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SOLID STATE PLASMA

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A solid state plasma is a system of positive and negative carriers (electrons and holes) in solids.

Solid state plasma can be charged (electron plasma of metals, electron or hole plasma of semiconductors, plasma with unequal electron or hole concentration in alloys) or neutral (electron-hole plasma of semiconductors and semimetals); the plasma particle density varies in different solids over a wide range (from 0 to 10^{22} cm^{-3} for a charged plasma and to 10^{17} cm^{-3} for a neutral plasma).

Some properties of solid state plasma (thermodynamic properties, kinetic coefficients) are closely related to the type and peculiarities of the crystal lattice of the solid and to the interaction between the carriers and the lattice; on the other hand, in many cases solid state plasma can be regarded as an almost isolated subsystem of the solid (which interacts weakly with the lattice), and the properties of this subsystem can be studied separately.

I. PLASMA OSCILLATIONS IN A SOLID

A characteristic property of solid state plasma is the presence of collective excitations—plasma oscillations.

1. Plasma in a metal. Langmuir oscillations can exist in the electron plasma of a metal[1]. In the presence of these oscillations, a plane layer of electrons with density n_0, shifted a distance δx from the equilibrium position, is acted upon by a restoring force

$$\delta F = e\,\delta E = -e\cdot 4\pi n_0 e\,\delta x,$$

which gives rise to oscillations about the equilibrium position, with plasma frequency

$$\omega_p^2 = -\frac{\delta F}{m\,\delta x} = \frac{4\pi n_0 e^2}{m}. \qquad (1)$$

Formula (1) is the correct expression for the frequency only for sufficiently long Langmuir waves (whose phase velocity ω/k is much larger than the electron Fermi velocity v_F); the correction to the frequency, needed to account for the finite ratio $v_F/(\omega/k)$, is due to the electron pressure and is equal to

$$\frac{\delta\omega}{\omega_p} = \frac{1}{2}\frac{\delta\omega^2}{\omega_p^2} = \frac{1}{2}\frac{\frac{\nabla p}{n}}{eE} \approx \frac{k^2 v_F^2}{\omega_p^2}.$$

In an electron plasma at the boundary between a metal and vacuum, there can propagate surface waves [2] with an electric field potential φ that varies harmonically along the boundary and in time, and decreases exponentially on both sides of the boundary:

$$\varphi = \varphi_0 e^{-k|x|}\cos(kz - \omega t).$$

Inasmuch as the electric-induction-vector component normal to the boundary

$$D = -\varepsilon\frac{\partial\varphi}{\partial x}$$

is continuous, the frequency of the surface wave should satisfy the $\epsilon(\omega) = -1$. For waves with phase velocity much larger than the Fermi velocity, the dielectric permittivity constant of the electron plasma is

$$\varepsilon = 1 + \frac{\frac{4\pi}{c}j}{\frac{1}{c}\frac{\partial E}{\partial t}} = 1 - \frac{\omega_p^2}{\omega^2}$$

and consequently the frequency of the surface waves is

$$\omega = \frac{1}{\sqrt{2}}\omega_p.$$

It must be noted that this expression holds true only for waves whose phase velocity ω/k is much smaller than the velocity of light c (the electric field of the wave is potential only in this case); if ω/k and c are commensurate, then the wave vector k and the frequency ω of the surface wave are related by

$$c^2k^2 = \omega^2\frac{\omega_p^2 - \omega^2}{\omega_p^2 - 2\omega^2}.$$

Oscillations with plasma frequency ω_p and with a decreased plasma frequency $\omega_p/\sqrt{2}$ are observed indirectly in experiments in which electrons[1] exciting such oscillations pass through a thin foil and lose thereby an energy $\Delta E = \hbar\omega_p, 2\hbar\omega_p, \ldots$ or $\hbar\omega_p/\sqrt{2}, \ldots$ (the order of magnitude of which is several dozen electron volts, Fig. 1). Comparison of the theoretical and experimental values of ω_p shows that the electron plasma of many metals is made up of the valence electrons of the lattice atoms.

The plasma of frequency ω_p determines the boundary between the regions of transparency and specular reflection of light from metals[52]: when $\omega > \omega_p$ we have $\epsilon > 0$ and the metallic film is transparent to the light, and if $\omega < \omega_p$, $\epsilon > 0$ so that the light is completely reflected from the surface of the metal. Investigations of the reflection and absorption[3] of light of a frequency close to the plasma frequency of a film is

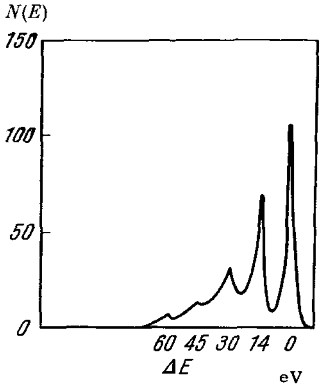

FIG. 1

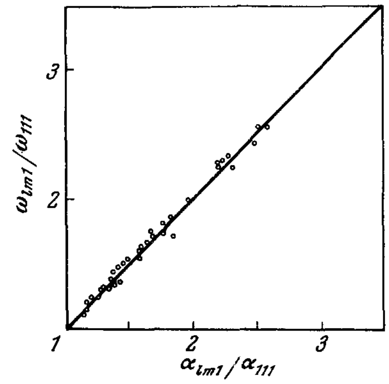

FIG. 2

an indirect method of observing Langmuir oscillations.

In the presence of a strong magnetic field H_0, helical waves, constituting electromagnetic oscillations in matter[4,5], can propagate in an electron plasma of density n_0. If the wave propagates along the force lines of the field H_0 (which is directed along the z axis), then the plasma electrons drift under the influence of the electric field of the wave* $E = -i\omega H/ck$ with a velocity $U = -ic(E/H_0)$, producing a current $j = n_0eU$, which supports an alternating magnetic field H:

$$-kH = \frac{4\pi}{c} j = \frac{4\pi}{c} n_0 e \left(-ic\frac{1}{H_0}\right)\left(-i\frac{\omega}{ck}H\right).$$

Thus, the frequency ω of the helical wave propagating along the magnetic field is connected with the wave number k by the following dispersion relation:

$$\omega = \frac{cH_0}{4\pi n_0 e} k^2.$$

The dispersion relation for helical waves propagating at an angle to the magnetic field is determined from the system of equations of motion of the plasma and electrodynamic equations, which in this case takes the form

$$i\,[\mathbf{kH}] = \frac{4\pi}{c} n_0 e\left(\frac{c\,[\mathbf{EH}_0]}{H_0^2} + \mathbf{v}_{\parallel}\right), \quad \frac{\omega}{c}\mathbf{H} = [\mathbf{kE}] \qquad (2)†$$

(E, H—electric and magnetic fields of the wave, $v_{\parallel}$—velocity component along the external magnetic field H_0). Eliminating v and H from the system (2) and assuming that the electron plasma in the low-frequency helical waves is incompressible ($\mathbf{k}\cdot\mathbf{v} = 0$), we obtain

$$\omega = \frac{cH_0}{4\pi n_0 e} k k_z, \qquad (3)$$

where k is the modulus of the wave vector and k_z its projection on the external magnetic field.

The dispersion relation (3) was experimentally verified by investigating low-frequency magnetoplasma resonance[6] in samples in the form of a parallelepiped.[7] In this case the wave vector of the helical wave is

$$\mathbf{k}_{lmn} = \pi\left(\frac{l}{X}, \frac{m}{Y}, \frac{n}{Z}\right),$$

where l, m, n—integers and X, Y, Z—lengths of the parallelepiped edges; the resonant frequency is therefore

$$\omega_{lmn} = \frac{cH_0}{4\pi n_0 e}\frac{n^2}{Z^2}\left(1 + \frac{l^2}{n^2}\frac{Z^2}{X^2} + \frac{m^2}{n^2}\frac{Z^2}{Y^2}\right)^{1/2} = \omega_{00n}\alpha_{lmn}.$$

The theoretical and experimental values for the ratio $\omega_{lm1}/\omega_{111}$ are in good agreement (Fig. 2).

