ON THE 400TH ANNIVERSARY OF THE BIRTH OF GALILEO GALILEI

In 1964, 400 years have elapsed from the birth of Galileo Galilei. On this occasion we are printing a paper by V. A. Fock that he has written for the celebrations taking place in Italy in September of this year. We are also reprinting the article by S. I. Vavilov, "Galileo in the History of Optics". This article was published in 1943 during the Second World War in the collected volume Galileo Galilei published by the Academy of Sciences of the USSR on the occasion of 300th anniversary of the death of Galileo. Since this collected volume was published in wartime in a very small edition, it has been a collector's item for a long time, and S. I. Vavilov's valuable article is not widely known.

GALILEO'S PRINCIPLES OF MECHANICS AND EINSTEIN'S THEORY

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We are indebted to the great Italian scientist Galileo Galilei for two principles of mechanics that have exerted an exceedingly great influence on the development not only of mechanics itself, but also of all of physics.

We have in mind, first, Galileo's principle of relativity for rectilinear uniform motion. In a laboratory in rectilinear uniform motion with respect to another stationary laboratory, everything will occur in the same manner as in the latter. To explain his principle, Galileo compared the physical phenomena within a stationary ship with those within a ship moving uniformly; he spoke of two ships, rather than of two laboratories. However, the numerous examples of physical phenomena cited by Galileo that can occur within the ship clearly show that Galileo here had in mind exactly what we now call a laboratory. We should emphasize the purely physical and experimentally-testable nature of Galileo's assertions.

Galileo's second principle can be associated with his observations on the falling of bodies in vacuo. Galileo established that all bodies fall with the same velocity, whatever their weights, provided that they do not experience the resistance of a medium. If, following Newton, we introduce the concept of acceleration, this principle can be formulated in words: all bodies falling in vacuo show the same acceleration.

From Galileo's time to the present, these principles have undergone development in a highly interesting way.

First of all, they have been made more precise on the basis of the laws of motion established by Newton. Newton's first law, which relates to rectilinear uniform motion, just as Galileo's first principle does, indicates the system of reference for which this principle is valid. This is Newton's inertial system. In fact, Newton's first law can be interpreted as defining an inertial system. We can interpret it thus: there exists a system of reference in which a body not subject to the action of forces moves rectilinearly and uniformly. It is precisely in such a system of reference that Newton's other laws hold. In such an interpretation, it becomes understandable why Newton formulated his first law of motion as a separate law, rather than as a consequence of the second law, which refers to the more complex case of motion under the action of forces. Thus, Galileo's principle of relativity is related to the concept of an inertial system of reference. On the other hand, Galileo's second principle, which refers to the free fall of bodies, is related to the concepts of inertial and gravitational mass, also based on Newton's ideas. This principle can be formulated as the equality of both types of mass (or as the equivalence of the corresponding concepts).

Three centuries after Galileo, his principles of mechanics have become generalized in Einstein's theory of relativity. However, it is more correct to speak of two different directions in which the generalization has proceeded; only one of these seems to us to be thoroughly justified.

First, the principle of relativity for rectilinear uniform motion has been extended to all physical phenomena, including the electromagnetic ones, in particular the propagation of light. In his theory of relativity created in 1905, Einstein derived from this principle a consequence involving the nature of space-time relations between various events in the physical universe, or in other words, involving the nature of space and time. An essential point to mention here is that the properties of space and time thus derived, of which the most important is the existence of a limiting value for the velocity of propagation of any type of action, characterize space in an absolute manner, rather than only with respect to a given observer. The relative has led to the absolute. In order to emphasize this fact, we might, following Fokker's felicitous suggestion, call Einstein's theory chronogeometry (rather than the theory of relativity).

Thus, the first direction of generalization extends Galileo's principle of relativity to all physical phenomena (rather than to mechanical ones alone), but as before, it applies it only to rectilinear uniform motion of the system of reference. The equations of transformation for the space-time variables (the Cartesian coordinates and the time) in two systems of reference in mutual motion remain linear, as in Galileo's transformation, but the coefficients therein now differ: Galileo's transformation is replaced by the Lorentz transformation.

