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# ASYMPTOTIC RELATIONS BETWEEN SCATTERING AMPLITUDES IN LOCAL FIELD THEORY

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## INTRODUCTION

MOST investigations of the asymptotic properties of matrix elements at high energies either start out from semi-phenomenological assumptions or are based on certain hypotheses of particular character. Thus, I. Ya. Pomeranchuk et al.[1] start from the premise (based on analysis of experimental data) that the differential cross sections of scattering processes with charge exchange vanish at high energies. Some advance the hypothesis that elastic processes are diffractive at high energies (see, for example,[2]). Others[3,4] start from the premise that new symmetry properties exist in strong interactions and should become manifest only at energies much higher than the particle mass. Finally, mention must be made of studies in which the starting point is the hypothesis that the asymptotic behavior of the scattering amplitude is determined at high energies by a single Regge pole[5-7]. The significance of each of these investigations depends on the results of the experimental verification. An experimental confirmation of the predictions contained in papers of this kind would be most valuable, since it would signify a discovery of a new law in strong interactions. On the other hand, an experimental refutation of these results would signify only the incorrectness of the special hypothesis or of the theoretical speculations, and would not affect the main principles of quantum theory. Since it is the latter tendency, the tendency to refutation (especially as regards papers on the "single pole Reggistics"), that has been predominant lately, interest has naturally increased in statements that are derivable only from the general principles of local field theory. The derivation of rigorous asymptotic relations between the scattering amplitudes, and consequently relations between the cross sections, polarizations, etc., at high energies, is the subject of most of the present paper. We pay particular attention to a discussion of hypotheses on the basis of which the derived relations can be proved.

We recall that basic among the principles of relativistic local quantum field theory are the following[8,9]:

1. Invariance relative to the inhomogeneous Lorentz group.

2. The microcausality principle, which in the form given by N. N. Bogolyubov states that:

$$\frac{\delta}{\delta\varphi_1(x)}\left[\frac{\delta S}{\delta\varphi_2(y)}S^+\right]=0 \qquad \text{for} \quad x \leqslant y,$$

with $\varphi_1(x)$ and $\varphi_2(x)$ standing either for one and the same field or for different fields.

3. The spectrality condition, according to which there exists a complete system of physical states with positive energy.

4. The unitarity condition

$$SS^+=1.$$

If we put $S = 1 + iR$, then this condition can be rewritten on the basis of postulate 3 in the form

$$\frac{1}{i}\langle f\,|R-R^+|\,i\rangle=\sum_n\langle f\,|R\,|\,n\rangle\langle n\,|R^+\,|\,i\rangle,$$

where $\sum_n$ denotes summation over the complete system of intermediate states.

Besides these principles there is also a certain requirement of mathematical nature.

5. It is required that the elements of the scattering matrix be generalized functions with moderate growth (that is, generalized functions on the class S)[8-11]. This condition causes the Fourier transforms of the retarded amplitudes to be polynomially bounded functions. Since this requirement is essential in the study of asymptotic properties of the scattering amplitude, we shall discuss in Sec. 1 the connection between this assumption and the micro-causality and unitarity principles.

If we add to the general principles of local field theory the assumption that the scattering amplitudes do not oscillate but have a definite (power-law or logarithmic) growth when the energy tends to infinity at a fixed momentum transfer, then we can obtain several experimentally verifiable relations between the amplitudes of the different processes. The first relation of this kind—the equality of the total particle and antiparticle interaction cross sections at high energies—was obtained by I. Ya. Pomeranchuk[12]. Different generalizations and refinements of the Pomeranchuk theorem were given in[13-18]. To prove the Pomeranchuk theorem, Sugawara and Kanazawa[16] rediscovered and proved again the following theorem: If the function f(z) is analytic in the upper half-plane and grows at infinity not faster than some power $z^n$, then it cannot tend to different limits along the positive and negative semi-axes. Meĭman[18] called attention to the fact that this statement is the well-known classical Phragmen-Lindelof theorem in the theory of analytic functions[19]. On this basis Sugawara et al.[16,17] and Meĭman[18] proved the Pomeranchuk theorem under rather general conditions.

Several asymptotic relations are established in[20-23] on the basis of the Phragmen-Lindelof theorem not only between the total but between the differential cross sections of different processes, and also between different polarization effects. The present article is devoted principally to a systematic exposition of the results of these papers and also some of the results are published here for the first time. Besides these results, we present a brief review of the work of Greenberg and Low[24], Froissart[25], Martin et al.[26], Meĭman[18], and Nambu and Sugawara[17] on related problems, our work[27] on asymptotic relations in the process of higher symmetries in strong interactions, and a paper[28] in which processes with particle production are considered. The asymptotic equality of the differential scattering cross sections of a particle and its antiparticle was also proved by Van Hove[29].

## 1. UPPER ESTIMATES FOR THE GROWTH OF THE CROSS SECTION AT HIGH ENERGIES

### 1. Limitations Imposed by the Microcausality Conditions on the Growth of the Amplitude

As indicated in the introduction, in the existing local field theory it is assumed that the elements of the scattering matrix are generalized functions with moderate growth. It follows therefore that the scattering amplitude is polynomially bounded in momentum space, including also for complex momenta in the region of analyticity of the amplitude. The postulated moderate growth is essential for the derivation of the usual dispersion relations with a finite number of subtractions. This raises the natural questions: To what degree is this requirement independent of the remaining postulates of local theory, primarily the microcausality postulate? What properties must be possessed by an amplitude satisfying all the postulates of relativistic quantum theory with the exception of the moderate-growth requirement? These questions are of interest from the point of view of investigations of the renormalized theory (see, for example,[30]). The behavior of the remaining terms of the perturbation theory in nonrenormalized theory shows that the amplitude is either nonanalytic with respect to the coupling constant, or increases more rapidly than any polynomial when the energy tends to infinity, at least on some sequence of points in the complex E plane.

It is easy to note that the microcausality condition is not compatible with an arbitrary growth of the amplitude in the plane of the energy E[31]. This circumstance is simplest to illustrate with a one-dimensional model in which the amplitude depends only on the time (energy) and the dependence on the spatial coordinates (spatial components of the momentum) is omitted. In this model the microcausality condition takes the form

$$F(t) = \int_{-\infty}^{\infty} e^{-iEt} T(E)\, dE = 0 \qquad \text{for } t < 0. \qquad (1.1)$$

Condition (1.1) is equivalent to the condition of analyticity and polynomial boundedness of the amplitude T(E) in the upper half of the energy plane (that is, for Im E > 0), if it is assumed that F(t) is a generalized function of moderate growth (that is, $F \in S^*$, where S is the space of rapidly decreasing infinitely differentiable Schwartz functions)[10]. If we retain the assumption that the amplitude T(E) is analytic in the upper half-plane but assume that in some direction in the upper half of the E plane the function T(E) increases faster than some exponential function (even if it still remains bounded on the real axis), then the microcausality condition (1.1) becomes violated. This can be verified using as an example the functions $\exp(-iaE)$ or $\exp(-a^2E^2)$. In[32] there were advanced certain arguments of more general character in favor of the statement that the Fourier transform of the causal amplitude should increase more slowly than any exponential function in the complex energy plane

$$|T(s,t)| \leqslant A e^{\epsilon |s|} \qquad \text{for } \frac{\operatorname{Im} s}{|s|} \geqslant \delta > 0, \qquad (1.2)$$

where $\epsilon$ is an arbitrarily small positive number. In this paper we employ inequality (1.2) as a constituent part of the microcausality postulates.

It will be shown in the next subsection that by making certain supplementary (sufficiently natural) assumptions and by using the unitarity condition we obtain from (1.2) polynomial boundedness of the amplitude. This points therefore to the place of the possible microcausal theory.

### 2. Condition of Polynomial Boundedness of the Causal Elastic-scattering Amplitude

Let us assume that a cross section of a certain process is polynomially bounded in energy in the physical region (in fact, the observed differential cross sections do not even increase at all with energy). Then the amplitude T(s, t) is polynomially bounded in the entire complex s plane.

Indeed, if a function $T(s, t)/(s + i)^n$ which is analytic in the upper half-plane is bounded on the real axis, then according to the Phragmen-Lindelof theorem it is either bounded in the entire upper half of the s-plane or increases more rapidly than some exponential on some sequence of points that tends to infinity. The second possibility, however, drops out since it contradicts the condition (1.2). We have thus proved the polynomially boundedness of the amplitude T(s, t) in the upper half of the s-plane. The polynomial boundedness of the function T(s, t) in the lower half plane follows from this if we take into consideration the condition that the amplitude be real

$$T(s^*, t) = T(s, t)^*.$$

We shall now show that the assumed polynomial boundedness of the differential cross sections in the physical region can be obtained from a weaker and quite natural hypothesis. To this end it is necessary to employ the unitary condition of the scattering amplitude and the analytic properties in the momentum transfer t (or in the cosine of the scattering angles $z = \cos\theta$). We recall that Lehmann[35] proved on the basis of the general principles of the local field theory, and with the aid of the Jost-Lehmann-Dyson integral representation,[33,34] that the $\pi$N-scattering amplitude regarded as a function of the momentum transfer at a fixed energy is analytic in some ellipse (the Lehmann ellipse). The Jost-Lehmann-Dyson interval representation was obtained by using the assumption that the scattering amplitude is polynomially bounded. Subsequently, however, in[36,37], the analyticity of the scattering amplitudes in the momentum transfer was proved without any assumption concerning the degree of growth.

Let us assume that when t runs over the Lehmann ellipse and $s \to +\infty$ (along the real axis), we have

$$|T(s, t)| < Be^{as^N} \tag{1.3}$$

for a certain choice of the positive numbers B, a, and N. The arguments presented to justify the inequality (1.2) make quite probable the assumption that the Fourier transform of the retarded amplitude satisfies for complex t in the Lehmann ellipse the even stronger inequality (1.2). Therefore the condition (1.3), if it can be regarded at all as a hypothesis, imposes rather weak limitations on the scattering amplitude. We shall show that this hypothesis and the unitarity condition lead to the polynomial boundedness of the amplitude on the real s axis for physical t. From this, by virtue of the Phragmen-Lindelof theorem and inequality (1.2), follows the polynomial boundedness of the amplitudes for all complex s.

We shall follow the reasoning of Greenberg and Low [24]. The small Lehmann ellipse in which the elastic-scattering amplitude of a particle of mass m by a particle of mass M is analytic (for example, $\pi$N scattering) has for $s > (M+m)^2$ a center at the point $t_0 = -2k^2$ where k is the c.m.s. momentum.

$$k^2 = \frac{[s-(M+m)^2][s-(M-m)^2]}{4s}, \tag{1.4}$$

and has semi-axes $x_0$ and $\sqrt{x_0 + 4k^2}$, where

$$x_0 = 2k\left[k^2 + \frac{(m_1^2-m^2)(m_2^2-M^2)}{s-(m_1-m_2)^2}\right]^{1/2}. \tag{1.5}$$

Here $m_1$ and $m_2$ are the masses of the lowest many-particle states with quantum numbers of particles with masses m and M, respectively, (in the case of $\pi$N scattering, $m_1 = 3m$ and $m_2 = M + m$). We employ the Cauchy theorem for some smaller ellipse D with a minor semi-axis

$$c = k\left[\frac{(m_1^2-m^2)(m_2^2-M^2)}{s-(m_1-m_2)^2}\right]^{1/2} \tag{1.6}$$

and a major semi-axis $\sqrt{c^2+4k^2}$ (in order not to assume continuity of amplitude on the boundary of the Lehmann ellipse). Using the formula

$$\frac{1}{z'-z} = \sum_{l=0}^{\infty}(2l+1)P_l(z)Q_l(z'),$$

we obtain

$$T(s, t) = \frac{1}{\pi^2}\frac{s}{k^2}\sum_{l=0}^{\infty}(2l+1)a_l(s)P_l\left(1+\frac{t}{2k^2}\right), \tag{1.7}$$

where

$$a_l(s) = \frac{1}{2\pi^3 i}\oint_D T(s, t')Q_l\left(1+\frac{t'}{2k^2}\right)dt'. \tag{1.8}$$

By virtue of (1.8) and assumption (1.3) we have

$$|a_l(s)| \leqslant \frac{1}{\pi^{5/2}}\frac{A}{\sqrt{l}}\frac{e^{as^N}}{[c+\sqrt{1+c^2}]^l}, \tag{1.9}$$

where c is given by (1.6). On the other hand, from the unitarity condition and from the fact that the metric is positive definite in the state-vector space it follows that

$$|a_l(s)| \leqslant 1. \tag{1.10}$$

Fixing s, we choose $l_0$ as the largest integer which does not exceed

$$\frac{as^N}{\ln[c+\sqrt{1+c^2}]}.$$

For the first $l_0$ terms in the sum (1.7) we make use of the estimate (1.10) and for the remainder of inequality (1.9). As a result we obtain

$$|T(s, 0)| \leqslant \text{const}\cdot s^{N+2} \tag{1.11}$$

and an analogous condition for $t < 0$. This proves the polynomial boundedness of the amplitude.

The results can be summarized as follows. If inequality (1.3) holds in the microcausal relativistic theory when t belongs to the Lehmann ellipse, then the scattering amplitude is polynomially bounded in the complex plane of the energy s (as $s \to \infty$).

## 3. Estimates of the Growth of the Cross Sections

We now assume that the scattering amplitude is polynomially bounded for all t in the Lehmann ellipse, that is, we can represent the exponential exp $(as^N)$ in the right side of inequality (1.3) by the power $s^N$. We then obtain in lieu of (1.11) the estimates of Greenberg and Low[24]:

$$|T(s, 0)| \leqslant \text{const}\cdot s^2(\ln s)^2,$$
$$|T(s, t)| \leqslant \text{const}\cdot\frac{s^{7/4}(\ln s)^{3/2}}{|t|^{1/4}}, \quad t < 0, \tag{1.12}$$

and consequently,

$$\sigma_{tot}(s) \leqslant \text{const}\cdot s(\ln s)^2,$$
$$\frac{d\sigma(s, t)}{dt} \leqslant \text{const}\cdot\frac{s^{3/2}(\ln s)^3}{|t|^{1/2}}, \quad t < 0. \tag{1.13}$$

If we assume, for example, that the semi-axes of the ellipse in the t plane in which the amplitude is analytic, do not change with increasing energy (this takes place in the case of the Mandelstam representation), then we obtain in (1.12) and (1.13) the results of Froissart[25]

$$|T(s, 0)| \leqslant \text{const} \cdot s(\ln s)^2,$$
$$|T(s, t)| \leqslant \text{const} \cdot \frac{s(\ln s)^{3/2}}{|t|^{1/4}}, \qquad t<0, \tag{1.14}$$

and consequently

$$\sigma_{tot}(s) \leqslant \text{const} \cdot (\ln s)^2,$$
$$\frac{d\sigma(s, t)}{dt} \leqslant \text{const} \cdot \frac{(\ln s)^3}{|t|^{1/2}}, \qquad t<0. \tag{1.15}$$

Under the same assumptions, Kinoshita, Loeffel, and Martin[26] obtained a different estimate for the limitation of the growth of the amplitude for nonzero momentum transfer:

$$|T(s, t)| \leqslant \text{const} \cdot \frac{s(\ln s)^{3/2}}{|t|}, \qquad t<0, \tag{1.16}$$

and

$$\frac{d\sigma(s, t)}{dt} \leqslant \text{const} \cdot \frac{(\ln s)^3}{t^2}, \qquad t<0. \tag{1.17}$$

If we consider the asymptotic behavior of the amplitude for a fixed value of the angle (and not for a fixed momentum transfer), then the estimate (1.16) is stronger than (1.14).

## 2. ASYMPTOTIC PROPERTIES OF THE SCATTERING AMPLITUDE OF SCALAR PARTICLES

### 1. The Phragmen-Lindelof Theorem and the Asymptotic Equality of Differential Cross Sections

We consider crossing scattering processes of scalar particles

$$a_1 + b_1 \rightarrow a_2 + b_2 \tag{I}$$

and

$$\bar{a}_2 + b_1 \rightarrow \bar{a}_1 + b_2. \tag{II}$$

Let $q_1$ and $p_1$ be the momenta of the particles $a_i$ and $b_i$ in the initial state of each process, and $q_2$ and $p_2$ the same quantities in the final state. The masses of the particles $a_i$ and $b_i$ are denoted by $m_i$ and $M_i$ respectively. For the first process $q_i^2 = -m_i^2$ and $p_i^2 = -M_i^2$ and for the second, $-q_1^2 = -m_2^2$, $q_2^2 = -m_1^2$, and $p_i^2 = -M_i^2$. We put $s = (p_1 + q_1)^2$, $u = -(p_1 - q_2)^2$, $t = -(p_1 - p_2)^2$. The matrix elements of processes (I) and (II) are of the form

$$\langle f | R | i \rangle = (2\pi)^4 \delta^4(p_1 + q_1 - p_2 - q_2) \frac{1}{\sqrt{16 p_1^0 q_1^0 p_2^0 q_2^0}} T^J(s, t), \tag{2.1}$$

and the differential cross sections are equal to

$$\frac{d\sigma^J(s, t)}{dt} = \frac{1}{64\pi s k_J^2} |T^J(s, t)|^2, \tag{2.2}$$

where J = I, II denote one of the reactions under consideration and $k_J$ is the value of the three-dimensional momentum in the initial state of the process J in the center-of-mass system. The amplitudes of the processes (I) and (II) for real s and t are connected by the crossing symmetry relation

$$T^I(u, t) = T^{II}(s, t)^*, \qquad s + t + u = M_1^2 + M_2^2 + m_1^2 + m_2^2 \tag{2.3}$$

(the asterisk denotes complex conjugation). Relation (2.3) is obtained from the equality

$$T^I(u, t) = T^{II}(s, t), \tag{2.4}$$

which is valid for s and u from the analyticity region of the functions $T^J$ by taking the limit.

