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STATISTICAL ACCELERATION OF PARTICLES IN A TURBULENT PLASMA

V. N. TSYTOVICH

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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INTRODUCTION

THE most unique characteristic of a plasma is the comparatively rapid generation of oscillations even under the influence of relatively weak perturbations. The presence of diverse instabilities in a plasma* rapidly converts ordered motions in a plasma into disordered oscillations. The plasma then goes over into an excited state, frequently referred to as turbulent. Unstabilized turbulent plasma is encountered very frequently under laboratory conditions. In outer space, the bulk of matter is ionized, i.e., it is a plasma. Cosmic plasma is turbulent both as a consequence of nonstationary conditions and as a result of other factors[2].

The idea that a small fraction of particles can be accelerated in any turbulent medium to high energies has been prevalent for a relatively long time. Such an acceleration originates in the fluctuation fields of the turbulent motion of the medium and is therefore statistical. Statistical acceleration of particles in a turbulent plasma is apparently a very widespread phenomenon. Without dwelling here on the various manifestations of the acceleration effect, which will be treated separately later, we call attention only to the fact that acceleration of cosmic rays can be associated precisely with the presence of plasma turbulence[2,3]. This had been the starting point of almost all investigations of the acceleration of cosmic rays[2].

They were based essentially on the acceleration mechanism proposed by Fermi[4], and the plasma turbulence was considered in analogy with the turbulence of a liquid. The limited character of these assumptions became clear recently. Thus, it was indicated in [5] that particles can be statistically accelerated by electromagnetic waves. Further development of the theory[6-10] disclosed the relative role of different waves in statistical-acceleration effects, especially waves propagating in a plasma both in the absence of external fields and in the presence of electric and magnetic fields. On the other hand, a connection was established between the acceleration mechanisms in question and the Fermi statistical acceleration[4], which turned out to be a particular limiting case of acceleration by low-frequency oscillations in a magnetoactive plasma[12]. Different astrophysical applications were considered in [12-16], while the acceleration of particles in powerful pulsed discharges and others was considered in [17].

Recently, especially as a result of [18, 24], the theory of plasma turbulence has received further development. The success attained in this direction is connected with the use of the notion of a weakly-turbulent plasma regarded as the excited state of a plasma with a large number of superthermal plasmons that interact weakly with one another[19]. It must be emphasized that the weak-turbulence approximation has a broad range of applicability and can describe in many cases the entire dynamics of the plasma turbulization process (for example, when beams of low density pass through a plasma[20]). Effects of wave interaction in a weakly turbulent plasma have by now been treated in a larger number of papers[25-37]. Interest in this question is connected in part with the fact that nonlinear effects can determine the spectra of the stationary turbulence of a weakly turbulent plasma*. The approximation in which the nonlinear effects of wave interaction are neglected is called quasilinear[19].

It is shown in [19] that the quasilinear approximation incorporates, in particular, the effect of static acceleration by longitudinal Langmuir oscillations of a plasma. The nonlinear effects in the acceleration of charged particles by longitudinal plasma oscillations were considered in [37].

The research on turbulent plasma has led to the

*See the review[1] concerning possible plasma instabilities.

*It must borne in mind, however, that the approach used in this case can be adequate only for relatively weak instabilities (small increments).

conclusion that Fermi acceleration is not the most effective of all possible acceleration mechanisms in a turbulent plasma. Even the presence of a magnetic field, which has previously been regarded as necessary for the acceleration, is not essential and may in many cases have in general a weak influence on the acceleration.

On the other hand, the turbulence of a collisionless plasma can differ in many respects from the turbulence of liquids. This is connected, in particular, with the fact that the plasma has, owing to the possibility of charge separation, a larger number of degrees of freedom and can support not only the hydrodynamic-type turbulence that is possible also in liquids, but also a high-frequency turbulence connected with generation of electron oscillations relative to the ions. Furthermore, the development of turbulence can proceed in an entirely different direction in the absence of collisions. A striking difference, for example, is that hydrodynamic turbulence leads in the presence of collisions to fractionalization of the turbulence scale, whereas in a collisionless weakly turbulent plasma the characteristic dimensions of the turbulence may increase.

We report in this article the progress made in the statistical-acceleration theory that results from further development of the theory of weakly turbulent plasma, and describe various applications connected with astrophysical problems and with the possible interpretation of many laboratory experiments *.

The first chapter is devoted to general characteristics of the statistical acceleration. It can be understood by anyone with general knowledge of methods of statistical physics as covered in university courses.

The second chapter is devoted to the theory of statistical acceleration and can be used for a deeper insight into the theory of statistical acceleration, on the one hand, and for reference purposes, on the other. The third and last chapter, devoted to the application of acceleration mechanisms, is aimed at presenting a general picture of the extensive possibilities of applying acceleration mechanisms to vital problems of turbulent-plasma physics.

I. GENERAL CHARACTERISTICS OF STATISTICAL ACCELERATION

1. Some Information on Plasma Turbulence, Mechanisms of Plasma Turbulence Generation

As already mentioned, plasma turbulence is much more varied than the turbulence of liquids, which has been well known for a relatively long time. Therefore the very term "turbulence" calls for a definition when applied to a plasma. A plasma is usually called turbulent if its motion is characterized by excitation of a large number of collective degrees of freedom. As a rule, excitation of collective degrees of freedom in the plasma is accompanied by excitation of random oscillations of the electric field. For example, it is usually easy to excite in a plasma oscillations of electrons relative to the ions. If at some initial instant of time the charges in the plasma become separated, then a force is produced and tends to restore the neutrality of the plasma. The resultant motion of the electrons relative to the ions leads in final analysis to oscillations of the electron density, with Langmuir frequency $\omega_{0e}^2 = 4\pi n_0 e^2/m_e$. The Langmuir oscillations have finite phase velocities $v_{ph} = \omega/k$ (ω —frequency, k —wave number) in the interval $v_{Te} < v_{ph} < \infty$, where v_{Te} is the average electron thermal velocity. Another example may be ion-sound oscillations, in which both ions and electrons participate, and whose phase velocities lie in the interval $v_{Ti} < v_{ph} < v_{Te}\sqrt{m_e/m_i}$ ($T_e \gg T_i$). In the presence of magnetic fields, the spectra of the plasma oscillations become more complicated, and the number of different types of waves increases. If the turbulence is not strong, we can say that different types of waves are excited in the plasma, and that the correlation and interaction of these waves are relatively weak. By virtue of the weak interaction between the waves, they can be regarded in the first approximation as linear.

As usual, an effective means of describing turbulence is to use statistical methods. This is connected with the complicated and involved actual motion of the particles when a large number of waves is excited. The statistical approach makes it possible to trace only the average characteristics of the turbulent motions, to which in fact the greatest interest attaches. The statistical approach is always fruitful when it is required to obtain limited information on the behavior of complicated systems.

As applied to weakly interacting waves in a plasma, the use of the statistical approach leads to the concept of random oscillations or, more accurately, oscillations characterized only by wave amplitudes and random phases. The square $|E_k|^2$ of the amplitude of a wave with wave vector k in a plasma characterizes the intensity (energy) of the oscillations. In quantum language, the wave energy is proportional to the product $\hbar\omega_k$ by the number of quanta N_k. Excitation of plasma oscillations can be regarded as the creation of quanta —plasmons. By way of an example we present the connection between $|E_k|^2$ and N_k for Langmuir quanta.

$$|E_k|^2 = \frac{\hbar N_k}{4\pi^2\omega_{0e}}. \quad (1.1)$$

We note that this representation of a turbulent plasma as an excited state with a large number of super-

*We do not touch on the various applications of the theory of weakly turbulent plasma which are connected with magnetic confinement of plasma, with anomalous diffusion, with shock waves, etc.[22-34]. We consider only those questions which are important for the mechanisms of statistical acceleration. Any references made to papers on nonlinear effects are confined, naturally, to the circle of discussed problems and do not pretend to be complete.

thermal plasmons (under conditions of thermal equilibrium, N_k corresponds to thermodynamic equilibrium values) is not always possible, but only when, on the one hand, in the transparency regions of the plasma, where the plasma damping is small, and, on the other hand, in excitation regions, where the generation increments γ are not too large, $\gamma \ll \omega$. In regions of strong excitation, $\gamma \sim \omega$, the plasmon energy $\hbar\omega_k$ does not have a precise meaning, since the width of the spectrum γ_k is of the order of the frequency ω_k.

In many cases, states with large increments break up rapidly because of the instability itself. At the same time, the case $\gamma \sim \omega$ can lead to the occurrence of strong turbulence in which even nonlinear interaction of the waves is no longer small.

Let us stop to discuss the mechanisms of plasma turbulization. It is obvious that when the plasma is subjected to pulsed action, for example when it is abruptly compressed or when pulses of external fields are rapidly applied, various oscillations should become excited in the plasma, that is, the plasma should become turbulent.

At first glance, the simplest turbulization mechanism is the direct creation of plasmons. Naturally, the most effective is the case when the number of plasmons increases in avalanche fashion. Such a situation is possible when the plasmons are coherently excited, say by a beam of charged particles[88-90]. Let us illustrate how this takes place.

For an avalanche-like increase in the number of plasmons it is necessary that each beam particle be capable of radiating a plasmon. The plasma is a medium in which the particle interaction is weak. This means that the acceleration attainable by an individual particle is small and therefore the radiation from an individual particle due to acceleration is negligible. The most important is the possibility of radiation in the absence of acceleration, under uniform straight-line motion, that is, as a result of the Cerenkov effect. For this purpose it is necessary that the particle velocity exceed the phase velocity of the plasma waves. Langmuir and sound waves in a plasma can have, as noted above, very small phase velocities. Therefore, if the particle velocity is $v > v_{Ti}$ ($T_e \gg T_i$), the emission of ion-sound plasmons becomes possible, and Langmuir plasmons can be emitted when $v > v_{Te}$. A plasmon radiated by one of the beam particles will be further multiplied in avalanche fashion. Indeed, it can induce radiation from another beam particle; as a result two plasmons appear, etc. We have here a complete analogy with the radiation produced in quantum generators, and the beam in the plasma corresponds to the same particle energy (velocity) distribution that corresponds to negative temperature in the terminology of quantum radiophysics.

The increment of the development of the beam instability can be obtained from simple balance considerations. Let $u_p^l(k)$ be the probability of Cerenkov radiation of a plasmon with momentum **k** by a particle having a momentum **p**. The probability of absorption of a plasmon **k** by a particle of momentum **p** − **k** is, according to the principle of detailed balancing, $u_p^l(k)$, and the probability of absorption by a particle of momentum **p** is $u_{p+k}^l(k)$. Consequently, the change in the number of plasmons N_k^l due to induced radiation and absorption is described by the relation

$$\frac{\partial N_k^l}{\partial t} = N_k^l \int [u_p^l(\mathbf{k}) - u_{p+k}^l(\mathbf{k})] f_p \, d\mathbf{p}. \quad (1.2)$$

Here f_p is the plasma particle distribution function; in formula (12) we took account of the fact that the induced radiation differs from the spontaneous radiation by a factor N_k^l.[110] Since $k \ll p$, we have

$$\frac{\partial N_k^l}{\partial t} = \gamma_k^l N_k^l, \quad \gamma_k^l = \int u_p^l(\mathbf{k}) \left(\mathbf{k} \frac{\partial f_p}{\partial \mathbf{p}}\right) d\mathbf{p}. \quad (1.3)$$

We see that the increment is proportional to the derivative of the distribution function with respect to the momentum.

We must stipulate here that this mechanism of beam instability occurs in the case of beams that are sufficiently smeared out with respect to energy. In the case of "monoenergetic" beams a distinction is made between two stages of the development of the beam instability—hydrodynamic and quasilinear[90,18]. The first is characterized by increments of the order of[90]

$$\gamma \sim \omega_{0e} \left(\frac{n_1}{n_0}\right)^{1/3}, \qquad n_1 \ll n_0, \quad (1.4)$$

where n_1 is the beam density, and n_0 the plasma density. The principal role is played by the slower quasilinear stage[90], characterized by increments (1.3) of the order of

$$\gamma \sim \omega_{0e} \frac{n_1}{n_0}. \quad (1.5)$$

During this stage, the growth of the plasma waves can be obtained as a result of processes of induced Cerenkov radiation and absorption of waves by the beam particles.

Along with generation of longitudinal oscillations, the beam can also generate transverse oscillations due to the induced scattering effects[35]. In this case waves with $\omega \gg \omega_{0e}$ can be generated.

Similarly, turbulence is excited by collision of two plasmoids, which are equivalent under certain conditions to interpenetrating beams.

Along with the opposing motions of the plasma currents and the beams, when a major role is played by the motion of the beam electrons relative to the plasma electrons, instability is produced also if a directional current of electrons relative to the ions is produced in the plasma, for example, in a constant external electric field[36]. In this case the increment is of the order of

$$\gamma \sim \omega_{0e}\left(\frac{m_e}{m_i}\right)^{1/3}. \qquad (1.6)$$

Turbulence can also be generated in a plasma by means of shock waves[102]. Turbulization of plasma is possible also in when strong nonlinear waves propagate through the plasma[93,34,94-97]. If, for example, the waves propagate perpendicular to the magnetic field, then current, or motion of the electrons relative to the ions, is produced on its front. If the velocity of the electrons in the wave exceeds v_{Te}, two-stream instability sets in. Usually the development of such an instability is accompanied by a decrease in the electric conductivity of the plasma[98-102]. This is caused by the fact that, figuratively speaking, the electrons are braked by the oscillations which they generate. An upper estimate of the magnitude of such an electric conductivity, which can be called "turbulent," can be obtained by substituting for the characteristic collision time in the expression for the conductivity

$$\sigma = (4\pi)^{-1}\frac{\omega_{0e}^2}{\nu_{col}} \qquad (1.7)$$

the time of development of the two-stream instability $u_{col} \sim \gamma$. Hence

$$\sigma \sim \omega_{0e}/10. \qquad (1.8)$$

This value of σ corresponds approximately to that obtained in experiment[101] and is much smaller than the electric conductivity due to collisions.

The question of the structure of shock waves in a collisionless plasma is closely related to the question of the dissipation mechanism.

The existence of "turbulent" electric conductivity consequently allows us to state that collisionless shock waves can really exist in a plasma. The first hypothesis concerning the possible existence of collisionless shock waves was advanced by R. Z. Sagdeev[93,102]. Collisionless dissipation in shock waves can be connected with other forms of plasma instability[103-105], for example with anisotropy of the distribution function[106].

In a recently published interesting paper, [166] S. I. Syrovatskii considers effects arising near zero points of the magnetic field. It is well known that near such points there occur intense chromospheric flares on the Sun, accompanied by rapid rearrangement of the magnetic field and strong acceleration of the particles. The zero points arising during the motion of sun spots, can be analyzed using a model of two parallel currents. According to Syrovatskii, regions of plasma rarefaction with large magnetic field gradients are produced near the neutral point when the currents are displaced. This induces electric fields. We note here that if these fields exceed the Dreicer field $E_D \approx eL/r^2_{De}$, then intense buildup of oscillations takes place, accompanied by rapid heating of the plasma, In view of the fact that according to (1.8) the electric conductivity of the plasma, decreases rapidly, fast collisionless dissipation of the magnetic fields is possible. The "dynamic" dissipation referred to in[166] calls for very large characteristic fields, which exceed the Dreicer field by many times, in a ratio

$$\eta \approx \frac{v_{Te}^2}{c^2}\frac{1}{L}\frac{l}{r_0},$$

where L is the Coulomb logarithm, $r_0 \approx e^2/m_e c^2 \approx 10^{-13}$ cm is the characteristic dimension of the strong-field region, and v_{Te} the average thermal velocity of the electrons; when $v_{Te}/c \approx 10^{-3}$, $L \approx 10$, and $l \approx 10^6$ we have $\eta \approx 10^{12}$. We can conclude from this that turbulent dissipation of the magnetic fields is produced first. The mechanism considered in[166] must be regarded as an effective plasma-turbulization mechanism. We note that according to [167] intense conversion of the low-frequency sound waves generated in fields $E > E_D$, into high-frequency Langmuir and transverse waves radiated by the plasma takes place when the electron drift velocities relative to the ions exceed $v_{Te}\sqrt{T_i/T_e}$ ($T_i \ll T_e$), The question of the possible dynamic dissipation of magnetic fields, connected with uninhibited acceleration by induced electric fields, must be regarded as still open, since it calls for the analysis of the role of turbulent processes not accounted for in[166].

It must also be noted that turbulence can arise when an intense high frequency field acts on a plasma, namely, if the average velocity of the plasma particle oscillations (say electrons), which is of the order of $v_E = eE_0/m_e\Omega$ (E_0 —amplitude of the high frequency field, Ω —its frequency, m —particle mass) is larger than v_{Te} —the average thermal velocity—then an instability is produced whose maximum increment is of the same order as in a constant electric field $\gamma_{max} \sim \omega_{0e}(m_e/m_i)^{1/3}$.[160] This instability constitutes parametric resonance in the plasma. A somewhat more complicated picture arises when a strong pulsed electric field is applied to the plasma (for example, in different plasma sources). In this case the electric field in the plasma has a complicated time dependence and, roughly speaking, effects similar to those occurring in a high-frequency field can arise in addition to effects that take place in a constant field.

The instability does not disappear if the high frequency field, in spite of having sufficiently large intensity, is still such that $v_E \ll v_T$. The mechanism of plasmon generation then becomes very similar to the Cerenkov effect, namely, the electromagnetic wave of the high frequency field can radiate plasmons because of the nonlinear effects of the decay[76,86,87]. The laws of energy and momentum conservation for the decay, say, of a transverse wave into a longitudinal one, are perfectly analogous to the Cerenkov conditions (we put henceforth c = 1):

$$\omega^t = \omega'^t + \omega^l, \qquad \mathbf{k} = \mathbf{k}' + \mathbf{k}^l; \qquad (1.9)$$

hence

$$\cos\theta = \frac{\mathbf{k}\mathbf{k}^l}{kk^l} = \frac{\omega^t\omega^l}{k^t k^l} + \frac{(k^l)^2 - (\omega^l)^2}{2k^t k^l}. \qquad (1.10)$$

The generation of plasmons as a result of decays is possible not only for the high frequency field proper, that is, for example, at frequencies of the order of plasma frequencies, but also when $\omega \gg \omega_{0e}$, that is, at frequencies which can be close to the optical band. It is interesting to note that in the weak-nonlinearity approximation the nonlinear current describing the generation of waves by two transverse waves has in the absence of a magnetic field only a longitudinal component.

$$j_{\mathbf{k}} = \mathbf{k}\hat{L}\,(\mathbf{E}_1^{\mathrm{t}}\mathbf{E}_2^{\mathrm{t}}). \tag{1.11}$$

This shows that only longitudinal waves can essentially be generated.*

The characteristic time scale of development of the turbulence in a plasma in the presence of an intense high frequency radiation can be obtained in the same manner as that used above for a beam. When $k^t \gg k^l$, the increment takes the form[76] (see (1.3))

$$\gamma_{\mathbf{k}^{\mathrm{l}}} = \int u_{\mathbf{k}^{\mathrm{t}}}(\mathbf{k}^{\mathrm{l}}) \left(\mathbf{k}^{\mathrm{l}} \frac{\partial}{\partial \mathbf{k}^{\mathrm{t}}}\right) N_{\mathbf{k}^{\mathrm{t}}}^{\mathrm{t}}\, d\mathbf{k}^{\mathrm{t}}, \tag{1.12}$$

where $u_{\mathbf{k}^t}(\mathbf{k}^l)$ is the decay probability, and $N_{\mathbf{k}^t}^t$ is the number of transverse quanta. The generation is especially effective in the presence of directed beams of transverse waves[86] ($\Delta\theta \ll (\omega_{0e}/\omega^t)^{3/2}$), and then the growth increment for generation of Langmuir plasmons at $\omega^t \gg \omega_{0e}$ turns out to be proportional to the derivative, with respect to frequency, of the spectral distribution function of the transverse-wave energy $W^t(\omega)$, namely

$$\gamma_{k^{\mathrm{t}}} \approx \omega_{0e} \frac{\pi}{4} \frac{\omega_{0e}^2}{n_0 m_e c^2} \frac{dW^{\mathrm{t}}(\omega)}{d\omega}. \tag{1.13}$$

The generation efficiency increases if the width $\Delta\omega$ of the spectrum of the high-frequency waves is smaller than ω_{0e} ($\Delta\omega \ll \omega_{0e}$); then the development of the instability is accompanied by the appearance in the spectrum of the transverse high frequency waves of satellites that differ from the initial frequency by ω_{0e}.

This effect corresponds to induced Raman scattering of high frequency radiation from the plasma oscillations generated by the high frequency field.

A similar effect takes place when ion-sound oscillations are generated by Langmuir oscillations[126]; in this case the satellites are separated from ω_{0e} by distances of the order of ω_{0i}. The appearance of satellites can serve as a qualitative criterion of turbulence development in a plasma. We note that from the conservation laws (1.9) and (1.10) it follows that for frequencies of the order of ω_{0e} the decay is allowed only when $\omega^t > 2\omega_{0e}$, and at the threshold, when $\omega^t = 2\omega_{0e}$, the phase velocity of the longitudinal waves is equal to $1/\sqrt{3}$. With increasing ω^t the spectrum of the phase velocities of the longitudinal waves broadens and its lower limit can reach v_{Te}. For frequencies ω^t of the order of $2\omega_{0e}$ and larger, in the case of a well collimated beam of transverse waves and when $\delta\omega^t \ll \omega^t$, the increment can be estimated by means of the formula

$$\gamma_k^{\mathrm{l}} = \frac{\pi}{8} \frac{W^{\mathrm{t}}}{n_0 m_e c^2} \frac{(k^{\mathrm{l}})^2 \omega_{0e}^2}{\Delta\omega^{\mathrm{t}} \left\{\omega_{0e}\left[(k^{\mathrm{l}})^2 - \omega_{0e}^2\right] + k^{\mathrm{l}} \cos\theta \sqrt{\left[(k^{\mathrm{l}})^2 - \omega_{0e}^2\right]^2 + 4\omega_{0e}^2\left[(k^{\mathrm{l}})^2 \cos^2\theta - \omega_{0e}^{\mathrm{l}}\right]}\right\}}, \tag{1.14}$$

where θ is the angle between the directions of the longitudinal and transverse waves and W^t is the energy of the transverse waves. The Langmuir waves can also disintegrate into low-frequency sound waves. The increment of the buildup of ion-sound oscillations by Langmuir oscillations can be obtained from simple balance considerations[81,82] (solution of the self-consistent problem[80]):

$$\frac{\partial N_{\mathbf{k}^{\mathrm{s}}}^{\mathrm{s}}}{\partial t} \approx N_{\mathbf{k}^{\mathrm{s}}}^{\mathrm{s}} \int N_{\mathbf{k}^{\mathrm{l}}}^{\mathrm{l}} \{u_{\mathbf{k}^{\mathrm{l}}}(\mathbf{k}^{\mathrm{s}}) - u_{\mathbf{k}^{\mathrm{l}}+\mathbf{k}^{\mathrm{s}}}(\mathbf{k}^{\mathrm{s}})\}\, d\mathbf{k}^{\mathrm{l}}. \tag{1.15}$$

$u_{\mathbf{k}^l}(\mathbf{k}^s)$ is the probability of disintegration of a Langmuir plasmon l into a sound plasmon s. Estimates show that in this case it is relatively easy to satisfy the condition that the decays and coalescences occur in different regions of the wave numbers of the longitudinal waves. This provides an estimate of the increment in the case of a one-dimensional directional spectrum of Langmuir oscillations of width Δk

$$\gamma_k^{\mathrm{s}} \approx \omega_{0e} \frac{W^{\mathrm{l}}}{n_0 m_e v_{Te}^2} \frac{k^{\mathrm{l}}}{\Delta k} \left(\frac{v_{\mathrm{ph}}}{v_{Te}}\right) \sqrt{\frac{m_e}{m_i}}. \tag{1.16}$$

In the case of an isotropic distribution of Langmuir oscillations, when a difference effect is obtained, generation occurs in a limited region of wave phase velocities

$$3\sqrt{\frac{m_i}{m_e}}\, v_{Te} > v_{\mathrm{ph}} > \frac{3}{\sqrt{2}} \sqrt{\frac{m_i}{m_e}}\, v_{Te}. \tag{1.17}$$

Owing to the nonlinear interaction between the plasma waves, this narrow spectrum can become sufficiently broad after relatively short time intervals (see also [25,161]). We note that direct conversion of transverse waves into sound waves is less probable than multistep conversion,* when the transverse waves are consecutively transformed first into Langmuir waves, which in turn are transformed into ion-sound waves. The last process is possible only at phase velocities $v_{\mathrm{ph}}^{l} > 3v_{Te}(m_i/m_e)^{1/2}$, so that when $v_{\mathrm{ph}} \approx 1/\sqrt{3}$ we get for a hydrogen plasma $T_e > 5$ eV. The ratio of the characteristic $t \to l$ and $l \to s$ transformation times is of the order of

$$\left(\frac{c}{v_{Te}}\right)^3 \left(\frac{m_e}{m_i}\right)^{1/2} \gg 1.$$

*The Cerenkov generation of transverse waves by transverse ones, considered by Askar'yan[77], has therefore a higher order in terms of the wave amplitude.

