SOVIET PHYSICS USPEKHI VOLUME 7, NUMBER 4 JANUARY-FEBRUARY 1965
621.316.729

EFFECT OF FLUCTUATIONS ON THE OPERATING ACCURACY OF SYNCHRONIZATION EQUIPMENT

V. I. TIKHONOV

Usp. Fiz. Nauk 83, 665-694 (August, 1964)

INTRODUCTION

THE meaning of synchronization can be explained in the following way. Assume that an external periodic oscillation (in the simplest case, sinusoidal) is applied to a self-oscillating system either directly or through a special device. Then the driving force causes the self-oscillation frequency to assume, within a definite interval called the synchronization band, a value equal to the frequency of the external oscillation. The synchronization band is determined essentially by the parameters of the system and depends also on the amplitude of the external oscillation. Outside the synchronization band, the frequencies of the external oscillation and the natural oscillations do not coincide.

The synchronization phenomenon is widely used in radiophysics and engineering, and has therefore been the subject of many investigations. We present several examples showing the practical utilization of synchronization.

In physical measurements frequent use is made of phase methods in which some physical quantity is estimated by measuring the phase difference between two oscillations. This method is used in particular to measure accurately the propagation speed of electromagnetic energy. The same principle is used in radio range finders[1] with coherent synchronous heterodynes.

Investigations of nuclear reactions by means of cyclotrons presuppose high stability of the parameters of the deflected accelerated-ion beam and of the frequency of the accelerating voltage. Frequency deviations of the resonant system lead to a considerable reduction in the voltage on the dees. To eliminate this it is necessary either to adjust the master generator automatically to the natural frequency of the dee circuit, or to adjust the dee circuit to the frequency of the generator. The control system frequently incorporates automatic phase control of the frequency [2].

A very important factor in launching of artificial satellites and rockets is knowledge of their orbit parameters. These parameters can be measured by various methods (interference, Doppler, etc.), but all are based on the use of either a highly stabilized or a synchronous generator. Among the main elements of such devices are automatic phase control systems[3].

To solve many radiophysical problems it is necessary to generate highly stabilized microwave oscillations. The existing klystron and magnetron generators as a rule do not have enough stability. To increase their frequency stability, such generators are synchronized by means of low-power oscillations from highly stabilized standard generators (for example, quartz-controlled or quantum generators).

In television, synchronization devices are widely used to synchronize the horizontal and vertical sweeps. In color television systems synchronization is essential to restore the subcarrier frequencies in the receiver[4,5].

Communication systems using synchronous detection have come into extensive use recently. The use of synchronous detectors in amplitude, frequency, and phase telegraphy makes it possible to attain near-maximum interference immunity[6]. Promising radio communication systems with single-band modulation, or with two-band modulation with suppressed carrier, call for the presence of synchronization devices on the receiving end[7,8]

Among other applications of synchronization we can point to time-service systems, synchronous radio broadcasting, coherent radar, some types of phase-controlled radio navigation, etc.

The synchronization of self-oscillating systems is among the most complicated problems in nonlinear oscillation theory, owing to the variety and subtlety of the effects observed even if the synchronization is by means of external sinusoidal oscillations, and also owing to the essentially nonlinear character of the phenomena.

The fundamentals of synchronization theory were established in the early thirties by Van der Pol[9] and by A. A. Andronov and A. A. Vitt[10,11]. Subsequently a major contribution to the solution of this problem was made by the Soviet scientists L. I. Mandel'shtam, N. D. Papaleksi, N. N. Bogolyubov, S. M. Rytov, V. V. Migulin, Yu. B. Kobzarev, S. I. Evtyanov, R. V. Khokhlov, and others. A complete review of this research is given in [12].

Interest in the study of the influence of fluctuating signals on the synchronization process has increased recently. This problem includes an analysis of the effect of fluctuations on the operation of an autonomous generator. The result of such an analysis helps explain the statistical character of the autonomous oscillations themselves. The fluctuations are generated either in the self-oscillator circuit elements (shot and flicker noise of electron tubes, thermal fluctuations of loss resistances) or by external random signals (changes in the temperature, pressure, humidity, etc.).

A qualitative manifestation of fluctuations is, for example, that the self-oscillations cease to be strictly sinusoidal but are modulated in amplitude and in frequency in random fashion. The energy spectrum of such a stochastic oscillation is continuous.

The basic work dealing with problems of this type was that of A. A. Andronov, L. S. Pontryagin, and A. A. Vitt[13], who solved in general form the behavior of dynamic systems in the presence of random signals. Their assumption that the process occurring in such a system is a Markov process has made it possible to apply the statistical formalism of the Fokker-Planck-Kolmogorov equation to the solution of the problem.

A direct continuation of [13] was a series of theoretical[14,15] and experimental[16] papers by I. L. Bershteĭn on the amplitude and phase fluctuations of self-oscillations of a vacuum-tube oscillator. Since it is not our intent to present here a complete review of all the work in this field, we shall cite only some of the papers. In 1955 S. M. Rytov[17] developed a general theory of amplitude and phase fluctuations in weakly-linear self-oscillating systems, for which the small-parameter method can be used to determine the periodic stationary mode. Assuming the fluctuations to be delta-correlated and to be of second-order smallness, expressions were obtained in general form for the correlation functions of the fluctuations of the amplitude and phase in both autonomous and non-autonomous self-oscillating systems.

In a fundamental paper published in 1958, R. L. Stratonovich[18] investigated, on the basis of the Fokker-Planck-Kolmogorov equation, the stationary phase and amplitude fluctuations of a vacuum-tube oscillator synchronized by a sinusoidal signal at the fundamental frequency, under the assumption that the correlation time of the applied fluctuations is much shorter than the phase and amplitude transient time of the self-oscillations. In addition, the synchronizing signal and the intensity of the fluctuation were assumed to be sufficiently small to make it possible to solve separately the abbreviated equations for the phase and amplitude.