We assume that the masses $m_i$ and $M_i$ and the interactions of particles $a_i$ and $b_i$ are such that the principles of local theory lead to analyticity of the amplitude $T^J(s, t)$ for fixed t in the complex s-plane with cuts along the real axis. In addition to the cuts, $T^J$ has as a rule a finite number of poles on the real s axis. In investigating the asymptotic behavior of the amplitude as $s \rightarrow \infty$ it is convenient to subtract first from the amplitude the pole terms, which have a known asymptotic behavior, like 1/s, and investigate the asymptotic behavior of the function bounded at finite points of the real axis. We shall henceforth use $T^J$ to denote the amplitude after subtraction of the pole terms.

To encompass the class of amplitudes with sufficiently general asymptotic behavior, we introduce one other auxiliary concept. We call a function $\varphi(s, t)$ admissible, if for fixed t (from a certain interval) the function $1/\varphi(s, t)$ is analytic and is smaller than any exponential $\exp(\epsilon |s|)$, $\epsilon > 0$, when $s \rightarrow \infty$ in the upper half plane, continuous on the real axis, and in addition if

$$\lim_{s\rightarrow\infty} \frac{\varphi(s, t)}{\varphi(-s, t)} = e^{-i\pi\alpha(t)}, \tag{2.5}$$

where $\alpha(t)$ is an arbitrary real function. An example of an admissible function is

$$\varphi(s, t) = (s+i)^{\alpha(t)} [\ln(s+i)]^{\beta(t)} [\ln\ln(s+i)]^{\gamma(t)},$$

where $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are real.

The following theorem holds true:

Theorem I. Assume that for a certain admissible function there exist finite limits

$$V^I(t) = \lim_{s\rightarrow\infty} \frac{T^I(s, t)}{\varphi(s, t)}, \qquad V^{II}(t) = \lim_{s\rightarrow\infty} \frac{T^{II}(s, t)^*}{\varphi(-s, t)}. \tag{2.6}$$

Then in the local theory these limits are equal to each other:

$$V^I(t) = V^{II}(t). \tag{2.7}$$

From this follows also the asymptotic equality of the differential cross sections of the processes (I) and (II):

$$\lim_{s\rightarrow\infty} \frac{d\sigma^I(s, t)/dt}{d\sigma^{II}(s, t)/dt} = 1, \quad \text{or} \quad \frac{d\sigma^I(s, t)}{dt} \sim \frac{d\sigma^{II}(s, t)}{dt}. \tag{2.8}$$

Proof. By a virtue of the assumptions made relative to the amplitude $T^J(s, t)$, the function

$$V(s, t) = \frac{T^I(s, t)}{\varphi(s, t)} \tag{2.9}$$

is analytic, does not exceed $e^{\epsilon|s|}$ in the upper half-plane of s, and is bounded on the real axis. In addition, it follows from (2.3) and (2.6) that

$$\lim_{s\to+\infty} V(s, t) = V^I(t), \qquad \lim_{s\to-\infty} V(s, t) = V^{II}(t). \tag{2.10}$$

Therefore we can apply the following Phragmen-Lindelof theorem[19] to the function V(s, t).

Theorem II. Let f(z) be an analytic function of $t = re^{i\theta}$ regular in the domain D enclosed between two straight lines $L_1$ and $L_2$ forming an angle $\pi/\tau$ with the vertex at the origin, and bounded on these lines ($|f(z)| \leq C$ along $L_1$ and $L_2$). Then the following alternatives exist: either $|f(z)| \leq C$ at all points of the domain D, or else there exists a sequence $z_n \to \infty$ such that

$$\max_{|z_n|=r_n} |f(z_n)| \geqslant e^{\nu r_n^\tau}, \qquad \nu > 0. \tag{2.11}$$

On the other hand, if the function f(z) is smaller in absolute value than any exponential $\exp(\epsilon r^\tau)$ in the angle D, then the first possibility should be realized, that is, f(z) is bounded by a constant in the entire domain D.

Let the function w = f(z) be regular and bounded in the angle D. We denote by $E_i$ (i = 1, 2) the set of limiting values of w as $z \to \infty$ along the line $L_i$. Then either the sets $E_1$ and $E_2$ have a common point, or else one surrounds the other and separates it from the circle $|w| = C$. In particular, if these exist finite limits $a^I$ and $a^{II}$ when $z \to \infty$ along $L_1$ and $L_2$, then $a^I = a^{II} = a$ so that $f(z) \to a$ as $z \to \infty$ uniformly in D.

The function V(s, t) satisfies all the conditions of the theorem II (in our case D is the upper half-plane, $\tau = 1$). Since this function is polynomially bounded, its limiting values as $s \to \pm\infty$ should coincide. This proves (2.7).

If the discarded pole terms decrease as $s \to \infty$ more rapidly than the function $T^J(s, t)$ itself, then formula (2.1) remains valid as $s \to \infty$ for that part of the amplitude, and we obtain the asymptotic equality (2.8) of the differential cross sections. On the other hand, if the amplitude behaves at infinity like 1/s, then a direct account of the pole terms shows that the equality (2.8) remains in force in this case too. This proves theorem I.

## 2. Case of Elastic Scattering. Equality of the Total Cross Sections

In the particular case of elastic scattering (in this case $m_1 = m_2 = m$, and $M_1 = M_2 = M$) the corollary of theorem I regarding the asymptotic equality of the differential cross sections can be obtained under less stringent requirements. We assume that there exist only the limits

$$\lim_{s\to\infty} \frac{64\pi s k^2}{|\varphi(s, t)|^2} \frac{d\sigma^J(s, t)}{dt} = [a^J(t)]^2, \qquad J = \text{I, II}, \tag{2.12}$$

and that the imaginary part of the ratio $T^J(s, t)/\varphi(s, t)$ is not negative at fixed t and $s \to \infty$. This second assumption is natural since in the expansion of the imaginary part of V(s, t) in Legendre polynomials

$$\text{Im}\, V(s, t) = 4\pi \frac{\sqrt{s}}{k} \sum_{l=0}^{\infty} (2l+1) \frac{\text{Im}\, a_l(s)}{\varphi(s, t)} P_l(z) \tag{2.13}$$

all the coefficients $\text{Im}\, a_l(s)/\varphi(s, t)$ are non-negative by virtue of the unitarity condition and the condition that the metric is positive definite in the space of the state vectors. In addition, $z \to 1$ when $s \to \infty$ and P(1) = 1, so that each term of the series (2.13) becomes non-negative for sufficiently low s. It does not follow from this, however, that Im V(s, t) becomes non-negative at high energies, since the series (2.13) converges non-uniformly with respect to s, and the non-negative nature of Im V(s, t) as $s \to \infty$ is hypothetical.

We note also that this assumption that Im V(s,t) is non-negative is equivalent to the assumption that when $s \to \infty$ the main contribution to the series (2.13) is made by partial waves with momentum $l \sim ck$. Indeed, for large k and $l \sim ck$ we have

$$P_l\left(1 + \frac{t}{2k^2}\right) \approx J_0\left(\frac{l}{k}\sqrt{-t}\right).$$

The first root of $J_0(x)$ is equal to $x_0 = 2.4048$. In the region $\frac{l}{k} \times \sqrt{-t}x_0$ the function $J_0(\frac{l}{k}\sqrt{-t})$ and consequently the $P_l(z)$ are positive. If we assume that the main contribution is made in (2.13) by terms with $l \approx ck$, then these terms will be non-negative when $|t| < x_0^2/c^2$, that is, for sufficiently small t.*

We shall show that under the assumptions made the limits $A^I(t)$ and $A^{II}(t)$ coincide. Indeed, the assumption that the limits (2.12) exist signifies that the modulus of the function (2.9) tends to definite limits when $s \to \pm\infty$. The limiting sets $E_1$ and $E_2$ for the same function V(s, t) therefore lie on two concentric circles: $|V(s, t)| = a^I(t)$ and $|V(s, t)| = a^{II}(t)$. By virtue of the second part of theorem II, either the sets $E_1$ and $E_2$ cross, which is possible only if $a^I(t) = a^{II}(t)$, or one of them surrounds the other, that is, consists of all the points of the circle $|V(s, t)| = \max(a^I, a^{II})$. In the case in question, however, the second possibility is not realized, for by virtue of the non-negativity of Im V(s, t) when $s \to \infty$ each of the sets $E_i$ can occupy not more than a semicircle, meaning that it cannot surround the second set. It follows therefore that $a^I(t) = a^{II}(t)$ and consequently the differential cross sections (2.8) are equal.

Other conditions under which the differential cross sections are asymptotically equal are given in Meĭman's paper [38]. The results of this paper can be formulated in the following manner. Let the elastic scattering amplitudes $T^J(s, t)$ have no real zeroes, and let

*It is useful to note that since $P_l(x) \geq 1$ for real $x \geq 1$, the imaginary part of the amplitude is non-negative in this part of the physical region, where the series (2.13) converges and $t \geq 0$.

the following integrals converge

$$\int_{-\infty}^{\infty} |\ln|T^J(s,\,t)||\frac{ds}{1+s^2}<\infty, \qquad J=\mathrm{I},\ \mathrm{II}.$$

This leads to the convergence of the products

$$\pi^J(s,\,t)=\prod_k\left[\left(1-\frac{s}{s_k^J(t)}\right)\Big/\left(1-\frac{s}{s_k^J(t)^*}\right)\right],$$

where $s_k^J(t)$ are the zeroes of the amplitude $T^J(s, t)$ in the upper half of the s plane. Assume further that the ratio of the absolute values of the amplitudes of processes (I) and (II) has a limit as $s \to \infty$:

$$\lim_{s\to\infty}\left|\frac{T^{\mathrm{I}}(s,\,t)}{T^{\mathrm{II}}(s,\,t)}\right|=\gamma.$$

It is assumed that this ratio is bounded from above and from below on the entire real s axis by positive functions of t. Then $\gamma = 1$, if the argument (phase) of the ratio

$$\frac{T^{\mathrm{I}}(s,\,t)/\pi^{\mathrm{I}}(s,\,t)}{T^{\mathrm{II}}(s,\,t)/\pi^{\mathrm{II}}(s,\,t)}$$

increases (or decreases) more slowly than ln s (or respectively, −ln s) when $s \to \infty$; $\gamma$ has a finite positive value different from unity if the phase of this ratio increases(decreases) like ln s (−ln s), $\gamma = 0$ or $\infty$ if the phase of this ratio increases (decreases) more rapidly than ln s (−ln s).

We now proceed to a study of the forward scattering. If $\alpha(t)$ in (2.5) satisfies the condition

$$\alpha(0)=1 \tag{2.14}$$

and the real part of the amplitude increases no more rapidly than its imaginary part, then theorem I leads to the statement that the total cross sections of the interaction of the particles and antiparticles are equal. For this purpose it is sufficient to note that by virtue of the optical theorem the total cross sections $\sigma^J_{tot}(s)$ corresponding to processes (I) and (II) are expressed in terms of the imaginary parts of the amplitudes of these processes by means of the formula

$$\sigma^J_{\mathrm{tot}}(s)=\frac{1}{2k\sqrt{s}}\,\mathrm{Im}\,T^J(s,\,0), \qquad J=\mathrm{I},\ \mathrm{II}. \tag{2.15}$$

Nambu and Sugawara[17] have proved the asymptotic equality of the total cross sections of the interaction of a particle and antiparticle without assuming the existence of limits of the cross sections. Here, however, they have assumed that the elastic forward scattering amplitudes become pure imaginary at high energies. A second proof of the Pomeranchuk theorem was given in[18] under the assumption that when $s \to \infty$ the total cross sections tend to constant limits, and the differential cross sections forward are bounded. Asymptotic equality of the total cross sections for the interaction of a particle and an antiparticle was proved also in the case when these cross sections increase logarithmically[15,18].

If a is a neutral, scalar (or pseudoscalar) particle which coincides with its own antiparticle, then the amplitudes of the processes (I) and (II) coincide:

$$T^{\mathrm{I}}(s,\,t)=T^{\mathrm{II}}(s,\,t)=T(s,\,t). \tag{2.16}$$

On the other hand, in the case of forward scattering (with $\alpha(0) = 1$) the theorem I leads to the relation

$$\lim_{s\to\infty}\frac{T(s,\,0)}{T(s,\,0)^*}=-1. \tag{2.17}$$

It follows therefore that in the asymptotic case (as $s \to \infty$) the amplitude is pure imaginary:

$$\lim_{s\to\infty}\frac{\mathrm{Re}\,T(s,\,0)}{\mathrm{Im}\,T(s,\,0)}=0. \tag{2.18}$$

From (2.1), (2.15) and (2.18) we get the following asymptotic connection between the differential and total cross sections of the process under consideration:

$$\left.\frac{d\sigma(s,\,t)}{dt}\right|_{t=0}\sim\frac{1}{16\pi}[\sigma_{\mathrm{tot}}(s)]^2. \tag{2.19}$$

## 3. ASYMPTOTIC PROPERTIES OF THE AMPLITUDES OF MESON-BARYON SCATTERING

### 1. Symmetry Properties of the Amplitude

Let us consider the processes (I) and (II) for the case when the particles $a_i$ have spin 0 and particles $b_i$ have spin 1/2. Then the amplitudes of the processes can be written in the form

$$T^J(p_1,\,q_1;\ p_2,\,q_2)=\bar{u}_b(p_2)\left[F_1^J(s,\,t)+i\,\frac{\hat{q}_1+\hat{q}_2}{2}\,F_2^J(s,\,t)\right]u_b(p_1), \tag{3.1}$$

if the relative parity of the particles in the initial state $I_i$ coincides with the relative parity of the particles in the final state $I_f$, $I_i = I_f$, or in the form

$$T^J(p_1,\,q_1;\ p_2,\,q_2)=\bar{u}_b(p_2)\left[F_1^J(s,\,t)+i\,\frac{\hat{q}_1+\hat{q}_2}{2}\,F_2^J(s,\,t)\right]\gamma_5u_b(p_1), \tag{3.2}$$

if $I_i = -I_f$. In expressions (3.1) and (3.2) $p_1$ and $q_1$ are the 4-momenta of the fermion and boson in the initial state ($b_1$ and $a_1$ for process (I) and $b_1$ and $\bar{a}_2$ for process (II)) and $p_2$ and $q_2$ are the 4-momenta in the final state.

The invariant amplitudes $F_1^J(s, t)$ and $F_2^J(s, t)$ of the processes (I) and (II) are related by equations of the crossing-symmetry type.