The second direction of generalization that Einstein followed starts from the interpretation of Galileo's second principle as being the principle of equivalence between kinematic acceleration and the gravitational force field, or in other words, between inertial and gravitational forces. Einstein combined this kinematic interpretation of the force of gravity, called simply the "principle of equivalence," with the principle of relativity, and the union of these principles led Einstein to the idea of "general relativity." In spite of the very indefinite and controversial nature of this idea, it seemed sufficient to Einstein's genius to lead him to his remarkable theory of gravitation, which he called the general theory of relativity.

We shall state outright that Einstein's theory of gravitation, which is at the same time a theory of space and time, is a work of genius. It is convincing in the beauty of its conception and in its mathematical treatment. It provides the long-awaited solution of a number of problems posed by Newton's theory (e.g., problems of action at a distance). It admits of astronomical verification and even of verification in the laboratory. However, it is not a general theory of relativity, since general relativity does not exist.

If we accept unreservedly Einstein's final result as expressed by his famous tensor covariant equations of gravitation, we must still determine whether the considerations that led him to these equations are completely logical. We must examine the question whether it is admissible to speak of general relativity and of the complete equivalence of accelerational and gravitational fields, or whether we should adhere to a more cautious viewpoint, which, furthermore, is closer to Galileo's. This viewpoint accepts the principle of relativity only for rectilinear uniform motion, and also makes use of the principle of equality of inertial and gravitational mass. In adopting this viewpoint, we will have to acknowledge that the concepts of general relativity and of complete equivalence are not, in spite of Einstein, the true principles of his theory. Then we have to answer the question of what are the true principles that Einstein's theory of gravitation is based upon.

In order to have a reliable starting point for our analysis, we shall try to formulate more precisely what we mean by the principle of relativity. This sharpening of the concept is all the more necessary, in that completely different meanings have been ascribed to this term. Basically, these amount to a mathematical and a physical meaning.

We shall take the given term in the physical sense, and take relativity to mean <u>the existence of corresponding physical processes</u> (or phenomena) in two systems of reference (in two laboratories). This means that <u>any given</u> process in the one system corresponds to the <u>same</u> process in the other system. (The word "same" can be defined more precisely as follows: let a phenomenon occurring in each of the two laboratory systems be described by functions of the variables pertinent to this system; then the phenomena will be identical if the mathematical form of these functions is identical for both systems.)

When both systems of reference are inertial, we return to Galileo's principle of relativity in his precise formulation. In this case, the definition of a system of reference associated with the laboratory presents no difficulties, and we can take the Cartesian coordinates and the time as such a system of reference. Two such systems of reference are related by the Lorentz transformation. Hence the relation is evident between relativity in the physical sense and the behavior of the equations upon transformations of the coordinates.

On the contrary, every attempt to apply these definitions to non-inertial systems of reference and to non-uniform motion has come up against insuperable difficulties. This means that every attempt to develop the idea of "general relativity" has turned out unsound.

First of all, for accelerated motions, the behavior of the laboratory (and even its geometric shape) depends on the arrangement of this laboratory. Thus we can no longer make a general definition of the laboratory system. However, this is not the worst of the difficulties. The decisive one is the simple fact that the principle of relativity in the proper meaning of the word, i.e., understood as a physical principle, obviously does not hold for accelerated systems. To convince ourselves of this, we need only consider a clock with weights and a pendulum on the Earth and on an artificial satellite. Such a clock might run very well in a laboratory on Earth (it might serve there as a very precise means of measuring time), but it wouldn't run at all on the satellite. Furthermore: there is no physical phenomenon on the satellite that would correspond to the way that such a clock runs on Earth. The fundamental assumption of the principle of relativity, that corresponding physical processes exist, is not satisfied in this case.