Unlike the earlier papers, no limitation was imposed in [18] on the smallness of the phase fluctuations, and a clear mathematical picture was presented for the case of phase fluctuations that are commensurate with the period of the oscillations. The stationary distributions were obtained for the phase and amplitude fluctuations of the self-oscillations, and the shift of the average oscillator frequency resulting from phase "jumps" amounting to an integral number of periods and due to the action of the fluctuation noise, was calculated. The results of [18] were substantially expanded and experimentally verified by I. G. Akopyan[12,19] and others[20-22].

The main purpose of this article is to generalize and systematize the principal results of the analysis of the effect of fluctuation noise on typical synchronization devices [synchronized self-oscillation and automatic phase control of the frequency (see [51])], and also to present a few estimates of the effect of fluctuations on the operation of relaxation oscillators, as well as some ideas concerning rational methods of pulsed synchronization.

1. PRINCIPAL STATISTICAL CHARACTERISTICS OF SELF-OSCILLATIONS

In analyzing the operation of self-oscillators used in many radiophysics problems it is necessary to take into account the internal fluctuations (noise in the tubes and resistances) as well as external random signals (random variations of the power-supply voltage, oscillations in the ambient temperature, vibrations, etc.).

The effect of fluctuations and random signals is manifest in the fact that the self-oscillations cease to be strictly sinusoidal, being randomly modulated in amplitude and in frequency. The amplitude and frequency fluctuations due only to the internal noise of the self-oscillator are customarily called natural fluctuations[23]. These fluctuations can not be eliminated in principle and determine the limit beyond which the frequency and amplitude stability of a given self-oscillator can no longer be increased. The frequency and amplitude fluctuations due to external random action are called technical fluctuations, and can be eliminated in principle by parametric stabilization (thermostating, shock absorption, etc.) and by stabilizing the supply voltages.

In spite of the fact that under real conditions technical instabilities greatly exceed the natural instabilities, we present in what follows the main results pertaining to natural fluctuations, since they are of basic interest.

The operation of a self-oscillator subject to internal fluctuations was analyzed by many authors. With regards to the mathematical formalism employed, the main papers can be divided into two groups, one[14-16,25] using the Fokker-Planck-Kolmogorov equation and the other[17] using the small-parameter method. Although the applicability of neither methods is subject to doubt, the method of linearization relative to the fluctuation corrections is simpler and more natural in this case.

The equation of a vacuum-tube oscillator (Fig. 1) and of a few other self-oscillating systems reduces to the form

$$\ddot{x} + \omega^2 x = \varepsilon f(x, \dot{x}, t), \tag{1}$$

where the dot denotes the time derivative, $x(t)$—some

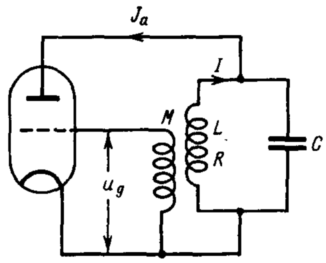

FIG. 1. Diagram of self-oscillator.

coordinate (for example the current in the inductive branch), ω—resonant frequency of the oscillating system, ϵ—small parameter, and $f(x, \dot{x}, t)$—a function whose form is determined by the circuit under consideration.

To study the solution of (1) it is expedient to go over from one second-order equation to two first-order equations that describe the behavior of the amplitude and of the phase. Although the concepts of amplitude and phase are intuitively clear, an attempt to define them for oscillations that are not strictly sinusoidal, no matter how close to sinusoidal they may be, involves a certain arbitrariness. It is appropriate to recall here the non-uniqueness of the definition of the envelope and phase of quasiharmonic fluctuations[26].

Taking into account the smallness of the parameter ϵ and the quasiharmonic character of the self-oscillations, we define the amplitude and phase by the relations

$$x = A\cos(\omega t + \theta), \qquad \dot{x} = -\omega A \sin(\omega t + \theta). \tag{2}$$

From this we get*

$$A^2 = \left(x^2 + \frac{\dot{x}^2}{\omega^2}\right), \qquad \theta = -\omega t - \operatorname{arctg}\frac{\dot{x}}{\omega x}.$$

Differentiating these equations with respect to the time and taking (1) into account we obtain

$$\dot{A} = \frac{\dot{x}}{\omega^2 A}\varepsilon f(x, \dot{x}, t), \qquad \dot{\theta} = -\frac{x}{\omega A^2}\varepsilon f(x, \dot{x}, t). \tag{3}$$

Let us apply these equations to the generator circuit in question (Fig. 1) It is easy to verify that the differential equation for the current I flowing through the inductive branch of the tank circuit is

$$\ddot{I} + \omega^2 RC\dot{I} + \omega^2 I = \omega^2 J_a, \qquad \omega^2 = \frac{1}{LC}. \tag{4}$$

We represent the tube plate current J_a in the form of a sum of two components: regularly varying $I_a(t)$ and fluctuating $i(t)$, the latter taking into account the shot noise of the plate current:

$$J_a(t) = I_a(t) + i(t).$$

To simplify the formulas we assume that there is no grid current; we can neglect the anode reaction and we can approximate the characteristic of the tube over some range by the cubic parabola

$$I_a = Su_g - \frac{\gamma}{3}u^3_g.$$

Since $u_g = M\dot{I}$, where M—mutual-inductance coefficient of the plate and grid coils, we can write

$$\ddot{I} + \omega^2 I = \omega^2\dot{I}\left[(SM - RC) - \frac{\gamma}{3}M^3\dot{I}^2\right] + \omega^2 i(t). \tag{5}$$

In spite of the large number of assumptions made in the derivation of the fundamental equation (5), it describes satisfactorily the qualitative aspects of the processes occurring in the oscillator.