Let us proceed to derive these equations. The amplitudes $T^J$ are expressed in terms of the variational derivatives of the S matrix by the equations

$$T^{\mathrm{I}}(p_1,\,q_1;\ p_2,\,q_2)=\int d^4x\left\langle b_2(p_2)\left|\frac{\delta j_{a_1}\left(-\frac{x}{2}\right)}{\delta\varphi_{a_2}\left(\frac{x}{2}\right)}\right|b_1(p_1)\right\rangle e^{-i\frac{q_1+q_2}{2}x}, \tag{3.3}$$

$$T^{\mathrm{II}}(p_1,\,q_1;\ p_2,\,q_2)=\int d^4x\left\langle b_2(p_2)\left|\frac{\delta j^+_{a_2}\left(-\frac{x}{2}\right)}{\delta\varphi^+_{a_1}\left(\frac{x}{2}\right)}\right|b_1(p_1)\right\rangle e^{-i\frac{q_1+q_2}{2}x}, \tag{3.4}$$

where

$$j_{a_i}(x)=i\,\frac{\delta S}{\delta\varphi^+_{a_i}(x)}\,S^+.$$

In the physical region each of the integrals (3.3) and (3.4) must be understood as a limit when the vector $q_1 + q_2$ has a negative increment that tends to zero and belongs to the future light cone. Therefore, when the vectors $p_i$ and $q_i$ are in the physical region of any of the processes (I) or (II), expressions (3.3) and (3.4) are Hermitian adjoints of each other, that is,

$$T^{II}(p_1, q_1; p_2, q_2) = T^{I'}(p_2, -q_1; p_1, -q_2)^*, \quad (3.5)$$

where (I′) is the process inverse to (I):

$$a_2 + b_2 \to a_1 + b_1.$$

Processes (I) and (I′) are related by space-time reflection. If

$$T^I(p_1, q_1; p_2, q_2) = \bar{u}(p_2) M^I(p_1, q_1; p_2, q_2) u(p_1), \quad (3.6)$$

then

$$T^{I'}(p_2, q_2; p_1, q_1) = \eta \bar{u}(p_2) [BM^I(p_1, q_1; p_2, q_2) B^{-1}]^T u(p_1), \quad (3.7)$$

where the matrix B has the properties

$$B\gamma_\mu B^{-1} = \gamma_\mu^T, \qquad B^T = -B, \quad (3.8)$$

the upper index T denotes the transpose of the matrix, and $\eta$ denotes the phase factor, $|\eta| = 1$. In the case of elastic scattering $\eta = 1$. From (3.5) and (3.7) we obtain

$$M^{II}(p_1, q_1; p_2, q_2) = \eta\gamma_4 [BM^I(p_1, -q_2; p_2, -q_1) B^{-1}]^* \gamma_4. \quad (3.9)$$

Relation (3.9) can be written in a more conventional form by expressing B in terms of the charge-conjugation matrix C:

$$B = C^{-1}\gamma_5, \qquad C^{-1}\gamma_\mu C = -\gamma_\mu^T, \qquad C^T = -C. \quad (3.10)$$

By virtue of relativistic invariance we have

$$\gamma_5 M^I(p_1, -q_2; p_2, -q_1)\gamma_5 = M^I(-p_1, q_2; -p_2, q_1). \quad (3.11)$$

From (3.9)—(3.11) follows a crossing-symmetry relation for the total amplitude

$$M^{II}(p_1, q_1; p_2, q_2) = \eta\gamma_4 [C^{-1} M^I(-p_1, q_2; -p_2, q_1) C]^* \gamma_4. \quad (3.12)$$

From this we can easily obtain crossing relations between the invariant amplitudes of $F_i^J(s, t)$ of processes (I) and (II). Depending on whether the relative parities of the particles in the initial and final states are identical or opposite, we obtain

$$F_i^I(u, t) = (-1)^{i+1} F_i^{II}(s, t)^*, \qquad \text{if} \quad I_i = I_f,$$
$$F_i^I(u, t) = (-1)^i F_i^{II}(s, t)^*, \qquad \text{if} \quad I_i = -I_f. \quad (3.13)$$

We shall henceforth assume that $\eta = 1$, which does not lead to a change in the final results, since the expressions for the differential cross sections and polarizations always contain the product $\eta\eta^*$.

Along with processes (I) and (II), we shall consider for the case when $b_i$ are particles with spin, also the process

$$a_1 + \bar{b}_2 \to a_2 + \bar{b}_1. \quad \text{(III)}$$

The amplitude of this process will be written in the form

$$T^{III}(p_1, q_1; p_2, q_2) = \bar{u}(p_2) M^{III}(p_1, q_1; p_2, q_2) u(p_1). \quad (3.14)$$

In analogy with the preceding, we can obtain the following crossing-symmetry relation between the amplitudes $M^I$ and $M^{III}$:

$$M^{III}(p_1, q_1; p_2, q_2) = \gamma_4 M^I(p_2, -q_1; p_1, -q_2)^+ \gamma_4, \quad (3.15)$$

or, in terms of the invariant amplitudes,

$$F_i^I(u, t) = (-1)^{i+1} F_i^{III}(s, t)^*, \qquad \text{if} \quad I_i = I_f,$$
$$F_i^I(u, t) = -F_i^{III}(s, t)^*, \qquad \text{if} \quad I_i = -I_f. \quad (3.16)$$

## 2. Asymptotic Equality of the Differential Cross Sections

The differential cross section of the process (I) is equal to

$$\frac{d\sigma^I(s, t)}{dt} = \frac{1}{64\pi s k_I^2} G^I(s, t), \quad (3.17)$$

where

$$G^I(s, t) = [(M_2 \pm M_1)^2 - t]\,|F_1^I(s, t)|^2 + \frac{1}{4}\{(s-u)^2 - (m_1^2 - m_2^2) - [t - 2(m_1^2 + m_2^2)][t - (M_2 \pm M_1)^2]\}\,|F_2^I(s, t)|^2 + [(M_2 \pm M_1)(u - s) + (M_2 \pm M_1)(m_1^2 - m_2^2)] \operatorname{Re} F_1^I(s, t) F_2^I(s, t)^*, \quad (3.18)$$

with the upper sign corresponding to the case of identical relative parities, $I_i = I_f$, and the lower to the case of opposite parities, $I_i = -I_f$.

For fixed t and $s \to \infty$, (3.18) takes the form

$$G^I(s, t) \sim |(M_2 \pm M_1) F_1^I(s, t) - s F_2^I(s, t)|^2 - t\,|F_1^I(s, t)|^2. \quad (3.19)$$

The differential cross section of the process (II) is obtained from (3.17) and (3.18) by replacing $F_1^I(s, t)$ and $F_2^I(s, t)$ by $F_1^{II}(s, t)$ and $F_2^{II}(s, t)$ and by interchanging the positions of $m_1$ and $m_2$, while the differential cross section of process (III) is obtained by replacing $F_1^I(s, t)$ and $F_2^I(s, t)$ by $F_1^{III}(s, t)$ and $F_2^{III}(s, t)$ and interchanging the places of $M_1$ and $M_2$.

Let us prove the asymptotic equality of the cross sections of processes (I), (II), and (III) for fixed t and $s \to \infty$. If only one of two amplitudes $F_1^J(s, t)$ and $F_2^J(s, t)$ makes the main contribution to the asymptotic value of the cross section, then it is sufficient to consider this amplitude, and the asymptotic equality of the cross sections follows directly from theorem I. We must therefore consider the general case when both amplitudes $F_1^J(s, t)$ and $F_2^J(s, t)$ make contributions of equal order to the asymptotic values of the cross section. In this case, for some choice of the admissible function $\varphi(s, t)$ there exist finite limits

$$U_1^\pm(t) = \lim_{s \to \pm\infty} \frac{F_1^I(s, t)}{\varphi(s, t)}, \qquad U_2^\pm(t) = \lim_{s \to \pm\infty} \frac{s F_2^I(s, t)}{\varphi(s, t)}. \quad (3.20)$$

By virtue of theorem II the limiting values of (3.20) are equal to each other:

$$U_i^+(t) = U_i^-(t), \quad (3.21)$$

and, consequently, taking account of the crossing-symmetry relations (3.13) and (3.16), we get

$$\lim_{s\to\infty}\frac{F_1^{I}(s,t)}{F_1^{II}(s,t)^*}=\lim\frac{F_2^{I}(s,t)}{F_2^{II}(s,t)^*}=\pm e^{-i\pi\alpha(t)},$$
$$\lim_{s\to\infty}\frac{F_1^{I}(s,t)}{F_1^{III}(s,t)^*}=\pm\lim\frac{F_2^{I}(s,t)}{F_2^{III}(s,t)^*}=\pm e^{-i\pi\alpha(t)}, \quad (3.22)$$

where again the upper sign corresponds to $I_i = I_f$, and the lower to $I_i = -I_f$.

From these asymptotic relations between the amplitudes of processes (I), (II), and (III) follows the asymptotic equality of the cross sections of these processes for fixed t and $s\to\infty$. For example, from this follow the asymptotic equalities of the differential processes

$$\left.\begin{array}{lll} \pi^+ + p \to \pi^- + p & \text{and} & \pi^- + p \to \pi^- + p, \\ K^+ + p \to K^+ + p & \text{and} & K^- + p \to K^- + p, \\ \pi^+ + p \to K^+ + \Sigma^+ & \text{and} & K^- + p \to \pi^- + \Sigma^+, \\ \pi^- + p \to K^0 + \lambda & \text{and} & \bar{K}^0 + p \to \pi^+ + \lambda, \\ \Sigma^+ + \text{He} \to p + \text{He}_\lambda & \text{and} & \bar{p} + \text{He} \to \bar{\Sigma}^+ + \text{He}_\lambda. \end{array}\right\} \quad (3.23)$$

## 3. Asymptotic Relations Between the Polarizations of Fermions in the Final State

We assume that the fermions are not polarized in the initial state. We denote by $n_\mu$ a unit space like 4-vector proportional to $i\epsilon_{\mu\alpha\beta\gamma}\, p_{1\alpha}\, q_{1\beta}\, p_{2\gamma}$, where $\epsilon_{\alpha\beta\gamma\mu}$ is a completely antisymmetrical tensor. In the c.m.s. we have $n_4 = 0$, $\mathbf{n} = [\mathbf{p}_1\times\mathbf{p}_2]/|[\mathbf{p}_1\times\mathbf{p}_2]|$. The polarization state of the fermions in the final state is characterized by a polarization 4-vector $\xi_\mu^J$ proportional to the unit vector

$$\xi_\mu^J = P^J(s,t)\, n_\mu. \quad (3.24)$$

Calculating $P^J(s,t)$, we obtain

$$P^J(s,t) = 2s\sqrt{-t}\, C^J(s,t)\,\frac{\operatorname{Im} F_1^J(s,t)\, F_2^J(s,t)}{G^J(s,t)}, \quad (3.25)$$

where the function $C^J(s,t)$ tends to unity for fixed t and $s\to\infty$ (see [22] and formula (2.35) below). For the process (I) the function $G^I$ is determined by formula (3.18), and in the asymptotic case by formula (3.19), while for the processes (II) and (III) the functions $G^J$ are obtained from formula (3.18) by means of the method described following formula (3.19).

As already shown, for fixed t and $s\to\infty$ the functions $G^J(s,t)$ are equal to each other for all processes (I), (II), and (III). Therefore, in investigating the polarization it is sufficient to consider the quantity $\operatorname{Im} F_1^J(s,t)\, F_2^J(s,t)^*$. From the asymptotic relations (3.22) it follows that

$$\lim_{s\to\infty}\frac{\operatorname{Im} F_1^{I}(s,t)\,F_2^{I}(s,t)^*}{\operatorname{Im} F_1^{II}(s,t)\,F_2^{II}(s,t)^*}=\pm\lim_{s\to\infty}\frac{\operatorname{Im} F_1^{I}(s,t)\,F_2^{I}(s,t)^*}{\operatorname{Im} F_1^{III}(s,t)\,F_2^{III}(s,t)^*}=-1, \quad (3.26)$$

where, as in (3.22), the plus sign corresponds to the case $I_i = I_f$ and the minus sign to the case $I_i = -I_f$.

Thus, for fixed t and $s\to\infty$ the polarizations of the fermions in the final states of processes (I), (II) and (III) are connected by the asymptotic relations

$$P^{I}(s,t)\sim -P^{II}(s,t) \quad \text{for both cases} \quad (3.27)$$

and

$$\begin{array}{lll} P^{I}(s,t)\sim -P^{III}(s,t), & \text{if} & I_i = I_f, \\ P^{I}(s,t)\sim P^{III}(s,t), & \text{if} & I_i = -I_f. \end{array} \quad (3.28)$$

The results obtained are applicable, in particular, to the processes (3.22) and (3.23). For example, the polarizations of the recoil protons in the processes

$$\pi^+ + p \to \pi^+ + p \quad\text{and}\quad \pi^- + p \to \pi^- + p,$$
$$K^+ + p \to K^+ + p \quad\text{and}\quad K^- + p \to K^- + p$$

are equal in magnitude and opposite in sign for identical values of the energy s and of the momentum transfer.* For the polarizations of the $\Sigma^+$-hyperons in the processes

$$\pi^+ + p \to K^+ + \Sigma^+ \quad\text{and}\quad K^- + p \to \pi^- + \Sigma^+$$

there is also an analogous asymptotic relation regardless of the relative parities of the particles. However, the asymptotic relations between the polarizations of the nucleon and antihyperon in the last pair of processes (3.23) depend on the relative parity $I_{\Sigma\lambda}$: the polarizations of p and $\bar{\Sigma}^+$ are equal in magnitude and opposite in sign if this parity is +1, and equal both in magnitude and in sign if the parity of $I_{\Sigma\lambda}$ is −1.

We have considered a general case when both invariant amplitudes contribute to the asymptotic values of the cross sections. It is easily seen from (3.25) that in this case the polarizations P tend to differ from zero when $t \neq 0$. If only one of these two amplitudes contributes to the asymptotic values of the cross sections, then the polarization $P^J$ tends to zero.

We shall now show that there are several processes in which the polarizations of the fermions in the final states tend to zero as $s\to\infty$ and for fixed non-vanishing t, independently of the relative behavior of the invariant amplitudes. These are processes that go over into themselves in the crossing transformation, that is, processes for which $\bar{a}_2 = a_1$. In this case processes (I) and (II) coincide, that is, $P^I(s,t) = P^{II}(s,t)$. On the other hand, according to (3.27), $P^I(s,t) \sim -P^{II}(s,t)$ independently of the relative parities of the particles. Consequently, in this case $P^I \sim P^{II} \sim 0$.

Thus, for example, the polarization of the fermions in the final states in the processes

$$K_2^0 + p \to K_2^0 + p \quad (3.29)$$

and

$$K^- + p \to K^+ + \Xi^- \quad (3.30)$$

(if the spin of the Ξ-hyperon is equal to 1/2) tends to zero when $s\to\infty$ and t is fixed, independently of the relative parities of the particles, even when both invariant amplitudes contribute to the asymptotic values

*The asymptotic connection between the polarizations of the protons in these processes was first obtained in [39] under the assumption that $F_1^J$ and $sF_2^J$ behave for $s\to\infty$ like s when $t \le 0$.

of the cross sections. If isotopic invariance is satisfied, then the polarization of the recoil neutron likewise tends to zero in the charge exchange process

$$\pi^- + p \to \pi^0 + n. \qquad (3.31)$$

Indeed, we have seen that the polarization of the recoil neutrons in this process and in the process for

$$\pi^0 + p \to \pi^+ + n \qquad (3.32)$$

are opposite when $s \to \infty$. However, it follows from the isotopic invariance that the matrix elements of the processes coincide (apart from the sign) and therefore the polarizations of the neutrons should be equal in both processes. It follows therefore that when $s \to \infty$ the polarizations of the recoil neutrons of the processes under consideration tend to zero.

For an experimental verification of the obtained asymptotic relations between the polarizations, it is probably simplest to use a polarized nucleon target and to measure the parameter of the left-right asymmetry for the different processes, inasmuch as in the case of equal parity ($I_i = I_f$) the asymmetry parameter is equal to the polarization, and in the case of different parities ($I_i = -I_f$) the asymmetry parameter differs from the polarization only in sign[39].

4. Complete Experiment in the Case of Elastic Scattering of a Meson by a Nucleon

In the case considered of the scattering of a meson (with zero spin) by a nucleon (with spin 1/2) the complete *experiment should yield three real quantities* (for each value of the variables s and t)*. We can choose these quantities to be the differential cross sections (3.17)–(3.18), the polarization $P^J(s, t)$ of the fermion in the final state when scattered by an unpolarized target (3.25), and the polarization when scattered by a polarized target with a polarization vector $\boldsymbol{\xi}$.

In elastic scattering of a meson by a polarized nucleon target, the three-dimensional nucleon-polarization vector in the final state is given in the c.m.s. by

$$\xi^J = P^J(s, t)\,\mathbf{n} + \xi + Q^J(s, t)\,[(\xi\mathbf{n})\,\mathbf{n} - \xi], \qquad (3.33)$$

where $P^J(s, t)$ is given by (3.25), and

$$Q^J(s, t) = -32t\,[C(s, t)]^2\,\frac{s^2}{(\sqrt{s}+M)^2 - m^2} \times \frac{|F_1^J(s, t) + (\sqrt{s}+M)\,F_2^J(s, t)|^2}{G^J(s, t)}; \qquad (3.34)$$

$G^J(s, t)$ for $J = I$ is given by formula (3.18). ($G^J(s, t)$ for other values of J is obtained by the method indicated following formula (3.19)),

$$[C(s, t)]^2 = -\frac{su - (M^2 - m^2)^2}{s^2} \to 1. \qquad (3.35)$$

*In [40] the complete experiment is confined to determination of two quantities, for when the energy is lower than the threshold of the inelastic processes the remaining quantities can be determined from the two-particle unitarity condition.

For the third measured quantity (alongside with $d\sigma^J/dt$ and $P^J(s,t)$) we can take, by virtue of (3.33), the quantity $Q^J(s, t)$. It is easily seen that under the assumptions made above the asymptotic value of the quantities $Q^J(s, t)$ as $s \to \infty$ should coincide for processes (I) and (II).

