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THE JOSEPHSON TUNNELING EFFECT IN SUPERCONDUCTORS

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1. INTRODUCTION

THE discovery and detailed study of tunneling effects in superconductors in low temperature physics is surely one of the most interesting developments of recent years.

The fact that two normal metals separated by a thin insulating layer will have an electric current flowing between them because of ordinary quantum mechanical effects was already known in the early years after development of quantum mechanics.[1,2] But the study of this phenomenon in superconductors has revealed a whole series of features which make it a very convenient and effective device for studying many characteristics of superconducting metals.

We shall consider only one, but possibly the most interesting of the tunneling effects in superconductors —the so-called Josephson effect (for other tunneling effects cf. the detailed survey of Douglas and Falicov,[3] which also gives an extensive bibliography).

We may assume that in a superconductor at T = 0 all the electrons with energies near the Fermi surface are paired so that the total momentum of each pair is equal to zero. (A detailed presentation of the microscopic theory of superconductivity can be found in various books and reviews, for example, [4-6].) The presence of such a "long-range order" in momentum space results in a correlation in coordinate space also. The characteristic length ξ_0, over which there is an effective smearing of the wave functions of individual pairs is called the coherence length (or the size of a pair); for most superconductors it is of the order of $\xi_0 \sim 10^{-4}$ cm. The usual tunneling effects are associated with the breaking up of a pair and the transfer of the individual electrons from one metal to the other. Since the breaking of a pair requires a finite energy, equal to the binding energy of the pair, the usual single-particle tunneling effects have a threshold: a tunneling current begins to flow only when a finite voltage is applied. By this means one can measure directly the magnitude of the so-called energy gap in a superconductor.[3]

The Josephson effect,[7] on the other hand, is a tunneling of coupled electron pairs from the ground state (the Fermi surface) of one superconductor to the Fermi surface of another. Since no energy need be expended to break up the pair, the current can flow with zero difference of potential between the metals (Fig. 1,a). It is intuitively clear that occurrence of this effect requires that the thickness of the insulating layer be substantially less than the coherence length ξ_0. There is then an overlap of the electron wave functions in the two metals, so that exchange of superconducting pairs can occur. The required thickness is of order 10—20 Å.

Experimentally the effect was first observed by Anderson and Rowell,[8] who passed a current through a very thin sheet of dielectric (tin oxide with thickness $\sim 10^{-7}$ cm) between two superconductors. They found that if the total current through the barrier did not exceed some value (several milliamps), the current flowed through the dielectric without producing any voltage drop in the insulating layer. The resistance of the layer is obviously zero, as is characteristic for superconductors. (We mention that the resistance of the same layer when the metals on both sides of the layer were in the normal state was 0.4 ohm.) The superconducting nature of the current through the barrier is also demonstrated by experiments[9-11] in which one observes a stationary undamped current in a ring of superconductor split by a tunneling layer. Sometimes these properties of the passage of a current through a thin insulating layer are described as "weak superconductivity,"[12] as contrasted to the properties of true superconductors.

Coupled electron pairs can also go over from one superconductor to the other when there is a nonzero voltage across the layer. The only requirement is that the excess energy 2 eV gained or lost by the pair of electrons in crossing the potential difference V be emitted (or absorbed) as, for example, electromagnetic radiation of frequency ν (cf. Fig. 1,b), where $h\nu$ = 2 eV, or more generally,

$$nh\nu = 2eV \quad (n = 1, 2, 3, \ldots), \tag{1}$$

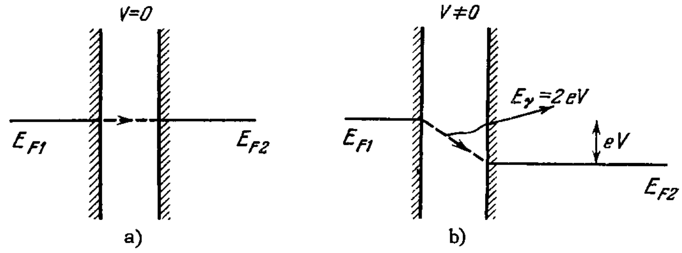

FIG. 1. Quasiparticle picture of the Josephson current. a) Constant current of superconducting pairs when V = 0; b) Variable current when V ≠ 0, accompanied by radiation of photons (or absorption of photons when external microwave radiation is present).

if more than one photon is absorbed or emitted. It has been observed experimentally[13,14] that when microwave radiation of a given frequency ν is incident on the tunneling layer, a voltage drop is established across the barrier through which the Josephson current flows, with constant voltage steps as given by formula (1) (cf. also [53–55]). Recently there has also been a direct observation of the corresponding electromagnetic radiation from a barrier[15–17] subjected to a voltage V. We shall see later that when $V \neq 0$ and when electromagnetic radiation is present, the current of superconducting pairs through the barrier should have a nonstationary, time-dependent component, oscillating at frequency $\nu' = 2eV/h$. A direct measurement of the variable component of the current has not been achieved as yet. The Josephson current also depends on the magnetic field across the barrier in a very remarkable way. We shall discuss this effect in detail.

Before proceeding to the theoretical description of the Josephson effect, we mention one fact that we shall need. In the Ginzburg–Landau phenomenological theory of superconductivity,[18] which also follows from a microscopic treatment of the problem,[19] the superconducting electrons are described by a wave function ψ, and the expression for the current in the superconductor has the form

$$\mathbf{J} = \frac{iq\hbar}{2m}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{q^2}{mc}\mathbf{A}\,|\psi|^2, \tag{2}$$

where q is the effective charge of the current carriers (in our case $q = 2e$ is the charge of the coupled pair of electrons, m is the effective mass, and $\mathbf{A}$ is the vector potential of the electromagnetic field. Writing ψ as $\psi = \sqrt{\rho}\,e^{i\theta}$ where ρ represents the density of superconducting electrons, and θ is a phase, we rewrite (2) as

$$\mathbf{J} = \frac{\hbar}{m} q \left(\nabla\theta - \frac{q}{\hbar}\mathbf{A}\right)\varrho. \tag{3}$$

If we imagine a closed contour to be drawn inside the bulk superconductor, we must satisfy the condition $J = 0$ or $\hbar\nabla\theta = q\mathbf{A}$ along it. Then by integrating along the contour we get

$$\hbar \oint \nabla\theta ds = q \oint \mathbf{A}\, ds = q\Phi, \tag{4}$$

where Φ is the magnetic flux through the contour. The requirement that the wave function be single-valued forces the phase θ at any point to change only by amounts $2\pi n$ after circling the contour. We then get from (4) the condition

$$\Phi = n\Phi_0, \quad \Phi_0 = \pi\hbar c/e \cong 2\cdot 10^{-7}\ \mathrm{G/cm^2} \tag{5}$$

where Φ_0 is the magnetic flux quantum. In a simply-connected bulk superconductor the field inside the metal is zero; in formula (5) this case corresponds to $n = 0$.

