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POLARIZED PROTON TARGET IN EXPERIMENTS WITH HIGH-ENERGY PARTICLES

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1. INTRODUCTION

REPORTS of experiments with a target containing polarized protons have been recently published[1-3]. In all these experiments the target used was single-crystal $La_2Mg_3(NO_3)_{12}\cdot 24H_2O$ in which approximately 1% of La^{3+} was replaced by paramagnetic Ce^{3+} or Md^{3+} ions. The protons of the water of crystallization, which constitute about 3% of the weight of the crystal, are polarized by the so-called dynamic polarization method. We describe here briefly a simplified scheme by which the protons become polarized. The reader who wishes a more detailed knowledge of the polarization mechanism is referred to [4].

We consider the energy levels of a system comprising an electron (more accurately, the electron shell of a paramagnetic ion with effective spin $s = 1/2$) and a proton, situated in a constant magnetic field H (Fig. 1).

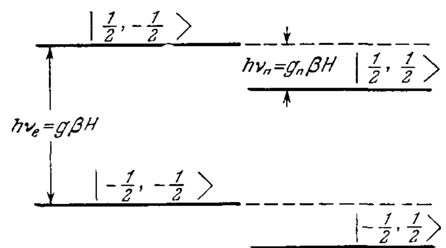

FIG. 1.

In Fig. 1, $|M, m\rangle$ —state with electron and nucleus spin projections M and m, β —Bohr magneton, g and g_n —electron and proton Lande factors. Owing to the dipole-dipole coupling of the magnetic moments of the electron and proton, the unperturbed states $|M, m\rangle$ are mixed and in an alternating magnetic field perpendicular to the field H, transitions are produced with simultaneous change of the electron and proton spin projections ("forbidden" transitions). Upon saturation, say, of the transition $|-1/2, -1/2\rangle \rightleftarrows |1/2, 1/2\rangle$, the populations of these states become equal*:

$$N_+ n_+ = N_- n_-, \tag{1.1}$$

where $M_\pm$ and $n_\pm$ are the numbers of the electrons and protons with spin projections $\pm 1/2$, respectively. If the relaxation time of the allowed transitions $|M, m\rangle \rightleftarrows |M\pm 1, m\rangle$ is much smaller than the time of the "forbidden" transition induced by the alternating field, then the electrons have a near-Boltzmann distribution

$$\frac{N_+}{N_-} \approx e^{-\frac{g\beta H}{kT}}. \tag{1.2}$$

From (1.1) and (1.2) we find that

$$\frac{n_+}{n_-} = e^{\frac{g\beta H}{kT}}, \tag{1.3}$$

which yields for the proton polarization

$$P = \frac{\frac{n_+}{n_-} - 1}{\frac{n_+}{n_-} + 1} = \mathrm{th}\,\frac{g\beta H}{2kT}. \tag{1.4†}$$

It is obvious that in the absence of an alternating field the proton polarization is equal to

$$P_{\text{stat}} = \mathrm{th}\,\frac{g_n\beta H}{2kT}. \tag{1.5}$$

Thus, saturation of the paramagnetic resonance gives rise to a proton polarization appreciably larger than the static polarization. It follows from (1.4) and (1.5) that for $g\beta H/2kT \ll 1$ we have

$$P \simeq \frac{g\beta H}{2kT} \simeq \frac{g}{g_n} P_{\text{stat}} \simeq 6.6\cdot 10^2 P_{\text{stat}}.$$

*Saturation of the "forbidden" paramagnetic resonance means that the proton relaxation time is much larger than the time of the transition induced by the external alternating magnetic field.

†th = tanh.

This method was used in [5] to obtain a 60% polarization in a crystal having a volume of approximately 15 cm^3, at T = 1.2°K, a constant field H = 18.5 kG, and an alternating-field frequency 70 Gc.

In this review we consider possible applications of a polarized proton target in high-energy physics. The use of a polarized proton target allows noticeable progress to be made in the measurement of polarization effects in various reactions. For example, proton polarization in π-p and p-p scattering can be determined by measuring the left-right asymmetry of the pions (nucleons) in the scattering of pions (unpolarized nucleons) by a polarized target. If the target is not polarized, then the proton polarization is determined by means of a double experiment using the asymmetry in the secondary scattering of the recoil protons. We note that even in the first experiment on π-p scattering from a polarized target [2] the proton polarization was determined with higher accuracy than in the double experiment [6-8], and at those angles at which the measurement of the left-right asymmetry is made difficult by the low analyzing ability of the analyzer target. The same method was used to measure nucleon polarization in p-p scattering in the energy range 1.7—6.1 GeV. In the case of nucleon-nucleon scattering it becomes possible, for example, to replace the triple experiment for determining the polarization correlation by a measurement of the cross section for the scattering of a polarized beam by a polarized target, etc.

A polarized target makes possible several experiments not otherwise feasible. An example is the determination of the spin-flip parameters in π-p scattering.

It is clear that the performance of these experiments will lead to noticeable progress in the determination of the scattering matrices and to a deeper understanding of the spin dependence of the interactions between elementary particles.

The use of a polarized proton target offers more possibilities in the study of inelastic processes, too.

The study of the reactions

$$\pi(\overline{K}) + p \to Y + K(\pi),$$
$$\overline{K} + p \to \Xi + K$$

with a polarized target permits a unique determination of the parity of the strange particles [10-14].

Our analysis of the possible applications of a polarized target will be based only on general requirements of invariance under space rotations and reflections and under time reversal. These requirements will be treated in a separate section. We shall formulate on their basis the main theorems used in the study of polarization phenomena in strong interactions.

The authors hope that the review can serve as an introduction to many problems connected with studies of polarization phenomena.

2. THE "POLARIZATION-ASYMMETRY" RELATION IN THE CASE OF POLARIZATION PERPENDICULAR TO THE REACTION PLANE. THE BOHR RULE

We start with an elementary consideration of the important relations that will be needed later on.

We consider first the scattering of spinless particles by polarized particles with spin $1/2$, for example the scattering of pions by a polarized hydrogen target. Polarization of particles with spin $1/2$ is defined as the mean value of the operator $\boldsymbol{\sigma} = 2\mathbf{s}$ ($\mathbf{s}$ —spin operator, σ —Pauli matrices). This quantity describes completely the spin state of particles with spin $1/2$. Let $\mathbf{P} = P\mathbf{n}_0$ denote the initial polarization of the target particles. The initial and final relative momenta $\mathbf{p}$ and $\mathbf{p}'$ (in the c.m.s.) define the scattering plane. We assume that the vector $\mathbf{n}_0$ is orthogonal to this plane. We shall agree to say that the particles are scattered to the left if the vector product $\mathbf{p} \times \mathbf{p}'$ is parallel to $\mathbf{n}_0$, and to the right if $\mathbf{p} \times \mathbf{p}'$ is antiparallel to $\mathbf{n}_0$ (Fig. 2). If we align the z axis with $\mathbf{p}$ and the y axis with $\mathbf{n}_0$, then the azimuthal angle φ is zero for left scattering and $\varphi = \pi$ for right scattering.

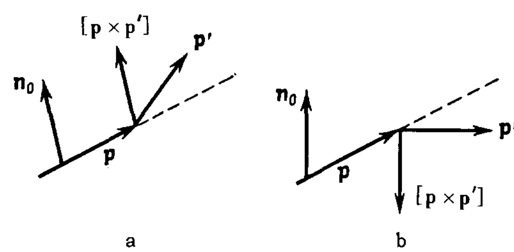

FIG. 2. a – Scattering to the left; b – scattering to the right.

The state with polarization $\mathbf{P} = P\mathbf{n}_0$ is described by an incoherent "mixture" of two eigenfunctions χ_+ and χ_- of the operator $(\boldsymbol{\sigma}, \mathbf{n}_0)$. If w_+ and w_- are normalized probabilities with which χ_+ and χ_- enter into the mixture, then

$$P = w_+ - w_-. \tag{2.1}$$

From (2.1) and from the normalization conditions we get

$$\left.\begin{aligned} w_+ &= \frac{1}{2}(1+P), \\ w_- &= \frac{1}{2}(1-P). \end{aligned}\right\} \tag{2.2}$$

Let $\sigma^{L}_{m';m}(\theta)$ and $\sigma^{R}_{m';m}(\theta)$ —differential cross sections for right and left scattering through an angle θ and m and m′ projection of the spin on $\mathbf{n}_0$ in the initial and final states. We then obtain for the differential cross sections for scattering by a target with polarization P

$$\begin{aligned} \sigma^{L,R}(\theta) &= w_+(\sigma^{L,R}_{++}(\theta) + \sigma^{L,R}_{-+}(\theta)) + w_-(\sigma^{L,R}_{+-}(\theta) + \sigma^{L,R}_{--}(\theta)) \\ &= \frac{1}{2}(\sigma^{L,R}_{++}(\theta) + \sigma^{L,R}_{-+}(\theta) + \sigma^{L,R}_{+-}(\theta) + \sigma^{L,R}_{--}(\theta)) \\ &+ \frac{1}{2}P(\sigma^{L,R}_{++}(\theta) + \sigma^{L,R}_{-+}(\theta) - \sigma^{L,R}_{+-}(\theta) - \sigma^{L,R}_{--}(\theta)). \end{aligned} \tag{2.3}$$

The following relations can be readily established between the cross sections for left and right scattering:

$$\sigma^L_{m';\,m}(\theta) = \sigma^R_{-m';\,-m}(\theta). \tag{2.4}$$

To this end we must use invariance under rotation. Indeed, rotation through π about the initial momentum transforms the final momentum of the right-hand scattering into the final momentum of the left-hand scattering and vice versa, with the spin projections reversing sign (Fig. 3). The first term in (2.3), which we denote by $\sigma_0(\theta)$, is the cross section for scattering by an unpolarized target. It is seen from (2.4) that this cross section is the same for the left and right scattering.

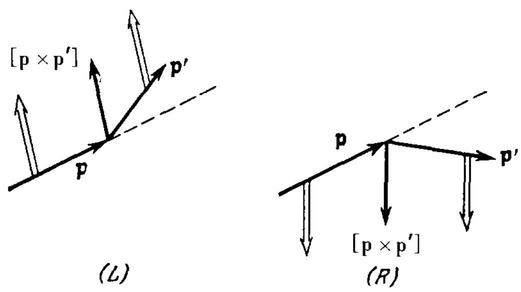

FIG. 3. The scattering (L) is obtained from (R) by a rotation of π and **p** (the double arrows indicate the spin directions).

From (2.4) it follows also that the sign of the term in (2.3) proportional to the target polarization P depends on whether the scattering is to the left or to the right. The effect of target polarization can be consequently investigated by measuring the difference in the cross sections for left and right scattering. It is customary to introduce the quantity

$$e^{LR}(\theta) = \frac{\sigma^L(\theta) - \sigma^R(\theta)}{\sigma^L(\theta) + \sigma^R(\theta)}, \tag{2.5}$$

called the left-right asymmetry. From (2.3) and (2.4) we obtain for the left-right asymmetry the expression:

$$e^{LR}(\theta) = P\,\frac{\frac{1}{2}\left[\sigma^L_{++}(\theta) + \sigma^L_{-+}(\theta) - \sigma^L_{+-}(\theta) - \sigma^L_{--}(\theta)\right]}{\sigma_0(\theta)}. \tag{2.6}$$

It was shown first by Wolfenstein[15] that the coefficient of P in this expression is equal to the polarization of the recoil particles due to left-hand scattering by an unpolarized target. In the case that we are considering, scattering of particles with spin 0 and $1/2$, this is a consequence of the invariance under space rotations and reflections. From the foregoing invariance requirements it follows, first, that the polarization, which is the mean value of the spin operator, is a pseudovector. If the target is not polarized, then the vector product $\mathbf{p} \times \mathbf{p}'$ is a unique pseudovector which can be constructed from the physical quantities of the problem. Thus, the final-particle polarization resulting from scattering by an unpolarized target is orthogonal to the scattering plane. In the case of left-hand scattering the polarization is obviously equal to

$$P^L_0(\theta) = \frac{\frac{1}{2}\left(\sigma^L_{++}(\theta) + \sigma^L_{+-}(\theta) - \sigma^L_{-+}(\theta) - \sigma^L_{--}(\theta)\right)}{\sigma_0(\theta)}. \tag{2.7}$$

At first glance this expression differs from that in (2.6) [the cross sections $\sigma^L_{+-}(\theta)$ and $\sigma^L_{-+}(\theta)$ have different signs in (2.6) and (2.7)]. However, from the invariance against rotations and reflections it follows that in the case of elastic scattering $\sigma_{+-}(\theta) = \sigma_{-+}(\theta) = 0$. This is easiest to establish by means of the rule formulated by O. Bohr[14].

We derive this rule in the general case of a two-particle reaction

$$a + b \rightarrow c + d. \tag{2.8}$$

Bohr's rule follows from the invariance against reflection in the reaction plane. Let us assume that the initial and final wave functions $|\mathbf{p}, M\rangle$ and $|\mathbf{p}', M'\rangle$ are eigenfunctions of the operator of projection of the total spin on the direction normal to the reaction plane (M and M' are the corresponding projections). It is obvious that the momenta **p** and **p'** remain unchanged under reflection R in the reaction plane. Since the spin is a pseudovector, its projections on the normal direction likewise remain unchanged. Therefore $|\mathbf{p}, M\rangle$ and $|\mathbf{p}', M'\rangle$ are the eigenfunctions of the reflection operator R. In order to determine the corresponding eigenvalues, we note that the operation of reflection in the reaction plane can be represented by a product of the inversion operation I (reflection at the origin) and rotation $R_n(\pi)$ through an angle π about the normal. Letting the operator R act on the initial state, we obtain

$$R\,|\mathbf{p},\ M\rangle = R_n(\pi)\, I\,|\mathbf{p},\ M\rangle = I_i R_n(\pi)\,|-\mathbf{p},\ M\rangle$$
$$= I_i e^{i\pi(\mathbf{sn})}\,|\mathbf{p},\ M\rangle = I_i e^{i\pi M}\,|\mathbf{p},\ M\rangle, \tag{2.9}$$

where $I_i = I_a I_b$ —product of the intrinsic parities of the initial particles, and **s** —total spin operator. We obtain analogously for the final-state function

$$R\,|\mathbf{p}',\ M'\rangle = I_f e^{i\pi M'}\,|\mathbf{p}',\ M'\rangle, \tag{2.10}$$

where $I_f = I_c I_d$. Owing to invariance under reflections, the eigenvalues of R in the initial and final states should coincide, and we arrive at the following selection rule:

$$I_i e^{i\pi M} = I_f e^{i\pi M'}. \tag{2.11}$$

This is the Bohr rule. It relates the intrinsic parities of the particles participating in the reactions with the projections of the total spin on the direction of the normal to the reaction plane.

We now consider elastic scattering of particles with spins 0 and $1/2$. In this case $I_i = I_f$, and the possible values of $M = m$ and $M' = m'$ are $\pm 1/2$. As can be seen from (2.11), this means that $m = m'$ and scattering with spin-flip is forbidden: $\sigma_{+-} = \sigma_{-+} = 0$. Thus, the left-right asymmetry (2.6) and the polarization (2.7) are equal to

$$e^{LR}(\theta)=P\frac{\frac{1}{2}(\sigma^L_{++}(\theta)-\sigma^L_{--}(\theta))}{\sigma_0(\theta)}, \tag{2.12}$$

$$P_0^L(\theta)=\frac{\frac{1}{2}(\sigma^L_{++}(\theta)-\sigma^L_{--}(\theta))}{\sigma_0(\theta)}. \tag{2.13}$$

It is therefore evident that

$$e^{LR}(\theta)=PP_0^L(\theta). \tag{2.14}$$

This relation plays an important role in the analysis of experiments with polarized particles. We shall show below that (2.14) is valid also in the general case of elastic collisions of polarized particles with spin ½ and unpolarized particles with arbitrary spin. In deriving (2.14) for our very simple case we make use only of the requirement of invariance under rotations and reflections. It is therefore clear that (2.14) will hold true also for arbitrary reactions of the type

$$0+\tfrac{1}{2}\to 0+\tfrac{1}{2} \tag{2.15}$$

(0 and ½ —particle spins) if $I_i = I_f$.

It turns out that even if the total intrinsic parity does change ($I_i = -I_f$), the left-right asymmetry in reactions of the type (2.15) is completely determined by the polarizations P and $P_0^L(\theta)$. This can be readily understood, as in the preceding case, by using Bohr's rule. Indeed, if $I_i = -I_f$ it follows from (2.11) that $m = -m'$, and reaction without spin-flip is forbidden: $\sigma_{++} = \sigma_{--} = 0$. It then follows from (2.6) and (2.7) that

$$e^{LR}(\theta)=P\frac{\frac{1}{2}(\sigma^L_{-+}(\theta)-\sigma^L_{+-}(\theta))}{\sigma_0(\theta)}, \tag{2.16}$$

$$P_0^L(\theta)=\frac{\frac{1}{2}(\sigma^L_{+-}(\theta)-\sigma^L_{-+}(\theta))}{\sigma_0(\theta)}. \tag{2.17}$$

From this we get

$$e^{LR}(\theta)=-PP_0^L(\theta). \tag{2.18}$$

Thus, for arbitrary reactions of the type (2.15), the left-right asymmetry is[10]

$$e^{LR}(\theta)=\pm PP_0^L(\theta), \tag{2.19}$$

where the "+" sign corresponds to $I_i = I_f$ and the "−" sign pertains to the case $I_i = -I_f$. This relation, which will be discussed in detail in what follows, makes it possible to determine the intrinsic parities of particles in experiments with polarized hydrogen targets.

Let us proceed to consider elastic collisions between unpolarized particles with spin j and polarized particles with spin ½. We shall assume, as before, that the initial polarization is orthogonal to the scattering plane. Obviously, the unpolarized particles with spin j can be described by an incoherent "mixture" of (2j + 1) eigenfunctions of the operator of spin projection on the $\mathbf{n}_0$ direction. The weight with which each function enters into the "mixture" is $1/(2j+1)$. The cross section for scattering to the left (to the right) through an angle θ will be denoted by $\sigma^{L(R)}_{\mu'm;\mu m}(\theta)$ (μ' and μ —projections of the spin j in the final and initial states). Then the differential cross sections $\sigma^{L(R)}(\theta)$, averaged over the initial and summed over the final spin states, are equal to

$$\sigma^{L,R}(\theta)=\frac{1}{2}\frac{1}{2j+1}\sum_{\substack{\mu,\mu'\\ m,m'}}\sigma^{L,R}_{\mu'm';\mu m}(\theta)+\frac{1}{2(2j+1)}P\sum_{\mu,\mu',m'}[\sigma^{L,R}_{\mu'm';\mu+}(\theta)-\sigma^{L,R}_{\mu'm';\mu-}(\theta)]. \tag{2.20}$$

Invariance under rotations leads to the relation

$$\sigma^L_{\mu'm';\mu m}(\theta)=\sigma^R_{-\mu'-m';-\mu-m}(\theta). \tag{2.21}$$

Summing these relations over the spin projections, we get

$$\sum_{\substack{\mu',\mu\\ m',m}}\sigma^L_{\mu'm';\mu m}(\theta)=\sum_{\substack{\mu',\mu\\ m',m}}\sigma^R_{\mu'm';\mu m}(\theta), \tag{2.22}$$

$$\sum_{\mu',\mu}\sigma^L_{\mu'm';\mu m}(\theta)=\sum_{\mu',\mu}\sigma^R_{\mu'(-m');\mu(-m)}(\theta). \tag{2.23}$$

The first relation signifies that the differential cross sections for the scattering of unpolarized particles to the left and to the right are equal to each other. From the second relation it follows that the coefficient of P in (2.20) reverses sign when the scattering direction is changed. Taking this into account, we obtain the following expression for the left-right asymmetry

$$e^{LR}(\theta)=P\frac{1}{2(2j+1)\sigma_0(\theta)}\times\sum_{\mu',\mu}[\sigma^L_{\mu'+;\mu+}(\theta)+\sigma^L_{\mu'-;\mu+}(\theta)-\sigma^L_{\mu'+;\mu-}(\theta)-\sigma^L_{\mu'-;\mu-}(\theta)]. \tag{2.24}$$

When unpolarized particles are scattered the polarization of the spin ½ particles is equal to

$$P_0^L(\theta)=\frac{1}{2(2j+1)\sigma_0(\theta)}\times\sum_{\mu',\mu}[\sigma^L_{\mu+;\mu+}(\theta)+\sigma^L_{\mu'+;\mu-}(\theta)-\sigma^L_{\mu'-;\mu+}(\theta)-\sigma^L_{\mu'-;\mu-}(\theta)]. \tag{2.25}$$

It is easy to see that when j differs from zero the Bohr rule does not forbid transitions with changes in the spin-½ projection, that is, $\sum_{\mu',\mu}\sigma_{\mu'\pm,\mu\mp}(\theta)$ are generally speaking different from zero. In this case, however, as was shown by Wolfenstein and Ashkin[16] and by Dalitz[17], relation (2.14) holds for the left-right asymmetry. To prove this it is necessary to use besides invariance under rotations and reflections also invariance under time reversal. As is well known, in the case of the two-particle reaction (2.8), the invariance under time reversal relates the probability of the transition from the state $|\alpha, \mathbf{p}, \mu, m\rangle$ into the state $|\beta, \mathbf{p}', \mu', m'\rangle$ with the probability of the inverse transition from $|\beta, -\mathbf{p}', -\mu', -m'\rangle$ into $|\alpha, -\mathbf{p}, -\mu, -m\rangle$. Here α and β characterize the type of particle participating in the reaction. In the case of elastic scattering, the direct and the inverse processes are essentially identical ($\alpha = \beta$). They are

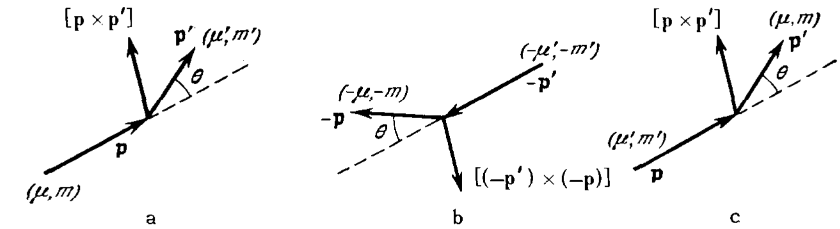

FIG. 4.

shown in Figs. 4a and b. The cross sections for the scatterings shown in Figs. 4b and c are equal as a result of invariance under rotations (it is obvious that Fig. 4c can be obtained from Fig. 4b by rotating through an angle π around p' and subsequent rotation through $\pi-\theta$ around the normal to the scattering plane). On the other hand, the cross sections of the scatterings in Figs. 4a and b are equal because of the invariance under time reversal. We arrive thus at equality of the cross sections for the scatterings shown in Figs. 4a and c:

$$\sigma_{\mu' m';\, \mu m}(\theta)=\sigma_{\mu m;\, \mu' m'}(\theta). \tag{2.26}$$

It follows from this that

$$\sum_{\mu',\mu} \sigma^{L}_{\mu'+;\, \mu-}(\theta)=\sum_{\mu',\mu} \sigma^{L}_{\mu'-;\, \mu+}(\theta), \tag{2.27}$$

Thus, the cross sections for scattering with spin-½ flip drop out from expressions (2.14) and (2.25) for the asymmetry and polarization, and we arrive again at (2.14).

In the general case of inelastic processes, equality (2.27), together with relation (2.14), does not hold true. We shall show below, however, that the left-right asymmetry can be related in this case to the polarization produced in the inverse reaction. Let us consider the process (2.8) under the assumption that the spin of particle b is equal to ½, and the spins of the remaining particles are arbitrary. Denoting them by j_a, j_c, and j_d respectively and proceeding as before, we obtain for the left-right asymmetry in the reaction with polarized particles b and unpolarized particles a the following expression:

$$e^{LR}(\theta)=P_b\frac{1}{2(2j_a+1)\,\sigma_0^{cd;\,ab}(\theta)} \times \sum_{m_a, m_c, m_d} [\sigma^{L}_{m_d m_c;\, (+)m_a}(\theta)-\sigma^{L}_{m_d m_c;\, (-)m_a}(\theta)], \tag{2.28}$$

where $\sigma_0^{cd;ab}(\theta)$ —differential cross section of the process (2.8) in the case of unpolarized particles, and P_b —polarization of particles b. From the invariance under rotations and time reversal we see that the cross sections of the direct process (2.8) and the inverse process

$$c+d\rightarrow a+b \tag{2.29}$$

are connected by the following relation:

$$\sigma_{m_d m_c;\, m_b m_a}(\theta)=\sigma_{m_b m_a;\, m_d m_c}(\theta)\frac{p'^2}{p^2}. \tag{2.30}$$

With the aid of these relations we get from (2.28)

$$e^{LR}(\theta)=P_b\frac{1}{(2j_c+1)(2j_d+1)\,\sigma_0^{ab;\,cd}(\theta)} \times \sum_{m_c, m_d, m_a} (\sigma^{L}_{(+)m_a;\, m_d m_c}(\theta)-\sigma^{L}_{(-)m_a;\, m_d m_c}(\theta)), \tag{2.31}$$

where $\sigma_0^{ab;cd}(\theta)$ —differential cross section of the reaction (2.29) with unpolarized particles. It is obvious that the coefficient of P_b represents in this expression the polarization $P_b^{inv}(\theta)$ of the particles b, resulting from the inverse reaction (2.29), when particles c and d are not polarized. We obtain therefore the following relation[18-21]

$$e^{LR}(\theta)=P_b P_b^{inv}(\theta), \tag{2.32}$$

which is a generalization of (2.14) to include the case of inelastic reactions.