*A similar qualitative deduction was obtained for waves with fixed phase in[83].

Thus, the mechanisms of turbulence generation are quite diverse even in a homogeneous plasma without external magnetic fields. It is important that the different branches of the oscillations turn out to be interrelated as a result of nonlinear effects. In the presence of inhomogeneities and external magnetic fields, new oscillations appear, and on the whole the turbulence development picture becomes quite complicated. A very important role is played, for example, by drift instabilities of an inhomogeneous plasma (see [161]) and by cyclotron buildup of magnetoactive-plasma oscillations. To conclude this section, we refer the reader to reviews [1,161] in which possible plasma instabilities are considered in detail.

2. Statistical Change of Particle State

We present here general considerations concerning statistical acceleration of arbitrary particles (even uncharged ones) without regard to the concrete turbulent states that may actually be responsible for the effects of statistical acceleration in a plasma. The general relations are illustrated by means of two examples: a) acceleration of Langmuir oscillations in a weakly turbulent plasma; b) acceleration by high frequency transverse waves with induced Compton effect.

It is best to speak of the statistical change in the state of a particle in the case when noticeable changes in the parameters are the results of a large number of independent interactions, in each of which the particle can acquire or lose energy in small batches. If, for example, $\Delta\epsilon_+$ is the energy acquired by the particle in an individual interaction act and $\Delta\epsilon_-$ is the energy, approximately equal to $\Delta\epsilon_+$, lost in an individual act, that is, $\Delta\epsilon_+ \approx \Delta\epsilon_- = \Delta\epsilon$, then the particle has a probability W_ν of acquiring as a result of s interaction acts an energy $\nu\Delta\epsilon$, with $0 < \nu < s$. In particular, the probability exists that the particle will acquire a maximum energy $s\Delta\epsilon$. In the general case the particle can acquire an arbitrary energy in the interval $-s\Delta\epsilon < \epsilon < s\Delta\epsilon$. The accelerated particles will consequently have a broad energy spectrum. Systematic acceleration of particles can be the result of two causes: either $\Delta\epsilon_+$ is somewhat larger than $\Delta\epsilon_-$, or the number of interaction acts per unit time, leading to an increase in the particle energy ν_+, slightly exceeds the number of acts involving an energy decrease ν_-.

The foregoing general description of the statistical acceleration corresponds, for Fermi acceleration in particular, to $\Delta\epsilon_+ = \Delta\epsilon_-$ and $\nu_+ > \nu_-$. The individual interaction event in this case is a collision between a charge and a moving magnetic wall. From the elementary kinematics of the collision it follows that in the case of head-on collisions the charge acquires an energy $\Delta\epsilon_+ \approx 2\epsilon uv$, where u is the velocity of the wall, v the velocity of the charge ($c = 1$), and ϵ its energy, and in the case of rear-end collisions the charge loses an equal amount of energy. If l is the average distance between magnetic walls, then the number of head-on collisions is $\nu_+ = (v + u)/l$, whereas that of rear-end collisions is $\nu_- = (v - u)/l$. Therefore the average particle energy increases systematically

$$\left\langle \frac{d\varepsilon}{dt} \right\rangle = \Delta\varepsilon\,(\nu_+ - \nu_-) = \frac{4\varepsilon v u^2}{l}. \tag{2.1}$$

This is the formula for Fermi acceleration, used in a large number of papers[2,3], with l assumed equal to the mean distance between two turbulent elements that carry the magnetic field.

A characteristic feature is that the smaller the turbulence scale l, the more effective the acceleration. This is a general consequence of the increase in the number ν of interactions per unit time, and is not at all connected with the concrete expression (2.1). Thus, when the frequency of the interactions is large, the acquired energy can be much larger than when ν is small, even if in the latter case the particle acquires a greater energy increment in each individual interaction. To determine the efficiency of the statistical acceleration in a plasma, it is important to ascertain the possible existence of small-scale turbulence.

We shall show that it is precisely in a plasma, owing to the possible charge separation, that a turbulence having characteristic scales much smaller than the particle mean free paths can be excited. Even in relatively weak electromagnetic fields, a small-scale turbulence can accelerate particles much more effectively than a large-scale turbulence.*

Statistical acceleration of particles is not necessarily connected with turbulence, and can be realized by means of specially produced fields that vary in random fashion[38,39]. It is possible to construct on this principle accelerators which are called stochatrons[40].

After these general qualitative considerations, let us stop to describe the acceleration process quantiatively. (The theory of random process is covered in an extensive literature[41].) It is simplest to start from intuitive physical considerations. Let $w_p(k)$ be the probability that a particle with momentum p will experience as a result of an interaction a momentum change $\Delta p = \mathbf{k}$, and that the particle energy will change by $\Delta\epsilon = \omega(\mathbf{k})$. Let f_p be the distribution function of the accelerated particles, and let the dependence of the particle energy on the momentum p be described by ϵ_p. After the interaction the particle energy is ϵ_{p-k}. For concreteness we put $\epsilon_{p-k} < \epsilon_p$ and $\epsilon_p - \epsilon_{p-k} = \omega(\mathbf{k}) > 0$. From the principle of detailed balancing we can easily obtain the probability that the particle will acquire an energy $\omega(\mathbf{k})$, namely, $w_p(\mathbf{k})$ is by virtue of the detailed balancing principle the probability that a particle with momentum $\mathbf{p} - \mathbf{k}$ will acquire an energy $\omega(\mathbf{k})$, and consequently $w_{p+k}(\mathbf{k})$ is

*An important question is also the injection in the acceleration mode (see below).

the sought-for probability. As a result of the processes described by the probabilities $w_p(\mathbf{k})$ and $w_{p+k}(\mathbf{k})$, the number of particles with momentum **p** decreases, the inverse processes leading to an increase in the number of particles with momentum **p**. This yields an equation for f_p:

$$\frac{\partial f_{\mathbf{p}}}{\partial t} = -\int [w_{\mathbf{p}}(\mathbf{k})(f_{\mathbf{p}} - f_{\mathbf{p}-\mathbf{k}}) + w_{\mathbf{p}+\mathbf{k}}(\mathbf{k})(f_{\mathbf{p}} - f_{\mathbf{p}+\mathbf{k}})]\, d\mathbf{k}. \quad (2.2)$$

From this equation we readily obtain the Fokker-Planck equation in the case when $|\mathbf{k}| \ll |\mathbf{p}|$. Elementary expansion in terms of the momentum **k** transferred to the particle in the interaction yields

$$\frac{\partial f_{\mathbf{p}}}{\partial t} = \frac{\partial}{\partial p_i} D_{ij} \frac{\partial f_{\mathbf{p}}}{\partial p_j}, \quad (2.3)$$

$$D_{ij} = \int w_{\mathbf{p}}(\mathbf{k})\, k_i k_j\, dk. \quad (2.4)$$

The systematic change in the average energy of the particles is best determined from (2.3). Let $\langle L_p \rangle = \int L_p f_p \, dp$ be the mean value of the arbitrary quantity L. Then we obtain from (2.3) and (2.4), integrating by parts,

$$\frac{\partial}{\partial t} \langle \varepsilon_{\mathbf{p}} \rangle = \langle \dot{E}_{\mathbf{p}} \rangle, \quad (2.5)$$

$$\dot{E}_{\mathbf{p}} \equiv \frac{\partial}{\partial p_j} \int k_j w_{\mathbf{p}}(\mathbf{k}) \left(\mathbf{k} \frac{\partial \varepsilon_{\mathbf{p}}}{\partial \mathbf{p}} \right) dk. \quad (2.6)$$

In the case of "isotropic" collisions, when the probability $w_p(\mathbf{k})$ does not depend on the directions of **k**, the diffusion coefficient is expressed in terms of two invariant tensors:

$$D_{ij} = D^l(p^2) \frac{p_i p_j}{p^2} + D^t(p^2) \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right). \quad (2.7)$$

If the particle distribution function depends furthermore only on the modulus $|\mathbf{p}| = p$, then Eq. (1.5) takes the form

$$\frac{\partial f_{\mathbf{p}}}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D^l(p^2) \frac{\partial f_{\mathbf{p}}}{\partial p}. \quad (2.8)$$

We note that (2.8) is valid also in the case when the energy changes in small batches during the interaction, whereas the momentum direction, and consequently the momentum itself, vary appreciably. In this case Eq. (2.3) is not suitable. This is the situation for Fermi acceleration, since the particle reflected from the magnetic barrier experiences a large change (2p) in its momentum* (see [42]).

In those cases when the particle energy depends only on the modulus of the momentum †, the quantity D^l is a physical characteristic of the particle-energy change. This follows from the form of (2.6); in this case

$$\dot{E}_{\mathbf{p}} = \frac{\partial}{\partial p_i} \frac{\partial \varepsilon_{\mathbf{p}}}{\partial |\mathbf{p}|} \frac{p_j}{|\mathbf{p}|} D_{ij} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D^l(p^2) \frac{\partial \varepsilon_{\mathbf{p}}}{\partial p}. \quad (2.9)$$

If the diffusion coefficient is a power-law function, $D^l = D_0/p^\alpha$, then the acceleration takes place for nonrelativistic particles when $\alpha < 3$, and for relativistic ones for $\alpha < 2$. All the mechanisms described below for statistically changing the particle state, including the Fermi mechanism, satisfy this requirement.

The physical meaning of the coefficient $D^t(p^2)$ is best explained by recognizing that for $\epsilon_p = \epsilon(|\mathbf{p}|)$ it does not lead to a change in the particle energy, but merely changes the particle momentum, that is, it characterizes the particle scattering.

As applied to acceleration in a turbulent medium, the turbulence should be regarded as isotropic if (2.7) is satisfied.

Isotropic turbulence with anisotropic distribution of the accelerated particles (for example, in the presence of particle beams in an isotropically turbulent plasma) can lead to two effects—statistical acceleration D^l and scattering D^t*.

In describing acceleration effects, frequent use is made of the energy distribution function [2]. We define one such function $f(\epsilon)$ such that $f(\epsilon)d\epsilon$ is the number of particles in the interval $d\epsilon$. Obviously,

$$f_{\mathbf{p}} = f(|\mathbf{p}|) = \frac{1}{4\pi p^2} \frac{\partial \varepsilon}{\partial p} f(\varepsilon). \quad (2.10)$$

In the equation

$$\frac{\partial}{\partial t} f(\varepsilon) = \frac{\partial^2}{\partial \varepsilon^2} D^l \left(\frac{\partial \varepsilon}{\partial p} \right)^2 f(\varepsilon) - \frac{\partial}{\partial \varepsilon} \left[\frac{1}{p^2} \frac{\partial}{\partial p} p^2 D^l \frac{\partial \varepsilon}{\partial p} \right] f(\varepsilon) \quad (2.11)$$

which is derived from (2.8) we can see clearly the physical meaning of the individual terms. The last term of (2.11) describes systematic acceleration, and can be expressed in terms of the average increase in energy (2.9):

$$-\frac{\partial}{\partial \varepsilon} \dot{E}_{\mathbf{p}} f(\varepsilon).$$

The first term of (2.11) describes the fluctuation acceleration. Indeed, using this term we can readily obtain for $f(\epsilon) \sim \delta(\epsilon - \epsilon_0)$

$$\frac{d}{dt} \left[\langle \varepsilon^2 \rangle - (\langle \varepsilon \rangle)^2 \right] = 2D^l(p_0^2) \left(\frac{d\varepsilon_0}{dp_0} \right)^2 \equiv G(\varepsilon_0). \quad (2.12)$$

*The result is presented for the case when the magnetic "clouds" or "walls" have an isotropic distribution[42],

$$D^l = \frac{2}{3} \frac{u^2}{l} p\varepsilon.$$

† So far, the considerations advanced were independent of the concrete form of ϵ_p and could correspond, for example, to particles in crystals, where such a dependence can be anisotropic [43]. We make this remark in view of the interest in problems of the so called solid-state plasma.

*This takes place for beams whose energy density $n_1 m v_1^2$ is much smaller than the turbulence energy per cm³ (n_1 – beam density, v_1 – beam velocity). If this condition is not satisfied then, as is well known [19, 20], the particle beams generate the plasma turbulence.

Formula (2.12) describes the increase in the energy scatter following acceleration. A characteristic feature of the statistical acceleration is the appearance of an energy scatter of the accelerated particles.

Equation (2.11) can be written in terms of quantities characterizing the time variation of the average parameters[44]:

$$\frac{\partial f(\varepsilon)}{\partial t}=\frac{\partial^2}{\partial \varepsilon^2}\left[\frac{1}{2}G(\varepsilon)f(\varepsilon)\right]-\frac{\partial}{\partial \varepsilon}[\dot{E}_p f(\varepsilon)]. \quad (2.13)$$

We note that the effect of the average change in particle energy can be obtained directly from (2.2) without assuming that the energy and momentum transferred during the interaction are small compared with the energy and momentum of the particle. We have seen that this case is realized for Fermi acceleration, when the momentum transfer is not small.

For these purposes it is convenient to use the law of energy conservation in the interaction, $\epsilon_p - \epsilon_{p-k} = \omega(\mathbf{k})$. Multiplying (2.2) by ϵ_p and integrating with respect to $\mathbf{p}$ the right side of the resultant expression, we can transform it, by replacing the variable $\mathbf{p}$, to a form containing only f_p:

$$\frac{\partial}{\partial t}\int \varepsilon_p f_p\, dp=-\int [(\varepsilon_p-\varepsilon_{p-k})w_p(\mathbf{k}) + (\varepsilon_p-\varepsilon_{p+k})w_{p+k}]\, f_p\, d\mathbf{p}\, dk. \quad (2.14)$$

Hence[5]

$$\dot{E}_p=\int \omega(\mathbf{k})\,(w_{p+k}(\mathbf{k})-w_p(\mathbf{k}))\,dk. \quad (2.15)$$

This shows that the average change in the particle energy is equal to sum, over all possible interaction events with different momentum transfer, of the product of the transferred energy by the probability of energy acquisition, less the probability of energy loss[5].

We must emphasize finally that the probability $w_p(\mathbf{k})$ can be regarded as a specified external characteristic only if the number of accelerated particles is sufficiently small.

Assuming a small number of accelerated particles (see below), the formulas presented are valid for particles with arbitrary mass, charge, spin, etc. We can thus speak not only of acceleration of charged particles but, for example, of acceleration of photons[45], neutrinos[46], and other neutral particles. In order to illustrate the general relations, we present two examples.

a) Acceleration by Langmuir oscillations. Let us assume that Langmuir waves are excited in a plasma and have a distribution which is isotropic in space and stationary. A possible interaction between a particle and such a turbulent plasma, in which the particle loses or acquires energy, is induced Cerenkov radiation or absorption of a Langmuir quantum. The probability $w_p(\mathbf{k})$ of this process is (for more details see Sec. 4)

$$w_p(\mathbf{k})=\frac{e^2\omega_{0e}}{2\pi\hbar k^2}\,\delta(\omega^l-\mathbf{kv})\,N_\mathbf{k}^l,\qquad \omega^l=\omega_{0e}+\frac{3}{2}\frac{k^2v_{Te}^2}{\omega_{0e}}, \quad (2.16)$$

where N_k^l is given by (1.1) (see Sec. 4). The change in particle momentum during the radiation act is $\hbar\mathbf{k}$. Substituting (2.16) in (2.6) we obtain for nonrelativistic particles

$$\dot{E}_p\approx\frac{\sqrt{m}e^2\omega_{0e}^3}{2\sqrt{2}\varepsilon^{3/2}}\,T_{\text{eff}}(\varepsilon). \quad (2.17)$$

Here $\epsilon = mv^2/2$ is the energy of the accelerated particles, and for the effective plasmon temperature we put

$$T_{\text{eff}}(\varepsilon)=\hbar\omega_{0e}N^l\Big|_{v_{ph}=v=\sqrt{\frac{2\varepsilon}{m}}}. \quad (2.18)$$

In those cases when T_{eff} depends little on ϵ, the acceleration effect decreases with increasing ϵ like $\epsilon^{-3/2}$.* Induced Cerenkov acceleration and cyclotron acceleration play an important role in a plasma. A detailed examination of the different effects of Cerenkov and cyclotron acceleration will be presented in Chapter II.

b) Acceleration by high frequency transverse waves in the induced Compton effect. Assume that intense electromagnetic radiation, consisting of N_k^t transverse quanta, is present in vacuum. The probability of induced Thomson scattering is

$$w_p(\mathbf{kk}')=\frac{N_\mathbf{k}^t N_{\mathbf{k}'}^t}{(2\pi)^3}\frac{r_0^2}{2kk'}\frac{m^2}{\varepsilon_p^2}(1+\cos^2\theta')\,\delta(k-k'-(\mathbf{k}-\mathbf{k}')\mathbf{v}),$$

$$r_0^2=\frac{e^2}{mc^2}. \quad (2.19)$$

Here θ' is the angle between $\mathbf{k}$ and $\mathbf{k}'$ in the reference frame in which the charge is at rest:

$$\cos\theta'=1+\frac{m^2(\mathbf{kk}'-kk')}{\varepsilon_p^2(k-\mathbf{kv})(k'-\mathbf{k}'\mathbf{v})}, \quad (2.20)$$

$\epsilon_p = \sqrt{p^2+m^2}$ is the particle energy. The change in particle momentum during the scattering act is $\hbar(\mathbf{k}-\mathbf{k}')$, and consequently the diffusion coefficient (2.4) is

$$D_{ij}=\int w_p(\mathbf{k},\mathbf{k}')\,(k_i-k_i')\,(k_j-k_j')\,d\mathbf{k}\,d\mathbf{k}'\cdot\hbar^2. \quad (2.21)$$

The quantity $D^l = (v_iv_j/v^2)D_{ij}$, which determines the acceleration effect, is by virtue of (2.19) equal to

$$D^l=\frac{1}{v^2}\int (k-k')^2\,w_p(\mathbf{k},\mathbf{k}')\,d\mathbf{k}\,d\mathbf{k}'\cdot\hbar^2. \quad (2.22)$$

For isotropic radiation, that is, when N_k^t depends only on the modulus of $\mathbf{k}$, we can easily integrate (2.22) and (2.19) over the angles. For nonrelativistic velocities $v \ll 1$ we have

*In the case of intense nonlinear wave transfer, $T_{\text{eff}}(\epsilon)$ can be a rapidly growing function of ϵ, and E_p may increase with ϵ (Sec. 6b).

$$D^1 \approx \frac{16}{9} r_0^2 \int \frac{N_k^2 k^4\, dk}{2\pi} \hbar^2. \qquad (2.23)$$

From (2.9) we obtain the systematic acceleration

$$\dot{E}_p = \frac{16}{3m} r_0^2 \int \frac{N_k^2 k^4\, dk}{2\pi} \cdot \hbar^2. \qquad (2.24)$$

For almost monochromatic radiation with a spectral width $\Delta k \ll k$ we get from (2.24) the order-of-magnitude relation

$$\dot{E}_p \approx \omega \cdot \frac{8\pi^3}{3} \left(\frac{k}{\Delta k}\right) \frac{W^t \lambda\!\!\!^{-3}}{mc^2} W^t r_0^2 \lambda\!\!\!^{-}, \qquad (2.25)$$

where $W^t \sim (E^2)/4\pi$ is the energy of the transverse waves per cm^3, and $\lambda\!\!\!^{-} = c/\omega = 1/k$.

Acceleration due to induced scattering also plays an important role in a plasma. Different types of waves can be scattered in this case, particularly Langmuir waves (Sec. 6). It must be emphasized that in those regions where the conservation laws allow Cerenkov or synchrotron-radiation acceleration, the effects of acceleration due to induced scattering describe only corrections, which are usually small, but which help clarify a very important question, that of the maximum oscillation intensities up to which it is still meaningful to speak of Cerenkov acceleration. As a rule, these intensities are quite high (see Sec. 6). However, acceleration due to induced scattering plays an important role in those cases when the Cerenkov and cyclotron accelerations are forbidden by conservation laws. For definite types of waves, for example transverse waves, which have in an isotropic plasma phase velocities larger than the speed of light, such forbiddenness always exists. For Langmuir waves, on the other hand, the Cerenkov condition is not satisfied for particles whose velocities are much smaller than v_{Te}, as is quite frequently the case for ions (Sec. 6). It must be noted that in a plasma the scattering cross sections are appreciably altered by the fact that any charge is surrounded by a screening cloud of charges of opposite sign, and this screening charge also produces scattering. As a result, besides the usual Compton scattering due to the vibrations of the charge, nonlinear scattering is produced in the field of the scattered waves[64] (see Secs. 6 and 9).

The two foregoing examples are typical of different accelerating mechanisms in a weakly turbulent plasma.

3. Spectra and Average Energies of Accelerated Particles

In order to disclose the main characteristics of statistical acceleration in a plasma, let us consider the spectrum of accelerated particles and its time variation. We note that the form of the spectrum depends on the possible energy losses, which under real conditions can be quite varied (bremsstrahlung, synchrotron radiation, ionization losses, escape of particle from the acceleration region, etc[15]). Equilibrium may be established between the acceleration and the energy losses, and this leads to certain stationary spectra of the accelerated particles. An appreciable role can be played by nonlinear effects of wave interaction in a plasma, and by other factors which lead to a time variation of the plasma oscillation spectrum. We note that the general relations of Sec. 6 are not suitable for arbitrary nonstationary oscillation spectra, that is, for nonstationary turbulence in the general case. We shall illustrate by means of two examples the roles of the energy losses and of the nonlinear effects. We consider by way of an example the acceleration of nonrelativistic particles by plasma oscillations in the presence of collisions[10,16]. The systematic change of particle energy because of the "ionization" loss of energy to collisions can be written in the form

$$\dot{E}_p^{col} = \frac{\sqrt{m}}{\sqrt{2}} \frac{e^2\omega_{0e}^2}{\varepsilon_p^{1/2}} \ln \frac{\varepsilon}{\hbar\omega_{0e}}, \qquad (3.1)$$

whereas the acceleration by plasma oscillations is described by the formula

$$\dot{E}_p^{acc} = \frac{\sqrt{m}}{2\sqrt{2}} \frac{e^2\omega_{0e}^2}{\varepsilon^{3/2}} T_{eff}. \qquad (3.2)$$

Figure 1 shows plots of both (3.1) and (3.2) with T_{eff} constant. The bending of curve 1 at $\epsilon \sim mv_{Te}^2$ is connected with the fact that in this region the plasma oscillations begin to attenuate noticeably as a result of the Landau absorption[49], and consequently the intensity of the oscillation decreases.

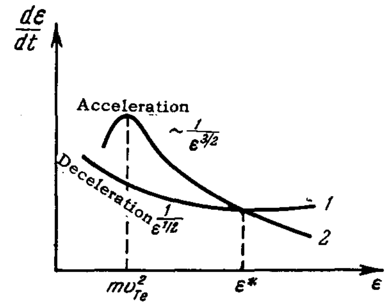

FIG. 1

We note that the point of intersection of curves 1 and 2 is stable—an increase in the particle energy, $\epsilon > \epsilon^*$, causes the deceleration forces to prevail over the acceleration forces and the particle energy decreases, and if $\epsilon < \epsilon^*$ acceleration predominates over deceleration, thus returning the particle to $\epsilon = \epsilon^*$. There is no stability if the acceleration is produced by a force that is constant or increases with ϵ, or even if E_p decreases more slowly than $1/\epsilon^{1/2}$. Thus, the statistical acceleration should lead to an appreciable decrease in the number of accelerated particles when $\epsilon > \epsilon^*$. It must be mentioned that Fig. 1 shows only average characteristics, and that owing to the fluctuation acceleration there is always an energy scatter. The spectra of the accelerated particles are expected to be different, depending on whether the particle energy can reach a value ϵ^* within the acceleration time or not. In the latter case there is no stable point or a maximum near $\epsilon = \epsilon^*$. The value of ϵ^* in (3.1) and (3.2) can be estimated from

$$\varepsilon^* \ln \frac{\varepsilon^*}{\hbar\omega_{0e}} \approx 2T_{\text{eff}}, \tag{3.3}$$

that is, it is of the order of T_{eff}. This result is quite illustrative and indicates that the accelerated particles gain energy up to a temperature of the order of effective oscillation temperature. We recall that the Fermi acceleration of particles by reflection from magnetic clouds is frequently interpreted as heating of a gas of light particles by collision with heavy "magnetic cloud particles." The tendency to equipartition leads in this case to an effective acceleration (heating) of the light particles. Such an interpretation is a particular case of the general relation (3.3)*.

