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# BASIS FOR SPECIAL RELATIVITY THEORY PROVIDED BY EXPERIMENTS IN HIGH ENERGY PHYSICS

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#### 1. INTRODUCTION

The interpretation of geometry advocated here cannot be directly applied to submolecular (smaller than a molecule) spaces ... it might turn out that such an extrapolation is just as incorrect as an extension of the concept of temperature to particles of a solid of molecular dimensions\*.

A. Einstein

THE present state of the theory of elementary particles seems to be very analogous to the one which prevailed in the theory of the atom at the beginning of the twenties of the present century.

Many facts of atomic physics had not only a qualitative but a quantitative explanation on the basis of the ideas of N. Bohr on the quantization of orbits and on the principle of correspondence. However, an investigation of the two-body problem (He atom) already led to a suspicion of the existence of a "non-mechanical" coupling, and there were many other "inconsistencies" which indicated that the theory of N. Bohr was not yet the key to the understanding of the secrets of the intraatomic world.

The solution of the problems and of the riddles of the atom was provided by wave mechanics which fundamentally altered our concepts of the laws of motion of microparticles.

The present theory of elementary particles is based on quantum field theory and on the special theory of relativity. Remarkable successes of quantum field theory are well known in the explanation of such subtle phenomena as the Lamb shift, or, more recently, in the explanation of the systematics of elementary particles on the basis of the theory of unitary multiplets.

On the other hand it is still a fact that, as yet, no complete and exhaustive theory of elementary particles has been created which would be as complete and perfect as quantum mechanics is perfect in the world of atoms and molecules.

In such a situation an investigation of the question of the degree to which the most fundamental principles of the contemporary theory are supported by experimental data can turn out to be very useful.

The present review has as its aim an analysis of the degree of support which experiments in high-energy physics give to the special theory of relativity. The principal reason for concentrating our attention spe-

cifically on the theory of relativity is, of course, not any sympathy with ignorant attacks on this theory, but the circumstance that its basic postulates touch upon the deepest foundations of the physics-geometry of space-time.

At first sight it might appear that for such a critical analysis one might have selected other concepts and ideas of contemporary theory which might, perhaps, be more susceptible to attack, for example: the concept of a field, the concept of a particle, the laws of quantization of a field, etc. However, at the present stage of development of our knowledge it turns out to be very difficult to produce any predictions of results of experiments based on an analysis of some possible modification of these important concepts. Moreover, these concepts are in fact bound up in the closest possible manner with the geometry of space-time. Therefore, the direction of analysis selected by us is, apparently, the most general one.

### 2. THEORETICAL FOUNDATIONS FOR CHECKING THE SPECIAL THEORY OF RELATIVITY IN THE DOMAIN OF HIGH ENERGIES AND SMALL DI-MENSIONS

It is natural that in the period of development of the special theory of relativity A. Einstein showed little interest in the space-time inside elementary particles. In his basic work "On the Electrodynamics of Moving Bodies" A. Einstein regarded the concept of simul-taneity at a given point to be self-evident, emphasizing by this that his main interest was concentrated on the rules for the transformation of physical laws in going over from one reference system to another moving with respect to it  $[2^2]$ .

Another important idea of A. Einstein which we would like to recall in connection with the problems posed by us above is the idea that in reality we are given only the sum "geometry-physics," and not each of the two components individually [1].

In particular, at the basis of the definitions adopted in the special theory of relativity lies the principle of the constancy of the speed of light in vacuo. This principle determines the form of causality and the geometry of the four-dimensional continuum in which microscopic phenomena are situated. Contemporary theory carries over these ideas of the theory of relativity to a different world which is different in prin-

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<sup>\*</sup>Cf. reference [1].

ciple, the world of elementary particles. At the same time the history of science teaches us that a change of scale is usually accompanied by qualitative changes in the physical laws. Therefore, one might expect that there exists a certain elementary length a which serves as a scale for that region of space-time in which the structure of space-time can, in principle, turn out to be different from the one with which we are familiar in the macroworld<sup>[3]</sup>.

However, an elementary length a is by itself insufficient to describe that "magic circle" within which the laws of causality and the metric might turn out to be different from those adopted in the special theory of relativity; we also need a certain four-dimensional time-like vector n (which can be taken to be a unit vector,  $n^2 = 1$ ). The point is that if we start moving from the direction of large scale space-time (and we believe that in the domain of large dimensions the usual relations hold) toward smaller dimensions, then due to the indefiniteness of the metric of the Einstein-Minkowski space-time we shall never be able to specify these dimensions in an invariant manner: in the Einstein-Minkowski space-time the distance between two world points P(x') and P(x'') (x' = t', x'; x''= t'', x'') is measured, as is well known, by the interval  $x^2 \equiv (x' - x'')^2 = (t' - t'')^2 - (x' - x'')^2$ . Therefore, "nearness" of two points P(x') and P(x'') in this case would merely mean that they lie on the light cone (or near it), so that  $x^2 \cong 0$  and that, consequently, they can be connected by a light signal. The interval  $x^2$ "does not know" whether it is small because the differences |t'-t''| and |x'-x''| are separately small (nearness in the usual sense of this word) or because  $|t'-t''| \cong |x'-x''|$ , even though individually these differences are not small.