In the case under consideration, that of elastic scattering of a meson by a nucleon, the results obtained can be derived under weaker assumptions: in place of the existence of two complex limits (3.20) (i.e., four real limits) it is sufficient to assume the existence of limits for the three indicated experimentally-measured quantities or, what is the same, the existence of limits (as $s \to \infty$) of the quantities

$$\left|\frac{H^J(s, t)}{\varphi(s, t)}\right|, \quad \left|\frac{F_1^J(s, t)}{\varphi(s, t)}\right| \quad \text{and} \quad \arg\left[\frac{F_1^J(s, t)}{\varphi(s, t)}\right],$$

where

$$H^J(s, t) = 2MF_1^J(s, t) - (s - M^2 - m^2)\,F_2^J(s, t), \qquad (3.36)$$

under the condition that $\mathrm{Im}\,[H^J(s, t)/\varphi(s, t)] \geq 0$ as $s \to \infty$. The latter assumption is natural since

$$H^J(s, t) = \sum_{l=0}^{\infty} [(l+1)\,f_l^{(+)} + l f_l^{(-)}]\,P_l(z) - \frac{t}{2k^2}\sum_{l=1}^{\infty} (f_l^{(+)} - f_l^{(-)})\,P_l'(z), \qquad (3.37)$$

and furthermore by virtue of the unitarity condition

$$\mathrm{Im}\, f_l^{(\pm)} \geqslant 0. \qquad (3.38)$$

Under these assumptions the conclusions concerning the asymptotic equality of the differential cross sections, the opposite signs of the polarizations (3.27), and the equality of the quantities (3.34) for processes (I) and (II) remain in force. This statement is proved in analogy with the proof of the similar statements for the case of scalar particles (Sec. 2).

## 4. ASYMPTOTIC PROPERTIES OF THE BARYON-BARYON SCATTERING AMPLITUDES

1. Symmetry Properties of the Amplitudes

We now proceed to study processes (I) and (II) in the case when all particles $a_i$ and $b_i$ have spin 1/2. The matrix elements of these processes are of the form

$$T^J(p_1, q_1; p_2, q_2) \qquad (4.1)$$

$$= \sum \bar{u}_a(q_2)\,\Gamma_i^{(a)}(p)\,u_a(q_1)\,\bar{u}_b(p_2)\,\Gamma_i^{(b)}(q)\,u_b(p_1)\,F_i^J(s, t),$$

$$q = \frac{q_1 + q_2}{2}, \qquad p = \frac{p_1 + p_2}{2},$$

where $\Gamma_i^{(a)}(p)$ can be chosen independently of the relative parities of the particle:

$$\Gamma_i^{(a)}(p) = \{1,\ 1,\ i\hat{p},\ i\hat{p},\ \gamma_5,\ \gamma_5,\ i\hat{p}\gamma_5,\ i\hat{p}\gamma_5\}, \qquad (4.2)$$

and $\Gamma_i^{(b)}(p)$ depend on the parities:

$$\Gamma_i^{(b)}(q)=\{1,\ i\hat{q},\ i\hat{q},\ 1,\ \gamma_5,\ i\hat{q}\gamma_5,\ \gamma_5,\ i\hat{q}\gamma_5\},\qquad I_i=I_f, \quad (4.3)$$

$$\Gamma_i^{(b)}(q)=\{\gamma_5,\ i\hat{q}\gamma_5,\ i\hat{q}\gamma_5,\ \gamma_5,\ 1,\ i\hat{q},\ 1,\ i\hat{q}\},\qquad I_i=-I_f. \quad (4.4)$$

The invariant amplitudes $F_i^J(s, t)$ satisfy the crossing-symmetry relations

$$F_i^{I}(u, t)=\pm(-1)^{i+1}F_i^{II}(s, t)^*. \quad (4.5)$$

2. Asymptotic Equality of the Differential Cross Sections

The differential cross sections of the processes in question can be expressed in terms of the invariant amplitudes $F_i^J(s, t)$. From these expressions and from the crossing-symmetry relations (4.5) we can prove the asymptotic quality of these differential cross sections. To this end it is sufficient to apply the Phragmen-Lindelof theorem and to repeat the entire reasoning given in the preceding section. In particular, for fixed t and $s\to\infty$ the differential cross sections of the following processes are equal

$$p+p\to p+p \quad\text{and}\quad \bar{p}+p\to\bar{p}+p, \quad (4.6)$$
$$\Sigma^++p\to\Sigma^++p \quad\text{and}\quad \bar{\Sigma}^++p\to\bar{\Sigma}^++p, \quad (4.7)$$
$$\Sigma^-+p\to\lambda+n \quad\text{and}\quad \bar{\lambda}+p\to\bar{\Sigma}^-+n, \quad (4.8)$$
$$\Sigma^++p\to p+\Sigma^+ \quad\text{and}\quad \bar{p}+p\to\bar{\Sigma}^++\Sigma^+, \quad (4.9)$$
$$\Sigma^-+p\to n+\lambda \quad\text{and}\quad \bar{n}+p\to\bar{\Sigma}^-+\lambda. \quad (4.10)$$

We note that in the last two processes (4.9) and (4.10) the momentum transfer t is measured between the initial proton and the final hyperon. This transfer is usually denoted by u. Therefore in these cases the differential cross section of the elastic or inelastic backward scattering of a hyperon by a proton is in fact equal to the differential cross section for proton-antiproton pair annihilation by a hyperon-antihyperon pair.

3. Asymptotic Properties of Polarization Effects

We denote by $P^{Ja}$ or $P^{Jb}$ the polarizations of the particles $a_2$, $\bar{a}_1$, or $b_2$, respectively, in the final state of the process J = I, II with unpolarized initial particles, and by $\eta^{Ja}$ or $\eta^{Jb}$ the parameters of the left-right asymmetry in processes with polarized particles $a_1$, $\bar{a}_2$, or $b_1$, respectively. Then, expressing $P^{Ja}(s, t)$, $P^{Jb}(s, t)$ and $\eta^{Ja}(s, t)$, $\eta^{Jb}(s, t)$ in terms of the invariant amplitudes and using the crossing-symmetry relations (4.5), we can prove the following asymptotic relations

$$P^{Ia}(s, t)\sim-\eta^{IIa}(s, t),\qquad P^{IIa}(s, t)\sim-\eta^{Ia}(s, t), \quad (4.11)$$
$$P^{Ib}(s, t)\sim-P^{IIb}(s, t),\qquad \eta^{Ib}(s, t)\sim-\eta^{IIb}(s, t). \quad (4.12)$$

If (I) and (II) are elastic-scattering processes, then it follows from T-invariance that

$$P^{Ja}(s, t)=\eta^{Ja}(s, t),\qquad P^{Jb}(s, t)=\eta^{Jb}(s, t).$$

In this case $P^{Jb}(s, t)$ and $\eta^{Jb}(s, t)$ also satisfy the asymptotic relation (4.11), while $P^{Ja}(s, t)$ and $\eta^{Ja}(s, t)$ satisfy (4.12).

Let us present some examples. For fixed t and $s\to\infty$ the polarizations of the recoil protons in processes (4.6) and (4.7), and also of the recoil neutrons in (4.8), have equal magnitudes and opposite signs. The polarizations of the hyperons in the final state of the processes (4.9) and (4.10), of the proton and antiproton in (4.6), and of the hyperon and antihyperon in (4.7) are also opposite. In concluding this section, we note that whereas the dispersion relations for the scattering of the pion by a nucleon have been proved on the basis of general principles of local theory[8], the analytic properties of the nucleon-nucleon scattering amplitudes, which are necessary to apply theorem II, have been proved only in arbitrary order of perturbation theory[41,42]. As to the scattering of a hyperon by a nucleon, as shown in[43] the general dispersion relations are not valid in the lower orders of perturbation theory even in the case of forward scattering. It turns out, however, that in the physical region the amplitudes of the processes in question tend as $s\to\infty$ to asymptotic amplitudes that are analytic functions of the variable s, and the Phragmen-Lindelof theorem is applicable to these functions. Therefore the asymptotic relations obtained can be regarded as proved (for details see Sec. 8).

## 5. ASYMPTOTIC PROPERTIES OF THE PHOTOPRODUCTION AND COMPTON EFFECT AMPLITUDES

1. Photoproduction of a Meson on a Baryon

We consider the crossing processes of photoproduction of a meson on a baryon:

$$\gamma+b_1\to a+b_2, \quad (I)$$
$$\gamma+b_2\to\bar{a}+b_1, \quad (II)$$

where $b_1$ and $b_2$ are baryons, and a and $\bar{a}$ are the meson and its antiparticle. We denote by $M_i$ the mass of the baryon $b_i$, by m the mass of the meson a, by k and $p_1$ the 4-momenta of $\gamma$ (photon) and the baryon, and by q and $p_2$ the 4-momenta of the final meson and baryon. The matrix elements take the form

$$T^J(k, p_1; q_1, p_2)=\sum_{i=1}^{4}\bar{u}_b(p_2)\Gamma_i u_b(p_1)F_i^J(s, t), \quad (5.1)$$

where the independent covariant matrices $\Gamma_i$ are equal to

$$\left.\begin{aligned}\Gamma_1&=i\gamma_5\hat{\varepsilon}\hat{k},\\ \Gamma_2&=2i\gamma_5[(P\varepsilon)\,qk)-(Pk)\,(q\varepsilon)],\\ \Gamma_3&=\gamma_5[\hat{\varepsilon}\,(qk)-\hat{k}\,(q\varepsilon)],\\ \Gamma_4&=2\gamma_5[\hat{\varepsilon}\,(Pk)-\hat{k}\,(P\varepsilon)],\end{aligned}\right\} \quad (5.2)$$

$P = 1/2(p_1 + p_2)$, $\epsilon_\mu$ is the photon-polarization 4-vector.

The invariant amplitudes $F_i^J(s, t)$ of the photoproduction processes (I) and (II) satisfy the crossing-symmetry relations

$$F_i^{I}(u, t) = F_i^{II}(s, t)^*, \quad i = 1, 2, 4,$$
$$F_i^{I}(u, t) = -F_i^{II}(s, t)^*, \quad i = 3. \tag{5.3}$$

The differential cross sections of processes (I) and (II) can be expressed in terms of the invariant amplitude $F_i^J(s, t)$. From these expressions and from the crossing-symmetry relations (5.3) we can prove, with the aid of the Phragmen-Lindelof theorem, the asymptotic equality of the cross sections of processes (I) and (II) for fixed t and $s \to \infty$. Thus, for example, the cross sections of the processes

$$\gamma + p \to \pi^+ + n, \tag{5.4}$$

$$\gamma + n \to \pi^- + p \tag{5.5}$$

are asymptotically equal. We note that this equality was obtained without using the isotopic invariance of strong interactions.

We denote by $P^J(s, t)$ the polarizations of baryons in the final states of the processes with unpolarized initial particles and by $\eta^J(s, t)$ the parameters of the left-right asymmetry in processes with unpolarized photon and initial polarized baryon. From the expressions for $P^J(s, t)$ and $\eta^J(s, t)$ and from relations (5.3) we can prove the following asymptotic relation between the polarization $P^J(s, t)$ in one process and the asymmetry parameter $\eta^J(s, t)$ in the other

$$P^{I}(s, t) \sim -\eta^{II}(s, t), \qquad P^{II}(s, t) \sim -\eta^{I}(s, t). \tag{5.6}$$

For the photoproduction processes which go over into themselves in the crossing transformation ($b_1 = b_2$, $\bar{a} = a$), we have in lieu of (5.6)

$$P(s, t) \sim -\eta(s, t). \tag{5.7}$$

Examples of such processes are

$$\gamma + p \to \pi^0 + p,$$
$$\gamma + n \to \pi^0 + n.$$

2. Compton Effect

We now consider the elastic scattering of a photon by a nucleon. This process goes over into itself under the crossing transformation. When we investigated the scattering of scalar particles by spinor particles we have shown that in processes of this type the polarization of the recoil fermions tends to zero as $s \to \infty$ and fixed t. We shall prove that this takes place in this case, too.

The amplitude of the process under consideration can be written in the form

$$\left.\begin{aligned} &T(k_1, p_1; k_2, p_2) = \sum_{i=1}^{6} \bar{u}(p_2)\Gamma_i u(p_1) F_i(s, t),\\ &\Gamma_1 = \frac{(\varepsilon_1 P')(\varepsilon_2 P')}{P'^2}, \quad \Gamma_2 = \frac{(\varepsilon_1 P')(\varepsilon_2 P')}{P'^2} i\hat{k}, \quad \Gamma_3 = \frac{(\varepsilon_1 N)(\varepsilon_2 N)}{N^2},\\ &\Gamma_4 = \frac{(\varepsilon_1 N)(\varepsilon_2 N)}{N^2} i\hat{k}, \quad \Gamma_5 = \frac{(\varepsilon_1 N)(\varepsilon_2 P') - (\varepsilon_1 P')(\varepsilon_2 N)}{\sqrt{2P'^2N^2}} i\gamma_5,\\ &\Gamma_6 = \frac{(\varepsilon_1 N)(\varepsilon_2 P') + (\varepsilon_1 P')(\varepsilon_2 N)}{\sqrt{2P'^2N^2}} i\gamma_5\hat{k}, \end{aligned}\right\} \tag{5.8}$$

where $p_1$ and $p_2$ are the 4-momenta of the nucleons in the initial and final states, respectively, $k_1$ and $\epsilon_1$ and the 4-momentum and 4-vector of the photon polarization for scattering, and $k_2$ and $\epsilon_2$ the same quantities after scattering:

$$k = \frac{k_1 + k_2}{2}, \quad p = \frac{p_1 + p_2}{2}, \quad P'_\mu = P_\mu - \frac{(Pk)}{k^2} k_\mu,$$
$$N_\alpha = i\varepsilon_{\alpha\beta\gamma\delta} P'_\beta k_\gamma (k_1 - k_2)_\delta.$$

The amplitudes $F_i$ have the following crossing-symmetry properties:

$$F_i(u, t) = (-1)^{i+1} F_i(s, t)^*. \tag{5.9}$$

Using the expressions for the polarization of the recoil nucleon, relation (5.9), and the Phragmen-Lindelof theorem, we can easily show that the polarization of the recoil nucleon tends to zero when $s \to \infty$ and t is fixed even in the general case when all the independent invariant amplitudes make a contribution to the asymptotic cross section.

## 6. ASYMPTOTIC RELATIONS BETWEEN FORWARD ELASTIC SCATTERING AMPLITUDES

We have seen, using the processes between the scalar particles as an example (Sec. 2), that in the case of elastic forward scattering, under the additional assumption $\alpha(0) = 1$, we can obtain equality of the total cross sections and also a few other asymptotic relations of the type (2.19). Let us show that in the case of elastic zero-angle scattering of particles with spin it is also possible (assuming $\alpha(0) = 1$) to obtain some new relations (in addition to the equality of the differential cross sections). The number of the experimentally verified relations increases if account is taken of the isotopic invariance of strong interactions.