It can be stated that the particles accelerated in a turbulent plasma draw energy from the macroscopic-motion energy contained in the plasma oscillations.

It is easy to compare the characteristic acceleration time $\tau_{\text{acc}} \sim p^2/D^l$ with the wave relaxation time $\tau_{\text{rel}} \sim 1/\gamma_{\text{Land}}$. It turns out that $\tau_{\text{acc}} \ll \tau_{\text{rel}}$ when $W^l \gg n\epsilon$, that is, when the energy of the waves greatly exceeds the energy of the particles. This indicates that only a small fraction of the total number of particles can be accelerated.

One might think that a time should come when all the waves are attenuated as a result of the acceleration. Actually this may not be true, at least for two reasons. First, different sources of plasma oscillations may exist in the system and may be located both in the region where particle acceleration takes place, and outside this region (in the latter case the plasma oscillations "pass" through the boundary of the region, which can be regarded as a source). Second, oscillations may be generated by the accelerated particles themselves[162]. This possibility is quite important both for the determination of the injection energy of the accelerated particles, and for the question of the partition of the energy between the accelerated particles and the turbulence (this question, as applied to cosmic-ray acceleration, was raised in [24]). The mechanism of oscillation generation can arise, even for isotropically distributed particles, as a result of induced scattering effects[35].

It must be specially emphasized that only when a small fraction of all the plasma particles is accelerated is it advantageous to speak of acceleration of particles in a plasma. The opposite case, when the bulk of the particles draws energy from the plasma oscillations, corresponds to turbulent heating of the plasma. In fact, to realize turbulent heating it is necessary to have continuous pumping of oscillations from the external sources, (for example, from an external electric field). In such a situation the turbulence of the plasma is not weak. At the same time, it is just the condition for weak turbulence, $W^l \ll n_0T$, which is satisfied in practice for high frequency turbulence (see Ch. III). The energy can then be acquired only by a small fraction of the total number of plasma particles, that is, we are dealing with acceleration of particles in a plasma.

Let us ascertain the nature of the stationary spectrum of the accelerated particles in the case of acceleration by stationary Langmuir oscillations and deceleration by collisions. The diffusion coefficient for isotropically distributed Langmuir oscillations is (see (2.4), (2.16), and Sec. 5)

$$D^l(v) = \frac{e^2\omega_{0e}^3}{v^3} \int_{v_{\text{ph}_1}}^{v} \frac{\hbar N^l(v_{\text{ph}})\, dv_{\text{ph}}}{v_{\text{ph}}}, \tag{3.4}$$

where $v_{\text{ph}1}$ is the lower value of v_{ph}, for which $N^l(v_{\text{ph}}) \neq 0$. The equation describing the accleration is (2.8). The change in the distribution function due to the collisions can be described by a collision integral

$$\frac{\partial f_p}{\partial \tau} \approx \frac{1}{u^2} \frac{\partial}{\partial u} \left(\frac{1}{u} \frac{\partial f}{\partial u} + f \right), \tag{3.5}$$

$u = v/v_{\text{Te}}$, $\tau = \nu_{\text{col}}t$, and ν_{col} is the collision frequency.

The equations describing particle acceleration by the plasma waves and deceleration by collisions take the form[10,42]

$$\frac{\partial}{\partial \tau} f(u, \tau) = \frac{1}{u^2} \frac{\partial}{\partial u} \left(\frac{1+\alpha}{u} \frac{\partial f(u, \tau)}{\partial u} + f(u, \tau) \right),$$

$$u = \frac{v}{v_{\text{Te}}}, \quad \tau = t\nu_{\text{col}},$$

$$\alpha = \frac{e^2\omega_{0e}^3 \hbar}{m\nu_{\text{col}} v_{\text{Te}}^5} \int_{u_{\text{ph}1}}^{u} N^l(u_{\text{ph}}, \tau) \frac{du_{\text{ph}}}{u_{\text{ph}}}. \tag{3.6}$$

From this we have the equilibrium solution

$$\varphi(\varepsilon)\frac{u}{2} = u^2 f$$

$$= \begin{cases} n\sqrt{\frac{2}{\pi}}\, e^{-\frac{u^2}{2}} u^2 & \text{for } u < u_{\text{ph}_1}, \\ n\sqrt{\frac{2}{\pi}}\, e^{-\frac{u_{\text{ph}1}^2}{2}} u^2 \exp\left\{ -\int_{u_{\text{ph}1}}^{u} \frac{u\, du}{1+\alpha(u)} \right\} & \text{for } u > u_{\text{ph}1}, \end{cases} \tag{3.7}$$

Thus, up to $u = u_{\text{ph}1}$ the distribution is Maxwellian ($u_{\text{ph}1}$ is the lowest phase velocity of the waves). The form of the spectrum for $u > u_{\text{ph}1}$, where the accelerating mechanisms is effective, depends essentially on the form of the wave spectrum. A general feature, however, is the presence of a section in which the distribution function grows, followed by a descending section. This is the result of the fact that $\alpha \gg 1$, which takes place when the effective temperature of the oscillations greatly exceeds the electron temperature (Fig. 2).

*We have illustrated this by means of a very simple example. However, the same qualitative conclusion holds for other variants of acceleration.

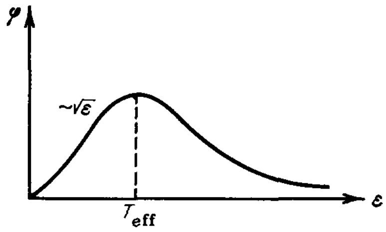

FIG. 2

If $\alpha(u) \sim u^2$, as is the case when $N^1(u_{ph}) \sim u_{ph}^2$, then the descending part of the spectrum obeys a power law, $f \sim 1/\epsilon^{\alpha}$ $(u \gg u_{ph\,1})$.

The different forms of the spectrum in the case of acceleration by Langmuir oscillations, with allowance made for other losses (synchrotron radiation for relativistic electrons etc.), were obtained in [15,75]. Of important significance may also be nonlinear interactions of Langmuir oscillations with one another. The mechanism of such interaction consists in induced scattering of Langmuir oscillations by thermal electrons and ions of the plasma (Sec. 6); the oscillation spectrum narrows down and shifts towards larger phase velocities, with insignificant change of the total oscillation energy and with conservation of the total number of Langmuir quanta N_k (see Sec. 6). We note that the efficiency of acceleration increases in this case noticeably. Indeed, the diffusion coefficient (3.4) becomes, in the case when the oscillations are concentrated in a narrow interval Δv_{ph} near $v_{ph} < v$,

$$D^1 \approx \frac{e^2}{\omega_{0e} v^3} v_{ph}^3 \cdot 2\pi^2 W^1, \tag{3.8}$$

that is, it increases like v_{ph}^3. By way of illustration let us consider the case when the characteristic phase velocity of the waves increases linearly with the time, assuming the initial spectrum of the oscillations to be sufficiently narrow. The diffusion coefficient is in this case equal to (3.8) when $v_{ph} < v$, and vanishes when $v_{ph} > v$. The larger the particle energy, the larger the time interval during which the particle experiences acceleration. This leads to a "drawing out" of the distribution function on the region of high energies. Thus, the solution of the diffusion equation (2.11) for (3.8) and $v_{ph} = v_{ph\,0}(1 + t/t_0)$ shows that the group of particles, which had at the initial instant of time a small velocity scatter about $v_0 \gg v_{ph}$, is described after the acceleration process (the acceleration stops if $v_{ph} > c$) by the distribution

$$f(\varepsilon) = \frac{4n_1\sqrt{\varepsilon_0}}{5\beta} I_{2/5}\left(\frac{16}{25}\frac{\varepsilon_0^{5/4}}{\beta\varepsilon^{3/4}}\right) \exp\left\{-\frac{8}{25\beta}\frac{(\varepsilon_0^{5/2}+\varepsilon^{5/2})}{\varepsilon^2}\right\}. \tag{3.9}$$

Here $\epsilon = u^2/2$ $(u = v/v_{Te})$ —particle energy in mass units, $\epsilon_0 = u_0^2/2$, $u_0 = v_0/v_{Te}$ —initial particle energy,

$$\beta = \omega_{0e} t_0 \frac{W^1 \pi}{n_0 m v_{ph0}^2}, \tag{3.10}$$

n_1 —number of accelerated particles, $I_{2/5}$ —Bessel function of imaginary argument, and t_0 —characteristic "transfer time." In the region of small phase velocities of the waves, $v_{ph\,0}/v_{Te} \lesssim (m_i/m_e)^{1/3}$, the pumping over of the oscillation is due to the scattering by electrons and β has an order of magnitude [31,56] $\beta \approx v_{ph\,0}/v_{Te}$. The pumping connected with scattering by ions is significant in the case when $v_{ph} \gtrsim v_{Te}^2/v_{Ti}$, and then $\beta \approx v_{Ti}/v_{ph\,0}$. Attention must be called, however, to the relatively small decrease of the distribution function (3.9) with energy, like $\exp(-\sqrt{\epsilon})$, and not like $\exp(-\epsilon)$, as in (3.7) for α = const. In the case of other initial particle spectra, of course, the charged particles have more complicated distributions. It is important to emphasize the growth of the pumping efficiency with increase v_{ph}, on the one hand, and the possibility of establishment of stationary turbulence spectra with large v_{ph}, on the other. The latter may be due to four-plasmon interactions, as suggested in [163]. An important role is played in the problem of the spectrum of accelerated particles by the energy partition among the turbulence, magnetic field, and the particles [2].

II. THEORY OF STATISTICAL ACCELERATION

We proceed to a consecutive exposition of the theory of statistical acceleration, which was outlined in general features in the preceding section. Along with reporting concrete results on different acceleration mechanisms in a turbulent plasma, both isotropic and containing external fields, we shall discuss the relative role of low-frequency and high-frequency turbulent pulsations, the question of limitations on the oscillation intensity, the question of the limiting transition of the statistical acceleration in the particular case of Fermi acceleration, the role of nonstationary behavior of the turbulent pulsations, and other questions.

4. Induced Cerenkov Acceleration

The change in the energy and momentum of charged particles is brought about by interaction with electromagnetic fields. In a plasma, which is a system of weakly interacting particles, the prevalent type of interaction is Cerenkov absorption and emission of electromagnetic waves. This is connected with the fact that the particles can move freely in the plasma for a long time without experiencing collision or acceleration, and also with the possibility of propagation of electromagnetic waves with very small phase velocities in the plasma. The processes of induced absorption and emission of the accelerated particles are the elementary acts whereby their statistical state is changed.

The probabilities of spontaneous Cerenkov radiation in an isotropic transparent medium can be obtained from the expression for the charge-energy loss W, determined by the Landau method [48-52].

To present the complete picture, we shall illustrate here the method of calculating W by using as a simple example an isotropic medium. The power radiated by an extraneous current flowing in the medium is equal to the work performed by the current in the field E(r,t) produced by it, averaged over a unit of time:

$$W=-\lim_{T\to\infty}\int_{-T/2}^{T/2} dr\,\frac{dt}{T}\,\mathbf{E}(\mathbf{r},t)\,\mathbf{j}(\mathbf{r},t)=-\lim_{T\to\infty}\frac{(2\pi)^4}{T}\int d\mathbf{k}\,d\omega\,\mathbf{j}^*_{\mathbf{k}\omega}\mathbf{E}_{\mathbf{k}\omega},$$

where $\mathbf{E}_{\mathbf{k}\omega}$ and $\mathbf{j}_{\mathbf{k}\omega}$ are the Fourier components of the field and of the current. Averaging over the time has been carried out in order to eliminate the oscillating terms from W. For a linear medium the field produced by the current can be readily obtained from Maxwell's equations.

$$(k^2\delta_{ij}-k_ik_j-\omega^2\varepsilon_{ij}(\omega,\mathbf{k}))\,E_{j\mathbf{k}\omega}=4\pi i\omega j_{i\mathbf{k}\omega},$$

where ϵ_{ij} is the dielectric constant with allowance for spatial disperson. We have

$$E_{j\mathbf{k}\omega}=\Pi_{ij\mathbf{k}\omega}j_{i\mathbf{k}\omega};$$

Π_{ij} is the reciprocal Maxwell operator. Hence

$$W=-\lim\frac{(2\pi)^4}{T}\int dk\,d\omega\,j_i^*(\mathbf{k},\omega)\,\Pi_{ij}(\mathbf{k},\omega)\,j_i(\mathbf{k},\omega).$$

A charge moving uniformly with velocity v produces a current

$$\mathbf{j}=e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t),$$

whose Fourier components are

$$\mathbf{j}_{\mathbf{k}\omega}=\frac{e\mathbf{v}}{(2\pi)^3}\,\delta(\omega-\mathbf{kv}).$$

When this expression is substituted in W we must take into account the fact that (see[157])

$$\delta^2(\omega-\mathbf{kv})\to\delta(\omega-\mathbf{kv})\,\frac{T}{2\pi}.$$

This enables us to rewrite the expression for the power loss in the form

$$W=-\frac{e^2}{(2\pi)^3}\int dk\,d\omega\,v_i\Pi_{ij}(\mathbf{k},\omega)\,v_j\delta(\omega-\mathbf{kv}).$$

In an isotropic medium we can readily obtain the expression for the inverse operator

$$\Pi_{ij}=-\frac{4\pi i\omega}{k^2}\left\{\frac{k_ik_j}{\omega^2}\,\frac{1}{\varepsilon^{l}(\omega,\mathbf{k})}-\frac{k^2\delta_{ij}-k_ik_j}{k^2-\omega^2\varepsilon^{t}(\omega,\mathbf{k})}\right\};$$

ϵ^t and ϵ^l are the longitudinal and transverse dielectric constants which enter in the expression for ϵ_{ij}:

$$\varepsilon_{ij}(\omega,\mathbf{k})=\varepsilon^{l}(\omega,\mathbf{k})\,\frac{k_ik_j}{k^2}+\varepsilon^{t}(\omega,\mathbf{k})\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right).$$

Substituting this expression in W, we can conveniently recast this result in a form in which the integration is only over positive frequencies†

$$W=\frac{e^2}{\pi^2}\int_0^\infty d\omega\int d\mathbf{k}\omega\left\{\frac{[\mathbf{kv}]^2}{k^2}\,\mathrm{Im}\,\frac{1}{k^2-\omega^2\varepsilon^{t}(\omega,\mathbf{k})}-\frac{1}{k^2}\,\mathrm{Im}\,\frac{1}{\varepsilon^{l}(\omega,\mathbf{k})}\right\}\delta(\omega-\mathbf{kv}).$$

We took account here of the fact

$$\varepsilon^{t}(-\omega,-\mathbf{k})=\varepsilon^{t*}(\omega,\mathbf{k}),\quad \varepsilon^{l}(-\omega,-\mathbf{k})=(\varepsilon^{l}(\omega,\mathbf{k}))^*.$$

In the region of transparency of the medium

$$\mathrm{Im}\,\frac{1}{k^2-\omega^2\varepsilon^{t}(\omega,\mathbf{k})}=\pi\delta(k^2-\omega^2\varepsilon^{t}(\omega,\mathbf{k})),$$

$$\mathrm{Im}\,\frac{1}{\varepsilon^{l}(\omega,\mathbf{k})}=-\pi\delta(\varepsilon^{l}(\omega,\mathbf{k})).$$

Thus, in the region of transparency of an isotropic medium we have

$$W=\frac{e^2}{\pi}\int_0^\infty d\omega\int d\mathbf{k}\omega\left\{\frac{[\mathbf{kv}]^2}{\mathbf{k}^2}\,\delta(\omega-\mathbf{kv})\,\delta(\mathbf{k}^2-\omega^2\varepsilon^{t}(\omega,\mathbf{k}))+\delta(\omega-\mathbf{kv})\,\frac{1}{\mathbf{k}^2}\,\delta(\varepsilon^{l}(\omega,k))\right\}.$$

On the other hand

$$W=\int\hbar\omega(\mathbf{k})\,u_{\mathrm{p}}(\mathbf{k})\,dk,$$

where $u_p(\mathbf{k})$ is the probability of spontaneous Cerenkov emission.

The equations $k^2=\omega^2\epsilon^t(\omega,\mathbf{k})$ and $\epsilon^l(\omega,\mathbf{k})=0$, which determine the wave dispersion law, can have several branches $\omega_\sigma(\mathbf{k})$.

Accordingly, the probabilities of the Cerenkov radiation by transverse and longitudinal waves will contain sums over the positive branches*:

$$u_{\mathrm{p}}^{t}(\mathbf{k})=\frac{e^2}{\hbar\pi}\sum_\sigma\frac{[\mathbf{kv}]^2}{k^2}\,\frac{\delta(\omega_\sigma-\mathbf{kv})}{\frac{\partial}{\partial\omega_\sigma}\,\omega^2\varepsilon^{t}(\omega,\mathbf{k})},\qquad(4.1)†$$

$$u_{\mathrm{p}}^{l}(\mathbf{k})=\frac{e^2}{\hbar\pi}\sum_\sigma\frac{1}{\frac{\partial\varepsilon^{l}(\omega,\mathbf{k})}{\partial\omega_\sigma}}\,\frac{1}{k^2}\,\delta(\omega_\sigma-\mathbf{kv}).\qquad(4.2)$$

It remains for us only to take account of the fact that the probability of the induced process differs from the probability of the spontaneous process by a factor equal to the number of waves N (number of quanta), that is, the quantity $w_p(\mathbf{k})$ derived above (see text preceding Eq. (2.2)) takes the form

$$w_{\mathrm{p}}^{t}(\mathbf{k})=u_{\mathrm{p}}^{t}(\mathbf{k})\,N_{\mathbf{k}}^{t},$$
$$w_{\mathrm{p}}^{l}(\mathbf{k})=u_{\mathrm{p}}^{l}(\mathbf{k})\,N_{\mathbf{k}}^{l}.\qquad(4.3)$$

Using the results of the preceding section, we can readily obtain the characteristics of the accelerated particles.

Let us consider the simplest example of the action of isotopic transverse Cerenkov radiation, assuming the spatial dispersion to be negligibly small

$$D^{l}(\mathbf{p}^2)=\frac{1}{v^2}\int w_{\mathrm{p}}^{t}(\mathbf{k})\,(\mathbf{kv})^2\,dk$$
$$=\frac{1}{v^2}\,\frac{e^2}{\pi}\sum_\sigma\int\hbar N_{|\mathbf{k}|}^{t}\,dk\,\frac{[\mathbf{kv}]^2}{k^2}\,\frac{\omega_\sigma^2(\mathbf{k})}{\frac{\partial}{\partial\omega_\sigma}(\omega^2\varepsilon^{t})}\,\delta(\omega_\sigma-\mathbf{kv})$$
$$=\frac{e^2}{v}\int_{nv>1}^{\infty}\hbar N_\omega^{t}\,d\omega\,\omega^2\left(1-\frac{1}{\varepsilon^{t}(\omega)\,v^2}\right),$$
$$N_\omega^{t}=N_{|\mathbf{k}|}^{t}\quad\text{for}\quad k^2=\omega^2\varepsilon^{t}(\omega).\qquad(4.4)$$

In the last expression it is convenient to go over to integration over the frequencies. From (4.4) and (2.9) we obtain the effect of systematic acceleration

$$\dot{E}_{\mathrm{p}}=\frac{2e^2}{p}\int_{nv>1}\hbar N_\omega^{t}\omega^2\left(1-\frac{1}{\varepsilon^{t}(\omega)}\right)d\omega\qquad(4.5)$$

and of fluctuation acceleration (see (2.12))

$$G(\varepsilon)=e^2v\int_{nv>1}\hbar N_\omega^{t}\omega^2\left(1-\frac{1}{\varepsilon^{t}(\omega)\,v^2}\right)d\omega.\qquad(4.6)$$

The result is proportional to $\hbar\omega N_\omega^t$ – the energy of the waves in the specified frequency interval, which is proportional to $E_\omega{}^2$ –

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*The derivative $\partial/\partial\omega_\sigma$ signifies the operation $\partial/\partial\omega$ followed by the substitution $\omega=\omega_\sigma(\mathbf{k})$.

†$[\mathbf{k}\,\mathbf{v}]\equiv\mathbf{k}\times\mathbf{v}$.

the square of the amplitude of the fluctuating field of the wave.

The result (4.5) can be obtained by directly averaging the motion of the particle in the fluctuating fields[47,53-55]. This derivation is of independent interest, since it can be readily generalized to include the case of unstable systems. Let

$$\mathbf{E}(\mathbf{r}, t)=\sum_{\sigma} \int \mathbf{E}(\mathbf{k})\, e^{i\mathbf{kr}-i\omega_\sigma(\mathbf{k})t}\, d\mathbf{k}, \quad (4.7)$$

and the mean value of the quadratic combinations[48,49]

$$\langle E_i(\mathbf{k}) E_j(\mathbf{k}')\rangle=\left(\delta_{ij}-\frac{k_i k_j}{k^2}\right) E^2_{|\mathbf{k}|}\delta(\mathbf{k}+\mathbf{k}') \quad (4.8)$$

define E_k^2, which in this case of isotropic radiation depends only on the modulus $|\mathbf{k}|$. The factor $\delta_{ij}-k_ik_j/k^2$ takes into account the transversality of the waves. The motion of the particle in the field (4.7) is described by the equation

$$\frac{d}{dt}\frac{m\mathbf{v}(t)}{\sqrt{1-v^2(t)}}=\mathbf{f}(t)=e\sum_{\sigma}\int\left[\left(1-\frac{\mathbf{kv}}{\omega_\sigma(\mathbf{k})}\right)\mathbf{E}_\mathbf{k} + \frac{\mathbf{k}}{\omega_\sigma(\mathbf{k})}(\mathbf{v}(t)\mathbf{E}_\mathbf{k})\right] e^{i\mathbf{kr}(t)-i\omega_\sigma(\mathbf{k})t}\, dk. \quad (4.9)$$

This equation can be solved by perturbation theory, expanding in powers of $\mathbf{E}_k$:

$$\mathbf{v}=\mathbf{v}_0+\mathbf{v}_1, \quad \mathbf{v}_1=\frac{1}{\varepsilon_p}\int_0^t [\mathbf{f}_0(t)-\mathbf{v}_0(\mathbf{v}_0\mathbf{f}_0(t))]\, dt,$$

$$\mathbf{r}(t)=\mathbf{v}_0 t+\mathbf{r}_1, \quad \mathbf{r}_1=\frac{1}{\varepsilon_p}\int_0^t dt_1\int_0^{t_1}[\mathbf{f}_0(t')-\mathbf{v}_0(\mathbf{v}_0\mathbf{f}_0(t'))]\, dt',$$

$$\varepsilon_p=\frac{m}{\sqrt{1-v_0^2}}.$$

Here $\mathbf{v}_0$ is the constant particle velocity in the absence of a wave field, $\mathbf{f}_0(t)=\mathbf{f}(t)$ for $\mathbf{v}=\mathbf{v}_0$ and $\mathbf{r}=\mathbf{v}_0 t$. We find the average change in particle energy, retaining only terms quadratic in $\mathbf{E}_k$:

$$\dot{E}_p=\left\langle\frac{d\varepsilon_p}{dt}\right\rangle=\langle\mathbf{f}(t)\,\mathbf{v}(t)\rangle=\langle\mathbf{f}_0(t)\,\mathbf{v}_1+\mathbf{v}_0\mathbf{f}_1(t)\rangle,$$

where

$$\mathbf{f}_1(t)=e\sum_{\sigma}\int\left\{\left[\frac{\mathbf{k}}{\omega_\sigma(\mathbf{k})}(\mathbf{v}_1(t)\mathbf{E}_\mathbf{k})-\frac{\mathbf{kv}_1(t)}{\omega_\sigma(\mathbf{k})}\mathbf{E}_\mathbf{k}\right] + i(\mathbf{kr}_1)\left[\left(1-\frac{\mathbf{kv}_0}{\omega_\sigma(\mathbf{k})}\right)\mathbf{E}_\mathbf{k}+\frac{\mathbf{k}}{\omega_\sigma(\mathbf{k})}(\mathbf{v}_0\mathbf{E}_\mathbf{k})\right]\right\} e^{i(\mathbf{kv}_0-\omega_\sigma(\mathbf{k}))t}\, dk$$

The averaging can be readily carried out with the aid of (4.8):

$$\dot{E}_p^{\,\mathrm{t}}=\frac{2e^2\pi^2}{\varepsilon_p v_0}\int E_\omega^2\left(1-\frac{1}{\varepsilon^{\mathrm{t}}(\omega)}\right)\frac{\partial}{\partial\omega}\,\omega^2\varepsilon^{\mathrm{t}}\, d\omega,$$

$$E_\omega^2=|E_k|^2 \text{ for } k^2=\omega^2\varepsilon^{\mathrm{t}}(\omega). \quad (4.10)$$

From a comparison with (2.5) we see that the number of waves is connected with E_ω^2 by the relation $\hbar N_\omega = (\pi^2 E_\omega^2/\omega^2)\frac{\partial}{\partial\omega}\omega^2\epsilon^{\mathrm{t}}$, which incidentally could have been obtained by averaging the expression for the energy of the electromagnetic field directly over the fluctuations

$$\frac{\mathbf{B}^2}{8\pi}+\frac{1}{4\pi}\int_{-\infty}^{t}\mathbf{E}\frac{\partial\mathbf{D}}{\partial t}\, dt.$$

The foregoing derivation discloses the approximate character of the formulas for induced Cerenkov acceleration, namely, in (4.5) and (4.10) account is taken only of the effect of the first power in E_k^2—the square of the field amplitude. If we continue the procedure of expanding in powers of the fields, which leads to (4.10), we obtain the next-order corrections $\sim E_k^4$, etc. We shall not do so here, since it is much simpler to start from the simple intuitive physical considerations presented below.

