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VERY-LOW-FREQUENCY RADIO EMISSION OF THE UPPER ATMOSPHERE AND ITS RELATIONSHIP WITH OTHER GEOPHYSICAL PHENOMENA

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1. INTRODUCTION

RECENTLY, the scale of investigations of the upper atmosphere of the Earth has been extended considerably. In particular, there has been much interest in the problems associated with the structure of the exosphere and magnetosphere and with the nature of the processes taking place in these regions. In spite of the great importance of the direct experiments carried out in space vehicles, the role of the indirect methods of investigating the plasma at large distances from the Earth is still considerable. Among such indirect methods are, for example, observations of the short-period pulsations of the geomagnetic field.[1] The recordings of such perturbations are currently supplemented by analyses of the conditions of generation[2] and calculations of the effectiveness of the transport of these perturbations from the excitation region to the surface of the Earth.[3]

Another, no less important, source of information on the state of the exosphere is the low-frequency range of radio signals, whose spectra have frequencies f ranging from about 1 to 20 kc. In this range of frequencies, the radiation may be excited by atmospheric discharges near the surface of the Earth. After passing through the ionospheric and exospheric plasma,* the radiation is usually received in the form of sound of varying frequency. Such signals are known as whistling atmospherics (whistlers). In this range of frequencies, radio waves may also be generated directly in the exosphere. This form of radio radiation is known as the very-low-frequency (VLF) radiation.

The problems of whistlers, VLF radiation and the concomitant phenomena have been considered in a number of reviews,[4-8] one of which was published in Uspekhi Fizicheskikh Nauk in 1960.[6] However, new facts have been established in recent years and valuable information has been obtained about the exo spheric plasma. Consequently, various aspects of the relationship between whistlers, VLF radiation, and other geophysical phenomena have become clear.

In 1964, R. A. Helliwell published in the U.S.A. the first monograph on the subjects of interest to us here —"Whistlers and Related Ionospheric Phenomena."[9] He presented full details of all the experimental and theoretical results on whistling atmospherics. However, Helliwell gave little prominence to the VLF radiation and did not deal at all with many important problems. Consequently, it seemed desirable to concentrate the present review on the problems associated with the VLF radiation.

Before considering the main subject of the review, it will be useful to present, even in very concise form, the most important results of recent years obtained in the treatment and analysis of the observational data on whistlers. Full details can be found in Helliwell's monograph.[9] The analysis of many observations has led to the conclusion that the possibility of the transmission of whistlers or the VLF radiation through the upper atmosphere is possibly due to the formation in the exosphere of characteristic plasma waveguides (channels) oriented along the direction of the lines of force of the geomagnetic field H.[10,11] The propagation along such channels has been observed also by sounding the ionosphere with high-frequency pulses.[12-14] Next, an analysis of a number of whistler spectrograms has shown that, during certain periods, the electron concentration N decreases at distances $(3-4)R_0$ from the center of the Earth (R_0 is the radius of the Earth).[15] This conclusion is in agreement with the results of some direct measurements of N.[16] The recordings of whistlers above the ionosphere, obtained by means of artificial earth satellites (AES), have been of great interest.[17,18] The latest results include the detection of whistlers with an anomalously low dispersion. The trajectories of these whistlers should differ from the standard trajectories.[19] Their nature is not yet very clear.[20] (Interesting effects of the interaction between whistlers and the VLF radiation will be dealt with later.)

We shall consider first the results of systematic observations of the VLF radiation carried out in the recent past. Then, we shall move on to the problems of the generation of various types of this radiation and analyze the importance of the VLF noise in the dynamics of the outer radiation belt of the Earth. Because of the large number of publications involved, we shall consider relatively briefly the problems of an observational and morphological nature.

*Here and later, we shall divide the upper atmosphere into the ionosphere (height h from 80 to 1000 km) and the exosphere (h > 1000 km).

2. MAIN RESULTS OF SYSTEMATIC OBSERVATIONS OF THE VLF RADIATION

There are many types of spectrum of the VLF radiation. They were classified first by Gallet[5] (see also reviews [6,8]) who divided all types of the VLF radiation into two groups: continuous and discrete. On the basis of the data now available from several years of observation, it is more convenient to divide all types of VLF radiation into three main groups:

1) *continuous* radio emission of the hiss type covering a wide range of frequencies and lasting about an hour;

2) numerous short-duration bursts of increasing frequency, known as the chorus;

3) various forms of discrete VLF radiation, characterized by duration of the order of 1 sec and a narrow spectrum varying with time (rising, steady, and falling tones, hooks, etc.).

The chorus occupies an intermediate position as far as its spectra are concerned. It shares the properties of the continuous and the discrete VLF radiation. We shall consider now in detail the characteristics of various types of VLF signal.

The hiss is a form of noise and usually does not have any clear fine structure. Its spectrum covers a wide range of frequencies Δf of the order of 5 kc. The noise has the highest intensity in the range $f = 1-5$ kc. Dowden[21] described simultaneous measurements of the hiss at four discrete frequencies: $f \approx 4$, 9, 70, and 230 kc. He found that the intensity of the received radiation I_f decreased rapidly with increasing frequency. Thus, for example, at $f \approx 9$ kc, $I_f \approx 6 \times 10^{-16}$ W.m^{-2}(cps)$^{-1}$, while at $f \approx 230$ kc, $I_f \approx 10^{-19}$ W.m^{-2}(cps)$^{-1}$. The data on the intensity of the VLF radiation received by terrestrial stations can be found also in [22,23]. According to [23], the maximum intensity of the hiss at middle latitudes is 10^{-13}–10^{-14} W.m^{-2}(cps)$^{-1}$. Satellite measurements gave considerably higher values. Thus, the intensity measured by "Injun-III" reaches values $I_f \approx 10^{-11}$ W.m^{-2}(cps)$^{-1}$.[24]

As already mentioned, the hiss signals last about an hour. However, during geomagnetic storms the hiss-type radiation may last up to 10 or more hours.[5,22,25] The time dependence is usually characterized by a gradual increase in the intensity followed by a slow decay. However, there are cases when VLF radiation bursts have steep fronts.[26] The frequency of occurrence of the hiss does not show any seasonal variation. The daily variation at middle latitudes exhibits a minimum near the local noon, which may be associated with an increase in the ionospheric absorption. The hiss received in the auroral zone behaves differently, exhibiting a maximum during day-time.[27]

The chorus consists of very many closely spaced signals. In each signal, the frequency f increases typically from 1.5 to 3.5 kc.[28,29] The duration τ of one signal is usually 0.1–0.5 sec; the repetition frequency of signals varies from 1 to 10 sec^{-1}.[28] The

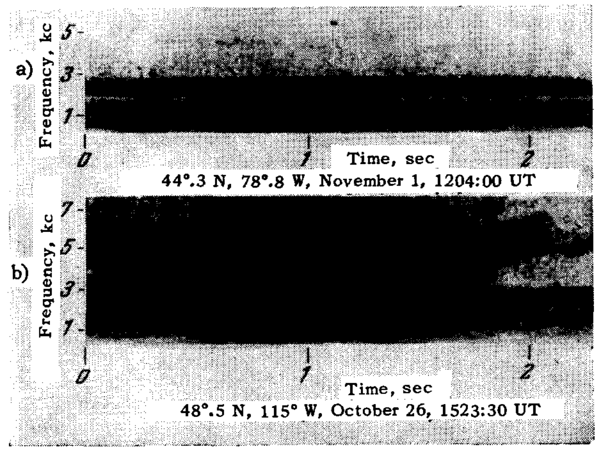

FIG. 1. Hiss spectra.

duration of the chorus bursts is of the order of one hour. The chorus intensity is approximately the same as the intensity of the hiss. The chorus appears most frequently early in the morning (about 6 a.m. local time). At higher latitudes, the situation is somewhat more complicated because this dawn maximum may be masked by the so-called polar chorus.[28,30-32] The latter may differ from the normal chorus and is obviously of different origin. The seasonal variations of the normal and polar choruses may be very different. The seasonal variation of the normal chorus is most likely to be associated with the variation of the ionospheric absorption, while the frequency of the occurrence of the polar chorus may have a marked summer maximum.[27-29,33]

Typical spectrograms of the hiss and chorus are given in Figs. 1 and 2, respectively. The spectrograms, obtained by frequency analysis of VLF signal recordings, are the dynamic spectra. Using these spectrograms, we can follow the time dependences of the spectra.

The spectrograms of the discrete VLF signals are such that at each moment there is a fairly definite frequency at which the intensity is maximal (Fig. 3). Sometimes there are many repetitions of a given spectrum.[5] For example, during magnetic storms the hook-type signals are repeated every few minutes or seconds. The durations of individual discrete VLF signals of various types, shown in Fig. 3, are of the order of a second.