In order to avoid these difficulties and to make it possible to apply the concept of relativity to nonuniform motion, Einstein introduced two very essential changes in the meaning of the terms "system of reference" and "principle of relativity," although in masked form. First, he changed the definition of a system of reference, and began to take this term to mean a system of coordinates, rather than a physical laboratory system. Since a coordinate transformation that is non-linear in time implies the introduction into the equations of motion of a fictitious gravitational field as well as Coriolis forces, this new definition of the system of reference now implies, strictly speaking, a rejection of the physical principle of relativity. Einstein "corrected" this principle by means of the principle of equivalence, which was taken from Galileo's second principle of mechanics. Even in the simplest case of purely translational (rotationless) motion, the existence of corresponding processes in two laboratories is gained at the cost of introducing into one of them a fictitious gravitational field (which must be compensated when necessary by a real gravitational field). Thus, for example, a real laboratory within the satellite is compared, not with a real laboratory on Earth, but with a fictitious laboratory which differs from a real laboratory on Earth by the absence of a gravitational field therein, even though it is stationary with respect to the Earth.

Let us return to the principle of equivalence. Undoubtedly, this principle permits us in the general case to eliminate a gravitational field within a sufficiently small region of space (e.g., inside a satellite). However, even the application of the principle of equivalence cannot save the idea of general relativity. (We are not even speaking of electric forces, which cannot be eliminated thus, and which might in principle indicate the existence of an absolute acceleration or rotation.) In order to make it possible to speak of general relativity, Einstein had to deprive this term of its physical content, and ascribe to it a completely different, purely mathematical meaning. At first he gave this term the somewhat hazy interpretation of "identical form of the laws of nature" in different systems of reference; then he replaced the systems of reference by systems of coordinates, and the laws of nature by the differential equations of the field or the equations of motion (although the differential equations alone are insufficient to determine the course of a physical process). Consequently, Einstein arrived at the interpretation of "general relativity" as being the "covariance of the differential equations under an arbitrary transformation of coordinates." However, this new interpretation is something quite different from a physical principle of relativity. It is a purely logical (and not at all physical) requirement that must always be satisfied, even when the physical principle of relativity does not hold. In fact, until a coordinate system is selected, it is necessary for obtaining of unequivocal results that the differential equations as written for all permissible coordinate systems should be equivalent. An example of a mathematical apparatus satisfying this requirement has been known for a long time: we have in mind Lagrange's equations of the second kind.

In attacking the idea of general relativity and emphasizing the limited character of the principle of equivalence, however, we are far from denying the heuristic value of these principles and their historic role in the creation of Einstein's theory of gravitation. Whatever the logical gaps in Einstein's considerations, we are ready to admit that the principles of relativity and equivalence indicated to him the advantages to be gained from comparison of the kinematic space-time relations with the phenomenon of gravitation, and also helped him to find a mathematical form for his gravitational equations that was sufficiently simple, and covariant as well.

Nevertheless, while agreeing with this, we must state that the cited two principles do not comprise the true logical basis of Einstein's theory. Hence, we are faced with the problem of finding other principles that are actually contained in Einstein's theory, and can now be considered as its logical basis. Since Einstein gave us a complete mathematical formulation of his theory of gravitation, it is not hard to point out these true principles.

The first fundamental idea of the theory is the combination of space and time into a single chronogeometric manifold of four dimensions, of which one has the character of time, and the other three of space. This latter fact is expressed verbally thus: the metric of the space-time manifold is indefinite and has the signature $(+, -, -, -)$. This form of the metric is related to the law of propagation of any action proceeding at the limiting velocity, e.g., a light wave front. This limiting velocity is always equal to the velocity of light. The existence of a limiting velocity is a fundamental fact of the physical universe, and the recognition of it has profoundly changed our notions of space and time. We note that, according to Robb and Aleksandrov, this fact is associated with the concepts of cause and effect.