We note here only that the expansion becomes inadequate, as follows from (4.9), when $|\mathbf{k}|r_1(t) \ll 1$, that is, in the case when the amplitude of the particle oscillations in the wave field becomes of the order of the wavelength. We note that in addition to the condition $|\mathbf{k}|r_1(t) \ll 1$ it is necessary also to satisfy the inequality $|v_1| \ll v_0$ or the equivalent inequality $r_1 \ll (v_0 k/\omega)/k$. By virtue of the Cerenkov condition $v_0 k/\omega > 1$, the second condition will be satisfied if the first is.

5. Acceleration by Longitudinal Plasma Oscillations

Before we proceed to a more detailed analysis of the corrections to the Cerenkov acceleration, it is advisable to discuss Cerenkov acceleration by longitudinal waves.

This effect is of special interest. The point is that the longitudinal oscillations usually have relatively small phase velocities, that is, they can accelerate relatively slow particles. It is well known[49] that in an isotropic plasma two types of longitudinal oscillations are possible, satisfying the dispersion equation

$$\varepsilon^{\mathrm{l}}(\omega, \mathbf{k})=0.$$

These are, first, Langmuir oscillations with spectrum $\omega_e^2 \approx \omega_{0e}^2+3v_{Te}^2k^2$, where $\omega_{0e}^2=4\pi ne^2/m_e$, and $v_{Te}=\sqrt{T_e/m_e}$. The phase velocities ω/k of the Langmuir waves can be arbitrary and range from infinity to quantities of the order of v_{Te}. Second, these may be ion-sound oscillations, which can exist in a non-isothermal plasma with $T_e \gg T_i$, and whose phase velocities can approach v_{Ti}. Both types of oscillation can play an important role in a plasma when both electrons and ions are accelerated.

It is advantageous here to present also relations between the number of longitudinal quanta N_k^{l} with the Fourier amplitudes of the electric field of the oscillations

$$\langle E_i^{\sigma\pm}(\mathbf{k},\omega),\, E_j^{\sigma\pm}(\mathbf{k},\omega)\rangle=|E^2_{\mathbf{k}\sigma}|\,\delta(\omega+\omega')\,\delta(\mathbf{k}+\mathbf{k}')\,\delta(\omega) - \omega_{\sigma_\pm}(\mathbf{k})\frac{k_ik_j}{k^2}. \quad (5.1)$$

Here

$$\omega_{\sigma_\pm}(\mathbf{k})=\pm|\omega_\sigma(\mathbf{k})|,$$

$$\mathbf{E}(\mathbf{r}, t)=\sum_{\sigma_\pm}\int dk\, \mathbf{E}_{\sigma_\pm}(\mathbf{k})\, e^{i\mathbf{kr}-i\omega_{\sigma_\pm}(\mathbf{k})t}. \quad (5.1')$$

The foregoing relations serve as a definition of $|E_{k\sigma}|^2$. With the aid of (5.1') we can readily obtain the energy of the electromagnetic field of the oscillations

$$W\equiv\frac{1}{4\pi}\int_{\infty}^{t}\mathbf{E}^{\mathrm{l}}\frac{\partial\mathbf{D}^{\mathrm{l}}}{\partial t}\, dt=\sum_{\sigma_+}\int\frac{|\mathbf{E}_{k_{\sigma+}}|^2\, dk}{4\pi}\,\omega_{\sigma+}(\mathbf{k})\frac{\partial\varepsilon^{\mathrm{l}}(\omega,\mathbf{k})}{\partial\omega}\bigg|_{\omega=\omega_{\sigma+}(\mathbf{k})}.$$

Comparing the obtained value of W with the expression

$$W=\sum_{\sigma}\int\frac{\hbar\omega_{\sigma}N_{\mathbf{k}}^{\sigma}\,dk}{(2\pi)^3},$$

We find

$$N_{\mathbf{k}}^{\sigma}=\frac{2\pi^2}{\hbar}\left|\frac{\partial}{\partial\omega}\varepsilon^{l}(\omega,\mathbf{k})\right|_{\omega=\omega_{\sigma+}(\mathbf{k})}|E_{\mathbf{k}_{\sigma+}}|^2. \tag{5.1"}$$

A plasma in which intense oscillations are excited is called turbulent. For an isotropically turbulent plasma we can readily obtain from (4.1) expression for the diffusion coefficients D^l and D^t, which determine the acceleration and the scattering:

$$D^{l}=\frac{2e^2}{v^3}\sum_{\sigma}\int\limits_{v>\frac{\omega_{\sigma}(\mathbf{k})}{k}}\frac{\hbar N_k^l\,dk\,\omega_{\sigma}^2(\mathbf{k})}{k\frac{\partial\varepsilon^l(\omega,\mathbf{k})}{\partial\omega_{\sigma}}}, \tag{5.2}$$

$$D^{t}=\frac{e^2}{v}\sum_{\sigma}\int\limits_{v>\frac{\omega_{\sigma}(\mathbf{k})}{k}}k\,dk\,\frac{\hbar N_k^l}{\frac{\partial\varepsilon^l(\omega,k)}{\partial\omega_{\sigma}}}\left(1-\frac{\omega_{\sigma}^2(\mathbf{k})}{k^2v^2}\right). \tag{5.3}$$

From (5.2) and (5.3) we see that if the particle velocity greatly exceeds the characteristic phase velocity of the waves, $\omega_{\sigma}(\mathbf{k})/k \ll v$, then the scattering effects turn to have an order of magnitude k^2v^2/ω_{σ}^2 times larger than the acceleration effects. This occurs when the accelerated particles have a non-isotropic distribution. In this case the scattering first leads to isotropization of the angular distribution of the particles. Acceleration, on the other hand, will occur even at an isotropic distribution of the particles. This deduction, we must specially emphasize, has been obtained for the case of isotropic wave distribution.

If the characteristic velocity of the waves is quite close to the particle velocity, then the acceleration prevails over scattering. However, with increasing particle velocity in the acceleration process the scattering will again begin to prevail over acceleration.

For Langmuir oscillations we can assume that $\omega_{\sigma}\approx\omega_{0e}$. It is inconvenient to introduce in (5.2) integration over the phase velocities of the waves $v_{ph} = \omega_{0e}/k$.[10] Then $N_k^l = N^l(\omega_{0e}/k) = N^l(v_{ph})$ and

$$D^{l}=\frac{e^2\omega_{0e}^3}{v^3}\int\limits_{v_{ph1}}^{v}\frac{\hbar N^l(v_{ph})\,dv_{ph}}{v_{ph}}. \tag{5.4}$$

We have indicated here the lower limit v_{ph}, since plasma waves with phase velocities of the order of or smaller than thermal are strongly damped. Actually it is not necessary to introduce v_{ph1}, and one can assume that $N^l(v_{ph})$ vanishes when v_{ph} is of the order of or smaller than v_{ph1}.

From (5.2) we obtain the effect of systematic acceleration[10,57,12]:

$$\dot{E}_p=\frac{e^2\omega_{0e}^3}{p}\left\{\frac{1-v^2}{v^2}\hbar N^l(v)+2\int\limits_{v_{ph1}}^{v}\frac{\hbar N^l(v_{ph})\,dv_{ph}}{v_{ph}}\right\}. \tag{5.5}$$

We note that for ultrarelativistic particles $\dot{E}_p = 2D^l/\epsilon_p$, and $G(\epsilon) = D^l$, so that the equations describing the variation of the particle energy distribution function (2.13) takes the form[10]

$$\frac{\partial f(\varepsilon)}{\partial t}=D^l\frac{\partial^2 f(\varepsilon)}{\partial\varepsilon^2}-\frac{\partial}{\partial\varepsilon}\frac{2D^l}{\varepsilon}f(\varepsilon). \tag{5.6}$$

For nonrelativistic particles the effect of systematic change of energy can be written in the form

$$\dot{E}_p=\frac{e^2\omega_{0e}^3\sqrt{m}}{2\sqrt{2}\,\varepsilon^{3/2}}N^l(v)\,\hbar. \tag{5.7}$$

If $N^l \approx T_{eff}/\hbar\omega_{0e}$ = const, then it follows from (5.7) that the time necessary for the particle to acquire a specified energy is the smaller, the larger the particle mass, that is, ions with velocity larger than v_{Te} acquire the specified energy faster than electrons.

We now consider acceleration by low frequency ion-sound oscillations. It is well known that when $T_e \gg T_i$ such oscillations are possible (are weakly damped) if their phase velocity ω/k lies in the interval $v_{Ti} \ll \omega/k \ll v_{Te}$. Their spectrum has in this case the form

$$\omega_s(\mathbf{k})=\frac{kv_s}{\sqrt{1+k^2\lambda_{De}^2}}, \tag{5.8}$$

where $v_s = \sqrt{T_e/m_i} = v_{Te}\sqrt{m_e/m_i}$ is the speed of sound and $\lambda_{De} = v_{Te}/\omega_{0e}$ is the Debye radius of the electrons. The expression for ϵ^l, leading to (5.8), is

$$\varepsilon^l=1-\frac{\omega_{0i}^2}{\omega^2}+\frac{1}{k^2\lambda_{De}^2}. \tag{5.9}$$

In the region $\omega \ll \omega_{0i}$ the spectrum has a sonic character, $\omega_s = kv_s$, whereas when $k\lambda_{De} \gg 1$ we have $\omega_s \to \omega_{0i}$. With the aid of (5.2) we obtain the effect of systematic acceleration by ion-sound oscillations:

$$D^{l}=\frac{e^2\omega_{0e}^2}{v^3}\int\limits_{v_{ph2}}^{\tilde{v}}\frac{dv_{ph}}{v_{ph}}\hbar N^s(v_{ph})\left(\frac{v_s^2-v_{ph}^2}{v_{Te}^2}\right)^{3/2}. \tag{5.10}$$

Here v_{ph2} is of the order of v_{Ti}, and $\tilde{v} = \min(v, v_s)$. In this respect there is an essential difference from the case of Langmuir oscillations, manifest in the fact that the phase velocities of the ion-sound oscillations are bounded from above by the value of v_s and lie in the interval $v_{Ti} \ll v_{ph} \leq v_s$. This causes the acceleration of particles with $v < v_s$ and $v > v_s$ to be different, namely, the result can be represented in the form of a sum of two terms, one of which vanishes when $v > v_s$:

$$\dot{E}_p^s=\frac{e^2\omega_{0e}^3}{p}\left\{2\int\limits_{v_{ph2}}^{\tilde{v}}\frac{dv_{ph}}{v_{ph}}\hbar N^s(v_{ph})\left(\frac{v_s^2-v_{ph}^2}{v_{Te}^2}\right)^{3/2}+\frac{(1-v^2)}{v^2}\left(\frac{v_s^2-v^2}{v_{Te}^2}\right)^{3/2}\cdot\frac{1}{2}\left(1+\frac{v_s-v}{|v_s-v|}\right)\hbar N^s(v)\right\}. \tag{5.11}$$

When $v \ll v_s \ll 1$ we have*

$$\dot{E}_p^s=\frac{e^2\omega_{0e}^3\sqrt{m}}{2\sqrt{2}\,\varepsilon^{3/2}}N^s(v)\left(\frac{m_e}{m_i}\right)^{3/2}, \tag{5.12}$$

*We note that when $v > v_s$ the effect of systematic acceleration decreases sharply, but the fluctuation acceleration has the same order of magnitude as when $v < v_s$.

and for ultrarelativistic particles in the optimal case, when the characteristic value of the phase velocities of the ion-sound oscillations is much smaller than v_s, we get

$$\dot{E}_p^s = \frac{2e^2\omega_{0e}^3}{\varepsilon}\left(\frac{m_e}{m_i}\right)^{3/2}\int_{v_{ph1}}^{v_{ph\,max}}\frac{dv_{ph}}{v_{ph}}N^s(v_{ph}). \qquad (5.13)$$

We have presented here an example with acceleration by low-frequency oscillations, in order to illustrate by means of a concrete example the relative role of acceleration by high-frequency and low-frequency oscillations. Questions of acceleration by plasma oscillations in the presence of magnetic fields will be considered in greater detail below.

Frequently a situation is possible wherein one can assume from energy considerations that the oscillation energy does not exceed a certain specified value.*

It is therefore advantageous to compare the effects of acceleration by low-frequency and high-frequency oscillations in the case when the energies of these oscillations are of the same order of magnitude. If, in addition, the phase velocity of the Langmuir oscillations have an order v_{ph}^l, and those of the ion-sound oscillations are $v_{ph}^s \ll v_{ph}^l$, we obtain the following order-of-magnitude relations

$$\frac{N^l}{N^s} \approx \left(\frac{v_{ph}^l}{v_{ph}^s}\right)^3\left(\frac{m_e}{m_i}\right)^2, \qquad (5.14)$$

$$\frac{\dot{E}_p^l}{\dot{E}_p^s} \sim \sqrt{\frac{m_e}{m_i}}\left(\frac{v_{ph}^l}{v_{ph}^s}\right)^3 \gg \frac{m_i}{m_e} \gg 1. \qquad (5.15)$$

The latter inequality follows from $v_{ph}^l \gg v_{Te}$ and $v_{ph}^s \ll v_s$. Thus, we obtain the important qualitative deduction that the acceleration by low-frequency oscillations is less effective than acceleration by high-frequency oscillations. This deduction illustrates the statement made above, that acceleration by hydrodynamic turbulence is less effective than acceleration by high-frequency turbulence.

Attention must be called to a very important circumstance, concerning injection in the acceleration mode. Langmuir oscillations accelerate only particles with velocities $v > v_{ph} > v_{Te}$. For electrons, this condition is not stringent, and they can be accelerated from the tail of the Maxwellian distribution. For ions, however, this condition is stringent. The low-frequency ion-sound oscillations, on the other hand, can accelerate particles to velocities of the order of v_{Ti} and effect injection into the acceleration mode by means of high-frequency oscillations (see Sec. 4).

We note finally that, for a specified oscillation energy W^l, the higher the characteristic phase velocity of the waves the more effective the acceleration. Thus, for acceleration of relativistic particles by Langmuir oscillations we have the order-of-magnitude relation

$$\dot{E}_p \approx \frac{4\pi^2e^2}{\varepsilon}\frac{v_{ph}^3}{\omega_0}W^l. \qquad (5.16)$$

Therefore, for fixed oscillation energy, processes that lead to an increase of the phase velocities of the plasma waves increase the acceleration efficiency. We note that for a fixed value of W^l nonlinear effects can increase v_{ph}.

The turbulence spectra can be anisotropic, as is frequently the case, for example, in the development of two-stream instability of a plasma. To calculate the change of the particle energy in this case it is convenient to use (2.15); we obtain[11]

$$\dot{E}_p = \frac{d}{dp}j_p, \quad j_p = \int \omega k u_p(k)\hbar N_k\,dk. \qquad (5.17)$$

Let us consider an example when the noise spectrum N_k depends on k only via $(k\cdot n)$, where n is a unit vector characterizing the anisotropy of the noise distribution. Then for Langmuir oscillations, if we take account of the fact that the maximum $|k|$ cannot exceed ω_{0e}/v_{Te}, we obtain when $v \ll 1$ [11]

$$\dot{E}_p = \frac{e^2\omega_{0e}^2}{\pi m(v^2-(nv)^2)}\int d(kn)\,N^l(kn)\frac{\xi}{\sqrt{\xi_0^2-\xi^2}}, \qquad (5.18)$$

$$\xi = \frac{[\omega_{0e}-(kn)(vn)]}{(kn)(v^2-(nv)^2)^{1/2}}, \quad \xi_0^2 = \frac{\omega_{0e}^2-(kn)^2v_{Te}^2}{(kn)^2v_{Te}^2}, \quad \xi < \xi_0. \qquad (5.19)$$

We call attention to the fact that in this case the sign of $\dot{E}_p$ depends on the direction of v. If $(n\cdot v)$ is small, that is, the particle moves essentially perpendicular to the direction of n, then acceleration, $\dot{E}_p > 0$, always takes place. Formula (5.18) is not suitable when $v^2-(n\cdot v)^2 \to 0$. In this case it is convenient to use (5.17) directly, assuming that v and j are directed along n:

$$j_p n = \frac{e^2\omega_{0e}}{2\pi}N_1^l\left(\frac{\omega_0}{nv}\right), \quad N_1^l(kn) = \int N_k^l\,d(k-n(kn)); \qquad (5.20)$$

$N_1^l(k\cdot n)$ is the one-dimensional distribution function of the waves. We note that in this case the only particles interacting with the noise N^l are those for which $n\cdot v = v_{ph}$, where $v_{ph} = \omega_{0e}/(k\cdot n)$ is the phase velocity of the waves.

During the quasilinear stage, development of two-stream instability can lead in accord with [20], to spectra of the type

$$N^l(v_{ph}) \approx \frac{8\pi^3 m_e n_1 v_{ph}^4}{\omega_{0e}^2 v_1}, \quad v_1 > v_{ph} \gg v_{Te}; \qquad (5.21)$$

n_1 and v_1 are the density and the initial velocity of the beam, and n_0 is the plasma density. From this we obtain an acceleration effect for $v \ll 1$:

$$\dot{E}_p = 4\pi\omega_{0e}\left(\frac{m_e}{m_i}\right)^2 mv^2\left(\frac{v}{v_1}\frac{n_1}{n_0}\right). \qquad (5.22)$$

Acceleration takes place up to particle velocities equal to the initial velocity of the beam particle v_1. The characteristic time of acceleration from velocities of the order of v_{Te} is determined by the formula

$$\tau \approx \frac{1}{8\pi\omega_{0e}}\left(\frac{m}{m_e}\right)^2\frac{v_1}{v_{Te}}\frac{n_0}{n_1}. \qquad (5.23)$$

We note that the deceleration of the beam particle following generation of the oscillations[20] is connected

*For example, when oscillations are excited by beams, the oscillation energy cannot exceed the beam energy.

both with the anisotropy of the distribution of the generated oscillations and with the nonstationary nature of the wave spectrum.

In analogy with (5.18), we can investigate effects of acceleration by ion-sound oscillations. For $(\mathbf{n}\cdot\mathbf{v}) = 0$ in the region of the sound spectrum[11] $\omega_s = kv_s$ we have

$$\dot{E}_p=\frac{2e^2v_s^3v_{Te}^2}{\pi\omega_{0e}^2mv^3}\frac{1}{\sqrt{1-\frac{v_s^2}{v^2}}}\int\limits_0^\infty k^4\left\langle\frac{\partial N^s}{\partial\cos\vartheta}\right\rangle dk, \qquad (5.24)$$

where $\langle\partial N/\partial\cos\vartheta\rangle$ is the average value of the derivative in the interval $0 < \cos\vartheta < \sqrt{1 - v_s^2/v^2}$. The particle is accelerated if N has in k-space the form of a spheroid elongated along the polar axis, and is decelerated in the case of an oblate spheriod. In the case when $\mathbf{n} \parallel \mathbf{v}$ we have deceleration for a spheroid prolate along n and acceleration in the case of an oblate spheroid. The order of magnitude of the accelerating force is the same for $v > v_s$ as for the isotropic distribution when $v < v_s$.

For the one-dimensional spectrum (5.2) we readily obtain from (5.17)

$$\dot{E}_p=\frac{1}{m}\frac{d}{av}\frac{e^2}{2\pi}N_0^s(\omega)\Big|_{\omega=\omega_{0i}\sqrt{1-\frac{v^2}{v_s^2}}},$$

$$N_0^s(\omega)=N_s(\mathbf{kn})\Big|_{\mathbf{kn}=\frac{\omega}{v_{Te}\left(\frac{m_e}{m_i}-\frac{\omega^2}{\omega_{0e}^2}\right)}},\quad N_s(\mathbf{kn})=\int N_s(\mathbf{k})\,d[\mathbf{k}-\mathbf{n}(\mathbf{kn})]. \qquad (5.25)$$

The interaction of the particles and the waves takes place when $v < v_s$. For a one-dimensional spectrum which depends only on $(\mathbf{k}\cdot\mathbf{n})$ we have when $\mathbf{v} \parallel \mathbf{n}$

$$\dot{E}_p=\frac{1}{m}\frac{d}{dv}\left[\frac{e^2\omega_{0i}^3}{v^2}\left(1-\frac{v^2}{v_s^2}\right)^{3/2}N^s(\mathbf{kn})\Big|_{\mathbf{kn}=\frac{\omega_{0i}}{v}\sqrt{1-\frac{v^2}{v_s^2}}}\times\ln\frac{v}{v_{Ti}\left(1-\frac{v^2}{v_s^2}\right)}\right]. \qquad (5.26)$$

The stationary spectrum $N^s(\omega)$ following development of two-stream instability was obtained in[26]. It must be borne in mind that the mechanism of plasma turbulization, and consequently the stationary-turbulence spectra, can be quite different (Sec. 1).

In many cases, in the case of anisotropic distribution of the oscillations, it is useful to know the mean change of the particle energy, averaged over the directions of their momenta. Let $d\Omega_p$ be the solid angle of these directions and

$$\langle L\rangle_{\Omega_p}=\frac{1}{4\pi}\int L\,d\Omega_p. \qquad (5.27)$$

From (3.27) we readily obtain

$$\langle\dot{E}_p\rangle_{\Omega_p}=\frac{1}{p^2}\frac{d}{dp}\left[\frac{p^2}{\partial\varepsilon/\partial p}\langle\mathbf{j}_p\mathbf{v}\rangle_{\Omega_p}\right]=\frac{1}{p^2}\frac{d}{dp}\left[p^2\langle D^1\rangle_{\Omega_p}\frac{d\varepsilon}{dp}\right],$$

where

$$\langle D^1\rangle_{\Omega_p}=\frac{1}{v^2}\int\omega^2N_{\mathbf{k}}\langle u_p\rangle_{\Omega_p}dk.$$

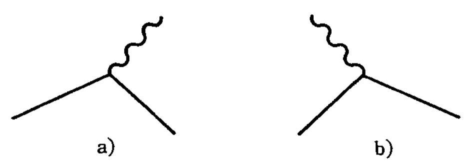
This result indicates that in this case the acceleration is determined by the value of D^1 averaged over the particle-momentum directions. For Langmuir oscillations, for example, we obtain in lieu of (3.5)

$$\langle\dot{E}_p\rangle_{\Omega_p}=\frac{e^2\omega_{0e}^3}{p}\left\{\frac{1-v^2}{v^2}\langle N^1(v)\rangle_{\Omega_k}+2\int\limits_{v_{ph1}}^{v}\frac{\langle N^1(v_{ph})\rangle_{\Omega_k}dv_{ph}}{v_{ph}}\right\}. \qquad (5.28)$$

The right side of (5.28) contains the wave distribution function averaged over the quantum momenta Ω_k. We note, finally, that nonlinear effects can contribute to isotropization of the quantum directions[31,56].