We shall consider now the latitude dependences and their relationships with the magnetic activity. As a rule, VLF radiation is closely associated with the magnetic activity. The relationship is different at middle latitudes and in the auroral zone. At middle latitudes, the VLF radiation activity increases rapidly during magnetic storms. According to [22,34], during quiet periods the hiss is observed for 3–24% of the total time, while during storms the VLF noise is heard for 80–90% of the time. On magnetically quiet days, the frequency of occurrence of the hiss and its intensity have a maximum at the geomagnetic latitude (g.l.)

FIG. 2. Spectrogram of the chorus. Single tones of rising frequency can be seen quite clearly.

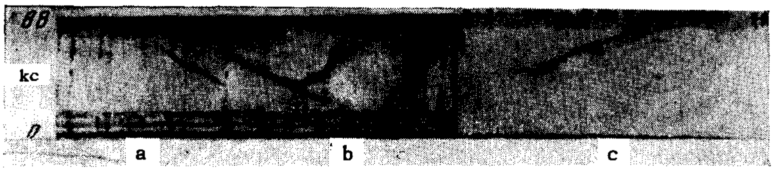

FIG. 3. Examples of the discrete VLF radiation. The spectrograms show tones with falling (a) and rising (b) frequencies, as well as hooks (c).

$\varphi \approx 60°$.[22,35] The chorus maximum is shifted somewhat toward higher latitudes ($\varphi = 60-70°$).[24] During magnetic storms, these VLF radiation activity maxima shift toward lower latitudes.[36-38] The maximum frequency of the radio waves and the width of the frequency spread increase during a storm.[5,39] The lowest latitude at which the hiss is still observed is $\varphi = 34°3'$ northern g.l.[40] In the southern hemisphere, weak hiss has been observed in Brisbane ($\varphi = 35°7'$ southern g.l.).[41]

Interesting observations have been obtained by the satellite "Alouette."[42,43] The measurements were carried out in the geographical latitude range from 80°N to 80°S. When "Alouette" traversed regions with $L > 9$ (L is the magnetic shell parameter representing the ratio of the maximum distance of a given line of force from the center of the Earth R_m to the radius of the Earth R_0), it recorded a stable radio noise in the frequency range from 2.5 kc to 400 cps. The lower limit was determined by the pass band of the receiver. At lower values of the magnetic shell parameter L, $9 > L > 4$, the VLF noise became very variable both in frequency and amplitude. In the region $L \leq 4$, one or several stable VLF noise bands were observed. Near $L = 3.5$, the spectrum contracted to a single band, which narrowed as L decreased further and disappeared completely at $L \approx 2.6$. The decrease in the VLF radiation intensity as L decreases has been observed also on the Earth. Thus, according to [22], the intensity drops by two orders of magnitude when g.l. goes φ from 51 to 42°. It should be mentioned that the data of the terrestrial and satellite observations do not agree completely, probably because of the absorption and reflection of the VLF waves in the lower ionosphere.

The repeated transits of the "Alouette" satellite revealed one characteristic feature of the hiss. It was found that the lower frequency limit in the hiss spectrum was, to a high degree of accuracy, proportional to ω_{HL}. Here and later, the subscript L will be used to denote the values on a fixed line of force with a given value of L for $\varphi = 0$.

A correlation of the VLF noise with the appearance of negative magnetic field bays has been reported in a number of investigations. The start of the noise bursts lags somewhat behind the bays.[44] The close relationship between the hiss and the middle-latitude auroras was reported in [45,46]. The time dependence of the brightness of the aurora corresponds to the variation in the VLF noise intensity.

The chorus at middle latitudes does not correlate with auroras. However, it closely correlates with the short-period geomagnetic pulsations (SPP).[47,48] Frequently, a curve representing the chorus intensity is found to be modulated by the SPP with periods of the order of 1 sec. Modulation with magnetic field pulsations of periods lasting several minutes has also been observed.[48]

At high latitudes, the VLF radiation is closely related to the aurora.[24,29-52] This is particularly true of the hiss. According to the "Injun-III" observations, the appearance and duration of energetic electron fluxes in the lower ionosphere coincide with the corresponding characteristics of the VLF hiss.[24] The intensities and spectra of the polar bursts are very variable. Frequently, particularly when the magnetic activity is increasing, the radiation spreads over two or several frequency bands.[49-52] There is an interesting negative correlation between the polar VLF radiation and the K_p index, which represents the perturbation of the geomagnetic field,[27,28] contrasting with the activity of the middle-latitude radiation, which increases as K_p increases.[27,28] The common feature of the polar and the middle-lattitude VLF radiation is an activity maximum during the decaying phase of a magnetic storm (about 9 hours after the sudden commencement of a storm).[5,27,53] This effect is particularly clear in the case of the hiss.

The recordings of the chorus by the "Injun-III" satellite established the stationary nature of the generation of this type of VLF radiation. Repeated transits of this satellite indicated a stable maximum at $\varphi = 63-67°$ ($L \approx 5-6$).[24]

Recently, new interesting properties of the VLF radiation have been discovered. A periodic (pulsating) VLF radiation of the hiss type has been observed with the intensity and the range of frequencies modulated partly or completely over the whole spectrum by periods of tens of seconds.[34,54]

Equally interesting is the induced VLF radiation. It consists of discrete signals (rising or falling tones, hooks) or short-duration bursts with a wide spectrum,

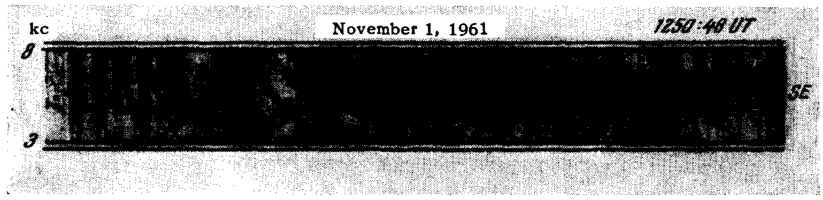

FIG. 4. Spectrogram of a sequence of whistler echoes, showing the gradual appearance of the induced VLF radiation of the noise burst type (upper part of the spectrogram) and discrete tones of falling frequencies.

appearing simultaneously with strong whistler echoes (Fig. 4). As a rule, the discrete VLF signals are at first weaker than the whistlers but then they become stronger than the whistlers.[55] Right from the beginning, the continuous short-duration bursts are stronger than the whistlers exciting them.[55] The induced radiation is a fairly frequent occurrence in the low-frequency range. This follows implicitly from the well-known paper of Storey,[56] who started a systematic investigation of whistling atmospherics.

Important data have been obtained by recording artificial echo signals from long-wavelength radio stations ($f = 18.6$ kc) when the propagation followed paths close to the whistler trajectories. Echoes of quasi-monochromatic signals, coded by the Morse alphabet, were received.[57] It was found that the Morse-code dashes (duration $\tau = 0.145$ sec) induced discrete VLF signals of various types. The transmission of the dots ($\tau = 0.045$ sec) did not generate the radiation. The VLF radiation was generated only by strong signals whose duration was greater than some critical value.

The induced radiation has been received also by AES.[58,59] In some cases, the signals received by a satellite were not heard at terrestrial stations. Such signals usually had a narrow spectrum with a sharp lower limit. The direction of the maximum signal was close to the normal to the geomagnetic field. The excitation may have occurred in the vicinity of AES (at heights $h \approx 1000$ km).

There is one further group of discrete signals which are received in a restricted range of frequencies together with whistlers.[60] The frequency of these signals increases at first and then approaches asymptotically the value of the gyrofrequency ω_{Hi} of the ions at the point of reception. The appearance of these signals is associated with the existence of two normal wave modes at frequencies $\omega < \omega_{Hi}$, for which the square of the refractive index is greater than zero.[60,61]

3. MECHANISMS OF GENERATION OF VLF RADIATION

It is premature to speak of a final theory of the origin of even the individual types of the VLF radiation. The most fully developed is the theory of hiss generation but even in this case there are many unsolved problems. The difficulties are partly due to the multiplicity of the observations requiring theoretical interpretation. Moreover, many phenomena cannot be explained on the basis of the linear approximation and therefore it is necessary to solve complex problems in the nonlinear theory of generation.