The superconductor may also be multiply connected, i.e., there may be holes through it containing a magnetic field. Relation (5) says that the magnetic field in a hole through a bulk superconductor can take on only discrete values, a fact that was first noted by London[20] and later confirmed experimentally.* If the superconductor is simply connected, from the condition div $\mathbf{J} = 0$ (in the gauge div $\mathbf{A} = 0$) we find from (3) the relation $\nabla^2\theta = 0$, from which, by using (4), we get $\theta = \text{const}$. In a multiply connected superconductor the phase in general depends on the coordinates.

2. PHENOMENOLOGICAL DESCRIPTION OF THE JOSEPHSON EFFECT

The wave function of the superconductor plays a very important role in the phenomenological description of the Josephson effect. Its importance is emphasized by the fact, following from the general principles of quantum mechanics, that the phase of the wave function θ and the number of electrons N are canonically conjugate variables. In particular, from gauge invariance (i.e., from invariance with respect to a change of phase), there follows the conservation of the particle number, just as invariance under rotations leads to conservation of angular momentum. We also have the usual indeterminacy relations between canonically conjugate variables†

$$\Delta N \cdot \Delta\theta \geqslant 1. \tag{6}$$

From our discussion it follows, in particular, that the number of superconducting pairs and the phase of the wave function of the superconductor also satisfy relation (6). If we have a piece of superconducting metal with a fixed number of electron pairs, it follows from (6) that the absolute value of the phase is undetermined. If we imagine the superconductor split into two parts, the number of superconducting pairs in each half cannot be fixed exactly, since fluctuations may cause a certain number of pairs to go over from one half to the other or vice versa. Consequently the difference in phase of the wave functions in the two halves of the superconductor has a definite meaning, according to (6), even though the overall phase for the superconductor was not defined before hand.

As already stated, in the Josephson effect one observes a tunneling current of coupled electron pairs between two superconductors separated by a thin insulating layer (Fig. 2).

Following Feynman,[25] it is convenient to write the

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*We restrict our treatment to bulk superconductors. Concerning the nature of the quantization of the field in hollow cylindrical superconductors with wall thickness comparable to the penetration depth, cf, for example,[20,23].

†Cf. also[24], where the meaning of the uncertainty relations for particle number and phase is made more precise and where the whole question is discussed.

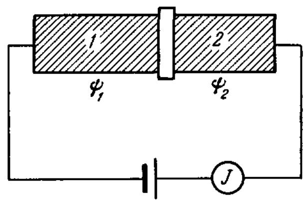

FIG. 2

following system of equations, giving a phenomenological description of the Josephson effect:

$$i\hbar\frac{\partial\psi_1}{\partial t}=U_1\psi_1+K\psi_2,\quad i\hbar\frac{\partial\psi_2}{\partial t}=U_2\psi_2+K\psi_1. \qquad (7)$$

Equation (7) is the Schrödinger equation for a coupled quantum mechanical system with two states. Here ψ_1 and ψ_2 are the wave functions of superconductors 1 and 2 (cf. Fig. 2); U_1 and U_2 are the energy terms which act as Hamiltonians for the individual superconductors, K is some matrix element providing the coupling between the wave functions of the system.

Suppose that there is some difference of potential V between the superconductors, so that $U_1 - U_2 = qV$, where q is the charge of the current carriers. Choosing the zero of energy appropriately, we can write (7) as

$$i\hbar\frac{\partial\psi_1}{\partial t}=\frac{qV}{2}\psi_1+K\psi_2,\quad i\hbar\frac{\partial\psi_2}{\partial t}=-\frac{qV}{2}\psi_2+K\psi_1. \qquad (8)$$

Introducing the notation $\psi_{1,2}=\sqrt{\rho_{1,2}}\,e^{i\theta_{1,2}}$, $\varphi=\theta_2-\theta_1$, we find from (8) four equations for the quantities ρ_1, ρ_2, θ_1, θ_2:

$$\dot\rho_1=-\dot\rho_2=\frac{2}{\hbar}K\sqrt{\rho_2\rho_1}\sin\varphi,\quad \dot\theta_{1,2}=\frac{K}{\hbar}\sqrt{\frac{\rho_2}{\rho_1}}\cos\varphi\mp\frac{qV}{2\hbar}. \qquad (9)$$

Since the current J from 1 to 2 is $\dot\rho_1$ (or $-\dot\rho_2$), and noting that $\dot\theta_2-\dot\theta_1=\dot\varphi$, we get from (9)

$$J=J_{\max}\sin\varphi,\quad J_{\max}=\frac{2K}{\hbar}\sqrt{\rho_1\rho_2},\quad \varphi=\varphi_0+\frac{q}{\hbar}\int V(t)\,dt. \qquad (10)$$

Here φ_0 is some random phase difference.

Formula (10) describes the essential features of the Josephson effect. When V = 0 the value of the current J is related to the initial phase difference φ_0, which in turn depends on external conditions (the material of the superconductor, the presence of a magnetic field, etc). Since $|\sin\varphi_0|\le 1$ always, it follows from (10) that the Josephson current is bounded by the value J_{max}. Unfortunately the phenomenological approach does not enable us to calculate J_{max}, since we do not know the value of K.

Figure 3 shows a diagram of the experiment for observing the tunneling current, while Fig. 4 shows the characteristic dependence of J on V, observed on an oscillograph when a low frequency ac voltage is applied to the sample. The central solid line corresponds to the stationary Josephson current passing through the sample when V = 0. In the experiment one measures J_{max} directly.

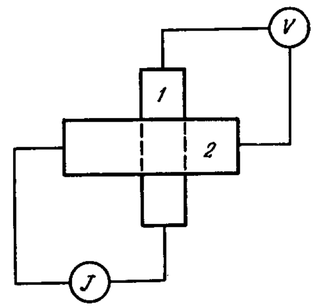

FIG. 3. Diagram of tunneling experiment. The superconducting film 1 is deposited on a backing and then oxidized in an oxygen atmosphere, forming a dielectric layer 10-20 Å thick. A second superconducting layer 2 is deposited perpendicular to 1. Current leads are attached to two ends of the sample, while the difference of potential across the barrier is taken from the other two.

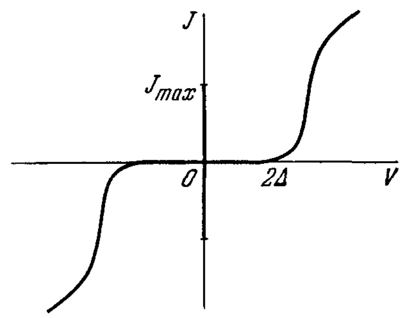

FIG. 4. Schematic of oscillogram of J vs. V obtained when a low frequency voltage is applied to the sample. The central vertical line gives the Josephson current J_{max}. The curve for $V \neq 0$ corresponds to the usual single-particle tunneling current. The scale is arbitrary.