In deriving the fundamental relations (2.14), (2.19), and (2.32) of this section, we have assumed that the polarization of the initial particles is perpendicular to the reaction plane. Later on, when we formulate the invariance requirements for the reaction amplitude, these relations will be generalized to include the case of an arbitrary polarization direction.

3. SYMMETRY PRINCIPLES AND LIMITATIONS ON THE FORM OF THE REACTION AMPLITUDE

We shall formulate first the invariance conditions in the form of requirements which must be satisfied by the S matrix.* The S matrix, as is well known, is defined as the operator which transforms the initial wave function of the system ($t\rightarrow-\infty$) into the wave function for $t\rightarrow+\infty$:

$$|\psi(+\infty)\rangle=S\,|\psi(-\infty)\rangle. \tag{3.1}$$

The form of this operator is determined by the interaction. If there is no interaction, then the wave function is independent of the time** and the S matrix becomes equal to unity.

*The reader who wishes to obtain a better knowledge of invariance principles and their use in nuclear physics is referred to the reviews[22-24].

**The S-matrix (3.1) is defined in the interaction representation. In this representation the wave function $|\psi(t)\rangle$ satisfies the equation

$$i\frac{\partial\,|\psi(t)\rangle}{\partial t}=H(t)\,|\psi(t)\rangle,$$

where H(t) is the interaction Hamiltonian. In the absence of the interaction.

$$\frac{\partial\,|\psi(t)\rangle}{\partial t}=0.$$

We consider first space rotations. Let $|\psi(t)\rangle$ be the wave function of an arbitrary state in some reference frame, and $|\psi(t)\rangle_R$ —the wave function describing the same state in a reference frame turned relative to the initial system. The wave functions $|\psi(t)\rangle$ and $|\psi(t)\rangle_R$ are related by the unitary transformation

$$|\psi(t)\rangle_R = U_R|\psi(t)\rangle. \tag{3.2}$$

The unitary operator U_R ($U_R^+ U_R = 1$) depends, naturally, on the angles characterizing the rotation of the reference frame. We multiply (3.1) by U_R from the left. Taking (3.2) into account, we obtain

$$|\psi(+\infty)\rangle_R = U_R S U_R^{-1}|\psi(-\infty)\rangle_R. \tag{3.3}$$

The wave functions $|\psi(+\infty)\rangle_R$ and $|\psi(-\infty)\rangle_R$ describe the final and initial states in the rotated reference frame, and consequently by definition the operator $U_R S U_R^{-1}$ is the scattering matrix in the new frame.

The postulate that the interactions are invariant under rotations consists in stating that for arbitrary rotations

$$U_R S U_R^{-1} = S. \tag{3.4}$$

Inasmuch as the S matrix is determined by the dynamics, we presuppose by the same token the independence of the dynamics of rotations of the coordinate frame. The invariance conditions (3.4) can, obviously, be written also in the form

$$U_R^{-1} S U_R = S. \tag{3.5}$$

In order to formulate the requirement of invariance under space inversion, we consider, besides the initial reference frame, a system whose axes are directed opposite to the axes of the first system (if the initial system is a right-hand system the second system is left-hand). Let U_I be a unitary operator that acts on the initial-state wave function $|\psi(t)\rangle$ in the first system to produce the wave function $|\psi(t)\rangle_I$ describing the same state in the reflected system

$$|\psi(t)\rangle_I = U_I|\psi(t)\rangle. \tag{3.6}$$

The operator $U_I S U_I^{-1}$ is the S matrix in the reflected reference frame, and the invariance condition with respect to reflections has consequently the form

$$U_I^{-1} S U_I = S. \tag{3.7}$$

We now proceed to formulate the conditions of invariance under time reversal. To explain the meaning of the corresponding invariance requirements, we turn first to the equation of motion. Let $|\psi(t)\rangle$ be the solution of the Schrödinger equation in the interaction representation

$$i\frac{\partial|\psi(t)\rangle}{\partial t} = H(t)|\psi(t)\rangle, \tag{3.8}$$

where $H(t)$ —interaction Hamiltonian. We take the complex conjugate of (3.8), in which we replace t by $-t$. We obtain

$$i\frac{\partial|\psi(-t)\rangle^*}{\partial t} = H^*(-t)|\psi(-t)\rangle^*. \tag{3.9}$$

In the general case of particles with spin, $H^*(-t) \neq H(t)$, and consequently the function $|\psi(-t)\rangle^*$ does not satisfy the Schrödinger equation. Let us assume, however, that the interaction Hamiltonian is such that there exists a unitary operator U_T that ensures the fulfillment of a condition

$$U_T H^*(-t) U_T^{-1} = H(t). \tag{3.10}$$

It is then obvious that the function $|\psi(t)\rangle_T = U_T|\psi(-t)\rangle^*$ is also a solution of (3.8)

$$i\frac{\partial|\psi(t)\rangle_T}{\partial t} = H(t)|\psi(t)\rangle_T. \tag{3.11}$$

Thus, if the interaction Hamiltonian satisfies (3.10), then alongside with the solution of the Schrödinger equation $|\psi(t)\rangle$ there always exists a solution $|\psi(t)\rangle_T = U_T|\psi(-t)\rangle^*$. This solution describes the "inverted" motion of the system. We shall explain the physical meaning of the "inverted" solution later. Relation (3.10) is the condition for the invariance under time reversal. Let us explain now what conditions are imposed on the S matrix by invariance under time reversal. The S matrix for the second solution is the same as for the first, since it is determined only by the interaction operator and does not depend on the initial state. We therefore have besides (3.1)

$$|\psi(+\infty)\rangle_T = S|\psi(-\infty)\rangle_T, \tag{3.12}$$

that is,

$$U_T|\psi(-\infty)\rangle^* = SU_T|\psi(+\infty)\rangle^*.$$

Multiplying this relation from the left by U_T^{-1}, we obtain

$$|\psi(-\infty)\rangle^* = U_T^{-1} S U_T|\psi(+\infty)\rangle^*. \tag{3.13}$$

On the other hand, multiplying (3.1) from the left by S^+, using the unitarity of the S matrix

$$S^+S = I$$

and taking the complex conjugate, we obtain

$$|\psi(-\infty)\rangle^* = \tilde{S}|\psi(+\infty)\rangle^*. \tag{3.14}$$

The symbol $\sim$ denotes in this case the transpose. Comparing (3.13) with (3.14) we obtain finally

$$U_T^{-1} S U_T = \tilde{S}. \tag{3.15}$$

From conditions (3.5), (3.7), and (3.15) follow rather stringent limitations on the possible form of the S-matrix elements.

Before we formulate these limitations, we note that in the problems which we consider (scattering or reaction) the particles do not interact and have definite momenta as $t \to -\infty$. We denote such a state by $|i\rangle$. We are interested in the probability of observing particles with definite momenta as $t \to +\infty$ (state $|f\rangle$). In order to separate that part of the matrix element

$\langle f|S|I\rangle$ which is caused by the interaction, we represent it in the form

$$\langle f|S|i\rangle = \langle f|i\rangle - 2\pi i\delta(\mathbf{Q}'-\mathbf{Q})\,\delta(E'-E)\,T_{fi}. \qquad (3.16)$$

Here $\mathbf{Q}'$, E' and $\mathbf{Q}$, E are the total momentum and energy of the final and initial states; the δ-functions ensure conservation of the total momentum and energy. The square of the modulus of the second term gives the probability of the transition induced by the interaction from the state $|i\rangle$ into the state $|f\rangle$. The matrix element T_{fi} can be represented in the form

$$T_{fi} = (\chi'^{*}T(\mathbf{p}',\ \mathbf{p})\,\chi). \qquad (3.17)$$

Here $\mathbf{p}$ and $\mathbf{p}'$ —initial and final relative momenta, χ and χ'—spin functions of the initial and final particles, and $T(\mathbf{p}', \mathbf{p})$ —matrix acting on the spin variables. The matrix $T(\mathbf{p}', \mathbf{p})$ is determined, naturally, by the dynamics of the process. Let us ascertain what general limitations are imposed by the invariance requirements on the form of this matrix. We start with rotations. From (3.5) we obtain

$$\langle f|S|i\rangle = \langle f|U_R^{-1}SU_R|i\rangle = {}_R\langle f|S|i\rangle_R, \qquad (3.18)$$

where $|i\rangle_R$ and $|f\rangle_R$ are the wave functions of the initial and final particles in the rotated reference frame. It is obvious that if the state $|i\rangle$ describes particles with total momentum $\mathbf{Q}$ and relative momentum $\mathbf{p}$, then the state $|i\rangle_R$ describes the same particles with momenta $\mathbf{Q}_R$ and $\mathbf{p}_R$:

$$\left.\begin{aligned}(\mathbf{Q}_R)_i &= a_{il}(\mathbf{Q})_l,\\ (\mathbf{p}_R)_i &= a_{il}(\mathbf{p})_l.\end{aligned}\right\} \qquad (3.19)$$

Here a_{il} —cosine of the angle between the new axis i and the old axis l.

Relation (3.18) states the equality of different matrix elements of the same operator. From (3.16), (3.17), and (3.18) we obtain

$$(\chi_R'^{+}T(\mathbf{p}_R',\ \mathbf{p}_R)\,\chi_R) = (\chi'^{+}T(\mathbf{p}',\ \mathbf{p})\,\chi). \qquad (3.20)$$

Here χ_R and χ_R' are the initial and final spin functions in the new reference frame. The functions χ and χ_R (χ' and χ_R') describe the same spin state in different systems. They are therefore related by the unitary transformation

$$\left.\begin{aligned}\chi_R &= u_R\chi,\\ \chi_R' &= u_R'\chi'.\end{aligned}\right\} \qquad (3.21)$$

The unitary matrixes u_R and u_R' act on the spin variables and depend on the angles of rotation. If the spins of both initial (final) particles differ from zero, then the function $\chi(\chi')$ is the product (or the sum of products) of the spin functions of the individual particles. The matrix $u_R(u_R')$ is in this case a direct product of matrices acting on each spin function separately.

The mean value of the spin operator should transform under rotation like a vector, that is,

$$\chi_R^{+}s_i\chi_R = a_{ik}\chi^{+}s_k\chi,$$

where s_i —spin operator of any of the initial particles. It follows therefore that the matrix u_R satisfies the relations

$$u_R^{-1}s_iu_R = a_{ik}s_k. \qquad (3.22)$$

It is also clear that the matrix u_R' satisfies analogous relations.

From (3.20) and (3.21) we obtain

$$u_R'^{-1}T(\mathbf{p}_R',\ \mathbf{p}_R)\,u_R = T(\mathbf{p}',\ \mathbf{p}). \qquad (3.23)$$

This is the condition which must be satisfied by the T matrix as a result of invariance under space rotations.

We now proceed to consider the inversion of the coordinate frame. If $\mathbf{p}_I$ and $\mathbf{p}_I'$ are the initial and final relative momenta in the new reference frame and χ_I and χ_I' are the initial and final spin functions in the same frame, then

$$\begin{gathered}\mathbf{p}_I = -\mathbf{p},\quad \mathbf{p}_I' = -\mathbf{p}',\\ \chi_I = u_I\chi,\ \ \chi_I' = u_I'\chi',\end{gathered} \qquad (3.24)$$

where u_I is a unitary matrix satisfying the relations

$$u_I^{-1}s_iu_I = s_i. \qquad (3.25)$$

Analogous relations hold true for the matrix u_I'. The conditions (3.25) follow from the fact that the mean value of the spin operator should transform like a pseudovector.

The action of the operator U_I on the function $|i\rangle$ consists in the substitutions $\mathbf{p} \to \mathbf{p}_I$, $\mathbf{Q} \to \mathbf{Q}_I$, and $\chi \to \chi_I$ and multiplication of this function by a factor whose modulus is equal to unity. From the superposition principle it follows that this factor is the same for different states of the given particles. Consequently, it characterizes their intrinsic properties. It is customary to call this operator the intrinsic parity. If the reference frame is inverted twice, we return to the initial system. The wave function of the particle with integer spin then coincides with the initial wave function, and the wave function of particles with half-integer spin either coincides with the initial one, or differs from it in sign (the latter is connected with the fact that two-fold inversion can be regarded as an identical transformation and a rotation through an angle 2π, which, as is well known, leads to sign reversal for particles with half-integer spin). This means that the intrinsic parity of Bose particles can be equal to ± 1, while the intrinsic parity of Fermi particles can be equal to ± 1 or $\pm i$. From (3.7) we obtain for the matrix elements

$$\langle f|S|i\rangle = {}_I\langle f|S|i\rangle_I. \qquad (3.26)$$

From this we get

$$(\chi'^{+}T(\mathbf{p}',\ \mathbf{p})\,\chi) = I_iI_f^{*}(\chi_I'^{+}T(-\mathbf{p}',\ -\mathbf{p})\,\chi_I), \qquad (3.27)$$

where $I_i(I_f)$ —product of the intrinsic parities of the initial (final) particles.

Thus, we ultimately obtain from the invariance under reflections

$$T(\mathbf{p}', \mathbf{p}) = I_i I_f^* u_I^{-1} T(-\mathbf{p}', -\mathbf{p}) u_I. \qquad (3.28)$$

We note that the product $I_i I_f^*$ can be equal only to ± 1.

We now proceed to clarify the consequences of invariance under time reversal. Multiplying (3.15) from the right by $|f\rangle^*$ and from the left by $|i\rangle$, we obtain

$$_T\langle i|S|f\rangle_T = \langle f|S|i\rangle. \qquad (3.29)$$

When t is replaced by $-t$, the momentum and the angular momentum reverse sign. Therefore the "time inverted" state $|i\rangle_T = U_T|i\rangle^*$ ($|f\rangle_T = U_T|f\rangle^*$) differs from the state $|i\rangle$ ($|f\rangle$) in the signs of the momenta and of the spin projections. From (3.29) we obtain

$$(\chi'^{+} T(\mathbf{p}', \mathbf{p}) \chi) = \eta_T \eta_T'^* (\chi_T^+ T_{o6}(-\mathbf{p}, -\mathbf{p}') \chi_T'). \qquad (3.30)$$

Here $T_{inv}(-\mathbf{p}, -\mathbf{p}')$ —matrix of the inverse process ($f \to i$), $\chi_T = u_T\chi^*$, $\chi_T' = u_T'\chi'^*$, u_T and u_T' —unitary matrices, and η_T and η_T' —factors with unity absolute value (the time parities of the initial and final states). The matrix u_T is defined by the requirement that the mean value of the spin reverse sign under time reversal:

$$\chi_T^+ s_i \chi_T = -\chi^+ s_i \chi. \qquad (3.31)$$

It follows therefore that u_T satisfies the condition

$$u_T \tilde{s}_i u_T^{-1} = -s_i. \qquad (3.32)$$

Analogous relations are satisfied by the matrix u_T'. From (3.30) we obtain

$$\tilde{T}(\mathbf{p}', \mathbf{p}) = u_T^{-1} T_{o6}(-\mathbf{p}, -\mathbf{p}') u_T' \eta_T \eta_T'^*. \qquad (3.33)$$

Relation (3.33) connects the matrices of the direct and inverse processes. For the elastic scattering process $T_{inv} = T$, $u_T' = u_T$, $\eta_T\eta_T'^* = 1$, and (3.33) represents in this case a limitation on the form of the scattering matrix*:

$$\tilde{T}(\mathbf{p}', \mathbf{p}) = u_T^{-1} T(-\mathbf{p}, -\mathbf{p}') u_T. \qquad (3.34)$$

4. SPIN DENSITY MATRIX

So far we have assumed that the spin state of the initial and final particles is described by wave functions. Under ordinary experimental conditions, however, the spin state cannot be described by a wave function, or more accurately speaking it cannot be described by a single wave function, but is described by a density matrix or by a "mixture" of wave functions. Let us proceed to determine the density matrix [25-26]. We consider a beam of particles with definite momentum. It is obtained as a result of interaction with some subsystem. If we assume that the entire system as a whole is described by a wave function, then after the interaction that leads to the formation of the beam the wave function of the entire system can always be represented in the form

$$\psi(x, \sigma, \xi) = \frac{1}{(2\pi)^{3/2}} \times e^{ipx} \sum_{\mu n} a_{\mu n} \chi_\mu(\sigma) \psi_n(\xi) = \frac{1}{(2\pi)^{3/2}} e^{ipx} \varphi(\sigma, \xi). \qquad (4.1)$$

Here $\chi_\mu(\sigma)$ —spin functions of the beam particles, $\psi_n(\xi)$ —wave functions of the subsystem, and the coefficients $a_{\mu n}$ do not depend on the time and are determined by those interactions which have led to the formation of the beam. We assume that the functions $\chi_\mu(\sigma)$ and $\psi_n(\xi)$ are orthonormal and comprise a complete system. The state of the subsystem is of no interest to us. Therefore, in calculating the mean values of the operators acting on the variable σ, it is necessary to carry out integration (summation) with respect to ξ. Let O be an arbitrary operator acting on the variable σ. Integrating with respect to ξ and assuming that the function $\varphi(\sigma, \xi)$ is normalized, we obtain for the mean value $\langle O\rangle$

$$\langle O\rangle = \sum_{\substack{\mu, \mu' \\ \sigma, \sigma'}} \chi_{\mu'}^*(\sigma') O_{\sigma'\sigma} \chi_\mu(\sigma) c_{\mu\mu'}, \qquad (4.2)$$

where

$$c_{\mu\mu'} = \sum_n a_{\mu n} a_{\mu' n}^*. \qquad (4.3)$$

If we introduce a matrix ρ with elements

$$\rho_{\sigma\sigma'} = \sum_{\mu, \mu'} \chi_\mu(\sigma) \chi_{\mu'}^*(\sigma') c_{\mu\mu'}, \qquad (4.4)$$

then the average value of the operator O is equal to

$$\langle O\rangle = \sum_{\sigma, \sigma'} O_{\sigma'\sigma} \rho_{\sigma\sigma'} = \mathrm{Sp}\, O\rho. \qquad (4.5)$$

The matrix ρ is called the density matrix. The condition for normalization of the function $\varphi(\sigma, \xi)$ leads to

$$\mathrm{Sp}\,\rho = 1. \qquad (4.6)$$

In the case when $\varphi(\sigma, \xi)$ is not normalized, the mean value of the operator O is

$$\langle O\rangle = \frac{\mathrm{Sp}\, O\rho}{\mathrm{Sp}\,\rho}. \qquad (4.7)$$

The mean value of any operator acting on the variable σ can be obtained if the matrix ρ is given. Thus, the density matrix (4.4) describes completely the spin state of the beam particles.

We note that the spin state of the beam can be described by a wave function only when the wave function of the entire system $\varphi(\sigma, \xi)$ is represented by a prod-

*If we carry out a rotation by an angle π around the direction $\mathbf{p}' - \mathbf{p}$, then, using (3.22), we can rewrite (3.34) in the form

$$\tilde{T}(\mathbf{p}', \mathbf{p}) = u_T^{-1} u_R^{-1} T(\mathbf{p}', \mathbf{p}) u_T u_R.$$

In the usual Pauli-matrix representation $u_T = -i\sigma_2 = \exp\left(-\frac{i}{2}\sigma_2\pi\right)$ and coincides with the operator of rotation through an angle $(-\pi)$ about the y axis. If the y axis is aligned with $\mathbf{p}' - \mathbf{p}$, then $\mu_T\mu_R = 1$ together with condition (3.34) is transformed into the T-matrix symmetry requirement: $\tilde{T}(\mathbf{p}'\mathbf{p}) = T(\mathbf{p}', \mathbf{p})$. This formulation of the T-invariance condition belongs to the late L. D. Puzikov.

uct of the wave functions of the parts of the system [one term in the expansion (4.1)]:

$$\varphi(\sigma, \xi) = \chi(\sigma)\psi(\xi). \tag{4.1a}$$

The density matrix of such a state is

$$\varrho_{\sigma\sigma'} = \chi(\sigma)\chi^*(\sigma'), \tag{4.8}$$

and the mean value of the operator O is

$$\langle O\rangle = \chi^+ O\chi. \tag{4.9}$$

It is obvious from (4.9) that the spin state of the beam is completely described by the wave function χ. Such states are called pure.

Let us list the main properties of the density matrix, which follow from (4.4), (4.3), and (4.8).

1. The density matrix is Hermitian:

$$\varrho = \varrho^+, \tag{4.10}$$

thus guaranteeing the reality of the mean values of the Hermitian operators.

2. The density matrix of the pure state (4.8) satisfies the relation

$$\varrho^2 = \varrho\,\mathrm{Sp}\,\varrho. \tag{4.11}$$

We therefore obtain for a pure state

$$\mathrm{Sp}\,\varrho^2 = (\mathrm{Sp}\,\varrho)^2. \tag{4.12}$$

3. In the general case

$$\mathrm{Sp}\,\varrho^2 \leqslant (\mathrm{Sp}\,\varrho)^2. \tag{4.13}$$

As already noted, the ground state of the beam is described by a single wave function in the case when the wave function of the entire system is of the form (4.1a). We shall now show that in the general case the spin state can always be described also by a set of several incoherent spin functions (mixture). To this end we write down the equation for the eigenfunctions and eigenvalues of the density matrix

$$\sum_{\sigma''} \varrho_{\sigma\sigma''} u_\mu(\sigma'') = \varrho_\mu u_\mu(\sigma). \tag{4.14}$$

Multiplying (4.14) by $u_\mu^*(\sigma')$, summing over μ, and using the completeness of the system of functions

$$\sum_\mu u_\mu(\sigma'') u_\mu^*(\sigma') = \delta_{\sigma''\sigma'},$$

we obtain

$$\varrho_{\sigma\sigma'} = \sum_\mu \varrho_\mu u_\mu(\sigma) u_\mu^*(\sigma'). \tag{4.15}$$

The new value of the operator O, calculated with the aid of (4.15), is

$$\langle O\rangle = \sum_\mu \varrho_\mu (u_\mu^+ O u_\mu). \tag{4.16}$$

From the condition for the normalization of ρ it follows that

$$\sum_\mu \varrho_\mu = 1. \tag{4.17}$$

In addition, we can show with the aid of (4.14), (4.3), and (4.4) that the eigenvalues are positive:

$$\varrho_\mu \geqslant 0. \tag{4.18}$$

Relations (4.16), (4.17), and (4.18) signify that the spin state of the beam is described in the general case by an incoherent mixture of functions u_μ, which enter into the mixture with probability ρ_μ.

By way of an example let us consider the simplest case of particles with spin $1/2$. The density matrix is in this case a 2×2 matrix, and consequently can be expanded in four basis matrices. It is convenient to choose as the basis the matrices I and σ_i. We obtain the expansion

$$\varrho = c + \sum_i d_i\sigma_i, \tag{4.19}$$

where c and d_i —real numbers (hermiticity of ρ). Normalizing the matrix (4.19) we find that $c = 1/2$. On the basis of (4.5) and (4.19) we get

$$\frac{1}{2}\langle\sigma_i\rangle = \frac{1}{2}\mathrm{Sp}\,\sigma_i\varrho = d_i. \tag{4.20}$$

The mean values $\langle\sigma_i\rangle$ form a pseudovector which is called the polarization. Introducing the notation $\mathbf{P} = \langle\boldsymbol{\sigma}\rangle$, we write down the matrix (4.19) in the form

$$\varrho = \frac{1}{2}(I + (\boldsymbol{\sigma}\mathbf{P})). \tag{4.21}$$

Thus, the spin state of particles with spin $1/2$ is completely specified by the polarization $\mathbf{P}$. From the inequality (4.13) we find that $P = |\mathbf{P}|$ (degree of polarization) does not exceed unity:

$$P \leqslant 1. \tag{4.22}$$

According to (4.12), the degree of polarization is equal to unity for a pure state. The state with $P = 0$ is called unpolarized.