6. Effects of Acceleration Under Induced Scattering

a) Comparison of scattering acceleration with Cerenkov acceleration. We proceed now to determine the region where induced Cerenkov acceleration predominates over acceleration due to scattering. We shall consider this question by using as an example acceleration by means of longitudinal oscillations, since this is of greatest interest for possible applications. It is simplest to use intuitive physical concepts borrowed from the quantum approach. The plasma constitutes a system of particles that interact weakly with one another. The interaction can therefore be expanded in terms of the number of interactions between the particle and the electromagnetic fields. Such an interaction act corresponds to emission or absorption of a wave by the particle. It is convenient to represent the corresponding processes by means of diagrams. Cerenkov radiation and absorption corresponds to the diagrams of Fig. 3. The wavy lines show the emitted (Fig. 3a) in the absorbed (Fig. 3b) waves. The solid lines shows the particle motion. The effect of acceleration, according (2.5), is the difference of the contributions of the induced absorption and induced radiation. Neglecting the spontaneous processes, we can state that to each external "photon" line there corresponds a factor N_k in the probability. Therefore, if we are interested in the term that follows N_k in the probability (proportional to N_k^2), we must consider the possible processes represented by diagrams with two external photon lines. It can be stated that a turbulent plasma is a medium in which the interaction between the particles is realized essentially not by collisions but by means of really (not virtually) radiated and absorbed particles. It is appropriate to recall here that any interaction between particles, in accordance with the modern notions originating in the works of Tamm[58], Yukawa[59], and Feynman[60], is realized by transfer of waves from one particle to another. A turbulent plasma is analogous to a nonstationary liquid in which waves propagate and transfer momentum from one particle to another. These waves



a) b)

FIG. 3

are capable of completely "breaking away" from one particle (that is, "going off" into the wave zone) before they interact with another particle. The presence of an aggregate of particles leads to the possibility of Cerenkov emission and absorption of waves, something forbidden in vacuum. In a weakly turbulent plasma, the processes containing two external photon lines are small compared with Cerenkov processes (see Fig. 3). Processes with two photon lines correspond either to the absorption of one of the waves with subsequent emission of another, that is, to a scattering process, or to the emission or absorption of two waves [61]. Let the probability of scattering be $u_-(\mathbf{k}_1, \mathbf{k}_2)$ ($\mathbf{k}_1$ is absorbed and $\mathbf{k}_2$ emitted), and let $u_+(\mathbf{k}_1, \mathbf{k}_2)$ be the probability of emission of two waves.* The statistical change of the state of the accelerated particle is determined by induced processes, the probabilities of which can be written in the form

$$w_+ = u_+ N(\mathbf{k}_1) N(\mathbf{k}_2), \qquad w_- = u_- N(\mathbf{k}_1) N(\mathbf{k}_2). \tag{6.1}$$

We note further that the change in momentum of the particle amounts to $\mathbf{k}_1 - \mathbf{k}_2$ in scattering and $\mathbf{k}_1 + \mathbf{k}_2$ in absorption.

Thus, according to the general formula (2.4), which determines the diffusion coefficient D_{ij} in terms of the probability of the process and the change of particle momentum, we obtain additional diffusion, over and above that obtained from the induced Cerenkov acceleration processes, due to the induced scattering and to induced emission (absorption) of two quanta:

$$D_{ij\pm} = \int u_\pm(\mathbf{k}_1, \mathbf{k}_2) N(\mathbf{k}_1) N(\mathbf{k}_2) (\mathbf{k}_{1i} \pm \mathbf{k}_{2i}) \times (\mathbf{k}_{1j} \pm \mathbf{k}_{2j})\, d\mathbf{k}_1\, d\mathbf{k}_2 \cdot \hbar^2. \tag{6.2}$$

However, (6.2) is not the only nonlinear correction to the induced Cerenkov acceleration. A change takes place also in the probability of Cerenkov emission [62,63]. We can write in place of (4.1)

$$u'_p(\mathbf{k}) = u_p(\mathbf{k}) - \int \delta w_p(\mathbf{k}_1 \mathbf{k}_2) N(\mathbf{k}_2)\, dk_2, \tag{6.3}$$

where $u_p(\mathbf{k})$ is the probability of spontaneous Cerenkov emission in the absence of waves. The additional diffusion, due to these corrections, is

$$\delta D_{ij} = -\int \delta w_p(\mathbf{k}_1, \mathbf{k}_2) N(\mathbf{k}_1) N(\mathbf{k}_2) k_{1i} k_{1j}\, d\mathbf{k}_1\, d\mathbf{k}_2. \tag{6.4}$$

In (6.3) we have substituted the minus sign in connection with the fact that the indicated corrections usually lead to a reduction in the intensity of the Cerenkov radiation. The physical meaning of the corrections (6.3) can be readily explained from a simple analogy with the scattering of electromagnetic waves by a radiating oscillator. Modulation by means of the external wave leads, as is well known, to the appearance of scattered frequencies and to a reduction in the intensity of the fundamental frequency [62]. In the case under consideration, the spontaneous emission in the system is due to the Cerenkov mechanism. The change of intensity of the Cerenkov radiation is connected with the change in the law of motion of the charge in the presence of waves. This will be the only correction to the probability of the Cerenkov emission solely in an electrodynamically linear radium.

The very important role of the corrections (6.3) can be illustrated for the case when the spectrum of the scattering waves includes waves incident at an angle that is quite close to the Cerenkov angle. This case, for example, is similar to resonant scattering, when the frequency of the incident radiation is close to the frequency ω_s of the radiating oscillator. The probability of scattering in this case increases like $1/(\omega - \mathbf{k}\cdot\mathbf{v})^4$. However, a similar rate of growth holds for the corrections to the Cerenkov radiation, so that the two effects cancel each other [62,63].

In a plasma there are also additional corrections to the probability of Cerenkov radiation, connected with the fact that the plasma is a nonlinear medium. This follows to an approximate degree quantitatively from the fact that in the presence of waves the plasma can be characterized an effective dielectric constant that depends on the wave intensity [25].

$$\varepsilon_{\text{eff}} = \varepsilon(\omega, \mathbf{k}) + \int w, \omega(\mathbf{k}, \mathbf{k}') N_{\mathbf{k}}\, dk, \tag{6.5}$$

and consequently the probability of Cerenkov radiation, which depends functionally on ϵ, turns out to be different in an electrodynamically nonlinear medium.

The nonlinearities affect strongly also scattering probability [63,64]. Within the framework of the linear approximation, the waves do not interact with each other. The simplest nonlinear interaction corresponds to emission of a wave by a wave, something that can be represented graphically (Fig. 4a).

Whereas the usual scattering connected with charge oscillations can be represented graphically as in Fig. 4b, the nonlinear scattering corresponds to inclusion of the element of Fig. 4a (see Fig. 4c). It is important that the scattered radiations due to the processes of Figs. 4b and 4c interfere and can very

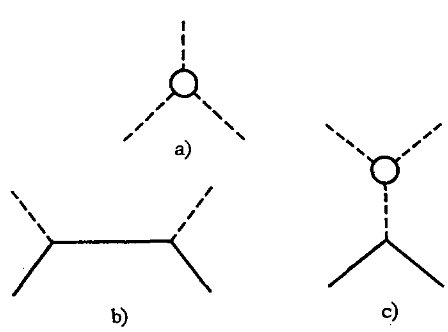


FIG. 4

*The probability of absorption of two waves is expressed in terms of u_+ on the basis of the principle of detailed balancing.

strongly suppress each other. The physical meaning of this is explained in the following manner. Whereas the scattering of Fig. 4b corresponds to dipole radiation from a charge oscillating under the influence of an incident electromagnetic wave, the scattering of Fig. 4c, as shown in [64], is connected with the fact that the charge passes through the plasma-density inhomogeneities produced by the incident wave with a velocity which in first approximation is constant. This gives rise to radiation of the transition-radiation type [65–68] in a medium having a layered structure. The radiation connected with the charge oscillations is therefore quite accurately phased with the radiation of the transition type. It must be noted that large nonlinearities occur in the case when the incident wave is longitudinal. For transverse waves with high frequencies $\omega \gg \omega_0$, the contribution of the nonlinearity is quite small. Finally, the indicated compensation takes place for electrons (this is connected with the fact that in first approximation the nonlinear element (Fig. 4a) is due to the interaction of the plasma electrons). For heavy particles, the oscillations in the field of the incident wave become negligibly small (Fig. 4a) compared with the nonlinear scattering (Fig. 4b) and therefore the probability of scattering by heavy particles can greatly exceed the probability of scattering by electrons [63,64].

We can now consider the question of the maximum intensities of the oscillations, up to which the corrections to the effects of induced Čerenkov acceleration by, say, Langmuir oscillations remain small. For this purpose we make use of the coefficients $D_{ij\pm}$ (6.2) and δD_{ij} (6.4) and the expressions for the scattering probabilities and for the corrections to the Cerenkov radiation [63]. Following [63], we shall consider corrections that arise for ultrarelativistic particles. This case is of interest, first, for astrophysical applications in the problem of cosmic-ray acceleration [2,12,13]. Second, in this limit, only one of the two types of effects (particle oscillations in the wave field and nonlinearities) is significant. Owing to the relativistic mass increase, the role of the electron oscillations in the wave field become negligibly small compared with the nonlinearities. Thus, the electrons behave like the heavy ions. The corrections in this case are independent of the particle mass. The probabilities of nonlinear scattering and the corrections to Cerenkov radiation of Langmuir waves are given in Appendix 1. This Appendix also presents the results of calculation of the systematic acceleration for an isotropic wave distribution, obtained by using the indicated probabilities as well as (2.9). A particularly clear result is obtained when the wave distribution $N^1(v_{ph})$ has a narrow spectrum about

$$v = v^0_{ph}, \quad \Delta v_{ph} \ll v^0_{ph}, \quad N^1(v^0_{ph}) = N^1_0 \Delta v^1_{ph}.$$

Then

$$\dot{E}_p = \frac{2e^2\omega^3_{0e}}{\varepsilon}\frac{\hbar N^1_0}{v^0_{ph}}\left(1 + \frac{e^2}{2\pi m^2_e}\frac{179}{1080}\frac{N^1_0}{(v^0_{ph})^6}\right). \tag{6.6}$$

The criterion for the smallness of the correction term can be conveniently expressed in terms of the mean square of the oscillation field intensity

$$W^1 = \frac{\overline{E}^2}{4\pi} = \int \frac{dk\omega_{0e}N^1_k}{(2\pi)^3} \approx \frac{1}{2\pi^2}\frac{\omega^4_{0e}}{(v^0_{ph})^4}N^1_0. \tag{6.7}$$

The sought criterion is

$$\frac{e^2}{m^2_e}\frac{1}{(v^0_{ph})^2}\frac{\overline{E}^2}{\omega^2_{0e}} \ll 1. \tag{6.8}$$

Its physical meaning can be readily understood by recognizing that the amplitude of the plasma-electron oscillations in the wave field is $x = eE_0/m\omega_0^2$, and consequently (6.8) signifies that this amplitude can be small compared with the wavelength, $k^2x^2 \ll 1$. We note that the motion of a trial charge of plasma in the field of the wave is determined in the general case by a nonlinear equation of the type

$$m\ddot{x} = eE_0 \cos(kx - \omega t) \tag{6.9}$$

and the condition of weak nonlinearity corresponds to $kx \ll 1$.

From the foregoing calculation we see that the main contribution to the corrections is made by the plasma nonlinearity, whereas the changes in the velocity of the accelerating particle are small. This follows already from the fact that the oscillations of the accelerating electron are ϵ/m times smaller than the oscillations of the plasma electron. It should be noted, however, that (6.8), owing to the small factor e^2/m, admits of rather large values of E^2, which frequently cannot be attained under conditions of possible applications.

The situation is different when a magnetic field is present in the plasma. In this case it may turn out that the influence of the change in the velocity of the accelerating particle will be larger than the effect of the plasma nonlinearities.

By way of illustration we present a simple example, where the corrections to the acceleration are due only to charge oscillations, and the medium can be regarded as electrodynamically linear. The nonlinearities in the medium are determined by the amplitude of the oscillations, for example, of the electrons of the medium under the influence of the external wave. These amplitudes are small if the electrons are bound and $\omega \ll \omega_s$ (ω_s is the natural frequency of the coupling). In this case it is natural to turn to the effect of Cerenkov acceleration by transverse electromagnetic waves, which was considered in Sec. 2. Owing to the presence of ω_s, we can, on the one hand, regard the medium as electrodynamically linear, and on the other the Cerenkov condition for transverse waves can be satisfied. For an isotropic medium with natural frequencies ω_s, the sought criterion is best obtained by considering the corrections to the induced Cerenkov radiation of the transverse waves [12]. Qualitatively this criterion follows from the conditions under which the expansion is possible

$$p \gg eA \sim \frac{eE}{\omega}, \quad W = \frac{E^2 n^2}{4\pi} \ll \frac{n^2\omega^2 \varepsilon_p^2}{4\pi e^2} \equiv W_{\max}, \quad \varepsilon_p \gg m.$$

It is meaningful to estimate the acceleration effect when $n^2 = \epsilon^t(\omega) \gg 1$, that is, for very slow waves. This case is of interest because it makes it possible to compare the considered acceleration effect with the Fermi effect (Sec. 2), inasmuch as the assumption that the waves are slow corresponds to some degree to the assumption that the magnetic wall velocity u in (2.1) is small compared with the velocity of light. It is obvious that in this case the role of u will be played by the quantity $1/n$. From (4.5) we obtain in order of magnitude

$$\dot{E}_p \sim 2\pi \frac{e^2}{\varepsilon_p} \frac{1}{\omega} \frac{1}{n^3} W \tag{6.10}$$

On the borderline of applicability of the theory, that is, for $W \sim W_{\max}$, we have

$$\dot{E}_p \approx \frac{u^2 \varepsilon_p}{\lambda}, \quad u = \frac{1}{n}, \quad \lambda = \frac{2\pi u}{\omega}. \tag{6.11}$$

Comparing (6.11) with (2.1) and recognizing that $v \sim 1$ ($\epsilon \gg m$), we see that in the limit the acceleration becomes of the Fermi type.* The foregoing analysis shows that a magnetic field must be present in the plasma to make the Fermi mechanism possible, and the wave frequency must be quite low. Thus, Fermi acceleration can arise only for the particular case of low-frequency hydrodynamic turbulence. Therefore Fermi acceleration is a very particular case corresponding to low-efficiency acceleration by a large-scale turbulence. We note also that acceleration by high-frequency turbulence increases with increasing wave intensity E^2, in other words with increasing energy of turbulent motion, whereas the Fermi acceleration corresponds to "saturation," that is, it is no longer dependent on the energy of the turbulent motion. It must also be noted that the limiting value (6.11) corresponding to saturation depends very strongly on the turbulence scale λ and in the case of high-frequency turbulence it greatly exceeds the corresponding value for low-frequency turbulence. Therefore even the below-limiting value of the acceleration by high-frequency turbulence can greatly exceed the limiting Fermi acceleration by low-frequency turbulence.

Furthermore, there are grounds for assuming that in the absence of the magnetic field the limiting value (6.11) for acceleration by plasma longitudinal oscillations is never attained. Indeed, the criterion (6.8) is equivalent to stating that the plasma oscillations become nonlinear. Acceleration by nonlinear oscillations would have the same character, something also indicated by formula (6.6), which has the same dependence on the particle energy as for induced Cerenkov acceleration. However, large-amplitude nonlinear plasma waves will be unstable. It is known that the criterion for the appearance of one of the fast instability mechanisms of multivelocity streams in nonlinear waves is

$$\frac{e^2}{m_e^2} \frac{1}{v_{\text{ph}}^2} \frac{\overline{E}^2}{\omega_{0e}^2} > 1. \tag{6.12}$$

The stability condition differs from (6.8) only in that the strong inequality is replaced by an inequality. Therefore the question of the transition from acceleration by plasma oscillations at H = 0 to Fermi acceleration reduces to the question whether waves of large amplitude can exist in the plasma for a long time.

b) Problem of time variation of the turbulence spectrum. Let us discuss briefly the role of induced scattering of the waves by the particles of the plasma itself, and not by accelerated particles. The probability of scattering by electrons, in the particular case when $v_{\text{ph}} \ll v_{Te}(m_i/m_e)^{1/3}$, is of the form[64,56]*

$$w_p(\mathbf{k}_1\mathbf{k}_1') = \frac{4e^4}{m_e^2\omega_{0e}^2} \frac{(\mathbf{k}_1\mathbf{k}_1')^2(\mathbf{k}_1\mathbf{v})^2}{k_1^2 k_1'^2} \delta(\mathbf{k}_1\mathbf{v} - \mathbf{k}_1'\mathbf{v} - \omega_1 + \omega_1'). \tag{6.13}$$

It is easy to show that the interaction described by (6.13) corresponds to conservation of the oscillation energy and to spectral pumping-over of the oscillations into the region of lower phase velocities with characteristic time of the order of[31,56]

$$\tau \sim \frac{1}{\omega_{0e}} \frac{m_e n v_{Te}^2}{W^l} \left(\frac{v_{\text{ph}}^l}{v_{Te}}\right)^3, \quad v_{\text{ph}}^l = \frac{\omega_{0e}}{|\mathbf{k}|}. \tag{6.14}$$

An important role may be played also by the spectral pumping-over connected with the scattering by the plasma ions[64,56,69,70]. The presence of the indicated spectral pumping-over is very important for the acceleration effect since, for a given oscillation energy, the acceleration efficiency increases like v_{ph}^3.

Finally, induced scattering by the plasma particles can lead to the transformation of longitudinal oscillations into transverse ones[56,69,72]. Transverse oscillations can accelerate the charged particles in the plasma only in the presence of external magnetic fields[10,14] (see Sec. 9 below). Finally, the oscillation spectrum can change as a result of decay processes[27,24,34,25] etc.

c) The injection problem. The problem of injection as part of the general problem of statistical particle acceleration was discussed quite long ago[2,148]. Principal attention was paid there to problems connected with the Fermi statistical acceleration mechanism. Very important results in this direction were obtained

*It is shown in[12], using "magnetic" clouds as an example, that Fermi acceleration takes place when the dimension of the "cloud" and the magnitude of the magnetic field are sufficient for reflection of the particle.

*The scattering of electromagnetic waves in a plasma is the subject of many papers[72,73]. The transition from transverse to longitudinal waves in scattering was first considered in[72].

by Syrovat-skiĭ and Korchak[148], who have shown that for heavy ions the injection conditions are easier in the Fermi acceleration. This is very important for the problem of the origin of cosmic rays.

In the here-considered version of statistical acceleration by high-frequency turbulence (for example, by plasma waves), the situation is radically changed. Namely, in the presence of sufficiently large high-frequency turbulence the electrons can be accelerated from the tail of the Maxwellian distribution $v > v_{Te}$, that is, "without injection" (actually the injection is determined by the self-regulation process [158]).

For ions, however, the situation is different, since their average thermal velocity is usually much smaller than v_{Te} and the injection conditions are not satisfied for ions.

It must be noted that the induced-scattering effects considered above can lead to acceleration effects, just as effects of induced emission and absorption.

Particle acceleration due to induced scattering is particularly important if the effects of Cerenkov-acceleration are forbidden, as is the case, for example, when high-frequency turbulence acts on ions with $v < v_{Te}$. Nonlinear effects of induced scattering by Langmuir oscillations can give rise to injection and Cerenkov acceleration if

$$(\mathbf{k}_1-\mathbf{k}_2)\,\mathbf{v}=\omega_1-\omega_2=\frac{3v_{Te}^2}{2\omega_{0e}}\,(k_1^2-k_2^2). \qquad (6.15)$$

It is also important that the probability of scattering by the ions is quite high (compared with the probability of scattering by electrons). The reason is that there is no cancellation here of two types of scattering (due to the charge oscillations in the wave field and due to transition radiation from the inhomogeneities produced by the wave). Because of the large mass of ion, only nonlinear scattering of the transition-radiation type occurs, and[70]

$$w_{\mathrm{p}}(\mathbf{k}_1,\mathbf{k}_2)=\frac{e^4(\mathbf{k}_1\mathbf{k}_2)^2}{m_e^2\omega_{0e}^4k_1^2k_2^2}\,\delta\left((\mathbf{k}_1-\mathbf{k}_2)\,\mathbf{v}-\omega_1+\omega_2\right)(2\pi)^{-3}. \qquad (6.16)$$

With the aid of the diffusion coefficient (6.2) and the formula for the systematic acceleration (2.9), we obtain in the case of $v \ll v_{Te} \ll 1$ and $v \ll v_{Te}^2/v_{ph}$,* for isotropic turbulence,

$$\dot{E}_{\mathrm{p}}=\frac{31}{45\pi^2}\,\frac{e^2}{m_e}\,\frac{\omega_{0e}}{nv_{Te}^2m}\int\limits_0^\infty k^5N_k^2\,dk. \qquad (6.17)$$

In order of magnitude we have the following estimate ($\Delta k \sim k$):

$$\dot{E}_{\mathrm{p}}=\frac{W^{\mathrm{l}}}{nm_ev_{Te}^2}\,\frac{31}{90\pi}\,\frac{e^2}{m\omega_{0e}}\,W^{\mathrm{l}}(2\pi)^3. \qquad (6.18)$$

From a comparison of (6.18) with induced Cerenkov acceleration of the ions

$$\dot{E}_{\mathrm{p}}\approx 2\pi^2\left(\frac{v_{\mathrm{ph}}}{v}\right)^3\frac{e^2}{m\omega_{0e}}\,W^{\mathrm{l}}\approx 2\pi^2\frac{e^2}{m\omega_{0e}}\,W^{\mathrm{l}} \text{ when } v\sim v_{\mathrm{ph}}$$

we see that (6.18) contains an additional factor of the order of

$$\eta=\frac{W^{\mathrm{l}}}{nm_ev_{Te}^2}<1. \qquad (6.19)$$

Thus, under intense turbulence the rate of injection can be close to statistical Cerenkov acceleration.

d) The problem of ion acceleration. A special distinction must be made in the case of the acceleration of plasma ions, which is of interest when it comes to turbulent heating of plasma. The ions accelerated in a turbulent plasma are usually observed experimentally (see ch. III), and their velocities are much lower than v_{Te}. In the case of intense turbulence (when η is not too small compared with unity) it follows from (6.18) that the characteristic time required for the ion to acquire an energy ϵ_i is the smaller, the higher the electron temperature:

$$\tau\approx\frac{1}{\omega_{0e}}\,\frac{m_i}{m_e}\,\frac{\varepsilon_i}{T_e}. \qquad (6.20)$$

Along with induced scattering, a possible mechanism of ion acceleration is Cerenkov acceleration by low-frequency oscillations generated by the high-frequency ones because of nonlinear decay effects (see Sec. 1).

7. Acceleration of Photons[45] and Plasmons

We have dealt so far with acceleration of charged particles. In order to emphasize the generality of the mechanism of statistical acceleration in a turbulent plasma, we present two examples: acceleration of photons and accelerations of neutrinos in a turbulent plasma under the influence of plasma oscillations.

1. The distribution function of the photons (the number of quanta) is denoted $N_{\mathbf{k}}^{\mathrm{t}}$. In the presence of a statistical mechanism that causes the photons to acquire or lose energy in small batches, we must write in place of (2.3)

$$\frac{\partial N_{\mathbf{k}^{\mathrm{t}}}}{\partial t}=\frac{\partial}{\partial k_i^{\mathrm{t}}}\,D_{ij}\,\frac{\partial N_{\mathbf{k}^{\mathrm{t}}}}{\partial k_j^{\mathrm{t}}}, \qquad (7.1)$$

$$D_{ij}=\int w_{\mathbf{k}^{\mathrm{t}}}(\mathbf{k}^{\mathrm{l}})\,k_i^{\mathrm{l}}k_j^{\mathrm{l}}\,d\mathbf{k}^{\mathrm{l}}. \qquad (7.2)$$

A process similar to Cerenkov radiation for transverse waves is decay of a transverse wave, for example, into a Langmuir wave, as shown in Fig. 5. The probability of this process was calculated in [76] in the approximation $\omega^{\mathrm{t}} \gg \omega_{0e}$. We present here the exact expression for the probability, which is valid when ω^{t} is of the order of ω_{0e} [45]:

$$u_{\mathbf{k}^{\mathrm{t}}}(\mathbf{k}^{\mathrm{l}})=$$

$$\frac{e^2\omega_{0e}^4(\mathbf{k}^{\mathrm{l}})^2\left(1+\dfrac{(\mathbf{k}^{\mathrm{t}},(\mathbf{k}^{\mathrm{t}}-\mathbf{k}^{\mathrm{l}}))^2}{(\mathbf{k}^{\mathrm{t}})^2(\mathbf{k}^{\mathrm{t}}-\mathbf{k}^{\mathrm{l}})^2}\right)\delta\left(\sqrt{(\mathbf{k}^{\mathrm{t}})^2+\omega_{0e}^2}-\sqrt{(\mathbf{k}^{\mathrm{t}}-\mathbf{k}^{\mathrm{l}})^2+\omega_{0e}^2}-\omega_0^{\mathrm{l}}\right)}{8\pi m_e^2\,[\omega^{\mathrm{l}}(\mathbf{k}^{\mathrm{l}})]^4\sqrt{(\mathbf{k}^{\mathrm{t}})^2+\omega_{0e}^2}\,\sqrt{(\mathbf{k}^{\mathrm{t}}-\mathbf{k}^{\mathrm{l}})^2+\omega_{0e}^2}\,\dfrac{\partial\varepsilon^{\mathrm{l}}(\omega^{\mathrm{l}},\mathbf{k}^{\mathrm{l}})}{\partial\omega^{\mathrm{l}}}}. \qquad (7.3)$$

*When $v \gg v_{Te}^2/v_{ph}$, the systematic acceleration decreases rapidly:

$$\dot{E}_p\approx\frac{189\pi}{4(2\pi)^3m}\left(\frac{v_{Te}}{v}\right)^5\frac{v_{Te}^3e^2}{\omega_0^4m_en}\int\limits_0^\infty N_{\mathbf{k}_2}^{\mathrm{l}}\,\frac{dk_2}{k_2}\int\limits_0^{k_2}dk_1N_{\mathbf{k}_1}^{\mathrm{l}}k_1^2(k_1^2-k_2^2)^4.$$

The fluctuation acceleration, however, has the same order of magnitude as when $v < v_{Te}^2/v_{ph}$.