When there is a constant difference of potential V_0 between the superconductors, the phase φ in (10) depends linearly on t, and the Josephson current J shows an oscillatory dependence on time, so that the average value of J is zero. Let us assume the dependence

$$V(t)=V_0+v\cos\omega t,$$

where V_0 corresponds to a constant voltage component across the barrier, while the second term corresponds to some additional periodic electric field (for example, an external electromagnetic wave). Assuming $v \ll V_0$ and expanding $\sin\varphi$ in (10) in series: $\sin(x+\alpha) \cong \sin x + \alpha\cos x + \ldots$, $\alpha \ll 1$, we get

$$J=J_{\max}\left[\sin\left(\varphi_0+\frac{q}{\hbar}V_0t\right)+\frac{q}{\hbar}\frac{v}{\omega}\sin\omega t\cos\left(\varphi_0+\frac{q}{\hbar}V_0t\right)\right].$$

The time average of the first term gives zero, while the second gives a finite result when the resonance condition $\hbar\omega = qV_0$ is satisfied, corresponding, as pointed out above, to an exchange of energy between the superconductor and the external electromagnetic radiation. In the experiment,[13] when external radiation of frequency $\nu = \omega/2\pi$ fell on the sample, one actually observed currents when the voltage across the barrier had values $V = nh\nu/2e$.

The effects associated with the presence of a magnetic field can be included in this treatment by generalizing the expression for the current $J = J_{max} \sin\varphi$ so that the resulting expression is gauge invariant. This reduces to adding to the phase the term $(q/\hbar c)\int \mathbf{A}\cdot d\mathbf{s}$, where the integration is along some path joining the two superconductors. Later we shall consider the problems related to the presence of a magnetic field.

So far we have given some qualitative arguments enabling us to give an intuitive approach to the description of the Josephson effect. It is important to give the derivation of the fundamental relations, starting from a more detailed microscopic picture. It is convenient to start from the following expression for that part of the Hamiltonian for the system of two superconductors that describes the tunneling transitions between them:[26]

$$H_t = \sum_{\mathbf{k},\,\mathbf{q},\,\sigma} (T_{\mathbf{kq}} a^+_{\mathbf{k}\sigma} c_{\mathbf{q}\sigma} + T^*_{\mathbf{qk}} c^+_{\mathbf{q}\sigma} a_{\mathbf{k}\sigma}), \tag{11}$$

where $a_{\mathbf{k},\sigma}$ and $c_{\mathbf{q},\sigma}$ are the usual electron operators for the two metals ($\mathbf{k}$ and $\mathbf{q}$ are the momenta, and σ the spin), and T is some matrix element describing transitions of electrons from one metal to the other (a more general expression for H_T is used in [27]).

Because of the interaction (11) between the two superconductors, the total energy of the system is less than the sum of the energies of the separate superconductors. The binding energy can be found from ordinary second order perturbation theory:

$$\Delta\mathscr{E}^{(2)} = \sum_{n\neq 0} \frac{|H_t|^2_{n0}}{\mathscr{E}_0 - \mathscr{E}_n}. \tag{12}$$

Here $\mathscr{E}_0$ is the ground state energy, and $\mathscr{E}_n$ the energies of the excited states of the system. When $T \cong 0$ the important states are the virtual states in which one of the electrons of a pair has gone over from one metal to the other, so that there is one "excitation" in each of the metals, with $\mathscr{E}_n - \mathscr{E}_0 = E_\mathbf{k} + E_\mathbf{q}$ where $E_\mathbf{p} = \sqrt{\epsilon_\mathbf{p}^2 + \Delta_\mathbf{p}^2}$ is the energy of a single particle excitation, $\Delta_\mathbf{p}$ is the energy gap parameter, $\epsilon_\mathbf{p}$ is the Bloch energy of the electron measured from the Fermi surface.[28]

In calculating the matrix element appearing in (12) it is convenient to express the operators $a_{\mathbf{k}\sigma}$ and $c_{\mathbf{q}\sigma}$ in terms of the Bogolyubov quasiparticle operators.[29] A simple computation, which we shall not reproduce here (cf. [12]) gives (for $T = 0$)

$$\Delta\mathscr{E}^{(2)} = -\sum_{\mathbf{k},\mathbf{q}} \frac{|T_{\mathbf{kq}}|^2}{E_\mathbf{k} + E_\mathbf{q}} \left(1 - \frac{\varepsilon_\mathbf{k}\varepsilon_\mathbf{q}}{E_\mathbf{k}E_\mathbf{q}} + \operatorname{Re}\frac{\Delta_\mathbf{k}\Delta^*_\mathbf{q}}{E_\mathbf{k}E_\mathbf{q}}\right). \tag{13}$$

All of the basic features of the Josephson effect already appear for the case of $T = 0$, to which we shall restrict our treatment. The generalization to finite temperatures involves no difficulties (cf., for example,[30] where the expression for the Josephson current for finite T is derived using the Green function method, and also [31] and [12]).

We know that in the microscopic approach the gap parameter Δ plays the part of the wave function ψ of the Ginzburg-Landau theory. If we assume that $\Delta_\mathbf{k}$ and $\Delta_\mathbf{q}$ do not depend on $\mathbf{k}$ and $\mathbf{q}$, and set

$$\Delta_\mathbf{k} = \Delta_1 \exp(i\theta_1),\quad \Delta_\mathbf{q} = \Delta_2 \exp(i\theta_2),$$

where Δ_1 and Δ_2 are real constants, we can write the term in (13) that depends on the phase difference in the form

$$\Delta\mathscr{E} = -N_1 N_2 \Delta_1\Delta_2 \langle |T_{\mathbf{kq}}|^2\rangle_{av} \cos(\theta_2 - \theta_1) \iint\limits_{-\infty}^{\infty} \frac{d\varepsilon_1\, d\varepsilon_2}{E_1 E_2 (E_1 + E_2)}.$$

Here N_1 and N_2 are the densities of electron states on the Fermi surfaces of the two superconductors.* Calculating the integrals, we find[12,20] when $\Delta_1/\Delta_2 \lesssim 0.5$,

$$\Delta\mathscr{E} \cong -N_1 N_2 \langle |T_{\mathbf{kq}}|^2\rangle_{av} \cos(\theta_2 - \theta_1)\cdot 2\pi^2 \frac{\Delta_1\Delta_2}{\Delta_1 + \Delta_2}, \tag{14}$$

or

$$\Delta\mathscr{E} = -\mathscr{E}_1 \cos\varphi_0,\quad \mathscr{E}_1 = \frac{h}{2e}\frac{\pi\Delta_1\Delta_2}{(\Delta_1 + \Delta_2) R_N}, \tag{15}$$

where $\varphi_0 = \theta_2 - \theta_1$, and R_N is the barrier resistance in the normal state. We shall see later that the magnitude of the Josephson current is closely related to the value of the binding energy (15).

3. THE EFFECT IN THE PRESENCE OF MAGNETIC AND ELECTRIC FIELDS

From arguments on gauge invariance[7,8,12,25] one can generalize the dependence (15), writing in the general case

$$\Delta\mathscr{E} = -\mathscr{E}_1 \cos\left(\varphi_0 - \frac{2e}{\hbar c}\int_1^2 \mathbf{A}\, d\mathbf{s} + \frac{2e}{\hbar}\int^t V(t)\, dt\right) = -\mathscr{E}_1 \cos\varphi, \tag{16}$$

where φ_0 is some initial difference of phase between the superconductors 1 and 2, $\mathbf{A}$ is the vector potential, and $V(t)$ is the potential difference across the tunneling layer.