We now turn to consider collision processes and obtain a connection between the spin density matrices of the initial and final states. For what follows it is convenient to use the representation of the initial density matrix in the form of a mixture

$$\varrho_0 = \sum_\mu \varrho_\mu\chi_\mu\chi_\mu^+. \tag{4.23}$$

On the basis of (3.16), (3.17), and (4.23) we obtain, after averaging over the initial spin states and summing over the final spin states, the following expression for the cross section:

$$d\sigma = \frac{1}{(2\pi)^2}\frac{1}{j_0}\sum_{\mu,\mu'}(\chi_{\mu'}'^{+}T(\mathbf{p}', \mathbf{p})\chi_\mu)\varrho_\mu(\chi_\mu^+ T^+(\mathbf{p}', \mathbf{p})\chi_{\mu'}')\,\delta(E'-E)\,d\mathbf{p}'$$
$$= \frac{1}{(2\pi)^2}\frac{1}{j_0}\mathrm{Sp}\,T(\mathbf{p}', \mathbf{p})\varrho_0 T^+(\mathbf{p}', \mathbf{p})\,\delta(E'-E)\,p'^2\,dp'\,d\Omega'. \tag{4.24}$$

Here j_0 —current density of the initial particles. In (4.24) we can easily integrate with respect to p′. After integration we obtain the following expression for the differential cross section:

$$\sigma \equiv \frac{d\sigma}{d\Omega'} = (2\pi)^4 \frac{v'}{v} \left(\frac{E_1' E_2'}{E}\right) \mathrm{Sp}\, T(\mathbf{p}', \mathbf{p}) \varrho_0 T^+(\mathbf{p}', \mathbf{p}), \quad (4.25)$$

where v′ and v —differences in the velocities of the final and initial particles in the c.m.s. It is convenient to introduce a matrix $M(\mathbf{p}', \mathbf{p})$, which differs only by a factor from $T(\mathbf{p}', \mathbf{p})$ and is defined in such a way that the differential cross section is

$$\sigma = \mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) \varrho_0 M^+(\mathbf{p}', \mathbf{p}). \quad (4.26)$$

We now proceed to construct the final-state density matrix. If the initial particles are described by a function χ_μ, then the spin function of the final particles is equal to

$$\chi' = \sum_{\mu'} \chi_{\mu'}'^{+} (\chi_{\mu'}' M(\mathbf{p}', \mathbf{p}) \chi_\mu). \quad (4.27)$$

Using the completeness of the functions $\chi'_{\mu'}$, we can write down the function χ' in the following fashion:

$$\chi' = M(\mathbf{p}', \mathbf{p}) \chi_\mu. \quad (4.28)$$

On the other hand, if the initial state is described by the density matrix (4.23), then the final density matrix is

$$\varrho = \sum_\mu (M\chi_\mu) \varrho_\mu (M\chi_\mu)^+ = M\varrho_0 M^+. \quad (4.29)$$

The final density matrix is normalized in such a way [see (4.26)] that the sum of its diagonal elements is equal to the differential cross section. On the other hand, the mean value of the operator O in the final state is, in accordance with (4.7), (4.29), and (4.26),

$$\langle O \rangle = \frac{\mathrm{Sp}\, M\varrho_0 M^+ O}{\sigma}. \quad (4.30)$$

5. THE "POLARIZATION-ASYMMETRY" RELATION IN THE CASE OF ARBITRARY POLARIZATION DIRECTION

From conditions (3.23), (3.28), and (3.33) we can obtain a general expression for the M matrix of any process. Wolfenstein and Ashkin[16] and Dalitz[17] have shown, on the basis of the general expression for the elastic-scattering amplitude, that the left-right asymmetry in the scattering of polarized particles with spin $\frac{1}{2}$ by unpolarized particles with arbitrary spin is equal to the scalar product of the initial polarization by the polarization of the particles with spin $\frac{1}{2}$ occurring in the scattering of unpolarized particles. As was already emphasized, this relation is the basis of the analysis of polarization phenomena and enables us, in particular, to determine the polarization of particles by measuring the left-right asymmetry. We obtain the "polarization-asymmetry" relation in the case of the two-particle reactions (2.8) by using directly the invariance requirements (3.23), (3.28), and (3.33). We assume that the spin of the initial particles is equal to $\frac{1}{2}$ and the spins of all the remaining particles are arbitrary. If only the particle with spin $\frac{1}{2}$ is polarized in the initial state (with polarization **P**), then the normalized initial density matrix is equal to

$$\varrho_0 = \frac{1}{2}(I + (\sigma\mathbf{P})) \frac{1}{2s+1}, \quad (5.1)$$

where s —spin of the second initial particle.* With the aid of (4.26) we obtain the following expression for the differential cross section of the process:

$$\sigma = \sigma_0 \left(1 + \frac{(\mathbf{P}\, \mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) \sigma M^+(\mathbf{p}', \mathbf{p}))}{\mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) M^+(\mathbf{p}', \mathbf{p})}\right), \quad (5.2)$$

where

$$\sigma_0 = \frac{1}{2(2s+1)} \mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) M^+(\mathbf{p}', \mathbf{p}) \quad (5.3)$$

is the differential cross section in the case of unpolarized initial particles. We now show that by virtue of the invariance requirements with respect to time reversal, rotations, and reflections, the ratio

$$\frac{\mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) \sigma M^+(\mathbf{p}', \mathbf{p})}{\mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) M^+(\mathbf{p}', \mathbf{p})} \quad (5.4)$$

is equal to the polarization of a spin-$\frac{1}{2}$ particle produced by the inverse reaction with unpolarized particles. We use first the requirement (3.33). If we take into account the proportionality coefficient between the matrices $M(\mathbf{p}', \mathbf{p})$ and $T(\mathbf{p}', \mathbf{p})$, then we can rewrite (3.33) in the form

$$p\widetilde{M}(\mathbf{p}', \mathbf{p}) = p' u_T^{-1} M_{o6}(-\mathbf{p}, -\mathbf{p}') u_T' \eta_T \eta_T'^*.$$

From this we get

$$\frac{\mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) \sigma M^+(\mathbf{p}', \mathbf{p})}{\mathrm{Sp}\, M(\mathbf{p}', \mathbf{p}) M^+(\mathbf{p}', \mathbf{p})} = \frac{\mathrm{Sp}\, \widetilde{M}^+(\mathbf{p}', \mathbf{p}) \widetilde{\sigma} \widetilde{M}(\mathbf{p}', \mathbf{p})}{\mathrm{Sp}\, \widetilde{M}^+(\mathbf{p}', \mathbf{p}) \widetilde{M}(\mathbf{p}', \mathbf{p})}$$

$$= -\frac{\mathrm{Sp}\, M_{\mathrm{inv}}(-\mathbf{p}, -\mathbf{p}') M_{\mathrm{inv}}^+(-\mathbf{p}, -\mathbf{p}') \sigma}{\mathrm{Sp}\, M_{\mathrm{inv}}(-\mathbf{p}, -\mathbf{p}') M_{\mathrm{inv}}^+(-\mathbf{p}, -\mathbf{p}')}. \quad (5.5)$$

Thus, relation (5.4), taken with a negative sign, is equal to the polarization of particles with spin $\frac{1}{2}$, produced in the inverse reaction upon collision with unpolarized particles (the initial and final momenta in the inverse reaction are $-\mathbf{p}'$ and $-\mathbf{p}$, respectively). We introduce the unit vectors $\mathbf{k} = \mathbf{p}/|\mathbf{p}|$ and $\mathbf{k}' = \mathbf{p}/|\mathbf{p}'|$ and denote the polarization produced in the inverse reaction by

$$\frac{\mathrm{Sp}\, M_{\mathrm{inv}}(-\mathbf{p}, -\mathbf{p}') M_{\mathrm{inv}}^+(-\mathbf{p}, -\mathbf{p}') \sigma}{\mathrm{Sp}\, M_{\mathrm{inv}}(-\mathbf{p}, -\mathbf{p}') M_{\mathrm{inv}}^+(-\mathbf{p}, -\mathbf{p}')} = \mathbf{P}_0^{\mathrm{inv}}(-\mathbf{k}, -\mathbf{k}'). \quad (5.6)$$

From the invariance of the reaction matrix under rotations and reflections [relations (3.23), (3.28), and (3.22), (3.25)] it follows that the polarization (5.6) is a pseudovector, that is,

*The beam and the target are usually prepared independently of each other. Therefore the density matrix of the initial state will be the direct product $\rho_b \times \rho_t$ of the density matrices of the beam and of the target. In our case the target is not polarized and the target density matrix is proportional to the unit matrix $\rho_t = [1/(2s+1)]I$. The proportionality coefficient in this expression is determined by the normalization condition. The beam density matrix ρ_t is given by the polarization p and its form is (4.21).

$$\mathbf{P}_0^{\text{inv}}(-\mathbf{k},\ -\mathbf{k}') = P_0^{\text{inv}}((\mathbf{k}\mathbf{k}'))\,[\mathbf{k}'\mathbf{k}]. \qquad (5.7)$$

We therefore get

$$\mathbf{P}_0^{\text{inv}}(-\mathbf{k},\ -\mathbf{k}') = -\mathbf{P}_0^{\text{inv}}(\mathbf{k}',\ \mathbf{k}). \qquad (5.8)$$

Thus, the ratio (5.4) is equal to $+\mathbf{P}_0^{\text{inv}}(\mathbf{k}',\mathbf{k})$ and we obtain ultimately for the differential cross section of the direct reaction

$$\sigma = \sigma_0(1 + (\mathbf{P}\mathbf{P}_0^{\text{inv}}(\mathbf{k}',\ \mathbf{k}))). \qquad (5.9)$$

Consequently the left-right asymmetry is equal to [18-29]

$$e^{LR} = \frac{\sigma^L - \sigma^R}{\sigma^L + \sigma^R} = (\mathbf{P}\mathbf{P}_0^{\text{inv}}). \qquad (5.10)$$

In the case of elastic scattering, the direct and inverse reactions coincide and $\mathbf{P}_0^{\text{inv}}(\mathbf{k}',\mathbf{k})$ is the polarization that would be produced if the incident particles were not polarized.

6. POSSIBLE METHODS OF DETERMINING THE PARITIES OF STRANGE PARTICLES IN POLARIZED-TARGET EXPERIMENTS

The determination of the intrinsic parities of K mesons and hyperons is one of the important problems in elementary-particle physics. In connection with parity non-conservation in all weak processes, including strange-particle decays, the intrinsic parity of the hyperons and K mesons can be determined only by investigating the strong and electromagnetic interactions responsible for their production and mutual transformations. Perhaps one of the most interesting applications of a polarized hydrogen target in high-energy physics is in possible experiments on the determination of the intrinsic parities of strange particles in the reactions

$$\left.\begin{array}{l} \pi + p \to Y + K, \\ \overline{K} + p \to Y + \pi, \qquad Y = (\Lambda,\ \Sigma) \end{array}\right\} \qquad (6.1)$$

with polarized protons. This possibility was pointed out in [10-13]. The connection between the intrinsic parities and the polarizations of the particles participating in the reaction was discussed in general form in an already cited paper [14]. The possibility of determining the parities of strange particles in investigations of reactions (6.1) with polarized protons is based only on the general requirements of invariance under rotations and reflections, and is not connected with any concrete assumptions concerning the dynamics of the interactions.

One of the possible formulations of the experiment is based on the relation between the left-right asymmetry $e^{LR}(\theta)$ in a reaction with a polarized target, and the polarization P_0^L produced upon collision of unpolarized particles, a relation which holds in the case of reactions of the type $0 + 1/2 \to 0 + 1/2$ (this type includes the reactions (6.1) of interest to us, since the K-meson spin is zero and the Σ and Λ hyperons have spin $1/2$). We recall that in the case of a target polarized perpendicular to the plane of the reaction this relation takes the form [see (2.19)]

$$e^{LR}(\theta) = \pm PP_0^L(\theta) \qquad (I_\pi I_p = \pm I_K I_Y), \qquad (6.2)$$

where P —degree of polarization of the target. By determining which of the possible alternatives actually takes place we can determine the parity of the (KY) pair relative to the proton. To this end it is necessary to compare the results of two independent experiments performed at the same energy. One experiment should yield the polarization of the proton in a reaction with an unpolarized target. In the second it is necessary to measure the asymmetry in a reaction with a polarized target. In principle it is sufficient to compare only the signs of $e^{LR}(\theta)$ and $P_0^L(\theta)$. If the signs coincide $I_\pi I_p = I_K I_Y$, if they are opposite $I_\pi I_p = -I_K I_Y$.

It is also possible to use a modified formulation of the experiment. To understand the latter, we assume first that we have at our disposal protons that are fully polarized in the direction $\mathbf{n}_0$. As before, we consider the reaction (6.1) in a plane perpendicular to the polarization direction. Then from the Bohr rule [see (2.11)] it follows immediately that the hyperons emitted at an arbitrary angle θ will be completely polarized and their polarization will coincide in direction with the proton polarization, if $I_K I_Y = I_\pi I_p$ (forbidden reaction with spin flip), and will be opposite to that direction if $I_K I_Y = -I_\pi I_p$ (forbidden reaction without spin flip). Thus, a determination of the direction of hyperon polarization occurring in a reaction with completely polarized protons, and a comparison of this direction with the direction of polarization of the target, also leads to an unambiguous conclusion concerning the parity of the (KY) pair. The situation does not change essentially if the protons are only partially polarized: $\mathbf{P} = P\mathbf{n}_0$. It is merely necessary to carry out a suitable averaging. Indeed, the average polarization of the hyperons* emitted at an angle θ in a plane perpendicular to $\mathbf{n}_0$, either to the right or to the left, can be readily verified with the aid of (2.4) to be

$$\langle P(\theta)\rangle = P\,\frac{\sigma_{++}^L(\theta) + \sigma_{--}^L(\theta) - \sigma_{-+}^L(\theta) - \sigma_{+-}^L(\theta)}{\sigma_{++}^L(\theta) + \sigma_{--}^L(\theta) + \sigma_{-+}^L(\theta) + \sigma_{+-}^L(\theta)}. \qquad (6.3)$$

Using the Bohr rule (2.11) we obtain from (6.3)

$$\langle P(\theta)\rangle = \pm P, \qquad I_K I_Y = \pm I_\pi I_p. \qquad (6.4)$$

As in the case of fully polarized protons, the average

*The average hyperon polarization is determined by the equation

$$\langle P(\theta)\rangle = \frac{N_+^L(\theta) + N_+^R(\theta) - N_-^L(\theta) - N_-^R(\theta)}{N_+^L(\theta) + N_+^R(\theta) + N_-^L(\theta) + N_-^R(\theta)},$$

where $N_+^L(\theta)$ – number of hyperons emitted at an angle θ to the left with a spin parallel to $\mathbf{n}_0$, etc. With such an averaging, the terms corresponding to the polarization due to collision of unpolarized particles drop out from the expression for the final polarization.

polarization of the hyperons is either equal to the proton polarization ($I_K I_Y = I_\pi I_p$), or is equal in magnitude but opposite in direction ($I_K K_Y = -I_\pi I_p$).

Let us forego now the assumption that the target is polarized in a direction orthogonal to the reaction plane, and let us derive formulas that are valid in the general case. To this end it is simplest to use the formalism of the density matrix and scattering amplitude, developed in Secs. 3–5. The density matrix of the initial state in reactions (6.1) takes the form

$$\varrho = \frac{1}{2}(I + (\sigma\mathbf{P})), \qquad (6.5)$$

where $\mathbf{P}$ —polarization of the hydrogen target.

The differential cross section of the reaction on the polarized target is

$$\sigma(\theta, \varphi) = \mathrm{Sp}\, M\varrho M^+ = \sigma_0(\theta)\left(1 + \left(\mathbf{P}\frac{\mathrm{Sp}\, M\sigma M^+}{\mathrm{Sp}\, MM^+}\right)\right), \qquad (6.6)$$

where σ_0 —cross section of the reaction with unpolarized target. On the other hand, the hyperon polarization $\mathbf{P}_0(\theta, \varphi)$, produced upon collision of unpolarized particles, is connected with the reaction amplitude by the relation

$$\mathbf{P}_0(\theta, \varphi) = \frac{\mathrm{Sp}\,\sigma MM^+}{\mathrm{Sp}\, MM^+}. \qquad (6.7)$$

The matrices σ and M in general do not commute. Therefore the traces $\mathrm{Sp}\, M\sigma M^+$ and $\mathrm{Sp}\,\sigma MM^+$, which enter into the expressions for the cross section and the polarization, are generally speaking not equal to each other. However, they are simply related by the requirements of invariance under reflections and rotations. These requirements signify above all that these traces are pseudovectors. Consequently

$$\left.\begin{aligned}\mathrm{Sp}\, M\sigma M^+ &= \mathbf{n}\,\mathrm{Sp}\, M(\sigma\mathbf{n})M^+,\\ \mathrm{Sp}\,\sigma MM^+ &= \mathbf{n}\,\mathrm{Sp}\,(\sigma\mathbf{n})MM^+,\end{aligned}\right\} \qquad (6.8)$$

where $\mathbf{n} = \mathbf{p}\times\mathbf{p}'/|\mathbf{p}\times\mathbf{p}'|$ —normal to the reaction plane. We now effect a reflection in the reaction plane. The vectors p and p′ go over into themselves, and the condition for the invariance of the M matrix takes the form

$$R^{-1}M(\mathbf{p}', \mathbf{p})R = \pm M(\mathbf{p}', \mathbf{p}) \qquad (I_K I_Y = \pm I_\pi I_p). \qquad (6.9)$$

Inasmuch as in the case of particles with spin $1/2$ the reflection operator R is equal to $(\sigma\cdot\mathbf{n})$, relation (6.9) signifies that the operator $(\sigma\cdot\mathbf{n})$ commutes with the M matrix in the case when $I_K I_Y = I_\pi I_p$, and anticommutes in the case when $I_K I_Y = -I_\pi I_p$. It follows from this and from (6.8) that

$$\mathrm{Sp}\, M\sigma M^+ = \pm\,\mathrm{Sp}\,\sigma MM^+ \qquad (I_K I_Y = \pm I_\pi I_p). \qquad (6.10)$$

For the differential cross section of the reaction we obtain from (6.6), (6.7), and (6.10)

$$\sigma = \sigma_0(1 \pm (\mathbf{P}\mathbf{P}_0)) \qquad (I_K I_Y = \pm I_\pi I_p). \qquad (6.11)$$

The left-right asymmetry is thus equal to

$$e^{LR} = \pm(\mathbf{P}\mathbf{P}_0) \qquad (I_K I_Y = \pm I_\pi I_p), \qquad (6.12)$$

where $\mathbf{P}_0$ is the polarization produced in scattering to the left. From (6.7), (6.8), and (6.12) we see that the absolute value of the asymmetry is maximal if the polarization of the target is parallel (or antiparallel) to the normal to the reaction plane.

Thus, the case considered previously, that of initial polarization orthogonal to the reaction time, is the most convenient from the point of view of asymmetry measurement.

We now proceed to consider hyperon polarization for arbitrary target polarization direction. We consider only the connection between the hyperon polarization, averaged over all the emission directions (over the directions p′), and the target polarization. The averaged polarization $\langle \mathbf{P}_Y \rangle$ is determined by the relation

$$\langle \mathbf{P}_Y \rangle = \frac{\int \mathrm{Sp}\, M\varrho M^+\sigma\, d\Omega}{\int \mathrm{Sp}\, M\varrho M^+\, d\Omega}. \qquad (6.13)$$

The integration is carried out here over all the directions p′, and dΩ is the corresponding solid-angle element. The general expression for $\langle \mathbf{P}_Y \rangle$ can be readily written out on the basis of the invariance requirements. Indeed, $\langle \mathbf{P}_Y \rangle$ should be a pseudovector constructed from the initial momentum p and P. As can be seen from (6.13), $\langle \mathbf{P}_Y \rangle$ can depend on P only linearly. Consequently

$$\langle \mathbf{P}_Y \rangle = \alpha\mathbf{P} + \beta(\mathbf{P}\mathbf{k})\,\mathbf{k}, \qquad (6.14)$$

where α and β —functions of the energies of the incoming particles, the specific form of which depends on the dynamics of the interaction. However, the sign of α does not depend on the dynamics. It turns out that α is positive when $I_K I_Y = I_\pi I_p$ and negative when $I_K I_Y = -I_\pi I_p$. Consequently, if the target polarization is perpendicular to the momentum of the incoming particles, then comparison of the signs of P and $\langle \mathbf{P}_Y \rangle$ makes it possible to determine the parity of the (KY) pair.

To prove this, we shall use the general expressions for the reaction matrix, which can be obtained from the invariance conditions formulated in Sec. 3. In the case when the intrinsic parity does not change, the M matrix is a scalar

$$M = a + b(\sigma\mathbf{n}), \qquad \mathbf{n} = \frac{[\mathbf{k}\mathbf{k}']}{|[\mathbf{k}\mathbf{k}']|}. \qquad (6.15)$$

When the intrinsic parity changes, the reaction matrix is pseudoscalar

$$M = c(\sigma\mathbf{k}) + d(\sigma\varkappa), \qquad \varkappa = \frac{\mathbf{k}' - (\mathbf{k}\mathbf{k}')\,\mathbf{k}}{|[\mathbf{k}\mathbf{k}']|}. \qquad (6.16)$$

In (6.15) and (6.16) k and k′ are unit vectors in the directions of the initial and final relative momenta, while a, b, c, and d are functions of the energy and $(\mathbf{k}\cdot\mathbf{k}') = \cos\theta$. Calculating with the aid of these expressions the traces contained in (6.13), and integrating over the directions k′, we arrive at the following expressions for the coefficient α:

I. $I_K I_Y = I_\pi I_p$,

$$\alpha = \frac{\int |a|^2 d\Omega}{\int (|a|^2 + |b|^2)\, d\Omega} = \frac{\int |a|^2 d\Omega}{\int \sigma_0\, d\Omega} \geqslant 0, \quad (6.17a)$$

II. $I_K I_Y = -I_\pi I_p$,

$$\alpha = -\frac{\int |c|^2 d\Omega}{\int (|c|^2 + |d|^2)\, d\Omega} = -\frac{\int |c|^2 d\Omega}{\int \sigma_0\, d\Omega} \leqslant 0. \quad (6.17b)$$

We now turn to a discussion of an experiment consisting of a comparison of hyperon asymetry and polarization measured in independent experiments. At the present time one of these experiments has already been carried out: the hyperon polarization produced in collisions between unpolarized particles was determined for several values of the energy.

Thus, for example, the reaction

$$\pi^- + p \to \Lambda + K^0 \quad (6.18)$$

was investigated over a wide range of incoming-pion momenta. Information on the magnitude and sign of the polarization of the Λ hyperons was obtained by studying their decays. The angular distribution of the protons from the decay of a polarized hyperon is of the form*

$$W(\vartheta) = W_0 (1 - \alpha_\Lambda P_\Lambda \cos\vartheta), \quad (6.19)$$

*Owing to parity nonconservation in Λ-hyperon decay

$$\Lambda \to \pi^- + p$$

the amplitude of the decay is a combination of a scalar and a pseudoscalar ($s_{1/2}$ and $p_{1/2}$ waves in the final state) and is of the form

$$M = a + b(\boldsymbol{\sigma}\mathbf{k}),$$

where a and b are constants characterizing the decay, and k is a unit vector in the direction of the proton momentum. If the hyperon is polarized, the initial density matrix is

$$\varrho_\Lambda = \frac{1}{2}(I + (\boldsymbol{\sigma}\mathbf{P}_\Lambda)),$$

where $\mathbf{p}_\Lambda = P_\Lambda \mathbf{n}$ – hyperon polarization ($\mathbf{n}$ – unit vector in the polarization direction). For the decay probability W we obtain

$$W = \mathrm{Sp}\, M \varrho_\Lambda M^+ = W_0 (1 - \alpha_\Lambda P_\Lambda \cos\vartheta),$$

where $W_0 = |a|^2 + |b|^2$ – probability of decay of an unpolarized hyperon, $\cos\vartheta = (\mathbf{n}.\mathbf{k})$, and the asymmetry parameter α_Λ is determined by the interference of the $s_{1/2}$ and $p_{1/2}$ waves, being equal to

$$\alpha_\Lambda = -\frac{2\,\mathrm{Re}\, ab^*}{|a|^2 + |b|^2}.$$

In the decay of unpolarized hyperons, the neutrons will be longitudinally polarized and the degree of their polarization is ($-\alpha_\Lambda$). Indeed, the proton polarization $\mathbf{P}_p$ is

$$\mathbf{P}_p = \frac{\mathrm{Sp}\, \boldsymbol{\sigma} MM^+}{\mathrm{Sp}\, MM^+} = \frac{2\mathrm{Re}\, ab^*}{|a|^2 + |b|^2}\mathbf{k} = -\alpha_\Lambda \mathbf{k}.$$

As can be seen from these arguments, the degree of polarization of the hyperons can be determined by investigating the asymmetry of the decay product, provided the parameter α_Λ is known. Its determination calls in turn for a measurement of the longitudinal polarization of the recoil protons in the decay of the unpolarized Λ hyperon. These arguments obviously apply to non-lepton decay of any hyperon with spin 1/2.

where ϑ —angle between the proton momentum and the direction of the hyperon polarization $\mathbf{P}_\Lambda$ (we recall that the hyperons are polarized perpendicular to the plane of their production). It is seen from this that a study of the asymmetry of the angular distribution yields the product $\alpha_\Lambda P_\Lambda$. The coefficient of asymmetry ($-\alpha_\Lambda$) is connected with the hyperon decay mechanism and is equal (see the last footnote) to the longitudinal proton polarization produced in the decay of unpolarized hyperons. The longitudinal polarization of the protons, and consequently also α_Λ, have already been determined experimentally. The method of determining the longitudinal polarization is based on the fact that protons polarized in the direction of their momentum in the hyperon rest system will, generally speaking, have a transverse polarization component in the laboratory frame (relative to the direction of their momentum in the laboratory frame). This transverse component can be determined by investigating the azimuthal asymmetry in the scattering of protons by a nucleus of known analyzing ability, for example carbon. Since the Λ particles obtained in the reaction (6.18) are polarized, to obtain unpolarized hyperons it is necessary to carry out averaging over all the orientations of the plane of their production. The most exact value $\alpha_\Lambda = -0.60 \pm 0.07$ was obtained by Cronin and Overseth[27] from an analysis of 1156 cases of the scattering of protons in carbon plates placed in a spark chamber. The values of α_Λ obtained by other authors are listed in Table I, which is borrowed from the review article delivered by Crawford[28] at the 1962 Geneva Conference.