The introduction of the vector n which for the time being we treat in a purely formal manner enables us to distinguish between these two possibilities and to define the concept of the nearness of two points. Indeed, with the aid of n we can form the invariant [4]

$$R^2 = 2(nx)^2 - x^2 \ge 0 \tag{1}$$

(here  $nx = n_t t - nx$  is the scalar product of n and x) which, in contrast to  $x^2$ , is positive definite and in the system of coordinates in which (n = 1, 0, 0, 0) reduces to

$$R^2 = t^2 + \mathbf{x}^2.$$

Because of its definiteness the quantity R can be adopted as a measure of nearness of two points P(x')and P(x''), and one can assume that for  $R \gg a$  the usual theory holds, while for  $R \cong a$  some type of deviation from the geometry of the special theory of relativity occurs.

In principle the vector n can be of two types: a) it can be associated with the matter of interacting elementary particles (for example, the vector n can be parallel to the total momentum of the colliding particles); in this case we shall refer to n as an internal vector (of the particles or of the system of particles); b) the vector n can be associated with the physical vacuum (this situation occurs, for example, in some theories of quantized space-time); in this case we shall say that n is an external vector. The physical consequences arising from these two possibilities are quite different.

In case a) space-time remains homogeneous and isotropic. Therefore, the laws of conservation of energy-momentum must remain valid. The Lorentz invariance of all the laws referring to the motion of free particles must also be preserved. In this case we must expect deviations from special theory of relativity only inside elementary particles or in their immediate neighborhood. The elementary length a in this case characterizes that region of space-time near a particle or near a system of strongly interacting particles (for example, at the instant of their energetic collision) within which deviations occur from the principles of the theory of relativity.

The nonlinear field theory proposed in the thirties by M. Born<sup>[5]</sup> can serve as a theoretical model for the case under consideration. The homogeneity and isotropy of the physical vacuum is preserved in this theory. However, near particles where the fields and their gradients are large the signals in nonlinear electrodynamics are propagated in accordance with laws different from those assumed in the special theory of relativity: the speed of light becomes variable and can, in general, be greater than the speed of light in vacuo c. The vector n which in this case allows us to define the meaning of the expression "near the particles" is associated with the particles themselves. It should be noted that the space-time metric must be made to agree with the new law of propagation of signals. Physically this means that near the particles the metric tensor  $g_{\mu\nu}$  becomes a function of the field [7]. In a quantum theory of a nonlinear field this could lead to the quantization of the metric tensor itself\*.

It is clear that no direct methods of investigating the metric relations within small regions of spacetime associated with the matter of elementary particles are realizable.

However, it is possible to check whether microcausality coincides with macrocausality. The basis for such a check is the circumstance that the assumption about the identity of microcausality and macrocausality adopted in the theory of relativity leads to definite analytic properties of the amplitudes describing the processes of collision of elementary particles (dispersion relations, relations between the asymptotic

<sup>\*</sup>In this connection one should draw attention to the interesting paper [\*] in which for the first time the author succeeded in obtaining for a two dimensional case the general solutions of the nonlinear theory of M. Born.

cross sections  $\sigma_{\infty}$  which can be checked experimentally).

We now turn to case b) when the vector n is an external one. Here a situation occurs which is quite different from the one which we have just discussed. In this case the vector n is no longer associated with the system of interacting elementary particles but with the physical vacuum. This violates the isotropy of spacetime and, perhaps, also its homogeneity. Therefore, the laws of conservation of energy-momentum and of angular momentum which are consequences of the symmetry of space-time must turn out to be approximate ones.

Further, the existence of a vector n associated with the vacuum selects a particular system of coordinates. Therefore, relativistic invariance will be violated. Microcausality will also be violated. The nature of the possible violations can be explicitly illustrated by an example from crystal optics. For wavelengths  $\star \gg d$  (d is the lattice constant) a cubic crystal is a homogeneous and isotropic medium. The equations for the propagation of an electromagnetic wave in such a crystal are invariant with respect to arbitrary displacements and arbitrary rotations of the coordinates. However for  $\lambda \sim d$  the well-known phenomena of diffraction come into play and the equations for the propagation of the waves retain their invariance only under transformations which reflect the cubic symmetry of the crystal<sup>[9]</sup>. The theory of quantized space-time developed in <sup>[10]</sup>† can serve as a theoretical model illustrating the case under discussion. In this paper it is assumed that the law of addition of momenta  $p_1 + p_2 = p_3$  must be altered in such a manner that the momentum of the particle should not exceed a certain quantity  $2\pi\hbar/l$ , where l is a small elementary length. This can be achieved retaining the validity of the group properties of the theory by replacing the straight line  $-\infty by a circle$ of radius  $\hbar/2\pi l$ . Then we have

$$p_1 + p_2 = p_3 + \operatorname{mod}\left(\frac{2\pi\hbar}{l}\right).$$

This theory leads to a layered structure of spacetime: the time coordinate parallel to a certain unit time-like vector  $\lambda$  (our vector n) assumes only discrete values:  $\tau = ml$ . In this scheme the homogeneity of space-time is violated and a particular system of coordinates is picked out (the system in which  $\lambda = 1, 0, 0, 0$ ).