The VLF signals represent the radiation, in the exosphere of the Earth, of electrons or protons of solar or exospheric origin. In the range of frequencies of interest to us, the synchrotron and Cerenkov mechanisms of radiation are of the greatest importance. The resonance condition, which governs the frequencies $\omega = 2\pi f$ radiated by a nonrelativistic charge moving at a velocity $\mathbf{v}$ in an external constant magnetic field $\mathbf{H}$, is given by the well-known expression:[62]

$$\omega = s\omega_{He,\,i} + kv_z \cos\alpha \qquad (s = 0,\ \pm 1, \pm 2, \ldots). \tag{1}$$

In Eq. (1), $\omega_{He,i}$ is the gyrofrequency of electrons (subscript e) or ions (subscript i); k is the wave number; α is the angle between $\mathbf{H}$ and the wave vector $\mathbf{k}$. The magnetic field $\mathbf{H}$ is assumed to be directed along the z axis. We recall that $k = \omega n/c$, where c is the velocity of light in vacuum and n is the refractive index (it is assumed that there is no absorption). The value $s = 0$ in Eq. (1) corresponds to the Cerenkov radiation, $s = \pm 1$ corresponds to the first harmonic of the magnetodamping radiation. Here, $s = 1$ represents the normal Doppler effect and $s = -1$ represents the anomalous effect. The values $|s| > 1$ are associated with the synchrotron radiation losses at higher harmonics.

For the refractive index n, we can use the well-known expression[6,7]

$$n^2 = \frac{\omega_{0e}^2}{\omega(\omega_{He}\cos\alpha - \omega)}, \tag{2}$$

where ω_{0e} is the Langmuir frequency of electrons. The relationship (2) has been deduced neglecting the motion of ions and is inapplicable at very low frequencies. It is permissible to use Eq. (2) for the electron-proton plasma under exospheric conditions for frequencies, roughly speaking, from $f \gtrsim 1$ kc.[63] The contribution of the motion of heavier ions is important only at much lower frequencies, which are of no interest in the VLF emission.

The relationships (1) and (2) allow us to determine the range of emitted frequencies for given parameters of the exosphere and known velocities of charges. The converse problems can be solved also. Such analyses have been carried out in a number of papers.[5,64-72] Here, we shall note only the main results. Putting

$x = \omega/\omega_{He}$ in Eqs. (1) and (2), we obtain the relationship

$$\beta_z = \frac{v_z}{c} = \frac{\omega_{He}}{\omega_{0e}\cos\alpha}\sqrt{\frac{\cos\alpha - x}{x}}\left(x - \frac{s\omega_{He,\,i}}{\omega_{He}}\right). \qquad (3)$$

If we consider only the most frequent types of the VLF radiation, we can restrict the frequency range to $f = 1.5-20$ kc. Then, we can use the conditions

$$\omega_{Hi} \ll \omega < \omega_{He}. \qquad (4)$$

We note that these inequalities in fact determine the applicability of the relationship (2) because when $\omega \sim \omega_{Hi}$ it is necessary to allow for the motion of ions,[62,63] and when $\omega > \omega_{He}$ the plasma becomes opaque.

When Eq. (3) is applied to the radiation of ions, we can omit the term $s(\omega_{Hi}/\omega_{He})$, at not too high values of s ($s \ll \omega/\omega_{Hi}$) on the basis of Eq. (4). Then, the frequencies ω of the Cerenkov and synchrotron radiations become equal.[6] Consequently, we have for ω

$$\omega = \frac{\omega_{Hc}\cos\alpha}{2}\left\{1 \pm \left[1 - \left(\frac{2v_z\omega_{0e}}{c\omega_{He}}\right)^2\right]^{1/2}\right\}. \qquad (5)$$

This formula is valid also for the Cerenkov radiation of electrons ($s = 0$). According to Eq. (5), we can speak of the emission only when the following inequality is satisfied

$$2\frac{\omega_{0e}}{\omega_{He}}\frac{v_z}{c} < 1. \qquad (6)$$

In the ionosphere, this condition is satisfied with a large margin, and the following simple relationships are obtained:

$$\omega = \omega_{He}\cos\alpha, \qquad (7a)$$

$$\omega = \frac{v_z^2}{c^2}\frac{\omega_{0e}^2}{\omega_{He}}\cos\alpha. \qquad (7b)$$

In the case of the synchrotron radiation of electrons, a simple formula follows from Eq. (3) provided $\omega \ll \omega_{He}\cos\alpha$:

$$\omega = \frac{c^2}{v_z^2}\frac{\omega_{He}^3}{\omega_{0e}^2\cos\alpha} \quad (s = \pm 1). \qquad (8)$$

It is desirable to supplement this discussion with a graphical representation of the function $\beta_z(x)$. The curves shown in Fig. 5 are plotted for $\alpha = 45°$ and $\omega_{He}/\omega_{0e} = 0.1$. By altering the ordinate scale, Fig. 5 can be used for other values of the ratio ω_{He}/ω_{0e}. The curves for $s = 0$ and $s = \pm 1$ correspond to the electron radiation. As already shown, for ions the values of the emitted frequencies are approximately the same for $s = \pm 1$ and $s = 0$. There is no difference either in the excitation conditions for electrons and ions if $s = 0$. As shown earlier,[73,74] the intensity of the higher harmonics of the synchrotron radiation of ions may in some cases be considerable (higher than for $s = 0$ and $s = \pm 1$). Because of this,

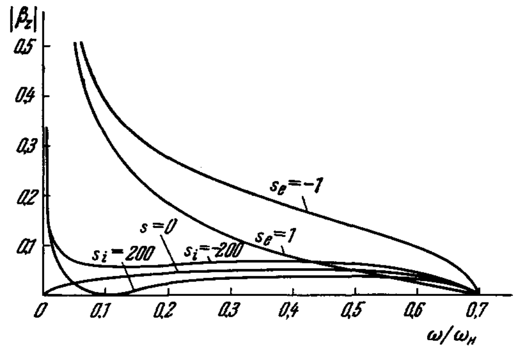

FIG. 5. Characteristic velocities of charged particles contributing to the VLF radiation in the upper atmosphere ($\omega_H = \omega_{He}$).

Fig. 5 gives the $\beta_z(x)$ curve for ions in the case $|s| \gg 1$.

Drawing horizontal straight lines whose positions are governed by the velocity v_z, we can find, from the points of intersection with the curves $\beta_z(x)$ in Fig. 5, the values of ω/ω_{He} and determine the frequencies excited at a given height in the ionosphere or exosphere. In the ionosphere, when the frequency ω_{He} is very high (for example, the frequency for $x \approx 10^{-2}$), the VLF radiation may be excited only by sufficiently slow ions (also by electrons if $s = 0$). This conclusion is valid if we exclude the case of small values of the quantity $|\cos\alpha|$. The same result can be obtained by analysis of the relationships (7) and (8). In the $f \approx 2-20$ kc range, the frequencies given by Eqs. (7a) and (8) for $\cos\alpha \approx 1$ cannot be excited at all in the ionosphere. The frequencies satisfying the condition (7b) lie in this range only if $v_z/c \ll 1$ (at least for $\beta_z < 0.1$ and heights $h \approx 1000-2000$ km). At greater heights (for example, $h \gtrsim R_0$), the resonance condition (1) may be satisfied in a wide range of velocities beginning from $v \approx v_z \approx 5 \times 10^7$ cm/sec (solar corpuscular streams) and ending at velocities comparable with c (energetic electrons and protons in the outer radiation belt). In addition to the electron radiation, the synchrotron radiation of ions may also be excited. This can be seen both from Fig. 5 and from the relationships (7) and (8).

This analysis is of interest not only when applied to the radiation but also from the point of view of the determination of the conditions of interaction between charged particles and waves in a magnetoactive plasma. This interaction will be considered later (mainly in Sec. 4). We note also that the excitation conditions are considered here without allowance for the spatial dispersion. Because of this, the emission of certain waves (plasma, acoustic, etc.) cannot be considered. Such waves do not affect the composition of the VLF radiation. However, one must bear in mind the possible interaction between various waves in an inhomogeneous plasma as well as the transformation of some normal waves into others.[62,75] Therefore, an analysis of the possibility of excitation of such waves is also of interest.

In considering the actual mechanisms of the VLF emission, we note that incoherent processes cannot produce the observed intensities. Thus, for example, calculations of the incoherent Cerenkov radiation of electrons in a radiation belt give the value $I_f \approx 10^{-19}$ W.m^{-2}(cps)$^{-1}$ in the $f \approx 5$ kc range.[76] Similar values are obtained for the gyroresonance radiation. Values of I_f up to 10^{-11} W.m^{-2}(cps)$^{-1}$ have been recorded by AES (cf. Sec. 2). Therefore, any theory of the generation should be based on an analysis of instability conditions ensuring the coherence of the radiation from a system of charged particles.

We shall now consider the possible mechanisms of the generation of the chorus. We have already pointed out that the generation of the VLF signals in the ionosphere is fairly difficult. It is more likely that the chorus is generated in the exosphere. It can be excited by the injection and simultaneous acceleration of solar plasma streams at the boundary of the magnetosphere. Another possibility is associated with the two-stream instability in the fronts of magnetohydrodynamic or shock waves.