From (16) it follows that in the general case the phase difference φ depends on the magnetic field be-

*The density of electron states $N(\epsilon)$ appears automatically when we go over from a summation over momenta and spins to an integral over the energy:

$$\sum_{\mathbf{k},\sigma} \to \frac{2}{(2\pi)^3}\int dk = \int N(\varepsilon)\, d\varepsilon,$$

where, for a free electron gas $N(\varepsilon) = \pi^{-2} m\sqrt{2m(E_F - \varepsilon)}$. Since only the electrons lying near the Fermi surface (with energies $\epsilon \sim 0$) participate in superconductivity, one usually assumes the density of states to be constant $N = N(0)$ and removes it from under the integral sign.

tween the superconductors and on the resulting voltage between them: $\varphi = \varphi(\mathbf{A}, V)$. Differentiating the phase φ with respect to t, we get the relation

$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar}. \qquad (17)$$

From this equation it follows that the presence of a potential difference across the tunneling layer is associated with a change of the phase with time, i.e., corresponds to the nonstationary case.

For the case of uniform magnetic field **H** in the plane of the barrier and perpendicular to the direction of the current, we find from (16) (cf. also[32])

$$\nabla\varphi = \frac{2ed}{\hbar c}[\mathbf{Hn}]. \qquad (18)^*$$

Here **n** is a unit vector in the direction of the current, d is the effective thickness of the layer in which the field **H** is different from zero, $d = l + \lambda_1 + \lambda_2$ where λ is the depth of penetration of the field into the superconductor, and l is the thickness of the insulating layer. Equation (18) shows that the presence of a magnetic field in the barrier is accompanied by a change in the phase from point to point.

We have seen that the presence of a finite transition amplitude $T_{\mathbf{kq}}$ for the electrons on both sides of the barrier leads to the appearance of an additional "binding energy" and the establishing of a corresponding phase difference φ_0 between the superconductors. It is not hard to see that the binding energy $\Delta\mathscr{E}$ in (16) actually represents the phase-dependent part of the free energy of the system, taken per unit area of the barrier. There is also another additional energy that depends on the phase, namely the energy associated with the passage of current through the barrier. This part of the energy has the form (cf. [9])

$$-\int^{t} \bar{J}V\,dt = -\bar{J}\,\frac{\hbar}{2e}\varphi(t), \qquad (19)$$

where $\bar{J}$ is the current through unit area of the barrier, and the integration constant is taken so that, in accordance with (17), when $V = 0$, we have $\varphi = \text{const} = \varphi(0)$. So the total energy of the system as a function of the phase φ has the form

$$\mathscr{E} = \int \left(-\mathscr{E}_1\cos\varphi - \frac{\hbar}{2e}\bar{J}\varphi\right)\frac{d\sigma}{S_b}, \qquad (20)$$

where the integration is over the surface of the barrier and S_b is the area of the barrier.

The phase difference φ_0 obviously must adjust so that the total energy of the system is a minimum. The condition $d\mathscr{E}/d\varphi_0 = 0$ leads to a relation between φ and the current through the barrier,

$$\bar{J} = \frac{2e}{\hbar}\mathscr{E}_1\int \sin\varphi\,\frac{d\sigma}{S_b}. \qquad (21)$$

It is clear that we can write the current density as

$$J = \frac{2e}{\hbar}\mathscr{E}_1\sin\varphi, \qquad (22)$$

where φ and also J in general depend on the coordinates in the plane of the barrier. The maximum current value is

$$J_{\max} = \frac{2e}{\hbar}\mathscr{E}_1 = \frac{\pi\Delta_1\Delta_2}{(\Delta_1+\Delta_2)R_N}, \qquad (23)$$

where we have used (15). In other words, the maximum Josephson current J_{max} is equal to the current that flows through the barrier in the normal state when the difference of potential is $V = \pi\Delta_1\Delta_2/(\Delta_1+\Delta_2)$.[30]

The measured values of the maximum current are always less than the theoretical values, sometimes being 95% of the predicted value and sometimes much less (by a sizable factor). One possible cause of the discrepancy is the presence of a magnetic field on the barrier and the resulting "effect of self-limiting of the tunneling current."[32] Let us discuss this point briefly.

To get the estimate (23) of the maximum Josephson current we set $\sin\varphi = 1$ in (22). We are obviously neglecting the spatial variation of the phase in the plane of the barrier if we assume $\varphi = \text{const}$. Actually one cannot neglect the dependence of the phase on the coordinates and the corresponding distribution of the current over the area of the barrier, so that integration of $\sin\varphi$ over the barrier surface in (21) gives a smaller effective value of the current per unit area than follows from (23).

In fact, suppose that we are dealing with the stationary case $V = 0$, and suppose that there is no external field on the barrier. There will nevertheless be a magnetic field in the barrier, associated with the current J flowing through the barrier. The usual Maxwell equation*

$$\operatorname{rot}\mathbf{H} = 4\pi c^{-1}\mathbf{J},$$

where $\mathbf{J} = \mathbf{J}_{\max}\sin\varphi$, together with Eq. (18), which gives the law of variation of the phase difference φ in the plane of the barrier, leads to the following equation for the phase:

$$\nabla^2\varphi = \lambda_J^{-2}\sin\varphi, \quad \lambda_J = \left(\frac{\hbar c^2}{8\pi e J_{\max} d}\right)^{1/2}. \qquad (24)$$

Here ∇^2 is the two-dimensional Laplace operator, acting in the plane of the barrier, and $d = l + \lambda_1 + \lambda_2$. The length λ_J plays the role of a penetration depth, which is clearly seen for small φ, when Eq. (24) takes the form $\nabla^2\varphi = \lambda_J^{-2}\varphi$, analogous to the equation describing the penetration of a magnetic field into a superconductor. A typical value is $\lambda_J \sim 1$ mm. These equations were solved by Ferrell and Prange[32] for the case of a tunneling barrier between superconductors, in the form of a layer of thickness d, finite in

*$[\mathbf{H\,n}] \equiv \mathbf{H}\times\mathbf{n}$.

*rot ≡ curl.

width and infintely long. The qualitative conclusion from this analysis is that in the case of a thin layer the current and the associated field are uniformly distributed over the cross section, but if the width of the layer begins to exceed λ_J, the current and field are restricted to a region of width λ_J from the edges of the barrier. Thus the whole area of the barrier does not contribute to the total current, but only an effective area with the characteristic size λ_J. The consequence is that the maximum current J_{max} in actual tunneling layers, which frequently have transverse dimensions ~ 1 mm, is less than the value found theoretically.

It should be mentioned that the model of Ferrell and Prange is quite idealized and that in actual experiments the distribution of current may be even more unfavorable as compared to this model. In principle, in order to get a realistic estimate of the effect of self-limiting of the current, one would have to solve the differential equations for the actual geometry of the sample, find the space dependence of the phase difference, after which an integration of $\sin\varphi$ in (11) over the barrier surface gives the total current.