Table I

Authors*	Λ-particle source	Detector	α_Λ
Cronin and Overseth[27]	$\pi^- + p$	Spark chamber	-0.62 ± 0.05
Gray et al[29]	$K^- + \mathrm{He}$	Helium bubble chamber	-0.66 ± 0.25
Beall et al[30]	$\pi^- + p$	Spark chamber	$-0.67^{+0.13}_{-0.17}$
Birge and Fowler[31]	π^- + propane	Propane bubble chamber	-0.45 ± 0.40
Boldt et al[32]	π^- + iron	Cloud chamber	$0.85^{+0.15}_{-0.21}$

*The articles are arranged in reversed chronological order. The earliest data of Boldt et al are now considered incorrect.

A negative value of α_Λ signifies that the protons have longitudinal helicity and the protons produced in the decay of a polarized Λ hyperon are emitted predominantly in the direction of the polarization $\mathbf{P}_\Lambda$.

Knowing α_Λ we can determine P_Λ on the basis of the measured values of $\alpha_\Lambda P_\Lambda$. Thus, at a pion energy $T_\pi = 783$ MeV, the average polarization P_Λ is approximately 70% and is directed along $(-\mathbf{p}_\pi) \times \mathbf{p}_\Lambda$. It is close to this value also when T_π = 871 MeV. The total cross sections of the reaction (6.18) at these energies are equal to $(0.14 \pm 0.01) \times 10^{-27}$ cm^2 and $(0.56 \pm 0.04) \times 10^{-27}$ cm^2. Details on the cross sections and polarizations at these energies are given in [33] where earlier papers are cited. According to the data of the Crawford group[34] at a pion momentum 1035 MeV the cross section amounts to $(0.73 \pm 0.028) \times 10^{-27}$ cm^2, and the polarization at 90° (c.m.s.) is close to 100%. This indicates the pion energies at which experiments on the parity of the (ΛK) system are best carried out.

A study of the reaction

$$\pi^+ + p \rightarrow \Sigma^+ + K^+ \tag{6.20}$$

with a polarized hydrogen target makes it possible not only tc determine the parity of the (Σ^+K^+) pair, but also the relative parity of the Σ and Λ hyperons, if the results of experiments on the reaction (6.18) are known. From among the various Σ-hyperon decays, only the process $\Sigma^+ \rightarrow p + \pi^0$ is characterized by an essentially nonvanishing asymmetry coefficient. According to the latest data[28,30]

$$\alpha_0 = 0.78 \begin{array}{l} +0.08 \\ -0.09. \end{array}$$

Analogous coefficients for other Σ-hyperon decays are[28]

$$\alpha_+ (\Sigma^+ \rightarrow n + \pi^+) = 0.03 \pm 0.08,$$
$$\alpha_- (\Sigma^- \rightarrow n + \pi^-) = 0.16 \pm 0.21.$$

According to Cork et al[35] the polarization of the hyperons in (6.20) at T_π = 990 MeV is close to 100%. However, the hyperon polarization in the same reaction is very small at T_π = 1090 MeV[36].

If the spin of the Ξ^- hyperon is equal to $1/2$, then the parity of the $(\Xi^- p)$ pair can be determined from the reaction

$$K^- + p \rightarrow \Xi^- + K^+. \tag{6.21}$$

Assuming that the spin of Ξ^- is $1/2$, we know both the asymmetry coefficient α_Ξ (the decay $\Xi^- \rightarrow \Lambda + \pi^-$) and the energy at which the Ξ^--hyperon polarization is large. The most accurate value of α_Ξ is[28]

$$\alpha_\Xi = 0.62 \pm 0.11.$$

The polarization of Ξ^- hyperons is close to 100%, for example, at a K^--meson momentum 1800 MeV/c.[37]

So far we have considered possible methods of determining the parity in reactions of the type

$$0 + \frac{1}{2} \rightarrow 0 + \frac{1}{2} \tag{6.22}$$

with a polarized target (0 and $1/2$ —particle spins).

We now consider briefly reactions of the type

$$\frac{1}{2} + \frac{1}{2} \rightarrow 0 + 0. \tag{6.23}$$

We shall show that the measurement of the total cross section of a reaction such as (6.23) with a polarized beam and a polarized target, and a comparison of the result with the total cross section of the same reaction with unpolarized particles, also make it possible to determine the intrinsic parities of the particles[38].

Indeed, the total cross section of the reaction (6.23) is of the form (see Sec. 7)

$$\sigma = \sigma_0 + \frac{1}{4}(\sigma_0^t - \sigma^s)(\mathbf{P}_1\mathbf{P}_2) + \frac{1}{2}(\sigma_+^t - \sigma_0^t)(\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k}), \tag{6.24}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ —polarizations of the beam and of the target, σ_0^t, σ_+^t, and σ^s —total cross sections of the reaction from the triplet state with projection 0, triplet state with projection +1, and singlet state respectively. Let the product of the intrinsic parities of all particles be $I = -1$. Then the parity conservation law yields

$$(-1)^{l_i} = -(-1)^{l_f}, \tag{6.25}$$

where l_i and l_f are the orbital angular momenta of the initial and final states. If the initial state is a singlet, then obviously the conservation of the total angular momentum leads to

$$l_i = l_f. \tag{6.26}$$

Relations (6.25) and (6.26) signify that when $I = -1$ the reaction from the singlet state is forbidden:

$$\sigma^s = 0, \quad I = -1. \tag{6.27}$$

For $I = +1$, the reaction from the triplet state with zero projection is forbidden:

$$\sigma_0^t = 0, \quad I = +1. \tag{6.28}$$

In this case it follows from the parity conservation law that

$$(-1)^{l_i} = (-1)^{l_f}, \tag{6.29}$$

whereas the law of conservation of the angular momentum and its projection require that l_i and l_f differ by unity.

We see therefore that for the reactions in question the sign of the coefficient of $(\mathbf{P}_1 \cdot \mathbf{P}_2)$ in the expression for the total cross section is uniquely related with the intrinsic parity. In the case when the target polarization is orthogonal to $\mathbf{k}$, the cross sections take on the form

$$\left.\begin{array}{ll} \sigma = \sigma_0 - \frac{1}{4}\sigma^s(\mathbf{P}_1\mathbf{P}_2), & I = 1, \\ \sigma = \sigma_0 + \frac{1}{4}\sigma_0^t(\mathbf{P}_1\mathbf{P}_2), & I = -1. \end{array}\right\} \tag{6.30}$$

Thus, comparison of the total cross sections σ and σ_0 makes it possible to determine the sign of the coefficient of $(\mathbf{P}_1 \cdot \mathbf{P}_2)$, and consequently the intrinsic parity of the particles. To this end we can investigate, for example, the reactions

$$\overline{\Lambda}(\overline{\Sigma}) + p \rightarrow K + \pi,$$
$$\overline{\Xi} + p \rightarrow K + K,$$
$$\Lambda + He^3 \rightarrow He^4 + \overline{K}^0.$$

We note that determination of the intrinsic parities in this fashion is at present more difficult than in reactions of the type (6.22).

Let us now discuss briefly the information on the intrinsic parities of strange particles that were obtained from experiments performed to date.

Indication that the parity of (ΛK) is negative were obtained in studies of the reactions

$$K^- + He^4 \begin{cases} \nearrow {}_\Lambda H^4 + \pi^0, \\ \searrow {}_\Lambda He^4 + \pi^-, \end{cases} \qquad (6.31)$$

considered by Dalitz[39]. If the spins of the hypernuclei are equal to zero, then conservation of the total angular momentum leads to equality of the orbital angular momenta in the initial and final states of the reactions (6.31):

$$l_i = l_f. \qquad (6.32)$$

The law of parity conservation yields in turn

$$I_{\bar{K}} I_N (-1)^{l_i} = I_\Lambda I_\pi (-1)^{l_f}, \qquad (6.33)$$

from which it follows that the reaction is allowed only if

$$I_{\bar{K}} I_N = I_\Lambda I_\pi. \qquad (6.34)$$

At the present time it has been experimentally confirmed that the spin of ${}_\Lambda H^4$ in the ground state is equal to zero (see, for example, [40]). Therefore, the observation on the part of the Block-Pevsner group[41] of several dozen cases of reactions (6.31) is weighty evidence in favor of $I_{\bar{K}} I_N = I_\Lambda I_\pi$. However, observation of reaction (6.31) does not determine the parity of the (KΛ) pair, if the hypernucleus has an excited state with unity spin and with a binding energy of several hundred keV[42]. Indeed, in this case and if $I_{\bar{K}} I_N = -I_\Lambda I_\pi$, the following sequence of reactions will take place

$$K^- + He^4 \to ({}_\Lambda H^4)^* + \pi^0, \quad ({}_\Lambda H^4)^* \to {}_\Lambda H^4 + \gamma, \qquad (6.35)$$

which imitates the reaction (6.31). Therefore observations of hypernuclei in the interactions between K^- mesons and He^4 can be regarded as proof of (6.34) only if it is established that there is no gamma radiation from the decays $({}_\Lambda H^4)^* \to {}_\Lambda H^4 + \gamma$ or $({}_\Lambda He^4)^* \to {}_\Lambda He^4 + \gamma$.

Very weighty evidence in favor of a positive relative parity of Σ and Λ particles was obtained most recently[43,44]. The ($\Sigma\Lambda$) parity, was determined by measuring the mass spectrum of the (e^+e^-) pairs from the decay of unpolarized Σ^0 hyperons:

$$\Sigma^0 \to \Lambda^0 + e^+ + e^-. \qquad (6.36)$$

A study of this decay with an aim at determining the ($\Sigma\Lambda$) parity was proposed in [45,46]. The problem consists in establishing whether the dipole transition is electric (negative ($\Sigma\Lambda$) parity) or magnetic (positive ($\Sigma\Lambda$) parity). At a given parity, the matrix element is characterized by two form factors. However, the form factor that vanishes in the case of emission of a real gamma quantum is reasonably assumed to be small at those values of momentum transfer which take place in the decay (6.36). It is precisely under this assumption that the authors of [43,44] conclude that the ($\Sigma\Lambda$) parity is positive. A similar conclusion is arrived at by the authors of [47,48] on the basis of a phenomenological analysis of K^--p interactions at K^--meson momenta near 400 MeV/c.

7. NUCLEON-NUCLEON SCATTERING

Experiments on nucleon-nucleon scattering are an important source of information on interactions between these particles. The direct purpose of these experiments is to obtain information on the asymptotic behavior of the wave functions of the colliding nucleons or, in other words, to reconstruct the nucleon-nucleon scattering matrix.

Considerable progress was made in recent years in the study of nucleon-nucleon collisions and in the solution of the problem of reconstructing the scattering matrix. This has become possible by the availability of polarized nucleon beams and by performance of double- and triple-scattering experiments. Polarized beams of fast protons were obtained by scattering protons from nuclei. If the beam obtained in this manner is scattered again by a target identical to the first target, we can determine the degree of polarization of the beam. At energies of several hundred MeV the polarization of protons elastically scattered by nuclei is close to 70—100% at certain scattering angles. The polarization produced in proton-proton collisions was determined by measuring the azimuthal asymmetry of scattering of the beam protons with known degree of polarization by hydrogen (double scattering). In triple scattering, where the first and third targets served as a polarizer and analyzer, the change in polarization upon scattering of polarized protons by hydrogen (second target) was determined.

The first such experimental program was carried out at 310 MeV[49] (Berkeley). This was followed by a detailed study of nucleon-nucleon scattering at 150 MeV[50] (Harvard, Harwell) and 210 MeV[51] (Rochester), 650 MeV[52] (Dubna), and close to 50 MeV[53].

In this section we consider the possible use of a polarized proton target, and also of a polarized beam and a polarized target, to solve the problem of reconstructing the nucleon-nucleon scattering matrix.

1. N-N scattering matrix. Within the framework of the requirements of isotopic invariance, the three nucleon-nucleon scattering processes (p-p, n-p, and n-n) are described by a matrix which acts on the spin and isotopic variables of the nucleons:

$$M(\mathbf{k}', \mathbf{k}) = M_0(\mathbf{k}', \mathbf{k}) \frac{1-(\tau_1\tau_2)}{4} + M_1(\mathbf{k}', \mathbf{k}) \frac{3+(\tau_1\tau_2)}{4}. \qquad (7.1)$$

Here τ_1 and τ_2 —isotopic matrices of the nucleons, **k** and **k'** —unit vectors in the directions of the initial and final relative momenta (c.m.s.), and M_0 and M_1 describe the scattering in states with isotopic spin T equal to zero or unity. As can be seen from (7.1), the p-p and n-n scattering matrices coincide with M_1, while n-p scattering is described by the half-sum of M_1 and M_0. More accurately, the amplitudes of p-p, n-n, and n-p scattering are expressed in terms of M_1 and M_0 as follows:

$$\langle pp | M | pp\rangle = \langle nn | M | nn\rangle = M_1,$$
$$\langle np | M | np\rangle = \frac{1}{2}(M_1 + M_0),$$
$$\langle pn | M | np\rangle = \frac{1}{2}(M_1 - M_0).$$

The general expression for the matrix $M_T(\mathbf{k}', \mathbf{k})$ can be obtained from the requirements of invariance under spatial rotations and reflections, and also time reversal [16,17]. We introduce a triplet of mutually orthogonal vectors

$$\mathbf{n} = \frac{[\mathbf{kk}']}{|[\mathbf{kk}']|}, \quad \mathbf{m} = \frac{\mathbf{k}-\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|}, \quad \mathbf{l} = \frac{\mathbf{k}+\mathbf{k}'}{|\mathbf{k}+\mathbf{k}'|}. \quad (7.2)^*$$

This triplet is convenient in that in the non-relativistic approximation the vectors **l** and **m** coincide in direction with the laboratory momenta of the scattered nucleon and of the recoil nucleon, respectively. Expanding $M_T(\mathbf{k}', \mathbf{k})$ in a complete system of 16 spin matrices I, σ_{1i}, σ_{2k}, σ_{1i}, σ_{2k}, and using the triplet vectors (7.2), we arrived with the aid of (3.23), (3.28), and (3.34) at the following general expression:

$$M_T(\mathbf{k}', \mathbf{k}) = a_T + b_T(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n}) + c_T[(\boldsymbol{\sigma}_1\mathbf{n}) + (\boldsymbol{\sigma}_2\mathbf{n})]$$
$$+ d_T[(\boldsymbol{\sigma}_1\mathbf{n}) - (\boldsymbol{\sigma}_2\mathbf{n})] + e_T(\boldsymbol{\sigma}_1\mathbf{m})(\boldsymbol{\sigma}_2\mathbf{m}) + f_T(\boldsymbol{\sigma}_1\mathbf{l})(\boldsymbol{\sigma}_2\mathbf{l}). \quad (7.3)$$

The coefficients a_T, b_T, c_T, d_T, e_T, and f_T are complex functions of the energies of the colliding particles and $(\mathbf{k}\cdot\mathbf{k}') = \cos\theta$.

We now show that $d_T = 0$ in the case of nucleon-nucleon scattering. This is a consequence of the identity of the particles in p-p and n-n scattering. For n-p scattering this is true only within the framework of the isotopic-invariance hypothesis. To prove this statement, we consider the initial (final) state of two nucleons with definite values of parity $(-1)^l$ or $(-1)^{l'}$, total spin s(s'), and total isotopic spin T(T'). It follows from the Pauli principle that

$$(-1)^l(-1)^{s+1}(-1)^{T+1} = -1,$$
$$(-1)^{l'}(-1)^{s'+1}(-1)^{T'+1} = -1.$$

Taking into account the conservation of the parity and of the total isotopic spin, we obtain from these equations

$$(-1)^s = (-1)^{s'}.$$

Inasmuch as the possible values of s and s' are 0 and 1, it follows that s = s' and the singlet-triplet transitions are forbidden. The only term in (7.3) which does not commute with the operator $S^2 = (\sigma_1 + \sigma_1)^2$ and which leads to singlet-triplet transitions is

$$d_T[(\boldsymbol{\sigma}_1\mathbf{n}) - (\boldsymbol{\sigma}_2\mathbf{n})].$$

Thus, $d_T = 0$ and the nucleon-nucleon scattering matrix is symmetrical with respect to the substitution $\sigma_1 \rightleftarrows \sigma_2$. We note that the absence of singlet-triplet transitions in nucleon-antinucleon scattering is the result of the G-invariance requirement.

The matrix $M_T(\mathbf{k}', \mathbf{k})$ $(d_T = 0)$ can be rewritten in a somewhat different form, separating the explicitly singlet and triplet scatterings:

$$M_T(\mathbf{k}', \mathbf{k}) = B_T\hat{S} + C_T[(\boldsymbol{\sigma}_1\mathbf{n}) + (\boldsymbol{\sigma}_2\mathbf{n})]$$
$$+ \frac{1}{2}G_T[(\boldsymbol{\sigma}_1\mathbf{m})(\boldsymbol{\sigma}_2\mathbf{m}) + (\boldsymbol{\sigma}_1\mathbf{l})(\boldsymbol{\sigma}_2\mathbf{l})]\hat{T} + \frac{1}{2}H_T[(\boldsymbol{\sigma}_1\mathbf{m})(\boldsymbol{\sigma}_2\mathbf{m})$$
$$- (\boldsymbol{\sigma}_1\mathbf{l})(\boldsymbol{\sigma}_2\mathbf{l})]\hat{T} + N_T(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})\hat{T}. \quad (7.4)$$

Here

$$\hat{S} = \frac{1}{4}[1 - (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)] \text{ and } \hat{T} = \frac{1}{4}[3 + (\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)]$$

—singlet and triplet projection operators, and

$$B = a - b - e - f, \quad C = c, \quad G = 2a + e + f,$$
$$H = e - f, \quad N = a + b. \quad (7.5)$$

The amplitude B describes singlet scattering, the remainder—triplet.

The requirement of antisymmetry of the final wave function $M(\mathbf{k}', \mathbf{k})\chi_S\chi_T$ (χ_S and χ_T —spin and isospin functions of the initial state) relative to permutation of the spatial $(\mathbf{k}' \to -\mathbf{k}')$, spin, and isotopic variables leads, as can be readily seen with the aid of (7.1) and (7.4), to amplitudes $B_1(\theta)$, $C_1(\theta)$, $H_1(\theta)$, $G_0(\theta)$, and $N_0(\theta)$ which remain unchanged under the substitution $\theta \to \pi - \theta$, whereas $B_0(\theta)$, $C_0(\theta)$, $H_0(\theta)$, $G_1(\theta)$, and $N_1(\theta)$ reverse sign. Using these symmetry properties, we can also readily establish with the aid of (7.5) the behavior of the amplitudes a_T, b_T, etc., under the substitution $\theta \to \pi - \theta$.

Thus, in the study of p-p and n-n scattering it is possible to confine oneself to measurements in the angle interval $0 \le \theta \le \pi/2$, since the value of the amplitude in the interval $\pi/2 < \theta < \pi$ is determined by the indicated symmetry properties. In the case of n-p scattering the interval of measurements doubles, $0 < \theta < \pi$, corresponding to the doubling the number of states in this system.

So far we have disregarded the limitations imposed on $M_T(\mathbf{k}', \mathbf{k})$ by the requirement that the S-matrix be unitary, $S^+S = 1$. As shown in [54], this requirement leads in the region of energies up to meson-production threshold to the following integral relation:

$$\frac{2\pi}{ik}[M_T(\mathbf{k}', \mathbf{k}) - M_T^+(\mathbf{k}, \mathbf{k}')] = \int M_T^+(\mathbf{k}'', \mathbf{k}')M_T(\mathbf{k}'', \mathbf{k})\,d\Omega_{\mathbf{k}''}, \quad (7.6)$$

where $d\Omega_{\mathbf{k}''}$ —*solid-angle element in the direction of* **k''**. This matrix element is equivalent for each value of T to five integral relations between the ten real functions of the angle and energy (real and imaginary

*$[\mathbf{kk}'] = \mathbf{k} \times \mathbf{k}'$.

parts of the coefficients a_T, b_T, etc., or B_T, C_T, etc.). If five of these functions are known for a given energy in the entire angle interval $(0 \le \theta \le \pi/2)$, then the other five functions are determined in the energy region up to pion-production threshold by the five relations (7.6). We emphasize that these relations determine also the general phase shift of the scattering matrix. We see therefore that in order to determine $M_T(\mathbf{k}', \mathbf{k})$ it is necessary in principle to carry out five experiments at all angles. Since the p-p scattering matrix coincides with M_1, in order to reconstitute this matrix it is sufficient to perform five experiments in the angle interval $0 \le \theta \le \pi/2$. In the case of n-p collisions, the scattering matrix is $(M_1 \pm M_0)/2$, and to determine it we must perform five experiments in the angle interval $0 \le \theta \le \pi$. If M_1 is known for a given energy from p-p scattering experiments, then M_0 can be determined with only five experiments on n-p scattering and the same energy in the interval $0 \le \theta \le \pi/2$.

In practice, to determine the scattering matrix one uses the phase-shift analysis method (see, for example [55]), which takes automatic account of the unitarity conditions (reality of the phase shifts and of the mixing parameters below the pion-production threshold).

The development of experimental techniques, and especially the availability of a polarized proton target and polarized nucleon beams, makes it possible to consider a direct determination of the scattering matrix elements, accurate to within a common phase factor for given values of the angle and energy, without the use of unitarity. At first glance this calls for nine independent experiments. However, Schumacher and Bethe [56] have shown that owing to the bilinear character of the dependence of the observed quantities on the scattering matrix elements, more independent experiments must be carried out for each value of angle and energy if the matrix is to be determined uniquely. This method is particularly useful at high energies, when the need of taking into account a large number of states and of considering the influence of inelastic processes makes the phase shift analysis difficult. The presence of a polarized target and polarized nucleon beams makes this program realizable even now.

2. Possible experiments. 1) The possible experiments on nucleon-nucleon scattering* differ both in the state of polarization of the initial beam and target and in the character of the measured quantities (cross section, polarization of the scattered particle, polarization of the recoil particle, polarization correlation). The parameters measured in experiments with an unpolarized target are usually called the parameters of single, double, and triple scattering. These names originated with the conditions under which the corresponding experiments were made in the absence of injectors of polarized particles. We shall describe these experiments briefly and show how they are modified when polarized targets are used.

We shall consider first collisions between non-identical particles, and discuss the changes necessitated by the Pauli principle later. The index 1 of a spin matrix will pertain to the incident (scattered) particles, while the index 2 will pertain to the target (recoil) particles.

The cross section for the scattering of an unpolarized beam by an unpolarized target

$$\sigma_0 = \frac{1}{4}\,\mathrm{Sp}\, MM^+ \tag{7.7}$$

is the parameter of single scattering. Expressions for the cross section and other measured quantities in terms of the amplitudes a, b, etc., are listed in Table II.

