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From the Current Literature

CHARGE TRANSPORT MECHANISM IN LIQUID HELIUM

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THE nature of the charge carriers in liquid helium has long remained unclear. At first, one might think that owing to the low density and the small polarizability of the liquid helium the electrons in the helium should move like free electrons, and in particular, their effective mass should be of the order of the mass of the free electron, while their velocity of thermal motion $\sim \sqrt{kT/m}$ should exceed the velocity of sound in liquid helium. One might also think that the transport of positive charge is realized not by the drift of He^+ ions but by the jump of a hole from one atom to another. If the probability of such a jump is sufficiently high, then the effective mass of the hole can be of the order of the mass of the free electron. The calculations of the carrier mobility made within the framework of this model[1] turned out to disagree strongly with the experimental data[2-8], which indicated beyond any doubt that the charge transport is effected by macroscopic formations whose effective mass exceeds by many times not only the mass of the free electron but also the mass of the helium atom. A hypothesis was advanced[2,7,8] that the charges ride on some macroscopic inclusions in the liquid helium, but measurements of the charge mobility, carried out by Meyer and Reif[3,4] by the velocity-selector method, have disclosed incidentally a very high homogeneity of the composition of the carriers. The selector recorded up to ten maxima.

A satisfactory explanation of the structure of the positive carriers was proposed in 1959 by Atkins[9]. According to his model, the hole is fixed sufficiently well in the He^+ ion or in molecular ions of the He_2^+ type, the existence of which has been reliably established.

The electric field $\mathcal{E} = e/r$ of a point charge polarizes the liquid surrounding the charge. From the condition that the chemical potential is constant

$$d\mu = \frac{V}{N}\,dp - \boldsymbol{\mathcal{E}}\,d\mathbf{P} = 0,$$

where $\mathbf{P} = N\alpha\boldsymbol{\mathcal{E}}$ is the polarization and $N\alpha/V_0 = 4.55 \times 10^{-3}$ is the polarizability of the liquid helium, we find that the pressure p increases in the direction towards the charge like $p = N\alpha e^2/V_0 r^4$, and that at each point of the liquid it produces an excess density

$$\delta\varrho_e = \frac{\partial\varrho}{\partial p}\,p = \frac{p}{c^2}.$$

At a pressure p_m = 25 atm. corresponding to $r_m = 7$ Å, the liquid helium should solidify. The effective mass of the positive carrier consists of the mass of the solid nucleus

$$M_s = \frac{4\pi r_m^3}{3}\,\varrho_s = 32 M_{He^4},$$

the mass due to the excess density of the liquid,

$$\delta M_e = 4\pi \int_{r_m}^{\infty} \delta\varrho_l r^2\,dr = 28 M_{He^4}$$

and the attached mass

$$M_e^* = \frac{2\pi}{3}\,r_m^3 \varrho_l = 15 M_{He^4}.$$

The total effective mass is 75 M_{He^4}.

The physical basis for the Atkins model is the small value of the pressure at which helium solidifies, such that even weak polarization effects make possible the formation of a solid core with dimensions of the order of ten atomic radii. This model is not suitable for negative carriers, since the interaction between the electron and the atoms at the distances of interest to us is strongly repulsive. There apparently are no formations of the type of ions He^-, He_2^-, etc., which could localize the electron in a small region with dimensions of the order r_m; no solid core can then be produced.

From the experimental point of view, the question of the nature of the carriers was investigated by Careri, Fasoli, and Gaeta[6], who analyzed the data on the flow of the current through the liquid-gas boundary in helium and concluded that the structures of the negative and positive carriers differ radically. They propose that in liquid helium the electron is located in the center of a certain spherical cavity of macroscopic dimensions. A similar model was previously proposed by Ferrel[10] to explain the anomalously large lifetime of the positronium in liquid helium.

Recent papers are devoted to the determination of the repulsion potential which prevents the electron from penetrating into liquid helium. Sommer[11], observing the entry of gas-discharge electrons into liquid helium, estimated the potential barrier is approximately 1.3 eV. Boyce[12] calculated U_0 —the lower energy of the excess-electron band in the helium crystal—using a Fermi pseudopotential (in the form of a sum of δ-functions) obtained from data on the scattering of electrons by helium atoms. Inasmuch as for different lattice types the value of U_0 depends only on the average density of the atoms, Boyce assumes for the liquid a value $U_0 = 1.4 \pm 0.3$ eV, which he obtains as a result of calculations for crystals.