Interesting effects occur when there is an external field **H** in the tunneling layer. Suppose that we have the stationary case V = 0, and that the current flows perpendicular to the plane of the barrier, and that a homogeneous field **H** is in the plane of the insulating layer, with the coordinate x giving the position in the plane of the layer perpendicular to the direction of **H**. According to (16) and (20) the free energy density of the system that depends on the phase difference has the form

$$\mathscr{E}(x) = -\mathscr{E}_1 \cos\left(\varphi_0 - \frac{2ed}{\hbar c} Hx\right) - \frac{\hbar}{2e} \bar{J} \left(\varphi_0 - \frac{2ed}{\hbar c} Hx\right), \quad (25)$$

where φ_0 is the phase difference when H = 0. The total energy associated with the barrier and the current flowing through it is obtained by integrating (25) over the surface of the tunneling layer. If the layer has the form of a strip of width w and length L (Fig. 5), the total energy takes the form

$$\mathscr{E}_{tot} = -L\mathscr{E}_1 \int_{-w/2}^{w/2} \cos\left(\varphi_0 - \frac{2ed}{\hbar c} Hx\right) dx$$
$$- L\frac{\hbar}{2e} \bar{J} \int_{-w/2}^{w/2} \left(\varphi_0 - \frac{2ed}{\hbar c} Hx\right) dx.$$

An elementary integration gives

$$\mathscr{E}_{tot} = -wL\mathscr{E}_1 \cos\varphi_0 \frac{\sin\left(\frac{2ed}{\hbar c} H \frac{w}{2}\right)}{\frac{2ed}{\hbar c} H \frac{w}{2}} - \frac{\hbar \bar{J} \varphi_0 w L}{2e}.$$

To obtain the expression for the total current we again equate to zero the derivative $\partial\mathscr{E}_{tot}/\partial\varphi_0 = 0$, and defining $I_{tot} = \bar{J}wL$, we find*

*Of course one gets this same result by integrating the current density (22) over the surface of the barrier.

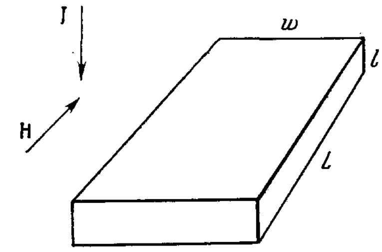

FIG. 5

$$I_{tot} = wL \cdot \frac{2e}{\hbar} \mathscr{E}_1 \sin\varphi_0 \left| \frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0} \right|, \quad (26)$$

where Φ = Hwd is the magnetic flux through the cross section of the barrier, $\Phi_0 = hc/2e$ is the quantity (5) introduced earlier, which is called the magnetic flux quantum. Since in the stationary case the total current has a definite sign, it is convenient to introduce the sign of the modulus in (26) and take $0 \le \varphi_0 \le \pi$. From (26) we see that the presence of the external field on the barrier leads to a modulation of the total current through the tunneling layer and that for values of the magnetic flux equal to $\Phi = n\Phi_0$, when there are an integral number of flux quanta in the barrier, the current goes to zero.

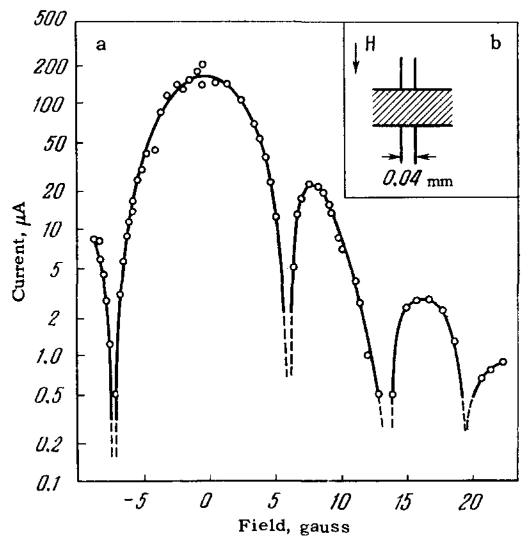

FIG. 6. a) Dependence of maximum Josephson current on field for Pb-I-Pb barrier at 1.3°K;[33] b) schematic of the tunneling barrier.

Figure 6 shows the experimental results of Rowell,[33] who studied the dependence of the Josephson current on magnetic field. The periodic variation appears most clearly for barriers with very small dimensions transverse to the field, when the field in the barrier may be assumed to be uniform. The data of Fig. 6 correspond to a barrier between two thick layers of lead, one of which, parallel to the field, had a width of 0.04 mm. The size of the region within the barrier carrying the field is $S_b = 0.04\ \text{mm} \times 2\lambda \cong 3 \times 10^{-8}\,\text{cm}^2$, where $\lambda = 390\,\text{Å}$ is the depth of penetration of the field in lead. The minima on the curve of Fig. 6 at H = 6.5, 13 and 19.5 are in very good agreement with the values of the flux through the barrier $\Phi = n\Phi_0$ for n = 1, 2, 3, showing good agreement with formula (26). The dependence of the Josephson current on magnetic field and temperature has also been studied in [34,51,58,59].

Let us present the interpretation of the results following from Eq. (18), giving the differential law of variation of the phase difference in the plane of the barrier, by using the notion of quantized magnetic flux.[35] The external field penetrates into the space between the superconductors in the form of a magnetic sheet of thickness $d = l + \lambda_1 + \lambda_2$. Let us consider two points P and Q on opposite sides of the barrier (outside the region of penetration of the field) and join them by two curves that intersect the barrier at points A and B (Fig. 7). According to (18) the change in the phase difference φ from point A to point B is proportional to the magnetic flux between the two curves, where one flux quantum corresponds to a change of phase of 2π.

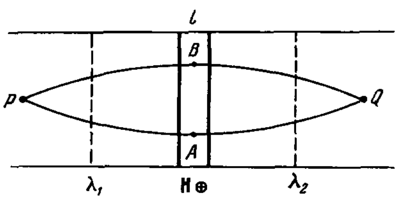

FIG. 7

The detailed behavior of the two superconductors coupled by the tunneling layer depends on the transverse dimensions of the barrier. Barriers with dimensions exceeding λ_J behave like superconductors of the second kind.[36] In a weak magnetic field the diamagnetic currents screen the field, and it penetrates into the barrier only along the edges, over a distance of order λ_J from the edges. When a critical field (cf. [32,35]) which is usually of order 1 gauss is reached, there is a phase transition of the second kind, and quantized lines of magnetic flux begin to penetrate into the barrier, the separation between the lines decreasing as the field increases.

The behavior of barriers with dimensions less than λ_J is somewhat different. As in the case of very thin superconducting films, the magnetic field in the barrier is almost uniform. In this case, as we have seen above, the most interesting point is the dependence of the maximum Josephson current on field. The presence of the field causes a change of the phase difference φ from point to point. Consequently, because of the dependence $J = J_1 \sin\varphi$, at sufficiently large fields the barrier splits up into regions in which the currents have opposite signs, and the total current through the barrier is sharply reduced. The dependence of the critical current on field is given by a formula like (26), which was obtained for a rectangular barrier:

$$I \sim \left| \frac{\sin(H/H_0)}{H/H_0} \right|, \qquad (26')$$

where $H_0/2\pi$ is the field at which there is one flux quantum $hc/2e$ in the barrier.