We introduce the following notation:

$$A_0 = 2\sqrt{\frac{SM - RC}{M^3\omega^2\gamma}}, \quad \varepsilon = \omega(SM - RC), \quad \frac{I}{A_0} = x, \quad \zeta(t) = \frac{\omega^2}{A_0}i(t). \tag{6}$$

A_0 is the stationary amplitude of the current self-oscillations in the tank circuit.

Equation (5) assumes in the new notation the form (1), in which we must put

$$\varepsilon f(x, \dot{x}, t) = \varepsilon\omega\dot{x}\left(1 - \frac{4\dot{x}^2}{3\omega^2}\right) + \zeta(t). \tag{7}$$

Substituting this expression in (3), going over in these expressions from the variables x and $\dot{x}$ to A and θ, and discarding the sinusoidal components with frequencies 2ω and 4ω, we obtain the following abbreviated equations for the amplitude and phase:

$$\left.\begin{aligned} \dot{A} &= \frac{\varepsilon\omega A}{2}(1 - A^2) - \frac{\omega}{A_0}i(t)\sin(\omega t + \theta), \\ \dot{\theta} &= -\frac{\omega}{AA_0}i(t)\cos(\omega t + \theta). \end{aligned}\right\} \tag{7'}$$

As is well known[27], the effect of the higher harmonics, which we failed to take into account, reduces to a certain frequency correction, but this correction is regular. Therefore, in spite of the fact that the frequency correction due to the higher harmonics exceeds the frequency fluctuations, we must assume that this simplification is justified.

From (7) we obtain the stationary operating mode of the oscillator in the absence of fluctuations. Thus, putting $\dot{A} = 0$ and $i(t) = 0$, we obtain $A_{st} = 1 = \text{const}$. Analogously, putting $i(t) = 0$, we obtain $\dot{\theta} = 0$, $\theta_{st} = \theta_0 = \text{const}$. No value of the initial phase can be given preference here. It must therefore be regarded as a random quantity, uniformly distributed over the interval $(-\pi, \pi)$.

Thus, the stationary current in the tank circuit is determined by the formula

$$I = A_0 x = A_0 A_{st}\cos(\omega t + \theta_0) = A_0\cos(\omega t + \theta_0). \tag{8}$$

An estimate of the frequency and amplitude fluctuations of the self-oscillations, due to the shot noise of the tube current, can be obtained by using a method wherein the equations in (7) are linearized in the vicinity of the stationary state. The use of the linearization method is justified by the fact that the shot

*arctg = $\tan^{-1}$

noise is small. It therefore causes small deviations of the amplitude A and of the frequency ω (and not of the phase) from their stationary values.

We denote the amplitude and phase fluctuations due to the noise by

$$a = A - A_{st}, \quad \psi = \theta - \theta_0. \tag{9}$$

By definition, a and ψ are small deviations.

We substitute (9) in the initial equations (7) and retain in the latter only the terms of first order in a. The fluctuation current i(t) we must assume here to be a quantity of first order of smallness, while the quantities A and θ, which remain as coefficients in the equations, must be replaced by their stationary values ($A_{st} = 1$, $\theta_{st} = \theta_0$). All these transformations yield

$$\dot{a} + \varepsilon\omega a = -\frac{\omega}{A_0} i(t) \sin(\omega t + \theta_0), \tag{10}$$

$$\dot{\psi} = -\frac{\omega}{A_0} i(t) \cos(\omega t + \theta_0). \tag{11}$$

The fluctuations of the tube plate current can be regarded, in many practical cases, as a normal white noise with a correlation function

$$\langle i(t_1)\, i(t_2)\rangle = \frac{N}{2}\delta(t_2 - t_1), \qquad N = 2eI_s.$$

Here and throughout angle brackets will denote statistical averaging, e = 1.6×10^{-19} C—electron charge, I_s—equivalent current of the diode in the saturation mode.

If the statistical characteristics of i(t) are known, Eqs. (10) and (11) enable us to calculate all the statistical characteristics of the amplitude and phase fluctuations. We shall henceforth pay principal attention to phase fluctuations.

From (11) we obtain an expression for the random phase deviation after a certain time T:

$$\Delta\psi = \psi(t_0 + T) - \psi(t_0) = -\frac{\omega}{A_0}\int_0^T i(t_0 + x)\cos(\omega t_0 + \omega x + \theta_0)\, dx. \tag{12}$$

It follows therefore that the mean value of the phase deviation is equal to zero: $\langle\Delta\psi\rangle = 0$. For the variance of the phase deviation we obtain

$$\sigma_{\Delta\psi}^2 = DT, \qquad D = \frac{N\omega}{4A_0^2} = \frac{eI_s\omega^2}{2A_0^2}. \tag{13}$$

The variance of the phase deviation increases in proportion to the observation time, that is, the phase variation has a diffusion character.

The phase fluctuations lead to a random spread of the instantaneous value of the self-oscillation frequency relative to its nominal value. In practice we cannot propose any measures for eliminating the natural instability without appreciably changing the operating principle of the generator itself.

We note incidentally that if a broadband normal stationary noise is applied to a tank circuit with high Q, then the abbreviated equation for the phase of the output oscillations coincides exactly with (11)[28]. The result can therefore be extended to include this case, too.