In view of the smallness of the length l the violations of the conservation laws must be quite considerable, but in virtue of the principle of correspondence with the present theory they must be rare.

In connection with the violation of CP-invariance in the decay of the  $K_2^0$ -meson (cf. Sec. 5 of this review) assumptions were proposed regarding the exis-

tence of a "cosmic field" which might violate the homogeneity of space-time (cf., for example, <sup>[13]</sup>). In view of the smallness of such a field and of its great degree of homogeneity the violations of the conservation laws associated with such a field could be very small.

Thus, in the case under consideration of an "external" vector in addition to checking microcausality one should also check experimentally the homogeneity and isotropy of space-time. Practically such a check can be carried out by verifying the laws of conservation of energy-momentum and of angular momentum, and also by studying the domain of applicability of Lorentz transformations.

#### 3. VERIFICATION OF MICROCAUSALITY. DISPERSION RELATIONS

The dispersion relations constitute a linear integral relation between the real part D and the imaginary part A of the scattering amplitude:

$$T(s, t) = D + iA \tag{2}$$

(here, as usual,  $s = p^2$  is the square of the total momentum,  $t = -q^2$ , where q is the transferred momentum). The dispersion relations are derived on the basis of the analytic properties of the amplitude T in the plane of the complex variable s (for a given t).

These analytic properties follow from the fundamental principles of a local theory [14]:

I. The validity of microcausality, which means the absence of an influence by one region of space-time  $(x_1, t_1)$  on another one  $(x_2, t_2)$  if they are separated by a space-like integral

$$x^{2} = (t_{2} - t_{1})^{2} - (\mathbf{x}_{2} - \mathbf{x}_{1})^{2} < 0$$
(3)

or if the interval is time-like:

$$x^{2} = (t_{2} - t_{1})^{2} - (\mathbf{x}_{2} - \mathbf{x}_{1})^{2} > 0, \qquad (3')$$

but  $t_2 < t_1$  (the effect should follow the cause!).

II. The existence of a spectrum of stable particles of positive energy

$$E = p_0 = + \sqrt{p^2 + m_k^2};$$
 (4)

 $m_k$  is the mass of the particles, k = 1, 2, ...

III. The amplitude T(s,t) for  $s \rightarrow \infty$  has a growth majorized by the polynomial

$$T(s, t)_{|s| \to \infty} < a \mid s \mid^m, \tag{5}$$

where m is a positive integer.

Experimental verification of the dispersion relations amounts to a verification of the basic assumptions of a local theory quoted above. The most significant one is statement I concerning microcausality.

Any violation of this principle leads to the appearance of additional singularities in the complex plane of s or violates restriction III. However, there exists no general proof that restriction III is a consequence of

<sup>\*</sup>Cf. also [11] and the collection of articles [12].

the first two hypotheses of the local theory. From the known experimental facts it follows that the cross sections do not increase for large s and do not oscillate. This can be regarded as a definite indication of the validity of hypothesis III.

For an experimental verification of the dispersion relations it is important that they should contain only quantities which are known experimentally and should not contain unobservable quantities.

This necessary property is possessed by the dispersion relations for the amplitude for the scattering of  $\pi$  mesons by nucleons for forward scattering at  $t = 0^{\lfloor 14 \rfloor}$ . Therefore, only this scattering is suitable for checking such a point of principle of the theory as microcausality. The dispersion relations for the scattering of mesons by nucleons have the form

$$D_{\pm}(E) = \frac{1}{2} [D_{+}(m) - D_{-}(m)] \pm \frac{E}{2m} [D_{+}(m) + D_{-}(m)]$$

$$+ \frac{g^{2}}{M} \frac{1}{\frac{4m^{2}}{M^{2}} - 1} \frac{k^{2}}{\frac{m^{2}}{2M} \mp E} + \frac{k^{2}}{4\pi^{2}} P \int_{m}^{\infty} \frac{dE'}{k'} \left[ \frac{\sigma_{\pm}(E')}{E' - E} + \frac{\sigma_{\mp}(E')}{E' + E} \right]$$
(6a)

$$D_{0}(E) = D_{0}(m) + \frac{g}{M^{2}} \left(\frac{m}{2M}\right)^{2} \frac{k^{2}}{\left[1 - \frac{m^{2}}{4M^{2}}\right] \left[E^{2} - \left(\frac{m^{2}}{4M}\right)^{2}\right]} + \frac{k^{2}}{\pi^{2}} P \int_{m}^{\infty} \frac{E' \, dE'}{k'} \frac{\sigma_{0}(E')}{(E'^{2} - E^{2})} , \qquad (6b)$$

where  $D_{\pm}$ ,  $D_0$  are the real parts of the amplitude for  $\pi^{\pm}$  and  $\pi^0$  mesons, m is the meson mass, M is the nucleon mass,  $g^2$  is the coupling constant (the usually adopted constant  $f^2 = 0.08$  is related to  $g^2$  by the equation  $f^2 = m^2 g^2/4M^2$ ), and  $k^2 = E^2 - m^2$ . The imaginary parts of the amplitudes are expressed with the aid of the optical theorem in terms of the total cross sections  $\sigma_{\pm}$  and  $\sigma_0$  for the interaction of  $\pi^{\pm}$  and  $\pi^0$  mesons with nucleons.