The last formula is similar to the one in optics giving the Fraunhofer diffraction by a slit. It is clear from our argument that this is not an accident, since the Josephson effect owes its origin to the quantum mechanical interference of the phases of the wave functions of the individual superconductors, so the analogy with interference phenomena in optics is entirely correct. This analogy goes much further in the example given next of the Josephson effect in two tunneling barriers connected in parallel.

4. JOSEPHSON EFFECT IN BARRIERS CONNECTED IN PARALLEL

An extremely interesting effect was discovered recently by Jaklevič, Mercereau, et al.[37] They studied the Josephson current through two tunneling barriers connected in parallel, as a function of the applied magnetic field. To understand the origin of the effect we consider the diagram of the experiment shown in Fig. 8.

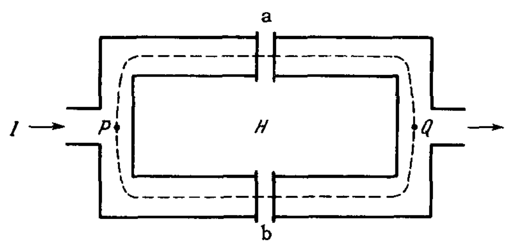

FIG. 8

We assume that the cross sections of the barriers a and b are small compared to the cross section of the whole superconducting ring. This assumption allows us to neglect the spatial variation of the phase in the insulating layer. We shall also assume that the transverse dimensions of the barriers are small compared to the characteristic Josephson length λ_J. This means that the current density through the barrier is uniform. For simplicity we further assume that the two insulating layers are identical and arranged symmetrically, and that the field H is perpendicular to the plane of Fig. 8. We shall assume that the main body of the ring is sufficiently thick so that we can choose a continuous contour inside the ring on which the superconducting current J = 0. Then according to (4) and (5) there can only be an integral number of magnetic flux quanta inside the contour.

The total currents through each of the individual barriers a and b are

$$J_a = J \sin\varphi_a, \qquad J_b = J_1 \sin\varphi_b,$$

where φ_a and φ_b are the jumps in phase at the respective barriers. The total external current I passing through the system is the sum of the currents through the barriers. The wave function of the system must be single-valued when we go around the ring. Consequently, the phase difference when we go from point P to point Q along the contour PaQ, passing through barrier a, must be the same (to within a term $2\pi n$) as the phase difference along the contour PbQ, passing through barrier b. In other words,

$$\varphi_a+\frac{2e}{\hbar c}\int\limits_a \mathbf{A}\,ds=\varphi_b+\frac{2e}{\hbar c}\int\limits_b \mathbf{A}\,ds+2\pi n.$$

or, rewriting the terms in a different order,

$$\varphi_a-\varphi_b=\frac{2e}{\hbar c}\oint \mathbf{A}\,ds+2\pi n.$$

To give the result a symmetric form, we write

$$\varphi_a=\varphi_0+\frac{e}{\hbar c}\Phi+n\pi,\quad \varphi_b=\varphi_0-\frac{e}{\hbar c}\Phi-n\pi,$$

$$\Phi=\oint \mathbf{A}\,ds=\int \mathbf{H}\,d\sigma;$$

then

$$I_{\text{tot}}=J_1\sin\left(\varphi_0+\frac{e}{\hbar c}\Phi+n\pi\right)+J_1\sin\left(\varphi_0-\frac{e}{\hbar c}\Phi-n\pi\right)$$
$$=2J_1\sin\varphi_0\left|\cos\frac{\pi\Phi}{\Phi_0}\right|,\qquad 0\leqslant\varphi_0\leqslant\pi, \tag{27}$$

where φ_0 is a phase jump which depends only on the total current through the system, $\Phi_0 = hc/2e$ is the flux quantum and Φ is the magnetic flux through the ring.

If the dimensions of the barriers transverse to the field are not small compared to the area of the ring, which is closer to the actual experimental case, the result (27) must be changed somewhat. We must sum the currents in different parts of the barrier, in accordance with the local phase difference, as was done for the case of a single tunneling layer (cf. (26)). For the case of two symmetrical identical barriers the result obtained for the dependence of maximum current on magnetic field is analogous to "interference modulated diffraction" in optics:

$$I=2J_1\sin\varphi_0\left|\frac{\sin(\pi H/H_0)}{\pi H/H_0}\right|\left|\cos\frac{\pi H}{H_1}\right|, \tag{28}$$

where H_1 is the field corresponding to one quantum of flux through a contour lying in the ring and passing through the centers of the barriers, while H_0 is the field corresponding to one flux quantum in each of the barriers, taking account of the depth of penetration of the field.

Figure 9 shows schematically one of the experimental curves,[37] in which both of these periodic variations of the field can be seen. The large-scale minima and maxima correspond to modulation of the phase within each barrier, while the fine ones are due to interference effects. In principle these effects allow one to use a double Josephson junction as a sensitive device for precise measurements of magnetic field.

In the original experiment[37] the distance ΔH between successive fine maxima was 10^{-2}–10^{-3} gauss. Obviously the greater the area S lying within the ring shown schematically in Fig. 8, the closer together are the maxima and minima and thus the greater the accuracy with which one can measure the magnetic field. In one of the experiments[38] they succeeded in reaching a resolution of $\Delta H \cong 3\times10^{-4}$ gauss (cf. also [39,52]). The main difficulty to date has been the preparation of tunneling samples of large size. But, as pointed out by Feynman,[25] if one has a sample with a ring area S $\sim 1\ \text{cm}^2$, the resolution will be $\Delta H = \Phi_0/S \cong 10^{-7}$ gauss. If, however, one considers the possibility of preparing not two, but 10 or 20 or 100 parallel Josephson barriers, the resolving power of the apparatus will be even greater. One is struck by the analogy with the optical diffraction grating, which allows very accurate measurements of wave lengths of light. We may hope that development of techniques for preparing tunneling samples will allow the attainment of precisions in magnetic field measurement comparable to those of optical measurements.[25]

Another interesting experiment done by the same authors[38] was to bring the magnetic flux into the ring (cf. Fig. 8) by means of a long thin magnetic solenoid, so that the field outside the solenoid was negligibly small. Despite the fact that the magnetic field in the superconducting leads and tunneling barriers remained equal to zero, the Josephson current through the system showed the characteristic periodic dependence on the total flux $\Phi = \oint \mathbf{A}\cdot d\mathbf{s}$ in the ring. In other words, the magnitude of the current was determined by the values of the vector potential along the contour. The authors of [38] remark that this experiment seems to allow one to count the vector potential **A** among the truly observable physical quanti-

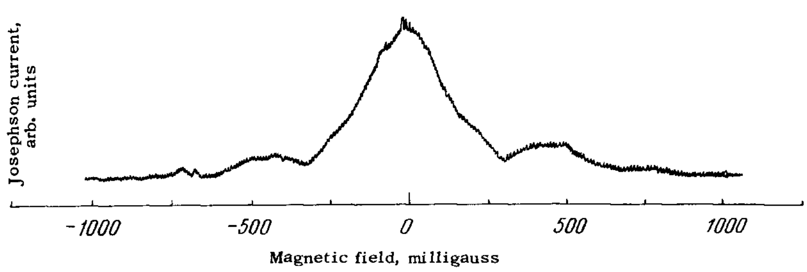

FIG. 9. Dependence of Josephson current on magnetic field for two barriers connected in parallel, showing the interference effects. The interference maxima are separated by an amount $\Delta H = 4.8\times10^{-3}$ gauss. The maximum Josephson current is about 10^{-3} amp.

ties, like the magnetic and electric fields, which are derivatives of **A** (concerning this, cf. [40,52]). It is also clear that we are not falling into a contradiction with the gauge invariance of the electromagnetic field. Actually in this experiment one observes the dependence of a physical quantity (the current) not on the vector potential **A** itself, but on its integral around a closed contour, which, because of the relation

$$\oint \mathbf{A}\, ds = \int \mathbf{H}\, d\sigma = \Phi$$

makes the gauge invariance obvious.