We begin with a consideration of the elastic scattering of a pion by a nucleon. The amplitudes $F_1(s, t)$ and $F_2(s, t)$ of this process, (3.1) and (3.2), have the following isotopic structure:

$$F_i^{\beta\alpha}(s, t) = F_i^{(+)}(s, t)\delta_{\beta\alpha} + F_i^{(-)}(s, t)\frac{1}{2}[\tau_\beta, \tau_\alpha]. \tag{6.1}$$

The amplitudes of the physical processes

$$\left.\begin{aligned} &a_I)\ \pi^+ + p \to \pi^+ + p, \quad a_{II})\ \pi^- + p \to \pi^- + p,\\ &b_I)\ \pi^- + p \to \pi^0 + n, \quad b_{II})\ \pi^0 + p \to \pi^+ + n,\\ &c)\ \pi^0 + p \to \pi^0 + n \end{aligned}\right\} \tag{6.2}$$

and the processes obtained from (6.2) by making the substitutions p ↔ n and $\pi^+ \leftrightarrow \pi^-$ are connected with the amplitudes (6.1) by the relations

$$\left.\begin{aligned} F_i^{a_I}(s, t) = F_i^{(+)}(s, t) - F_i^{(-)}(s, t), \quad F_i^{a_{II}}(s, t) = F_i^{(+)}(s, t) + F_i^{(-)}(s, t), \\ F_i^{b_I}(s, t) = -F_i^{b_{II}}(s, t) = -\sqrt{2}F_i^{(-)}(s, t), \quad F_i^c(s, t) = F_i^{(+)}(s, t). \end{aligned}\right\} \tag{6.3}$$

We note that in the case of forward scattering, the differential and total cross sections are expressed in terms of the same function

$$\left.\frac{d\sigma^i(s, t)}{dt}\right|_{t=0} = \frac{1}{64\pi s k^2}|H^i(s)|^2, \qquad i = a_I, a_{II}, b_I, b_{II}, c,$$
$$\sigma_{tot}(s) = \frac{1}{\sqrt{2ks}} \operatorname{Im} H^i(s), \qquad i = a_I, a_{II}, c, \tag{6.4}$$

where $H^i(s)$ is given by formula (3.36) with t = 0:

$$H^i(s) = 2MF_1^i(s, 0) - (s - M^2 - m^2) F_2^i(s, 0). \tag{6.5}$$

We can apply to the functions $H^i(s)$ the theorem I, which with allowance for $\alpha(0) = 1$ leads to the asymptotic relation

$$H^{a_I}(s) \sim -H^{a_{II}}(s)^*, \qquad H^{b_I}(s) \sim -H^{b_{II}}(s)^*, \; H^c(s) \sim -H^c(s)^*. \tag{6.6}$$

It follows from (6.3) and (6.6) that

$$\operatorname{Im} H^{(-)}(s) = 0, \qquad \operatorname{Re} H^{(+)}(s) = 0, \tag{6.7}$$

meaning that

$$\begin{gathered} \operatorname{Im} H^{a_I} \sim \operatorname{Im} H^{a_{II}} \sim \operatorname{Im} H^c, \qquad \operatorname{Im} H^{b_I} \sim \operatorname{Im} H^{b_{II}} \sim 0, \\ \operatorname{Re} H^{a_I} \sim -\operatorname{Re} H^{a_{II}} \sim \frac{1}{\sqrt{2}} \operatorname{Re} H^{b_I} \sim -\frac{1}{\sqrt{2}} \operatorname{Re} H^{b_{II}}, \\ \operatorname{Re} H^c \sim 0. \end{gathered} \tag{6.8}$$

All the relations (6.8) can be experimentally verified. Indeed, the equality of the imaginary parts for the processes $a_I$, $a_{II}$, and c leads to the asymptotic equality of the total cross sections

$$\sigma_{tot}(\pi^+ p) \sim \sigma_{tot}(\pi^- p) \sim \sigma_{tot}(\pi^0 p). \tag{6.9}$$

The first of these is the theorem of I. Ya. Pomeranchuk, while the second was proposed in [1] on the basis of the analysis of the experimental data. Further, noting that the amplitudes of scattering with charge exchange $H^{b_I}$ and $H^{b_{II}}$ are real at large values of s, and using (6.8), we can obtain the following interesting relation between the differential and total cross sections:

$$\left[\frac{d\sigma^{a_I}(s, t)}{dt} - \frac{1}{2}\frac{d\sigma^{b_I}(s, t)}{dt}\right]_{t=0} \sim \frac{1}{16\pi}[\sigma_{tot}^{a_I}(s)]^2. \tag{6.10}$$

The relation (6.10) is a generalization of (2.19) and includes the case of scattering of charged pions. We see from (6.8) that (2.19) remains in force without modification for the cross sections of the scattering of a pion by a proton (process c); see (6.2). We can demonstrate in the same manner the correctness of (2.19) for the cross sections of the scattering of $K_1^0$ and $K_2^0$ mesons by a nucleon (neglecting weak interactions the amplitudes of these two processes are equal). We note that if $\alpha(0) = 1$, then the amplitude of the process $K_2^0 + p \to K_1^0 + p$ for t = 0 and $s \to \infty$ is also pure imaginary.

The situation is somewhat more complicated in the case of elastic particles with spin 1/2, when we deal with six independent invariant functions (or with five functions in the case of nucleon-nucleon scattering). In this case instead of using formulas (4.1)–(4.4) with $F_6^J(s, t) = F_7^J(s, t) = 0$, it is more convenient to write out the amplitudes of processes (I) and (II) in the form

$$T^J(p_1, q_1; p_2, q_2) = \sum_{i=1}^{6} \bar{u}_b(p_2)\Gamma_i^{(b)}(q) u_b(p_1) \bar{u}_a(q_2) \Gamma_i^{(a)}(p) u_a(q_1) F_i^{(J)}(s, t), \tag{6.11}$$

where

$$\Gamma_i^{(a)} = \{1, \gamma_\alpha, \gamma_5, 1, i\hat{p}, i\hat{p}\}, \tag{6.12}$$

$$\Gamma_i^{(b)} = \{1, \gamma_\alpha, \gamma_5, i\hat{q}, i\hat{q}, 1\}. \tag{6.13}$$

In the case of nucleon-nucleon scattering $F_4^J = F_6^J$. The crossing-symmetry relation remains in the same form (in this case $I_i = I_f$) and in addition, the hermitian and antihermitian parts are given by

$$\begin{gathered} D^J(p_1, q_1; p_2, q_2) = \sum_{i=1}^{6} \bar{u}_b(p_2)\Gamma_i^{(b)}(q) u_b(p_1) \bar{u}_a(q_2) \Gamma_i^{(a)}(p) u_a(q_1) \operatorname{Re} F_i^J(s, t), \\ A^J(p_1, q_1; p_2, q_2) = \sum_{i=1}^{6} \bar{u}_b(p_2)\Gamma_i^{(b)}(q) u_b(p_1) \bar{u}_a(q_2) \Gamma_i^{(a)}(p) u_a(q_1) \operatorname{Im} F_i^J(s, t). \end{gathered} \tag{6.14}$$

The choice of the matrices (6.13) is convenient, in particular, because it separates the amplitude $F_2^J$ with the same matrix structure as the Coulomb amplitude. In the case of forward scattering (under the asumption that $\alpha(0) = 1$), theorem II enables us to prove the following asymptotic relations:

$$\begin{gathered} D^I(p_1, q_1; p_1, q_1) \sim -D^{II}(p_1, q_1; p_1, q_1), \\ A^I(p_1, q_1; p_1, q_1) \sim -A^{II}(p_1, q_1; p_1, q_1). \end{gathered} \tag{6.15}$$

The second equality leads to the asymptotic equality of the total cross sections:

$$\sigma_{tot}(pp) \sim \sigma_{tot}(\bar{p}p), \qquad \sigma_{tot}(\Sigma p) \sim \sigma_{tot}(\bar{\Sigma}p) \text{ etc.} \tag{6.16}$$

## 7. HIGHER SYMMETRIES OF STRONG INTERACTIONS AND ASYMPTOTIC RELATIONS BETWEEN THE AMPLITUDES OF MESON-BARYON SCATTERING AND PHOTOPRODUCTION

### 1. Meson-baryon Scattering and Photoproduction in Schemes with Higher Symmetry

Following the development of the Gell-Mann and Nishijima scheme for the classification of elementary particles, in which isotopic invariance holds, attempts were made to construct schemes for strong interactions with higher symmetries. At present much attention is being paid to unitary symmetry and to the symmetry of group $G_2$ (see [3]). The possibility of experimentally verifying these symmetries was already discussed in several papers. In particular, some relations were obtained for these models between the cross section of the meson-baryon and baryon-baryon scattering processes [44-47], and also between the meson-nucleon photoproduction amplitudes [44,48].

In the discussed schemes of strong interactions with higher symmetries, all the mesons and their antiparticles belong to one and the same multiplet. If the total cross sections of the meson-baryon interaction of the particles and antiparticles tend to constant limits when $s \to \infty$, then, according to the Pomeranchuk theorem, these limits are equal to each other. This circumstance reduces a number of independent scattering amplitudes in the model with higher symmetries[49,50].

As shown in the preceding sections, the differential cross sections of the crossing processes for a fixed momentum transfer and for high energies are asymptotically equal, and certain asymptotic relations hold for their amplitudes. Inasmuch as in models with higher symmetries the meson and baryon antiparticles belong to the same multiplet and there are crossing processes between the meson-baryon scattering in processes, the asymptotic relations between the amplitudes of the crossing processes together with the symmetry properties of the interactions lead to additional asymptotic relations between the cross sections of the processes under consideration.

We shall establish in this section several asymptotic relations between the cross sections of the meson-baryon scattering processes and the triplet and octet models of unitary symmetry and in the model with symmetry group $G_2$, and also asymptotic relations for the polarization effects in meson-baryon scattering and photoproduction of a meson on a nucleon. At low energies, the higher symmetries under consideration are broken. However, in all papers on higher symmetries of strong interactions, the hope is expressed that this symmetry is restored at high energies and large momentum transfers, or, at least, when one of these quantities is large. The obtained relations between the cross sections and the polarizations are the consequence of higher symmetries at high energies. Comparison of these relations with the experimental data can shed light on the discussed symmetry properties of strong interactions at high energies.

We shall consider the following meson-baryon scattering processes:

$$\pi^+ + p \to \pi^+ + p \quad (7.1a), \qquad \pi^- + p \to \pi^- + p \quad (7.1b),$$
$$\pi^0 + p \to \pi^+ + n \quad (7.2a), \qquad \pi^- + p \to \pi^0 + n \quad (7.2b),$$
$$K^+ + p \to K^+ + p \quad (7.3a), \qquad K^- + p \to K^- + p \quad (7.3b),$$
$$K^0 + p \to K^0 + p \quad (7.4a), \qquad \bar{K}^0 + p \to \bar{K}^0 + p \quad (7.4b),$$
$$K^0 + p \to K^+ + n \quad (7.5a), \qquad K^- + p \to \bar{K}^0 + n \quad (7.5b),$$
$$\pi^+ + p \to K^+ + \Sigma^+ \quad (7.6a), \qquad K^- + p \to \pi^- + \Sigma^+ \quad (7.6b),$$
$$\pi^- + p \to K^+ + \Sigma^- \quad (7.7a), \qquad K^- + p \to \pi^+ + \Sigma^- \quad (7.7b),$$
$$\pi^- + p \to K^0 + \Sigma^0 \quad (7.8a), \qquad \bar{K}^0 + p \to \pi^+ + \Sigma^0 \quad (7.8b),$$
$$\pi^- + p \to K^0 + \lambda \quad (7.9a), \qquad \bar{K}^0 + p \to \pi^+ + \lambda \quad (7.9b),$$
$$\pi^0 + p \to K^+ + \Sigma^0 \quad (7.10a), \qquad K^- + p \to \pi^0 + \Sigma^0 \quad (7.10b),$$
$$\pi^0 + p \to K^+ + \lambda \quad (7.11a), \qquad K^- + p \to \pi^0 + \lambda \quad (7.11b),$$
$$\pi^0 + p \to K^0 + \Sigma^+ \quad (7.12a), \qquad \bar{K}^0 + p \to \pi^0 + \Sigma^+ \quad (7.12b),$$
$$K^0 + p \to K^+ + \Xi^0 \quad (7.13a), \qquad K^- + p \to K^0 + \Xi^0 \quad (7.13b),$$
$$K^- + p \to K^+ + \Xi^- \quad (7.14),$$

and also the following processes of photoproduction of mesons on nucleons:

$$\gamma + p \to K^+ + \lambda, \qquad (7.15)$$
$$\gamma + n \to K^0 + \lambda, \qquad (7.16)$$
$$\gamma + p \to K^0 + \Sigma^+. \qquad (7.17)$$

We note that in the crossing transformation the process (7.ja) go over into processes (7.jb), while process (7.14) goes over into itself. For convenience we consider also some unphysical processes with $\pi^0$ mesons in initial states.

In addition to processes (7.1a, b)—(7.14), there exist the processes obtained by means of the substitutions $n \leftrightarrow p$, $\pi^+ \leftrightarrow \pi^-$, $K^+ \leftrightarrow K^0$, $\Sigma^+ \leftrightarrow \Sigma^-$ and $\Xi^- \leftrightarrow \Xi^0$, whose amplitudes coincide, apart from the sign, with the amplitudes of the corresponding processes in (7.1a, b)—(7.14). The relations derived below are valid for these processes, too. Exceptions are the processes with neutral K mesons in initial states, for example processes (7.4a) and (7.4c) (see below). In place of these processes we shall consider the corresponding mirror processes

$$K^+ + n \to K^+ + n, \qquad (7.4a')$$
$$K^- + n \to K^- + n, \qquad (7.4b')$$

the amplitudes of which are equal to the amplitudes of processes (5.4a) and (5.4b), respectively, and the physical processes

$$K_2^0 + p \to K_2^0 + p, \qquad (7.4c)$$
$$K_2^0 + p \to K_1^0 + p. \qquad (7.4d)$$

If we denote the invariant amplitudes of the processes (7.4a) ... (7.4d) by $F_i^{4a}(s, t)$, ..., $F_i^{4d}(s, t)$ respectively, ($i = 1, 2$), and neglect weak interactions, then

$$F_i^{4c}(s, t) = \frac{1}{2}\,[F_i^{4a}(s, t) + F_i^{4b}(s, t)], \qquad (7.18)$$

$$F_i^{4d}(s, t) = \frac{1}{2i}\,[F_i^{4a}(s, t) - F_i^{4b}(s, t)]. \qquad (7.19)$$

In place of processes (7.5a), (7.8b), (7.9b), (7.12c), and (7.13a) one observes experimentally only the corresponding physical processes with $K_2^0$ mesons in initial states:

$$K_2^0 + p \to K^+ + n, \qquad (7.5c)$$
$$K_2^0 + p \to \pi^+ + \Sigma^0, \qquad (7.8c)$$
$$K_2^0 + p \to \pi^+ + \lambda, \qquad (7.9c)$$
$$K_2^0 + p \to \pi^0 + \Sigma^+, \qquad (7.12c)$$
$$K_2^0 + p \to K^+ + \Xi^0, \qquad (7.13c)$$

the cross sections of which are equal to the cross sections of corresponding processes with neutral $K^0$ or $\bar{K}^0$ mesons in the initial state, multiplied by 1/2. We note that all the discussed symmetries include isotopic invariance, which leads to the following relations between the amplitudes of the processes in question:

$$F_i^{1a}(s,t)-F_i^{1b}(s,t)=-\sqrt{2}\,F_i^{2a}(s,t)=\sqrt{2}\,F_i^{2b}(s,t), \quad (7.20)$$

$$F_i^{4a,b}(s,t)+F_i^{5a,b}(s,t)=F_i^{3a,b}(s,t), \quad (7.21)$$

$$F_i^{6a,b}(s,t)-F_i^{7a,b}(s,t)=\sqrt{2}\,F_i^{8a,b}(s,t)=-\sqrt{2}\,F_i^{12a,b}(s,t), \quad (7.22)$$

$$F_i^{6a,b}(s,t)+F_i^{7a,b}(s,t)=2F_i^{10a,b}(s,t), \quad (7.23)$$

$$F_i^{9a,b}(s,t)=\sqrt{2}\,F_i^{11a,b}(s,t), \quad (7.24)$$

$$F_i^{13a}(s,t)+F_i^{13b}(s,t)=-F_i^{14}(s,t). \quad (7.25)$$

These relations take place in all the models under consideration.

2. Higher Symmetries and Relations Between Amplitudes

We now proceed to establish relations between amplitudes of processes (7.1a, b)—(7.14). We consider first the octet model. In this model the wave functions of the initial and final state in processes (7.1a, b)—(7.14) are products of two representations $D^8(1, 1)$. These products are resolved into irreducible representations in the following manner:

$$D^8(1,1)\times D^8(1,1)=D^1(0,0)+D^8(1,1)_s + D^8(1,1)_a+D^{10}(3,0)+D^{10+}(0,3)+D^{27}(2,2).$$

In the product under consideration there are two $D^8(1, 1)$ representations. It will be convenient in what follows to choose these representations in such a way that the wave functions of one of them are even relative to R-reflection, where the wave functions of the other are odd. We denote these representations by $D^8(1, 1)_s$ and $D^8(1, 1)_a$. Invariance of the interactions relative to unitary transformations requires that the matrix elements between the states belonging to non-equivalent irreducible representations vanish. Since R-reflection does not enter into the group, the matrix elements between the states of which one belongs to $D^8(1, 1)_s$ and the other to $D^8(1, 1)_a$ are not equal to zero. Moreover, from the invariance to time reflection it follows that the matrix element with initial state $D^8(1, 1)_a$ and final state $D^8(1, 1)_s$ is equal to the matrix element with initial state $D^8(1, 1)_s$ and final state $D^8(1, 1)_a$. Thus, the matrix elements of the processes under consideration are expressed in terms of seven different independent amplitudes

$$F_i^{(0,0)},\quad F_i^{(2,2)},\quad F_i^{(1,1)s},\quad F_i^{(1,1)a},\quad F_i^{(1,1)as},\quad F_i^{(3,0)} \text{ and } F_i^{(0,3)}.$$

The coefficients of all these independent amplitudes can be calculated with the aid of the Clebsch-Gordan coefficients of the $SU_3$ group. From this we can obtain the following relations between the amplitudes of the processes (7.1a, b)—(7.14)[45-47]:

$$F_i^{1a,b}(s,t)+F_i^{6a,b}(s,t)=F_i^{3a,b}(s,t), \quad (7.26)$$

$$\sqrt{6}\,F_i^{9a,b}(s,t)-\sqrt{2}\,F_i^{8a,b}(s,t)=2\,[F_i^{7a,b}(s,t)-F_i^{5a,b}(s,t)], \quad (7.27)$$

$$\sqrt{3}\,F_i^{11a,b}(s,t)+\sqrt{2}\,F_i^{2a,b}(s,t)=F_i^{10a,b}(s,t)-F_i^{5a,b}(s,t), \quad (7.28)$$

$$F_i^{13a,b}(s,t)=F_i^{7a,b}(s,t). \quad (7.29)$$

If R invariance holds, then

$$F_i^{(1,1)as}=0,\qquad F_i^{(3,0)}=F_i^{(0,3)},$$

and we obtain the additional relations:

$$F_i^{1a,b}(s,t)=F_i^{4a,b}(s,t), \quad (7.30)$$

$$F_i^{5a,b}(s,t)=F_i^{6a,b}(s,t), \quad (7.31)$$

$$F_i^{8a}(s,t)-F_i^{8b}(s,t)=F_i^{2b}(s,t), \quad (7.32)$$

$$F_i^{9a}(s,t)-F_i^{9b}(s,t)=\sqrt{3}\,F_i^{2b}(s,t), \quad (7.33)$$

$$F_i^{9a}(s,t)-F_i^{9b}(s,t)=\sqrt{3}\,[F_i^{8a}(s,t)-F_i^{8b}(s,t)], \quad (7.34)$$

$$F_i^{9a}(s,t)+F_i^{9b}(s,t)=-\frac{1}{\sqrt{3}}\,[F_i^{8a}(s,t)+F_i^{8b}(s,t)]. \quad (7.35)$$

The relations between the amplitudes of the processes in which nucleons and the λ hyperon participate in the Sakata triplet model with unitary symmetry can also be obtained in similar fashion. For these processes, the wave functions in initial and final states are products of irreducible representations $D^3(1, 0)$ and $D^8(1, 1)$. These products can be resolved into irreducible representations in the following manner:

$$D^3(1,0)\times D^8(1,1)=D^3(1,0)+D^6(0,2)+D^{15}. \quad (2.1)$$

Therefore the matrix elements of the meson-baryon scattering processes with participation of nucleons and λ hyperons in the triplet model of Sakata with unitary symmetry are expressed in terms of the independent amplitudes

$$F_i^{(1,0)},\quad F_i^{(0,2)} \text{ and } F_i^{(2,1)}.$$

The coefficients of these independent amplitudes can also be calculated with the aid of the Clebsch-Gordan coefficients.