The stationary solution of (10) is

$$a(t) = -\frac{\omega}{A_0}\int_{-\infty}^{t} e^{-\varepsilon\omega(t-x)} i(x)\sin(\omega x + \theta_0)\, dx.$$

From this we can readily obtain an expression for the variance of the amplitude fluctuations

$$\sigma_A^2 = \frac{eI_s\omega}{4\varepsilon A_0^2}.$$

Consequently, taking into account the amplitude and phase fluctuations, these self-oscillations can be represented in the form

$$I(t) = A_0[1 + a(t)]\cos[\omega t + \theta(t)], \qquad \theta(t) = \theta_0 + \psi(t). \tag{14}$$

If we recognize that in the linear approximation the amplitude and phase fluctuations are independent of each other and have normal distributions, we obtain, after making the necessary calculations, the following expression for the energy spectrum of the self-oscillations (14):

$$S(\Omega) = A_0^2\left[\frac{\frac{1}{2}D}{\frac{1}{4}D^2 + (\Omega - \omega)^2} + \frac{\varepsilon\omega\sigma_A^2}{(\varepsilon\omega)^2 + (\Omega - \omega)^2}\right].$$

It follows from (8) that in the absence of fluctuations [$i(t) \equiv 0$] the oscillator generates a sinusoidal signal with an energy spectrum represented by a discrete line of height $A_0^2/2$ and of frequency ω. The energy spectrum of the quasiharmonic oscillation (14), obtained in the presence of fluctuations, is continuous. It is symmetrical and has a maximum at the frequency ω. The spectrum consists of two terms, the first of which is due to the phase fluctuations and the second to the amplitude fluctuations.

Let us estimate the various quantities. Let $\epsilon = 5 \times 10^{-2}$, f = 10 Mc, $I_s \approx 1$ mA, and $A_0 \approx 100$ mA. Then $D \approx 3 \times 10^{-5}$ and $\sigma_A^2 \approx 5 \times 10^{-12}$. At these values we can neglect in the analysis of the energy spectrum of a quasiharmonic oscillation the second term, which carries insignificant power and which is more "broadband." We then obtain for the spectrum the simple formula

$$S(\Omega) = \frac{2DA_0^2}{D^2 + 4(\Omega - \omega)^2}.$$

Thus, the energy spectrum of the oscillation is transformed by the phase fluctuations from a discrete line into a continuous spectrum which has at the 0.5 level a width $\Delta\Omega$ = D. The natural instability of the oscillator frequency can be quantitatively characterized by the relative width of the energy spectrum

$$\frac{\Delta\Omega}{\omega} = \frac{D}{\omega} = \frac{eI_s\omega}{2A_0^2} \approx 10^{-12}.$$

2. TECHNICAL METHODS OF SYNCHRONIZATION

The natural fluctuations and the external random signals lead to frequency instability of self-oscillators. One of the effective means of maintaining the frequency constant is external synchronization of the self-oscillator by means of another more stable source of oscillations. Two ways are possible here.

In the first, a synchronizing signal is applied directly to the self-oscillator tank circuit. The resultant "captured" oscillator operates on the principle of frequency "entrainment."

The second is based on comparison of the frequency of the synchronized generator with that of a standard generator. Depending on the frequency difference, a control voltage is generated which adjusts the frequency of the stabilized generator (SG) via an actuating mechanism, until it coincides with the frequency of the highly stabilized reference generator (RG). Such systems contain as a rule a feedback loop and constitute automatic control systems. If the frequency comparison unit is a phase detector, the result is automatic phase control. On the other hand, if the comparison is with the aid of a frequency discriminator, we get an automatic frequency control system. In some applications it is convenient to have both phase and frequency detectors, and the result is a frequency-phase control system.

A feature of all the synchronization systems is that a reference generator of relatively low power can stabilize the frequency of a generator of considerably higher power.

The presence of fluctuation noise acting on the synchronization systems together with the synchronizing signal leads to disturbances to normal operation. Whereas in-phase operation of the generators can be attained in the absence of noise, the presence of noise can cause fluctuation oscillations of the phase difference. At a low noise level we can speak of synchronous generator operation, that is, we can say that the average frequencies of the reference and synchronized generators are equal. Large noise causes phase jumps amounting to an integer number of cycles, and synchronous operation of the two generators is impossible. A more rigorous albeit arbitrary delineation between these modes will be given below.

From among the foregoing synchronization methods, we shall consider in what follows the operation of a synchronized self-oscillator and phase-type automatic frequency control in the presence of noise, since the frequency-type automatic frequency control does not permit synchronous operation of generators in the absence of noise, owing to the presence of a residual frequency deviation[29].

1. Synchronized self-oscillator. It can be shown [12,18,25] that if a sinusoidal synchronizing signal $E \sin \omega_{syn}t$ is applied to the tank circuit of a self-oscillator (see Fig. 1) together with a relatively weak fluctuation noise $\xi(t)$, then the oscillations of the generator can be approximately represented under certain conditions in the form $A \cos(\omega_{syn}t + \varphi)$. The statistical characteristics of the phase φ are determined here by the equation

$$\dot{\varphi} = \Delta_0 - \Delta \sin \varphi + \frac{\omega}{A_0} \xi(t) \cos(\omega_{syn} t + \varphi), \tag{15}$$

where $\Delta_0 = \omega - \omega_{syn}$—initial detuning, $\Delta = \omega_{syn}E/A_0$—synchronization band, A_0—average value of the amplitude, and ω—natural frequency of the tank circuit.

A solution of this nonlinear stochastic equation and a discussion of the results will be given in the next two sections.

2. Phase-type automatic frequency control. Let us examine briefly the operation of a phase-type AFC circuit, the block diagram of which is shown in Fig. 2.

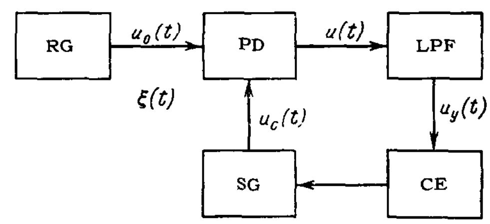

FIG. 2. Block diagram of phase-type AFC.