Owing to the friction between the electron plasma and the crystal lattice (the electron momentum is transferred to the phonons, impurities, etc.), the helical wave attenuates within a time approximately equal to $\Omega\tau$ periods of oscillation:

$$\frac{\gamma}{\omega} \approx (\Omega\tau)^{-1}$$

($\Omega = eH_0/mc$, τ—momentum relaxation time), since the wave energy dissipation per period of the oscillation is

$$\frac{1}{\omega} F_{\mathrm{fr}} v = \frac{1}{\omega}\frac{nmv}{\tau} v \approx \frac{H^2}{\Omega\tau}.$$

Another cause of attenuation of the helical waves is the viscosity of the electronic plasma; the ratio of viscous losses to friction losses is

$$\frac{F_{\mathrm{visc}}}{F_{\mathrm{fr}}} = \frac{m\nu\Delta v}{\frac{mv}{\tau}} \approx k^2 l^2$$

(ν—kinematic viscosity, and l—mean free path of the electron in the metal).

With the aid of low-frequency magnetoplasma resonance it is possible to determine the Hall coefficient $R = 1/n_0ec$ (by comparing the experimental value of the resonant frequency with the theoretical $\omega = k^2c^2H_0R/4\pi$) and the resistivity $\rho = m/ne^2\tau$ (by comparing the experimental resonance width with the theoretical $\gamma/\omega = 1/\Omega\tau = \rho/H_0R$ for $kl \ll 1$) of the

*Here $E = E_x + iE_y$, $H = H_x + iH_y$, etc.

†$[\mathbf{k}\,\mathbf{H}] = \mathbf{k} \times \mathbf{H}$.

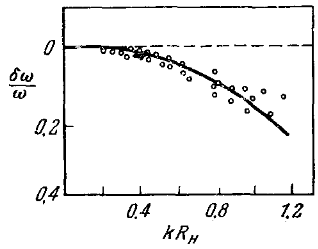

FIG. 3

material of the sample. Such investigations were made for Cu, Ag, Au, Pb, Sn, Zn, Cd, and Hg[8].

Relation (3) between the frequency and the wave vector of the helical wave is valid only for waves much longer than the Larmor radius of the electron $R_H = v_F/\Omega$. The correction to the helical-wave frequency, necessitated by the finite value of kR_H, can be obtained in the following fashion. For waves propagating along a magnetic field H_0 directed along the z axis we obtain

$$\frac{ck}{\omega}E = -iH, \quad kH = -\frac{4\pi}{c}j, \tag{4}$$

where

$$E = E_x + iE_y, \quad H = H_x + iH_y,$$

and the current density

$$j = e\int (v_x + iv_y) f_1 d\mathbf{v} \tag{5}$$

is determined by the perturbed distribution function f_1, which satisfies the linearized kinetic equation

$$-i\omega f_1 + ikv\cos\theta f_1 - \Omega\frac{\partial f_1}{\partial\varphi} = e\,(E_x v_x + E_y v_y)\frac{\partial f_0}{\partial\varepsilon} = e\frac{Ee^{-i\varphi} + E^* e^{i\varphi}}{2} v\sin\theta\frac{\partial f_0}{\partial\varepsilon} \tag{6}$$

(v, θ, φ—spherical coordinates in velocity space). Substituting in (5) the solution (6) and expressing E in terms of H with the aid of (4) we obtain for the case of a gas filling a Fermi sphere of radius v_F the following dispersion equation

$$\omega A = \frac{cH_0}{4\pi n_0 e}k^2,$$

where

$$A = \frac{3}{4}\int_0^\pi \frac{\sin^2\theta}{1 + \frac{\omega}{\Omega} + \frac{kv_F}{\Omega}\cos\theta}\,d(\cos\theta) = 1 - \frac{1}{5}k^2R_H^2 + \dots$$

Thus, the decrease in the frequency of the helical wave is equal to[9]

$$\frac{\delta\omega}{\omega} = -\frac{1}{5}k^2R_H^2. \tag{7}$$

This value is in good agreement with the experimental data[10] (Fig. 3).

2. Neutral plasma of semimetals and semiconductors. In such a plasma there can exist, besides the Langmuir oscillations, also longitudinal electron-hole sound waves (analogous to the ionic-sound waves in a gas plasma[53]). If, for example, the masses of the holes and electrons (m_+, m_-) and the average energies of their random motion (T_+, T_-) satisfy the inequalities $m_+ \gg m_-$ and $T_- \gg T_+$, then the holes in such a wave move under the influence of the alternating electric field $m\,dU_+/dt = -e\nabla\varphi$, produced by the space charge $-\nabla^2_\varphi = 4\pi e(n_+ - n_-)$; the hole density is connected with the hole velocity $\partial n_+/\partial t = -\nabla n_+ U_+$, while the electron density is determined, for a sufficiently high collision frequency, by the Boltzmann distribution

$$n_- = Ne^{-e\varphi/T_-}.$$

For a plane wave of low amplitude we obtain from this the following connection between the frequency in the wave vector:*

$$\omega^2 = \frac{c_s^2k^2}{1 + k^2D^2},$$

where

$$c_s = \sqrt{\frac{T_-}{m_+}}, \quad D = \left(\frac{4\pi Ne^2}{T_-}\right)^{-1/2}.$$

In the presence of an external magnetic field, so-called Alfven and magnetic-sound waves can propagate in a neutral plasma of semimetals and semiconductors. When a wave with frequency much lower than the cyclotron frequencies of the carriers propagates in a cold plasma ($nT \ll H_0^2$), the velocity of the particles in the wave is determined by the drift transversely to the magnetic field $\mathbf{H}_0$ under the influence of the electric field E and of the inertia force

$$\mathbf{F}_i = -m\frac{\partial}{\partial t}\frac{[\mathbf{EH}_0]}{H_0^2} = im\omega c\frac{[\mathbf{EH}_0]}{H_0^2}:$$

namely:

$$\mathbf{v} = c\left[\left(\mathbf{E} + \frac{\mathbf{F}_{tr}}{e}\right)\frac{\mathbf{H}_0}{H_0^2}\right].$$

The current due to the vibrational plasma-charge motion with such velocity produces the magnetic field of the wave

$$i\,[\mathbf{kH}] = [i\mathbf{k}[\frac{c\mathbf{k}}{\omega}\mathbf{E}]] = \frac{4\pi}{c}\sum ne\left(i\omega\frac{mc^2}{e}\frac{\mathbf{E}}{H_0^2}\right). \tag{8}$$

It follows from (8) that for an Alfven wave (the vector $\mathbf{E}$ lies in the $\mathbf{k}$, $\mathbf{H}_0$ plane)

$$\omega = v_A k\cos\vartheta \tag{9}$$

(ϑ—angle between the direction of wave propagation in the external magnetic field $\mathbf{H}_0$), we have for a magnetic-sound wave (vector E perpendicular to the $\mathbf{k}$, $\mathbf{H}_0$ plane)†

$$\omega = v_A k. \tag{10}$$

*A detailed analysis of the propagation of longitudinal waves in an electron-hole plasma can be found in [11].

†An analysis of the dispersion relations on the basis of the kinetic equation in the free-carrier model can be found in [12,13] and in the references cited there.

Alfven and magnetic-sound waves were observed in experiments with bismuth and its alloys with tin and tellurium[12] (in these experiments $H_0 \approx 10$ kG and $v_A = 10^7 - 10^8$ cm/sec).

In a sufficiently large external magnetic field, cyclotron absorption of the helical and Alfven waves by the carrier plasma is observed when the frequency of the wave, in the coordinate system moving together with the carrier

$$\omega' = \omega - \mathbf{kv}$$

coincides with the frequency of revolution of the carrier in the magnetic field $\omega' = \Omega$. If the external magnetic field is such that the inequality $\Omega > \omega + k(\omega) v_F$ is satisfied (v_F—Fermi velocity of the carriers), then the frequencies ω' and Ω cannot be equal and there is no cyclotron absorption. The propagation of Alfven waves in a solid-state plasma and cyclotron damping were observed in bismuth and in the alloys Bi + Sn and Bi + Te[14,15].

The results of experiments[59] on cyclotron absorption of helical waves in sodium are in good agreement with the theoretical relations for the critical magnetic field H_{cr}, at which cyclotron absorption of helical waves vanishes:

$$\frac{eH_{cr}}{mc} = k(\omega) v_F = \left(\frac{\omega}{\frac{cH_{cr}}{4\pi n_0 e}}\right)^{1/2} v_F$$

($\omega \ll \Omega$).