Thus, the penetration of an electron into liquid helium is energetically unfavored; a lower energy should be possessed by the already mentioned system comprising an electron in a spherical cavity (potential well) whose walls represent for the electron a potential barrier of height U_0. At the lowest values of the well radius (we assume the well to be square) the lowest energy of the electron is U_0. When the radius is increased to $a_{cr} = \sqrt{\pi h^2/8mU_0}$ a level appears whose energy begins to decrease rapidly with further increase of the radius, increasing asymptotically like $\pi^2 h^2/2ma^2$, where m is the electron mass. The dashed curve in Fig. 1 represents the energy of the lower state of the electron as a function of the cavity volume $V = 4\pi a^3/3$.

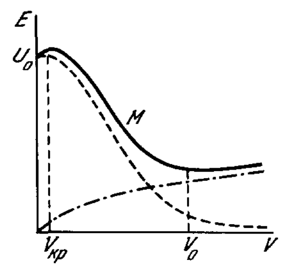

FIG. 1.

The total energy of the system includes also the energy lost to formation of the separation surface $4\pi\sigma a^2$, where $\sigma = 0.145$ dyne/cm^2 is the surface tension of helium. The corresponding plot in Fig. 1 is shown by the dash-dot curve. The total-energy curve (solid line) has a minimum which determines the dimension $a_0 = 17.4$ Å and the energy $E = 0.12$ eV of the system.*

The effective mass of the bubble is obviously simply equal to the "attached" mass of liquid:

$$M^* = \frac{2\pi}{3} \varrho_l a_0^3 = 245 M_{He^4}.$$

It is easy to calculate the carrier mobility by using the described models of positive and negative carriers.

At high temperatures ($T \sim T_\lambda$) it is necessary to use the Stokes formula for the force of resistance to the flow of a viscous liquid around a solid sphere

$$F = 6\pi\eta a_+ v \tag{1}$$

(positive charges) and the Rybchinskii-Hadamard formula for flow around an empty cavity

$$F = 4\pi\eta a_- v \tag{2}$$

(negative charges). Here $\eta = 2 \times 10^{-5}$ is the viscosity of the liquid helium.

At low temperatures the mean free path of the excitations (rotons and photons) is much larger than the dimension of the carrier (the so-called Knudsen case). The force exerted on the sphere by a gas moving with velocity v and excited in the rest system of the sphere can be obtained by calculating the momentum transferred by the excitations to the sphere per unit time.

The flux incident on a surface element of the sphere

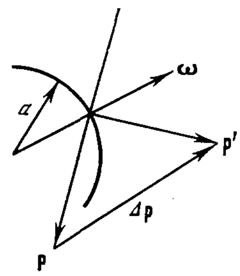

FIG. 2.

(Fig. 2) $a^2 d\omega$ (ω is a unit vector in the direction normal to the surface) is

$$\left(\boldsymbol{\omega} \frac{\partial \varepsilon}{\partial \mathbf{p}}\right) n\, d^3p.$$

Here n is the distribution function of the excitations with respect to the momenta, normalized such that $\int n d^3p = N$ is the total number of excitations per unit volume. In the case of an elastic collision, the momentum transferred is

$$\Delta \mathbf{p} = 2\boldsymbol{\omega}(\boldsymbol{\omega}\mathbf{p}).$$

Thus, the unknown force is equal to

$$\mathbf{F} = \int\int a^2 d\boldsymbol{\omega} \left(\boldsymbol{\omega} \frac{\partial \varepsilon}{\partial \mathbf{p}}\right) \cdot 2\boldsymbol{\omega}(\boldsymbol{\omega}\mathbf{p})\, n\, d^3p.$$

For each specified ω we should, as seen from Fig. 2, integrate with respect to d^3p only over the hemisphere from which the particles can reach the specified surface element. It is convenient to extend the limits of integration to include the entire sphere

$$\mathbf{F} = a^2 \int\int d\boldsymbol{\omega} \left| \boldsymbol{\omega} \frac{\partial \varepsilon}{\partial \mathbf{p}} \right| \boldsymbol{\omega}(\boldsymbol{\omega}\mathbf{p})\, n\, d^3p,$$

to integrate first with respect to $d\omega$, and separate the term linear in the velocity in the expansion of $n(\epsilon - \mathbf{p}\cdot\mathbf{v})$ with respect to $\mathbf{v}$:

$$\mathbf{F} = \frac{a^2}{4} 4\pi \int \frac{d\varepsilon}{\partial p} \frac{\partial n}{\partial \varepsilon} \mathbf{p}(\mathbf{p}\mathbf{v})\, d^3p.$$

Integrating once by parts, we obtain

$$\mathbf{F} = e\mathscr{E} = \frac{4\pi a^2}{3} \mathbf{v} \int pn\, d^3p$$

($\mathscr{E}$ is the external electric field). The integration must be carried out with respect to the phonon and roton parts of the energy spectrum in the helium, yielding

$$e\mathscr{E} = \mathbf{v}\, \frac{4\pi a_\pm^2}{3} \left(\frac{\mathscr{E}_{ph}}{c} + p_0 N_p\right), \tag{3$^\pm$}$$

where $p_0 = 1.72 \times 10^{-19}$ is the "momentum" of the roton, N_p is the number of rotons per unit volume,

*These values were obtained by numerical calculation for a square well of depth $U_0 = 1.3$ eV. We can use with good accuracy the asymptotic expression for the energy of the lower level which yields a value $a_0 = 19.2$ Å, independently of U_0.

and $\mathscr{E}_{ph}$ is the energy density of the phonon. The inelasticity of the collisions can be taken into account by introducing an accommodation coefficient.