The sinusoidal oscillations $u_0(t)$ and $u_{syn}(t)$ of the reference and of the synchronized generators

$$u_0(t) = A_1 \cos \Phi_1(t) = A_1 \cos[\omega_0 t + \theta_1(t)],$$
$$u_{syn}(t) = A_2 \sin \Phi_2(t) = A_2 \sin[\omega_{syn} t + \theta_2(t)]$$

will act together with the normal stationary fluctuation noise

$$\xi(t) = A(t) \cos \Phi(t) = A(t) \cos[\omega_0 t + \theta(t)]$$

on the phase detector (PD). The output of the PD is a voltage proportional to the phase difference of the applied oscillations. After going through a low pass filter (LPF) this voltage changes the frequency of the SG by means of a control element (CE), causing it to coincide with the frequency of the RG.

If we assume the LPF to be ideal, that is, its transfer function equal to unity at low frequencies $(\omega_{syn} - \omega_0)$ and zero at high frequencies $(\omega_{syn} + \omega_0)$, then we can obtain for the phase difference $\varphi = \Phi_2 - \Phi_1$ the following differential equation[20]:

$$\dot{\varphi} = \Delta_0 - \Delta \sin \varphi - \frac{\Delta}{A_1} (A_{syn} \sin \varphi - A_s \cos \varphi) + \dot{\psi}. \tag{16}$$

Here $\Delta_0 = \omega_{syn0} - \omega_0$—initial frequency detuning of the generators, Δ—synchronization band, which depends on the circuit parameters and on the amplitudes A_1 and A_2, and

$$\dot{\psi} = \dot{\theta}_2 - \dot{\theta}_1, \quad A_{syn} = A(t) \cos(\theta - \theta_1), \quad A_s = A(t) \sin(\theta - \theta_1). \tag{17}$$

Equation (16) is analogous to (15).

If we use for the LPF an RC integrating network, then the operation of the phase-type AFC (Fig. 2) is described by the following nonlinear differential equation[21]:

$$\ddot{\varphi} + \alpha\dot{\varphi} + \alpha\Delta \sin\varphi = \alpha\Delta_0 + \zeta(t), \tag{18}$$

where

$$\alpha = \frac{1}{RC}, \quad \zeta(t) = \ddot{\psi} + \alpha\dot{\psi} + \frac{\alpha\Delta}{A_1}(A_s \cos\varphi - A_{\text{syn}} \sin\varphi). \tag{19}$$

Inclusion of an RC filter increases the order of the equation, and we can assume intuitively that the influence of the noise is smaller in such a system than in the first-order systems indicated above. However, it must be borne in mind that the presence of a filter causes a narrowing of the lock-in band of the phase-type AFC, which is not always acceptable. Therefore in practice the tendency is to choose filters that ensure the required interference immunity for a specified lock-in band. These can be an RC filter with velocity correction (proportionally-integrating filter) [30], an LCR filter[31,32], and also a nonlinear filter [33,34].

We present below solutions of (15), (16), and (18), and consider separately two cases of small and large external fluctuation noise.

3. LINEAR THEORY OF OPERATION OF SYNCHRONIZATION DEVICES

When relatively low noise is applied to the synchronization system and when the initial frequency deviations are small, the linearization method can be used for the analysis.

We consider the calculation of the mean value of the phase increment and of the variance of the increment, using a tank circuit as an example. As will be shown below, in this case the derivations will be valid for a locked-in self oscillator (15) and for phase-type AFC with an ideal filter (16). In addition, the results obtained answer directly the question of the transformation of the phase fluctuations of the self-oscillations by means of resonant systems in the presence of additive fluctuation noise[35].

Assume that an LCR tank circuit (Fig. 3) is acted upon by a signal from a self-oscillator

$$s(t) = A(t)\cos[\omega t + \psi(t)]$$

and a weak additive normal white noise $\xi(t)$ with zero mean value and with a correlation function

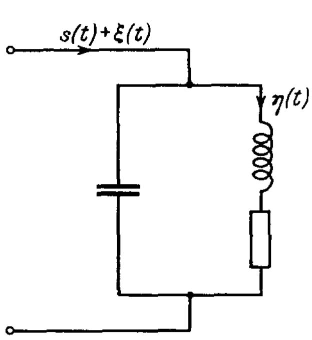

FIG. 3. Effect of self-oscillations and noise on a tank circuit.

$$\langle \xi(t)\xi(t+\tau)\rangle = \frac{N_0}{2}\delta(\tau), \tag{20}$$

where $\delta(t)$ is the delta-function.

Because of the internal fluctuations of the self-oscillator, the amplitude $A(t)$ and the phase $\psi(t)$ are random functions of the time, and the statistical characteristics of the phase $\psi(t)$ are determined by (11).

According to (11), the correlation function for the derivative of the phase is

$$K(\tau) = \langle\dot{\psi}(t)\dot{\psi}(t+\tau)\rangle = \frac{N\omega^2}{4A_0^2}\delta(\tau)\cos\omega\tau. \tag{21}$$

Since the internal fluctuations of the self-oscillator are usually small, we shall henceforth disregard the amplitude fluctuations of the self-oscillations, that is, we put $A(t) = A_0$, where A_0—the self-oscillation amplitude obtained in the absence of the internal noise.

The differential equation for the current in the inductive branch of the tank circuit is

$$\ddot{\eta} + 2\alpha\dot{\eta} + \omega_0^2\eta = \omega_0^2[s(t) + \xi(t)] \quad \left(\alpha = \frac{R}{2L}, \ \omega_0^2 = \frac{1}{LC}\right). \tag{22}$$

We confine ourselves to an examination of the following case of practical interest, in which the tank circuit has high Q and the deviation between the self-oscillation frequency ω and the tank-circuit resonant frequency ω_0 is small, that is, $|\omega - \omega_0| \ll \omega_0$.

