(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})} \langle\delta n^2\rangle_{\mathbf{q}\Delta\omega} \, d\omega \, do, \tag{6.1}$$

where η and $\mathbf{e}$ are determined by (2.12) and (2.14) in the case of scattering and by (2.5) and (2.6) in the case of transformation. When $H_0 = 0$ formula (6.1) goes over into formula (10.44) of [5].

The spectral distribution of the incoherent fluctuations is characterized by a sharp maximum in the region of small frequency shifts. The width of this maximum is determined by the thermal velocity of the ions. Formula (6.1) is valid only in the region of the incoherent maximum. The ratio of the coefficient of transformation of the Langmuir wave into a high-frequency electromagnetic wave to the coefficient of scattering of Langmuir waves with small change of frequency is s^3/c^3.

2. Scattering and Transformation by Coherent Fluctuations

The scattering and transformation of Langmuir waves by coherent fluctuations can be readily investigated on the basis of (5.7) and (5.15).

In the scattering and transformation of Langmuir waves by low-frequency fluctuation oscillations, only the fluctuations of the electron density and of the magnetic field are of importance. The cross sections for various types of scattering and transformation of Langmuir waves are similar to the corresponding cross sections for scattering and transformation of electromagnetic waves. We present the final expressions for the cross sections of different types of scattering and transformation of Langmuir waves.

The cross sections for the scattering and transformation of Langmuir waves by slow magnetic-sound fluctuations in a strongly nonisothermal plasma are

$$d\Sigma_{L\to\tilde{S}\genfrac{}{}{0pt}{}{\nearrow L}{\searrow E}} = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^4}{\Omega^4} \frac{\eta^3 |\mathbf{e}^*(\hat{\varepsilon}-1)\varkappa_0|^2}{\eta_0\xi_0(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times \{\delta(\Delta\omega - qv_s\cos\tilde{\vartheta}) + \delta(\Delta\omega + qv_s\cos\tilde{\vartheta})\} \, d\omega \, do. \tag{6.2}$$

The cross sections for the scattering and transformation of Langmuir waves by fast magnetic-sound fluctuations are

$$d\Sigma_{L\to\tilde{M}\genfrac{}{}{0pt}{}{\nearrow L}{\searrow E}} = \frac{1}{2} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^4}{\Omega^4} \frac{v_s^2}{v_A^2} \frac{\eta^3 |\mathbf{e}^*\hat{Q}^M\varkappa_0|^2}{\eta_0\xi_0(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times \{\delta(\Delta\omega - qv_A) + \delta(\Delta\omega + qv_A)\} \, d\omega \, do. \tag{6.3}$$

The cross sections for the scattering and transformation of Langmuir waves by Alfven fluctuations are equal to

$$d\Sigma_{L\to\tilde{A}\genfrac{}{}{0pt}{}{\nearrow L}{\searrow E}} = \frac{1}{6} n_0 \left(\frac{e^2}{mc^2}\right)^2 \frac{\omega_0^6}{\Omega^6} \frac{s^2}{c^2\cos^2\vartheta} \frac{\eta^3 |\mathbf{e}^*\hat{Q}^A\varkappa_0|^2}{\eta_0\xi_0(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times \{\delta(\Delta\omega - qv_A\cos\tilde{\vartheta}) + \delta(\Delta\omega + qv_A\cos\tilde{\vartheta})\} \, d\omega \, do. \tag{6.4}$$

The quantities $\hat{Q}^M$ and $\hat{Q}^A$ are defined by expressions of the same type as in the case of scattering of electromagnetic waves. We note that the ratio of the coefficient of transformation of the Langmuir wave to the coefficient of transformation of all types of low-frequency fluctuations is of the order of s^2/c^2.

In the case of interaction between a Langmuir wave and high-frequency fluctuations, it is essential to take into account the fluctuations of the electron density and of the electric field. The incident Langmuir wave can interact with either the Langmuir fluctuations or the high-frequency electromagnetic fluctuations.