We now turn to the experimental data. The real part of the scattering amplitude for strongly interacting particles for small angle scattering was first discovered in the study of pp-interactions by means of an original method developed at Dubna<sup>[15]</sup>. Later the real part of the scattering amplitude was also discovered and measured for  $\pi p$  scattering<sup>[16-19]</sup>. Particularly complete data were obtained at Brookhaven<sup>[19]</sup>.

A comparison of the theory with the experimental data at high energies was carried out in <sup>[20-23]</sup>. Analysis has shown that numerical results obtained from the dispersion relations for the real part of scattering amplitude D(E) depends on assumptions on the asymptotic behavior of the total cross section  $\sigma_{\pm}(E)$  for  $E \to \infty$ .

In the calculations of V. S. Barashenkov the following formula was utilized

$$\sigma_{\pm}(E) = 22.5 + \frac{c_{\pm}}{(E-m)^n}$$
 mb (7)

where the index n = 0.5,  $\sigma_{\pm}(\infty) = 22.5$  and the con-

stants  $c_{\pm}$  were selected on the basis of known data for  $E = 19 \text{ GeV}^{[24,25]}$ . The choice of the index was checked by means of the formula for the total cross section for the charge exchange process  $\pi^- + p \rightarrow \pi^0 + n$ , which is expressed by the formula<sup>[21]</sup>

$$\sigma_{\text{charge exch.}}(E) = 5 (D_{+} - D_{-})^{2} + \frac{0.05}{(4\pi k)^{2}} (\sigma_{+} - \sigma_{-})^{2}, \qquad (8)$$

where D and  $\lambda$  are measured in Fermi units, and the cross sections are measured in mb. Since this cross section contains differences of the real parts and differences of the total cross sections it is particularly convenient for checking the choice of the index n in the formula (7). n was taken equal to 0.5, and the results are altered very little if n lies in the range 0.3-0.7.

In the calculations both of [20,21] and of [22,23] it was assumed that the real part D(E) of the scattering amplitude increases for  $E \rightarrow \infty$  not faster than E. Physically this amounts to the very probable assumption that the real parts of the phases tend to zero for  $E \rightarrow \infty$ . Therefore, the coefficient B of E in the asymptotic expansion of the amplitude

$$D(E) = AE \ln E + BE \tag{9}$$

must vanish. This assumption leads to the correct value of the coupling constant  $f^2 = 0.08 + 0.003$  which is obtained independently by the method of G. Chew from measurements at low energies. It should be noted that the vanishing of the constant B can be regarded as a consequence of present experimental data: within the presently available limits of error B = 0.

In addition to the calculations already published at present there exists a more extensive comparison of experimental data with theory carried out in Dubna in the group of V. S. Barashenkov. In these calculations the latest data on the behavior of total cross sections at E = 20 GeV were utilized. In subsequent discussion we use the results of these calculations.

The results of the calculations of the real part D(E)of the forward scattering amplitude are shown in Fig. 1 which shows the ratio of the real part to the imaginary part  $\alpha_{\pm}(E) = D_{\pm}(E)/A_{\pm}(E)$  for  $\pi^{+}p$  and  $\pi^{-}p$  scattering. As can be seen from the diagram the experimental curve for  $\pi^{-}p$  scattering in the range 7-25 GeV approaches zero much more steeply than the theoretical curve.

The fact that the points for  $\pi^+ p$  scattering lie above the points for  $\pi^- p$  scattering is particularly unsatisfactory as this is even in qualitative disagreement with the theory which predicts just the opposite relationship which, moreover, does not strongly depend on assumptions about the asymptotic behavior of the total cross sections.

Recently in [26,27] the following inequality was derived from the dispersion relations by integrating them

$$\int_{E'}^{E} \frac{D(E') - D(m)}{E'^2} dE' > \frac{1}{\pi} \int_{m}^{E_2} A(E') \ln \left| \frac{(E_1 + E)(E' - E_1)}{(E' - E)(E' + E_1)} \right| dE', \quad (9')$$

where

$$D(E) = \frac{1}{2} [D_{+}(E) - D_{-}(E)] -$$

-the pole term, and

$$A(E) = \frac{k}{8\pi} [\sigma_+(E) - \sigma_-(E)].$$

This inequality is convenient because it contains no singularities in the integrand and does not require a knowledge of the imaginary part of the amplitude A for  $E \rightarrow \infty$ .