In conclusion we emphasize once more that the experiments described here demonstrate interference effects (and also phase coherence) in a quantum mechanical system over large distances (up to 3.5 mm;[38] cf. also [57,61–63]). They confirm the picture proposed by London[20] of the superconductor as a single quantum state, characterized by coherence over macroscopic distances.

5. THE NONSTATIONARY JOSEPHSON EFFECT

We now consider in somewhat more detail the nonstationary effects associated with the presence of a finite difference of potential V on a barrier and the corresponding time dependence of the phase. Most of the effects give only an indirect proof of the existence of an ac Josephson current. We have already mentioned the experiments[13,14] where a barrier is placed in an external microwave field and one observes voltage steps in accordance with Eqs. (1) and (17) (cf. also [58,60]).

Another indirect experiment,[41] illustrating the presence of a varying current component, is based on the fact that the presence of ac fields in the barrier leads to a typical resonant dependence of the constant component of the current on the voltage across the barrier. In order better to understand this effect, we give a simple computation[41] in which the tunneling sample is considered to be two semi-infinite superconductors separated by a thin oxide layer of thickness l. We shall also assume that there is an external field H_0 on the barrier, parallel to the y axis (Fig. 10).

We write the equations describing the Josephson current:

$$j(\mathbf{r}, t) = j_1 \sin\varphi(\mathbf{r}, t), \quad \frac{\partial\varphi}{\partial t} = \frac{2eV}{\hbar}, \quad \nabla\varphi = \frac{2ed}{\hbar c}[\mathbf{Hn}], \tag{29}$$

where **H** is the total magnetic field in the barrier, $V = V_0 + v(t)$, $d = l + 2\lambda$, j_1 is the current through the barrier in the normal state for voltage $\frac{1}{2}\pi\Delta$, φ is the phase difference in the barrier. For $V = V_0$ and $H = H_0$, (29) gives $\varphi = \omega t - kz$, where $\omega = 2eV_0/\hbar$, $k = 2eH_0 d/\hbar c$.

To take account of the effect of the varying electromagnetic field in the barrier on the Josephson current we must add to (29) the Maxwell equations

$$\operatorname{rot}\mathbf{E} = \frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \quad \operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{\varepsilon}{c}\frac{\partial\mathbf{E}}{\partial t} \tag{30}$$

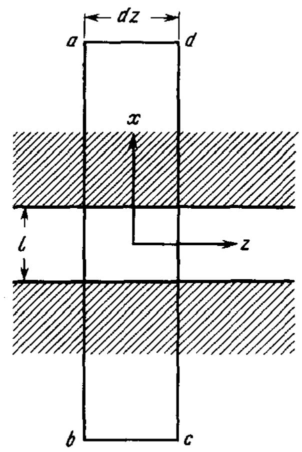

FIG. 10

(where ϵ is the dielectric constant of the oxide layer). The layer thickness is usually of order 10–20 Å, so that the dependence of the field in the layer on x can be neglected. The field component E_x is screened and does not penetrate into the superconductor. Integration of the first of Eqs. (30) over the surface abcd (Fig. 10) gives

$$l\frac{\partial E_x^0}{\partial z} = -\frac{2\lambda + l}{c}\frac{\partial H_y^0}{\partial t}, \tag{31}$$

where E_x^0 and H_y^0 are the fields in the oxide layer. Similarly, taking the normal to the surface parallel to the z axis, we find

$$l\frac{\partial E_x^0}{\partial y} = \frac{2\lambda + l}{c}\frac{\partial H_z^0}{\partial t}. \tag{32}$$

From the second equation in (30) we find

$$\frac{\partial H_z^0}{\partial y} - \frac{\partial H_y^0}{\partial z} = \frac{4\pi}{c}j_x + \frac{\varepsilon}{c}\frac{\partial E_x^0}{\partial t}. \tag{33}$$

From (31)–(33) one gets an equation for the variable component of the voltage across the barrier, $v = lE_x^0$:

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{\bar{c}^2}\frac{\partial^2}{\partial t^2}\right)v = \frac{4\pi l}{\varepsilon\bar{c}^2}\frac{\partial j_x}{\partial t}, \tag{34}$$

where $\bar{c} = c(l/\epsilon d)^{1/2}$ is the velocity of propagation of electromagnetic disturbances in the insulating layer. Since $v \ll V_0$ in the experiment, the right side of (34), which acts as a driving force, can be written as $(4\pi l/\epsilon\bar{c}^2)j_1\omega\cos(\omega t - kz)$. Introducing the quantity Q, which characterizes the losses in the system (to both dissipation and radiation), we find the solution:[7]

$$v = v_0\cos(\omega t - kz + \theta),$$
$$v_0 = \frac{(4\pi l/\varepsilon\omega)j_1}{\{[1-(k\bar{c}/\omega)^2]^2 + (1/Q)^2\}^{1/2}}, \quad \theta = \tan^{-1}\frac{1/Q}{1-(k\bar{c}/\omega)^2}. \tag{35}$$

Since the total $V = V_0 + v$ appears in (29), the phase shift φ is changed by the amount $(v_0/V)\sin(\omega t - kz)$ and the current density takes the form

$$j = j_1\sin\left[\omega t - kz + \left(\frac{v_0}{V}\right)\sin(\omega t - kz + \theta)\right]. \tag{36}$$

The appearance of the additional time dependence in (36) is caused by the interaction of the ac component

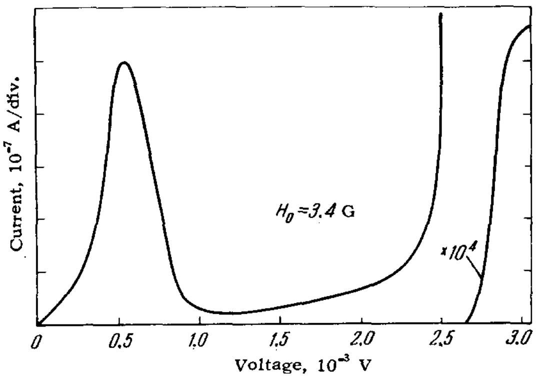

FIG. 11. One of the experimental curves,[41] showing the resonance structure of the dependence of I on V.