Under these assumptions, the solution of (22) can be sought in the form of a quasiharmonic oscillation

$$\eta(t) = B(t)\cos[\omega t + \varphi(t)]. \tag{23}$$

For a tank circuit with high Q, the functions $B(t)$ and $\varphi(t)$ will be slowly varying compared with $\cos[\omega t + \varphi(t)]$. Therefore we can put, with some approximation,

$$\dot{\eta}(t) = -\omega B(t)\sin[\omega t + \varphi(t)]. \tag{24}$$

From (23) and (24) we obtain

$$B^2 = \left(\eta^2 + \frac{\dot{\eta}^2}{\omega^2}\right), \qquad \varphi = -\omega t - \operatorname{arctg}\frac{\dot{\eta}}{\omega\eta}.$$

Differentiation of these expressions yields

$$\dot{B} = \frac{\dot{\eta}}{\omega^2 B}(\ddot{\eta} + \omega^2\eta), \qquad \dot{\varphi} = -\frac{\eta}{\omega B^2}(\ddot{\eta} + \omega^2\eta). \tag{25}$$

Substituting $s(t)$ in (22) and introducing the initial detuning $\Delta_0 = \omega - \omega_0$, we obtain

$$\ddot{\eta} + \omega^2\eta = -2\omega\Delta_0\eta - 2\alpha\dot{\eta} + \omega_0^2\{A_0\cos(\omega t + \psi) + \xi(t)\}, \tag{26}$$

where we put

$$\omega^2 - \omega_0^2 = (\omega - \omega_0)(\omega + \omega_0) = 2\omega\Delta_0.$$

If we substitute (26) in (25), carry out the trigonometric transformations, and discard in the right sides of the resultant equations the terms containing the harmonics of frequency 2ω, we obtain the following abbreviated equations for the amplitude and the phase:

$$\left.\begin{aligned}\dot{B} &= -\alpha B - \frac{\omega A_0}{2}\sin(\varphi-\psi) - \omega\xi(t)\sin(\omega t+\varphi),\\ \dot{\varphi} &= \Delta_0 - \frac{\omega A_0}{2B}\cos(\varphi-\psi) - \frac{\omega}{B}\xi(t)\cos(\omega t+\varphi).\end{aligned}\right\} \quad (27)$$

From the second equation we can readily obtain the stationary values of the phase difference in the absence of noise ($\xi(t) = 0$) by putting $\dot{\varphi} = 0$. We get

$$\varphi - \psi = \arccos\left(\frac{2B\Delta_0}{\omega A_0}\right). \quad (28)$$

In the particular case when there is no initial detuning ($\Delta_0 = 0$), we have $\varphi = \psi - \pi/2$. Consequently in the absence of external noise, or for zero initial detuning, the mean value of the phase of the quasiharmonic current in the tank circuit differs from the mean value of the phase of the self-oscillations by $\pi/2$; all of their other statistical characteristics coincide. When $\Delta_0 \neq 0$, owing to the reactive properties of the tank circuit, a deviation not equal to $\pi/2$ is produced between the phases.

To find the statistical characteristics of the phase $\dot{\varphi}$, we change in the second equation of (27) from φ to a new variable

$$\chi = \varphi - \psi + \frac{\pi}{2}. \quad (29)$$

We obtain

$$\dot{\chi} = \Delta_0 - \frac{\omega A_0}{2B}\sin\chi - \dot{\psi} - \frac{\omega}{B}\xi(t)\cos(\omega t+\varphi). \quad (30)$$

Although the "amplitude" $B(t)$ in this equation is a random function of the time, we replace it by a constant quantity B_0—the stationary amplitude of the current oscillations in the tank circuit, due only to the signal $s(t)$. In the first equation of (27), putting $\xi(t) = 0$ and $\varphi - \psi = -\pi/2$, we get

$$B(t) = B_0 = \frac{\omega A_0}{2\alpha} = \frac{\omega_0 A_0}{2\alpha}. \quad (31)$$

It is obvious that replacement of B with B_0 is valid if the variance σ_η^2 of the noise current in the inductive branch of the tank circuit is much smaller than B_0^2. Using the known formula[36]

$$\sigma_\eta^2 = \frac{\omega_0^2}{8\alpha} N_0,$$

we obtain the initially indicated condition for the smallness of the noise $\xi(t)$:

$$N_0 \ll \frac{2A_0^2}{\alpha}.$$

When applied to the cases for which this inequality is not satisfied, the results obtained below must be regarded as a crude approximation.

Substituting in (30) the value of B_0 from (31), we can write ultimately

$$\dot{\chi} = \Delta_0 - \alpha\sin\chi - \dot{\psi}(t) - \zeta_1(t), \quad (32)$$

where

$$\zeta_1(t) = \frac{2\alpha}{A_0}\xi(t)\cos(\omega t+\varphi)$$

is a random process with zero mean value and with a correlation function

$$k_1(\tau) = \frac{\alpha^2 N_0}{A_0^2}\delta(\tau)\cos\omega\tau. \quad (33)$$

Comparing (32) with (15) and (16), we verify that (32) describes the synchronization processes in a vacuum-tube oscillator and in a simplest phase-type AFC circuit in the presence of a fluctuation noise. The only difference is that in the last two cases we must replace α in the right side of (32) by the synchronization band Δ. The analytical and experimental investigation of this equation will be considered in later sections.

We consider below the case of small noise, when there are practically no phase jumps, and the phase difference χ is sufficiently small, so that Eq. (32) can be linearized. Assuming that $\sin\chi = \chi$, we obtain the linear differential equation

$$\dot{\chi} + \alpha\chi = \Delta_0 - \dot{\psi}(t) - \zeta_1(t). \quad (34)$$

This equation shows that the phase fluctuations of the quasiharmonic current depend both on the phase of the self-oscillation and on the external noise.