The generation of the chorus by the injection of a stream of particles into the exosphere has been considered, for example, in [72]. The conditions of instability were analyzed for a plasma penetrated by streams of electrons and ions with parameters close to the properties of the solar corpuscular flux (velocity $v_0 \approx 5 \times 10^7 - 5 \times 10^8$ cm/sec). The stream was assumed to be weak so that the condition $N_s \ll N$ was satisfied; here, N_s is the concentration of charged particles in the stream and N is the concentration in the exosphere. The stability was investigated in relation to the excitation of electromagnetic and plasma waves for the emission frequencies given approximately by the relationship (7a). The necessary condition for the two-stream instability is the inequality

$$v_{ph} = \frac{\omega}{k} < v_0, \tag{9}$$

where v_{ph} is the phase velocity. The sufficient condition is related to allowance for the re-absorption and, according to [72] has the following form:

$$\frac{N_s}{N} > \frac{v_{ts}^2}{2v_t v_0}\left\{2\frac{v_0^2}{v_{ts}^2}\exp\left(-\frac{v_0^2}{2v_{ts}^2}\right) + \sin^2\alpha\cdot\exp\left[-\frac{v_0^2}{v_{ts}^2}\left(\frac{1-\cos\alpha}{\cos\alpha}\right)^2\right]\right\}, \tag{10}$$

where v_t and v_{ts} are the average velocities of the thermal motion of electrons in the exosphere and in the stream. The criterion (10) was obtained under certain restrictions, one of which was the inequality $v_0 \gg v_t$ (for details see [70]). Assuming, for example, $v_t = v_{ts}$, $v_0 = 7v_t$, $\alpha = 45°$ and $T = 10^3$ °K, Eq. (10) showed that an instability appeared when $N_s > 10^{-3}$ N.

It should be borne in mind that in the case of excitation at the frequencies $\omega = \omega_{He} \cos\alpha$, given by Eq. (7a), the absorption in the path of a wave propagated from the point of generation is important. According to [72], this absorption is slight if $v_0 > 1.2 \times 10^8$ cm/sec.

We have just considered the radiation from the electrons in a stream. The ions in that stream can excite magnetohydrodynamic waves[77] which are usually associated with the chorus.[71] However, this mechanism meets with a number of difficulties. First, the nature of the injection process of suprathermal particles into the exosphere is not clear. Moreover, the temperatures of a stream and of the exosphere may be quite high (up to $T \approx 10^5$ °K), which makes it more difficult for an instability to appear. These remarks apply to [78], in which an attempt is made to associate the appearance of the VLF signals with the space-charge waves in streams of charged particles penetrating the exosphere. In view of these difficulties, it is more promising to consider the mechanism of the generation of the chorus by isolated magnetohydrodynamic or shock waves, which can be excited at the boundary of the magnetosphere.[79-81]

In collisionless shock waves, an instability may appear in the plane of a front due to the relative motion of electrons and ions. Such an instability in waves propagated at right-angles to an external magnetic field H was used in [83,84] to explain the generation of type II solar radio bursts. Several workers have investigated[79,81,85] the excitation of ion-acoustic waves in the front of a shock wave traveling away from the magnetosphere. Although the perturbations due to an instability were within the VLF frequency range, considerable difficulties were encountered in the explanation of transmission of radio signals to the Earth from regions outside the magnetosphere at distances $L \gtrsim 10$. However, this mechanism could apply in the interior of the magnetosphere when shock waves, generated at the boundary by inhomogeneities of a corpuscular stream, penetrated into the inner regions. The possibility of the existence of shock waves in the exosphere and criteria of their instability were discussed in [86].

An instability appears in the front of a shock wave when the velocity of electrons relative to ions, v_0, is greater than the thermal velocity of electrons v_t. The relative velocity v_0 is due to the acceleration of electrons in the electric field generated in the wave front by the separation of charge. If v_0 is expressed in terms of the wave parameters, the instability condition for weak shock waves becomes

$$\frac{h}{H} \gg \left(\frac{8\pi N\kappa T}{H^2}\right)^{1/3}, \tag{11}$$

where κ is the Boltzmann constant and h is the magnetic field intensity in the weak wave ($h \ll H$).

The VLF chorus may be generated also by an instability in colliding streams of electrons in the Alfvén wave fronts.[82,87] The separation of charge, due to different conditions for the reflection of electrons and ions from an isolated finite-amplitude wave, gives rise to a potential difference φ in the wave front, whose maximum value can be estimated from the relationship obtained in [82]

$$e\varphi \cong \frac{h}{H}\frac{Mv_A^2}{2}, \qquad (12)$$

in which h is the wave amplitude, M is the mass of ions, and $v_A = H/\sqrt{4\pi NM}$ is the Alfvén velocity. The wave is assumed to be weak so that $h \ll H$. Electrons are accelerated by the potential difference, giving rise to two oppositely directed streams in a narrow region of the front. For an instability to appear, the electron velocity in a stream v_0 must be greater than the average thermal velocity v_t. Using Eq. (12), the condition $v_0 > v_t$, and the expression for the velocity $v_0 \approx \sqrt{2e\varphi/m}$, we obtain the inequality

$$\frac{h}{H} \gg \frac{8\pi N\varkappa T}{H^2} \qquad (13)$$

The condition (13) imposes less rigorous limitations on the amplitude h than the condition (11) in the shock-wave case and it may be easily satisfied in the exosphere. Such an instability may also appear in the front of a magnetic-sound wave propagated at an acute angle to a magnetic field **H**.

This mechanism accounts for the close relationship between the chorus and the geomagnetic field micropulsations. Moreover, the least stable magnetohydrodynamic waves govern the repetition frequency of individual tones in the chorus.

When $\omega_{0e} \gg \omega_{He}$, the frequencies excited both in magnetohydrodynamic and shock waves are $\omega \approx \omega_{0i}$, where ω_{0i} is the Langmuir frequency for ions. In the exosphere, where $\omega_{0i} \lesssim \omega_{He}$, these frequencies lie in the VLF range. However, in this case the quasi-acoustic and not the electromagnetic waves are amplified, the refractive index for the latter being given by Eq. (2). The acoustic waves can be obtained in a collisionless plasma only if the thermal motion of electrons and ions is taken into account. The question now arises how these waves are transformed into the electromagnetic waves. A detailed analysis of this problem has not yet been undertaken. Qualitative considerations indicate that a transformation of this type should take place without heavy losses.

The final selection of a generation mechanism requires the development of a nonlinear theory which would make it possible to estimate the duration and intensity of the emission; a careful analysis of the problem of the range of excited frequencies is also needed.

The origin of the polar chorus was considered specially by Ondoh,[70] who discussed the excitation of VLF signals by protons in the frequency range satisfying the criterion of the anomalous Doppler effect. The same protons can simultaneously excite magnetohydrodynamic waves and enhance the ionization in the ionospheric layers. This may lead to an anomalous absorption of cosmic radio waves. Ondoh mentioned the possibility of the coincidence of these three effects.[70]

Some types of discrete radiation may be associated with energetic electrons in the outer radiation belt. We shall mention here the paper[68] by Dowden, who associated the appearance of hooks and tones having rising (falling) frequencies with the radiation of electron bunches moving in the magnetic field of the Earth between the mirror points. The emission frequency was determined from Eq. (8). The hooks were explained by radiation over the whole trajectory between the mirror points. VLF signals with rising or falling frequencies were considered as components of the hooks. By selecting the particle energy and the angle between the direction of motion of a bunch of particles and a field **H**, as well as allowing for the time of flight, Dowden was able to explain satisfactorily the form of real spectrograms. The best agreement was obtained for electron energies in the 5–50 keV range.

It should be mentioned, however, that a stream of particles in the normal Doppler effect region can radiate coherently only for a fairly special electron-velocity distribution.[88] Calculations of the increment, reported in [68], were inaccurate because the re-absorption in a stream and in the plasma matrix was not allowed for. Moreover, the observations of the hook sequences indicate that the repetition frequency of the hooks is often equal to the repetition frequency of multiple whistlers rather than to the oscillation periods of electrons between the mirror points. Examples of the appearance of discrete VLF signals induced by whistlers have been given in Sec. 2. However, there is as yet no theory of these phenomena. The qualitative considerations given in [89] cannot be regarded as satisfactory. The nonlinear interaction between electromagnetic waves may play an important role in the generation of induced VLF signals.