As a result we obtain the following relations between the amplitudes in question[44,47]:

$$F_i^{1a,b}(s,t)=F_i^{3a,b}(s,t), \quad (7.36)$$

$$F_i^{4a}(s,t)=F_i^{4b}(s,t), \quad (7.37)$$

$$F_i^{5a,b}(s,t)=F_i^{9a,b}(s,t). \quad (7.38)$$

We now establish the relations between the amplitudes of the processes under consideration in the presence of the symmetry group $G_2$. Since the λ hyperon in this model is a singlet, the wave functions of the final states in the considered processes in which a λ hyperon participates, (7.9a, c) and (7.11b), belong to the representation $D^7(1, 0)$. The wave functions of the other states are products of two representations $D^7(1, 0)$. These products are resolved into irreducible representations in the following manner:

$$D^7(1,0)\times D^7(1,0)=D^1(0,0)+D^7(1,0)+D^{14}(0,1)+D^{27}(2,0).$$

Therefore the matrix elements of the processes with participation of the λ hyperon are expressed in terms of the same amplitude $F_i^{(1,0)}$. They are proportional to each other:

$$F_i^{9a}(s, t) = -F_i^{9b}(s, t), \tag{7.39}$$

$$F_i^{11a}(s, t) = -F_i^{11b}(s, t). \tag{7.40}$$

We recall that these amplitudes are connected by the isotopic relation (7.24). As to the amplitudes of the remaining processes, they are expressed in terms of the following four independent amplitudes:

$$F_i^{(0,0)}, \quad F_i^{(1;0)}, \quad F_i^{(0,1)} \text{ and } F_i^{(2,0)}.$$

The coefficients of these independent amplitudes can be calculated with the aid of the Clebsch-Gordan coefficients of the $G_2$ group[46]. As a result we obtain

$$F_i^{1a,b}(s, t) = F_i^{4a,b}(s, t), \tag{7.41}$$

$$F_i^{5a,b}(s, t) = -F_i^{6a,b}(s, t), \tag{7.42}$$

$$F_i^{8a,b}(s, t) = -F_i^{2a,b}(s, t), \tag{7.43}$$

$$F_i^{7a,b}(s, t) = F_i^{13a,b}(s, t), \tag{7.44}$$

$$F_i^{1a,b}(s, t) - F_i^{7b,a}(s, t) = F_i^{3b,a}(s, t). \tag{7.45}$$

3. Asymptotic Relations for the Cross Sections and Polarizations

On the basis of the relations that follow for the meson-baryon scattering amplitudes from the sym-

| Relations | Model and method | Literature |
|---|---|---|
| $\sigma(\pi^\pm p \to \pi^\pm p) - \frac{1}{2}\sigma(\pi^- p \to \pi^0 n) \geqslant 0$ | $I$, P-L | 27 |
| $\sigma(\pi^\pm p \to \pi^\pm p) - \frac{1}{2}\sigma(\pi^- p \to \pi^0 n) = \frac{1}{16\pi}[\sigma_{tot}(\pi^\pm p)]^2,\ t=0$ | $I$, P-L | 21, 27 |
| $\sigma(K^- p \to K^0\Xi^0) - \frac{1}{4}\sigma(K^- p \to K^+\Xi^-) > 0$ | $I$, P-L | 27 |
| $P(\pi^- p \to \pi^0 n) = \eta(\pi^- p \to \pi^0 n) = 0$ | $I$, P-L | 27 |
| $\sigma(K^- p \to \overline{K}^0 n) = \sigma(K^- p \to \pi^-\Sigma^+)$ | O, $G_2$ | 44–48 |
| $\sigma(K^0 p \to K^+ n) = \sigma(\pi^+ p \to K^+\Sigma^+)$ | O, $G_2$ | 44–48 |
| $\overline{\sigma(K^0} p \to K^+\Xi^0) = \sigma(\pi^- p \to K^+\Sigma^-)$ | O, $G_2$ | 44–48 |
| $\sigma(K^- p \to K^0\Xi^0) = \sigma(K^- p \to \pi^+\Sigma^-)$ | O, $G_2$ | 44–48 |
| $\sigma(\pi^\pm p \to \pi^\pm p) = \sigma(K^\pm n \to K^\pm n)$ | O, $G_2$ | 44–48 |
| $\sigma_{tot}(\pi^\pm p) = \sigma_{tot}(K^\pm n)$ | O, $G_2$ | 44–48 |
| $\sigma(K_2^0 p \to K_1^0 p) = \frac{1}{2}\sigma(\pi^- p \to \pi^0 n)$ | O, $G_2$, P-L | 27 |
| $\sigma(\pi^\pm p \to \pi^\pm p) = \sigma(K_2^0 p \to K_2^0 p) + \sigma(K_2^0 p \to K_1^0 p)$ | O, $G_2$, P-L | 27 |
| $\sigma(\pi^- p \to K^0\Sigma^0) - \frac{1}{4}\sigma(\pi^- p \to \pi^0 n) > 0$ | O, P-L | 27 |
| $\sigma(\pi^- p \to K^0\lambda) - \frac{3}{4}\sigma(\pi^- p \to \pi^0 n) > 0$ | O, P-L | 27 |
| $\sigma(\pi^\pm p \to \pi^\pm p) = \sigma(K^\pm p \to K^\pm p)$ | T | 44–48 |
| $\sigma_{tot}(\pi^\pm p) = \sigma(K^\pm p)$ | T | 44–48 |
| $\sigma(K^- p \to \overline{K}^0 n) = \sigma(\pi^- p \to K^0\lambda)$ | T | 44–48 |
| $\sigma(K^0 p \to K^+ n) = \sigma(\overline{K}^0 p \to \pi^+\lambda)$ | T | 44–48 |
| $\sigma(K^+ n \to K^+ n) = \sigma(K^- n \to K^- n)$ | T | 44–48 |
| $\sigma_{tot}(K^+ n) = \sigma_{tot}(K^- n)$ | T | 44–48 |
| $\sigma(K_{2p}^0 \to K_{2p}^0) = \sigma(K^\pm n \to K^\pm n)$ | T | 27 |
| $\sigma_{tot}(K_{2p}^0) = \sigma_{tot}(K^\pm n)$ | T | 27 |
| $\sigma_{tot}(K_2^0 p \to K_1^0 p) = 0$ | T | 27 |
| $P(K^\pm n \to K^\pm n) = \eta(K^\pm n \to K^\pm n) = 0$ | T, P-L | 27 |
| $\sigma(\pi^- p \to K^0\Sigma^0) = \sigma(\pi^- p \to \pi^0 n)$ | $G_2$ | 27 |
| $\sigma(\gamma p \to \pi^+ n) = \sigma(\gamma p \to K^+\lambda)$ | T | 44 |
| $P(\gamma n \to K^0\lambda) = -\eta(\gamma n \to K^0\lambda)$ | T, P-L | 23 |
| $P(\gamma p \to K^0\Sigma^+) = -\eta(\gamma p \to K^0\Sigma^+)$ | O, P-L | 23 |

metry properties of the interactions, and the asymptotic relations between the amplitudes of the crossing processes, we can obtain several simple asymptotic equations for the differential cross sections, polarizations, and asymmetry parameters of the processes. For comparison we have tabulated all the relations obtained in[27] and in other papers. By $\sigma(\pi^+p \to K^+\Sigma^+)$ we denote the differential cross section of the process $\pi^+ + p \to K^+ + \Sigma^+$ for a certain t, while $P(\pi^+p \to K^+\Sigma^+)$ denotes the polarization of the $\Sigma^+$ hyperon in this process with unpolarized proton, $P(\gamma p \to \pi^+ n)$ denotes the neutron polarization in the photoproduction process $\gamma + p \to \pi^+ + n$ with unpolarized initial particles, while $\eta(\gamma p \to \pi^+ n)$ denotes the asymmetry parameter in this process with a polarized proton. The table lists also the model in which the given relation holds true and shows whether the Phragmen-Lindelof (or the Pomeranchuk) theorem is used in the proof of this relation or not. We use the following symbols for the models: I—isotopic invariance, O—octet model with R-invariance, T—triplet model, $G_2$—model with symmetry group $G_2$, P-L—Phragmen-Lindelof theorem.

## 8. ASYMPTOTIC RELATIONS BETWEEN THE AMPLITUDES OF PROCESSES WITH A VARIABLE NUMBER OF PARTICLES

### 1. Kinematics

We have obtained above the asymptotic relations between the amplitudes of the binary crossing processes (scattering and photoproduction). On the basis of these relations we have established asymptotic relations for the differential cross sections and the polarizations. In this section we establish asymptotic relations between the amplitudes of processes with a variable number of particles, and obtain, in particular, asymptotic equality of the differential cross sections for these processes.

For simplicity let us consider the production of pions in meson-nucleon collisions:

$$\pi + N \to \pi' + \pi'' + N', \qquad \text{(I)}$$

$$\bar{\pi} + N' \to \bar{\pi}' + \bar{\pi}'' + N, \qquad \text{(II)}$$

where $\pi$, $\pi'$ or $\pi''$ stands for one of the pions, and N or N′ stand for p or n. The reasoning presented below can be applied, with slight modification of kinematic nature, to any other process, such as

$$a + b \to c + d + e, \qquad \text{(I}'\text{)}$$

$$\bar{a} + e \to \bar{c} + \bar{d} + b. \qquad \text{(II}'\text{)}$$

The processes (I) and (II) or (I′) and (II′) will henceforth be called crossing processes. An analogous definition of crossing processes in photoproduction was given in Sec. 5.

Let us consider, for example, process (I). We denote the 4-momenta of the nucleons in initial and final states by p and p′, respectively, and the 4-momenta of the mesons by q, q′, and q″:

$$p + q = p' + q' + q'', \qquad (8.1)$$

$$p^2 = p'^2 = -M^2, \quad q^2 = q'^2 = q''^2 = -m^2, \qquad (8.2)$$

where M and m are the masses of the nucleon and the meson. We introduce for convenience the following more symmetrical notation for the momenta and their squares:

$$p_1 = p, \quad p_2 = q, \quad p_3 = -p', \quad p_4 = -q', \qquad (8.3)$$
$$p_5 = -q'', \quad p_j^2 = -m_j^2.$$

The kinematics of the process in which five particles participate is characterized by five invariants, which can be chosen from among the following ten variables:

$$s_{ij} = -(p_i + p_j)^2, \qquad i, j = 1, 2, 3, 4, 5; \qquad (8.4)$$

the quantities $s_{ij}$ and $m_j^2$ are connected by five linear relations

$$\sum_{\substack{j=1 \\ j \neq i}}^{5} s_{ij} = m_i^2 + \sum_{j=1}^{5} m_j^2, \qquad i = 1, 2, 3, 4, 5, \qquad (8.5)$$

or

$$s_{ij} + s_{jk} + s_{ki} - s_{ln} = m_i^2 + m_j^2 + m_k^2, \qquad (8.6)$$

where i, j, k, $l$, or n is any permutation of the numbers 1, 2, 3, 4, 5 (only five of the 120 equations in (8.6) are independent; they are equivalent to (8.5)).

Following[51,52], we choose the following independent variables:

$$t \equiv s_{13} = -(p - p')^2, \qquad t'' \equiv s_{25} = -(q - q'')^2,$$
$$w^2 \equiv s_{45} = -(q' + q'')^2, \qquad e^{2\xi} = \frac{q'(p + p')}{q''(p + p')},$$
$$\omega = \frac{q(p + p')}{4 \operatorname{ch} \xi \sqrt{M^2 - \frac{t}{4}}}. \qquad (8.7)^*$$

This choice of variables is convenient because by fixing the variables $t \le 0$, $t'' \le 0$, $w^2 \ge 4m^2 \cosh \xi$, and $\xi$ we can let the energy variable $\omega$ go to infinity while remaining in the physical region. The three first variables, t, t″, and $w^2$, have a clear-cut physical meaning. The energy variable $\omega$ is connected with the total energy of the reaction in the c.m.s.:

$$s \equiv s_{12} = 4\omega \operatorname{ch} \xi \sqrt{M^2 - \frac{t}{4}} + M^2 + \frac{m^2 + w^2 - t}{2}. \qquad (8.8)$$

When $\omega$ is large the variable $\xi$ determines the ratio of the energy of the system comprising the nucleon and one of the mesons in the final state, for example $\sqrt{s_{34}}$, to the total energy of the process ($\sqrt{s}$). Indeed, it is easy to verify that

*ch ≡ cosh.

$$\left.\begin{aligned}
u &\equiv s_{23} = -(q-p')^2 = -4\omega\,\mathrm{ch}\,\xi\sqrt{M^2-\frac{t}{4}}+M^2+\frac{m^2+w^2-t}{2},\\
s' &\equiv s_{34} = -(q'+p')^2 = 2\omega e^{\xi}\sqrt{M^2-\frac{t}{4}}+M^2+\frac{m^2+t''-t}{2},\\
u' &\equiv s_{14} = -(q'-p)^2 = -2\omega e^{\xi}\sqrt{M^2-\frac{t}{4}}+M^2+\frac{m^2+t''-t}{2},\\
s'' &\equiv s_{35} = -(q''+p')^2 = 2\omega e^{-\xi}\sqrt{M^2-\frac{t}{4}}+M^2+2m^2-\frac{w^2+t''}{2},\\
u'' &\equiv s_{15} = -(q''-p)^2 = -2\omega e^{-\xi}\sqrt{M^2-\frac{t}{4}}+M^2+2m^2-\frac{w^2+t''}{2},\\
t' &\equiv s_{24} = -(q-q')^2 = 3m^2+t-t''+w^2,
\end{aligned}\right\} \quad (8.9)$$

and

$$e^{2\xi} = \frac{s'-u'}{s''-u''} \xrightarrow[s\to\infty]{} \frac{s'}{s-s'}. \quad (8.10)$$

In place of the vectors q′ and q″ it is convenient to introduce vectors Q and Δ which are orthogonal to each other and are defined by the formula

$$\Delta = \frac{1}{2}(e^{-\xi}q' - e^{\xi}q''),$$

$$Q = \frac{1}{2}(e^{-\xi}q' + e^{\xi}q'') + \frac{m^2}{2\Delta^2}\,\mathrm{sh}\,2\xi\Delta. \quad (8.11)^*$$

We shall consider the processes (I) and (II) in the Breit system for the nucleon

$$\mathbf{p} + \mathbf{p}' = 0. \quad (8.12)$$

Then, of all the vectors p, p′, Q and Δ, in terms of which any 4-vector can be expressed (linearly), only the vector Q depends on $\omega$.