Table II

$\sigma_0 = \lvert a\rvert^2 + \lvert b\rvert^2 + 2\lvert c\rvert^2 + \lvert e\rvert^2 + \lvert f\rvert^2$,	(1)
$\sigma_0 D_{nn} = \lvert a\rvert^2 + \lvert b\rvert^2 + 2\lvert c\rvert^2 - \lvert e\rvert^2 - \lvert f\rvert^2$,	(2)
$\sigma_0 D_{ll} = \lvert a\rvert^2 - \lvert b\rvert^2 - \lvert e\rvert^2 + \lvert f\rvert^2$,	(3)
$\sigma_0 D_{mm} = \lvert a\rvert^2 - \lvert b\rvert^2 + \lvert e\rvert^2 - \lvert f\rvert^2$,	(4)
$\sigma_0 D_{ml} = 2\,\mathrm{Im}\, c^* (a-b)$,	(5)
$\sigma_0 P^0 = 2\,\mathrm{Re}\, c^* (a+b)$,	(6)
$\sigma_0 C_{ml} = 2\,\mathrm{Im}\, c^* (e-f)$,	(7)
$\sigma_0 K_{ml} = 2\,\mathrm{Im}\, c^* (e+f)$,	(8)
$\frac{1}{2}\sigma_0 C_{nn} = \mathrm{Re}\, ab^* + \lvert c\rvert^2 - \mathrm{Re}\, ef^*$,	(9)
$\frac{1}{2}\sigma_0 K_{nn} = \mathrm{Re}\, ab^* + \lvert c\rvert^2 + \mathrm{Re}\, ef^*$,	(10)
$\frac{1}{2}\sigma_0 C_{ll} = \mathrm{Re}\, af^* - \mathrm{Re}\, be^*$,	(11)
$\frac{1}{2}\sigma_0 K_{ll} = \mathrm{Re}\, af^* + \mathrm{Re}\, be^*$,	(12)
$\frac{1}{2}\sigma_0 C_{mm} = \mathrm{Re}\, ae^* - \mathrm{Re}\, bf^*$,	(13)
$\frac{1}{2}\sigma_0 K_{mm} = \mathrm{Re}\, ae^* + \mathrm{Re}\, bf^*$.	(14)

The polarizations $\mathbf{P}_1^0$ and $\mathbf{P}_2^0$ of the scattered and recoil particles produced by collisions of unpolarized particles* are equal to each other†

$$\mathbf{P}_1^0 = \mathbf{P}_2^0 \equiv \mathbf{P}^0 = \frac{1}{4}\,\frac{\mathrm{Sp}\,\boldsymbol{\sigma}_1 MM^+}{\sigma_0}. \tag{7.8}$$

They are determined by measuring the asymmetry in a second scattering from a target of known analyzing ability, and are the parameters of double scattering. Usually a modified formulation of the experiment is used. The polarization produced upon collision of un-

*An analysis of all possible experiments on nucleon-nucleon scattering and of the connections established by invariance requirements between the experiments was made in[54].

*As already noted, owing to the invariance under reflections, the polarizations $\mathbf{P}_1^0$ and $\mathbf{P}_2^0$ are orthogonal to the scattering plane.

†The equality of the polarizations $\mathbf{P}_1^0$ and $\mathbf{P}_2^0$ is a consequence of the symmetrical dependence of M on σ_1 and σ_2 (d = 0). This statement is not correct in the case when singlet-triplet transitions exist.

polarized nucleons is determined from the asymmetry of scattering of a beam with known degree of polarization, obtained by collision with nuclei of known polarizing ability (first target), by hydrogen (second target).

It is obvious that the use of a polarized target makes it possible to forego the additional scattering that results in a beam with known degree of polarization, and thereby replace the experiment on determining **P** for measurements of the asymmetry in double scattering by *an experiment involving measurements of the* asymmetry of the scattering of an unpolarized beam by a polarized target.

The use of a polarized target will be very helpful in measurements of polarization in n-p scattering. For this purpose it is sufficient to observe the asymmetry of the recoil protons in experiments with unpolarized neutrons. Beams of fast neutrons are usually obtained from proton accelerators by exchange n-p scattering. Their polarization is small. Elastic scattering of neutrons by nuclei leads to polarized beams of neutrons having low intensity. As a result, the greater part of the data on polarization in n-p scattering has been obtained so far from an analysis of experiments on quasi-elastic scattering of polarized protons by neutrons contained in deuterons.

Experiments with a polarized proton target make it possible to obtain information on the polarization of particles at energies of several GeV, where investigations without a polarized target are very difficult.

2) We now consider the so-called depolarization tensor D_{ik}, defined by the relation

$$\sigma_0 D_{ik} = \frac{1}{4}\,\mathrm{Sp}\,\sigma_{1i} M \sigma_{1k} M^+. \tag{7.9}$$

In the scattering of a polarized beam with polarization P_1 by an unpolarized target, the tensor D_{ik} relates $\mathbf{P}_1$ with the i-th component of the polarization $\mathbf{P}'_1$ of the scattered particles by means of the relations

$$P'_{1i} = \frac{\mathrm{Sp}\,\sigma_i M \varrho M^+}{\mathrm{Sp}\,M \varrho M^+} = \frac{1}{4}\,\frac{\mathrm{Sp}\,\sigma_{1i} M\,[1+(\boldsymbol{\sigma}_1\mathbf{P}_1)]\,M^+}{\sigma_0\,[1+(\mathbf{P}^0\mathbf{P}_1)]} = \frac{P_i^0 + D_{ik}P_{1k}}{1+(\mathbf{P}^0\mathbf{P}_{11})}, \tag{7.10}$$

where, as before, $\mathbf{P}^0 = P^0\mathbf{n}$ —polarization of the particles produced by collision of unpolarized nucleons. Owing to the symmetrical dependence of M on $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, the tensor D_{ik} is also equal to $(1/4\sigma_0)\,\mathrm{Sp}\,\sigma_{2i}M\sigma_{2k}M^+$ and determines the polarization of the recoil particles $\mathbf{P}'_2$ in scattering of unpolarized particles by a target with polarization $\mathbf{P}_2$:

$$P'_{2i} = \frac{P_i^0 + D_{ik}P_{2k}}{1+(\mathbf{P}^0\mathbf{P}_2)}. \tag{7.11}$$

The definition of D_{ik} [see (7.9)], the requirements (3.23) and (3.28) of invariance under rotations and reflections, and the transformation properties of the spin matrices [see (3.22) and (3.25)], all show that D_{ik} is a second-rank tensor and has consequently the form

$$D_{ik}(\mathbf{k}', \mathbf{k}) = D_{nn}n_i n_k + D_{ll}l_i l_k + D_{mm}m_i m_k + D_{ml}m_i l_k + D_{lm}l_i m_k. \tag{7.12}$$

The coefficients D_{mn}, D_{ll}, etc., are functions of the energy, and $(\mathbf{k}\cdot\mathbf{k}') = \cos\theta$.

Further limitations on D_{ik} follow from the requirements of invariance under time reversal. Using (3.34) and (3.32) we obtain

$$\begin{aligned}\sigma_0 D_{ik}(\mathbf{k}', \mathbf{k}) &= \frac{1}{4}\,\mathrm{Sp}\,\sigma_{1i}M(\mathbf{k}', \mathbf{k})\,\sigma_{1k}M^+(\mathbf{k}', \mathbf{k})\\ &= \frac{1}{4}\,\mathrm{Sp}\,\widetilde{M}^+(\mathbf{k}', \mathbf{k})\,\widetilde{\sigma}_{1k}\widetilde{M}(\mathbf{k}', \mathbf{k})\,\widetilde{\sigma}_{1i}\\ &= \frac{1}{4}\,\mathrm{Sp}\,u_T^{-1}M^+(-\mathbf{k}, -\mathbf{k}')\,u_T\widetilde{\sigma}_{1k}u_T^{-1}M(-\mathbf{k}, -\mathbf{k}')\,u_T\widetilde{\sigma}_{1i}\\ &= \frac{1}{4}\,\mathrm{Sp}\,\sigma_{1k}M(-\mathbf{k}, -\mathbf{k}')\,\sigma_{1i}M^+(-\mathbf{k}, -\mathbf{k}')\\ &= \sigma_0 D_{ki}(-\mathbf{k}, -\mathbf{k}').\end{aligned} \tag{7.13}$$

Since the substitution $\mathbf{k} \rightleftarrows -\mathbf{k}'$ causes the vectors **l**, **m**, **n** to go over into $-\mathbf{l}$, **m**, and $-\mathbf{n}$, it follows from (7.12) and (7.13) that

$$D_{ml} = -D_{lm}. \tag{7.14}$$

Thus, D_{ik} is determined by four scalar functions:

$$D_{ik} = D_{nn}n_i n_k + D_{ll}l_i l_k + D_{mm}m_i m_k + D_{ml}(m_i l_k - l_i m_k). \tag{7.15}$$

In the discussion of triple scattering experiments [57], Wolfenstein introduced the following parameters:

$$\left.\begin{aligned}D &= (\mathbf{n}_{\rm lab})_i D_{ik}(\mathbf{n}_{\rm lab})_k = (\mathbf{n})_i D_{ik}(\mathbf{n})_k = D_{nn},\\ R &= [\mathbf{n}_{\rm lab}\mathbf{k}'_{\rm lab}]_i D_{ik}[\mathbf{n}_{\rm lab}\mathbf{k}_{\rm lab}]_k = -(\mathbf{m})_i D_{ik}[\mathbf{nk}]_k = D_{mm}\cos\frac{\theta}{2} - D_{ml}\sin\frac{\theta}{2},\\ A &= [\mathbf{n}_{\rm lab}\mathbf{k}'_{\rm lab}]_i D_{ik}(\mathbf{k}_{\rm lab})_k = -(\mathbf{m})_i D_{ik}(\mathbf{k})_k = D_{mm}\sin\frac{\theta}{2} - D_{ml}\cos\frac{\theta}{2},\\ R' &= (\mathbf{k}'_{\rm lab})_i D_{ik}[\mathbf{n}_{\rm lab}\mathbf{k}_{\rm lab}]_k = (\mathbf{l})_i D_{ik}[\mathbf{nk}]_k = D_{ll}\sin\frac{\theta}{2} + D_{ml}\cos\frac{\theta}{2},\\ A' &= (\mathbf{k}'_{\rm lab})_i D_{ik}(\mathbf{k}_{\rm lab})_k = (\mathbf{l})_i D_{ik}(\mathbf{k})_k = D_{ll}\cos\frac{\theta}{2} - D_{ml}\sin\frac{\theta}{2}.\end{aligned}\right\} \tag{7.16}$$

Here $\mathbf{n}_{\rm lab}$, $\mathbf{k}_{\rm lab}$, and $\mathbf{k}'_{\rm lab}$ —normal to the scattering plane and unit vectors in the directions of the momenta of the incident and scattered particles in the laboratory frame. To obtain the connection between the introduced parameters and D_{nn}, D_{mm}, etc., we made use of the fact that $\mathbf{n}_{\rm lab} = \mathbf{n}$, $\mathbf{k}_{\rm lab} = \mathbf{k}$, and $\mathbf{k}'_{\rm lab} = \mathbf{l}$ in the non-relativistic approximation. It follows from (7.16) that the parameters E, A, R′, and A′ are related by

$$(A+R') = (A'-R)\,\mathrm{tg}\,\frac{\theta}{2}. \qquad (7.17)^*$$

The fact that only three of the four triple-scattering parameters are independent remains in force, of course, for elastic scattering of spin-$\frac{1}{2}$ particles by particles of arbitrary spin s. The special case of the reaction $0 + \frac{1}{2} \rightarrow 0 + \frac{1}{2}$ is discussed in Sec. 8.

To measure the parameters (7.16) or, what is the same, the components D_{ik} in the absence of a polarized target, triple scattering experiments are necessary. As in double scattering, the first scattering plays the role of a polarizer—a beam is produced with known polarization $\mathbf{P}_1$. The second scattering is from the investigated (hydrogen) target. The third scattering by target analyzer serves to determine the polarization $\mathbf{P}_1'$. The geometry of the different experiments on triple scattering is determined by the fact that the polarization produced by collision of unpolarized particles is orthogonal to the scattering plane, and by the fact that the analyzing scattering again makes it possible to determine only the polarization component normal to its plane. Thus, to determine D, all three scatterings must be carried out in a single plane. To determine R, the plane of investigated scattering should be perpendicular to the plane of the polarizing and analyzing scatterings. To determine A, the plane of the analyzing scattering is perpendicular to the plane of the main scattering, and to obtain a longitudinally-polarized beam, a magnetic field must be placed between the polarizing and the investigated scatterings, etc. The use of a polarized target makes the scattering by the polarizer target† superfluous and makes it possible to replace a triple scattering experiment by a double scattering experiment. Additional advantages arise in the investigation of n-p scattering. Thus, for example, to measure D in n-p scattering it is sufficient to scatter unpolarized neutrons from a proton target polarized normally to the scattering plane, and then measure the left-right asymmetry of the recoil protons.

3) We now turn to the polarization-transfer tensor

$$K_{ik} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,\sigma_{2i} M \sigma_{1k} M^+. \qquad (7.18)$$

This tensor determines the polarization $\mathbf{P}_2'$ of the recoil particles when a polarized beam (with polarization $\mathbf{P}_1$) is scattered by an unpolarized target:

$$P_{2i}' = \frac{P_i^0 + K_{ik} P_{1k}}{1 + (\mathbf{P}^0 \mathbf{P}_1)}. \qquad (7.19)$$

Owing to the symmetrical dependence of M on σ_1 and σ_2, the same tensor determines the polarization P_1' of the scattered particles when an unpolarized beam is scattered by a polarized target (with polarization $\mathbf{P}_2$)

$$P_{1i}' = \frac{P_i^0 + K_{ik} P_{2k}}{1 + (\mathbf{P}^0 \mathbf{P}_2)}. \qquad (7.20)$$

The general expression for K_{ik} can be written by repeating the procedure that has led to the general expression (7.15) for D_{ik}. It is only necessary to bear in mind that the requirement of invariance under time reversal imposes on K_{ik} a limitation of the form

$$K_{ik}(\mathbf{k}', \mathbf{k}) = K_{ki}(-\mathbf{k}, -\mathbf{k}') \qquad (7.21)$$

only if M has a symmetrical dependence on σ_1 and σ_2. We finally obtain

$$K_{ik} = K_{nn} n_i n_k + K_{ll} l_i l_k + K_{mm} m_i m_k + K_{ml}(m_i l_k - l_i m_k). \quad (7.22)$$

As in the case of D_{ik}, the determination of the different components of the tensor K_{ik} in experiments with an unpolarized target necessitates the use of triple scattering. The geometry of these experiments is analogous to the geometry of experiments on the determination of D_{ik}, and will not be discussed here. The use of a polarized target makes superfluous, as before, the scattering by the polarizing target and reduces a triple experiment to a double one.

Before we proceed to discuss other applications in a polarized target, we shall make a few remarks concerning experiments on the determination of D_{ik} and K_{ik}. In experiments on the determination of D_{ik} with an unpolarized target it is necessary to measure the polarization of scattered particles following the investigated (second) scattering. If the particle is scattered through large angles (in the c.m.s.), then its laboratory energy will be low and the measurement of the polarization will be difficult because of the absence of good polarization analyzers. In measurements of D_{ik} in experiments with a polarized target, on the other hand, it is necessary to measure the polarization of the recoil particles. But at large scattering angles these particles will have a large energy, and the measurement of their polarization should not cause any special difficulty. Therefore experiments on the determination of D_{ik} with a polarized target complement, in the case of non-identical particles, the determination of D_{ik} in experiments with a polarized beam. The same considerations pertain to experiments on the determination of K_{ik}.

We now consider a few features of the measurement of the components of the tensors D_{ik} and K_{ik} in p-p scattering, resulting from the identity of the particles. In p-p scattering we define as the scattered particle the proton which falls in the angle interval $0 \leq \theta < \pi/2$. The proton which falls in the interval $\pi/2 < \theta \leq \pi$ is called the recoil particle. We recall that, for example in experiments with a polarized beam, the determination of the D_{ik} tensor components calls for the measurements of the polarization of the scattered particles, while the determination of the components of K_{ik} calls for the measurement of the polarization of the recoil particles. It follows therefore that measurement of K_{ik} in the case of identical particles is equivalent to extending the range of measurements of the components of D_{ik} to include the angels $\theta > \pi/2$. The components

*tg = tan.

†The same pertains to a magnetic field ahead of the investigated scattering.

D_{ik} are then related to the components K_{ik} by the following equations:

$$\left.\begin{array}{l} D_{nn}(\pi-\theta)=K_{nn}(\theta), \quad D_{mm}(\pi-\theta)=K_{ll}(\theta), \\ D_{ll}(\pi-\theta)=K_{mm}(\theta), \quad D_{ml}(\pi-\theta)=K_{lm}(\theta)=-K_{ml}(\theta). \end{array}\right\} \tag{7.23}$$

The fact that the component D_{nn} is connected by the foregoing relation with K_{nn}, D_{il} with K_{mm}, etc., can be readily understood by recognizing that the substitution $\mathbf{k}' \to -\mathbf{k}'$ ($\theta \to \pi-\theta$, $\varphi \to \varphi+\pi$) causes $\mathbf{m}$ to go over into $\mathbf{l}$, $\mathbf{l}$ into $\mathbf{m}$, and $\mathbf{n}$ into $-\mathbf{n}$.

From the foregoing definition of the recoil particles it follows that the recoil protons will always have a lower laboratory-system energy than the scattered protons. If the recoil proton falls in the energy interval from 20 to 100 MeV, the measurement of K_{ik} (or, what is the same, of D_{ik} at the complementary angles $\pi-\theta$) in experiments with a polarized beam is very difficult, for lack of analyzer targets suitable for the measurement of the polarization of recoil protons with such an energy (in the energy region below 20 MeV, the measurement of the polarization is again easier if a helium target is used as an analyzer). It is possible to get rid of this difficulty by using a polarized target, for in this case the determination of K_{ik} calls for the measurement of the polarization of scattered particles of higher energy.

4) We now proceed to consider the cross section for the scattering of polarized particles by a polarized target. The initial-state density matrix has in this case the form

$$\varrho=\frac{1}{4}\left[I+(\boldsymbol{\sigma}_1\mathbf{P}_1)\right]\left[I+(\boldsymbol{\sigma}_2\mathbf{P}_2)\right]. \tag{7.24}$$

The differential cross section is

$$\sigma_{P_1P_2}=\operatorname{Sp} M\varrho M^+=\sigma_0+\frac{1}{4}\operatorname{Sp} M(\boldsymbol{\sigma}_1\mathbf{P}_1)M^+ + \frac{1}{4}\operatorname{Sp} M(\boldsymbol{\sigma}_2\mathbf{P}_2)M^+ + \frac{1}{4}\operatorname{Sp} M(\boldsymbol{\sigma}_1\mathbf{P}_1)(\boldsymbol{\sigma}_2\mathbf{P}_2)M^+. \tag{7.25}$$

The traces $\frac{1}{4}\operatorname{Sp} M\sigma_{1,2}M^+$ represent the polarization $\sigma_0\mathbf{P}^0$ which is produced in collisions of unpolarized particles, and (7.25) can be rewritten in the form

$$\sigma_{P_1P_2}=\sigma_0\left(1+P_{1i}P_i^0+P_{2k}P_k^0+P_{ik}P_{1i}P_{2k}\right), \tag{7.26}$$

where

$$\sigma_0P_{ik}=\frac{1}{4}\operatorname{Sp} M\sigma_{1i}\sigma_{2k}M^+. \tag{7.27}$$

This tensor P_{ik} is a new characteristic, data on which can be obtained by measuring the cross section for the scattering of polarized particles by a polarized target. As will be presently shown, this tensor essentially coincides with the correlation tensor C_{ik}

$$\sigma_0C_{ik}=\frac{1}{4}\operatorname{Sp}\sigma_{1i}\sigma_{2k}MM^+, \tag{7.28}$$

of the polarization produced in scattering of unpolarized particles by an unpolarized target.

Owing to the symmetrical dependence of M on $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, the tensors P_{ik} and C_{ik} are symmetrical in the indices i and k. Taking this into account and using the invariance under rotations and reflections, we obtain

$$P_{ik}(\mathbf{k}',\mathbf{k})=P_{nn}n_in_k+P_{mm}m_im_k+P_{ll}l_il_k+P_{ml}(m_il_k+m_kl_i), \tag{7.29}$$

$$C_{ik}(\mathbf{k}',\mathbf{k})=C_{nn}n_in_k+C_{mm}m_im_k+C_{ll}l_il_k+C_{ml}(m_il_k+m_kl_i). \tag{7.30}$$

From the requirements of invariance under time reversal (3.34) we get

$$\begin{aligned} \sigma_0P_{ik}(\mathbf{k}',\mathbf{k}) &= \frac{1}{4}\operatorname{Sp} M(\mathbf{k}',\mathbf{k})\,\sigma_{1i}\sigma_{2k}M^+(\mathbf{k}',\mathbf{k}) \\ &= \frac{1}{4}\operatorname{Sp}\widetilde{M}^+(\mathbf{k}',\mathbf{k})\,\widetilde{\sigma}_{2k}\widetilde{\sigma}_{1i}\widetilde{M}(\mathbf{k}',\mathbf{k}) \\ &= \frac{1}{4}\operatorname{Sp}\sigma_{1i}\sigma_{2k}M(-\mathbf{k},-\mathbf{k}')M^+(-\mathbf{k},-\mathbf{k}') \\ &= \sigma_0C_{ik}(-\mathbf{k},-\mathbf{k}'). \end{aligned} \tag{7.31}$$

Substituting (7.29) and (7.30) in (7.31) we get

$$P_{nn}=C_{nn}, \quad P_{mm}=C_{mm}, \quad P_{ll}=C_{ll}, \quad P_{ml}=-C_{ml}, \tag{7.32}$$

which proves the foregoing statement.

Measurements of the cross section at different beam and target-polarization orientations make it possible to determine different components of P_{ik} or C_{ik}. For nucleon-nucleon scattering we have

$$\begin{aligned} \sigma_{P_1,P_2}=\sigma_0\{1&+P^0(\mathbf{P}_1\mathbf{n})+P^0(\mathbf{P}_2\mathbf{n})+P_{nn}(\mathbf{P}_1\mathbf{n})(\mathbf{P}_2\mathbf{n}) \\ &+P_{ll}(\mathbf{P}_1\mathbf{l})(\mathbf{P}_2\mathbf{l})+P_{mm}(\mathbf{P}_1\mathbf{m})(\mathbf{P}_2\mathbf{m})+P_{ml}[(\mathbf{P}_1\mathbf{l})(\mathbf{P}_2\mathbf{m}) \\ &+(\mathbf{P}_1\mathbf{m})(\mathbf{P}_2\mathbf{l})]\}. \end{aligned} \tag{7.33}$$

For the case of target and beam polarizations orthogonal to the scattering plane ($\mathbf{P}_1 = P_1\mathbf{n}$; $\mathbf{P}_2 = P_2\mathbf{n}$), Eq. (7.33) reduces to

$$\sigma_{nn}=\sigma_0\{1+P^0P_2+[P^0+P_2C_{nn}]P_1\}. \tag{7.34}$$

To obtain data on other components of the tensor P_{ik} it is necessary to measure the cross sections $\sigma_{P_1P_2}$ for other orientations of $\mathbf{P}_1$ and $\mathbf{P}_2$. It is sufficient to choose for the directions of $\mathbf{P}_1$ and $\mathbf{P}_2$ the directions of the vectors $\mathbf{k}_{\text{lab}} = \mathbf{k}$ and $\mathbf{k}_{\text{lab}} \times \mathbf{n}_{\text{lab}} = \mathbf{k}\times\mathbf{n}$ in the laboratory frame. If $\mathbf{P}_1 = P_1\mathbf{k}$ and $\mathbf{P}_2 = P_2\mathbf{k}$, then

$$\sigma_{k,k}=\sigma_0\left\{1+\frac{1}{2}P_1P_2\left[C_{ll}(1+\cos\theta)+C_{mm}(1-\cos\theta)-2C_{ml}\sin\theta\right]\right\}. \tag{7.35}$$

When $\mathbf{P}_1 = P_1\mathbf{k}$ and $P_2 = \mathbf{P}_2\mathbf{k}\times\mathbf{n}$, (7.33) goes over into

$$\sigma_{k,[\mathbf{k}\,\mathbf{n}]}=\sigma_0\{1+P_1P_2[(C_{ll}-C_{mm})\sin\theta+C_{ml}\cos\theta]\}, \tag{7.36}$$

which coincides with the expression for the cross section when the beam polarization is directed along the vector $\mathbf{k}\times\mathbf{n}$, and the target is polarized along $\mathbf{k}$.

For the case of beam and target polarization along the vector $\mathbf{k}\times\mathbf{n}$ we have

$$\sigma_{[kn],[kn]}=\sigma_0\left\{1+\frac{1}{2}P_1P_2\left[C_{ll}(1-\cos\theta)+C_{mm}(1+\cos\theta)+2C_{ml}\sin\theta\right]\right\}. \tag{7.37}$$

Thus, the simple relations (7.34)—(7.37) show how the use of polarized targets makes it possible to replace difficult measurements with three targets, necessary to determine the tensor C_{ik}, by simpler measurements of the cross sections for the scattering of polarized particles by a polarized target.

Owing to the difficulty of measuring the polarization of low-energy recoil particles, the greater part of the measurements performed so far on the parameters C_{nn} and C_{ml} pertain to the scattering angle $\theta = \pi/2$. Since no such difficulties arise in the measurement of the cross sections, polarized targets add greatly to our information on the tensor C_{ik}.