We shall now consider the problems of the theory of the generation of hiss. The most satisfactory is the mechanism associated with a kinetic (cyclotron) instability* in the outer radiation belt.[90-93] It has been shown in [94,95] that an anisotropic distribution of electrons in a magnetoactive plasma with $T_\perp > T_\parallel$ ($T_\perp$ and $T_\parallel$ are, respectively, the temperatures for the motion at right-angles to **H** and parallel to **H**) is unstable under the action of electromagnetic perturbations of frequencies $\omega < \omega_{He}$. Measurements have shown that the electron distribution in the radiation belt has such an anisotropy.

It is interesting that in the terrestrial radiation belts the increments are governed by the "hot" component (energetic electrons in a belt) while the phase

*In contrast to a hydrodynamic instability, a kinetic instability is due to the characteristics of the distribution function of particles in the momentum space.

velocity depends only on the parameters of the "cold" component of the plasma. The increments are not exponentially small for a kinetic instability. Calculations of the instability for $T_\perp > T_{\parallel}$ under the conditions obtaining in a radiation belt have been reported in [90-93]. We shall consider the simplest case when the electron distribution is approximated by a nonrelativistic Maxwellian function[91]

$$f_0 = \frac{N_n(z)}{\delta(2\pi m\varkappa T)^{3/2}} \exp\left\{-\frac{p_\perp^2}{2m\varkappa\delta T} - \frac{p_\parallel^2}{2m\varkappa T}\right\}, \qquad (14)$$

where m is the electron mass, δ is the degree of anisotropy, and $p_\perp$ and $p_\parallel$ are the transverse and longitudinal momenta of an electron. The concentration of energetic electrons $N_n(z)$ decreases away from the top of a radiation belt along the lines of force in accordance with the law

$$N_n(z) = N_{n0}\left[\frac{H_L(0)}{H(z)}\right]^{\delta-1}. \qquad (15)$$

Using Eqs. (14) and (15), we can obtain the following expression for the gain factor η and for the VLF noise energy ϵ_k in the linear approximation:

$$\eta(x) = \eta_{max}(x - x_0 + 1)\exp(x_0 - x), \qquad (16)$$

where

$$\eta_{max} = 8{,}9\cdot 10^{-12}\,\frac{\delta-1}{\omega_{He}}\,S_L \exp(-x_0),$$

$$\varepsilon_k = \frac{\varkappa T}{(2\pi)^3}\,x_0 \exp\left\{\int \eta\,dz\right\}. \qquad (17)$$

In Eqs. (16) and (17), the notation is*

$$x = \frac{\omega_H^3}{2\beta^2\omega_0^2\omega}\left(\beta^2 = \frac{\varkappa T}{mc^2}\right) \text{and } x_0 = 1 + \frac{\delta}{\delta-1}\,\frac{\omega_H^2}{2\beta^2\omega_0^2}.$$

S_L represents the density of the electron flux, in $cm^{-2}sec^{-1}$, at the top of a radiation belt. The frequency at which the emission is strongest is given by

$$\omega_{max} = \omega_H \frac{\delta-1}{\delta x_0}. \qquad (18)$$

We shall consider some consequences of Eqs. (16)–(18). According to Eqs. (15) and (16), the maximum gain factor decreases rapidly away from the equatorial plane (along a given line of force). Consequently, the excitation takes place mainly in the top of a radiation belt. From the condition $\eta(x) = 0$ [cf. Eq. (16)], we can find the limiting value ω_2 of the spectrum of excited frequencies:

$$\omega_2 = \frac{\delta-1}{\delta}\,\omega_{HL}, \qquad (19)$$

where ω_{HL} is the value of the gyrofrequency in the equatorial plane. The frequency ω_2 is governed by the anisotropy δ and by the gyrofrequency ω_{HL} at the belt top. For δ on quiet days, we can use the value $\delta = 1.5$ and on magnetically active days, $\delta = 2$.[96,97]

*Here and later, we shall omit, for brevity, the subscript e.

For $\kappa T \approx 10$ keV at the top of the outer radiation belt ($L = 3.5$, $N \approx 3 \times 10^2$ cm^{-3} and $\omega_{HL} = 1.2 \times 10^5$ cps), we obtain the following expressions for the energy flux $I_\omega = \epsilon_k k^2$ in $W.m^{-2}(cps)^{-1}$[91]

$$I_\omega = 2.26\cdot 10^{-23}\exp(3\cdot 10^{-9}S_L), \quad \delta = 1.5,$$
$$I_\omega = 3{,}4\cdot 10^{-23}\exp(5{,}1\cdot 10^{-9}S_L), \quad \delta = 2. \qquad (20)$$

The electron flux density for which the intensity of emission in the outer belt is $I_\omega \approx 10^{-11}$ $W.m^{-2}(cps)^{-1}$ for $L = 3.5$ should have the values $S_L \approx 9 \times 10^9$ $cm^{-2}sec^{-1}$ for $\delta = 1.5$ and $S_L = 5 \times 10^9$ $cm^{-2}sec^{-1}$ for $\delta = 2$.

Trakhtengerts[90,92] calculated the kinetic instability for various values of the magnetic shell parameter using electron distribution functions which were closer to real functions than Eq. (14). Trakhtengerts allowed for the anisotropy introduced by the loss cone. It was found that to excite the observed intensities of the hiss, trapped electron fluxes weaker than those given by Eq. (20) were sufficient.

The electron flux density needed for the generation of the observed VLF radiation decreases rapidly as L increases (see below, Sec. 4). In some cases, particularly when $\delta \approx 1$, the anisotropy associated with the loss cone may make a considerable contribution. The appearance of an anisotropy with $T_\perp > T_\parallel$ is related to electrons with high longitudinal velocities escaping from traps through magnetic mirrors.

The spectral characteristics of the VLF radiation, which follow from the expressions (16)–(19), are in quite good agreement with observations. Thus, for $L = 3.5$ the frequencies f_{max} of Eq. (18) and f_2 of Eq. (19) are $f_{max} = 3.2$ kc and $f_2 = 4.3$ kc for $\delta = 1.5$. For $\delta = 2$, we have $f_{max} \approx 6$ kc and $f_2 = 9.4$ kc. During periods of weak geomagnetic activity, the measurements of the VLF noise give[5] $f_{max} = 3.5$ kc and $f_2 \geq 5$ kc, while for magnetically active days we have $f_{max} = 6$ kc and $f_2 \geq 8$ kc.[39] This mechanism allows us to explain also the latitude dependence of the lower frequency limit f_1 in the VLF noise spectrum. In fact, if we assume that $f_1 \approx f_{max}/2$ [the gain factor of Eq. (16) then decreases by a factor of about 2], in the case $\omega_H^2\delta/2\omega_0^2\beta^2(\delta-1) \gg 1$ the frequency f_1 is found to be proportional to f_{HL}; which is indeed observed (cf. Sec. 2).

The quasi-linear theory, developed in [91-93], makes it possible to determine the change in the distribution function of the energetic electrons during the development of an instability and to find the duration of the VLF noise. The problem of the distribution function will be considered in the next section. Here, we should mention that, if the deformation of the distribution function is allowed for, the VLF radiation energy can be found at any given moment by means of the linear-theory formula (17), in which, however, the gain factor is a function of time. According to [91], the time dependence of ϵ_k without an external source of energetic electrons in a radiation belt is, for given initial conditions, described by the following expression:

$$\varepsilon_k = \varepsilon_{k0}\left(1 + D_0 t \int \eta_0\, dz\right)^{-1}, \qquad (21)$$

where ϵ_{k0} is the energy of waves governed by the electron distribution at the initial moment $t = 0$. The diffusion coefficient D_0 has the form

$$D_0(\alpha = 0) = \frac{\pi^2 \omega_0^5 \varepsilon_{k0} \omega^{3/2}}{2mc^5 N_0 \omega_H^{5/2}}. \qquad (22)$$

Another important point should be mentioned as well. We have assumed that the VLF radiation is absorbed completely in the lower ionospheric layers. Some fraction of the VLF noise leaks through the ionosphere and is received at terrestrial stations. However, a considerable fraction of the VLF wave energy may be reflected from the ionosphere and may be returned to a radiation belt. Multiple transits through a radiation belt may make the VLF noise intensity much higher at lower values of the trapped electron flux than those just indicated.

In Sec. 2 we have mentioned one characteristic property of the hiss (particularly with reference to the auroral zone), which is the appearance of additional noise bands at higher frequencies. These bands are frequently correlated with certain types of aurora. A probable mechanism of this effect is the excitation of VLF waves by the particle streams causing the aurora.[69] However, before drawing final conclusions, it is necessary to develop a more rigorous theory allowing for the re-absorption, the particle stream intensity, and inhomogeneities of the exosphere.