Einstein put into effect the idea of combining space and time in the two forms of his theory of relativity: in the theory of 1905, usually called the special theory, and in the theory of 1916, usually called the general theory. In the 1905 theory, the space-time metric admits of an expression with constant coefficients for an infinitesimally small interval. However, in the 1916 theory, the metric is assumed to be more general, and in particular, Riemannian (it is characterized directly by the values of the coefficients in the expression for the interval, or the components of the metric tensor).

The second fundamental idea of Einstein's theory of gravitation, an idea that distinguishes this theory from the so-called special theory of relativity, is the rejection of a fixed metric. According to this idea, the space-time metric is not assigned once and for all, it is not "fixed," but can depend on processes occurring in space and time, and above all on the distribution and motion of masses. This idea, which occurred to Einstein between 1908 and 1915 (probably in 1911, while he was in Prague) represents something completely new—here Einstein had no precursors. The significance of this idea is tremendous. On the basis of this idea, Einstein was able to establish a relation between the space-time metric and the phenomenon of gravitation, a relation so intimate that we can speak of the unity of the two. This unity is expressed formally in the fact that the components of the metric tensor completely determine the gravitational field as well, so that we need not introduce any other functions to characterize this field. As is known, if we start from this assumption and use the analogy with the equations of gravitation of Newton's theory, we can derive Einstein's equations almost uniquely.

We have seen how Galileo's principles, which Einstein did not always interpret systematically and logically, helped him to formulate his remarkable theory of gravitation, whose two fundamental ideas we have tried to present above. However, we can also consider the relation between Galileo's principles and Einstein's theory from another standpoint. We can ask ourselves, "Taking Einstein's theory in its final form, what can we say of the part taken in this theory by Galileo's principles?"

At first glance, it might seem that, since Galileo's principle of relativity is applicable only to rectilinear uniform motion, it plays no role in this theory, since space-time ceases to be uniform in the presence of masses and gravitational fields (the expression for the interval then cannot be reduced to constant coefficients). However, this is actually not the case. In fact, we can always impose four supplementary equations on the metric tensor simultaneously with Einstein's equations, giving a limitation imposed by the choice of coordinates. According to this condition, each of the space coordinates, as well as the time variable, must satisfy a wave equation. Moreover, for a system of masses like the solar system, we can subject the metric tensor to boundary conditions at infinity expressing the isolation of the given system of masses and its Euclidean character at infinity. Under these conditions we can state that the coordinate system (which is usually called "harmonic") is unequivocally defined within the accuracy of the Lorentz transformation. In Einstein's theory of gravitation, a harmonic coordinate system is the closest possible analog to the variables (the Cartesian coordinates and the time) that pertain to a certain inertial system in Newton's sense. Since the admissible transformations of harmonic coordinates are reduced to linear ones, rectilinear uniform motion retains the privileged place that it occupied in Galileo's principle of relativity. Consequently, this principle remains in force as a physical principle also in the case under discussion of Einstein's inhomogeneous space. It is self-evident that the concept of corresponding physical processes in two systems of reference must involve also corresponding positions and motions of the heavy masses. We note that the very possibility of introducing the concept of corresponding physical processes in inhomogeneous space exists only by virtue of the rejection of fixity of the metric.

As for Galileo's second principle, interpreted in Newton's spirit as the equality of inertial and gravitational mass, this principle remains in force to the precision with which we can make independent measurements of the two masses. In Einstein's theory this measurement is possible only as an approximation. However, if we solve Einstein's equations for a system of moving bodies, then for each body the same constant will appear in the solution as the inertial mass and as the gravitational mass. Hence we can say that in Einstein's theory the equality of masses is automatically satisfied. We note that this equality, along with Galileo's second principle, does not have the purely local character that the principle of equivalence has.

The mechanical principles of Galileo that we have analyzed here comprise only part of his gigantic scientific achievements, whose value for physics can be traced right down to the present. However brief and incomplete our analysis has been, it shows how profound the influence of Galileo has been on human thought. This influence continues even now—it continues for centuries.

Translated by M. V. King