The first experiment with a polarized proton target and a polarized beam [1] involved p-p scattering at 20 MeV, and the interaction occurred essentially in the $^1S_0$ state. By virtue of the symmetry properties of the p-p scattering amplitude under the substitution $\theta \to \pi - \theta$, we have

$$P^0(90^\circ) = 0 \text{ and } \sigma_{P_1P_2}(90^\circ) = \sigma_0(1 + P_1P_2C_{nn}),$$

if the polarization of the beam and of the target are perpendicular to the scattering plane. If $\sigma_\pm$ —cross section for the scattering by a target polarized "upward" (+) and "downward" (–), then

$$C_{nn} = \frac{1}{P_1P_2}\frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

and the measurement of the cross sections leads directly to a determination of $C_{nn}(90^\circ)$.

We note also that knowledge of the sum of C_{nn}, C_{ll}, and C_{mm} makes it possible to separate directly the singlet and the triplet contributions to the differential cross section of the scattering of an unpolarized beam by an unpolarized target. Indeed,

$$C_{nn} + C_{ll} + C_{mm} = \langle(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\rangle,$$

where $\langle(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\rangle$ denotes the mean value of the scalar product of the spins. On the other hand, the cross sections of the singlet and triplet scatterings are, respectively,

$$\left.\begin{aligned} \sigma_s &= \operatorname{Sp} MM^+\hat{S} = \sigma_0(1 - \langle(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\rangle), \\ \sigma_t &= \frac{1}{3}\operatorname{Sp} MM^+\hat{T} = \sigma_0\left(1 + \frac{1}{3}\langle(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)\rangle\right), \end{aligned}\right\} \quad (7.38)$$

where $\hat{S}$ and $\hat{T}$ —singlet and triplet projection operators.

5) Measurement of the cross section σ_0, of the polarization P^0, and components of the tensors D_{ik}, K_{ik}, and C_{ik} yields 14 equations for the determination of a, b, etc. (Table II). If the observed quantities are known with sufficient degree of accuracy, then we can, on the basis of these equations, following [56], reconstruct the scattering matrix for specified values of the scattering angle and energy. Since the observed quantities are quadratic in the matrix M, the common phase shift cannot be determined from measurements at a given angle and it becomes necessary to determine nine real quantities (real and imaginary parts of the parameters a, b, etc.) We choose as the common phase the phase of the amplitude c, and will accordingly assume henceforth that c is a real positive number.

From Eqs. (1) and (2) of Table II it follows that

$$2(|e|^2 + |f|^2) = \sigma_0(1 - D_{nn}). \quad (7.39)$$

Subtracting (4) from (3) we get

$$2(|f|^2 - |e|^2) = \sigma_0(D_{ll} - D_{mm}). \quad (7.40)$$

From (7.39) and (7.40) we obtain the moduli of e and f:

$$4|e|^2 = \sigma_0(1 - D_{nn} - D_{ll} + D_{mm}), \quad (7.41)$$

$$4|f|^2 = \sigma_0(1 - D_{nn} + D_{ll} - D_{mm}). \quad (7.42)$$

If we assume that c is known, we can obtain from (7) and (8) the imaginary parts of e and f:

$$\operatorname{Im} e = \frac{1}{4c}\sigma_0(C_{ml} + K_{ml}) \equiv \frac{N'}{c}, \quad (7.43)$$

$$\operatorname{Im} f = \frac{1}{4c}\sigma_0(K_{ml} - C_{ml}) \equiv \frac{M'}{c}, \quad (7.44)$$

where we introduced the notation

$$N' = \frac{1}{4}\sigma_0(C_{ml} + K_{ml}) \text{ and } M' = \frac{1}{4}\sigma_0(K_{ml} - C_{ml}).$$

Consequently, if c is known, then (7.41)—(7.44) determine e and f, apart from the signs of their real parts.

To determine c we consider Eqs. (9) and (10) of the table. Subtracting (9) from (10) we get

$$\operatorname{Re} ef^* \equiv \operatorname{Re} e \operatorname{Re} f + \operatorname{Im} e \operatorname{Im} f = \frac{1}{4}\sigma_0(K_{nn} - C_{nn}). \quad (7.45)$$

We denote $\frac{1}{4}\sigma_0(K_{nn} - C_{nn})$ by L. Transferring (Im e Im f) to the right side of (7.45), squaring the resultant equation, and using (7.41)—(7.44), we obtain after simple transformations the following expression for c in terms of the observed quantities L, M′, and N′, and the previously obtained $|e|^2$ and $|f|^2$:

$$c^2 = \frac{N'^2|f|^2 + M'^2|e|^2 - 2LM'N'}{|e|^2|f|^2 - L^2}. \quad (7.46)$$

Thus, (7.41)—(7.46) determine five parameters:

$$c, \ \operatorname{Im} e, \ \operatorname{Im} f, \ \operatorname{Re} e, \ \operatorname{Re} f,$$

accurate to within the sign of one of the real parts.

We now must determine a and b. It is more convenient to find the combinations

$$u = \frac{1}{2}(a + b), \quad v = \frac{1}{2}(a - b).$$

Adding (1) to (2), (3) to (4), and (9) to (10) we get

$$|a|^2 + |b|^2 = \frac{1}{2}\sigma_0(1 + D_{nn}) - 2c^2,$$

$$|a|^2 - |b|^2 = \frac{1}{2}\sigma_0(D_{ll} + D_{mm}),$$

$$2\operatorname{Re} ab^* = \frac{1}{2}\sigma_0(C_{nn} + K_{nn}) - 2c^2.$$

It follows therefore that

$$2|u|^2 = \frac{1}{4}\sigma_0(1 + D_{nn} + K_{nn} + C_{nn}) - 2c^2, \quad (7.47)$$

$$2|v|^2=\frac{1}{4}\sigma_0(1+D_{nn}-K_{nn}-C_{nn}), \tag{7.48}$$

$$\mathrm{Re}\, uv^*=\frac{1}{8}\sigma_0(D_{ll}+D_{mm}). \tag{7.49}$$

In addition, it follows from (5) and (6) that

$$\mathrm{Re}\, u=\frac{1}{4}\frac{\sigma_0 P^0}{c}, \tag{7.50}$$

$$\mathrm{Im}\, v=\frac{1}{4}\frac{\sigma_0 D_{ml}}{c}. \tag{7.50'}$$

From (7.47), (7.48), (7.50), and (7.50′) we can determine the four parameters Re u, Im u, Re v, and Im v accurate to within the signs of Im u and Re v. We can determined the signs from (7.49). If the experimental data are sufficiently accurate, this equation will give unambiguous information concerning the signs of Im u and Re v.

Thus, Eqs. (1)—(10) of Table II enable us to find all the parameters of the amplitude, apart from an arbitrarily selected sign of Re e (or Re f). This ambiguity in the choice of the sign can be eliminated by making use of any of the hitherto unused equations (11)—(14). These equations may also prove useful for the reconstruction of the scattering matrix, if the experimental data are insufficiently accurate.

6) Let us obtain, starting from the usual invariance requirements, general expression for the total cross section of any reaction channel in the case of a collision between a polarized beam and a polarized target [58,59].

The expression for the total cross section σ should be a scalar made up of quantities characterizing the state of the system before the collision: the polarizations $\mathbf{P}_1$ and $\mathbf{P}_2$ of the beam and the target, and the relative momentum $\mathbf{p}$ in the initial state. It is necessary merely to take account here of the fact that σ depends linearly on each of the polarizations. Thus,

$$\sigma=\sigma_0+\sigma_1(\mathbf{P}_1\mathbf{P}_2)+\sigma_2(\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k}). \tag{7.51}$$

Here, as before, $\mathbf{k}$ is a unit vector in the direction of $\mathbf{p}$, σ_0 is the total cross section of the reaction with unpolarized particles, and the coefficients σ_1 and σ_2 are certain functions of the energies of the colliding particles. Their meaning can be readily explained with the aid of the following simple considerations.

We choose the direction of $\mathbf{k}$ as the quantization axis. We denote by w^s and w_m^t the probabilities of observing in the initial state a singlet and a triplet with projection m. The scalar products $(\mathbf{P}_1\cdot\mathbf{P}_2)$ and $(\mathbf{P}_1\cdot\mathbf{k})(\mathbf{P}_2\cdot\mathbf{k})$, which enter in (7.51) and are equal respectively to the mean values of the operators $(\sigma_1\cdot\sigma_2)$ and $(\sigma_1\cdot\mathbf{k})(\sigma_2\cdot\mathbf{k})$ in the initial state, can be written in the form

$$(\mathbf{P}_1\mathbf{P}_2)=\langle(\sigma_1\sigma_2)\rangle=\langle 2S^2-3\rangle,$$
$$(\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k})=\langle(\sigma_1\mathbf{k})(\sigma_2\mathbf{k})\rangle=\langle 2(\mathbf{S}\mathbf{k})^2-1\rangle,$$

where $\mathbf{S}=(\sigma_1+\sigma_2)/2$ —system spin operator.

We obtain

$$\left.\begin{aligned}(\mathbf{P}_1\mathbf{P}_2)&=\sum_m w_m^t-3w^s,\\ (\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k})&=\sum_m(-1)^{m+1}w_m^t-w^s.\end{aligned}\right\} \tag{7.52}$$

From (7.52) and from the normalization condition $w^s+\sum_m w_m^t=1$, we obtain the following expressions for the probabilities:

$$\left.\begin{aligned}w^s=\frac{1-(\mathbf{P}_1\mathbf{P}_2)}{4},\quad w_0^t&=\frac{1+(\mathbf{P}_1\mathbf{P}_2)-2(\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k})}{4},\\ w_+^t+w_-^t&=\frac{1+(\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k})}{2}.\end{aligned}\right\} \tag{7.53}$$

If we denote by σ^s and σ_m^t the total cross sections of the reaction from the corresponding spin states, then

$$\sigma=w^s\sigma^s+\sum_m w_m^t\sigma_m^t. \tag{7.54}$$

Recognizing that $\sigma_+^t=\sigma_-^t$, because of the invariance under rotations and reflections,* we obtain from (7.54) and (7.53) after simple transformations

$$\sigma=\sigma_0+\frac{1}{4}(\sigma_0^t-\sigma^s)(\mathbf{P}_1\mathbf{P}_2)+\frac{1}{2}(\sigma_+^t-\sigma_0^t)(\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k}), \tag{7.55}$$

from which we get the connection between the cross section of the reactions from the triplet and singlet states and σ_1 and σ_2.

We can obtain complete information on the total cross section σ_0, σ_1, and σ_2 by carrying out experiments with an unpolarized beam and an unpolarized target, and also under conditions when: a) $\mathbf{P}_1$ and $\mathbf{P}_2$ are parallel to each other and perpendicular to the beam direction, and b) $\mathbf{P}_1$ and $\mathbf{P}_2$ are parallel to each other and also to the beam direction. By measuring the total cross section σ under these conditions, we can obtain σ_1 and σ_2, after which, using the obvious relation

$$\sigma_0=\frac{1}{4}\sigma^s+\frac{1}{4}\sigma_0^t+\frac{1}{2}\sigma_+^t,$$

we can determine the three independent cross sections of the reactions from the singlet and the triplet states.

The obtained relations pertain to any channel of a reaction with two particles having spin ½ in the initial state. Consequently, they are valid also for the total cross section σ_{tot} of all the processes. Using the unitarity of the S-matrix, we can easily relate the coefficients σ_{1tot} and σ_{2tot} in the expression for the total cross section with the coefficients of the forward elastic-scattering matrix [58-60]. In the case of forward scattering we have in (7.3) c(0) = d(0) = 0 and b(0) = e(0), so that the matrix (7.3) takes on the form

*The latter can be seen, for example, from (7.51). Indeed, the values of the total cross sections $\sigma_+{}^t$ and $\sigma_-{}^t$ can be obtained from the general expression (7.51), if we assume that the beam and the target are fully polarized either in the direction of k ($\sigma_+{}^t$), or in the direction of $-$k ($\sigma_-{}^t$). This means that in both cases $(\mathbf{P}_1\cdot\mathbf{P}_2)=1$ and $(\mathbf{P}_1\cdot\mathbf{k})(\mathbf{P}_2\cdot\mathbf{k})=1$. This leads to the result stated above.

$$M(\mathbf{k},\,\mathbf{k}) = a(0) + e(0)(\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2) + [f(0) - e(0)](\boldsymbol{\sigma}_1\mathbf{k})(\boldsymbol{\sigma}_2\mathbf{k}). \quad (7.56)$$

Along with the ordinary optical theorem

$$\operatorname{Im} a(0) = \frac{k}{4\pi}\sigma_{0\ \mathrm{tot}}, \quad (7.57)$$

the condition for the unitarity of the S matrix gives the following relations:

$$\left.\begin{aligned}\operatorname{Im} e(0) &= \frac{k}{4\pi}\sigma_{1\ \mathrm{tot}},\\ \operatorname{Im}[f(0) - e(0)] &= \frac{k}{4\pi}\sigma_{2\,\mathrm{tot}}.\end{aligned}\right\} \quad (7.58)$$

Thus, measurements of the total cross section of all the processes at different orientations of the beam and target polarizations enable us to determine the imaginary parts of all three forward elastic scattering amplitudes. These measurements make it also possible to improve the known estimate of the lower boundary of the differential forward elastic scattering cross section. Using (7.56)—(7.58) we obtain the following inequality:

$$\frac{d\sigma_{el}}{d\Omega} \geqslant \left(\frac{k}{4\pi}\right)^2 [\sigma_{0\ \mathrm{tot}}^2 + 2\sigma_{1\ \mathrm{tot}}^2 + (\sigma_{1\ \mathrm{tot}} + \sigma_{2\ \mathrm{tot}})^2], \quad (7.59)$$

where $d\sigma_{el}/d\Omega$ —cross section for the elastic scattering of the unpolarized particles.

7) Many of the results obtained above for nucleon-nucleon scattering also apply to elastic scattering of fermions with spin $1/2$ by each other (scattering of electrons by protons, hyperons by nucleons, etc.), and also to a more general class of reactions of the type $1/2 + 1/2 \to 1/2 + 1/2$, with both $I_i = I_f$ and $I_i = -I_f$.

Let us see how the results for N-N scattering are modified in the presence of singlet-triplet transitions. First of all, the polarizations $\mathbf{P}_1^0$ and $\mathbf{P}_2^0$ are different. To describe the change in the polarization in the collisions it is necessary to introduce, in place of the single tensor D_{ik}, the two tensors

$$D_{ik}^{(1)} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{1i}M\sigma_{1k}M^+ \quad (7.60)$$

and

$$D_{ik}^{(2)} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{2i}M\sigma_{2k}M^+. \quad (7.61)$$

The tensor $D_{ik}^{(1)}$ determines the polarization of the scattered beam $\mathbf{P}_1'$ when the polarized beam (polarization $\mathbf{P}_1$) is scattered by an unpolarized target. The tensor $D_{ik}^{(2)}$ determines the polarization of the recoil particles $\mathbf{P}_2'$ after scattering of unpolarized particles by a target with polarization $\mathbf{P}_2$.

We introduce analogously the polarization transfer tensors $K_{ik}^{(1)}$ and $K_{ik}^{(2)}$:

$$K_{ik}^{(1)} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{2i}M\sigma_{1k}M^+ \quad (7.62)$$

and

$$K_{ik}^{(2)} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{1i}M\sigma_{2k}M^+. \quad (7.63)$$

The tensor $K_{ik}^{(1)}$ determines the polarization $\mathbf{P}_2'$ of the recoil particles after scattering of a polarized beam (with polarization $\mathbf{P}_1$) by an unpolarized target. The tensor $K_{ik}^{(2)}$ determines the polarization $\mathbf{P}_1'$ of scattered particles after collision between an unpolarized beam and a polarized target (polarization $\mathbf{P}_2$).

In the presence of singlet-triplet transitions, the requirements of T-invariance for elastic-scattering processes lead to the following equations

$$\left.\begin{aligned}D_{ik}^{(1)}(\mathbf{k}',\,\mathbf{k}) &= D_{ki}^{(1)}(-\mathbf{k},\,-\mathbf{k}'),\\ D_{ik}^{(2)}(\mathbf{k}',\,\mathbf{k}) &= D_{ki}^{(2)}(-\mathbf{k},\,-\mathbf{k}')\end{aligned}\right\} \quad (7.13')$$

and

$$K_{ik}^{(1)}(\mathbf{k}',\,\mathbf{k}) = K_{ki}^{(2)}(-\mathbf{k},\,-\mathbf{k}'), \quad (7.21')$$

which replace (7.13) and (7.21).

In the general case of reactions of the type $1/2 + 1/2 \to 1/2 + 1/2$ with $I_i = \pm I_f$, many interesting relations are obtained from the requirements of invariance under space reflections. We note first that the operator (2.9) of reflection in the plane of the reaction has for this case the form

$$R = (\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n}). \quad (7.64)$$

Thus,

$$RMR^{-1} = (\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})M(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n}) = +M \quad (7.65)$$

for reactions with $I_i = +I_f$. We note that the validity of (7.65) is obvious if we use the explicit expression (7.3) for the M matrix. Relation (7.65) is replaced by

$$RMR^{-1} = (\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})M(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n}) = -M \quad (7.66)$$

for the case of a reaction with $I_i = -I_f$.

With the aid of (7.64) we can show once more that the polarization $\mathbf{P}^0$ is perpendicular to the scattering plane and that the tensors B_{ik}, K_{ik}, P_{ik}, and C_{ik} have no components of the type D_{nl} and C_{mn}.

It is easy to show with the aid of (7.64) that in the most general case the depolarization parameters of the tensors $D_{ik}^{(1)}$ and $D_{ik}^{(2)}$ are equal to each other (apart from the sign in the case of the reaction in question with $I_k = -I_f$). Indeed, considering simultaneously the case of both values $I_i = \pm I_f$, we have

$$\begin{aligned}\sigma_0 D_{nn}^{(1)} &= \frac{1}{4}\operatorname{Sp}(\boldsymbol{\sigma}_1\mathbf{n})M(\boldsymbol{\sigma}_1\mathbf{n})M^+\\ &= \pm\frac{1}{4}\operatorname{Sp}(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})M(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})(\boldsymbol{\sigma}_1\mathbf{n})M^+\\ &= \pm\frac{1}{4}\operatorname{Sp}(\boldsymbol{\sigma}_2\mathbf{n})M(\boldsymbol{\sigma}_2\mathbf{n})M^+ = \pm\,\sigma_0 D_{nn}^{(2)}.\end{aligned} \quad (7.67)$$

An analogous result holds also for the tensors $K_{ik}^{(1)}$ and $K_{ik}^{(2)}$. Indeed,

$$\begin{aligned}\sigma_0 K_{nn}^{(1)} &= \frac{1}{4}\operatorname{Sp}(\boldsymbol{\sigma}_2\mathbf{n})M(\boldsymbol{\sigma}_1\mathbf{n})M^+\\ &= \pm\frac{1}{4}\operatorname{Sp}(\boldsymbol{\sigma}_1\mathbf{n})M(\boldsymbol{\sigma}_2\mathbf{n})M^+ = \pm\,\sigma_0 K_{nn}^{(2)}.\end{aligned} \quad (7.68)$$

In the case of an inelastic reaction, the tensors C_{ik} and P_{ik}, which generally speaking yield different information, have identical components C_{nn} and P_{nn} (apart from the sign). Indeed,

$$\sigma_0 P_{nn} = \frac{1}{4}\,\mathrm{Sp}\,M(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})M^+$$

$$= \pm\frac{1}{4}\,\mathrm{Sp}(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})MM^+ = \pm\sigma_0 C_{nn}. \quad (7.69)$$

An experimental check on relations of the type (7.67)–(7.69) in inelastic reactions makes it possible to investigate experimentally the degree of parity conservation.

8) Maximum information can be obtained by carrying out experiments with simultaneous utilization of a polarized target and a polarized particle beam. We have already seen what information can be extracted in studies of the differential and total cross sections for the interaction between a polarized beam of nucleons and a polarized target. We can ask: what additional information is afforded by a study of the nucleon polarization and polarization correlation after the scattering of a polarized beam by a polarized target?

The general expression for the projection of the polarization vectors of the scattered particle 1 on the direction of an arbitrary vector $\mathbf{Q}$ can be represented in the form

$$\sigma_{P_1,P_2}(\mathbf{P}_1'\mathbf{Q}) = \sigma_0[P^0(\mathbf{Q}\mathbf{n}) + K_{Q,P_2} + D_{Q,P_1} + M_{QP_1P_2}], \quad (7.70)$$

where $K_{ab} = K_{ik}a_ib_k$, $D_{ab} = D_{ik}a_ib_k$, and D_{ik} and K_{ik} are the previously introduced depolarization and polarization-transfer tensors.

In the newly introduced tensor

$$M_{ikq} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,\sigma_{1i}M\sigma_{1k}\sigma_{2q}M^+ \quad (7.71)$$

there are only 13 nonvanishing elements, by virtue of parity conservation. The general expression for this pseudotensor of third rank is of the form

$$M_{ikq}(\mathbf{k}',\mathbf{k}) = M_{nnn}n_in_kn_q + M_{nll}n_il_kl_q + M_{lnl}l_in_kl_q$$
$$+ M_{lln}l_il_kn_q + M_{nmm}n_im_km_q + M_{mnm}m_in_km_q + M_{mmn}m_im_kn_q$$
$$+ M_{nlm}n_il_km_q + M_{lnm}l_in_km_q + M_{lmn}l_im_kn_q + M_{nml}n_im_kl_q$$
$$+ M_{mnl}m_in_kl_q + M_{mln}m_il_kn_q. \quad (7.72)$$

Not all the components of the tensor $N_{ikq}(k', k)$ contain new information (compared with the simpler experiments). Thus, the parameter M_{nnn} coincides with P_2^0. The proof is based on invariance under reflections and admits of generalization to the general case of reactions of the type $\frac{1}{2} + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2}$. Indeed, in the right part of the equation

$$\sigma_0 M_{nnn} = \frac{1}{4}\,\mathrm{Sp}(\sigma\mathbf{n})M(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})M^+$$

we can write under the trace symbol, using (7.65), (7.66), and the fact that $(\sigma_2\cdot\mathbf{n})^2 = 1$,

$$\sigma_0 M_{nnn} = \frac{1}{4}\,\mathrm{Sp}(\sigma_2\mathbf{n})(\sigma_2\mathbf{n})(\sigma_1\mathbf{n})M(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})M^+$$

$$= \pm\frac{1}{4}\,\mathrm{Sp}(\sigma_2\mathbf{n})MM^+ = \pm\sigma_0 P_2^0,$$

which proves the foregoing statement.

We can find analogously that the invariance requirements "reduce" the results of some more complicated experiments to simpler ones. Some examples will be given later.