of the current with the normal vibrations in the tunneling layer and is analogous to the effects of an external microwave field.[14] To first order in v_0/V the constant part of (36) is

$$j_{\text{const}} = j_1 \frac{v_0}{V} \sin\theta = j_1 \frac{4\pi l j_1}{\varepsilon\omega V_0} \frac{1/Q}{[1-(k\bar{c}/\omega)^2]^2+(1/Q)^2}, \tag{37}$$

i.e., the resulting constant current has a resonance structure (Fig. 11), where the current maximum occurs at $\omega/k = \bar{c}$. Using the values of k, $\bar{c}$ and ω given above, we get the following linear relation between the potential difference V_p, at which there is a current peak, and the applied magnetic field:

$$V_p = \left(\frac{ld}{\varepsilon}\right)^{1/2} H_0. \tag{38}$$

Figure 12a shows a comparison of formula (38) with the experimental data[41] for a tunneling barrier of lead-lead oxide-lead with $\lambda = 400$ Å, $l = 15$ Å, and $\epsilon = 3.8$.

From (37) we can also find the dependence of the peak current on voltage. At resonance the current density is

$$(j_{\text{const}})_p = \frac{j_1 Q \cdot 4\pi l j_1}{\varepsilon\omega V_p}, \tag{39}$$

and, since $\omega = 2eV_p/\hbar$, the right side of (39) varies like $1/V_p^2$. In Fig. 12b we give a comparison of (39) with the experimental data. The best agreement is gotten for Q = 3.5. The simple model used gives a surprisingly good description of the data. A more detailed discussion of these questions can be found in [41].

In the work of Giaever,[15] the presence of radiation accompanying the nonstationary Josephson current was demonstrated in the following elegant way. A special tunneling sample was prepared with the cross section shown schematically in Fig. 13. A layer of tin 1 was deposited on a backing; the surface was subjected to oxidation for a long time, forming a thick oxide layer (shaded region). Then another layer 2 of tin was deposited and subjected to a short-time oxidation, so that the dielectric layer was quite thin. Finally still another layer 3 was deposited, as shown in Fig. 13. The film between 1 and 2 was quite thick, to emphasize the Josephson effect, while one could observe the Josephson current on the film between 2 and 3.

With a voltage V_{12} applied between layers 1 and 2, a volt-ampere characteristic was taken for the usual one-particle tunneling current (curve 1 in Fig. 14). If now a Josephson current is passed through layers 2 and 3 so that there is a voltage V_{23} across the barrier, and we simultaneously take the volt-ampere characteristic between layers 1 and 2, the latter will have the form shown in Fig. 14, curve 2. The current steps on this curve occur at voltage values $V_{12} = (1/e)(2\Delta \pm 2neV_{23})$ where 2Δ is the gap width for tin (in Fig. 14 it corresponds to the start of the rapid rise in the usual one-particle current). The energy $2eV_{23}$ is just equal to the energy $\hbar\omega$ of the photons emitted from the Josephson barrier (2, 3) and absorbed in the tunneling structure (1, 2). Here we have the complete analog of the experiment of Dayem and Martin,[42] who found the same kind of steps in the one-particle current when the barrier was irradiated with external electromagnetic radiation.

Finally, in the experiment of Yanson, Svistunov, and

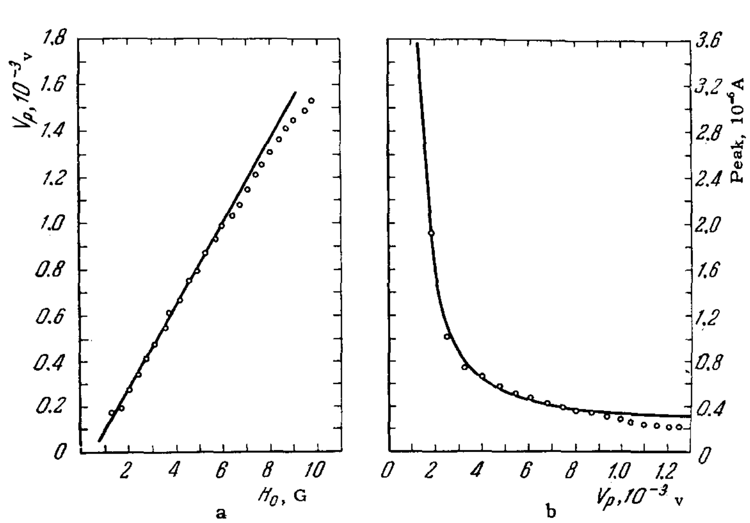

FIG. 12. Experimental data and theoretical curves,[41] giving: a) dependence of peak position V_p on applied magnetic field H^0; b) dependence of peak height on V_p.

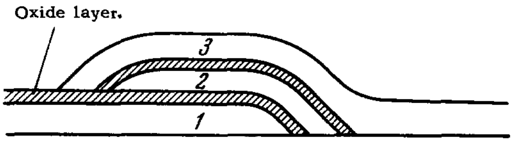

FIG. 13. Schematic diagram of tunneling sample.[15].

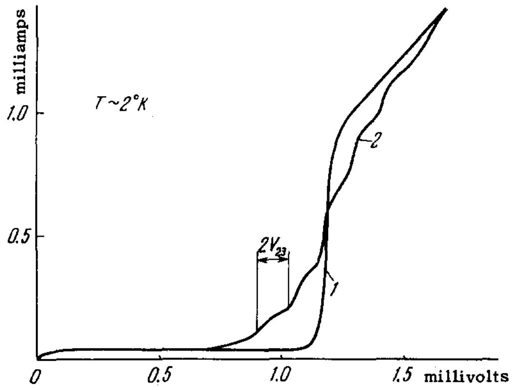

FIG. 14. Volt-ampere characteristic of the tunneling layer shown in Fig. 13. Curve 1—one-particle current I_{12} for $V_{23} = 0$; 2—one-particle current I_{12} for $V_{23} \neq 0$.

Dmitrenko[16] the Josephson tunneling sample was placed in a waveguide; using a sensitive receiver they recorded electromagnetic radiation at 3 cm with an output power of 10^{-14} watts, accompanying the ac Josephson current. In the work of Dmitrenko and Yanson,[56] an order of magnitude greater power was obtained. This paper contains interesting data about the width and shape of the Josephson line, by means of which one may possibly be able to study the energy distribution of the superconducting electron pairs. The investigation of the structure of the Josephson radiation is continuing.

Thus the occurrence of a nonstationary Josephson effect should be considered as definitely established. In principle one could try on the basis of this effect to construct variable-frequency microwave generators, but the power so far obtained is insufficient for practical purposes.

In conclusion we give some additional references to papers where the Josephson effect is treated within the framework of the Ginzburg-Landau theory (cf. [43,44]), where one uses the quasispin approach,[45] and where one considers the influence of paramagnetic impurities in the superconducting sample,[46] etc. A discussion of possibilities of practical applications of the tunneling effects can be found in the survey [47]. We also mention a number of papers that are not yet published.[48-50]

We can be sure that the number of papers devoted to this interesting effect will continue to grow.

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