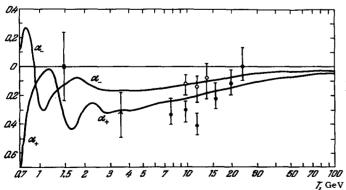
The authors of the inequality (9') give an estimate for it basing themselves on the known data in the range  $E_1 = 4$  GeV,  $E_2 = 30$  GeV adopting the experimental value  $\alpha = -0.2$ . They point out that  $\alpha = -0.33$  would already lead to a contradiction with the inequality (9'). They have also carried out an extrapolation in the region of 160 GeV and they note that a contradiction with the inequality (9') is obtained for  $\alpha = -0.2$ , and if it decreases not too rapidly at energies above 30 GeV. Thus, the situation is very critical but is not yet quite definite.

In concluding this section we note that in the case of violation of microcausality the dispersion relations can be preserved but they would be somewhat altered in form. In <sup>[28]</sup> dispersion relations are given for a definite type of an acausal theory. Specifically, it is assumed that all the local dependences of the radiation operators  $S_C(x_1 - x_2, x_2 - x_3, ...)$  on the points when the particles "enter" and "emerge"  $x_1, x_2, x_3...$  are replaced by nonlocal ones:

$$S_{a}(x_{1}-x_{2}, x_{2}-x_{3}, \ldots) = \int S_{c}(x_{1}-x_{2}-\xi_{1}, x_{2}-x_{3}-\xi_{2}, \ldots)$$

$$\times \varrho(\xi_{1}, n) \varrho(\xi_{2}, n) \ldots d^{4}\xi_{1} d^{4}\xi_{2} \ldots, \qquad (10)$$

where  $\rho(\xi, n)$  is a weighting function which depends on a certain timelike vector n and which vanishes for  $|\xi| \gg a$ , a is an elementary length (cf. Sec. 2). Further, it was assumed that: a) the spectral conditions remain



a. 7

unchanged (condition II of the local theory), b) definite symmetry conditions exist between retarded and advanced functions. The microcausality condition I is violated only in a small region.

In view of the properties of  $\rho(\xi, n)$  which follow from a), b), and I, condition III of the local theory is not violated.

In the simplest case in such a scheme poles appear on the imaginary axis at a distance  $\Omega = 1/a$  from the origin. This leads to the fact that the real part of the scattering amplitude  $D_{\pm}^{a}(E)$  for charged mesons which is expressed by formulas (6) in a local theory acquires an additional term  $\psi_{\pm}(E)$ :

$$D^{a}_{\pm}(E) = D_{\pm}(E) + \psi_{\pm}(E),$$
 (11)

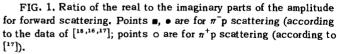
where we take  $D_{\pm}^{a}(E)$  to stand for the real part of the acausal amplitude, and we take  $D_{\pm}(E)$  to stand for its expression in terms of the usual causal dispersion realations (6), as if there were no violations of micro-causality, and, finally,  $\psi_{\pm}(E)$  denotes additional terms brought about by a violation of microcausality in the region  $\mathbf{x}^{2} + t^{2} \sim a^{2}$ . These additional terms are due to the singularities of the function  $\rho(\xi, n)$  and have the form

$$\psi_{\pm}(E) = \frac{k^2}{2\sqrt{\Omega^2 + m^2}} \frac{1}{1 + \frac{E^2}{\Omega^2}} \times \left\{ \frac{D_{\pm}^a(\Omega) + D_{\pm}^a(\Omega)}{\sqrt{\Omega^2 + m^2}} \pm \frac{E}{\Omega} \frac{\sigma_{\pm}(\Omega) - \sigma_{\pm}(\Omega)}{4\pi} \right\},$$
(12)

and for the charge exchange amplitude the additional term has the form

$$\psi_{\text{charge exch.}}(E) = \frac{1}{2\sqrt{2}} \frac{k^2}{\sqrt{\Omega^2 + m^2}} \frac{1}{1 + \frac{E^2}{\Omega^2}} \frac{E}{\Omega} \frac{\sigma_+(\Omega) - \sigma_-(\Omega)}{4\pi} .$$
(12')

From (12) and (12') it can be seen that due to the presence of the term proportional to  $\pm (E/\Omega)$  the real part  $D^{a}(E)$  can even change its sign for  $E > \Omega$  since  $\sigma_{-} < \sigma_{+}$ . If  $\Omega$  is large this will occur in the region of appropriately high energies E where the real part  $D^{a}_{+}(E)$  itself can already be very small.



# 4. VERIFICATION OF ASYMPTOTIC RELATIONS BETWEEN CROSS SECTIONS

In <sup>[29]</sup> a relation was established for the first time between the total cross sections for the interaction of  $\pi^{\pm}$  mesons with nucleons at high energy. Specifically, from the dispersion relations it was deduced that for  $E \rightarrow \infty$ 

$$\sigma_{\pi^+N} = \sigma_{\pi^-N},\tag{13}$$

independently of whether the scattering occurs on a proton or on a neutron. This statement which is known as the Pomeranchuk theorem, was extended in a series of papers also to cross sections of other particles and antiparticles (cf. [30-33]).