We note also that for the chosen variables the differential cross section will be

$$\frac{d^4\sigma^J(s,\,t,\,t'',\,w^2,\,\xi)}{dt\,dt''\,dw^2\,d\xi}.$$

## 2. Properties of the Asymptotic Amplitudes

The retarded amplitude of process (I) can be written in the form[51-53]

$$T^{\mathrm{I}}(p,q;\,p',q',q'')$$

$$= \int d^4x'\,d^4x''e^{-i(q'x'+q''x'')}\left\langle p'\left|\frac{\delta^2 j_\pi(0)}{\delta\varphi_{\pi'}(x')\,\delta\varphi_{\pi''}(x'')}\right|p\right\rangle$$

$$= T^{\mathrm{ret}}(\omega) = \int d^4x\,d^4y e^{i(Qx+\Delta y)}F^{\mathrm{ret}}(x,y), \quad (8.13)$$

where

$$j_\pi(x) = i\frac{\delta S}{\delta\varphi_\pi^+(x)}S^+. \quad (8.14)$$

By virtue of the microcausality condition we have

$$F^{\mathrm{ret}}(x,y) \neq 0$$

only in the region

$$x>0,\ -\left(1-\frac{m^2}{2\Delta^2}\mathrm{sh}\,2\xi\right)x<y<\left(1+\frac{m^2}{2\Delta^2}\mathrm{sh}\,2\xi\right)x. \quad (8.15)$$

Introducing

$$G^{\mathrm{ret}}(x_0,x_1) = \int dx_2\,dx_3\,d^4y e^{i(\Delta y-Q_2x_2-Q_3x_3)}F^{\mathrm{ret}}(x,y), \quad (8.16)$$

we can write $T^{\mathrm{ret}}(\omega)$ in the form

$$T^{\mathrm{ret}}(\omega) = \int dx_0\,dx_1\,e^{i\{\omega x_0-\sqrt{\omega^2-\omega_0^2}\,x_1\}}G^{\mathrm{ret}}(x_0,x_1), \quad (8.17)$$

where $\omega_0$ is given by formula (8.18)

$$4\,\mathrm{ch}^2\xi\,\omega_0^2 = w^2 + \frac{w^4}{4\Delta^2}\frac{\mathrm{sh}^2\xi}{\sin^2\alpha} - \frac{(w^2+t-m^2)^2}{4t\sin^2\alpha}$$

$$+\frac{w^2(w^2+t-m^2)}{4|\Delta||\mathbf{p}_1|}\cdot\frac{\mathrm{sh}\,\xi\cos\alpha}{\sin\alpha}, \quad (8.18)$$

$$\cos\alpha = \frac{(w^2-m^2)e^{\xi}-te^{-\xi}-2(m^2-t'')\,\mathrm{ch}\,\xi}{8|\Delta||\mathbf{p}|}. \quad (8.19)$$

Since the amplitude $T^{\mathrm{ret}}(\omega)$ is regarded as a function of $\omega$ with the remaining variables fixed, $\omega_0$ in (8.17) is a constant.

Further, following[54], we replace the amplitude (8.17) by means of the following asymptotic amplitude:

$$T_\infty^{\mathrm{ret}}(\omega) = \int dx_0\,dx_1\,e^{i\omega(x_0-x)}G^{\mathrm{ret}}(x_0,x_1). \quad (8.20)$$

Denchev established general conditions under which $T^{\mathrm{ret}}(\omega)$ and $T_\infty^{\mathrm{ret}}(\omega)$ coincide asymptotically, i.e.,

$$\lim_{\omega\to\infty}\frac{T^{\mathrm{ret}}(\omega)}{T_\infty^{\mathrm{ret}}(\omega)} = 1.$$

This equality is certainly satisfied if the amplitude does not oscillate rapidly, and for arbitrary $\epsilon > 0$ and sufficiently large $\omega$ it is bounded by the inequality

$$B_\varepsilon e^{-\varepsilon|\omega|} \leqslant |T^{\mathrm{ret}}(\omega)| \leqslant A_\varepsilon e^{\varepsilon|\omega|}. \quad (8.21)$$

By virtue of the microcausality condition $G^{\mathrm{ret}}(x_0, x_1) \neq 0$ only when $x_0 > |x_1|$. Therefore $T_\infty^{\mathrm{ret}}(\omega)$ is analytic only in the upper half of the $\omega$ plane (that is, when $\mathrm{Im}\,\omega > 0$) and the Phragmen-Lindelof theorem is applicable to it.

We now establish the crossing-symmetry relations between the amplitudes of processes (I) and (II). We denote by p and p′ the 4-momenta of the nucleons in the initial and final states of process (II), and by q, q′, and q″, respectively, the 4-momenta of the mesons. The amplitude of this process takes the form

$$T^{\mathrm{II}}(p,q;\,p',q',q'')$$

$$= \int d^4x'\,d^4x''\,e^{-i(q'x'+q''x'')}\left\langle p'\left|\frac{\delta^2 j_\pi^+(0)}{\delta\varphi_{\pi'}^+(x')\,\delta\varphi_{\pi''}^+(x'')}\right|p\right\rangle. \quad (8.22)$$

*sh ≡ sinh.

Let us consider the amplitude (8.22) in the unphysical region $T^{II}(p_1', -q;\ p, -q', -q'')$. For these values of the momenta we have also the conservation law (8.1). Then

$$T^{II}(p', -q;\ p, -q', -q'') = \left[\int d^4x'\, d^4x''\, e^{-i(q'x'+q''x'')} \left\langle p' \left| \frac{\delta^2 j_\pi(0)}{\delta\varphi_{\pi'}(x')\,\delta\varphi_{\pi''}(x'')} \right| p \right\rangle\right]^*, \tag{8.23}$$

that is, we obtain the crossing-symmetry relation between the amplitudes of processes (I) and (II)

$$T^{I}(p, q;\ p', q', q'') = T^{II}(p', -q;\ p, -q', -q'')^*. \tag{8.24}$$

### 3. Asymptotic Equality of the Differential Cross Sections

From invariance considerations, it follows that the amplitudes $T^J(p, q;\ p', q', q'')$ are of the form

$$T^J(p, q;\ p', q', q'') = \sum_{i=1}^{4} \bar{u}(p')\,\Gamma_i u(p)\, F_i^J(\omega, t, t'', w^2, \xi), \tag{8.25}$$

where

$$M_1 = \gamma_5,\quad M_2 = i(\hat{q}_1 + \hat{q}_2)\gamma_5,\quad M_3 = i(\hat{q}_1 - \hat{q}_2)\gamma_5,$$
$$M_4 = (\hat{q}_1 + \hat{q}_2)(\hat{q}_1 - \hat{q}_2)\gamma_5. \tag{8.26}$$

Substituting (8.25) and (8.26) in (8.24), we obtain the crossing-symmetry relations between the invariant amplitudes

$$\left.\begin{aligned} F_i^{I}(-\omega, t, t'', w^2, \xi) &= -F_i^{II}(\omega, t, t'', w^2, \xi)^*, \qquad i = 1, 2, 3, \\ F_i^{I}(-\omega, t, t'', w^2, \xi) &= F_i^{II}(\omega, t, t'', w^2, \xi)^*, \qquad i = 4. \end{aligned}\right\} \tag{8.27}$$

The differential cross sections of processes (I) and (II)

$$\frac{d^4\sigma^J(s, t, t'', w^2, \xi)}{dt\, dt''\, dw^2\, d\xi}$$

are proportional to the quantities

$$H^J = \sum_{i=1}^{4} A_i\, |F_i^J|^2 + \sum_{i<j} A_{ij}\, \mathrm{Re}\, F_i^J F_j^{J*}, \tag{8.28}$$

where asymptotically ($s, \omega \to \infty$)

$$\left.\begin{aligned} &A_1 = -2t,\quad A_2 = 2(s'+s'')^2,\quad A_3 = 2(s'-s'')^2, \\ &A_4 = 2(s'+s'')^2(w^2 - 4m^2) + 2(s'-s'')^2 w^2, \\ &A_{12} = 4M(t'+t''-2t),\quad A_{13} = 4M(t''-t'), \\ &A_{14} = -4[(t+m^2)(s'-s'') - t's' + t''s''], \\ &A_{23} = 4(s'^2 - s''^2),\quad A_{24} = 8M(s'-s'')\,w^2, \\ &A_{34} = 8M(s'+s'')(w^2 - 4m^2). \end{aligned}\right\} \tag{8.29}$$

Using expressions (8.28) and (8.29) for the cross sections and the crossing-symmetry relation (8.27), we can prove by means of the method developed above the asymptotic equality of the differential cross sections of processes (I) and (II). In particular, the differential cross sections of the processes

$$\pi^+ + p \to \pi^+ + \pi^0 + p \quad\text{and}\quad \pi^- + p \to \pi^- + \pi^0 + p \tag{8.30}$$

are constant when $s \to \infty$ and $t$, $t''$, $w^2$ and $\xi$ are fixed.

In the case when systems of two pions are produced in a resonant state with $J = 1$, $I = 1$ ($\rho$ meson), we have equality of the differential cross sections for the processes

$$\pi^+ + p \to \varrho^+ + p \quad\text{and}\quad \pi^- + p \to \varrho^- + p. \tag{8.31}$$

Let us discuss possible generalizations of the obtained results. We have considered above the case when the total energy $w$ of the system of two pions (in the c.m.s. of these two particles) is fixed, and the total energy $w_{\pi N}$ of the system comprising the nucleon and one of the mesons (in the c.m.s. of these particles) tends to infinity together with the total energy of the reaction $\sqrt{s}$ ($\xi$ is fixed). We can also consider in similar fashion the case when $w$ is fixed and the energy $w_{\pi N}$ tends to infinity together with $\sqrt{s}$, and prove the equality of the differential cross sections. When nucleon-meson systems are produced in the resonant state, we have equality of the differential cross sections of the processes, for example,

$$\pi^+ + p \to \pi^+ + \Delta^+ \ (\Delta^+ \to p + \pi^0) \quad\text{and}\quad \pi^- + p \to \pi^- + \Delta^+ \ (\Delta^+ \to p + \pi^0), \tag{8.32}$$

$$\pi^+ + n \to \pi^+ + \Delta^0 \ (\Delta^0 \to p + \pi^-) \quad\text{and}\quad \pi^- + p \to \pi^- + \Delta^0 \ (\Delta^0 \to p + \pi^-). \tag{8.33}$$

We can further consider also the case of mesons with different masses and obtain, in particular, asymptotic equality of the differential cross sections for the processes

$$K^+ + p \to K^+ + \pi^0 + p \quad\text{and}\quad K^- + p \to K^- + \pi^0 + p, \tag{8.34}$$

$$K^+ + p \to K^0 + \pi^+ + p \quad\text{and}\quad K^- + p \to \overline{K^0} + \pi^- + p, \tag{8.35}$$

$$\pi^- + p \to K^+ + K^- + n \quad\text{and}\quad \pi^+ + n \to K^- + K^+ + p. \tag{8.36}$$

In the case of production of two-meson systems or meson-nucleon systems in the resonant state, we have respectively the equality of the differential cross sections for the processes

$$K^+ + p \to K^{*+} + p \quad\text{and}\quad K^- + p \to K^{*-} + p, \tag{8.37}$$

$$\pi^- + p \to \varphi + n \quad\text{and}\quad \pi^+ + n \to \varphi + p \tag{8.38}$$

or

$$K^+ + p \to K^+ + \Delta^+ \ (\Delta^+ \to p + \pi^0) \quad\text{and}\quad K^- + p \to K^- + \Delta^+ \ (\Delta^+ \to p + \pi_0) \tag{8.39}$$

We can consider analogously the photoproduction of two mesons and the production of mesons in nucleon-nucleon collisions, for example,

$$\gamma + p \to \pi^+ + \pi^0 + n \quad\text{and}\quad \gamma + n \to \pi^- + \pi^0 + p, \tag{8.40}$$

$$p+p\to n+\pi^++p \quad \text{and} \quad \bar{n}+p\to \bar{p}+\pi^++p, \tag{8.41}$$

$$p+p\to p+\pi^0+p \quad \text{and} \quad \bar{p}+p\to \bar{p}+\pi^0+p, \tag{8.42}$$

and in the particular case

$$\gamma+p\to \varrho^++n \quad \text{and} \quad \gamma+n\to \bar{\varrho}+p, \tag{8.43}$$

$$p+p\to n+\Delta^{++}\ (\Delta^{++}\to p+\pi^+) \quad \text{and} \quad \bar{n}+p\to \bar{p}+\Delta^{++}\ (\Delta^{++}\to p+\pi^+), \tag{8.44}$$

$$p+p\to p+\Delta^+\ (\Delta^+\to p+\pi^0) \quad \text{and} \quad \bar{p}+p\to \bar{p}+\Delta^+\ (\Delta^+\to p+\pi^0). \tag{8.45}$$

## 9. ASYMPTOTIC BEHAVIOR OF THE FORM FACTORS

For the electromagnetic and weak interactions we can use perturbation theory and confine ourselves to the lowest order, since in these cases the interaction constants are small and the higher approximations are insignificant (at least at the energies now attainable). Allowance for the contribution of strong interactions leads to the appearance of form factors. The analytic properties of the form factors and the Phragmen-Lindelof theorem make it also possible to draw certain conclusions concerning the asymptotic behavior of these form factors.

We consider first electromagnetic scattering of an electron by a proton

$$e^-+p\to e^-+p$$

and the annihilation of a proton-antiproton pair into an electron-positron pair

$$\bar{p}+p\to e^++e^-.$$

In the $e^2$ approximation both processes are described by two form factors $F_1(t)$ and $F_2(t)$. In place of the form factors $F_1(t)$ and $F_2(t)$ it is convenient to use the form factors

$$G_E(t)=F_1(t)+\frac{t}{4M^2}\mu F_2(t),$$

$$G_\mu(t)=F_1(t)+\mu F_2(t),$$

where $t=-(p_1-p_2)^2$ is the momentum transfer for the first process and $t=-(p_1+p_2)^2$ is the square of the energy for the second. The differential cross sections of the processes in question are expressed in terms of the square of the moduli of the form factors $G_E$ and $G_\mu$.

A similar situation obtains in the theory of weak interactions. For example, in the lowest order of perturbation theory in weak interactions the matrix elements of the two processes

$$\bar{\nu}+p\to e^++n,$$

$$\bar{n}+p\to e^++\nu$$

are also expressed in terms of the same form factor $H_i(t)$.

The dispersion relations for the electromagnetic form factors and for the form factors of the weak interactions were established in several papers. These dispersion relations were proved by starting from the general principles of local quantum field theory. However, on the basis of perturbation theory it can be stated that the form factors $F_i(t)$ and $H_i(t)$ under consideration are analytic functions in the complex t plane, with a cut along the positive real axis[55]. In local quantum field theory the form factors increase in the complex t plane no faster than a polynomial. Let us assume that when $t\to\pm\infty$ the form factors have a definite growth rate and do not oscillate. Then from the Phragmen-Lindelof theorem it follows that the modulus of the ratio of the form factors $G_i(t)/G_i(-t)$ ($i=\mu$, E) or $E_i(t)/H_i(-t)$ tends to unity when $t\to\infty$. This makes it possible to relate the cross sections of the scattering and annihilation processes, if the lowest order of perturbation theory in electromagnetic weak interactions describes the processes in question well.

In connection with the asymptotic equality of the moduli of the form factors when $t\to\pm\infty$, special interest is attached to the case when the form factors tend to constant values when $t\to\pm\infty$. Let us assume that this takes place with a certain form factor $F_i(t)$. It follows from theorem II that the limiting values of this form factor when $t\to\pm\infty$ should coincide, that is,

$$\lim_{t\to+\infty}F_i(t)=\lim_{t\to-\infty}F_i(t). \tag{9.1}$$

But the form factor does not have an imaginary part on the negative real axis. Therefore when $t\to+\infty$ the imaginary part of $F_i(t)$ tends to zero and the integral

$$\frac{1}{\pi}\int_{t_0}^{\infty}\frac{\operatorname{Im}F_i(t')}{t'-t}\,dt' \tag{9.2}$$

converges. The integral along a large circle of radius R is equal to

$$\frac{1}{2\pi}\int_{C_R}\frac{F_i(e^{i\varphi}R)}{(e^{i\varphi}R-t)}\,Re^{i\varphi}\,d\varphi. \tag{9.3}$$

According to theorem II, $F_i(e^{i\varphi}R)$ tends uniformly to $F_i(\infty)$ when $R\to\infty$; therefore the interval along the large circle tends to a finite limit $F_i(\infty)$ and the dispersion relations take the form

$$F_i(t)=F_i(\infty)+\frac{1}{\pi}\int_{t_0}^{\infty}\frac{\operatorname{Im}F_i(t')}{t'-t}\,dt', \qquad \operatorname{Im}F_i(\infty)=0. \tag{9.4}$$

## 10. CONCLUSION

We now summarize our results. On the basis of the fundamental principles of local quantum field theory and some general assumptions concerning the behavior of the scattering amplitudes when $s\to\infty$ and t is fixed, we obtained asymptotic relations between the amplitudes of the crossing processes. From these relations for the amplitudes it is possible to establish similar asymptotic relations between the differential and total cross sections, and also between the polarizations, asymmetry parameters, etc.