3. Charged plasma of semiconductors. The presence of a charged (electron, hole, or electron-hole) plasma in semiconductors leads to the appearance of magnetoplasma resonance, which is observed when a homogeneous high-frequency electric field $\mathbf{E}$ acts on a semiconductor sample placed in a constant magnetic field. The magnetic field of the wave propagating along $\mathbf{H}_0$ is determined by the equality*

$$kH = -\frac{4\pi}{c} \sum neU + \varepsilon \frac{i\omega}{c} E,$$

where

$$U = -i \frac{eE}{m} (\Omega - \omega)^{-1}$$

is the velocity of the carriers in the electric field of the wave, $\mathbf{E} = -i(\omega/ck)\mathbf{H}$, and the summation is over the species of the carriers. It follows therefore that

$$k^2 = \frac{\omega^2}{c^2} \left(\varepsilon + \sum \frac{\omega_p^2}{\omega(-\omega + \Omega)} \right). \tag{11}$$

In the case of carriers of a single species and $\epsilon = 1$, it follows from (11) that (owing to the homogeneity of the field $k = 0$), resonance takes place when

$$\Omega = \omega - \frac{\omega_p^2}{\omega}$$

*$E = E_x + iE_y$, $H = H_x + iH_y$, $U = U_x + iU_y$.

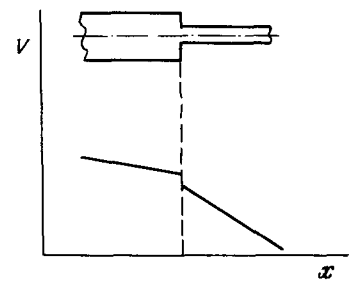

FIG. 4

(an account of the depolarization in the sample leads to replacement of ω_p by $A\omega_p^2$, where $A = L(1 + \chi L)^{-1}$, χ—electric susceptibility of the crystal lattice, L—depolarization factor of the sample[16,17]). Comparison of the theoretical and experimental data (made for Ge[17]) makes it possible to determine the plasma density, the effective carrier masses, and the collision frequency.

II. PLASMA FLOW

The flow of a plasma in a solid is the directional carrier motion arising in the presence of gradients of the electrical or chemical potential and of the temperature. This flow was thoroughly investigated long ago in experiments aimed at the determination of the resistance of solids (with or without a magnetic field), the thermoelectric coefficients, the Hall constant, etc.

We consider two cases of plasma flow in a solid: 1) flow of a charged plasma in a metallic conductor of variable cross section, leading to the appearance of the so-called configurational emf[18,19]; 2) flow of a neutral plasma in a semiconductor situated in an external magnetic field, leading to the appearance of the magnetic moment of the plasma[20].

1. When electric current flows through a metallic wire whose cross section decreases abruptly at some point, a potential jump is produced at this point (Fig. 4). The reason for the appearance of the potential jump is as follows. Since accumulation of charge is impossible in stationary flow, the total current flowing through regions 1 and 2 must be the same. Therefore the current density jumps abruptly on going into region 2. The charged plasma of the metal is practically incompressible, so that the increase in the current density $j = neU$ can be connected only with an increase in the velocity U. A sharp increase in the plasma velocity in the transition region presupposes the existence of a δ-like electric field

$$E = \frac{m}{e} \frac{dU}{dt}$$

on the boundary between regions 1 and 2; the potential jump

$$v_2 - v_1 = -\int_1^2 E\,dx$$

can be expressed in terms of the areas of the transverse cross sections of the conductor S_1 and S_2 and the total current I

$$v_2 - v_1 = -\int_1^2 \frac{m}{e}\frac{dU}{dt}\,dx = -\frac{m}{2e}(U_2^2 - U_1^2)$$

$$= -\frac{1}{2}\frac{m}{e}\left(\frac{1}{ne}\right)^2 I^2\left(\frac{1}{S_2^2} - \frac{1}{S_1^2}\right). \quad (12)$$

The configurational emf (12)* reverses sign when the sign of the charges is reversed, but does not depend on the direction of the current I.

2. Let us consider the radial diffusion of a neutral plasma in a solid, in a direction towards the surface of a cylindrical sample (radius R) situated in an external axial magnetic field. Inasmuch as there is no accumulation of charge under stationary conditions, the electrons and the holes should move with equal velocity $v_r^+ = v_r^-$ in the radial direction, and rotate in the azimuthal direction with velocities $v_\theta^\pm$, producing an azimuthal electric current $j_\theta = \Sigma nev_\theta$ (summation over the electrons and the holes), and an axial magnetic moment with average density

$$M = \frac{1}{2c}\frac{\int_0^R rj_\theta(r)\,2\pi r\,dr}{\pi R^2}. \quad (13)$$

Expressing with the aid of the equations of motion

$$e_\pm E_r + e_\pm \frac{v_\theta^\pm}{c} H - \frac{m_\pm v_r}{\tau_\pm} - \frac{T\nabla_r n}{n} = 0,$$

$$-e_\pm \frac{v_r}{c} H - \frac{m_\pm v_\theta^\pm}{\tau_\pm} = 0$$

the azimuthal velocities $v_\theta^\pm$ in terms of $\nabla_r n$ and substituting in (13) the current $j_\theta = n(e_+ v_\theta^+ + e_- v_\theta^-)$ and the density gradient $\nabla_r n = -n\delta(r - R + 0)$ (we thus assume that there is a sharp decrease in the density near the surface of the sample), we obtain

$$M = -\frac{2nT}{H}\frac{\left(\frac{\mu H}{c}\right)^2}{1 + \left(\frac{\mu H}{c}\right)^2} \quad (14)$$

where $\mu = (\mu_+/\mu_-)^{1/2}$—average mobility.

The results of an experimental investigation of the photomagnetic moment † of plasma in germanium[20] agree with formula (14) (Fig. 5).

If diffusion takes place in the plasma into the sample, then the radial density gradient reverses sign, and the diamagnetic moment (14) should give way to a paramagnetic moment |M|; such a paramagnetic moment was actually observed[20].

Thus, solid-state plasma can be either diamagnetic or paramagnetic, depending on the conditions under which the nonequilibrium carrier concentration is produced.

*Calculation of the configurational emf, based on the Boltzmann equation for free electrons scattered by immobile centers, leads to the appearance of a factor on the order of unity in (12) [19].

†This moment arises when a nonequilibrium plasma concentration is produced by illuminating a germanium sample.

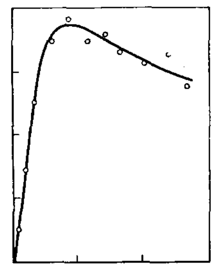

FIG. 5

III. FLOW STABILITY

Laminar flow of a solid-state plasma, due to the presence of potential, temperature, density gradient, etc., may turn out to be unstable. The instability of the plasma is manifest in two ways: first, the perturbation produced by the external source becomes intensified in the unstable plasma; second, even in the absence of an external stimulus, an increase takes place in the fluctuations which are always present in the plasma and changes the averaged flow characteristics.

One of the well-known flow instabilities is the so-called two-stream (or "sausage") instability arising in a neutral plasma in the presence of sufficiently rapid relative motion of oppositely charged plasma components (see, for example, the review of work on two-stream instability in a gas plasma[44]). Thus, from the hydrodynamic equations that describe the motion of a two-component plasma with sufficiently large relaxation times,*

$$\frac{\partial v_\pm}{\partial t} + (U_\pm \nabla) v_\pm = -s_\pm^2 \frac{\nabla n_\pm}{n_\pm} + e_\pm \frac{E}{m_\pm},$$

$$\frac{\partial n_\pm}{\partial t} + N_\pm \nabla v_\pm + U_\pm \nabla n_\pm = 0, \quad \nabla E = 4\pi e(n_+ - n_-)$$

($s_\pm$—average thermal velocities, $U_\pm$—velocities of ordered motion of the components, which for simplicity are assumed to be parallel, $v_\pm$ and $n_\pm$—perturbations of the velocities and densities, E—electric field) we obtain, for a plane wave $\exp(-i\omega t + ikx)$ propagating along the relative-motion velocity, the following connection between the frequency and the wave vector:

$$k^2 = \sum \frac{\omega_p^2}{\left(\frac{\omega}{k} - U\right)^2 - s^2} \equiv F\left(\frac{\omega}{k}\right) \quad (15)$$

(ω_p—plasma frequency; the summation is over the species of the carriers). It follows from (15) that

*A calculation of the conditions under which a two-stream instability is produced in a plasma in semiconductors with small relaxation times can be found, for example, in [21].