4. ROLE OF THE VLF RADIATION IN THE DYNAMICS OF THE [OUTER] RADIATION BELT OF THE EARTH

In the preceding section, we have shown that an instability of the electron distribution in the outer radiation belt may cause the generation of the continuous component of the VLF radiation. In the present section, we shall consider mainly the role of the VLF radiation in the dynamics of this belt. In particular, the VLF radiation may be important in the electron escape mechanisms.

First of all, we shall consider the problem qualitatively. It is now known that there are intense sources of energetic electrons in the outer radiation belt. This is indicated, in particular, by the existence of the stationary auroral zone. Streams of electrons with energies $E \gtrsim 10$ keV have been recorded systematically in AES observations.[98,99] It is interesting that, even during geomagnetic storms when the intensity of these sources increases considerably, there is no marked accumulation of trapped electrons but particles are precipitated into the dense layers of the atmosphere. These observations could be explained by assuming the existence, at the top of the radiation belt, of an anisotropic source with a preferential electron acceleration along the lines of force of the field H.[100] However, it is difficult to see how this could be realized in practice. Recently, there has been a discussion of the possibility of acceleration and escape of electrons and ions from a geomagnetic trap due to the action of electrostatic fields perpendicular to the geomagnetic field.[101] Obviously, this mechanism may play an important role in the auroral zone on the night-side of the Earth. On the day-side of the magnetosphere and in inner regions of the radiation belt the acceleration mechanisms giving rise either to an isotropic electron-velocity distribution or an anisotropic distribution with predominantly transverse velocities are more likely.* Among such mechanisms are the acceleration by electromagnetic radiation,[102] magnetohydrodynamic and shock waves,[82,86,87,103,104] and induced drift.[104] An effective source can be introduced also to describe the acceleration processes in the adiabatic compression of the geomagnetic field.

When the acceleration mechanism leads mainly to an accumulation of particles in a trap, the particle loss may be affected considerably by various plasma instabilities associated with an anisotropy of the electron (ion) velocity distribution. If the anisotropy is not too strong† and $T_\perp > T_\parallel$, an instability leading to the excitation of VLF-type electromagnetic waves appears very easily. As long as there are relatively few particles in a geomagnetic trap and the VLF noise intensity is low, the action of a source will increase the trapped electron concentration. Then, only a slight fraction of the particles will reach the dense layers of the atmosphere (for an isotropic source this fraction is of the order of $H_L/H_{cr} \approx L^{-3}$, where H_L and H_{cr} are the values of the geomagnetic field at the belt top and at the Earth's surface). The intensity of the VLF radiation will increase rapidly as the electron density increases. At the same time, the diffusion of particles by VLF waves into the loss cone will increase. Under steady-state conditions, the VLF radiation intensity will be such that the number of particles supplied by a source will be equal to the number of particles lost from the belt.

The spatial inhomogeneity of the radiation belt is important: because of it, a large fraction of the electrons in the belt interacts resonantly with the radiation extending over finite band of frequencies. This can easily be deduced from Fig. 6, which shows schematically a geomagnetic trap with mirrors. Lines AB and A'B' represent the levels at which electrons are absorbed. The closed curves in Fig. 6 represent

*As mentioned in Sec. 3, such an anisotropy is favored by the escape of electrons from a trap at low pitch angles Φ ($\tan\Phi = p_\perp/p_\parallel$).

†If the anisotropy is quite strong (say, $T_\perp > 10T_\parallel$), plasma wave excitation may take place[105-107] and this would lead to a rapid relaxation of the particle distribution to a state with a lower anisotropy.

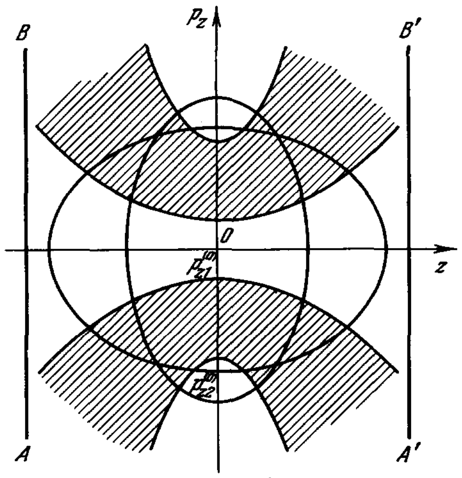

FIG. 6. Schematic representation of a geomagnetic trap. Shaded areas represent the region of interaction of particles in a radiation belt with the VLF radiation.

trapped electron trajectories conserving the particle energy E and the adiabatic invariant $\mu = p_\perp^2/2mH$. The equations of these trajectories have the form

$$p_\parallel^2 + 2\mu H(z) = 2mE.$$

The shaded regions represent the interaction of electrons with the VLF radiation. The boundaries of these regions are governed by the minimum ω_1 and the maximum ω_2 frequencies of the VLF noise spectrum and by the gyroresonance condition in the normal Doppler effect region, as given by Eq. (1) (s = 1). It is in this case that the interaction is strongest.[90] The equations for the boundary curves have the form

$$\omega_{1,2} - \omega_H(z) = \frac{k(\omega_{1,2})}{m} p_\parallel \cos\alpha. \tag{23}$$

It follows from Fig. 6 that there is no interaction between the radiation and those electrons whose longitudinal momentum in the equatorial plane z = 0 is less than the value

$$p_{\parallel 1} = \frac{\omega_1 - \omega_{HL}}{k(\omega_1)\cos\alpha} m.$$

For a strong anisotropy, the frequency ω_1 is close to ω_{HL},[92] and therefore the value of $p_{\parallel 1}$ becomes small. In this case, practically all the energetic electrons interact with the VLF radiation.

Let us consider also another property of the interaction between charged particles and the VLF radiation. Strong interaction takes place when the resonance condition (1) is satisfied. During the emission (absorption), the energy E and the pitch angle Φ of a particle vary. We can easily find the relationship between changes in $\Delta\Phi$ and ΔE if we recall that the interaction of a plane wave with a particle in a uniform magnetoactive plasma is characterized by an integral of motion $E - (cp_\parallel/n) = \text{const.}$ [108] Using this integral, we can obtain from Eq. (1) the relationship (for $\tan\Phi \approx 1$)

$$\Delta\Phi = \frac{1}{2}\left(\frac{2s\omega_{H_e,i}}{\omega} - 1\right)\frac{\Delta E}{E}. \tag{24}$$

Application of Eq. (24) to the interaction of electrons shows that in the emission, when $\Delta E < 0$, the pitch angle Φ will decrease only in the region of the normal Doppler effect ($s \gtrsim 1$). Then, if $\omega \ll \omega_H$, then

$$|\Delta\Phi| \cong \frac{\omega_H}{\omega}\frac{|\Delta E|}{E} \gg \frac{|\Delta E|}{E}.$$

Thus, an electron may escape from a trap through magnetic mirrors after a relatively small change in its energy E. Real values of the velocity of electron escape from a trap can be found only by simultaneous allowance for the emission and absorption.

From now on, we will not consider the mechanisms of electron loss from the outer radiation belt which are not related to the VLF radiation (collisions, scattering by magnetohydrodynamic waves, etc.). This is justified because such mechanisms give rise to electron lifetimes longer than tens of days. We are interested in processes with characteristic times of the order of several hours.

We shall now obtain some quantitative estimates of the role of a kinetic instability in the dynamics of the outer radiation belt. The most interesting is the problem of the electron distribution function f in the belt and of the streams of escaping particles. The main equation which describes the variation of f and allows for the diffusion of particles due to electromagnetic field oscillations (in our case, the VLF noise), is the quasi-linear equation for a weakly perturbed plasma[92,109]

$$\frac{\partial f}{\partial t} + \Lambda f = \frac{\partial}{\partial p_\parallel} p_\perp^2 D \frac{\partial f}{\partial p_\parallel} + I(p_\parallel, q, z), \tag{25}$$

where all the terms are functions of the variables $p_\parallel$ and q, which are related to μ and E as follows:

$$p_\parallel = \sqrt{2m(E - \mu H)} \quad \text{и} \quad q = E + c\int_{p_\parallel^{(1)}}^{p_\parallel} \frac{dp_\parallel}{n\cos\alpha}.$$

The first term on the right-hand side of Eq. (25) represents the contribution of the electron diffusion by VLF waves. The diffusion coefficient D is given by Eq. (22). The second term on the right-hand side of Eq. (25) is I, which is the source of energetic electrons in the radiation belt. The term Λf is due to the inhomogeneity of the geomagnetic trap. The equation $\Lambda f = 0$ makes it possible to find the equilibrium distribution of charged particles in an inhomogeneous magnetic field, which depends only on μ and E (in the absence of other fields and collisions).

Changes in the electron distribution in the radiation belt can be described fairly completely by considering the relaxation under given initial conditions without a source and then finding the steady-state conditions in the presence of a source.