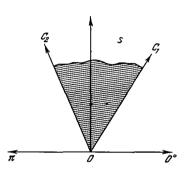


FIG. 2. The complex s-plane.  $C_1$  and  $C_2$  are the limits to which f(z) tends along the rays OC<sub>1</sub> and OC<sub>2</sub>. The segment of the real axis  $00^{\circ}$  is for the reaction a + b = c + d, while the segment  $O\pi$  is for the cross reaction  $\overline{c} + b = \overline{a} + d$ .

Recently in order to establish asymptotic relations use was made of the Phragmén-Lindelöf theorem which determines the behavior of an analytic function inside an angle, formed by two rays, depending on the limits  $C_1$  and  $C_2$  to which it tends along these rays (Fig. 2). From this theorem it follows that if this function had different limits along the boundaries of the angle then it would grow inside the angle not slower than an exponential. If for this angle we take the angle  $\pi$  (the upper half-plane) then for  $s \rightarrow \infty$  we will be concerned with the amplitude for the process

$$a+b=c+d, \tag{14}$$

while for  $s \rightarrow -\infty$  we will be concerned with the amplitude for the cross process

$$\overline{c} + b = \overline{a} + d. \tag{14'}$$

If the amplitude satisfies the requirements I-III of a local theory then the limits for  $s \rightarrow \pm \infty$  are equal, and, consequently, the cross section for processes (14) and (14') are also equal<sup>[34]</sup>. With the aid of this theorem not only a more rigorous proof was given for previously known asymptotic relations for the total cross sections of some processes, but many new relations were also obtained particularly for differential cross sections<sup>[35]</sup>. We reproduce here the most important relations. a) For total cross sections:

$$\pi^{+} + p = \dots \text{ and } \pi^{-} + p = \dots,$$

$$K^{+} + p = \dots \text{ and } K^{-} + p = \dots,$$

$$p + p = \dots \text{ and } \overline{p} + p = \dots,$$

$$\Sigma^{+} + p = \dots \text{ and } \overline{\Sigma}^{-} + p = \dots$$
(15)

b) For differential cross sections: for  $s \rightarrow \infty$  and for a given momentum transfer t the differential cross sections for the following processes are equal to one another:

$$\begin{array}{c} \pi^{+} + p \longrightarrow \pi^{+} + p \quad \text{and} \quad \pi^{-} + p \longrightarrow \pi^{-} + p, \\ K^{+} + p \longrightarrow K^{+} + p \quad \text{and} \quad K^{-} + p \longrightarrow K^{-} + p, \\ \pi^{+} + p \longrightarrow K^{+} + \Sigma^{+} \quad \text{and} \quad K^{-} + p \longrightarrow K^{-} + \Sigma^{+}, \\ \pi^{-} + p \longrightarrow K^{0} + \Lambda \quad \text{and} \quad \overline{K}^{0} + p \longrightarrow \pi^{+} + \Lambda \end{array} \right\}$$
(16)

etc. (We note that the superior bar is used to denote antiparticles.)

The theoretical significance of an experimental verification of these asymptotic relations is the same as that of a verification of the dispersion relations since the same assumptions of a local theory lie at the basis of the derivation of both sets of relations.

A comparison of the deductions from the theory with experimental data is made difficult by the fact that the theory does not predict the manner in which the cross sections approach their limiting value for  $E \rightarrow \infty$ . Therefore, it is not possible to indicate the energy E for which asymptotic relations of the type (13), (15) and (16) should hold with a good degree of accuracy. We can only investigate their tendency to do so.

According to the data which were reported at the conference on high energy physics in Dubna in the summer of  $1964^{[24]}$  (cf. also <sup>[25]</sup>) the total cross sections for cross processes in the neighborhood of E = 20 GeV still differ appreciably from each other.

Figure 3 shows curves for some processes from which it can be seen that cross sections for  $\pi^{\pm}p$  processes differ by 2 mb for E = 16-20 GeV, by 4 mb for K<sup>±</sup>p, and by 10 mb for pp and pp. In other words, at an energy of E = 16-20 GeV the asymptotic value of the cross section common to the cross processes has not yet been reached. Some curves (K<sup>±</sup>p) seem to

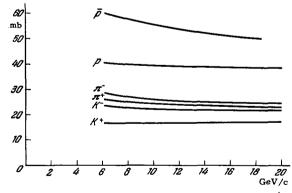


FIG. 3. Total cross sections for the processes pp,  $p\bar{p}$ ,  $\pi^{\pm}p$  and  $K^{\pm}p$  (according to the data of  $[^{24}, ^{25}]$ ).

emphasize that there exists a tendency to approach different limits.

The study of the differential cross sections at high energies is in a considerably less satisfactory state, the accuracy with which these quantities have been measured is significantly smaller than the accuracy with which the total cross sections have been measured, and, therefore, we shall not discuss these data.