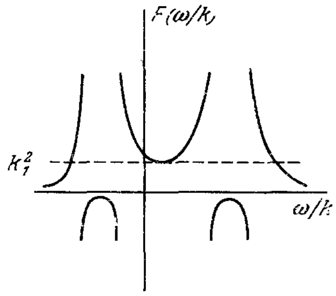

FIG. 6

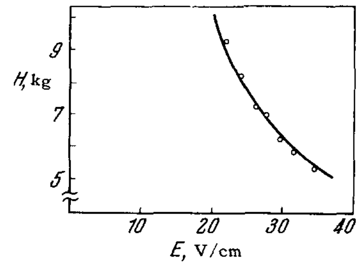

FIG. 7

when $|U_+ - U_-| > s_+ + s_-$ there are always sufficiently long waves ($k < k_l \approx \omega_p/U$) for which there are two real and two complex roots of the dispersion equation (15) (Fig. 6); the frequency of such waves is complex, and the perturbations with wavelength $\lambda > 2\pi/k_l$ will grow.

Thus, if the velocity of relative motion of the electrons and holes exceeds the sum of their thermal (or Fermi) velocities, the plasma flow in the solid will be unstable.

Another example of flow instability is the so-called screw instability of current in a longitudinal magnetic field. Screw instability arises when the plasma is acted upon simultaneously by sufficiently strong (parallel or almost parallel) electric and magnetic fields (**E** and **H**). The carrier plasma in a sample of finite dimensions is then inhomogeneous ($\nabla_0 N \neq 0$).

Let us examine the propagation of a plane wave in such a plasma (all the quantities in the wave vary like $\exp[-i\omega t + i\mathbf{k}\cdot\mathbf{x}]$), under the assumption that:
a) $\Omega\tau \ll 1$, b) the wavelength is much shorter than the characteristic dimensions over which the plasma parameters vary;

$$\frac{1}{k}\frac{\nabla_0 N}{N} \ll 1,$$

c) the oscillations are of low frequency ($\omega\tau \ll 1$) and potential ($E = -\nabla\Phi$), and the plasma remains quasineutral all the time ($N_+ = N_-$).

Under these conditions the equations of motion

$$-\frac{T\nabla N}{N} - \frac{m}{\tau}U + e\mathcal{E} + \frac{e}{c}UH = 0$$

yield

$$U_\alpha = \left(-\frac{D\nabla_\beta N}{N} + \mu\mathcal{E}_\beta\right)(\delta_{\alpha\beta} + \Omega\tau\varepsilon_{\alpha\beta\gamma}h_\gamma)$$

($D = T\tau/m$—diffusion coefficient, $\mu = e\tau/m$—mobility, $\mathbf{h} = \mathbf{H}/H$, $\epsilon_{\alpha\beta\gamma}$—completely antisymmetrical unit tensor); substituting this expression in the continuity equation

$$\frac{\partial N}{\partial t} + \nabla_\alpha N U_\alpha = 0,$$

we obtain an equation for the plasma charge density $N_\pm$

$$\frac{\partial N}{\partial t} + \nabla_\alpha(-D\nabla_\beta N + \mu\mathcal{E}_\beta N)(\delta_{\alpha\beta} + \Omega\tau\varepsilon_{\alpha\beta\gamma}h_\gamma) = 0. \qquad (16)$$

Linearizing this equation, that is, putting $N = N + n$ and $\mathcal{E} = E - ik\varphi$, $\nabla = \nabla_0 + ik$, retaining in (16) terms proportional to the amplitude of the alternating density n and to the potential φ, and recognizing that $\nabla_0 E = 0$, we obtain the following relation between n and φ:

$$-i\omega n + (D_\pm k_\alpha k_\beta n + \mu_\pm N k_\alpha k_\beta\varphi - \mu_\pm i k_\beta\varphi\nabla_\alpha N + \mu E_\beta i k_\alpha n)(\delta_{\alpha\beta} + \Omega_\pm\tau_\pm\varepsilon_{\alpha\beta\gamma}h_\gamma) = 0$$

(the subscript 0 of ∇_0 will henceforth be omitted).

Equating the determinant of this system of two algebraic equations for n and φ to zero, we arrive at the following dispersion equation, relating the frequency ω with the wave vector k:

$$A_+B_- - B_+A_- = 0, \qquad (17)$$

where

$$A = -\omega + \mu\mathbf{kE} - i\frac{\mu T}{e}k^2 + \mu\Omega\tau\mathbf{k}[\mathbf{Eh}],$$

$$B = \mu k^2 N - \mu i\mathbf{k}\nabla N - \mu\Omega\tau i\mathbf{k}[\nabla N\mathbf{h}].$$

From (17) we obtain the imaginary part γ of the complex frequency $\omega + i\gamma$

$$\gamma = \frac{\mu_+|\mu_-|}{\mu_+ + |\mu_-|}(k^2N)^{-1}\Big[-\frac{2T}{|e|}k^4N - (\Omega_+\tau_+ + |\Omega_-\tau_-|) \times[(\mathbf{kE})(\mathbf{k}[\nabla N\mathbf{h}]) - (\mathbf{k}\nabla N)(\mathbf{k}[\mathbf{Eh}])]\Big]. \qquad (18)$$

If, for example, the vectors **E** and **H** are directed along the z axis and ∇N directed along the x axis, then the waves will increase provided that

$$\frac{\Omega_+\tau_+ + |\Omega_-\tau_-|}{2}\frac{|e|E\lambda}{T}\frac{\lambda\nabla_x N}{N}\frac{k_y k_z}{k^2} > 1 \quad \left(\lambda \equiv \frac{1}{k}\right).$$

It follows therefore that for a specified k_z, only waves with one sign of k_i increase, that is, the growing perturbations are of the screw type.

An experimental investigation of screw instability shows that on the whole the theory describes the phenomenon correctly,* and the theoretical dependence of the critical magnetic field on the electric

*A more detailed analysis of the conditions under which screw instability arises in semiconductors and in semimetals is contained in [22,57] (the theory of screw instability was first developed as applied to a gas plasma in the positive column of a glow discharge [54]).

field ($H_{cr} \sim E^{-1}$, Fig. 7) of the critical frequency (at which attenuation gives way to amplification) on E were confirmed[23,57]. The character of the distortion of the current when the instability arises was investigated and the resultant perturbation was indeed shown to be of the screw type[24].

When a strong electric current passes through a solid-state plasma a pinch effect is observed in the plasma, namely the contraction of the plasma into a pinch under the influence of the ponderomotive force $j \times H/c$ due to the interaction between the current and its own magnetic field[26-29]. The temperature and density of such a compressed plasma state are determined by the balance of the forces acting in the plasma pinch (the equilibrium between the kinetic plasma pressure and the magnetic field pressure), and the balance of energy influx (Joule heat) and outflow (lattice heating, thermal conductivity). The formation of the plasma pinch can be accompanied by many secondary phenomena such as a considerable change in electric resistivity (that is, in the current-voltage characteristic)[27,28], excitation of standing sound waves in the sample[29], appearance of glow due to recombination of electrons and holes in a plasma with non-equilibrium carrier density[27], heating and melting of the crystal lattice in the region of the plasma pinch[26], incandescence of the sample under the influence of mechanical and thermal stresses arising during the pinch effect[26], etc. Along with the method of passing strong current through a p-n junction, the pinch effect in a solid-state plasma can be used in principle to produce a laser that makes use of recombination radiation of electron-hole pairs and transforms the electric energy directly into coherent light-emission energy.

IV. INTERACTION BETWEEN LATTICE VIBRATIONS AND A PLASMA

The interaction between the solid-state plasma and the crystal lattice leads to many effects which manifest themselves when sound propagates in solids. These effects include the variation in the velocity of sound, attenuation (or amplification) of sound as a result of interaction with the plasma, and the appearance of local anomalies on the dispersion curves.