From (34) we obtain the mean value

$$\langle\chi\rangle = \frac{\Delta_0}{\alpha}, \quad \langle\varphi\rangle = \langle\psi\rangle + \frac{\Delta_0}{\alpha} - \frac{\pi}{2} = \frac{\Delta_0}{\alpha} - \frac{\pi}{2}. \quad (35)$$

The general solution of (34) is of the form

$$\chi(t) = \chi_0 + e^{-\alpha t}\int_0^t e^{\alpha x}[\Delta_0 - \dot{\psi}(x) - \zeta_1(x)]\,dx.$$

Let us find the increment Δ_χ after a certain time $T > 0$:

$$\begin{aligned}\Delta\chi &= \chi(t+T) - \chi(t)\\ &= e^{-\alpha t}\left\{e^{-\alpha T}\int_0^{t+T} e^{\alpha x}[\Delta_0 - \dot{\psi}(x) - \zeta_1(x)]\,dx\right.\\ &\left.- \int_0^t e^{\alpha x}\left[\Delta_0 - \dot{\psi}(x) - \zeta_1(x)\right]dx.\right. \end{aligned} \quad (36)$$

The mean value of this increment is

$$\langle\Delta\chi\rangle = \frac{\Delta_0}{\alpha}e^{-\alpha t}(1 - e^{-\alpha T}). \quad (37)$$

We see therefore that the mean value of the increment of the quantity χ in the stationary state ($t \to \infty$) is equal to zero, that is, there is no systematic divergence of the phases $\varphi(t)$ and $\varphi(t)$ with time.

Let us calculate now the variance $\sigma_{\Delta\chi}^2$ of the increment after a time T.

According to (36), we can write

$$\sigma_{\Delta\chi}^2 = \langle\Delta\chi^2\rangle - \langle\Delta\chi\rangle^2 = e^{-2\alpha T}\Big\langle\Big\{e^{-\alpha t}\int_0^{t+T} e^{\alpha x}[\Delta_0 - \dot{\psi}(x) - \zeta_1(x)]\,dx$$

$$- \int_0^t e^{\alpha x}[\Delta_0 - \dot{\psi}(x) - \zeta_1(x)]\,dx\Big\}\Big\{e^{-\alpha T}\int_0^{t+T} e^{\alpha y}$$

$$\times [\Delta_0 - \dot{\psi}(y) - \zeta_1(y)]\, dy - \int_0^t e^{\alpha y} [\Delta_0 - \dot{\psi}(y) - \zeta_1(y)]\, dy \Big\} \Big\rangle - \langle \Delta\chi \rangle^2.$$

In the calculations we must take it into account that the random processes $\dot{\psi}(t_1)$ and $\zeta_1(t_2)$ are independent of each other, that their mean values are equal to zero, and that the autocorrelation functions are given respectively by formulas (21) and (33).

Leaving out the intermediate calculations, we present the final result:

$$\sigma_{\Delta\chi}^2 = \frac{1}{2A_0^2}\left(N_0\alpha + \frac{\omega^2 N}{4\alpha}\right)(1 - e^{-\alpha T})[2 - e^{-2\alpha t}(1 - e^{-\alpha T})]. \quad (38)$$

It is seen from the formula that the variance $\sigma_{\Delta\chi}^2$ is the sum of two terms, the first due to the external noise and proportional to the noise intensity N_0 and to the bandwidth $\Delta f = \alpha/\pi$ of the tank circuit (at the 0.5 level), and the second due to the fluctuations of the phase of the self-oscillations, is proportional to the phase diffusion coefficient $D = \omega^2 N/4A_0^2$, and is inversely proportional to the bandwidth of the tank circuit.

As applied to the stationary state ($t \to \infty$), formula (38) simplifies somewhat:

$$\sigma_{\Delta\chi}^2 = \frac{1}{A_0^2}\left(N_0\alpha + \frac{\omega^2 N}{4\alpha}\right)(1 - e^{-\alpha T}). \quad (39)$$

At small time intervals ($\alpha T < 1$) the phase deviation satisfies a diffusion equation:

$$\sigma_{\Delta\chi}^2 = \frac{1}{A_0^2}\left(N_0\alpha + \frac{\omega^2 N}{4\alpha}\right)\alpha T. \quad (40)$$

For large time intervals ($\alpha T \gg 1$), the variance of the phase difference has a constant value

$$\sigma_{\Delta\chi}^2 = \frac{1}{A_0^2}\left(N_0\alpha + \frac{\omega^2 N}{4\alpha}\right). \quad (41)$$

It is interesting to note that (41) makes it possible to determine the optimum value of the damping coefficient of the tank circuit α, at which the variance has a minimum value. It is easy to verify that

$$\sigma_{\Delta\chi\,\min}^2 = \frac{5}{4}\omega\frac{\sqrt{NN_0}}{A_0^2} \quad \text{if} \quad \alpha_0 = \omega\sqrt{\frac{N}{N_0}}. \quad (42)$$

An analogous relation is valid in the linear approximation for the optimal synchronization band (from the point of view of interference immunity) in a "locked-in" self-oscillator or in the simplest phase-type AFC circuit. We can note incidentally that the principles underlying this question are still unclear.

For the synchronization system described by (18), the statistical characteristics can be calculated in analogous fashion. We exclude from consideration the internal phase fluctuations ($\dot{\psi} = 0$) and linearize (18), putting $\sin\varphi = \varphi$ and $\cos\varphi = 1$. We then obtain

$$\ddot{\varphi} + \alpha\dot{\varphi} + \alpha\Delta\left(1 - \frac{A_{syn}}{A_1}\right)\varphi = \alpha\Delta_0 - \frac{\alpha\Delta}{A_1}A_s.$$

In view of the smallness of the noise, we can neglect the term $A_{syn}/A_1 \ll 1$ in the left side of this equation. We then obtain

$$\ddot{\varphi} + \alpha\dot{\varphi} + \alpha\Delta\varphi = \alpha\Delta_0 - \alpha\Delta\frac{A_s}{A_1}. \quad (43)$$

Neglect of the component A_{syn} alone has a perfectly defined physical meaning. The point is that the cosine component leads to amplitude fluctuations of the signal, which we neglect, while the sinusoidal component A_s leads to phase fluctuations, to which the phase detector responds.