It is clear that further investigations of the limiting cross sections would present a particularly important problem for future accelerators.

# 5. VERIFICATION OF THE HOMOGENEITY AND ISOTROPY OF SPACE-TIME

The conservation laws had a fundamental significance in the physics of the last century. Modern theory ascribes a more fundamental significance to symmetries and to group properties.

From this point of view the conservation laws are consequences of definite symmetries. In particular, the laws of conservation of energy-momentum and of angular momentum are a consequence of the homogeneity and isotropy of space-time.

Violations of such homogeneity and isotropy should lead to violations of the conservation laws. In this connection it is appropriate to recall that in the general theory of relativity which deals with inhomogeneous Riemann space there exists no local conservation law for energy-momentum.

But let us return to our topic. What is the degree of accuracy and detail with which proofs have been given of the laws of conservation of energy and momentum with reference to the world of elementary particles in the domain of very high energies? It turns out that it is not easy to answer this question since the validity of these fundamental laws is assumed to be self-evident, and for this reason no special experiments have been designed to verify these laws.

And yet one should clearly realize that a possible violation of these laws could be a consequence of a violation of homogeneity and isotropy of space-time in the microworld, and that it is doubtful that there is any basis for making the idea of homogeneity and isotropy of space-time an article of faith for the physicists.

What information regarding the conservation laws can be obtained from modern experiments in the high energy domain? It turns out that the most accurate data refer to elastic collisions of protons, viz.: the accuracy with which relativistic kinematics holds is based on the conservation laws for energy-momentum

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2,$$
 (17)

$$E(\mathbf{p}_{1}) + E(\mathbf{p}_{2}) = E(\mathbf{p}_{1}') + E(\mathbf{p}_{2}'),$$
 (17')

where  $E = +\sqrt{p^2 + m^2}$  amounts in the energy range E = 2-10 GeV to  $\sim 3\%$ .

The corresponding wavelength in the laboratory co-

ordinate system amounts to  $\lambda \sim 10^{-15}$  cm. Further, one can regard as convincing the fact that there are no sharp discontinuous violations of the kinematics (17)-(17') at a 10% level, but they could be present at a 3% level<sup>[36]</sup>. However, there are no experimental data on this subject.

And yet, if there does exist some inhomogeneity of space-time associated with ultrasmall dimensions, say a, then the expected violations of the conservation laws should be considerable:  $\Delta E \sim \hbar c/a$ ,  $\Delta p \sim \hbar/a$ , but they should also be rare. It is obvious that it would be difficult to distinguish between such violations and processes in which neutral particles participate. Particularly in those cases when there would be a "loss" of energy and momentum a possibility would arise of interpreting the events as a result of production of neutral particles (the kinematics in this case admits quite wide possibilities). Therefore, it would be of interest to observe cases of "spontaneous" increase in energy and momentum of particles.

In the case of inhomogeneity of space-time in the microworld one of the coordinate systems might turn out to be a preferred one. In particular, this could mean that phenomena proceed differently in the laboratory system of coordinates and in the center of mass system, naturally, in the domain of sufficiently high energies E comparable to a certain critical energy  $E_0 = \hbar c/a^{[9]}$ .

Very accurate data on this subject are now available in connection with the study of the now famous decay  $K_2^0 \rightarrow \pi^+ + \pi^-$  which has been measured in detail at two energies. The probability of such a decay dW in the usual theory is a function of two invariants:  $s = P^2$  and  $r = P_{p_1} = P^2 - P_{p_2}$ , where P is the fourmomentum of the  $K_2^0$  meson, while  $p_1$  and  $p_2$  are the momenta of the  $\pi^{\pm}$  mesons. Therefore, the probability dW can be recalculated in the usual manner from one reference system into another one, in particular, if it is known for a meson at rest it can be calculated for a meson in motion. The criterion for the validity of such a transformation based on the usual kinematics of the theory of relativity, and on the conservation laws, can be the invariance of the rest mass of the decaying particle, in the present case of the  $K_2^0$  meson.

Experiments carried out at  $K_2^0$ -meson energies of  $E = 1 \text{ GeV}^{[37]}$  and  $E = 10.7 \text{ GeV}^{[38]}$  show that the scatter  $\Delta M$  of the possible values of the meson rest mass M in the former case amounts to  $\sim 0.7\%$ , and in latter case to  $\sim 1\%$  for the same average value of M.

In other words, in this case, just as in the case of elastic scattering of nucleons, relativistic kinematics holds on the average with an accuracy of approximately 1%. One can say just as little about individual deviations as in the case of pp-scattering (they are  $\ll 10\%$ ).

The fact that the  $K_2^0$  meson decays into two pions was reported for the first time at the International Conference at Dubna in the summer of 1964 and created a sensation since it indicated nonconservation of the combined CP-parity.

At present the existence of two-pion decay of  $K_2^0$  mesons has been confirmed by other measurements (cf., for example, <sup>[38]</sup>) and a considerable number of theoretical papers has appeared which give an interpretation of this new phenomenon <sup>[39]</sup>. As yet no phenomena have been found other than the decay  $K_2^0 \rightarrow \pi^+ + \pi^-$  in which nonconservation of CP-parity is observed. Therefore, as yet there exists no sufficient basis for a choice between different theoretical ideas\*.