1. Motion of solid-state plasma in an alternating electric field, resulting from the deformation of the crystal lattice by the propagating sound wave, leads to a change Δs in the speed of sound in the magnetic field; this change was measured in many solid[30,31] (Cu, Ag, Au, Al, Ta, V) and liquid[32] (Hg, Ka + Na, Pb, Cu, Al) metals; the experimental and theoretical values of Δs are in good agreement with each other.

2. The character of the interaction of the solid-state plasma particles with the lattice vibrations is different in different bodies (although in all cases the force exerted on the plasma particle by the deformed lattice is electric in nature). Let us consider one of the frequently encountered types of interaction, when the Lagrangian of the system consisting of the lattice and the plasma has the following form[33]:

$$L=\frac{MN}{2}\int\left[\left(\frac{\partial\xi}{\partial t}\right)^2-s^2(\nabla\xi)^2\right]dx+\sum\frac{mv^2}{2}+\sum q\nabla\xi. \qquad (19)$$

The first term is the Lagrangian of the sound oscillations, the second the kinetic energy of the plasma particles, and the third the energy of interaction between these particles and the deformed lattice. The quantity ξ in (19) is the deformation, M and N the mass and density of the lattice atoms, m the carrier masses, and q the interaction constants. The summation is over all plasma particles.

Going over to a continuous description of the plasma and varying (19), we obtain an inhomogeneous d'Alambert equation for the deformation

$$\frac{\partial^2\xi}{\partial t^2}-s^2\Delta\xi=-\frac{1}{MN}\sum q\nabla n \qquad (20)$$

(n = plasma-particle density) and the equation of motion of the particles in the sound field

$$m\frac{dv}{dt}=+q\nabla^2\xi. \qquad (21)$$

Actually, Eq. (21) should contain in addition to the deformation force $q\nabla^2\xi$ the pressure gradient, the Lorentz force, and the friction force, so that the total equation of motion takes the form

$$m\frac{dv}{dt}=q\nabla^2\xi-\frac{\nabla p}{n}+e\left(E+\frac{v}{c}H\right)-\frac{mv}{\tau}. \qquad (22)$$

Propagation of the sound wave causes the plasma to go into motion under the influence of the deformation forces, and gives rise to a change in the properties of the wave itself. Let us consider for concreteness the case of the neutral plasma consisting of two species of particles and assume that the inertial forces and the friction force are small, and the plasma remains quasineutral when the sound propagates. We then obtain from (22)

$$\left.\begin{aligned}eE-T_1\frac{\nabla n}{n}+q_1\nabla^2\xi=0,\\ -eE-T_2\frac{\nabla n}{n}+q_2\nabla^2\xi=0\end{aligned}\right\} \qquad (23)$$

(where $T_{1,2} = (\partial p/\partial n)_{1,2}$ are the average energies of random motion of the particles), so that the change in the plasma density due to the deformation wave is equal to

$$\frac{\delta n}{n}=\frac{q_1+q_2}{T_1+T_2}\nabla\xi.$$

Substituting δn in (20), we obtain the altered speed of sound

$$s'^2=s^2+\frac{(q_1+q_2)^2}{T_1+T_2}\frac{n}{NM}. \qquad (24)$$

It turns out, however, that the interaction between the sound wave and a degenerate plasma can lead not only to a gross effect, namely the change in the slope of the line $\omega = sk$, but also to the appearance of a local singularity on the dispersion curve, at $k = 2k_F$[34]. To clarify the character of this singularity, let us use the equation for the plasma particle-density matrix

$$i\frac{\partial\varrho(y,z)}{\partial t}=\left(-\frac{\Delta_y}{2}+\frac{\Delta_z}{2}+\Psi(y)-\Psi(z)\right)\varrho(y,z). \quad (25)$$

Here

$$\Delta_y=\frac{\hbar^2}{m}\frac{\partial^2}{\partial y^2},$$

$\Psi = e\Phi - q\nabla\xi$—potential energy of the particle in the deformation and electric fields ($E = -\nabla\Phi$). Going over in (25) to the variables $x = (y + z)/2$ and $x' = y - z$, and introducing the quantum distribution function[35]

$$f_{xp}=\sum_{x'}e^{-ix'p}\varrho\left(x+\frac{x'}{2},\ x-\frac{x'}{2}\right), \quad (26)$$

we obtain for its Fourier component

$$f_{kp}=\sum_x f_{xp}e^{-ikx}$$

the quantum kinetic equation

$$\frac{\partial f_{kp}}{\partial t}+i\mathbf{kp}f_{kp}=\Psi_k\frac{f_{p+\frac{k}{2}}-f_{p-\frac{k}{2}}}{i}. \quad (27)$$

For a plane sound wave $\exp(-i\omega t + ikx)$ propagating in a two-component plasma, by linearizing (27) and substituting

$$\Psi_k=\frac{1}{ik}(-eE-q\nabla^2\xi),$$

we obtain

$$eE-\frac{1}{\beta_1}\frac{\nabla n}{n}+q_1\nabla^2\xi=0,$$

$$-eE-\frac{1}{\beta_2}\frac{\nabla n}{n}+q_2\nabla^2\xi=0,$$

where

$$\beta=\int\frac{f_{p+\frac{k}{2}}-f_{p-\frac{k}{2}}}{\omega-\mathbf{kp}}d\mathbf{p}, \quad (28)$$

so that we arrive again at Eq. (23), in which, however, the T are replaced by β^{-1}. In the case when the particles fill a Fermi sphere in momentum space, the quantity β has near $k = 2k_F$ a singular part $\beta(k) - \beta(k_F) \sim (k - 2k_F)\ln|k - 2k_F|$; consequently, a logarithmic singularity of the group velocity appears on the dispersion curve of the sound in the presence of a plasma:

$$\frac{d\omega}{dk}\sim\ln\left|1-\frac{k}{2k_F}\right|$$

(an analysis made in [26] for cylindrical and plane Fermi surfaces in the case of a metal, when there is only one species of carriers, shows that in this case the singularity of $d\omega/dk$ becomes stronger).

3. When the sound frequency is

$$\omega=\frac{1}{4}\frac{s^2}{c^2}\frac{\omega_p^2}{\Omega},$$

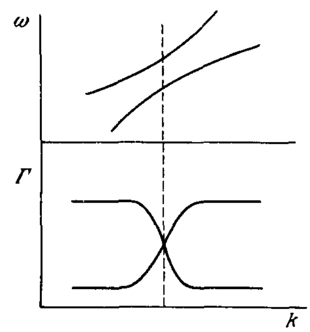

FIG. 8

corresponding to the point of intersection of the dispersion curves of the sound wave ($\omega = sk$) and the helical wave ($\omega = cH_0k^2/4\pi n_0e$), anomalous absorption should take place[37]. An analogous phenomenon should be observed at frequencies corresponding to the points of intersection of the dispersion curve of sound with the dispersion curves of the electron-hole sound, the Alfven wave, etc.

Indeed, near the points (ω_0, k_0) where the dispersion curves of two normal x and y oscillations intersect, the system of equations for the quantities x and y has, in the presence of coupling between x and y, the form

$$(\Omega-i\gamma-s_1q-i\Gamma_1)x+\Lambda y=0,$$
$$(\Omega-i\gamma-s_2q-i\Gamma_2)y+\Lambda x=0,$$

where $\Omega = \omega - \omega_0$, $q = k - k_0$, and the terms containing Λ describe the coupling between the oscillations. Assuming Λ to be real, we obtain the following expressions for the frequency $\Omega_{1,2}$ and damping $\Gamma_{1,2}$ (which is assumed small) of "mixed" oscillations (Fig. 8):

$$\left.\begin{aligned}\Omega_{1,2}&=\frac{s_1+s_2}{2}q\pm\sqrt{\left(\frac{s_1-s_2}{2}\right)^2q^2+\Lambda^2},\\ \gamma_{1,2}&=\frac{\Gamma_1+\Gamma_2}{2}\pm\frac{\Gamma_1-\Gamma_2}{2}\frac{\frac{s_1-s_2}{2}q}{\sqrt{\left(\frac{s_1-s_2}{2}\right)^2q^2+\Lambda^2}}.\end{aligned}\right\} \quad (29)$$

If the damping decrements Γ_1 and Γ_2 differ greatly then the oscillation, which attenuates weakly far away from the point of intersection, attenuates very strongly near the intersection point

$$\gamma_1=\gamma_2=\frac{\Gamma_1+\Gamma_2}{2}$$

at $q = 0$.