The scattering of Langmuir waves by Langmuir fluctuations is impossible, by virtue of the conservation laws (5.8). The transformation of a Langmuir wave by Langmuir fluctuations into a high-frequency ordinary or extraordinary electromagnetic wave is characterized by the cross section

$$d\Sigma_{L\to\widetilde{L}\to E}=\frac{1}{2}n_0\left(\frac{e^2}{mc^2}\right)^2\frac{\omega^4}{\Omega^4}a^2q^2\frac{\eta^3|\mathbf{e}^*\hat{Q}^L\varkappa_0|^2}{\eta_0\xi_0(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times\frac{|\Delta\omega^2-\omega_H^2|}{\widetilde{\omega}_+^2-\widetilde{\omega}_-^2}\{\delta(\Delta\omega-\widetilde{\omega}_+)+\delta(\Delta\omega+\widetilde{\omega}_+)+\delta(\Delta\omega-\widetilde{\omega}_-)$$

$$+\delta(\Delta\omega+\widetilde{\omega}_-)\}\,d\omega\,do,$$

$$Q_{ij}^L=(\varepsilon_{ik}-\delta_{ik})(\delta_{kj}+\widetilde{\varkappa}_k\varkappa_j^0). \tag{6.5}$$

This cross section is c/s times larger than (5.13). The frequencies of the waves produced in this process are close to the sum of the modified Langmuir frequencies $\omega_\pm(\vartheta_0)\pm\omega_\pm(\widetilde{\vartheta})$.

The cross sections for the scattering and transformation of Langmuir waves by electromagnetic fluctuations are

$$d\Sigma_{L\to\widetilde{E}\genfrac{}{}{0pt}{}{\nearrow L}{\searrow E}}=\frac{1}{2}n_0\left(\frac{e^2}{mc^2}\right)^2\frac{\omega^4}{\Omega^4}a^2k_0^2\frac{\eta^3|\mathbf{e}\hat{Q}^E\widetilde{\mathbf{e}}|^2}{\eta_0\xi_0(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})\widetilde{\eta}^2(|\widetilde{\mathbf{e}}|^2-|\widetilde{\varkappa}\widetilde{\mathbf{e}}|^2)}$$

$$\times\left\{\delta\left(\Delta\omega-\frac{qc}{\widetilde{\eta}}\right)+\delta\left(\Delta\omega+\frac{qc}{\widetilde{\eta}}\right)\right\}d\omega\,do, \tag{6.6}$$

where

$$Q_{ij}^E=\begin{cases}(\varepsilon_{ik}-\delta_{ik})\left\{\delta_{kj}+\frac{\omega_0\Delta\omega}{\Omega^2}[(\varepsilon_{kl}^0-\delta_{kl})\varkappa_l^0\varkappa_m(\widetilde{\varepsilon}_{mj}-\delta_{mj})\right.\\ \qquad\left.+(\widetilde{\varepsilon}_{kj}-\delta_{kj})\varkappa_l(\varepsilon_{lm}^0-\delta_{lm})\varkappa_m^0]\right\},\\ (\varepsilon_{ik}-\delta_{ik})\left\{\delta_{kj}-\frac{\Delta\omega}{\omega_0}\frac{\widetilde{\eta}}{\eta_0}(\widetilde{\varepsilon}_{lj}-\delta_{lj})\widetilde{\varkappa}_l\widetilde{\varkappa}_k^0\right\}.\end{cases}$$

The cross sections for the scattering and transformation of Langmuir waves in a nonequilibrium plasma are characterized by the same anomalies in the region of critical fluctuations as the corresponding cross sections for electromagnetic waves.

7. TRANSFORMATION OF LOW FREQUENCY WAVES BY LANGMUIR FLUCTUATIONS IN A MAGNETOACTIVE PLASMA

In conclusion let us discuss also the transformation of low-frequency waves into high-frequency waves by Langmuir fluctuations in a magnetoactive plasma. In the case of an incident low-frequency wave, the most essential terms in the expression (5.9) for the current are those connected with the magnetic field and with the electron density in the incident wave. Using formula (5.7) for the intensity of radiation of high-frequency waves, and dividing it by the energy flux density connected with the incident low-frequency wave, we can easily find explicit expressions for the cross sections of the different wave-transformation processes.