In the case when $k^l \ll k^t$, we can speak of a statistical change of the state of the photon, describing it by an equation of the type (7.1). Processes that lead to a change of the state of the photon correspond to induced processes of decay and coalescence of waves, the probabilities of which are written

$$w_{k^t}(k^l) = N^l_{k^l} u^{(0)}_{k^t}(k^l),$$

where $u^{(0)}_{k^t}(k^l)$ corresponds to (7.3) in the case of small recoil. Assuming approximately that $\omega^l \approx \omega_{0e}$ we obtain

$$u^{(0)}_{k^t}(k^l) = \frac{e^2\omega_{0e}(k^l)^2}{8\pi m_e^2[(k^t)^2+\omega_{0e}^2]}\delta(\omega_{0e} - k^l v^t_{gr}), \tag{7.3a}$$

where $v_{gr}{}^t$ is the group velocity of the transverse waves in the plasma. Formula (6.3a) describes "Cerenkov" radiation of longitudinal waves by transverse ones, the role of the "particle" velocity being played here by the group velocity of the waves.*

$$v^t_{gr} = \frac{k^t}{k^t}\frac{d\omega^t}{dk^t} = \frac{k^t}{\sqrt{(k^t)^2+\omega_{0e}^2}}.$$

It is instructive to compare expression (7.3a) with the probability of the Cerenkov radiation by a plasma wave

$$u_p(k^l) = \frac{e^2\omega_{0e}}{2\pi(k^l)^2}\delta(\omega_{0e} - k^l v).$$

The probability of Cerenkov radiation of the charge is the larger, the smaller the momentum of the radiated quantum, whereas the opposite holds for decay processes.

From (7.2) we obtain for $k^l \ll k^t$

$$D^l = \frac{e^2\omega_{0e}^3}{8\pi(k^t)^3 m_e^2}\int k^l N^l_{k^l}\,dk^l\sqrt{(k^t)^2+\omega_{0e}^2}. \tag{7.4}$$

According to the general formula (2.9), the systematic change in the average photon energy is zero, and

$$\frac{d}{dt}\langle\omega^2\rangle = D^l(v^t_{gr})^2.$$

The order of magnitude of the increase in the photon-energy scatter is

$$\frac{d}{dt}\frac{\omega^2}{\omega_{0e}^2} = \omega_{0e}\frac{\pi}{4}\frac{W^l}{nm_e v_{ph}}\left(\frac{\omega_{0e}}{\omega}\right)^2. \tag{7.5}$$

Consequently, in the approximation of small momentum transfer, the systematic acceleration is equal to zero and only fluctuation acceleration takes place. It is easy to see, however, that the vanishing of the systematic acceleration is only the consequence of the approximation employed. The exact expression for the systematic acceleration without expansion in powers of k^l can be obtained from (2.15):

$$\frac{d}{dt}\langle\omega\rangle = \int \omega^l(k^l)\left[w_{k^t+k^l}(k^l) - w_{k^t}(k^l)\right]dk^l. \tag{7.6}$$

Using the probability (7.3) for isotropic turbulence, we have

$$\frac{d}{dt}\langle\omega\rangle = \int\frac{dv_{ph}}{v_{ph}^7}\left\{N^l(v_{ph})\frac{e^2}{2m_e^2}\frac{\omega_{0e}^9}{\omega^2(\omega^2-4\omega_{0e}^2)(\omega^2-\omega_{0e}^2)^{3/2}}\times\left[2\omega^2(1-v_{ph}^2) - \frac{\omega_{0e}^2}{v_{ph}^2}(1+v_{ph}^2)^2\right]\right\}. \tag{7.7}$$

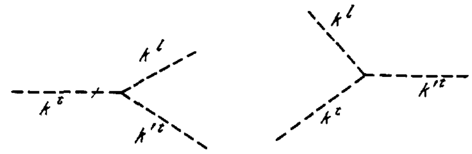


FIG. 5

Formula (7.7) is correct if decays and coalescences can occur simultaneously for a given frequency, something requiring simultaneous satisfaction of the inequalities for coalescence (7.8) and decay (7.9):

$$\omega > \frac{\omega_{0e}}{2}\left\{\frac{1}{v_{ph}}\sqrt{\frac{1+3v_{ph}^2}{1-v_{ph}^2}} - 1\right\}, \tag{7.8}$$

$$\omega > \frac{\omega_{0e}}{2}\left\{\frac{1}{v_{ph}}\sqrt{\frac{1+3v_{ph}^2}{1-v_{ph}^2}} + 1\right\}. \tag{7.9}$$

The right sides of (7.8) and 7.9) are minimal when $v_{ph}^2 = 1/3$. At this value, the coalescence is possible when $\omega > \omega_{0e}$, and decay when $\omega > 2\omega_{03}$.*

Thus, (7.7) is valid when $\omega > 2\omega_{0e}$ †. If $\omega \gg \omega_{0e}/v_{ph}$ and $v_{ph} \ll 1$, then the order of magnitude of the acceleration is

$$\frac{d}{dt}\langle\omega\rangle \approx \frac{\pi}{2}\omega_{0e}^2\frac{W^l}{nm_e v_{ph}^3}\left(\frac{\omega_{0e}}{\omega}\right)^5. \tag{7.10}$$

The systematic acceleration (7.10) is smaller in order of magnitude by a factor $\omega_{0e}^2/v_{ph}^2\omega^2 \ll 1$ than the fluctuation acceleration.

It must be noted that the described acceleration effect takes place at a sufficiently small number of photons, $N^t \ll N^l$ (for an exact equation see[76, 82]).

It must be specially emphasized that the photons experience besides the change in energy also a change of direction, that is, scattering, which in accordance with Sec. 1, defines D^t. From (7.2) it follows that

$$D^t = \frac{1}{2}\{\mathrm{Sp}\,D_{ij} - D^l\} = \frac{e^2\omega_{0e}}{16\pi m_e^2}\int N^l_{k^l}\,dk^l(k^l)^4 v^t_{gr}\left(1 - \frac{v_{ph}^2}{(v^t_{gr})^2}\right). \tag{7.11}$$

When $v_{ph} \ll 1$ the scattering is $1/v_{ph}{}^2$ times larger than the fluctuation change of frequency.

2. We consider further, for example, the acceleration of neutrinos** in a turbulent plasma. The absorption and emission of plasma waves by neutrinos is possible in the presence of weak $(e\nu)(e\nu)$ interaction, as shown in Fig. 6[78, 79]. The probability of the process was obtained in[46]. A formula such as (7.6) makes it possible to determine the systematic change in the neutrino energy

$$\frac{d}{dt}\langle\varepsilon_\nu\rangle = \int\frac{N^l_{k^l}\,dk^l g^2(\omega^l)^2}{(2\pi)^3 e^2\varepsilon_\nu}\left(1 - \frac{(\omega^l)^2}{(k^l)^2}\right)^2\frac{1}{\frac{\partial\varepsilon^l}{\partial\omega^l}}, \tag{7.12}$$

where g is the weak-interaction constant.

Although the $(e\nu)(e\nu)$ interaction is quite weak, it can play an important role under astrophysical conditions because of the in-

*Cerenkov radiation of transverse but not longitudinal waves from electromagnetic waves was considered by G. A. Askar'yan[77]. In particular, the Cerenkov condition $\omega = k \cdot v_{gr}$ was pointed out in[77]. The effect considered in[77] had a different dependence on either the frequency or the amplitude of the high-frequency field.

*The latter inequality is the result of the fact that the transverse wave resulting from the decay has a frequency $\omega > \omega_{0e}$, as does the produced longitudinal wave.

†When $v_{ph}^2 = 1/3$ and ω can reach $2\omega_{0e}$, the numerator of (7.7) is equal to $4/3(\omega^2 - 4\omega_0{}^2)$, that is, (7.7) has no singularity when $\omega = 2\omega_{0e}$.

**We present this example to illustrate the universality of the acceleration mechanism.

tense turbulence produced in explosive processes and because of the large penetrating ability of the neutrinos, which are capable of transporting energy from large masses of matter (for a detailed literature see[78]).

Besides acceleration, an important role may be played by the generation of neutrino pairs by the turbulence[46]. This process is allowed only for plasma waves with phase velocities larger than the speed of light in vacuum. Waves with such phase velocities can result from nonlinear effects[46].

The foregoing examples illustrate that not only charged particles are capable of statistical acceleration. We emphasize also that the increase in photon frequency, along with transformation of longitudinal waves into transverse ones of frequency ~ ω_{0e}, increases the efficiency of the acceleration of electrons by transverse waves (Sec. 9).

3. We now consider acceleration of Langmuir plasmons. The acceleration of Langmuir oscillations with the aid of low-frequency hydrodynamic oscillations is one of the possible mechanisms of the nonlinear coupling of different turbulence scales. Such a mechanism is similar to the photon acceleration considered above, if the role of the photons is assumed by Langmuir plasmons accelerated by low-frequency ion-sound oscillations.

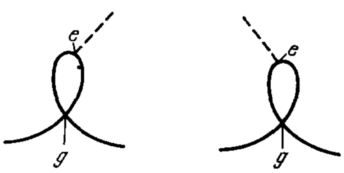


FIG. 6

The decay probability is of the form[80] (see [81, 82, 24, 35])

$$u_{\mathbf{k}^l}(\mathbf{k}^s)=\frac{e^2\omega_s^3 m_i}{16\pi m_e^3 v_{Te}^4 k_s^2}\,\frac{(\mathbf{k}^l,\mathbf{k}^l-\mathbf{k}^s)^2}{(\mathbf{k}^l)^2(\mathbf{k}^l-\mathbf{k}^s)^2}\,\delta(\omega^l(\mathbf{k}^l)-\omega^l(\mathbf{k}^l-\mathbf{k}^s)-\omega_s). \tag{7.13}$$

For isotropic turbulence

$$\frac{d}{dt}\langle\omega^l\rangle=\int N_{k_s}^s\,dk_s\,\frac{e^2 m_i\omega_{0e}^2\omega_s^5}{4\pi m_e^3 (k^l)^3 (k^s)^3}\,\frac{\left((k^l)^2(k^s)^2-\frac{(k^s)^4}{2}-\frac{2}{9}\frac{\omega_{0e}^2\omega_s^2}{v_{Te}^4}\right)}{9v_{Te}^8\left((k^l)^4-\frac{4\omega_{0e}^2\omega_s^2}{9v_{Te}^4}\right)}. \tag{7.14}$$

Simultaneous decays and coalescence (when formula (7.14) is valid) are possible only if

$$(k^l)^2>\frac{1}{(k^s)^2}\left(\frac{(k^s)^2}{2}+\frac{\omega_{0e}\omega_s}{3v_{Te}^2}\right)^2, \tag{7.15}$$

i.e., the phase velocity of the longitudinal oscillations should not exceed a quantity of the order of $v^*_{ph} = 3v_{Te}\sqrt{m_i/m_e}$. When $v^l_{ph} \ll v^*_{ph}$, the order of magnitude of the acceleration (7.14) is described by the expression

$$\frac{d}{dt}\langle\omega^l\rangle=\frac{\pi}{18}\,\omega_{0i}^2\left(\frac{v_{ph}^l}{v_{Te}}\right)^2\left(\frac{k^s}{k^l}\right)^3\frac{W^s}{nm_e v_{Te}^2}. \tag{7.16}$$

Since the total number of Langmuir quanta does not change in the approximation (7.16), the change in the total energy of the oscillations cannot be large compared with their initial energy.

8. Acceleration in a Plasma Situated in External Electric Fields

The electric field in the plasma is usually alternating. It is therefore necessary to distinguish between two cases: the case when the frequency of the external field is small compared with the characteristic frequency of the considered plasma oscillations, and the case when the frequency of the field is large compared with the plasma-oscillation frequency. In the first case one can speak of constant electric fields, whereas the second corresponds to the action of an intense high-frequency field on the plasma. We shall assume that the motion of the particles in the plasma is determined in first approximation by the external fields, and not by thermal motion of the plasma particles, for the opposite case does not differ much qualitatively from that considered above. It is necessary for this purpose that the particle velocity acquired in the external field exceed the average thermal velocity of the plasma particle, that is, that the external field intensity be sufficiently high.

It must be borne in mind that a plasma placed in an external field may become unstable. The evolution of the instability may lead to plasma heating. This in turn can cause the velocity acquired by the particle in the external field to become smaller than the thermal velocity of the particles. Thus, the approach employed may be applicable in many cases only during the initial time interval after "turning on" the external fields.

a) Constant electric field. It is known that oscillations that increase in time are produced in a plasma placed in a strong external constant electric field. The maximum increment and frequency of the oscillations are determined by the expression[98]

$$\gamma_{max}=\frac{\sqrt{3}\left(\frac{m_e}{m_i}\right)^{1/3}}{2^{4/3}}\,\omega_{0e},\qquad \omega\approx\frac{1}{2^{4/3}}\left(\frac{m_e}{m_i}\right)^{1/3}\omega_{0e}. \tag{8.1}$$

This formula is valid in the case when the change in particle velocity in the external field is small compared with the time of development of the instability. In view of the finite magnitude of the imaginary part of (8.1), it is impossible to use the probability approach.

It is most convenient to obtain the acceleration effect by directly averaging the power of the forces, as was done in Sec. 4. We shall assume the field of the electric oscillations to be longitudinal and to have random phases:

$$\mathbf{E}(\mathbf{r},t)=\int \mathbf{E}_{\mathbf{k}}e^{i\mathbf{kr}-i\omega(\mathbf{k})t+\gamma(\mathbf{k})t}\,d\mathbf{k}, \tag{8.2}$$

$$\langle E_{\mathbf{k}i}E_{\mathbf{k}'j}\rangle=\frac{k_ik_j}{k^2}\,\delta(\mathbf{k}+\mathbf{k}')\,|E_{\mathbf{k}}^0|^2,\quad |E_{\mathbf{k}}|^2=|E_{\mathbf{k}}^0|^2e^{2\gamma(\mathbf{k})t}. \tag{8.3}$$

We obtain*

$$\frac{d}{dt}\langle\varepsilon\rangle=\langle\mathbf{fv}\rangle=\frac{e^2}{m}\int |E_{\mathbf{k}}|^2\,\frac{2\gamma(\mathbf{k})(\mathbf{kv})(\mathbf{kv}-\omega)}{[(\mathbf{kv}-\omega)^2+\gamma^2]^2}\,dk$$
$$+\frac{e^2}{m}\int |E_{\mathbf{k}}|^2\,\frac{\gamma\,dk}{(\mathbf{kv}-\omega)^2+\gamma^2}. \tag{8.4}$$

*In the calculation we take into account only exponentially growing terms. This formula was derived in [99] for the plasma's own electrons accelerated by the electric field.

The first term of (8.4) greatly exceeds the second if $\mathbf{k}\cdot\mathbf{v} - \omega$ is of the order of γ, and contains $\mathbf{k}\, d/d\mathbf{v}(\omega - \mathbf{k}\cdot\mathbf{v})$ in the limit as $\gamma \to 0$, precisely corresponding to the formulas for induced Cerenkov acceleration.

Using (8.1), we obtain for $\gamma \sim \omega$ and $\mathbf{k}\cdot\mathbf{v} \ll \omega$

$$\frac{d}{dt}\langle\varepsilon\rangle \approx \frac{2^{4/3}\sqrt{3}\,e^2}{4m\omega_{0e}}\left(\frac{m_i}{m_e}\right)^{1/3}\int |E_{\mathbf{k}}|^2\,d\mathbf{k}. \qquad (8.5)$$

This result illustrates well the qualitative difference in the character of the acceleration in unstable and stable systems. Namely, in unstable systems there is no need to satisfy rigorously resonance conditions of the type $\omega = \mathbf{k}\cdot\mathbf{v}$, and $\delta(\omega - \mathbf{k}\cdot\mathbf{v})$ is replaced by $\gamma/[\gamma^2 + (\omega - \mathbf{k}\cdot\mathbf{v})^2]$. Thus, even nonresonant particles will be effectively accelerated. This question is of importance for the acceleration of ions having small initial velocities v. The larger the ratio γ/ω, the more effective the acceleration on the wings of the resonances. Therefore aperiodic instabilities are of interest for the question of acceleration of low-velocity particles*.

Aperiodic instabilities, as is well known, occur during the initial hydrodynamic stage of the development of two-stream instability [20]. For particles whose velocity is much smaller than the speed of the beam we have

$$\frac{d}{dt}\langle\varepsilon\rangle = \frac{e^2}{m}\frac{\sqrt{3}}{2^{2/3}}\left(\frac{n_0}{n_1}\right)^{1/3}\frac{1}{\omega_{0e}}\int |E_{\mathbf{k}}|^2\,d\mathbf{k}. \qquad (8.6)$$

Recognizing that $\int |E_{\mathbf{k}}|^2\,d\mathbf{k}$ does not exceed $4\pi n_1 m_e u^2 (n_1/n_0)^{1/3}$ (the quantity n_e enters for electron beams), and the acceleration time is $\tau \sim 1/\gamma$, we obtain an estimate of the possible average energy of the accelerated ions:

$$\langle\varepsilon_i\rangle \approx \frac{m_e}{m_i} m_e u^2 \left(\frac{n_1}{n_0}\right)^{2/3}. \qquad (8.7)$$

The instabilities connected with the anisotropy of the distribution function [106-108] can also be aperiodic.

Finally, in the case of slowly-growing periodic oscillations, $\gamma/\omega \ll 1$, the low-energy particles are also accelerated. This acceleration, however, is approximately γ/ω times less effective than that of resonant particles, $\omega = \mathbf{k}\cdot\mathbf{v}$. We note in conclusion that the development of aperiodic instabilities leads to heating of the electrons and to violation of the condition (8.1) $v_0 \ll v_{Te}$ for the existence of instabilities. If $T_e \gg T_i$, instability of ion-sound oscillations develops, for which the acceleration corresponds to (5.11).

b) Acceleration of particles in the presence of an intense high-frequency field in a plasma. We consider another limiting case, when the frequency of the external field Ω is much larger than the characteristic frequencies ω of the plasma oscillations. Assuming that the particle velocities in the wave field are of the order of $v_E = eE_0/m$ (E_0 is the amplitude of the external field) and greatly exceed the average thermal velocity v_{Te} [109], let us consider first the effects of acceleration in the case of in-phase oscillations of the particles in an external high-frequency field. The spectrum of the high-frequency Langmuir oscillations changes in the case, in accord with [109], when $kv_E \sim \Omega$ and $\omega \ll \Omega$:

$$\omega^2 = \omega_{0e}^2 + \omega_{0i}^2 I_0^2\left(\frac{kv_E}{\Omega}\right) + 3k^2 v_{Te}^2. \qquad (8.8)$$

The oscillation frequency remains close to ω_{0e}, and the change occurs only in the character of the spatial dispersion which, as before, has little influence on the oscillation spectrum. The acceleration of the particles by the oscillations (8.8) is qualitatively similar to the accelerations considered above. In particular, when the oscillations have a stationary spectrum, the necessary condition for the injection is $v > v_{Te}$.

Low-frequency oscillations in a strong high-frequency field are possible for arbitrary ratios of T_e and T_i:

$$\omega^2 = \omega_{0i}^2\left(1 - I_0^2\left(\frac{kv_E}{\Omega}\right)\right), \qquad (8.9)$$

and are described by a dielectric constant

$$\varepsilon^l = \frac{k_i k_j \varepsilon_{ij}}{k^2} = -\frac{\omega_{0e}^2}{\omega^2} + \frac{\omega_{0e}^2\omega_{0i}^2}{\omega^4}\left(1 - I_0^2\left(\frac{kv_E}{\Omega}\right)\right). \qquad (8.10)$$

When $kv_E \ll \Omega$, an acoustic spectrum is produced:

$$\omega = \frac{\omega_{0i}}{\Omega}(\mathbf{k}\mathbf{v}_E), \qquad (8.11)$$

but it is anisotropic. The anisotropy of the spectrum makes it possible for the phase velocities of the oscillations

$$\frac{\omega}{k} = \frac{\omega_{0i}}{\Omega} v_E \cos\theta \qquad (8.12)$$

to be quite small. The effect of particle acceleration by these oscillations can be estimated with the aid of (2.9) and (5.28). If we are interested in the result averaged over the directions of the particle momentum (over Ω_p), then

$$\langle D^l\rangle_{\Omega_p} = \frac{e^2\omega_{0i}^5 v_E^5}{v^2\omega_{0e}^2\Omega^2}\int_0^\infty k^4 \int_0^{\frac{v}{v_E}\frac{\Omega}{\omega_{0i}}} x^5 N(k, x)\,dx\,dk, \qquad (8.13)$$

where $x = \cos\theta$. Consequently, when $v \ll 1$ and $v < v_E\omega_{0i}/\Omega$,

$$\langle \dot{E}_p\rangle_{\Omega_p} = \frac{\Omega}{\omega_{0i}}\frac{v}{v_E}\frac{m_e}{m}\frac{1}{4\pi n_0}\int_0^\infty k^4 v^2\,dk N\left(k, \frac{v}{v_E}\frac{\Omega}{\omega_{0i}}\right). \qquad (8.14)$$

Let us stop to discuss the case when the phases of the external high-frequency field are not fixed. The simplest example may be an external high-frequency field consisting of a set of segments of sinusoids with characteristic length T, i.e., after a sharp jumplike change in the phase of the high frequency field takes place after a time T. Under such conditions, for example, instabilities and buildup of oscillations are possible, since a jumplike change in the phase corresponds, roughly speaking, to new initial conditions of the oscillations of the particles in an external field. This can produce in the plasma an average translation of the electrons relative to the ions, with a velocity (see Appendix 2)

$$v_E = -\frac{e}{m_e}\int \frac{E_\Omega}{i\Omega}\,d\Omega; \qquad (8.15)$$

here E_Ω is the Fourier component of the external high-frequency field. When $v_{Te} \gg v_{Ie}$, an instability sets in, with an increment of the order of $\omega_{0e}(m_e/m_i)^{1/3}$. Attention must be called to the fact that in the presence of high frequency fields collisions between electrons can lead to the appearance of "initial" velocities for the particles, equivalent to the appearance of translational particle velocities. Account must be taken here of the fact that in a strong high-frequency field the collision frequency can be much

*It must be noted that the indicated smearing of the δ-function can be due also to other causes, such as collisions or nonlinear interactions [159], and is not necessarily connected with instabilities.

lower than the frequency of the Coulomb collisions in the absence of the high-frequency field[111]. This can be readily demonstrated by elementary estimates of the Cross section $\sigma_{scat} \sim \pi r^2 \cdot e^2/r \sim mv^2$ and in this case the characteristic velocity of particle motion v must be regarded as equal not to the thermal velocity of the particles, but to the velocity in the high-frequency field $v \sim v_E$. This yields $\nu_{col} = n\pi e^4/m^2 v_E^3$ and $\sigma_{scat} \sim \pi e^4/m^2 v_E^4$. If the momentum of the high-frequency field is larger than the time between two collisions, then we can assume that mixing of the particle oscillation phases takes place in the case when the time of one collision τ is much smaller than the period of the high frequency field. Assuming that $\tau \sim r/v_E$, we obtain

$$v_E^3 \gg \frac{e^2\Omega}{m}, \tag{8.16}$$

which is usually satisfied at not too high frequencies.

The oscillations that arise during the development of instabilities in a high-frequency field can lead to statistical acceleration of the particles. During the initial stage of the instability, so long as the turbulence is weak, the acceleration effects are similar to those considered in item a) for a constant field. For a developed turbulence, the nonlinear interaction of the oscillations can lead to excitation of Langmuir oscillations of the electrons. Attention must be called to the fact that under these conditions the ions can be accelerated by induced-scattering acts (Sec. 7, item c).