For the polarization of the recoil particles we have, in analogy with (7.70)

$$\sigma_{P_1P_2}(\mathbf{P}_2'\mathbf{Q}) = \sigma_0[P^0(\mathbf{Q}\mathbf{n}) + K_{QP_1} + D_{QP_2} + N_{QP_1P_2}], \quad (7.73)$$

and the new information is contained in the tensor

$$N_{ikq} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\,\sigma_{2i}M\sigma_{1k}\sigma_{2q}M^+. \quad (7.74)$$

The structure of this tensor does not differ from (7.72). The set of symmetry properties is close to the symmetry properties of the tensor M_{ikq}. It is easy to verify, in particular, that

$$N_{nnn} = \pm P_1^0$$

for both cases of relative parities in the general case of reactions of the form $\frac{1}{2} + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2}$. In addition, since for example

$$\mathrm{Sp}(\sigma_1\mathbf{n})M(\sigma_1\mathbf{l})(\sigma_2\mathbf{l})M^+ = \mathrm{Sp}(\sigma_2\mathbf{n})[(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})M(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})$$
$$\times(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})(\sigma_1\mathbf{l})(\sigma_2\mathbf{l})M^+ = \pm\,\mathrm{Sp}(\sigma_2\mathbf{n})M(\sigma_1 m)(\sigma_2 m)M^+,$$

the requirements of invariance under reflections leads to the equations

$$\left.\begin{array}{ll} N_{nll} = \pm M_{nmm}, & N_{nmm} = \mp M_{nll}, \\ N_{nml} = \pm M_{nlm}, & N_{nlm} = \pm M_{nml}. \end{array}\right\} \quad (7.75)$$

We now consider the most complicated experiment—determination of the polarization correlation in experiments with simultaneous utilization of a polarized beam (polarization $\mathbf{P}_1$) and a polarized target (polarization $\mathbf{P}_2$). For the polarization correlations we can obtain in this case

$$\sigma_{P_1P_2}C_{ab}^{P_1P_2} = \frac{1}{4}\,\mathrm{Sp}(\sigma_1\mathbf{a})(\sigma_2\mathbf{b})M[I + (\mathbf{P}_1\sigma_1)][I + (\mathbf{P}_2\sigma_2)]M^+$$
$$= \sigma_0\{C_{ab} + C_{ikq}a_ib_kP_{1q} + P_{ikq}a_ib_kP_{2q} + C_{ikqp}a_il_kP_{1q}P_{2p}\}, \quad (7.76)$$

where $C_{ab} = C_{ik}a_ib_k$ is determined by the known tensor C_{ik}, and the possible new information is connected with the tensors C_{ikq}, P_{ikq}, and C_{ikqp}:

$$\left.\begin{array}{l} \sigma_0 C_{ikq} = \frac{1}{4}\,\mathrm{Sp}\,\sigma_{1i}\sigma_{2k}M\sigma_{1q}M^+, \\ \sigma_0 P_{ikq} = \frac{1}{4}\,\mathrm{Sp}\,\sigma_{1i}\sigma_{2k}M\sigma_{2q}M^+, \\ \sigma_0 C_{ikqp} = \frac{1}{4}\,\mathrm{Sp}\,\sigma_{1i}\sigma_{2k}M\sigma_{1q}\sigma_{2p}M^+. \end{array}\right\} \quad (7.77)$$

The tensor $C_{ikq}(P_{ikq})$ characterizes the correlation of the polarizations in experimentation with arbitrary (unpolarized) beam and unpolarized (polarized) target. A general form of the tensors C_{ikq} and P_{ikq} is given by an expression analogous to (7.72). The invariance under reflections leads to

$$C_{nnn} = \pm P_2^0, \qquad P_{nnn} = \pm P_1^0, \quad (7.78)$$

and certain components coincide with the components of the previously introduced third-rank tensors. For example,

$$\left.\begin{array}{lll} P_{mnm}=\pm M_{lnl}, & P_{lnl}=\pm M_{mnm}, & P_{mnl}=\mp M_{lnm}, \\ C_{mnn}=\pm M_{lln}, & C_{lnl}=\pm M_{mmn}, & P_{lnm}=\pm M_{mnl}, \\ C_{mnl}=\mp M_{lmn}, & C_{lnm}=\mp M_{mln}, & C_{nll}=\pm N_{mmn}, \\ C_{lln}=\mp P_{mmn}, & P_{nmm}=\pm N_{lnl}, & C_{nml}=\pm N_{lln}, \\ C_{mmn}=\mp P_{lln}, & C_{nlm}=\mp N_{mln}, & C_{nml}=\mp N_{lmn}, \\ C_{mln}=\pm P_{lmn}, & P_{nll}=\pm N_{mnm}, & P_{nlm}=\mp N_{mnl}, \\ P_{nml}=\mp N_{lnm}, & P_{mln}=\pm C_{lmn}. & \end{array}\right\} \quad (7.79)$$

The T-invariance requirements lead for elastic scattering to the relations

$$\left.\begin{array}{l} M_{ikq}(\mathbf{k}',\mathbf{k})=C_{kqi}(-\mathbf{k},-\mathbf{k}'), \\ N_{ikq}(\mathbf{k}',\mathbf{k})=P_{kqi}(-\mathbf{k},-\mathbf{k}'). \end{array}\right\} \quad (7.80)$$

We shall not discuss in detail the properties of the tensor C_{ikqp}. We note only that invariance under reflections leads to

$$\sigma_0 C_{nnnn}=\frac{1}{4}\,\mathrm{Sp}\,(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})\,M\,(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})\,M^+=\pm\,\mathrm{Sp}\,M^+M=\pm\,\sigma_0,$$

so that in the general case of reactions $\frac{1}{2}+\frac{1}{2}\to\frac{1}{2}+\frac{1}{2}$ with two parity values $I_i=\pm I_f$ we have

$$C_{nnnn}=\pm 1. \quad (7.81)$$

The requirement of invariance under reflections leads to the vanishing of the components C_{nnna}, C_{nabc} (a, b, c = l, m). The components C_{nnab} (a, b = l, m) coincide, as can be readily seen, with the components of the tensor C_{ik}. Indeed, examining simultaneously the general case of the reactions with both values $I_i = \pm I_f$, we have

$$\sigma_0 C_{nnab}=\frac{1}{4}\,\mathrm{Sp}\,(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})\,M\,(\sigma_1\mathbf{a})(\sigma_2\mathbf{b})\,M^+$$
$$=\pm\frac{1}{4}\,\mathrm{Sp}\,M\,(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})(\sigma_1\mathbf{a})(\sigma_2\mathbf{b})\,M^+=\pm\,\sigma_0 C_{[na],[bn]}$$

and

$$C_{nnab}=\pm C_{[na],[bn]}, \quad (7.82)$$

so that, for example,

$$C_{nnlm}=\pm C_{ml}.$$

Analogously we have

$$\begin{array}{lll} C_{nanb}=\pm D_{[an],[bn]}, & C_{nabn}=\pm K_{[an],[bn]}, & \\ C_{anbn}=\pm D_{[an],[bn]}, & C_{abnn}=\pm C_{[na],[bn]}, & \\ C_{llll}=\pm C_{mmmm}, & C_{mmll}=\pm C_{llmm}, & C_{mlml}=\pm C_{lmlm}, \\ C_{lmmm}=\mp C_{mlll}, & C_{mlmm}=\mp C_{lmll}, & C_{mllm}=\pm C_{lmml}, \\ C_{mmlm}=\mp C_{llml}, & C_{mmml}=\pm C_{lllm}. & \end{array}$$

The T-invariance requirement leads to the relation

$$C_{ikqp}(\mathbf{k}',\mathbf{k})=C_{qpik}(-\mathbf{k},-\mathbf{k}'). \quad (7.83)$$

Expressions for all tensors in terms of the coefficients of the amplitude (7.4) are given in [54].

9) The entire discussion above was based on the general assumption that none of the scalar functions in the amplitude (7.3) are small. The results of an investigation of nucleon-nucleon interaction in the energy region of several hundred MeV correspond to such a complicated picture. In the region of very high energies, when states with large values of orbital angular momenta participate effectively in the interaction, we can expect many simplifications in the general form of the amplitude. The latest investigations of p-p scattering in the region of very small scattering angles [61], in the energy range ~ 10 GeV, point to the need of investigating polarization phenomena even at such high energies.

Many simplifications can be made at low energies, when the interaction occurs essentially in S-states. The n-p scattering matrix has in this energy region the form

$$M=a_1\hat{T}+a_0\hat{S}=\alpha+\beta\,(\sigma_1\sigma_2), \quad (7.84)$$

where a_1 and a_0 —scattering lengths in the $^3S_1$ and $^1S_0$ states, respectively. The polarization P^0, of course, vanishes, and for other values it is easy to obtain

$$\left.\begin{array}{l} \sigma_0=\dfrac{\sigma^s+3\sigma^t}{4}=|\alpha|^2+3|\beta|^2,\quad \sigma^s=|\alpha-3\beta|^2,\quad \sigma^t=|\alpha+\beta|^2, \\ D_{ik}=\dfrac{|\alpha|^2-|\beta|^2}{|\alpha|^2+3|\beta|^2}\,\delta_{ik},\quad P_{ik}=\dfrac{2\,[\mathrm{Re}\,\alpha^*\beta-|\beta|^2]}{|\alpha|^2+3|\beta|^2}\,\delta_{ik}, \\ K_{ik}=\dfrac{2\,[\mathrm{Re}\,\alpha^*\beta+|\beta|^2]}{|\alpha|^2+3|\beta|^2}\,\delta_{ik}, \end{array}\right\} \quad (7.85)$$

where σ^s and σ^t —cross sections for scattering in the singlet and triplet states, respectively. As expected, all these tensors are proportional to δ_{ik}. The expression for the tensor P_{ik} can be reduced to the form

$$P_{ik}=\frac{\sigma^t-\sigma^s}{\sigma^s+3\sigma^t}\,\delta_{ik},$$

which is a particular case of the general relation (7.38). Inasmuch as in this case $D_{ll} = D_{mm}$, (7.17) goes over into two relations

$$A=-R', \quad A'=R.$$

An interesting application of the polarized proton target, for investigations in the region of rather low energies, was indicated by Yu. V. Taran and F. L. Shapiro [62]. In view of the large difference between σ^s and σ^t in n-p interactions, a neutron beam passing through a polarized target becomes strongly polarized. It is easy to show that the degree of polarization of a beam of neutrons passing through a polarized proton target of thickness d is

$$P_n=\tanh\frac{1}{4}\,[P_2 n\,(\sigma^s-\sigma^t)\,d],$$

where P_2 —polarization of the target and n —number of protons per cm^3. The use of polarized protons (and polarized targets) is very fruitful in investigations of nuclear levels and other problems.

For p-p scattering at low energies, the amplitude reduces to the singlet projection operator

$$M=\alpha\,[1-(\sigma_1\sigma_2)].$$

Thus, for p-p scattering at low energies, $D_{ik} = K_{ik} = 0$

and $P_{ik} = -1\delta_{ik}$. Deviations from these quantities are measures of higher states of the p-p system. Systematic polarization research at energies above 50 MeV is necessary to establish a unique set of phase shifts for the n-p system in this energy range. The presently available analysis results of the Yale [63] and Dubna [64] groups are particularly at variance in the energy dependence of the mixing coefficient of the states $^3S_1$ and $^3D_1$.

8. MESON-NUCLEON SCATTERING

In this section we shall consider scattering of pions and K mesons by nucleons. The general form of the scattering matrix can be obtained from the requirements of invariance under rotations and reflections (3.22) and (3.28). Invariance under time reversal does not impose in this case any additional limitations on the M matrix. As already noted (see Sec. 6), the scattering matrix of particles with zero spin or particles with spin $\frac{1}{2}$ is of the form

$$M = a + b\,(\boldsymbol{\sigma}\mathbf{n}). \tag{8.1}$$

The complete measurements include in this case a determination of the unpolarized cross section, the asymmetry in the scattering by a polarized target, and an investigation of the polarization of the recoil particles following scattering by unpolarized and polarized targets. Measurement of the asymmetry of elastic scattering by a polarized target is equivalent, owing to the polarization-asymmetry relation, to a determination of the polarization of the recoil particle in experiments with an unpolarized target. However, measurement of asymmetry with a polarized target makes it possible to determine the polarization of the recoil particles in an angle range such that its direct measurement in the double experiment is made difficult by the small analyzing ability of the analyzing target.

One of the first experiments with a polarized hydrogen target consisted of measuring the asymmetry of scattering of positive pions with energy 246 MeV by hydrogen [2]. The recoil proton polarization was measured in this case with a higher accuracy than in the double experiment, and at those angles for which the double experiment is very difficult to perform.

In the most extensively investigated energy region of the K-N system, the analysis presupposes that it is sufficient to take into account the interaction in the S state only. The cross section σ_0 for scattering by a polarized target is more sensitive to the contribution of the P states. It is precisely from this point of view that experiments with low-energy K mesons and a polarized target are of interest.

In the case of π(K)-nucleon scattering, a polarized proton target makes it possible to obtain quantities which cannot be determined without a polarized target. We have in mind here the depolarization tensor [9].

The depolarization tensor determines the polarization of the final nucleon following the scattering of mesons by a polarized target. If we denote by $\mathbf{P}'$ the polarization vector of the final nucleon and by $\mathbf{P}$ the polarization of the target, then

$$P'_i = \frac{\frac{1}{2}\operatorname{Sp}\sigma_i M\,[I + \boldsymbol{\sigma}\mathbf{P})]\,M^+}{\frac{1}{2}\operatorname{Sp} M\,[I + (\boldsymbol{\sigma}\mathbf{P})]\,M^+} = \frac{\sigma_0\,(P_i^0 + D_{ik}P_k)}{\sigma_0\,(1 + P_i P_i^0)}. \tag{8.2}$$

Here σ_0 and $\mathbf{P}^0$ are the differential cross section and the polarization of the recoil nucleon in the case when the meson is scattered by an unpolarized target, while the depolarization tensor is equal to

$$D_{ik} = \frac{1}{2\sigma_0}\operatorname{Sp}\sigma_i M \sigma_k M^+. \tag{8.3}$$

Let $\mathbf{k}_{lab} = \mathbf{k}$ and $\mathbf{k}'_{lab}$ be unit vectors in the directions of the momenta of the initial meson and recoil nucleon in the laboratory system. We resolve the polarization vector $\mathbf{P}$ of the target protons in the orthonormal system

$$\mathbf{n}_{lab} = \frac{[\mathbf{k}\mathbf{k}'_{lab}]}{|[\mathbf{k}\mathbf{k}'_{lab}]|} = -\mathbf{n}, \quad \mathbf{k} \text{ and } [\mathbf{n}_{lab}\mathbf{k}] = \mathbf{s},$$

and the recoil nucleon polarization vector $\mathbf{P}'$ in the system of vectors $\mathbf{n}_{lab}$, $\mathbf{k}'_{lab}$, and $\mathbf{n}_{lab} \times \mathbf{k}'_{lab} = \mathbf{s}'$. We obtain for the components of the vector $\mathbf{P}'$ the following expressions:

$$\left.\begin{aligned} \sigma\,(\mathbf{P}'\mathbf{n}_{lab}) &= \sigma_0\,(-P^0 + D\,(\mathbf{P}\mathbf{n}_{lab})),\\ \sigma\,(\mathbf{P}'\mathbf{k}'_{lab}) &= \sigma_0\,(A'\mathbf{k} + R'\,[\mathbf{n}_{lab}\mathbf{k}])\,\mathbf{P},\\ \sigma\,(\mathbf{P}'\,[\mathbf{n}_{lab}\mathbf{k}'_{lab}]) &= \sigma_0\,(A\mathbf{k} + R\,[\mathbf{n}_{lab}\mathbf{k}])\,\mathbf{P}, \end{aligned}\right\} \tag{8.4}$$

where

$$\left.\begin{aligned} \sigma &= \sigma_0\,(1 + P_i P_i^0),\\ \sigma_0 D &= \frac{1}{2}\operatorname{Sp}(\boldsymbol{\sigma}\mathbf{n}_{lab})\,M\,(\boldsymbol{\sigma}\mathbf{n}_{lab})\,M^+,\\ \sigma_0 A' &= \frac{1}{2}\operatorname{Sp}(\boldsymbol{\sigma}\mathbf{k}'_{lab})\,M\,(\boldsymbol{\sigma}\mathbf{k}_{lab})\,M^+,\\ \sigma_0 R' &= \frac{1}{2}\operatorname{Sp}(\boldsymbol{\sigma}\mathbf{k}'_{lab})\,M\,(\boldsymbol{\sigma}\,[\mathbf{n}_{lab}\mathbf{k}])\,M^+,\\ \sigma_0 A &= \frac{1}{2}\operatorname{Sp}(\boldsymbol{\sigma}\,[\mathbf{n}_{lab}\mathbf{k}'_{lab}])\,M\,(\boldsymbol{\sigma}\mathbf{k})\,M^+,\\ \sigma_0 R &= \frac{1}{2}\operatorname{Sp}(\boldsymbol{\sigma}\,[\mathbf{n}_{lab}\mathbf{k}'_{lab}])\,M\,(\boldsymbol{\sigma}\,[\mathbf{n}_{lab}\mathbf{k}])\,M^+. \end{aligned}\right\} \tag{8.5}$$

We note that the choice of the system of vectors in which $\mathbf{P}'$ is resolved follows naturally from the procedure of measuring the recoil particle polarization.

The essential feature of the processes considered here is that only two parameters in (8.4) are independent. We shall show that

$$D = 1, \quad A' = R, \quad A = -R'. \tag{8.6}$$

To prove this we use the relation

$$(\boldsymbol{\sigma}\mathbf{n})\,M\,(\boldsymbol{\sigma}\mathbf{n}) = M, \tag{8.7}$$

which follows from the invariance of the S matrix against reflections in the scattering plane [see (6.9)]. We note that the (8.7) becomes obvious if we make use of the explicit expression for the M matrix (8.1). Using formulas (8.5) and relation (8.7) we obtain

$$\left.\begin{aligned}\sigma_0 D &= \frac{1}{2}\,\mathrm{Sp}\, MM^+ = \sigma_0,\\ \sigma_0 A' &= \frac{1}{2}\,\mathrm{Sp}\,(\boldsymbol{\sigma}\mathbf{k}'_{\rm lab})\,(\boldsymbol{\sigma}\mathbf{n})\,M\,(\boldsymbol{\sigma}\mathbf{n})\,(\boldsymbol{\sigma}\mathbf{k})\,M^+\\ &= \frac{1}{2}\,\mathrm{Sp}\,(\boldsymbol{\sigma}\,[\mathbf{n}_{\rm lab}\mathbf{k}'_{\rm lab}])\,M\,(\boldsymbol{\sigma}\,[\mathbf{n}_{\rm lab}\mathbf{k}_{\rm lab}])\,M^+ = \sigma_0 R,\\ \sigma_0 R' &= \frac{1}{2}\,\mathrm{Sp}\,(\boldsymbol{\sigma}\mathbf{k}'_{\rm lab})\,(\boldsymbol{\sigma}\mathbf{n})\,M\,(\boldsymbol{\sigma}\mathbf{n})\,(\boldsymbol{\sigma}\,[\mathbf{n}_{\rm lab}\mathbf{k}])\,M^+\\ &= \frac{1}{2}\,\mathrm{Sp}\,(\boldsymbol{\sigma}\,[\mathbf{n}_{\rm lab}\mathbf{k}'_{\rm lab}])\,M\,(\boldsymbol{\sigma}\mathbf{k})\,M^+ = -\sigma_0 A.\end{aligned}\right\}\qquad(8.8)$$

Thus, in order to determine all the components of the polarization vector $\mathbf{P}'$ it is necessary to measure only two quantities. Naturally, it is simpler to measure the polarization component perpendicular to $\mathbf{k}'_{\rm lab}$, that is, the parameters A and R. To determine A(R) it is necessary to direct the target polarization along $\mathbf{k}$ (or $\mathbf{n}_{\rm lab}\times\mathbf{k}$) and to measure the recoil proton scattering asymmetry in a plane perpendicular to the plane of the initial scattering. The up-down asymmetry in the second scattering is obviously

$$e_R = \frac{\sigma\,(\mathrm{down}) - \sigma\,(\ \mathrm{up}\ )}{\sigma\,(\mathrm{down}) + \sigma\,(\ \mathrm{up}\ )} = P_c R P, \qquad (8.9)$$

if the polarization of the target is directed along $\mathbf{n}_{\rm lab}\times\mathbf{k}$, and

$$e_A = \frac{\sigma'\,(\mathrm{down}) - \sigma'\,(\ \mathrm{up}\ )}{\sigma'\,(\mathrm{down}) + \sigma'\,(\ \mathrm{up}\ )} = P_c A P, \qquad (8.10)$$

if the target polarization is $\mathbf{P} = P\mathbf{k}$. Here P_c —analyzing ability of the analyzer target, and the "up" direction is determined by the direction of the vector $\mathbf{k}\times\mathbf{k}'_{\rm lab}$.

Measurement of these parameters would make it possible to eliminate the ambiguities of the π-N scattering phase shift analysis, which still remain in the energy region from 200 to 400 MeV.

It must be emphasized that a study of boson-nucleon scattering with the aid of a polarized target permits a unique determination of the absolute values and the relative phase of the functions a and b, that is, to reconstruct the scattering matrix, apart from the common phase. Indeed, it is easy to obtain the following expressions for the observed quantities:

$$\left.\begin{aligned}\sigma_0 &= |a|^2 + |b|^2,\\ \sigma_0 P^0 &= 2\,\mathrm{Re}\,(ab^*),\\ \sigma_0 R &= (|a|^2 - |b|^2)\cos\theta_{\rm lab} - 2\,\mathrm{Im}\,(ab^*)\sin\theta_{\rm lab},\\ \sigma_0 A &= -(|a|^2 - |b|^2)\sin\theta_{\rm lab} - 2\,\mathrm{Im}\,(ab^*)\cos\theta_{\rm lab}\end{aligned}\right\}\qquad(8.11)$$

where $\theta_{\rm lab}$ —angle between the incident meson and the recoil nucleon in the laboratory system. From these relations we get

$$\left.\begin{aligned}|a|^2 &= \frac{1}{2}\,\sigma_0\,(1 + R\cos\theta_{\rm lab} - A\sin\theta_{\rm lab}),\\ |b|^2 &= \frac{1}{2}\,\sigma_0\,(1 - R\cos\theta_{\rm lab} + A\sin\theta_{\rm lab}),\\ ab^* &= \frac{1}{2}\,\sigma_0\,(P^0 - iR\sin\theta_{\rm lab} - iA\cos\theta_{\rm lab}).\end{aligned}\right\}\qquad(8.12)$$

The possibility of direct determination of the scattering matrix from the experimentally measured quantities is particularly essential at high meson energies, where a phase shift analysis is greatly hindered by a large number of states and by the need of taking into account the influence of inelastic processes.

To carry out a phase shift analysis of the π-N and K($\bar{\rm K}$)-N scattering data (or for a direct determination of the amplitude), further simplifications are obtained by using the requirements of isotopic invariance of strong interactions. Thus, the scattering of positive pions by protons (or negative pions by neutrons) corresponds to the V_3 amplitude of π-N scattering in a state with a system isospin $T = 3/2$. The amplitudes of the different processes initiated by the negative pions on protons are expressed in terms of V_3 and the scattering amplitude V_1 in states with system isospin T $= 1/2$ with the aid of the relations

$$M\,(\pi^- p \to \pi^- p) = \frac{1}{3}\,(V_3 + 2V_1), \quad M\,(\pi^- p \to \pi^0 n) = \frac{\sqrt{2}}{3}\,(V_3 - V_1).$$

For the K-N system we have two values of the system isospin, 1 and 0, with scattering amplitudes W_1 and W_0 respectively. Thus,

$$\begin{aligned}M\,(K^+ p \to K^+ p) &= W_1,\\ M\,(K^0 p \to K^0 p) &= \frac{1}{2}\,(W_1 + W_0),\\ M\,(K^0 p \to K^+ n) &= \frac{1}{2}\,(W_1 - W_0).\end{aligned}$$

Analogously, introducing the amplitudes z_1 and z_0, we have for the $\bar{\rm K}$-N system

$$\begin{aligned}M\,(K^- p \to K^- p) &= \frac{1}{2}\,(z_1 + z_0),\\ M\,(K^- p \to \bar{K}^0 n) &= \frac{1}{2}\,(z_1 - z_0),\\ M\,(\bar{K}^0 p \to \bar{K}^0 p) &= z_1.\end{aligned}$$

Each of the amplitudes introduced above has the structure (8.1).

We note in conclusion that the results obtained admit of a simple generalization. All the relations given above were based only on invariance under spatial rotations and reflections. This means that all the foregoing pertains also to inelastic reactions of the type $0 + 1/2 \to 0 + 1/2$, provided the total intrinsic parity of the initial particles I_i is equal to the intrinsic parity I_f of the final particles.

In the case of reactions of the same type with $I_i = -I_f$, it follows from the invariance under reflections in the plane of the reaction that (see Sec. 6)

$$(\boldsymbol{\sigma}\mathbf{n})\,M\,(\boldsymbol{\sigma}\mathbf{n}) = -M. \qquad (8.13)$$

Compared with (8.6) this leads to a reversal of sign in the right side

$$D = -1, \quad A' = -R, \quad R' = A. \qquad (8.6')$$

For reactions of the type $0 + 1/2 \to 0 + 1/2$, the general expression for the tensor D_{ik} (7.15) reduces by the same token to

$$D_{ik} = \pm\, n_i n_k + D_{ll}\,(l_i l_k \pm m_i m_k) + D_{ml}\,(m_i l_k \mp l_i m_k), \qquad (8.14)$$

where the upper and lower signs correspond to the two possible values $I_i = \pm I_f$.

Like the amplitude (8.1), expression (8.14) automatically satisfies the requirements of T-invariance for elastic scattering processes [see (7.19)]. An experimental verification of the relations obtained above, which are based on the requirements of invariance under reflections and rotations, allows us to proceed to an experimental investigation of parity conservation in strong interactions, including searches for the influence of parity nonconservation in weak interactions.

9. PHOTOPRODUCTION OF PIONS AND K MESONS

A study of the photoproduction of bosons from fermions is one of the most important sources of information on the produced particles. Along with the scattering of pions by nucleons, photoproduction of pions from nucleons was one of the main sources of information on pion-nucleon interaction at the very start of development of pion physics. The increased photon energy attainable with accelerators makes photoproduction processes an important tool in the investigation of higher pion-nucleon resonances. A study of the photoproduction far from threshold makes it possible to resolve with high accuracy many pion-physics problems that are still unclear, and carry out a reliable analysis of the pion photoproduction process itself. In addition to being of independent interest, detailed information on the mechanism of photoproduction of pions in a wide range of energies is the basis for the theory of the proton Compton effect.

Until recently photoproduction experiments were limited to measurements of the cross section for the production by unpolarized gamma quanta from an unpolarized target, and measurements of the polarization of the recoil nucleons from an initially unpolarized beam and target. Measurements were also made of the differential cross section for the production of pions by polarized gamma quanta from an unpolarized target. Progress in the production of polarized gamma beams and polarized proton targets allows us to undertake more complicated experiments.