Thus, the carrier mobility in helium, $\mu = v/\mathscr{E}$, can be calculated in the entire temperature range by using formulas (1)—(3). It must merely be borne in mind that the values of the carrier radii are meaningful only with accuracy to ±2 Å (the "dimension" of the roton), i.e., to 10—20%.

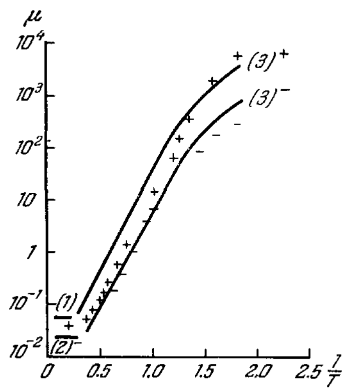

FIG. 3

Figure 3 shows the results of calculations by formulas (1), (2), $(3)^+$, and $(3)^-$ and the experimental points taken from the papers of Meyer and Reif[3,4]. The data of Careri, Scarmuzzi, and Thomson[5] agree with those of Meier and Reif. We see that the agreement with experiment is quite satisfactory. One can hardly expect better accuracy, since the objects under consideration are too small to be regarded as wholly macroscopic. This pertains in particular to the positive charges, whose radius is only 7 Å. (We recall that the distance between neighboring He atoms is 3.5 Å.)

A unique phenomenon is the motion of the free charges when the decelerating action of the thermal excitations is weak—at low temperatures or in strong electric fields. In this case, so long as the carrier energy is small, the velocity increases with increasing energy, in accordance with the equation

$$E = \frac{Mv^2}{2},$$

which connects the velocity and energy of the particle mass M. After the particle reaches a certain critical energy, an annular vortex of unit circulation is excited, and subsequently the energy acquired by the charge in the electric field goes to increase the dimensions of this vortex, which the particle drags with it[13].

The energy and velocity of the vortex are expressed in terms of its radius r by means of the formulas[13]

$$E = \frac{\rho h^2}{2M_{He^4}^2} r \left(\eta - \frac{7}{4}\right), \quad v = \frac{h}{4\pi M_{He^4} r}\left(\eta - \frac{1}{4}\right);$$

here $\eta = \ln 8r/a$, $a = 1.2$ Å.

It is possible to obtain the connection between the vortex energy and the velocity by eliminating from (4) the radius of the vortex:

$$Ev \approx \text{const}.$$

Thus, with further increase of the charge energy, its velocity decreases (Fig. 4). It is easy to obtain the critical value of the velocity and the corresponding vortex radius

$$v_{cr} \approx \frac{h}{M_{cr} a}\left(\frac{\eta^2}{4\pi}\right)^{1/3}, \qquad r_{cr} \approx a\left(\frac{\eta}{4\pi}\right)^{1/3}.$$

As expected, the radius of the vortex at the instant of its occurrence turns out to be of the order of the radius of the carrier. For negative charges $v_{cr} \approx 11$ m/sec and for positive ones $v_{cr} \approx 28$ m/sec.

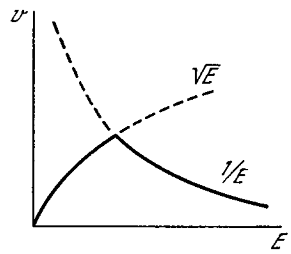

FIG. 4.

The negative carrier dimensions should depend strongly on the pressure. This dependence is determined by the modulus of hydrostatic compression $-V\frac{\partial p}{\partial V}$, shown by calculations to be equal to 9.3 atm. At sufficiently high pressures, corresponding to the inflection point M on Fig. 1, the bubble can "collapse"; this effect corresponds to $p \sim 20$ atm and $a \approx 3$ Å. This effect would be observable only at high temperatures, when the pressure cannot cause solidification of the liquid helium. Actually, a transition to the structure of the type proposed by Atkins for positive charges should occur already upon compression to $a \sim 7$ Å.

The dependence of the dimensions and of the characteristics of the Atkins structure on the pressure is much weaker; it is determined by the modulus of hydrostatic compression of the helium $K = \rho c^2 \sim 10$ atm.

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