The former problem has been considered in [91] in an approximation assuming that the field H is uniform. Then, $\Lambda f \equiv 0$, and Eq. (25) assumes the simple one-dimensional form of the diffusion equation:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial p_{\parallel}}p_{\perp}^{2}D\frac{\partial f}{\partial p_{\parallel}}. \tag{26}$$

This equation should be solved with the boundary conditions $\partial f/\partial p_{\parallel} = 0$, $p_{\parallel} = p_{\parallel}^{(1)}$, and $f = 0$ at the boundary with the loss cone ($\mu = \mu_{cr}$). The coefficient D depends on the VLF wave energy, which is found from the transport equation and is related in a complex manner (through an integro-differential operator) to f (cf. [91]). Consequently, the solution of Eq. (26) presents considerable difficulties. An approximate solution of Eq. (26) has been given in [91]. Some results which follow from this solution have been considered in the preceding section.

We shall now consider the time dependences of the electron fluxes captured and lost by the radiation belt. Let us assume that at $t = 0$ there is some anisotropic distribution of electrons with $T_{\perp} > T_{\parallel}$ and an associated initial VLF noise intensity. If the distribution is selected to be of the type given by Eq. (14), then the VLF noise intensity is found from Eqs. (16) and (17). The trapped particle flux density then decreases at $t > 0$ in accordance with the law

$$S_L=S_{L0}\left[1+\frac{\ln\left(1+D_0t\int\eta_0\,dz\right)}{\int\eta_0\,dz}\right]^{-1}. \tag{27}$$

Here and later, we shall use the subscript "0" to denote quantities whose values are taken at the time $t = 0$.

The density of the flux of electrons lost from the radiation belt, S_A, is also a maximum at $t = 0$ but it decreases with time much more rapidly than does S_L, following the dependence (21) for the VLF noise. The maximum density of the flux of escaping particles is approximately equal to

$$S_{A0}\cong\frac{a\sigma z_0}{v_T}S_{L0}D_0, \tag{28}$$

where $\sigma = H_{cr}/H_L$ is the mirror ratio, and $z_0 \approx R_0$ is the effective length of the radiation belt. The appearance of the small coefficient $a \ll 1$ in Eq. (28) is due to the fact that, if no allowance is made for the inhomogeneity of a trap, the VLF noise will interact only with a small fraction of the total number of electrons, which satisfies the resonance condition (1) for $s = 1$. Estimates based on Eq. (28) give values $S_A \approx 2\times10^{7}$ cm^{-2}sec^{-1} for $I_\omega \approx 10^{-11}$ W.m^{-2}(cps)$^{-1}$.

The steady states of the outer radiation belt with an inhomogeneous field H(z) and in the presence of an electron source I have been considered in [93]. For sufficiently weak sources, slow changes may be described by a distribution function f_0, depending only on μ and E, averaged out over the period of oscillations in a trap.

Let us consider a concrete case, taking I to be a source of the type

$$I=\begin{cases} I_0^{(\alpha)}E^{-\gamma-\alpha}\mu^{\alpha}, & E\geqslant E_0,\\ 0, & E<E_0.\end{cases} \tag{29}$$

Using the results published in [93] for $\alpha = 1$ and $\gamma = 3$, we have

$$f_0(\mu,E)=K(2mE)^{-2}\left[\frac{1}{5}+\frac{2}{3}\frac{\mu H_L}{E}+\left(\frac{\mu H_L}{E}\right)^2\right]\ln\frac{\mu H_{cr}}{E}. \tag{30}$$

The coefficient K in Eq. (30) depends weakly (like in I_0) on the power of the source and is approximately equal to

$$K\cong K_0=\frac{10mc^2\omega_{HL}N}{\omega_0^2}\left[\int\left(\frac{H_L}{H}\right)^3dz\right]^{-1}.$$

Using Eq. (30), we can easily find the distribution of the density of the trapped electrons S_L in a given range of energies (E_1, E_2):

$$S_L(E_1\leqslant E\leqslant E_2)=\frac{\pi K}{2m}\left\{\left[\frac{1}{5}+\frac{1}{3}\frac{H_L}{H}+\frac{1}{3}\left(\frac{H_L}{H}\right)^2\right]\ln\frac{H_{cr}}{H}-1+\frac{H}{H_{cr}}\right\}. \tag{31}$$

The density of the flux of electrons lost from the belt, S_A, is closely related to the VLF noise intensity I_ω and is given by

$$S_A\cong10^{-4}\Phi(\omega)\,\text{cm}^{-2}\,\text{sec}^{-1}\left(I_\omega=2\cdot10^{-23}\frac{\Phi(\omega)}{\ln\Phi(\omega)}\text{W}\cdot\text{m}^{-2}(\text{cps})^{-1}\right). \tag{32}$$

The spectral characteristic of VLF noise $\Phi(\omega)$ is proportional to the power of the source I_0 of Eq. (29) (cf. [93]).

From the relationships (30)—(32) and other results given in [93], we can deduce the main properties of the steady state of the outer radiation belt in the presence of a kinetic instability:

1. The distribution function depends weakly on the power of the source. The flux density at the top of the belt is $S_L \approx 3\times10^{8} - 6\times10^{9}$ cm^{-2}sec^{-1} for $3 \le L \le 8$.

2. A characteristic feature is the anisotropy in the distribution of electrons in the pitch angle with predominantly transverse velocities, the anisotropy depending on the parameters α and γ of the source function of Eq. (29). The concentration of energetic electrons with $E > E_0$ varies with E_0 in accordance with a power law: $N(E > E_0) \propto E_0^{-y}$, where $y \approx 1/2$. The values of y depend weakly on the properties of the source and on the parameter L.

3. The intensity of the VLF noise I_ω and the density of the flux of electrons lost from the belt S_A are both proportional to the power of the source I_0. For $S_A \approx 10^{5} - 10^{19}$ cm^{-2}sec^{-1}, the characteristic values are $I_\omega \approx 10^{-15} - 10^{-11}$ W.m^{-2}(cps)$^{-1}$. The density of the precipitated electrons S_A depends weakly on S_L. On the other hand, the energy spectrum of these electrons resembles closely the spectrum of the source.

These conclusions are valid for nonrelativistic electrons ($E \ll mc^2$) and for the case when the frequencies in the VLF spectrum are such that $\omega \ll \omega_H$. We again note that, if allowance is made for the possibility of repeated transmission of the VLF waves through a radiation belt, due to partial reflection from the ionospheric layers, the steady-state trapped particle fluxes will be less and the escaping electron fluxes will be greater than those in the case of total absorption of the VLF waves by the ionosphere, which has been considered earlier.

We shall now compare the theory with the experimental data for the outer radiation belt. As mentioned in Sec. 2, the VLF radiation and the electron fluxes in the lower ionosphere are positively correlated, the duration and the time dependence of the intensity being practically identical. Such observations lend impressive support to the electron loss mechanism described here.* The values of S_L, S_A, and I_ω, calculated by means of Eqs. (30)—(32) are in satisfactory agreement with the observational data.[110,111] The absence of a marked accumulation of particles in the radiation belt can be explained in a natural way. Further development of these problems requires information on the source over a wide range of energies.

So far, we have considered the diffusion of electrons by VLF waves, excited by the same electrons. Therefore, the presence of the VLF noise in the radiation belt depends strongly on the form of the electron distribution function and on the concentration of charged particles. However, similar particle precipitation from a radiation belt may be caused by the interaction with whistlers or with components of the VLF radiation generated by other sources (chorus, discrete radiation).