The cross sections for the transformation of a slow magnetic-sound wave by Langmuir fluctuations with excitation of ordinary, extraordinary, or Langmuir waves, are equal to

$$d\Sigma_{S\to\widetilde{L}\genfrac{}{}{0pt}{}{\nearrow E}{\searrow L}}=\frac{1}{2}n_0\left(\frac{e^2}{mc^2}\right)^2\frac{\omega^4}{\Omega^4}\frac{c\eta^3|\mathbf{e}^*(\hat{\varepsilon}-1)\widetilde{\varkappa}|^2}{v_s|\cos\vartheta_0|(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times\frac{|\Delta\omega^2-\omega_H^2|}{\widetilde{\omega}_+^2-\widetilde{\omega}_-^2}\{\delta(\Delta\omega-\widetilde{\omega}_+)+\delta(\Delta\omega+\widetilde{\omega}_+)+\delta(\Delta\omega-\widetilde{\omega}_-)$$

$$+\delta(\Delta\omega+\widetilde{\omega}_-)\}\,d\omega\,do. \tag{7.1}$$

The cross sections for the transformation of a fast magnetic-sound wave by Langmuir fluctuations with excitation of ordinary, extraordinary, or Langmuir waves are

$$d\Sigma_{M\to\widetilde{L}\genfrac{}{}{0pt}{}{\nearrow E}{\searrow L}}=\frac{1}{6}n_0\left(\frac{e^2}{mc^2}\right)^2\frac{\omega_0\omega^4}{\omega_H\Omega^4}\frac{s^2}{cv_A}\frac{\eta^3|\mathbf{e}^*\hat{P}^L\mathbf{e}_0|^2}{(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times\frac{|\Delta\omega^2-\omega_H^2|}{\widetilde{\omega}_+^2-\widetilde{\omega}_-^2}\{\delta(\Delta\omega+\widetilde{\omega}_+)+\delta(\Delta\omega+\widetilde{\omega}_+)+\delta(\Delta\omega-\widetilde{\omega}_-)$$

$$+\delta(\Delta\omega+\widetilde{\omega}_-)\}\,d\omega\,do,$$

$$P_{ij}^L=(\varepsilon_{ik}-\delta_{ik})\left\{\widetilde{\varkappa}_k\varkappa_l^0\varkappa_{lj}^{e0}+\frac{\Delta\omega}{\omega_0}(\varkappa_k^0\delta_{lj}-\varkappa_l^0\delta_{kj})(\widetilde{\varepsilon}_{lm}-\delta_{lm})\widetilde{\varkappa}_m\right\}. \tag{7.2}$$

The cross sections for the transformation of an Alfven wave by Langmuir fluctuations with excitation of ordinary, extraordinary, or Langmuir waves are

$$d\Sigma_{A\to\widetilde{L}\genfrac{}{}{0pt}{}{\nearrow E}{\searrow L}}=\frac{1}{6}n_0\left(\frac{e^2}{mc^2}\right)^2\frac{\omega^4\Delta\omega^2}{\Omega^6}\frac{s^2}{cv_A}\frac{\eta^3|\mathbf{e}^*\hat{R}^L\mathbf{e}_0|^2}{|\cos\vartheta_0|^3(\mathbf{e}^*\hat{\varepsilon}\mathbf{e})}$$

$$\times\frac{|\Delta\omega^2-\omega_H^2|}{\widetilde{\omega}_+^2-\widetilde{\omega}_-^2}\{\delta(\Delta\omega-\widetilde{\omega}_+)+\delta(\Delta\omega+\widetilde{\omega}_+)+\delta(\Delta\omega-\widetilde{\omega}_-)$$

$$+\delta(\Delta\omega+\widetilde{\omega}_-)\}\,d\omega\,do,$$

$$R_{ij}^L=(\varepsilon_{ik}-\delta_{ik})(\varkappa_k^0\delta_{lj}-\varkappa_l^0\delta_{kj})(\widetilde{\varepsilon}_{lm}-\delta_{lm})\widetilde{\varkappa}_m. \tag{7.3}$$

We note that the process with the largest cross section is the transformation of a slow magnetic-sound wave into a Langmuir wave. The ratio of the cross section of this process, integrated over the frequencies, to (5.13) is of the order of c^4/s^4.

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