9. Acceleration Mechanisms in a Plasma Situated in a Constant Magnetic Field

As is well known, a great variety of oscillations, which may also be unstable, can exist in the magnetic field in a plasma. We shall consider only some of them, the most important ones from our point of view, for possible applications.

a) General relations. Just as in the absence of the magnetic field, it is convenient to use for weakly damped oscillations a probability approach. We shall describe the motion of the particles quasiclassically with the aid of p_z and $n = p_\perp^2/2eH\hbar$, where p_z and $p_\perp$ are the particle-momentum components parallel and perpendicular to the magnetic field, respectively. The probability that a particle in the state p_z, n will emit a wave of frequency $\omega(\mathbf{k})$ with wave vector $\mathbf{k}$ followed by transition to $n' = n - \nu$ will be denoted by $w_{p_z, n, \nu}(\mathbf{k})$. Then we must write in lieu of (1.2)[113]

$$\frac{\partial f_{p_z, n}}{\partial t} = -\sum_\nu \int \{w_{p_z, n, \nu}(\mathbf{k})(f_{p_z, n} - f_{p_z - k_z, n-\nu}) + w_{p_z + k_z, n+\nu}(\mathbf{k})(f_{p_z, n} - f_{p_z+k_z, n+\nu})\}\, dk. \tag{9.1}$$

Recognizing that

$$\Delta p_\perp^2 = 2eH\hbar\Delta n = 2eH\nu\hbar = 2\varepsilon\nu\omega_H \ll p_\perp^2, \quad \omega_H = \frac{eH}{\varepsilon}$$

$k_z \ll p_z$, we obtain

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_z} D_{zz}\frac{\partial f}{\partial p_z} + \frac{\partial}{\partial p_z} D_{z\perp}\frac{1}{p_\perp}\frac{\partial f}{\partial p_\perp} + \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp} D_{\perp z}\frac{\partial f}{\partial p_z} + \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp} D_{\perp\perp}\frac{1}{p_\perp}\frac{\partial f}{\partial p_\perp}, \tag{9.2}$$

where

$$\left.\begin{aligned} D_{zz} &= \sum_\nu \int k_z^2 w_{p_z, p_\perp, \nu}\, dk, \\ D_{z\perp} = D_{\perp z} &= \sum_\nu \int k_z \nu\varepsilon\omega_H w_{p_z, p_\perp, \nu}\, dk, \\ D_{\perp\perp} &= \sum_\nu \int \nu^2\varepsilon^2\omega_H^2 w_{p_z, p_\perp, \nu}\, dk. \end{aligned}\right\} \tag{9.3}$$

From (9.2) and (9.3) we readily obtain the average change of the particle energy

$$\dot{E} = \sum_\nu \int dk\,\omega(\mathbf{k})\left(k_z\frac{\partial}{\partial p_z} + \frac{\omega_H\nu}{v_\perp}\frac{\partial}{\partial p_\perp}\right) w_{p_z, p_\perp, \nu}. \tag{9.4}$$

The probability $w_{p_z, p_\perp, \nu}$, just as in the absence of the field, can be obtained by comparison with the intensity of wave emission from an individual charge in the plasma. In the general case of media with spatial dispersion[6]

$$\left.\begin{aligned} w_{p_z, p_\perp, \nu}(\mathbf{k}) &= \int w_{p_z, p_\perp, \nu}(\omega, \mathbf{k})\, d\omega, \\ w_{p_z, p_\perp, \nu}(\omega, \mathbf{k}) &= \frac{1}{2}\frac{e^2}{(2\pi)^3\varepsilon^2}\Gamma_i^{\nu, p_z, p_\perp}(\Gamma_j^{\nu, p_z, p_\perp})^* \operatorname{Im} D_{ij}(\omega, k_z, k_\perp), \end{aligned}\right\} \tag{9.5}$$

$$\left.\begin{aligned} &\Gamma_1^{\nu, p_z, p_\perp} = 2p_\perp\nu\frac{J_\nu(z)}{z}, \quad \Gamma_2^{\nu, p_z, p_\perp} = -2ip_\perp J_\nu'(z), \\ &\Gamma_3^{\nu, p_z, p_\perp} = -2p_z(\operatorname{sign} k_x) J_\nu(z), \quad J_\nu'(z) = \frac{d}{dz}J_\nu(z), \\ &z = \frac{k_\perp v_\perp}{\omega_H}. \end{aligned}\right\} \tag{9.6}$$

The quantity $\operatorname{Im} D_{ij}(\omega, \mathbf{k})$ is the antihermitian part of the retarded Green's function of the electromagnetic field in the plasma:

$$(k^2\delta_{ij} - k_ik_j - \omega^2\varepsilon_{ij}(\omega, \mathbf{k}))D_{il}(\omega, \mathbf{k}) = 4\pi\delta_{il}. \tag{9.7}$$

b) Acceleration by plasma oscillations. In the presence of a magnetic field, a change takes place in the spectrum of the quasilongitudinal oscillations of the plasma, defined by the equation

$$k_ik_j\varepsilon_{ij} = 0, \quad k_\perp^2\varepsilon + k_z^2\varepsilon_z = 0, \tag{9.8}$$

where

$$\varepsilon \approx 1 - \frac{\omega_{0e}^2}{\omega^2 - \omega_{He}^2}, \quad \varepsilon_z = 1 - \frac{\omega_{0e}^2}{\omega^2} \quad \text{for small } \frac{kv_{Te}}{\omega}.$$

In the case of weak spatial dispersion, the effect of systematic variation in the particle velocity, for $v_z \gg v_\perp$, is[6]

$$\begin{aligned} \dot{E} = \frac{e^2\omega_{0e}^3}{2mv_z}\int u\,du \Big\{ &\left(\frac{m_e}{m}\dot{h} + \frac{m}{m_e}\frac{u^2}{h}\right)\frac{|\varepsilon_z|}{\varepsilon^2}(N_- - N_+) \\ &- \frac{|\varepsilon_z|}{\varepsilon^2}\cdot 2u(N_- + N_+) - \frac{4u}{|\varepsilon|}N_0 - 2u^2\frac{\omega_{0e}}{v_z}N_0'\frac{1}{|\varepsilon|}\Big\}, \end{aligned}$$

$$N_0' = \frac{dN_0}{dk_z}, \quad N_\nu = N_{\omega, k_z} \text{ for } k_z = \frac{\omega + \nu\omega_H}{v_z},$$

$$u = \frac{\omega}{\omega_{0e}}, \quad h = \frac{\omega_{He}}{\omega_{0e}}, \quad \nu = 0 \pm 1. \tag{9.9}$$

It must be noted that when $\omega_{He} \ll \omega_{0e}$ the acceleration effect (9.9) coincides with (5.5). We note that the latter inequality is always satisfied in astrophysical applications.

c) Acceleration by magnetic-sound and Alfven waves. Acceleration by low-frequency oscillations in a magnetic field was considered in [8]. We note that the phase velocities of Alfven and magnetic-sound waves can be quite large if

$$v_{A0}^2 = \frac{H^2}{4\pi n_i m_i} \gg 1.$$

In this case the Alfven velocity can be close to that of light[114]

$$v_A^2 = \frac{v_{A_0}^2}{1+v_{A_0}^2} \to 1 \quad \text{for} \quad v_{A_0} \to \infty.$$

This circumstance is important in that the acceleration effect, as shown by calculations [8], occurs when $v_z < v_A$, where v_z is the particle velocity parallel to the magnetic field. Without presenting the general formulas [8], we confine ourselves to an approximate formula for the acceleration by Alfven waves ($v_z \ll v_A$)

$$\dot{E}_p \sim \frac{2e^2\omega_{Hi}^2 v_A}{2\varepsilon v_{A_0}^2} \overline{N}_A,$$

where N_A is the average number of Alfven waves.

We note that Alfven waves can effect injection to the mechanism of acceleration by high frequency oscillations.

d) Cyclotron acceleration of relativistic electrons by transverse waves. The energy of a particle in a magnetic field can change as a result of loss or gain of energy through cyclotron emission and absorption of waves. This is most clearly pronounced in the possibility of acceleration by high-frequency transverse waves for which the Cerenkov condition cannot be satisfied. The statistical acceration is in this case a second-order difference effect. The first-order effect, similar to the Fermi effect, is in this case the self-resonant acceleration considered by Ya. B. Faĭnberg [95] and by A. A. Kolomenskiĭ and A. N. Lebedev [115]. Although the acceleration of relativistic particles, for example, is effected by frequencies $\omega \gg \omega_{0e}$, the deviation of the dielectric constant from unity is significant [116]. This is connected with the fact that $(v-1)$ can be of the order of $\epsilon - 1$. The result of the calculation is of the form [14]

$$\dot{E} = \frac{e^2\omega_H^2}{\pi\varepsilon\sqrt{3}} \int_0^\infty \nu^2 d\nu \left\{ 2(\xi+\eta)\zeta K_{5/3}(\zeta) - 3\eta\zeta K_{1/3}(\zeta) - 2\eta \int_\zeta^\infty K_{1/3}(\zeta')d\zeta' \right\} N(\nu)\hbar, \quad (9.10)$$

where N is the number of waves and

$$\xi = 1 - v_\perp^2 = \frac{m^2}{\varepsilon^2}, \quad \eta = 1 - \varepsilon^t(\omega_H \nu), \quad \zeta = \frac{2\nu}{3}(\xi+\eta)^{3/2}.$$

When $\nu \gg \omega_{0e} m/eH(\epsilon/m)$ we can put $\epsilon^t = 1$ and obtain [7]

$$\dot{E} = \frac{9\sqrt{3}e^2}{4\pi m}\left(\frac{eH}{m}\right)^3\left(\frac{\varepsilon}{m}\right)^3 \int_0^\infty \zeta^3 K_{5/3}(\zeta) N(\zeta)\, d(\zeta)\cdot\hbar. \quad (9.11)$$

For "temperature" radiation, when $N(\omega) = T_{eff}/\hbar\omega$, the acceleration is similar to the Fermi acceleration

$$\dot{E} = \alpha\varepsilon, \quad \alpha = \frac{8}{3}\frac{e^4H^2}{m^2}\frac{T_{eff}}{m^2}. \quad (9.12)$$

e) Cyclotron acceleration of ions by plasma waves. As was emphasized many times, the problem of acceleration of ions, which is important from the general point of view of their heating, is in particular a problem of injection into the mechanism of acceleration by high-frequency oscillations. The presence of a magnetic field greatly facilitates this problem, since it is known that very slow high-frequency waves can propagate perpendicular to the magnetic field. Cyclotron acceleration of ions can be the result of cyclotron absorption and emission of plasma waves [117] having small phase velocities as they propagate perpendicular to the magnetic field. The ions are then accelerated essentially perpendicular to the magnetic field. We confine ourselves to a formula for the acceleration of the ions in this case ($v_\perp \ll 1$):

$$\dot{E}_p = \frac{2e^2}{mv_\perp}\sum_\nu \omega_H^3\nu^4 \int_{-1}^{1} n^2(\omega_H\nu, x) J_\nu(\nu v_\perp n(\omega_H\nu, x)\sqrt{1-x^2}) \times J'_\nu(\nu v_\perp n(\omega_H\nu, x)\sqrt{1-x^2}) N(\omega_H\nu, x) \times |\varepsilon(\omega_H\nu, x)(1-x^2) + \varepsilon_z(\omega_H\nu, x)x^2|^{-1}(1-x^2)^{-\frac{1}{2}}dx; \quad (9.13)$$

$n(\omega, \cos\theta)$ is the refractive index. If the intensity of the oscillations is concentrated in the region of angles of the order of $x_0 \approx \sqrt{-\epsilon_z/\epsilon} \ll 1$, and the average velocity of the ions in the direction of the magnetic field is $\langle v_z \rangle$, we obtain an estimate for the characteristic acceleration time of the ion to an energy ϵ_i by the first harmonic ($\nu = 1$)

$$\tau = \frac{1}{\omega_{0i}}\frac{\varepsilon_i}{T_e}\langle v_z\rangle v_A \frac{nT_e}{W}, \quad v_A = \frac{H}{\sqrt{4\pi n m_i}}. \quad (9.14)$$

f) Cyclotron acceleration of ions at the combination frequency. Let us consider the effects of acceleration of ions in a magnetic field by induced scattering. The energy conservation law for scattering in the magnetic field is

$$\omega - \omega' - (k_z v_z - k'_z v_z) = \mu\omega_{Hi}. \quad (9.15)$$

It is pertinent to speak of scattering in a magnetic field when $\mu = 0$, and of cyclotron acceleration at the combination frequency $\omega - \omega'$ when $\mu = \pm 1$. An analysis presented in [164] shows that in a strong magnetic field the cyclotron acceleration at the combination frequency ($\mu = \pm 1$) greatly exceeds acceleration by induced scattering ($\mu = 0$). This increases essentially the ion-velocity components perpendicular to the external magnetic field. An estimate of the effect of plasma acceleration by plasma waves yields

$$\frac{d}{dt}\langle\varepsilon_\perp\rangle \approx \frac{\sqrt{2\pi}}{32}\frac{1}{m_i}\frac{k_\perp^2}{k^2}\frac{W^2k}{n_0^2 v_{gr}}, \quad (9.16)$$

where v_{gr} is the group velocity of the waves and W is their energy density. The order of magnitude of the effect (9.16) is the same as for induced scattering (Sec. 7).

We note also that effects of statistical acceleration in scattering of low-frequency magnetohydrodynamic and Alfven waves by plasma particles are considered in a paper by Galeev [165].

III. APPLICATIONS OF THE ACCELERATION MECHANISMS

10. Applications Connected with Particle Acceleration Under Laboratory Conditions

In this section we consider possible uses of the foregoing acceleration mechanisms to explain certain laboratory experiments. We shall describe briefly only some well known experimental results and compare them with the possible theoretical predictions.

Henceforth, unless specially stipulated, we shall describe predominantly the possibility of applying the statistical acceleration mechanism, without undertaking to analyze many other possibilities of data interpretation. In many cases such an analysis is premature because of the scarcity of data, and we speak only of a number of different possible explanations; in some cases the very use of the statistical acceleration is debatable. Therefore the exposition presented in this section is of necessity brief and fragmentary (some questions are treated in greater detail in the cited literature), and our aim is only to give a general idea of the broad possibilities of applying the discussed mechanisms of statistical particle acceleration in a plasma.

a) X ray and neutron radiation from powerful pulsed discharges[17]. In the investigation of powerful pulsed discharges in deuterium (of the pinch type) it was observed that under certain conditions such discharges are sources of powerful neutron and x-ray emission[118,119]. These experiments were the first attempts at plasma heating[118]. An analysis of the experimental results[120,119] shows that the neutrons appear in nuclear reactions produced by accelerated deuterium ions, and that the x rays are produced by accelerated electrons. The energy of the accelerated particles greatly exceeds their average thermal energy or the potential difference between the electrodes. This can indicate the presence of a particle-acceleration mechanism that leads to a gradual statistical acquisition of energy by the particles. This calls for satisfaction of a number of necessary conditions. The first is that the particles experience a sufficiently large number of events of energy changes within a relatively short discharge time, of the order of several microseconds. This singles out, from among the possible acceleration mechanisms, the high-frequency turbulence connected with the excitation of plasma oscillations. We recall that the shock wave accompanying powerful pinch discharges in a plasma can be the source of such a turbulence (Sec. 1). The existing experimental data offer evidence in favor of this hypothesis. The average energies of the accelerated electrons responsible for the hard x rays, and of the ions responsible for the neutron radiation, are in good agreement. According to Sec. 3 the average energy* does not depend on the mass and charge of the particle.

The energy spectra of the accelerated particles fall off rapidly at high energies, corresponding to stability of the statistical acceleration when $\epsilon = \epsilon^*$ (Sec. 3). Constant forces would have led to a smooth fall-off of the energy spectrum.

Finally, notice should be taken of the existing possibilities of explaining the dependence of the number of accelerated particles of the pressure, sort of gas, impurities, etc. (see [17]).

b) Acceleration of electrons during the development of two-stream instability in a plasma. The most effective mechanism of plasma turbulization is two-stream instability. It is therefore natural to expect acceleration effects to be observed when beams pass through a plasma or when beams are produced by the action of external fields on the plasma.

The effects of accelerated electrons and ions were actually observed in many experiments. Thus, Berezin, Faĭnberg, et al.[121,123] have observed that under conditions when the beam in the plasma loses an appreciable fraction of the energy, ~ 20% (which is accompanied in the experiments[122,123] by optimal excitation of high-frequency plasma oscillations) accelerated electrons are produced with energy 1.5 times larger than the beam-particle energy. It must be stated that the electrons belonging to the beam itself that generates the plasma oscillations are of course decelerated. Furthermore, the phase velocity of the generated oscillations is of the order of the beam velocity, and therefore the thermal electrons of the plasma cannot be effectively accelerated, since the injection conditions are not satisfied. As noted in Sec. 5, the particles of the beam itself can be accelerated only if the longitudinal waves are directed essentially perpendicular to the particles. A change in the direction of Langmuir-wave propagation is possible when the intensity of these waves is high, owing to nonlinear effects (see Sec. 7). As a possible interpretation of the experimental data we can propose that when the plasma is intensely turbulized by the beam the generated oscillations acquire components that are perpendicular to the beam, and accelerate a relatively small fraction of the beam particles. If W^l is of the order of the beam energy $n_1 m v_0^2$ and if $v_{ph} \sim v_0$, then the turnaround time of the direction of the longitudinal waves is

$$\tau^l \approx \frac{(2\pi)^{5/2}}{\omega_{0e}} \frac{n}{n_1} \left(\frac{v_0}{v_{Te}} \right); \qquad (10.1)$$

the time necessary to increase the phase velocities of the Langmuir oscillations generated by the beam is of the same order. The increase in the phase velocities of the waves can lead also to the diffusion of the beam particles (in velocity space) towards larger velocities, and this will lead to the appearance of particles whose velocities exceed the velocities of the injected beam.

Under the experimental conditions[122] the current of accelerated particles is several per cent of the beam current. We recall that in statistical acceleration the number of accelerated particles should be a small fraction of the total number of the particles.

c) Acceleration of ions during development of two-stream instability[124,121]. As already noted, in the acceleration of ions, problems of injection are important, and the acceleration by high frequency oscillations can be quite appreciable. Fedorchenko et al.[124] observed in their experiments excitation of low-frequency oscillations in the development of two-stream instabilities. The experimental results offer evidence that the low-frequency oscillations are brought about by the high frequency oscillations excited by the beam. According to [80,34,35] (see Sec. 1) the decay instabilities ensure effective transformation of the high frequency oscillations into low frequency ones, and in this case certain satellites should appear in the spectrum of the Langmuir oscillations, located at distances on the order of ω_{0i} from the frequency of the Langmuir waves. Later experiments (see [126]) have revealed this qualitative effect. A change takes place here in the intensity of the red and violet satellites, depending on the ratio of the energies of the low-frequency and high-frequency oscillations, namely, when the low-frequency oscillation energy is large the red satellites grow, and the intensity of the first satellites increases with increasing number of satellites. At high energy of the low-frequency oscillations, the violet satellites grow, in accord with the theory[80]. Accelerated ions were observed in the experiments of [123,127], and their occurrence was correlated with the occurrence of low-frequency oscillations. A broad spectrum of accelerated ions with more or less smooth decrease in the region of high energies was observed experimentally.

Neidigh[127] and Nezlin[128] also observed accelerated ions during the development of two-stream instability of a plasma. The beam of electrons was directed along the strong magnetic field. The acceleration can be ascribed either to cyclotron acceleration by Langmuir oscillations, or to low-frequency oscillations. The occurrence of the instability is apparently connected with the blocking of the electron beam and the appearance of a virtual cathode[133].

d) Acceleration of ions and turbulent heating of a plasma. In all cases when the accelerated particles are contained in a certain limited volume, it is natural to speak of their heating. This occurs, in particular, if the system is in a magnetic trap. A broad spectrum of high-energy electrons was observed in the experiments of Zavoĭskiĭ[129,130] et al. on turbulent heating of a plasma. The presence of a sufficiently broad spectrum is evidence in favor of the statistical nature of the acceleration. In the experiments of [130], the two-stream instability was produced by external electric quasistatistical fields of intense low-frequency waves. According to Sec. 8, the ions can also be accelerated under these conditions, owing to the nonstationary nature of the oscillation spectrum (see (8.5)). Induced scattering and cyclotron acceleration can apparently play a role here. The decay processes generate waves with small phase velocities; finally, the broadening of the oscillation spectrum as a result of nonlinear wave-interaction effects can contribute to the ion acceleration.

Effective heating of the electrons was observed in the experiments of [131] with prolonged injection of the particle beam.

It must be noted that a large duration of injection is essential, in view of the fact that statistical acceleration should occur within relatively long time intervals. A slow initial stage of acceleration, when the ion velocitity is much lower than the wave velocity, makes ion acceleration especially difficult. The acceleration is then less effective, but it can nevertheless take place as a result of high-frequency oscillations, of cyclotron acceleration, of the nonstationary nature of the process, and of induced scattering. When the velocity of the ions becomes of the order of the velocity of the Langmuir waves, the efficiency of the acceleration increases.

e) Acceleration of the particles with occurrence of runaway electrons in a plasma. When strong electric fields are applied, the electron velocities can exceed v_{Te}. The rapid development of instabilities can lead to the appearance of a large turbulent resistance, reaching $1/\sigma_{turb} \sim \omega_{0e}^{-1}$ (σ_{turb} is the turbulent electric conductivity, the resistance due to collisions being $1/\sigma_{col} \sim v_{col}/\omega_{0e}^2 \ll 1/\omega_{0e}$ (see [98,99] and the experiment in [132]). The plasma turbulization occurring under these conditions should lead to an effective acceleration of both electrons and ions in the plasma (see Sec. 7), which should be correlated with the current interruptions.

f) Acceleration of ions by action of an intense high-frequency field on a plasma. Accelerated ions of high energy, ~ 10 keV, were observed in experimental investigations of the radiation method of plasma acceleration[134,135]. It is characteristic that the plasma configuration is appreciably deformed by the interaction with the field, and in some cases it fills the entire volume of the waveguide. The spectrum of the accelerated particles is quite broad and falls off smoothly in the region of large energies. The acceleration was observed even when the plasma as a whole could not acquire an appreciable momentum from the high-frequency field. The entire aggregate of the experimental data apparently demonstrates that the interaction with a powerful high frequency field did not accelerate the plasma as a unit, but a small fraction of the total number of particles was accelerated through some mechanism. It is characteristic that under the conditions of [135] v_E is of the order of v_{Te}, that is, the high-frequency field is close to critical. It must also be borne in mind that the plasma can experience turbulent heating during the acceleration. Estimates show that acceleration by high-frequency turbulence resulting from nonlinear effects can be the most effective when $v_E \ll v_{Te}$, whereas when $v_E \gg v_{Te}$ the ions can acquire an energy of the order of 1–2 keV within 0.1 μsec, owing to the induced scattering effects (see Sec. 7). Strong turbulence can arise not only by direct action of a strong high frequency field, but also under conditions when $\omega < \omega_{0e}$, when opposing particle currents are produced by reflection of particles from a high-frequency field that does not penetrate into the plasma. Attention must also be called to the fact that nonlinear conversion of waves when $v_E \ll v_{Te}$ and the instability of the plasma in a strong high-frequency field lead to an increase in the absorption of power, and by the same token can contribute to the momentum transfer and to acceleration of the plasmoid. The instability in a strong high frequency field can lead to "turbulent" resistance which exceeds the collision-induced resistance considered in [111].

To conclude this section, we must emphasize that acceleration mechanisms other than the statistical one are also possible in a plasma: 1) generation of constant-phase waves and capture of particles by them, 2) appearance of space charge and the associated potential wells, virtual cathodes, etc.[133]*. We have touched upon only statistical mechanisms that are the most probable in the plasma, because random motion usually goes over rapidly into turbulent motion. On the other hand, we must emphasize the need for a detailed experimental analysis of all the possibilities in each of the experiments considered above.

11. Astrophysical Applications

Going over to possible astrophysical applications, we must point out that the accelerating mechanisms that are most prevalent under cosmic conditions most probably have a statistical character.