An experimental investigation of the dispersion of the oscillation near the point of intersection of the dispersion curves of the helical and sound waves[58] confirms the theory of [37].

4. The sound-wave damping due to the interaction with the solid-state plasma differs with the type of solid. The reason for the damping of sound in the charged lattice of a metal or semiconductor (when the space charge of the lattice is disturbed by its

motion), is the viscosity η of the charged plasma of the carriers.

If the wavelength of the sound becomes smaller than the mean free path of the electron, the viscous damping ($\sim\omega^2$) is replaced by Landau damping ($\sim\omega$), due to the interaction between the sound wave and the resonant electrons (the projections of whose velocities on the direction of wave propagation is equal to the speed of sound: $\omega = \mathbf{k}\cdot\mathbf{v}$). The magnitude of this damping can be obtained for longitudinal waves ($\mathbf{k} \parallel \mathbf{E} \parallel \mathbf{U}$) by linearizing the equations of continuity and motion of the lattice

$$-i\omega\delta N + ikNU = 0, \qquad -i\omega MU = eE,$$

and the kinetic equation for the electrons

$$(-i\omega + i\mathbf{k}\mathbf{v})\,\delta f = \frac{eE}{m}\frac{\partial f}{\partial \mathbf{v}}$$

and by substituting the values of δN and δf into the equality $\delta N = \int \delta f\, d\mathbf{v}$ (which is valid for long waves with $ka \ll 1$, where a is the interatomic distance). From the dispersion equation

$$\frac{k^2}{\omega^2} = \frac{M}{m}\frac{1}{N}\int \frac{\mathbf{k}\frac{\partial f}{\partial \mathbf{v}}}{\omega - \mathbf{k}\mathbf{v}}\, d\mathbf{v}$$

obtained in this manner it follows that $\omega = ks(1 - i\delta)$, where

$$s \approx \left(\frac{m}{M}\langle v^2\rangle\right)^{1/2}, \quad \delta \approx \frac{s}{\langle v^2\rangle^{1/2}}.$$

The sound wave, in which no ion space charge is produced (transverse wave in a dielectric or a semiconductor), attenuates as a result of the carrier Joule losses. In the presence of a strong external magnetic field ($\Omega\tau \gg 1$), the attenuation of the sound can reach a large value because of the appearance of a large transverse carrier velocity (the carriers oscillate with velocity $v_{\parallel}$ under the influence of the deformation of the lattice along the direction of propagation of the wave), namely $v_{\perp} = \Omega\tau v_{\parallel}$. Experimental observation of the attenuation of sound agrees with theory and makes it possible to determine several plasma characteristics[38,39].

The dissipative mechanisms which cause sound attenuation under ordinary conditions lead to amplification of the sound waves if the carrier plasma motion in the crystal lattice is fast enough[40].

Let us consider by way of an example the case when the interaction of sound with a plasma is described by a deformation potential, and let us assume for simplicity that the "deformation charge" q differs from zero only in one species of carriers. If we neglect the intrinsic pressure and the electric field of the carriers, their speed $\exp(-i\omega t + ikx)$ in a plane sound wave is determined by the equality of the deformation force and the friction force:

$$v = -k^2\frac{q\tau}{m}\xi.$$

The carrier density oscillations, as can be seen from the continuity equation

$$\frac{\partial n}{\partial t} + N\nabla v + U\nabla n = -i\omega n + ikNv + ikUn = 0,$$

are in phase with the oscillations of the velocity in the case of subsonic carrier translational velocity ($U < s$), and out of phase in the case of supersonic velocity ($U > s$):

$$\frac{n}{N} = \frac{v}{\frac{\omega}{k} - U}.$$

It follows therefore that the right side of the inhomogeneous d'Alambert equation (20) for the deformation, which by virtue of its imaginary nature describes the attenuation of the sound, reverses sign at $U = \omega/k \approx s$:

$$-\omega^2 + s^2k^2 = +i\,\frac{q^2k^3\tau}{Mm}\,\frac{1}{\frac{\omega}{k} - U},$$

i.e., the attenuation is replaced by amplification when $U > s$. Like the attenuation, the amplification of the sound waves by a plasma stream is due to different mechanisms in different solids.

a) When long-wave sound propagates in a charged lattice of a metal or semiconductor, the carriers execute oscillations under the influence of the force exerted by the lattice and under the influence of the viscosity force

$$\eta\frac{\partial^2 v_-}{\partial x^2} = -k^2\eta v_-.$$

Inasmuch as there should be no perturbation of space charge in the case of a high-density plasma, the perturbations of the carrier density

$$\delta n_- = kNv_-(\omega - \mathbf{k}\mathbf{U})^{-1}$$

(U—carrier drift velocity, v_-—carrier vibrational velocity), and of the lattice $\delta n_+ = kNv_+$ (v_+—lattice velocity) are equal, so that

$$v_- = \left(1 - \frac{\mathbf{k}\mathbf{U}}{ks}\right)v_+$$

and consequently the force exerted by the carriers on the lattice, $F = k^2\eta v_-$, is equal to

$$k^2\eta\left(1 - \frac{\mathbf{k}\mathbf{U}}{ks}\right)v_+.$$

When $U < s$, this force determines the attenuation of the ultrasound[38], and when $U > s$ it leads to amplification.

b) Sound waves in which no space charge of the lattice takes place, give rise to vibrational motion of the carriers with velocity $v = \tau F/m$, where F—force of interaction between the lattice and the carriers. The carrier-current oscillations resulting from this force are

$$i = \delta n U + Nv = \frac{N\tau}{m}F\left(1 - \frac{\mathbf{k}\mathbf{U}}{ks}\right)^{-1},$$

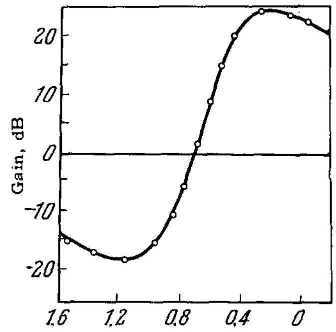

FIG. 9

so that the energy dissipation of the wave (mean value of the work performed by the force F) is proportional to

$$\langle iF\rangle \propto \frac{\frac{N\tau}{m}}{1-\frac{\mathbf{kU}}{ks}}\langle F^2\rangle.$$

The resonance width at the point $\mathbf{k}\cdot\mathbf{U}/ks = 1$ is

$$\Delta \approx \frac{4\pi\sigma}{\omega}(1+k^2D^2),$$

where σ—conductivity and D—Debye radius of the carrier plasma. This expression describes satisfactorily the attenuation (U < s) and amplification (U > s) of ultrasound in semiconductors[41] (Fig. 9).

c) When crossed electric and magnetic fields are applied to the plasma, the carriers drift with velocity

$$\mathbf{U}=\frac{c\,[\mathbf{E}_0\mathbf{H}_0]}{H_0^2}.$$

The interaction with a sound wave propagating along the vector $\mathbf{E}_0$ leads to the appearance in the plasma of a direct current proportional to the square of the amplitude of the oscillational carrier velocity in the wave:

$$\langle j\rangle = \langle ev_\perp\,\delta n\rangle = \frac{e\frac{\Omega\tau}{s}}{1-\frac{\mathbf{kU}}{ks}}\langle v_\parallel^2\rangle$$

(the resonance width at the point $\mathbf{k}\cdot\mathbf{U}/ks = 1$ is

$$\Delta \approx \frac{v_F}{s}kl\,(\Omega^2\tau^2+k^2l^2)^{-1},$$

where v_F—average velocity of random motion of the carriers and l—mean free path). When U < s the work of the electric field leads to attenuation of the ultrasound, which was thoroughly investigated experimentally[39]. When U > s the attenuation of sound is replaced by amplification.