The influence of whistlers on the lifetime of electrons in radiation belts has been analyzed by Dungey[112] and Cornwall.[113] A change in the electron momentum caused by a whistler can be determined from the equation of motion of particles in the field of a plane monochromatic wave.† Similar equations have been solved in a number of papers[114-117] and it has been found that "trapped particles" appear in the wave field. These particles are in resonance with the wave and exchange energy effectively with the wave. The particle momentum varies periodically with time. The amplitude of the momentum oscillates about a value corresponding to the exact resonance condition. The period of these oscillations τ^* is of the same order of magnitude as the time needed for the establishment of a plateau in the distribution function in the quasi-linear approximation.[117]‡ The width of the region of the resonant interaction with the wave is given by the following expressions which apply, respectively, to the longitudinal momentum $(\Delta p_\parallel)^*$ and the period τ^* of waves propagated along the direction of **H**:

$$(\Delta p_\parallel)^* = \left(2\frac{\omega_H}{\omega}\frac{h}{H}\frac{\beta_t}{n}\right)^{1/2} mc, \tag{33}$$

$$\tau^* = \frac{2\pi\sqrt{H}}{(nh\omega_H\omega\beta_t)^{1/2}}, \tag{34}$$

where h is the amplitude of the magnetic field of the wave, $\beta_t = v_t/c$. The expression (33) allows us to estimate the maximum change in the electron velocity. The lifetime of electrons in the belt can be estimated approximately on the basis of the following considerations. For an electron to escape from a geomagnetic trap, there must be a considerable change in the adiabatic invariant $\Delta\mu \approx \mu$. For $\omega \ll \omega_H$, $E \approx$ const and then

$$\left|\frac{\Delta\mu}{\mu}\right| \sim 2\left|\frac{\Delta p_\perp}{p_\perp}\right| \sim 2\left|\frac{\Delta p_\parallel}{p_\parallel}\right|.$$

For the interaction with one whistler $|\Delta p_\parallel/p_\parallel|_1 \ll 1$, and therefore an electron may escape by interacting with a large number of whistlers. Since the phases of different whistlers are random, μ will vary randomly. If ν is the frequency of "collisions" of a particle with whistlers, then, in a time t, μ will change by the relative amount

$$\left|\frac{\Delta\mu}{\mu}\right| \sim \sqrt{\nu t}\left|\frac{\Delta\mu}{\mu}\right|_1.$$

Using Eq. (33) and the condition $|\Delta\mu/\mu| \approx 1$, we obtain the following expression for the lifetime of an electron in a trap τ_e

$$\tau_e \sim \frac{\beta_t n\omega H}{8h\nu\omega_H}. \tag{35}$$

Selecting as typical values $h \approx 0.05\gamma$ and $\nu \approx 1$ whistlers/min for $L \approx 3.5$ and $E \approx 10$ keV, we find that $\tau_e \approx 22$ hours.

The inhomogeneity of the geomagnetic field and the wide range of whistler frequencies mean that practically all the electrons in the radiation belts interact with whistlers (cf. beginning of the present section). However, this reduces the resonance interaction time τ_{in} of a given electron with the wave. The time τ_{in} can be easily determined from the Doppler condition. During this time, the velocity of a particle should change, due to a change in the properties of the medium or a change in the wave frequency, from the resonance value $c(\omega - s\omega_H)/\omega n \cos\alpha$ by an amount greater than $(\Delta p_\parallel)^*/m$ [cf. Eq. (33) which defines the resonant interaction region]. This will destroy the electron-wave resonance. For example, if the field H depends on the coordinate z, then τ_{in} is

$$\tau_{in} \cong 2\sqrt{\frac{2n\omega h\beta_t}{\omega_H H}}\left[\frac{1}{H}\frac{dH}{dz}\left|\frac{d\omega}{dk}\right|\right]^{-1}.$$

For $\tau^* < \tau_{in}$, the lifetime is given by Eq. (35), as be-

*We should, however, bear in mind that particles causing auroras may make a contribution to the observed VLF radiation. In this case, there again should be a correlation between the precipitated electron flux and the VLF radiation.

†The phase of a wave must be allowed for in quasi-monochromatic signals (whistlers, discrete VLF radiation). The quasi-linear random phase approximation is then inapplicable.

‡The exception is the case of self-resonant acceleration (for n = 1), when the momentum of a particle in the wave field may increase without limit.[114]

fore. However, if $\tau^* > \tau_{in}$, then $\Delta p_{\parallel} < (\Delta p_{\parallel})^*$ and τ_e increases. A typical value of τ^* is $\approx 10^{-2}$ sec. Estimates show that, at the observed whistler amplitudes, τ_{in} may be larger or smaller than τ^*.

The relationship between τ_{in} and τ^* determines the limits of applicability of the linear approach in the calculation of the Landau damping. If $\tau_{in} \ll \tau^*$, then the decay of whistlers can be determined from the linear-theory formulas, using the geometrical optics approximation. If $\tau_{in} \gg \tau^*$, the distribution function has a plateau and the logarithmic decrement decreases considerably. This has not been allowed for in [118] where the gyroresonant absorption of whistlers in the presence of energetic electrons was considered. Moreover, the anisotropy of the velocity distribution function was not allowed for in [118]. If this anisotropy is taken into account, whistlers may be even amplified if their frequencies are such that $\omega \lesssim \omega_2 = \omega_{HL}[\delta - 1)/\delta]$ [cf. Eq. (19)]. This can partly explain the existence of multiple whistler echoes[5] as well as the cut-off of their spectra at high frequencies.[118] According to the observations, the upper frequency limit of whistlers is $\omega_{lim} \approx (0.4-0.55)\omega_{HL}$. If we substitute this value into Eq. (19), we obtain $\delta = 1.6-2.2$, which is in quite good agreement with some experimental data.[97]

The VLF radiation may, in principle, affect also the lifetime of the proton component of the radiation belts. This is due to the fact that the maximum in the gyroresonance radiation spectrum of slow waves (refractive index $n \gg 1$) shifts toward higher harmonics, even for nonrelativistic protons, and lies in the VLF range. Therefore, in some cases, the VLF radiation may have a greater effect than magnetohydrodynamic waves.[119-121] A detailed analysis of this problem would be very desirable.

In concluding this section, we must mention a number of papers on the role of the spatial variation of the parameters of the medium and of the wave frequency. It has been shown in these papers that in some cases this variation may favor the phasing of particles in the resonant interaction with a wave. Thus, Helliwell and Bell[122] considered the acceleration of relativistic electrons by whistlers. The decrease in the gyrofrequency ω_H due to an increase in the mass m was compensated by a reduction in the whistler frequency. Consequently, the resonance condition in the normal Doppler effect region was retained. Similar problems have been considered in [123,124] An analysis of analogous effect in the presence of spatial variation of the parameters of the medium has been given in [125,126].

5. CONCLUSIONS

From this review, we may conclude that the ways of solving the problems of the origin of various types of the VLF radiation have now become clear. There are sound reasons for assuming that the generation of the VLF radiation is associated with energetic particles in the outer radiation belt and in the auroral zone. Moreover, we may assume that the VLF radiation plays an important role in the dynamics of the radiation belts of the Earth. We shall formulate below a series of problems whose solutions will aid further development of the theory of the VLF radiation.

The development of a quasi-linear and a nonlinear theory of plasma oscillations, allowing for inhomogeneities of the medium, is of great interest. This would make it possible to estimate more correctly the values of the VLF radiation intensity (especially for the discrete types of radiation), as well as the duration of single tones. In the case of the chorus, detailed calculations of the excitation of the VLF radiation in the fronts of magnetohydrodynamic and shock waves, allowing for nonlinear effects, would be of considerable importance. Some progress has been made in this direction in connection with the theory of sporadic radio emission from the Sun. Several nonlinear problems arise in connection with the discovery of the induced VLF radiation. In particular, a nonlinear interaction of waves, in the form of induced scattering or decay, may be important.

In connection with the problems of the origin of the VLF radiation, it must be mentioned that, in addition to electromagnetic waves, there may be intense excitation of plasma waves in the exosphere. These waves may, on the one hand, be transformed into the VLF waves received at terrestrial stations, while on the other hand, they may accelerate charged particles in the exosphere. Investigation of these processes would be very desirable.

Investigation of the interaction of the VLF radiation not only with the electron but also with the proton component of the radiation belts of the Earth would be of definite interest in connection with the dynamics of these belts. As mentioned in Sec. 4, this interaction may be quite effective. Further investigations of the resonant acceleration of particles, allowing for the inhomogeneity of the medium or for slow variations of the wave frequency, also seem promising.

It is clear from the observational data given in Sec. 2 that the intensity of the VLF radiation should change markedly as the radiation is transmitted through the ionospheric layers, including the lower boundary of the ionosphere. Therefore, a theoretical analysis of the efficiency of the leakage of low-frequency waves through the ionosphere is of considerable interest. The published theoretical investigations[127-131] do not solve the problem of the leadage of the VLF waves sufficiently fully. Further calculations are necessary, allowing for all the characteristic features of the interaction between normal-mode waves appearing in connection with departures from the geometrical optics conditions.

On the experimental side, the most important are the investigations of the VLF radiation and of plasma waves by means of AES at great heights, including the central part of the [outer] radiation belt and the boundary of the magnetosphere. It would be desirable to extend the range of the investigated radio waves both toward low and high frequencies. There would be also great interest in combined observations of the VLF radiation, magnetic activity, auroras, charged particle fluxes, as well as further studies of the induced and periodic radiation.

We can mention the well-known similarity of the relationships characterizing the VLF radiation and the sporadic radio emission from the Sun. Thus, for example, the hiss is similar to the enhanced radio emission during magnetic storms, and the chorus is similar to a sequence of type I bursts. Time-varying, relatively narrow spectra are characteristic of both the discrete VLF radiation and type III bursts of the radio emission from the Sun.

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