The method used in the proof is described in detail in Sec. 2, where the crossing of scalar-particle scattering processes (I) and (II) is considered. These processes are characterized by invariant amplitudes $T^I(s, t)$ and $T^{II}(s, t)$, which are connected by the crossing-symmetry relation. If at fixed t and $s \to \infty$ these amplitudes do not oscillate, but have a definite (power-law or logarithmic) growth, then the asymptotic equality of the differential cross sections of processes (I) and (II) follows from the Phragmen-Lindelof theorem and from the crossing-symmetry relation.

In the case of elastic scattering this asymptotic equality can be obtained from weaker assumptions. In this case there are grounds for assuming that when $s \to \infty$, and t is fixed and belongs to a certain interval, the imaginary parts Im $T^J$ (s, t) are non-negative. Under this assumption it has been proved that if the differential cross sections of processes (I) and (II) do not oscillate, but have a definite growth when $s \to \infty$ and t is fixed, they are asymptotically equal.

In addition to asymptotic equality of the differential cross sections for the forward elastic-scattering amplitudes, there are several other asymptotic relations. For example, if for $s \to \infty$ the differential cross section for forward scattering (t = 0) and the total cross sections of the interaction tend to constant values, these total interaction cross sections are equal for the particle and the antiparticle. This is the well-known theorem of I. Ya. Pomeranchuk. Moreover, in this case the forward elastic-scattering amplitudes for the processes which go over into themselves in the crossing transformation (that is, the forward elastic-scattering amplitudes of truly neutral particles) are purely imaginary, and the differential cross section for forward scattering and the total cross section are connected by the asymptotic relation

$$\left.\frac{d\sigma(s,\ t)}{dt}\right|_{t=0} \sim \frac{1}{16\pi}\,[\sigma_{\text{tot}}(s)]^2. \qquad (10.1)$$

The Phragmen-Lindelof theorem also makes it possible to draw a certain conclusion concerning the behavior of the form factors $F_i(t)$ when $t \to \pm\infty$. If for $t \to \pm\infty$ they do not oscillate, then the moduli of the ratios $G_i(t)/G_i(-t)$ tend to unity. In particular, when $G_i(t)$ tend to constant values $G_i(\infty)$, on the positive axis the imaginary parts tend to zero and we can write the dispersion relations in the form

$$G_i(t) = G_i(\infty) + \frac{1}{\pi}\int_{t_0}^{\infty} \frac{\operatorname{Im} G_i(t')}{t'-t}\,dt', \qquad \operatorname{Im} G_i(\infty) = 0. \qquad (10.2)$$

The asymptotic relationship between the amplitudes of the crossing processes of meson-baryon and baryon-baryon scattering were considered in Secs. 3 and 4. From the obtained asymptotic relations between the invariant amplitudes of the crossing processes we established the asymptotic equality of the differential cross sections of these processes when $s \to \infty$ and t is fixed, the equality of the total interaction cross sections of the particle and antiparticle, and also some asymptotic relations between the polarizations and the asymmetry parameters.

It was shown, in particular, that at high energies and fixed momentum, the differential cross sections of the processes are asymptotically equal for each of the following pairs:

$$\pi^+ + p \to \pi^+ + p \quad \text{and} \quad \pi^- + p \to \pi^- + p, \qquad (10.3)$$
$$K^+ + p \to K^+ + p \quad \text{and} \quad K^- + p \to K^- + p, \qquad (10.4)$$
$$\pi^+ + p \to K^+ + \Sigma^+ \quad \text{and} \quad K^- + p \to \pi^- + \Sigma^+, \qquad (10.5)$$
$$\pi^- + p \to K^0 + \lambda \quad \text{and} \quad \bar{K}^0 + p \to \pi^+ + \lambda, \qquad (10.6)$$
$$K^- + p \to K^0 + \Xi^0 \quad \text{and} \quad \bar{K}^0 + p \to K^+ + \Xi^0, \qquad (10.7)$$
$$\Sigma^+ + \text{He} \to p + \text{He}_\lambda \quad \text{and} \quad \bar{p} + \text{He} \to \bar{\Sigma}^+ + \text{He}_\lambda, \qquad (10.8)$$
$$p + p \to p + p \quad \text{and} \quad \bar{p} + p \to \bar{p} + p, \qquad (10.9)$$
$$\Sigma^+ + p \to \Sigma^+ + p \quad \text{and} \quad \bar{\Sigma}^+ + p \to \bar{\Sigma}^+ + p, \qquad (10.10)$$
$$\Sigma^- + p \to \lambda + n \quad \text{and} \quad \bar{\lambda} + p \to \bar{\Sigma}^- + n, \qquad (10.11)$$
$$\Sigma^+ + p \to p + \Sigma^+ \quad \text{and} \quad \bar{p} + p \to \bar{\Sigma}^+ + \Sigma^+, \qquad (10.12)$$
$$\Sigma^- + p \to n + \lambda \quad \text{and} \quad \bar{n} + p \to \bar{\Sigma}^- + \lambda \qquad (10.13)$$

etc.

In all these processes t is defined as the momentum transferred between the first particle in the initial state and the first particle in the final state. For the last processes (10.12) and (10.13), this momentum transfer is frequently denoted by u. Here the differential cross section for the backward scattering of the hyperon and the differential cross section of the nucleon-antinucleon pair annihilation into a hyperon-hyperon pair are asymptotically equal.

If the total interaction cross sections tend to constant values on $s \to \infty$, and the differential cross sections for elastic scattering forward are bounded, then the following asymptotic equalities hold between the total interaction cross sections of the particles and antiparticles:

$$\sigma_{\text{tot}}(\pi^+p) \sim \sigma_{\text{tot}}(\pi^-p), \qquad (10.14)$$
$$\sigma_{\text{tot}}(K^+p) \sim \sigma_{\text{tot}}(K^-p), \qquad (10.15)$$
$$\sigma_{\text{tot}}(pp) \sim \sigma_{\text{tot}}(\bar{p}p), \qquad (10.16)$$
$$\sigma_{\text{tot}}(\Sigma^+p) \sim \sigma_{\text{tot}}(\bar{\Sigma}^+p). \qquad (10.17)$$

The differential cross section of the process

$$K_2^0 + p \to K_2^0 + p \qquad (10.18)$$

and the total cross sections for the interaction between a $K_2^0$ meson and a proton are asymptotically equal

$$\left.\frac{d\sigma(K_{2p}^0 \to K_{2p}^0)}{dt}\right|_{t=0} \sim \frac{1}{16\pi}\,[\sigma_{\text{tot}}(K_{2p}^0)]^2. \qquad (10.19)$$

If we take into account the isotopic invariance in the case of $\pi$N scattering, there is also the asymptotic equality

$$\left[\frac{d\sigma(\pi^\pm p\to\pi^\pm p)}{dt}-\frac{1}{2}\frac{d\sigma(\pi^- p\to\pi^0 n)}{dt}\right]_{t=0}\sim\frac{1}{16\pi}[\sigma_{tot}(\pi^\pm p)]^2. \quad (10.20)$$

Certain asymptotic relations were obtained also between the polarizations and the asymmetry parameters in the processes in question. If the protons are not polarized in the initial states, then the polarizations of the recoil baryons in the processes of each of the pairs (10.3)—(10.7) are equal in magnitude and opposite in sign, independently of the parities of the particles (for inelastic processes). The asymmetry parameters in the corresponding processes (10.3)—(10.7) with polarized protons in the initial state are also equal in magnitude and opposite in sign. It follows from this, in particular, that for processes which go over into themselves in the crossing transformation, for example, (10.18), the polarizations (for initially unpolarized particles) and the asymmetry parameters should tend *to zero as* $s\to\infty$ *and for fixed* t, *independently of the* relative behavior of the invariant amplitudes. This conclusion is valid also for the process

$$\pi^-+p\to\pi^0+n, \quad (10.21)$$

if charge symmetry holds. The relation between the polarizations of the baryons in the initial states depends on the relative parity of the $\lambda$ and $\Sigma$ hyperons; if this parity is positive, then the polarizations of the proton and the hyperon are opposite in sign and are equal in magnitude, and if this parity is negative, then the polarizations are equal both in magnitude and sign. A similar conclusion holds for the asymmetry parameters in the processes (10.8) with polarized hyperons and *antihyperons in the initial states.* In addition, for each process, the polarization in question and the asymmetry parameter are equal to each other if the parity is positive, and are of opposite sign if the parity is negative. Therefore the polarizations in one process and the asymmetry parameter in another, are subject to the following asymptotic relation, regardless of the relative parity of the particles: they are equal in magnitude and opposite in sign.

For meson elastic-scattering processes, the obtained asymptotic relations between the observed and physical quantities can be established on the basis of weaker assumptions. Indeed, it is sufficient to assume for this purpose that the observed physical quantities which enter in the complete experiment (differential cross section, asymmetry parameter, and correlations between polarizations) do not oscillate, and the imaginary parts of the amplitudes without spin flip are not negative when $s\to\infty$ and for certain values of t.

For the polarizations and the asymmetry parameters in baryon-baryon scattering processes, it is also possible to obtain certain relations. In particular, if the protons in the initial states are not polarized then the asymptotic polarizations of the recoil protons in the processes (10.9) and (10.10), the recoil neutrons in (10.11), the recoil $\Sigma^+$ hyperons in (10.12), the recoil $\lambda$ hyperons in (10.12), and the recoil $\lambda$ hyperons in (10.13) are equal to each other respectively in magnitude and opposite in sign. A similar conclusion is valid also for the asymmetry parameters in processes (10.9)—(10.13) with polarized targets. These asymptotic relations hold independently of the parities of the particles (for inelastic processes). In the elastic-scattering processes (10.9) and (10.10) there are analogous asymptotic relations between the polarizations of the first proton in the first process of (10.9) and the antiproton in the second (with unpolarized particles in the initial state), between the polarizations of the $\Sigma^+$ and $\overline{\Sigma}^+$ hyperons in (10.10), and between the asymmetry parameters in these processes with polarized incident particles. The polarizations of the first particles in the final states of the inelastic processes (10.11)—(10.13), namely, the $\lambda$ and $\overline{\Sigma}^-$ hyperons in (10.11), the proton and $\overline{\Sigma}^+$ hyperon in (10.12), and the *neutron and* $\overline{\Sigma}^-$ *hyperon in* (10.13) *(with unpolarized* initial particles) are related with the corresponding asymmetry parameters by the following asymptotic relations: the polarizations of the first particle in one process and the corresponding asymmetry parameter in the second process are equal in magnitude and opposite in sign for each pair, regardless of the parity of the particles. This relation holds true, of course, also for elastic processes (10.9) and (10.10). We note that they hold also for meson-baryon scattering and photoproduction.

The asymptotic relations between the amplitudes of the crossing processes of meson photoproduction on a baryon were considered in Sec. 5. From the relations obtained for the amplitudes we established that the differential cross sections of the processes are equal and found the following relations for the polarization effects: the polarization in one process with unpolarized initial particles, and the asymmetry parameter in the other process with polarized initial baryon, are asymptotically equal in magnitude and opposite in sign. These conclusions hold, in particular, for the processes

$$\gamma+p\to\pi^++n \quad (10.22)$$

and

$$\gamma+n\to\pi^-+p. \quad (10.23)$$

We note that we do not assume isotopic invariance here. For photoproduction processes of truly neutral mesons, that is, which go over into themselves in the crossing transformation, for example,

$$\gamma+p\to\pi^0+p \quad (10.24)$$

and

$$\gamma+n\to\pi^0+n, \quad (10.25)$$

the following asymptotic relations hold: the polarization (for unpolarized initial particles) and the asymmetry parameters (for a polarized initial nucleon) are equal in magnitude and opposite in sign.

The relations obtained in Sec. 3 can be applied to the study of the possibility of experimental verification of higher symmetries of strong interactions. In the strong-interaction model with higher symmetries, connections exist between the different scattering amplitudes. These connections, however, are in general rather complicated, and not much information can be obtained from them, generally speaking. In these models, all the mesons and their antiparticles belong to the same multiplet, and therefore there are several crossing processes among the meson-baryon scattering and photoproduction processes. With the aid of the asymptotic relations derived in Sec. 3 between the amplitudes of the crossing processes, it is possible to simplify the relations that follow from the higher symmetries, and obtain therefrom certain additional equalities between the cross sections of the meson-baryon scattering processes, and also relations for the polarization effects in these processes and in the photoproduction processes.

In Sec. 8 we developed a method of proving asymptotic equality of differential cross sections of processes with variable number of particles. By way of an example, we considered the case of creation of a pion in pion-nucleon collisions. In particular, we proved the asymptotic equality of the differential cross sections of the processes

$$\pi^+ + p \to \pi^+ + \pi^0 + p \quad \text{and} \quad \pi^- + p \to \pi^- + \pi^0 + p. \qquad (10.26)$$

In the case of production of two pions in a resonant state with J = 1, I = 1 ($\rho$ meson), we have equality of the differential cross sections for the processes

$$\pi^+ + p \to \varrho^+ + p \quad \text{and} \quad \pi^- + p \to \varrho^- + p, \qquad (10.27)$$

and in the case of formation of nucleon-meson systems in a resonant state we have equality of the differential cross sections, for example, of the processes

$$\pi^+ + p \to \pi^+ + \Delta^+ \quad \text{and} \quad \pi^- + p \to \pi^- + \Delta^+$$
$$\hookrightarrow p + \pi^0 \qquad\qquad \hookrightarrow p + \pi^0, \qquad (10.28)$$

and also

$$\pi^+ + n \to \pi^+ + \Delta^0 \quad \text{and} \quad \pi^- + n \to \pi^- + \Delta^0$$
$$\hookrightarrow p + \pi^- \qquad\qquad \hookrightarrow p + \pi^- \qquad (10.29)$$

We can consider further the case of mesons with different masses, and obtain, in particular, asymptotic equality of the differential cross sections of the processes

$$K^+ + p \to K^+ + \pi^0 + p \quad \text{and} \quad K^- + p \to K^- + \pi^0 + p, \qquad (10.30)$$
$$K^+ + p \to K^0 + \pi^+ + p \quad \text{and} \quad K^- + p \to \bar{K}^0 + \pi^- + p, \qquad (10.31)$$
$$\pi^- + p \to K^+ + K^- + n \quad \text{and} \quad \pi^+ + n \to K^- + K^+ + p. \qquad (10.32)$$

In the case when systems of two mesons are produced, or nucleon-meson systems in the resonant state, we have accordingly equality of the differential cross sections for the processes

$$K^+ + p \to K^{++} + p \quad \text{and} \quad K^- + p \to K^{+-} + p, \qquad (10.33)$$
$$\pi^- + p \to \varphi + n \quad \text{and} \quad \pi^+ + n \to \varphi + p, \qquad (10.34)$$
$$K^+ + p \to K^+ + \Delta^+ \quad \text{and} \quad K^- + p \to K^- + \Delta^+$$
$$\hookrightarrow p + \pi^0 \qquad\qquad \hookrightarrow p + \pi^0 \qquad (10.35)$$

We can obviously consider also the photoproduction of two mesons and the production of mesons in nucleon-nucleon collisions. For example,

$$\gamma + p \to \pi^+ + \pi^0 + n \quad \text{and} \quad \gamma + n \to \pi^- + \pi^0 + p, \qquad (10.36)$$
$$p + p \to n + \pi^+ + p \quad \text{and} \quad \bar{n} + p \to \bar{p} + \pi^+ + p, \qquad (10.37)$$
$$p + p \to p + \pi^0 + p \quad \text{and} \quad \bar{p} + p \to \bar{p} + \pi^0 + p, \qquad (10.38)$$

and in the particular case

$$\gamma + p \to \varrho^+ + n \quad \text{and} \quad \gamma + n \to \varrho^- + p, \qquad (10.39)$$
$$p + p \to n + \Delta^{++} \quad \text{and} \quad \bar{n} + p \to \bar{p} + \Delta^{++}$$
$$\hookrightarrow p + \pi^+ \qquad\qquad \hookrightarrow p + \pi^+, \qquad (10.40)$$
$$p + p \to p + \Delta^0 \quad \text{and} \quad \bar{p} + p \to \bar{p} + \Delta^0$$
$$\hookrightarrow p + \pi^0 \qquad\qquad \hookrightarrow p + \pi^0 \qquad (10.41)$$

It is natural to expect the asymptotic behavior to come into play in the region of energies above the masses of the particles and of the resonance. If we do not have equality of the cross sections of particles and antiparticles in this energy region, this will provide weighty arguments for assuming that the microcausality principle is violated at small distances. It is therefore very important to check experimentally the asymptotic relations with sufficiently high accuracy.


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