We shall consider here the photoproduction of $\pi(K)$ mesons from a polarized proton target. We confine ourselves to examination of unpolarized gamma quanta. The general expression for the reaction matrix can be obtained from the requirement of invariance under rotations and reflections (3.22) and (3.28). It can be shown that the reaction matrix is of the form[66]

$$M = A(\boldsymbol{\varepsilon}\mathbf{q}) + iB(\boldsymbol{\sigma}\mathbf{q})(\boldsymbol{\varepsilon}\mathbf{n}) + iC(\boldsymbol{\sigma}\mathbf{n})(\boldsymbol{\varepsilon}\mathbf{q}) + iD(\boldsymbol{\sigma}\mathbf{s})(\boldsymbol{\varepsilon}\mathbf{n}) \tag{9.1}$$

for the production of a scalar spinless boson from nucleons, and

$$M = a(\boldsymbol{\varepsilon}\mathbf{n}) + ib(\boldsymbol{\sigma}\mathbf{q})(\boldsymbol{\varepsilon}\mathbf{q}) + ic(\boldsymbol{\sigma}\mathbf{n})(\boldsymbol{\varepsilon}\mathbf{n}) + id(\boldsymbol{\sigma}\mathbf{s})(\boldsymbol{\varepsilon}\mathbf{q}) \tag{9.2}$$

for the photoproduction of a pseudoscalar particle.

In these formulas $\mathbf{q}$ and $\mathbf{k}$ are unit vectors in the direction of the boson and photon momenta in the c.m.s., respectively, $\mathbf{n} = \mathbf{k} \times \mathbf{q}/|\mathbf{k} \times \mathbf{q}|$, $\mathbf{s} = \mathbf{q} \times \mathbf{n}$, and $\boldsymbol{\epsilon}$—photon polarization vector satisfying the condition $(\boldsymbol{\epsilon} \cdot \mathbf{k}) = 0$.

If the target is polarized, then it follows from (4.26) and (4.21) that the cross section of the process is of the form

$$\sigma = \sigma_0 (1 + P_i A_i). \tag{9.3}$$

Here σ_0—cross section for the photoproduction from an unpolarized proton target, $\mathbf{P}$—proton and target polarization, and

$$\sigma_0 \mathbf{A} = \frac{1}{2} \operatorname{Sp} M \boldsymbol{\sigma} M^+.$$

From the general theorem proved in Sec. 5 [see (5.9)] it follows that $\mathbf{A}$ is the nucleon polarization produced in the inverse process of radiative capture of a boson by unpolarized baryons. In the general case this quantity does not coincide with the nucleon polarization resulting from photoproduction from unpolarized nucleons. Thus, measurement of these quantities gives independent information on the process. This circumstance is common to all inelastic reactions.

The polarization of the final baryon in photoproduction from a polarized target is obviously

$$\sigma P_i' = \sigma_0 (P_i^0 + D_{ik} P_k), \tag{9.4}$$

where

$$P_i^0 = \frac{1}{2\sigma_0} \operatorname{Sp} M M^+ \sigma_i$$

is the polarization of the final quantity for an unpolarized target, and the depolarization tensor D_{ik} is

$$D_{ik} = \frac{1}{2\sigma_0} \operatorname{Sp} \sigma_i M \sigma_k M^+. \tag{9.5}$$

We note that, just as in the case of scattering of pions by nucleons, the depolarization tensor can be measured during the photoproduction process only in the presence of a polarized target.

From considerations of invariance under spatial rotations and reflections it is seen that in the case of unpolarized gamma quanta five components of the tensor D_{ik} differ from zero. Unlike the process of the type $0 + \frac{1}{2} \to 0 + \frac{1}{2}$ and inelastic scattering of particles with spin $\frac{1}{2}$, all five components of the tensor D_{ik} are independent in the case of the photoproduction process (and in the case of other inelastic processes with two particles in the final state).

With the aid of (9.1) and (9.2) we can easily find expressions for the observed quantities. We present only the results of the calculations. (The photons are not polarized.)

1. Scalar boson:

$$\left.\begin{aligned}
2\sigma_0 &= (|A|^2 + |C|^2)\sin^2\theta + (|B|^2 + |D|^2),\\
\sigma_0(\mathbf{P}^0\mathbf{n}) &= -\operatorname{Im}(A^*C)\sin^2\theta - \operatorname{Im}(B^*D),\\
\sigma_0(\mathbf{A}\mathbf{n}) &= -\operatorname{Im}(A^*C)\sin^2\theta + \operatorname{Im}(B^*D),\\
2\sigma_0 D_{nn} &= (|A|^2 + |C|^2)\sin^2\theta - (|B|^2 + |D|^2),\\
2\sigma_0 D_{qq} &= (|A|^2 - |C|^2)\sin^2\theta + (|B|^2 - |D|^2),\\
2\sigma_0 D_{ss} &= (|A|^2 - |C|^2)\sin^2\theta + (|D|^2 - |B|^2),\\
\sigma_0 D_{sq} &= \operatorname{Re} A^*C \sin^2\theta + \operatorname{Re} B^*D,\\
\sigma_0 D_{qs} &= -\operatorname{Re} A^*C \sin^2\theta + \operatorname{Re} B^*D.
\end{aligned}\right\} \tag{9.6}$$

2. Pseudoscalar boson:

$$\left.\begin{aligned}2\sigma_0 &= (|a|^2+|c|^2)+(|b|^2+|d|^2)\sin^2\theta,\\ \sigma_0(\mathbf{P}^0\mathbf{n}) &= -\operatorname{Im} a^*c - \operatorname{Im} b^*d\sin^2\theta,\\ \sigma_0(\mathbf{An}) &= -\operatorname{Im} a^*c + \operatorname{Im} b^*d\sin^2\theta,\\ 2\sigma_0 D_{nn} &= |a|^2+|c|^2-(|b|^2+|d|^2)\sin^2\theta,\\ 2\sigma_0 D_{qq} &= |a|^2-|c|^2+(|b|^2-|d|^2\sin^2\theta,\\ 2\sigma_0 D_{ss} &= |a|^2-|c|^2+(|d|^2-|b|^2)\sin^2\theta,\\ \sigma_0 D_{sq} &= \operatorname{Re} a^*c + \operatorname{Re}(b^*d)\sin^2\theta,\\ \sigma_0 D_{qs} &= -\operatorname{Re} a^*c + \operatorname{Re}(b^*d)\sin^2\theta.\end{aligned}\right\} \quad (9.7)$$

Unlike processes of the type $0 + 1/2 \rightarrow 0 + 1/2$, the parameter D_{nn} is not equal to ± 1, for in the case of photoproduction processes the operator $(\boldsymbol{\sigma}\cdot\mathbf{n})$ is not the operator of reflection in the plane of reaction.

To determine the "plane" components of the tensor D_{ik} it is necessary to carry out four independent measurements.

Since the matrix of the photoproduction process is characterized by the four complex functions of the angle and of the energy, it is necessary to carry out at least seven independent experiments in order to reconstruct this matrix at a specified angle and for a fixed energy.

For unambiguous determination of the matrix of the process, as shown with meson-nucleon and nucleon-nucleon scattering as an example, it is necessary to have a larger number of experiments. This means, in particular, that the solution of the problem of reconstructing the photoproduction matrix calls for the performance of experiments with both polarized targets and polarized gamma quanta.

In conclusion we note that by virtue of the invariance under time reversal, the photoproduction of pions from nucleons can be investigated also by studying the inverse process, namely radiative capture of pions by nucleons [67].

Thus, for example, a study of the capture of a negative pion by protons

$$\pi^- + p \rightarrow n + \gamma \quad (9.8)$$

yields information on the production of pions from free neutrons by monochromatic gamma quanta. An investigation of the polarization of neutrons captured by an unpolarized target is equivalent to measurement of asymmetry with a polarized neutron target. The depolarization tensors $D_{ik}^{\gamma N}$ and $D_{ik}^{\pi N}$ in photoproduction and radiative capture respectively are connected by relation

$$D_{ik}^{\gamma N}(\mathbf{k},\mathbf{q}) = D_{ki}^{\pi N}(-\mathbf{q},-\mathbf{k}), \quad (9.9)$$

which is a generalization of the analogous relation for elastic scattering.

With increasing intensity of the pion beams, these relations make it possible to carry out detailed investigations that are equivalent to a study of photoproduction from a neutron target.

10. SCATTERING OF GAMMA QUANTA AND ELECTRONS BY NUCLEONS

As is well known, data on scattering of high-energy electrons by protons, when analyzed in the lowest order in the electromagnetic interaction constant, yield information on the electromagnetic form factors of strongly-interacting particles. The results of an investigation of γ-p scattering make possible an analysis based on the dispersion relations and on information concerning the mechanism of pion photoproduction.

We shall discuss here briefly the possibilities offered by a polarized proton target in such investigations. We start with the proton Compton effect. Most of the hitherto performed experiments on this process consisted of measurements of the scattering cross sections of unpolarized gamma quanta, with energies approximately up to 300 MeV, by protons [68]. Data pertaining to the energy region 800–900 MeV have been reported only recently.

An analysis of γ-p scattering at energies below 300 MeV leads to perfectly defined predictions with respect to the expected polarization of the recoil protons from unpolarized and polarized targets.

The amplitude of the proton Compton effect can be represented in the form [69]

$$\begin{aligned}M &= R_1(\mathbf{ee}') + R_2(\boldsymbol{\lambda}'\boldsymbol{\lambda}) + iR_3(\boldsymbol{\sigma}[\mathbf{e}'\mathbf{e}]) + iR_4(\boldsymbol{\sigma}[\boldsymbol{\lambda}'\boldsymbol{\lambda}])\\ &\quad + iR_5[(\boldsymbol{\sigma}\mathbf{k})(\boldsymbol{\lambda}'\mathbf{e}) - (\boldsymbol{\sigma}\mathbf{k}')(\boldsymbol{\lambda}\mathbf{e}')] + iR_6[\boldsymbol{\sigma}\mathbf{k}')(\boldsymbol{\lambda}'\mathbf{e}) - (\boldsymbol{\sigma}\mathbf{k})(\boldsymbol{\lambda}\mathbf{e}')],\end{aligned} \quad (10.1)$$

where $\boldsymbol{\lambda} = \mathbf{k}\times\mathbf{e}$, $\boldsymbol{\lambda}' = \mathbf{k}'\times\mathbf{e}'$, with $\mathbf{e}$, $\mathbf{k}$ and $\mathbf{e}'$, $\mathbf{k}'$ — unit vectors of polarization and momentum of the photon before and after scattering, respectively. For the cross section for the scattering of unpolarized gamma quanta by an unpolarized target we have [70]

$$\begin{aligned}4\sigma_0 &= |R_1+R_2|^2(1+\cos\theta)^2\\ &\quad + |R_1-R_2|^2(1-\cos\theta)^2 + |R_3+R_4|^2(3+2\cos\theta - \cos^2\theta)\\ &\quad + |R_3-R_4|^2(3-2\cos\theta-\cos^2\theta) + 2|R_5+R_6|^2(1+\cos\theta)^3\\ &\quad + 2|R_5-R_6|^2(1-\cos\theta)^3\\ &\quad + 4\operatorname{Re}(R_3+R_4)^*(R_5+R_6)(1+\cos\theta)^2\\ &\quad - 4\operatorname{Re}(R_3-R_4)^*(R_5-R_6)(1-\cos\theta)^2.\end{aligned} \quad (10.2)$$

The expression for the polarization of the recoil protons following interaction between initially unpolarized gamma quanta and nucleons

$$\begin{aligned}2\sigma_0\mathbf{P}^0 &= \mathbf{n}\sin\theta\operatorname{Im}[(R_3+R_4)(R_1+R_2)^*(1+\cos\theta)\\ &\quad - (R_3-R_4)(R_1-R_2)^*(1-\cos\theta)]\end{aligned} \quad (10.3)$$

coincides, of course, with the expression for the asymmetry of the cross section for the scattering of unpolarized photons by polarized protons.

Without the use of the polarized-target technique, measurements of proton polarization are difficult not only because of the small cross section but also because of the low nucleon recoil energy. The use of polarized targets eliminates this difficulty.

Below the pion production threshold, all the amplitudes R_i are real in the e^2 approximation, and the polarization P^0 vanishes. An experimental check of this statement is a check on the main assumptions of the analysis of the proton Compton effect. Above the pion production threshold, the polarization P^0 increases rapidly. Numerical estimates for the polarization in the energy region up to 300 MeV were obtained in [70-72] on the basis of an analysis based on the dispersion relations. It follows from these estimates that the maximum value of the polarization P^0 reaches about 30% near 250 MeV. A check on this prediction can be very important for the entire analysis. Measurement of P^0 at higher energies can serve as a test on the assumption of the diffraction character of γ-p scattering[73]. P^0 vanishes if the real parts of the amplitudes R_i are neglected compared with the imaginary parts.

In analogy with elastic meson-nucleon scattering and pion photoproduction, only the use of a polarized target makes it possible in this case to measure the components of the depolarization tensor D_{ik}. With the aid of (10.1) it is easy to obtain for the nonvanishing components of the tensor D_{ik} the following expressions:

$$\left.\begin{aligned}4\sigma_0 D_{nn} &= [|R_1+R_2|^2-|R_3+R_4|^2-4\,\mathrm{Re}\,(R_3+R_4)^*(R_5+R_6)]\,(1+\cos\theta)^2\\&+[|R_1-R_2|^2-|R_3-R_4|^2+4\,\mathrm{Re}\,(R_3-R_4)^*(R_5-R_6)]\,(1-\cos\theta)^2\\&-2\,|R_5-R_6|^2\,(1-\cos\theta)^3-2\,|R_5+R_6|^2\,(1+\cos\theta)^3,\\2\sigma_0(D_{ll}-D_{nn}) &= [2\,|R_3+R_4+(R_5+R_6)\,(1+\cos\theta)\,|^2\\&-(|R_3+R_4|^2+|R_3-R_4|^2)\,(1-\cos\theta)]\,(1+\cos\theta),\\2\sigma_0(D_{mm}-D_{nn}) &= [2\,|R_3-R_4-(R_5-R_6)\,(1-\cos\theta)\,|^2\\&-(|R_3+R_4|^2+|R_3-R_4|^2)\,(1+\cos\theta)]\,(1-\cos\theta),\\2\sigma_0 D_{lm} &= \mathrm{Re}\,[(R_1+R_2)^*(R_3+R_4)\,(1+\cos\theta)\\&+(R_1-R_2)^*(R_3-R_4)\,(1-\cos\theta)]\sin\theta.\end{aligned}\right\}\qquad(10.4)$$

Expressions for the polarization tensors in terms of the invariant amplitudes were obtained in [74]. In the low energy limit

$$D_{ik}\to 1\delta_{ik}.$$

With increasing energy, D_{nn}, D_{ll}, and D_{mm} decrease in magnitude. Estimates based on the results of [72] show that the parameter $D_{nn}(90°)$, which is approximately equal to 0.60 at a gamma-quantum energy 75 MeV, reverses sign somewhat below threshold, and reaches values near −0.30 in the 150—225 MeV energy region. In the gamma-quantum scattering-angle region near 180° the value of D_{nn} decreases with decreasing energy from an initial value equal to unity, reverses sign below the pion production threshold, and reaches values near −0.8 close to 200 MeV. The energy dependence of all the components of the tensor D_{ik}, especially in the region of small scattering angles, is characterized by a noticeable near-threshold effect.

Experimental investigations of the depolarization tensor components are essential for a more thorough understanding of the mechanism of the proton Compton effect. Measurement of the components D_{ik} in the region of large gamma-quantum energies is a good method for investigating nucleon isobars. If the difficulties with the small recoil particle energies are overcome, it will be possible to measure the components D_{ik} appreciably below threshold and to obtain additional information on the polarizability of the nucleons.

Investigations of the polarization effects in the scattering of gamma quanta by protons would make possible a complete phenomenological analysis of this process.

A similar situation obtains for the scattering of high-energy electrons by nucleons. The presently available experimental data can be reconciled within the framework of the single-photon approximation. Further increases in the particle energy and in the momentum transfer, made feasible by the new accelerators, will allow us to make further progress in the study of the nucleon structure. An investigation of the polarization effects in e-p scattering, by affording the possibility of direct verification of the assumed approximation, will make it possible, on the other hand, to increase the accuracy of the particle form-factor data.

In view of the absence of symmetry between the electron (particle 1) and the nucleon (particle 2), singlet-triplet transitions are allowed in the case of electron-nucleon scattering. In the e^2 approximation, an appreciable simplification is possible. Thus, the proton and electron polarizations $\mathbf{P}_2^0$ and $\mathbf{P}_1^0$ vanish. To describe the change in polarization after collision it is necessary to introduce the tensors $D_{ik}^{(1)}$, $D_{ik}^{(2)}$, $K_{ik}^{(1)}$, and $K_{ik}^{(2)}$, defined in (7.60)—(7.63). The tensor D_{ik} determines the polarization $\mathbf{P}_1'$ of the scattered electrons when a polarized electron beam (polarization $\mathbf{P}_1$) is scattered by an unpolarized target. The tensor D_{ik}^2 determines the polarization of the recoil nucleons $\mathbf{P}_2'$ after scattering of unpolarized electrons by a polarized proton target (polarization $\mathbf{P}_2$). The tensor $K_{ik}^{(1)}$ determines the polarization $\mathbf{P}_2'$ of the recoil nucleon after scattering of a polarized beam of electrons (with polarization $\mathbf{P}_1$) by an unpolarized target. The tensor $K_{ik}^{(2)}$ determines the polarization P_1' of the scattered electrons after collision between unpolarized electrons with a polarized proton target. By virtue of (7.67) and (7.68) we have $D_{nn}^{(1)} = D_{nn}^{(2)}$ and $K_{nn}^{(1)} = K_{nn}^{(2)}$.

Since P_2^0 vanishes in the e^2 approximation, measurement of the left-right asymmetry of recoil-proton scattering in the case of a proton target polarized normally to the scattering plane yields directly the value of D_{nn}. In the more general case (see Sec. 8) this asymmetry is given by an expression proportional to

$$P_2^0 + D_{nn}P_2.$$

The general structure of the tensors D_{ik} and K_{ik} is of the form

$$D_{ik}(\mathbf{k}', \mathbf{k}) = D_{nn}n_in_k + D_{ll}l_il_k + D_{mm}m_im_k + D_{lm}l_im_k + D_{ml}m_il_k. \tag{10.5}$$

The T-invariance requirements lead (in analogy with Sec. 7) to the relations

$$\left.\begin{aligned} D_{ik}^{(1)}(\mathbf{k}', \mathbf{k}) &= D_{ki}^{(2)}(-\mathbf{k}, -\mathbf{k}'), \\ K_{ik}^{(1)}(\mathbf{k}', \mathbf{k}) &= K_{ki}^{(2)}(-\mathbf{k}, -\mathbf{k}'), \end{aligned}\right\} \tag{10.6}$$

from which it follows that

$$\left.\begin{aligned} D_{lm}^{(1)(2)} = -D_{ml}^{(1)(2)}, \quad K_{ll}^{(1)} = K_{ll}^{(2)}, \quad K_{mm}^{(1)} = K_{mm}^{(2)}, \\ K_{lm}^{(1)} = -K_{ml}^{(2)}, \quad K_{lm}^{(2)} = -K_{ml}^{(1)}. \end{aligned}\right\} \tag{10.7}$$

Detailed calculations of the polarization effects in e-p scattering in the e^2 approximation were made in [75]. It follows from the results of this work that $K_{nn}^{(1)} = K_{nn}^{(2)} = 0$, and the parameter D_{nn} is equal to

$$D_{nn} = 1 - \frac{q^2}{2M^2}(1+\mu)^2 \operatorname{tg}^2\frac{\theta}{2}\left[1 + \frac{\mu^2q^2}{4M^2} + \frac{q^2}{2M^2}(1+\mu)^2\operatorname{tg}^2\frac{\theta}{2}\right]^{-1}, \tag{10.8}$$

where $\mu = F_2/F_1$, F_1 and F_2 are the Dirac and Pauli form factors of the nucleon, respectively, and the remaining notation is standard. Expressions were obtained in [73] for the other components of the tensors D_{ik} and K_{ik}, too.

We now proceed to consider the scattering of polarized electrons by polarized protons. In view of the vanishing of P_1^0 and P_2^0, the general expression (7.26) for the cross sections of electrons polarized orthogonally to the scattering plane ($\mathbf{P}_1 = P_1\mathbf{n}$) scattered by a proton target with polarization $\mathbf{P}_2 = P_2\mathbf{n}$ turns into

$$\sigma_{nn} = \sigma_0(1 + C_{nn}P_1P_2). \tag{10.9}$$

Measurement of the scattering cross sections for two target polarization directions yields C_{nn} directly for arbitrary scattering angles. In view of the fact that the ultrarelativistic electrons are longitudinally polarized, the quantity C_{nn} is of the order of the ratio of the masses of the colliding particles, so that the difference between σ_{nn} and σ_0 is, of course, more noticeable for scattering of muons by protons.

The scattering of polarized electrons by polarized protons in the e^2 approximation was considered theoretically in [76,77]. In view of the longitudinal nature of the high-energy electrons, especially noticeable effects are obtained by a combination of a longitudinally-polarized beam and a polarized target with orientation in the scattering plane (along the direction of the vectors $\mathbf{k}$ and $\mathbf{n}\times\mathbf{k}$). It is shown in [76] that the cross section for the scattering of a polarized beam of electrons by a polarized proton target has in the laboratory frame the following form:

$$\sigma_{p_1p_2} = \sigma_0[1 + (\mathbf{P}_1\mathbf{k})(\mathbf{P}_2\mathbf{k})M_{11} + (\mathbf{P}_1\mathbf{k})(\mathbf{P}_2[\mathbf{nk}])M_{13}], \tag{10.10}$$

where

$$\left.\begin{aligned} &\sigma_0 = \left(\frac{e^2}{4\pi}\right)^2 \frac{\left[1+\eta\mu^2+2\eta(1+\mu)^2\operatorname{tg}^2\frac{\theta}{2}\right]|F_1|^2\cos^2\frac{\theta}{2}}{4M^2\xi^2\left(1+2\xi\sin^2\frac{\theta}{2}\right)\sin^4\frac{\theta}{2}}, \\ &M_{11} = [\eta(1-\mu+\xi^{-1})-\xi]\,\varrho\operatorname{tg}\frac{\theta}{2}, \quad M_{13} = \varrho\eta(\xi^{-1}-\mu), \\ &\varrho = \frac{2(1+\mu)\operatorname{tg}\frac{\theta}{2}}{1+\eta\mu^2+2\eta(1+\mu)^2\operatorname{tg}^2\frac{\theta}{2}}, \quad \xi = \frac{\varepsilon_1}{M}, \quad \eta = \frac{q^2}{4M^2}. \end{aligned}\right\} \tag{10.10'}$$

Measurement of the cross sections for e-p scattering (in the region of applicability of the e^2 approximation) with polarized particles also makes it possible to increase the accuracy of the data on form factors (especially magnetic) and check the correctness of the approximation.

Experiments with muons increase the accuracy of comparison of the electromagnetic properties of leptons.

Measurement of the cross sections of π-p and N-N scattering by a polarized target in the region of extremely high energies is a most essential experiment for a verification of hypotheses on the behavior of the scattering matrix. Without the use of polarized targets, such research is extremely difficult.

By way of an example we can point to the possibility of clarifying the character of diffraction scattering by means of [78,79]. Diffraction with imaginary phase shifts, without wave refraction, is characterized by the fact that in the π-N scattering amplitude (8.1) the quantity $a(\theta)$ is imaginary and $b(\theta)$ is real. For N-N scattering, in the same approximation, all the amplitudes a, b, e, and f in (7.3) are imaginary, while c is real. Here $P^0 = 0$, and the cross section for the scattering of an unpolarized beam by a polarized target does not differ from the cross section for the scattering of unpolarized particles. The polarization P^0 vanishes only if the asymptotic γ_5 invariance is valid[80]. Definite predictions were obtained for the polarization effects within the framework of the Regge-pole theory[81]. In the asymptotic energy region, where the Pomeranchuk theorem is valid[82], relations should exist between the particle polarizations in crossing processes[83,84]. Thus, the polarization in π^--p scattering should equal the negative polarization in π^+-p scattering:

$$P^0(\pi^+p) = -P^0(\pi^-p).$$

Analogously, for example,

$$P_\Sigma^0(\pi^+p \to \Sigma^+K^+) = -P_\Sigma^0(K^-p \to \Sigma^+\pi^-),$$

and nucleon polarization in the nucleon Compton effect and in the pion charge-exchange process

$$\pi^- + p \rightarrow \pi^0 + n,$$

and also the polarization of the Ξ hyperon in the reaction

$$K^- + p \rightarrow \Xi^- + K^+$$

tend to zero.

Investigations of polarization effects at ultra high energies have barely begun. Here, too, the polarized target will make possible research whose results contain new information, presently unavailable without the use of this technique.

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