Acceleration mechanisms connected with generation of fast particles, especially cosmic rays, are not only widespread under cosmic conditions, but also take part in cosmological evolution. They influence the dynamics of supernova bursts[2], galaxies, and radio galaxies, and play an important role in galactic cores, the quasars discovered in 1962, etc. It is clear that the acceleration mechanism should be universal and simple. An appropriate universal property is the turbulence of outer-space plasma. Although the premise that particle acceleration is connected with turbulence is widespread, many difficulties have arisen[2], which frequently lead to difficulties with respect to energy. For example, it has been known for a long time that the number of radio-emitting and "optical" electrons in the most investigated radio source, Crab nebula, is much larger than the number expected in accordance with the Fermi acceleration mechanism. Serious theoretical difficulties arise when explaining the radiation of the radio galaxy A-Cygni. The large efficiency of acceleration of cosmic rays in chromospheric flares has led L. I. Dorman[3] to the hypothesis that acceleration by a first-order Fermi effect is possible, although it is clear that the case of systematic head-on motion of two magnetic walls is very improbable and in most cases the acceleration should have a statistical character. The acceleration of electrons in the Earth's radiation belts[137] is very effective. Difficulties arise in the interpretation of quasar radiation, whose surface flux turns out to be quite large; if this radiation is due to relativistic electrons, they must be accelerated by a very effective mechanism.

We could continue with similar examples. The total aggregate of the available information points to a very high efficiency of acceleration of fast particles under cosmic conditions, much greater than would follow from the Fermi Acceleration mechanism. On the other hand, the available data indicate efficiency of acceleration of the electrons, whereas in the case of Fermi acceleration they could be secondary products of generation of cosmic rays by heavy particles[2].

Effects of acceleration by high-frequency turbulence were used in [12-16] to explain many astrophysical data. We have already mentioned many times that acceleration by high-frequency turbulence is much less effective than by low-frequency turbulence, so that many of the old contradictions can be eliminated. It must be emphasized that the actual energy of the high-frequency turbulence is frequently not known directly, but can be estimated, in principle, from indirect considerations. The energy of high-frequency turbulence is frequently determined by the initial and boundary values of the fields. It is clear that the energy of the high-frequency turbulence should not exceed the energy of hydrodynamic turbulence and the quantity $H^2/8\pi$, which is of the same order under astrophysical conditions, that is,

$$W^l \leqslant \frac{H^2}{8\pi}.$$

Apparently, however, even $W^l \sim 10^{-6} H^2/8\pi$ is sufficient for the occurrence of very powerful acceleration effects, if the phase velocities of the plasma waves are close to c. It must be noted that owing to nonlinear effects the phase velocities of Langmuir oscillations approach rapidly the speed of light for the majority of radio sources[138]. It is necessary here that the time of nonlinear energy transfer be smaller than the collision time, giving the qualitative criterion

$$W^l > \frac{H^2}{8\pi} \frac{\omega_{0e}^3}{v_{Te}^3 n}. \tag{11.1}$$

The parameter $\omega_{0e}^3/v_{Te}^3 n \sim \sqrt{n}$ is quite small under astrophysical conditions (small n). High-frequency turbulence can be detected under outer-space conditions, in principle, by determining the change in the spectrum of the transverse waves passing through a turbulent plasma. In particular, it is advantageous to investigate the 21 cm line, for which satellites can appear. N. S. Kardashev and I. S. Shklovskiĭ have called attention to the fact that deformation of the 21 cm spectral line together with a red satellite, is

*The last effect is closely related with the blocking of an electron beam by instabilities[25, 26].

observed during passage of radio emission near the Sun. Quantitative estimates aimed at explaining the observed effect point to an appreciable high-frequency turbulence W^l, of the order of $H^2/8\pi$. When attempts are made to register similar effects from remote cosmic objects, it is necessary to take into account the possible modification of the spectrum by the Doppler effect etc. Contributing to an explanation of the role of high-frequency turbulence are many deductions that follow from the assumption that W^l is large; these deductions are amenable to experimental verification, especially the deductions concerning the time variation of the intensity of discrete radio-emission sources[15]. We shall present below several examples which illustrate the possible role of high-frequency turbulence under outer-space conditions.*

a) Acceleration of electrons and ions in the Earth's radiation belts[16]. As is well known[137,139], in the outer radiation belt the energy component contains electrons with energy $5 \times 10^4 - 2 \times 10^6$ eV, at a density $10^{-2} - 0.3 \times 10^{-5}$ cm^{-3}, moving in a cold plasma with $n \sim 10^3$ cm^{-3} and an average particle energy 10^3 eV/cm^3. It is customarily assumed that the origin of the fast electrons is connected with the action of an acceleration mechanism. An analysis carried out in [16] indicates that statistical acceleration of particles by plasma waves can explain many experimental data. Let us list the principal among them:

1. The radiation belt has relative stability. Thus, after a magnetic storm, the radiation level preceding the storm is restored. According to Sec. 7, the equilibrium between the accelerating forces of the plasma oscillations and the ionization losses is stable.

2. The electrons should be accelerated from energies of the order several eV to an energy $1-10^3$ keV, that is, "without injection." The acceleration by plasma oscillations ensures such a situation because the efficiency of acceleration increases with decreasing particle energy, $d\epsilon/dt \sim \epsilon^{-3/2}$.†

3. Acceleration by plasma oscillations fully offsets the rather appreciable ionization loss of low-energy particles, since the acceleration increases with decreasing energy of the accelerated particles more rapidly than the ionization losses. From the equality $(d\epsilon/dt)_{acc} = (d\epsilon/dt)_{ion}$ we obtain $T_{eff} \approx \epsilon \ln(\epsilon/\hbar\omega_0)$, where ϵ is the average energy of the accelerated particles. When $\epsilon \sim 50$ keV we have $T_{eff} \sim 10^9-10^{10}$ deg. On the one hand, this agrees with the value expected in the generation of plasma waves by the solar wind[16]; on the other hand, the energy of the plasma oscillations at the indicated value of T_{eff} is much smaller than the thermal energy of the plasma $nkT_{pl} \sim 10^3-10^4$ eV/cm^3.

4. Bursts of accelerated particles appear after a magnetic storm. The characteristic acceleration time of these particles can be explained by taking into account the fact that the magnetic storms connected with penetration of solar corpuscular streams generate a sufficiently large value of W^l. In addition, in the case of large turbulence the nonlinear interaction of the plasma waves increases rapidly their phase velocities, and this increases to a considerable degree the acceleration efficiency. Many additional data have been published recently, confirmed by some deductions of [16]. According to the statistical acceleration, the ions should have an average energy of the order of the electron energy[16], as was apparently observed in [140].

This was followed by a determination of the dependence of shapes of the radiation belts on the direction to the Sun[141]. There exists apparently a shock wave (Sagdeev[93,102,104]) that flows around the Earth's exosphere[141,142]. In [142,143] it is assumed that collisionless shock waves are the sources of plasma oscillations both under stationary conditions and at the instant of occurrence of the magnetic storms, which are connected in [142,143] with the arrival of the shock waves. However, direct excitation of oscillations by the two-stream instability of the corpuscular streams[144], which are connected both with the solar wind and with magnetic storms, is apparently more effective.

On the other hand, the effect of accumulation of plasma oscillations during continuous injection of corpuscular streams in the space near the Earth can produce a sharp transition layer[145], which to a certain degree is analogous to a shock wave.

The instability of corpuscular streams in the supercorona of the Sun[144] gives grounds for assuming the presence of high-frequency plasma turbulence in the supercorona. As shown in Sec. 7, if the characteristic phase velocity of the plasma waves v_{ph} is small compared with the velocity of light, the main effect of interaction of radio emission with a turbulent plasma should be scattering. Scattering of the 21-cm radiation was observed long ago[146]. It is not clear, however, what contribution can be made to it by high-frequency turbulence. Finally, interest attaches to research on the shape of the 21-cm line during passage near the Sun, as referred above.

b) Acceleration of cosmic rays. The origin of cosmic rays was considered in detail in papers by Ginzburg[2,147] and Shklovskiĭ[149], and has been discussed in various reviews and monographs[2,148,150]. It is interesting that the role of the statistical Fermi acceleration in the origin of cosmic rays has been continuously limited since the time of Fermi's paper. Thus, Fermi assumed that the acceleration occurs essentially in clouds of interstellar gas. It became clear subsequently[2] that acceleration has low efficiency in interstellar gas clouds, and Ginzburg[147] and Shklov-

*The injection is actually determined by a self-regulation process[16].

†We do not touch upon the well known acceleration mechanisms such as acceleration by hydrodynamic turbulence, Fermi acceleration, or acceleration by magnetohydrodynamic and shock waves, which are described in detail in[2,3].

skiĭ[149] advanced the hypothesis that acceleration by the Fermi mechanism is possible in supernova shells, in which vigorous motion occurs, and therefore the Fermi acceleration is more effective. It was shown later in [151,152] that no acceleration occurs at present in the most thoroughly investigated source, the Crab nebula, owing to the expansion effect, and the action of the Fermi acceleration was moved back to earlier stages, when the motions in the shell were more vigorous. The acceleration by high-frequency turbulence, proposed in [12,13], can eliminate a whole number of old contradictions. Thus, if the Fermi acceleration, in accord with [151], has in the Crab nebula at the present time an estimated value

$$\frac{d\varepsilon}{dt} \sim 10^{-18} \frac{\varepsilon}{mc^2} (\text{cgs esu}), \tag{11.2}$$

then the acceleration by plasma oscillations, with $W^l \sim 10^{-6}(H^2/8\pi)$ and $v_{ph} \sim c$, is*

$$\frac{d\varepsilon}{dt} \sim 10^{-8} \frac{mc^2}{\varepsilon}, \tag{11.3}$$

that is, 10^6 times more effective than the Fermi acceleration (for radio-emitting electrons $\epsilon/mc^2 \sim 10^2$). Plasma oscillations can at the present accelerate electrons to energies on the order of maximal, corresponding to the optical synchrotron radiation. The electron energy loss to synchrotron radiation is quite large[148], and an explanation of the cause of prolonged optical radiation, say of the Crab nebula, is a timely problem. Apparently an important role can also be played by cyclotron acceleration by trapped transverse radiation[13,14]. Without going into details (see[12-16]), we emphasize here only the following points.

1. Acceleration by high-frequency turbulence points to the possibility of predominant acceleration of electrons, that is, it corresponds to primary cosmic electrons. This circumstance is indicated by experimental data on the Crab nebula, for example, and on the radio source A-Cygni; Ginzburg and Syrovat-skiĭ have recently shown[153] that galactic relativistic electrons cannot be secondary. Finally, if the electrons of the Crab nebula are secondary, then gamma radiation should be observed from this nebula, yet experiments specially set up for this purpose by Chudakov[154] showed no gamma radiation from the Crab nebula.

2. Although acceleration of ions is less effective than that of electrons, heavy multiply-charged ions are accelerated more effectively than light ones.

3. The efficiency of acceleration by plasma oscillations is determined by the value of $v_{ph}^3 W^l$, that is, the acceleration can be effective for small W^l but large v_{ph}. Nonlinear interaction of plasma oscillations increases v_{ph} rapidly, and transformation into transverse oscillations, acceleration by which is also quite effective[10,14], takes place.

4. Vigorous motion in radio sources indicates the presence of turbulent motions of high velocity approaching the speed of light[138]. Thus, the condition for the excitation of plasma oscillations $v > v_{Te}$ will be satisfied. At the same time, ordinary hydrodynamic velocities can apparently also be larger than v_{Te}. On the other hand, at velocities smaller than v_{Te}, plasma oscillations can be excited in the presence of a magnetic field perpendicular to the direction of the magnetic field. Nonlinear interaction of the plasma waves can lead to isotropization of the high frequency turbulence that is generated in such cases. Finally, intense transverse waves occurring during explosions can cause, as a result of decay processes, intense plasma oscillations with phase velocities quite close to the speed of light[76].

5. The problem of acceleration by a turbulence, applied to the problem of quasar radiation[155], would eliminate[138] difficulties with the large outflow of energy and with the ejection of a considerable mass during gravitational collapse[156]. We emphasize that in final analysis the high-frequency turbulence is caused in this case by gravitational instability[155].

It should be noted in conclusion that we have emphasized above only the possibilities of applying the results to many problems. We do not claim a detailed analysis of the experiments and the possibilities of their explanation, for which purpose, it seems to us, there are still not enough data at present. The purpose of this article was, in particular, to stimulate such an analysis.

*Expression (11.3) is valid only when the spectrum of the plasma oscillations is stationary. If the oscillation spectrum changes because of nonlinear effects, the spectra of the accelerated particles can extend to high energies (Sec. 3).

APPENDIX 1

The probabilities of nonlinear scattering and the corrections of the Cerenkov radiation of Langmuir waves are of the form

$$w_+(\mathbf{k}_1, \mathbf{k}_2) = \frac{4}{9} \frac{e^4}{m_e^2} \left(\mathbf{k}_1\mathbf{k}_2 - \frac{4\,(k_1^2 + (\mathbf{k}_1\mathbf{k}_2))\,(k_2^2 + (\mathbf{k}_1\mathbf{k}_2))}{(\mathbf{k}_1+\mathbf{k}_2)^2} \right)^2 \times \frac{1}{k_1^2 k_2^2 \omega_{0e}^4} \frac{1}{\dfrac{\partial\varepsilon^l}{\partial\omega_1}\dfrac{\partial\varepsilon^l}{\partial\omega_2}} \delta\,((\mathbf{k}_1+\mathbf{k}_2)\,\mathbf{v} - (\omega_1+\omega_2)), \tag{A.1}$$

$$w_-(\mathbf{k}_1, \mathbf{k}_2) = \frac{4e^4}{m_e^2} \frac{(\mathbf{k}_1\mathbf{k}_2)^2\,\delta\,((\mathbf{k}_1-\mathbf{k}_2)\,\mathbf{v} - \omega_1 + \omega_2)}{\dfrac{\partial\varepsilon^l}{\partial\omega_1}\dfrac{\partial\varepsilon^l}{\partial\omega_2} k_1^2 k_2^2 \omega_{0e}^4}, \tag{A.2}$$

$$\delta w(\mathbf{k}, \mathbf{k}_1) = \frac{e^4\,\delta\,(\omega - \mathbf{k}\mathbf{v})}{\omega_{0e}^4 m_e^2 k^4 k_1^2 \dfrac{\partial\varepsilon^l}{\partial\omega}} \left\{ \frac{101}{36} k_1^6 - \frac{103}{36} k^6 - \frac{17}{72} k^4 k_1^2 + \frac{149}{108} k_1^4 k^2 + \frac{k^2 - k_1^2}{2kk_1} \left[\frac{103}{36} k^6 + \frac{101}{36} k_1^6 + \frac{37}{12} k^4 k_1^2 + \frac{13}{4} k_1^4 k^2 \right] \ln \left| \frac{k+k_1}{k-k_1} \right| \right\}. \tag{A.3}$$

The systematic acceleration for isotropic wave distribution can be readily obtained from (2.9) and A.1):

$$\dot{E}_p=\frac{2e^2\omega_{0e}^3}{\varepsilon}\left\{\int_0^\infty \frac{\hbar N^1(v_{ph})dv_{ph}}{v_{ph}}-\frac{\hbar e^2}{2\pi\omega_{0e}m_e^2}\right.$$
$$\times\int_0^\infty dv_{ph}\int_0^\infty dv'_{ph}\frac{v_{ph}}{(v'_{ph})^8}N^1(v_{ph})N^1(v'_{ph})\left[\frac{101}{36}-\frac{103}{36}\frac{v'^6_{ph}}{v^6_{ph}}-\right.$$
$$-\frac{17}{12}\frac{v'^4_{ph}}{v^4_{ph}}+\frac{149}{108}\frac{v'^2_{ph}}{v^2_{ph}}+\frac{1}{2}\left(\frac{v'_{ph}}{v_{ph}}-\frac{v_{ph}}{v'_{ph}}\right)$$
$$\times\left(\frac{101}{36}+\frac{103}{36}\frac{v'^6_{ph}}{v^6_{ph}}+\frac{37}{12}\frac{v'^4_{ph}}{v^4_{ph}}+\frac{13}{4}\frac{v'^2_{ph}}{v^2_{ph}}\right)\ln\left|\frac{v'_{ph}+v_{ph}}{v'_{ph}+v_{ph}}\right|\Bigg]$$
$$\left.+\frac{4}{9\pi}\frac{e^2}{m_e^2\omega_{0e}}\int_0^\infty\frac{dv_{ph}}{v_{ph}}\int_{v_{ph}}^\infty\frac{dv'_{ph}}{v'^6_{ph}}N^1(v_{ph})N^1(v'_{ph})\left(\frac{2}{15}+\frac{3v'^2_{ph}}{v^2_{ph}}\right)\right\}. \quad (A.4)$$

APPENDIX 2

Let us consider the oscillation of a plasma in an intense alternating electric field.

To reveal the characteristic instabilities it is advantageous to consider the initial problem, using the Laplace transformation (in the absence of a high frequency field—see Landau[71]). Let $E_0(t)$ be an arbitrary external high frequency field and let $f_0^\alpha(v)$ be the plasma particle distribution function at t = 0. Being interested only in potential oscillations, we can write the equation for $f_1^\alpha(r,v,t)$ and the corrections to $f_0^\alpha(v')$ in the form

$$\frac{\partial f_1^\alpha}{\partial t}+\mathbf{v}_\alpha\frac{\partial f_1^\alpha}{\partial \mathbf{r}}+\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t)\frac{\partial f_1^\alpha}{\partial \mathbf{v}}=-\frac{e_\alpha}{m_\alpha}\mathbf{E}(\mathbf{r},t)\frac{\partial f_0^\alpha(\mathbf{v}')}{\partial \mathbf{v}},$$
$$\mathbf{v}'=\mathbf{v}-\frac{e_\alpha}{m_\alpha}\int_0^t \mathbf{E}_0(t')\,dt', \quad (A.5)$$

where E is the electric field of the oscillations. By virtue of the spatial inhomogeneity of the problem, it is sufficient to consider the solution for (A.5) where one of the Fourier components, determined by $L_k = (2\pi)^{-3}\int \exp(-i\mathbf{k}\cdot\mathbf{r})L(\mathbf{r})d\mathbf{r}$:

$$L_{\mathbf{k}}=(2\pi)^{-3}\int e^{-i\mathbf{kr}}L(\mathbf{r})\,d\mathbf{r}:$$

$$\frac{\partial f_{1\mathbf{k}}^\alpha}{\partial t}+i\mathbf{kv}_\alpha f_{1\mathbf{k}}^\alpha+\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t)\frac{\partial f_{1\mathbf{k}}^\alpha}{\partial \mathbf{v}}=-\frac{e_\alpha}{m_\alpha}\mathbf{E}_{\mathbf{k}}(t)\frac{\partial f_0^\alpha(\mathbf{v}')}{\partial \mathbf{v}}. \quad (A.6)$$

This equation can be solved in elementary fashion and makes it possible to determine the space charge density $\rho_k^\alpha(t) = e\int f_{1k}^\alpha(v,t)dv$; which appears as a result of the perturbation f_{1k}^α:

$$\varrho_{\mathbf{k}}^\alpha(t)=e\int f_{1\mathbf{k}}^\alpha(\mathbf{v},t)\,d\mathbf{v}:$$

$$\varrho_{\mathbf{k}}^\alpha(t)=\int ef_1^\alpha(\mathbf{v},0)\exp\left\{-i\mathbf{kv}t-i\mathbf{k}\int_0^t dt'\int_0^{t'}dt''\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t'')\,dt''\right\}$$
$$+\frac{ie_\alpha k}{m_\alpha}\int_0^t dt'\,E_{\mathbf{k}}(t')(t'-t)f_0^\alpha(\mathbf{v})$$
$$\times\exp\left\{-i\mathbf{kv}(t'-t)-i\mathbf{k}\int_{t'}^t dt''\int_0^{t''}\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t''')\,dt'''\right\}. \quad (A.7)$$

Here $f_1^\alpha(v,0)$ is the perturbation at t = 0, $E_k(t) = (k/k)E_k(t)$, that is, we are considering longitudinal oscillations. The Poisson equation

$$ikE_{\mathbf{k}}(t)=4\pi\sum_\alpha\varrho_{\mathbf{k}}^\alpha(t) \quad (A.8)$$

gives in conjunction with (A.7) an integral equation for $E_k(t)$. The instability arises both in the case when the characteristic frequency Ω of the external high-frequency field is large compared with ω—the oscillation frequency—and in the case of comparable Ω and ω. It is simplest, however, to illustrate the gist of the matter using an example with $\Omega \gg \omega$, when Eq. (A.8) reduces to algebraic equation. We first consider the integral contained in (A.7)

$$I=\mathbf{k}\int_0^t dt'\int_0^{t'}dt''\frac{e_\alpha}{m_\alpha}\mathbf{E}_0(t'')\,dt''. \quad (A.9)$$

If

$$\mathbf{E}_0(t)=\mathbf{E}_0\cos(\Omega t+\varphi_0),$$

then

$$I=\mathbf{k}\frac{e_\alpha\mathbf{E}_0}{m_\alpha\Omega}t\sin\varphi_0 t-\mathbf{k}\frac{e_\alpha\mathbf{E}_0}{m_\alpha\Omega^2}\cos(\Omega t+\varphi_0).$$

Since t is of the order of ω^{-1}, the first term is Ω/ω times larger than the second and is of the order of $\mathbf{k}\cdot\mathbf{v}_E/\omega$, where $\mathbf{v}_E^\alpha = (e_\alpha E_0/m_\alpha\Omega)\sin\varphi_0$. Retaining only this term, we can readily solve (A.8) by using the Laplace transformation $E_{ks} = \int_0^\infty \exp(-st)E_k(t)dt$:

$$E_{\mathbf{ks}}=\frac{4\pi e^2}{ik}\frac{\sum_\alpha\int\frac{f_1^\alpha(\mathbf{v},0)\,d\mathbf{v}}{s+i\mathbf{kv}+i\mathbf{kv}_E^\alpha}}{1+\sum_\alpha\frac{\omega_{0\alpha}^2}{n}\int\frac{f_{0\alpha}(\mathbf{v})\,d\mathbf{v}}{(s+i\mathbf{kv}+i\mathbf{kv}_E^\alpha)^2}}. \quad (A.10)$$

This equation differs from that in[71] only in the constant term v_E in the denominator. Introducing v' $v + v_E^\alpha$, we verify that the presence of $v_{E\alpha}$ leas to the occurrence of a translational velocity $v_{E\alpha}$ for the plasma particles. In the absence of ions this does not lead to instability. The presence of ions, however, is quite significant. When $v_{Ee} \gg v_{Te}$ the translational motion of the electrons relative to the ions lead to instability with an increment of the order of $\omega_{0e}(m_e/m_i)^{1/3}$, that is, the same as in a constant external field. It must be noted that the occurrence of the term $\mathbf{k}\cdot\mathbf{v}_E/\omega$ in (A.9) is closely related to the method of turning on the field when t = 0. Thus, this term is missing in the case of adiabatic field application. The method of turning on the field (placing the plasma in the high frequency field) can therefore greatly influence its electromagnetic properties. The instability of the plasma in a random high-frequency field is closely related with this circumstance. Let the field be specified, $E_0(t) = E_0\cos(\Omega t + \varphi_0)$ for $0 < t < T$ and $E_0\cos(\omega t + \varphi_1)$ for $t > T$. Even if the field is turned on adiabatically, for $\Omega \gg \omega$ we obtain from (A.9)

$$I=\begin{cases}\mathbf{k}\frac{e_\alpha\mathbf{E}_0}{m_\alpha\Omega}(t-T)\cos(\Omega T+\varphi_1), & t>T,\\ 0, & t<T,\end{cases} \quad (A.10')$$

i.e., after the phases become randomized we can speak of the occurrence of translational motion of the electrons relative to the ions. It is now easy to consider the general case of an arbitrary $E_0(t)$, assuming that the characteristic frequencies of the high-frequency field are large:

$$\mathbf{E}_0(t)=\int\mathbf{E}_\Omega e^{i\Omega t}\,d\Omega, \qquad \Omega\gg\omega. \quad (A.11)$$

In this case we obtain Eq. (A.10), in which

$$\mathbf{v}_E^\alpha=-\frac{e_\alpha}{m_\alpha}\int\frac{\mathbf{E}_\Omega}{i\Omega}\,d\Omega. \quad (A.12)$$

In the hydrodynamic approximation, when $f_0(v) = n\delta(v)$ and $f_1^\alpha(v,0) = \Delta n_\alpha\delta(v_\alpha)$, we obtain in lieu of (A.10)

$$E_{ks}=\frac{4\pi e^2}{ik}\frac{\sum_\alpha \Delta n_\alpha \frac{1}{s+i\mathbf{k}\mathbf{v}_E^\alpha}}{1+\sum_\alpha \frac{\omega_{0\alpha}^2}{(s+i\mathbf{k}\mathbf{v}_E^\alpha)^2}}. \tag{A.13}$$

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