The aim of the present article does not include a review of the different theoretical explanations of two-pion decay.

In the aspect of interest to us it should be noted that nonconservation of CP-parity implies either a nonconservation of T-invariance (invariance under time reversal  $t \rightarrow -t$ ), or a nonconservation of CPT-invariance.

But CPT-invariance is a direct consequence of a local field theory<sup>[21]</sup> (but the converse has not been proven). Therefore, a violation of CPT-invariance would mean a violation of the basic principles of a local theory and could be related to a violation of the causality principle in the microworld or, speaking more generally, in the geometry of the microworld.

It should be noted that one of the consequences of the CPT-theorem is the equality of the masses m of the particles and  $\overline{m}$  of the antiparticles. At the present time the difference between the masses of the  $K_1^0$  and  $K_2^0$  mesons is known very accurately. It turns out that  $(\overline{m}_{k_1} - m_{k_2})/m_k \leq 10^{-14}$  and this indicates that the operator equation CPT = 1 holds very rigorously (cf. <sup>[39]</sup>).

A nonconservation of only T-invariance could denote a violation of the isotropy of time and would also present a serious difficulty in the logical structure of the present theory at the basis of which lies the concept of isotropic space-time.

If the violation of CP-parity observed in the twopion decay of  $K_2^0$  mesons will turn out to be a sufficiently general phenomenon, then the possibility is not excluded that it can be a consequence not so much of "anomalous" interaction laws, but rather a consequence of an alteration of space-time relationships on a small scale.

In this connection it is appropriate to recall that in some theoretical investigations violation of ordinary parity was associated with possible singularities in the geometry of the microworld [42-43].

# 6. SOME OTHER CHECKS OF THE THEORY OF RELATIVITY

In conclusion we draw attention to some experiments which have been carried out recently and which were designed to check the theory of relativity in the domain of high energies.

In 1957 a group of physicists at Dubna was checking the dependence of the proton mass on their velocity at a kinetic energy of T = 660 MeV. It turned out that the formula from the theory of relativity

$$m_p = \frac{m_0}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}, \qquad (18)$$

holds with an accuracy of  $\Delta m_\beta/m_\beta \sim 0.4\%$ . Electron accelerators of energy up to 6 GeV which are in operation at the present time confirm a still greater accuracy of formula (18). In particular, for  $\Delta m_\beta/m_\beta$ ~  $5 \times 10^{-4}$  a complete dephasing should already have occurred and, consequently, a complete breakdown of the acceleration process\*†.

Swedish physicists at CERN were recently checking one of the most important principles of the theory of relativity—the independence of the velocity of light on the velocity of the source<sup>[44]</sup>. As a source of  $\gamma$  radiation they utilized high energy  $\pi^0$  mesons with  $\beta$ = 0.99955. The velocity  $c_{\gamma}$  of  $\gamma$  rays of energy greater than 6 GeV was measured directly. The presently accepted value of the velocity of light is  $c_0$ = 299,793 ± 1 km/sec. The authors of the paper under discussion obtained  $c_{\gamma}$  = 299,790 ± 40 km/sec. Thus, within the limits of experimental error  $c_{\gamma} = c_0$ . Other similar measurements seem to be less accurate, but they are quoted in the review article<sup>[45]</sup>.

#### 7. CONCLUSION

Experimental data available to present day physics are restricted to dimensions  $a_0 > (\hbar/Mc)(Mc^2/E) \sim 10^{-15}$  cm in the laboratory coordinate system and correspondingly  $\sim 10^{-14}$  cm in the center of mass system.

1. The set of facts which are known in this domain does not contradict relativistic kinematics, and on the average this kinematics holds with an accuracy of approximately 1%.

The dependence of mass on velocity has been verified with considerably greater accuracy (up to 0.01%).

2. Possible large (but of low probability) deviations from relativistic kinematics remain uninvestigated. As has been explained earlier, they could be due to a violation of homogeneity or isotropy of space-time on a small scale. Such deviations could occur within limits of ~1%.

3. More disturbing is the situation with local field theory which is closely related to the assumed form of

<sup>\*</sup>Apparently, the most convincing theory is the one proposed by T. Lee who admits different operations C, P, T for different types of interactions (strong, electromagnetic, and weak)[<sup>40</sup>].

<sup>\*</sup>Private communication from V. P. Sarantsev.

geometry and to causality. Asymptotic cross sections in the region of 20 GeV approach one another "unwillingly," while a comparison of the presently known data on the forward scattering of high energy pions does not agree with the results of calculations by means of dispersion relations.

In view of the importance of this problem it is necessary to make the measurements still more accurate and to perfect the calculations.

If the indicated disagreement between theory and experiment is confirmed it would serve as a serious foundation for a radical revaluation of the basic postulates of contemporary theory.

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Translated by G. Volkoff

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