If the speed of the directional (drift) motion is so large that sound amplification is possible, the amplitude of the thermal sound oscillations can also build up; inasmuch as the waves that grow under such conditions are principally those propagating along the vector of the carrier translational velocity, additional translational momentum is transferred from the plasma to the lattice, i.e., an additional friction force is produced between the plasma and the crystal lattice. The appearance of such a force leads to a kink in the current-voltage characteristic of the investigated sample, that is, to a jumplike change in the slope of the current vs. voltage curve in the absence of a magnetic field[42] (an increase in friction reduces the current), and to an analogous jumplike increase of the slope in a strong magnetic field (a decrease in friction increases the current)[43,44].

In analogy with the amplification and generation of sound,* the flow of solid-state plasma can produce amplification (and generation) of magnetic excitation in ferromagnets, antiferromagnets, and ferrites, of Rayleigh waves, of flexural waves[47], etc.

In the case when the wave attenuation and the particle frequency collisions are small, so that the waves, electrons, and holes can be regarded as weakly interacting quasiparticles, we can indicate the following approximate condition for such an amplification: the translational velocity of the carriers must exceed the phase velocity of the excited waves. Indeed, the Hamiltonian of interaction of the individual electrons and holes (fermions) with the waves (bosons) has the same form as the interaction of a particle "current" with a wave "field":

$$H=\sum_{pp'}\lambda a_p^+ a_{p'} b_{p-p'} + \text{Herm. conj.}$$

where a^+, a, b^+, b—operators of creation and annihilation of the particles and waves, respectively, and λ—interaction amplitude. Consequently, the equation for the wave density N_k in the space of the wave numbers k is of the form (for a weak interaction)

$$\dot N_k=\sum_p w\,[(N_k+1)\,n_{p+\frac{k}{2}}(1-n_{p-\frac{k}{2}})-N_k n_{p-\frac{k}{2}}\times(1-n_{p+\frac{k}{2}})]\,\delta(\varepsilon_{p+\frac{k}{2}}-\varepsilon_{p-\frac{k}{2}}-\omega_k),$$

where ϵ and ω are the energies of the particles and waves, and n_p is the momentum distribution function of the Fermi particles; the transition probability w is proportional to λ^2. Assuming that $N_k \gg 1$ (which corresponds to the case of amplification or attenuation of a sufficiently intense wave packet), we obtain

$$\dot N_k = 2\gamma N_k,\quad \gamma=\frac{1}{2}\sum_p w\,(n_{p+\frac{k}{2}}-n_{p-\frac{k}{2}})\,\delta(\omega_k-\mathbf{kv}),$$

where $v = p/m^*$—velocity of the particles. For sufficiently long waves ($k \ll p$) we obtain therefore

$$\gamma \sim \left(\frac{\partial f}{\partial p_\parallel}\right)_{v_\parallel=\frac{\omega_k}{k}},$$

*The linear theory of amplification of sound by carrier flow is the subject of many papers (see, for example, [45]); the limitation of the amplitude of the amplified sound wave by nonlinear effects is treated in [46].

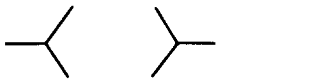

FIG. 10

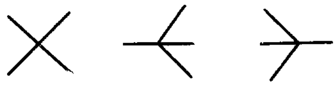

FIG. 11

where $f = \sum_{p_\perp} n_p$ (the summation is over the momentum components perpendicular to the wave vector of the excited oscillation). If the particle gas moves as a unit with velocity U, then

$$\frac{\partial f}{\partial p_\parallel} > 0 \text{ for } v < U \text{ and } \frac{\partial f}{\partial p_\parallel} < 0 \quad \text{for } v > U.$$

Therefore, when $U > \omega_k/k$ we should observe amplification of the waves with $\gamma > 0$.

V. WAVE INTERACTION

Waves propagating in a solid-state plasma interact with one another. The characteristic features of the interaction of plasma waves are easiest to explain by considering a case when the attenuation of these waves is sufficiently small, so that we can regard them in practice as quasiparticles obeying Bose statistics. The interaction of such quasiparticles is best described by a Hamiltonian containing the product of three, four, etc. field operators: $H = H^{(3)} + H^{(4)} + \ldots$ In this case, in the lowest order of perturbation theory, the Hamiltonian $H^{(3)}$ leads to two processes: a) decay of one wave into two waves and b) coalescence of two waves into one wave (Fig. 10). The Hamiltonian $H^{(4)}$ leads to three processes: a) scattering of two waves (transformation of two waves into two waves), b) decay of one wave into three, c) coalescence of three waves into one (Fig. 11). The waves present in the initial and final states of all these processes can belong to the same branch of the plasma oscillations, or to different branches*; phonons can also participate in the interaction with the plasma waves.

Knowing the Hamiltonian of the wave interaction, we can obtain the kinetic equation for the distribution functions of waves in the wave-vector space. For example, the part of the Hamiltonian $H^{(3)}$ describing the decay of a helical wave (helicon) into a helical and sound wave (phonon), and the inverse process, is of the form

$$H = \sum_{kq} \lambda_{kq} a_k^+ a_{k-q} b_q + \text{Herm. conj.}$$

(a^+, a; b^+, b—operators of creation and annihilation of helicons and phonons, λ_{qk}—interaction amplitude), the kinetic equation for the distribution function n_k of the helicons takes the form

*It must be noted that the transformation of one type of wave into another can be realized not only in processes of decay, coalescence, and scattering of waves, but also in the processes of induced scattering of waves by plasma particles.

$$\frac{dn_k}{dt} = \sum_{\substack{q \\ \omega_k = \omega_{k-q} + \Omega_q}} w\,[(n_k+1)\,n_{k-q}N_q - n_k(n_{k-q}+1)(N_q+1)] + \sum_{\substack{q \\ \omega_k + \Omega_q = \omega_{k+q}}} w\,[(n_k+1)\,n_{k+q}(N_q+1) - n_k(n_{k+q}+1)N_q]. \tag{30}$$

Here N_q—phonon distribution function, w—probability of the decay or coalescence process; the summation is over the values of the wave vector q satisfying the energy (frequency) conservation laws in the decay and in the coalescence $\omega_k = \omega_{k-q} + \Omega_q$ in the first sum and $\omega_k + \Omega_q = \omega_{k+q}$ in the second sum.

Inasmuch as the decay of a helicon into a helicon and a phonon is possible only if the energy conservation laws are satisfied, the decay of low frequency helicons is in general forbidden; when the frequency of the helicon rises above a threshold $\omega_1 = s^2(\pi n_0 e/cH_0)$ (for a wave propagating along the magnetic field), its group velocity exceeds the velocity of sound s and emission of a phonon by a helicon (decay) becomes possible, i.e., additional attenuation of the helicon sets in when $\omega > \omega_1$. An analogous attenuation, due to the decay and coalescence processes, is possible also for other waves in a plasma in a solid.

In some cases the interaction of plasma waves can have a different character. If, for example, high-frequency oscillations are produced in a plasma, then the low-frequency wave propagating in such a plasma will produce "compression" and "rarefaction" in the high-frequency oscillation gas, as a result of which its phase velocity, group velocity, and decrement (increment) can change noticeably. Such an "adiabatic" interaction of the waves can occur, in particular, in the development of two-stream instability in a plasma whose carriers have long relaxation times [48].

VI. OCCURRENCE OF TURBULENCE IN A PLASMA IN A SOLID

The amplitude of the small perturbations that develop in an unstable plasma in a solid increases exponentially with time, so that the square of the amplitude η satisfies the differential equation

$$\frac{d\eta}{dt} = 2\gamma\eta, \tag{31}$$

where γ—increment of the linear theory. As the perturbation develops, the rate of its growth changes and (31) is no longer valid. If η is small, and the resultant pulsations have a regular character, i.e., a definite frequency and wavelength, the rate of growth of η can be determined by expanding the equations describing the plasma dynamics in powers of the perturbation amplitude. We then obtain the equation

$$\frac{d\eta}{dt} = 2\gamma_H\eta, \quad \gamma_H = \gamma + a\eta + b\eta^2 + \ldots, \tag{32}$$

which differs from (31) in that the increment of the linear theory γ is replaced by a "nonlinear incre-

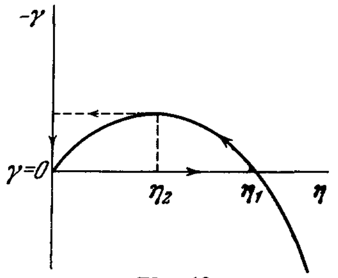

FIG. 12

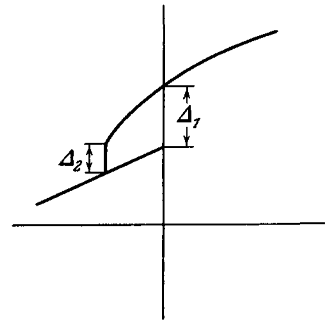

FIG. 13

ment'' γ_H, which depends on the square of the amplitude η. With the aid of (32) it is possible to describe several phenomena that occur in an unstable system in a slightly supercritical state, that is, in the case when the increment γ is small.

There are two modes of turbulence: ''soft''[55] and ''hard''[56]. The soft mode corresponds to the case $a < 0$ in (32); the square of the amplitude of the stationary motion arising in an unstable solid-state plasma is equal to

$$\eta = -\frac{\gamma}{a}$$

and increases smoothly from zero on going from stability ($\gamma < 0$) to instability ($\gamma > 0$) and as the supercriticality is increased. If the transition from the subcritical mode to the supercritical mode has been brought about by a change in some parameter X of the system (electric or magnetic field, temperature gradient, etc.), then a kink appears at the critical point $X = X_{cr}$ on the plot of any quantity Y, averaged over the pulsations, against this parameter.

Indeed, the expansion of any average quantity Y in powers of the amplitude of the perturbations that develop in an unstable plasma is of the form

$$Y = Y_0 + \alpha\eta + \ldots, \quad (33)$$

and since $\eta = 0$ when $X < X_{cr}$ and $\eta \sim X - X_{cr}$ when $X > X_{cr}$, the derivative of Y with respect to the parameter X has at the point $X = X_{cr}$ a finite discontinuity $\Delta(\partial Y/\partial X)$ (the function Y(X) itself is continuous).

In the case when two parameters $X_{1,2}$ vary, the transition to the unstable state occurs on some curve $\Phi(X_{1cr}, X_{2cr}) = 0$; the jumps of the derivatives of any two quantities that are averaged over the pulsations are then related by[49]:

$$\Delta\frac{\partial Y_1}{\partial X_1}\Delta\frac{\partial Y_2}{\partial X_2} = \Delta\frac{\partial Y_1}{\partial X_2}\Delta\frac{\partial Y_2}{\partial X_1}. \quad (34)$$

In the ''hard'' mode ($a > 0$) the turbulence develops in the following fashion. By varying some plasma parameter X it is possible to decrease the decrement, $\gamma \to 0$ (as $X \to X_{cr}$), and go over into the instability region, When $\gamma = +0$ the amplitude of the perturbations reaches jumpwise a finite value η_1, determined from the vanishing of the nonlinear increment γ_H:

$$a\eta_1 + b\eta_1^2 + \ldots = 0, \quad \eta_1 = -\frac{a}{b} \quad (35)$$

(Fig. 12). With further increase of supercriticality, the amplitude increases smoothly from a value η_1 to larger values η_0, defined by the equality $\gamma_H(\eta_0) = 0$. If we now reduce the parameter X below the critical value ($X < X_{cr}$), then the motion in the plasma does not stop; the amplitude $\sqrt{\eta}$ decreases to zero jumpwise only when $X = X'_{cr} < X_{cr}$. It follows from (32) that the solution $\eta = \eta_0$ is stable if

$$\left(\frac{\partial\gamma_H}{\partial\eta}\right)_{\eta=\eta_0} < 0,$$

and unstable if

$$\left(\frac{\partial\gamma_H}{\partial\eta}\right)_{\eta=\eta_0} > 0,$$

so that the perturbations are interrupted when $\partial\gamma_H/\partial\eta = 0$, that is, when

$$\eta = \eta_2 = -\frac{2a}{b}. \quad (36)$$

Thus, the hard mode of turbulence is characterized by a hysteresis, consisting in the occurrence and cessation of pulsations in the plasma at different values of the external parameters.

The plot of any observed quantity Y, averaged over the pulsations, against the parameter X has discontinuities and a hysteresis loop in the case of hard turbulence (Fig. 13). The jumps Δ_1 and Δ_2 on this curve (excitation and cessation of the pulsations) are connected by the simple relation

$$\frac{\Delta_1}{\Delta_2} = 2 \quad (37)$$

(which follows from (33) when account is taken of (35) and (36)). The kinks on the plot of $Y = Y(X)$ (soft mode) were observed on the voltage-current characteristics of piezoelectric semiconductors (CdS, CdSe)[42] and semimetals (Bi)[43]; the reason for the kink is considered to be the occurrence of instability of the plasma in the solid relative to excitation of sound oscillations, in the case when the velocity U of the plasma stream in an external electric field ($U = \mu E$) or in crossed electric and magnetic

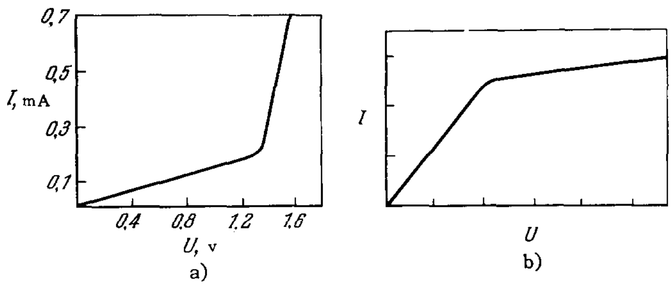

FIG. 14

fields ($U = cE \times H/H^2$) exceeds the speed of sound s. In the absence of a magnetic field, the pulsations occurring when U > s give rise to a decrease in the average plasma velocity, and dI/dE decreases (Fig. 14b). In the presence of a strong magnetic field, the pulsations reduce the average supersonic drift velocity; therefore the component of the Lorentz force in a direction opposite to the force due to the electric field decreases, and the current-voltage characteristic becomes steeper:

$$\Delta \frac{dI}{dE} > 0$$

(if the drift velocity in the turbulent plasma were to decrease to the speed of sound, the differential magnetoresistance of such a plasma would be equal to the ordinary resistance

$$\frac{dE}{dj} = \frac{m}{ne^2\tau},$$

in spite of the presence of a strong transverse magnetic field).

Hysteresis phenomena (hard mode) were observed during the course of development of screw perturbations and instabilities in a plasma in parallel electric and magnetic fields[50].

VI. THEORETICAL AND EXPERIMENTAL PROBLEMS

In conclusion let us point out several problems, both theoretical and experimental, the clarification of which would contribute to an understanding of the properties of solid-state plasma).

The most interesting problem of the theory is the clarification of the character of the plasma motion occurring during spontaneous growth of fluctuations in an unstable plasma (and also the following related problems: stabilization of instability by means of an external periodic perturbation, limitation of the amplification of an external signal in an unstable plasma, and generation of harmonics of this signal, and supersonic flow of solid-state plasma).

The transport of the energy released when radiation acts on the solid in an electronic plasma (in metals, organic crystals and amorphous substances, or large molecules) is not yet clear.

It would be of interest to investigate the effect exerted on the collective properties of a solid-state plasma by the spatial periodicity and anisotropy of the plasma.

One of the concrete problems which has arisen recently is the theory of the pinch effect in a solid-state plasma (equilibrium, stability, turbulent mode); additional experiments are needed here, however; another concrete problem is the theory of microplasma, namely small glowing regions (with diameter of several microns), observed during the course of breakdown in semiconductor n-p junctions, and characterized by a decreased breakdown voltage (see, for example, [51] and the literature cited there).

Interest attaches to experiments on the conditions of occurrence of different plasma instabilities, the investigation of the soft and hard modes of occurrence of turbulence, and particularly a check on relations (34) and (37) and a study of the character of variation of the averaged quantities during the occurrence of turbulence (see Fig. 13). Other experiments of interest are those on the reflection and scattering of electromagnetic waves by plasma in a near-unstable state, and also by a turbulent plasma, as well as the investigation of supersonic flow of solid-state plasma.

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