The right side of (43) contains the normal random function

$$\eta(t) = \alpha\Delta_0 - \alpha\Delta\frac{A_s(t)}{A_1}$$

with mean value $\langle\eta\rangle = \alpha\Delta_0$ and correlation function

$$k_\eta(\tau) = \left(\frac{\alpha\Delta}{A_1}\right)^2 k_s(\tau).$$

For the stationary state, the mean value of the phase difference can be obtained by averaging the right and left sides of (43):

$$\langle\varphi\rangle = \frac{\Delta_0}{\Delta}, \quad (44)$$

which coincides with the mean value of the phase difference for first-order systems.

Calculation of the phase-difference variance σ_φ^2 in the linearized second-order synchronization system leads to the following result:

$$\sigma_\varphi^2 = \frac{\Delta}{a^2\beta}, \quad (45)$$

where $a = A_1/\sigma$—signal/noise ratio at the input and $\beta = \pi\Delta f$—parameter characterizing the bandwidth of the selective system connected ahead of the phase detector.

It was assumed in the calculation that the bandwidth of the LPF is smaller than Δf. Such an assumption is satisfied in practice.

It is seen from (45) that the variance σ_φ^2 does not depend on the time constant of the LFP, if the assumption indicated above is satisfied. From this point of view it may turn out that the second-order synchronization system has the same interference immunity as the first-order system. However, it can be shown even with the aid of linear theory that this is actually not the case.

If we calculate with the aid of the procedure described above the mean value $\langle\Delta\varphi\rangle$ of the phase increment after a certain time T, then, as in the case of a first-order system, we find it to be equal to zero. On the other hand, the variance $\sigma_{\Delta\varphi}^2$ of the increment depends on the time constant of the LFP. For the case of practical interest when $\alpha \ll \beta$, the following approximate equality holds true:

$$\sigma_{\Delta\varphi}^2 \approx \frac{\Delta}{2a^2\beta}\alpha T. \quad (46)$$

We see that large time constants of the LPF correspond to a smaller variance of the phase-difference increment. This offers evidence of the better interference immunity of the synchronization system with low-pass filters, including RC filters.

4. NONLINEAR THEORY

In the case when the synchronization system operates with sufficiently large external noise or on the borderline of the synchronization band, the linear theory becomes unsuitable, since it does not yield even a qualitative picture. It is then necessary to consider the operation of synchronization devices in the nonlinear formulation, and it is expedient to make use of the Markov-process formalism.

It is known[25,37] that if some system with a time constant τ_{st} is acted upon by stationary random disturbances $\xi(t)$ with a small correlation time $\tau_c \ll \tau_{st}$, then the formalism of Markov processes can be used for the investigation of the behavior of such a system, particularly the Fokker-Planck-Kolmogorov equation. The first-order differential equation

$$\varphi = F[\varphi, \zeta(t)] \tag{47}$$

corresponds to the following Fokker-Planck-Kolmogorov equation for the univariate probability density of the function φ:

$$\frac{\partial w(\varphi, t)}{\partial t} = -\frac{\partial}{\partial\varphi}[K_1(\varphi) w(\varphi, t)] + \frac{1}{2}\frac{\partial^2}{\partial\varphi^2}[K_2(\varphi) w(\varphi, t)]. \tag{48}$$

The coefficients K_1 and K_2 are calculated by means of the formulas

$$K_1(\varphi) = \langle F[\varphi, \zeta(t)]\rangle,$$

$$K_2(\varphi) = \int_{-\infty}^{\infty} \{\langle F[\varphi, \zeta(t)] F[\varphi, \zeta(t+\tau)]\rangle - K_1^2(\varphi)\} d\tau. \tag{49}$$

If we are interested in the stationary distribution, then the right side of (48) must be equated to zero:

$$\frac{1}{2}\frac{d^2}{d\varphi^2}[K_2(\varphi) w(\varphi)] - \frac{d}{d\varphi}[K_1(\varphi) w(\varphi)] = 0. \tag{50}$$

The solution of (50) for a synchronized sélf-oscillator in the presence of an external fluctuation noise $\xi(t)$ with correlation function (20) is of the form[18,12]

$$w(\varphi) = \frac{1}{M}\exp(D_0\varphi + D\cos\varphi)\int_{\varphi}^{\varphi+2\pi}\exp(-D_0\gamma - D\cos\gamma)\, d\gamma. \tag{51}$$

Here M—normalization factor:

$$M = 4\pi^2 e^{-\pi D_0} |I_{iD_0}(D)|^2, \tag{52}$$

$I_{i\nu}(z)$—Bessel function of imaginary index and imaginary argument[38], the parameter D_0 characterizes the initial frequency deviation, and D characterizes the intensity of the acting fluctuations:

$$D_0 = \frac{8A_0^2\Delta_0}{N_0\omega^2}, \quad D = \frac{8A_0^2\Delta}{N_0\omega^2}. \tag{53}$$

It is seen from (51) that the probability density satisfies the natural periodicity condition $w(\varphi + 2\pi) = w(\varphi)$ and the condition of normalization over each period.

The integral in (51) cannot be expressed in terms of known functions. However, in the particular case of zero initial detuning ($D_0 = 0$) we obtain for $w(\varphi)$ the simple expression

$$w(\varphi) = \frac{1}{2\pi I_0(D)}\exp(D\cos\varphi). \tag{54}$$

Figure 4 shows plots of the probability densities for the phase of a synchronized self-oscillator in the presence of fluctuation noise. We see that the distribution has a symmetrical form with zero mean value. At large values of the noise (small D), the distribution tends to the uniform value over the interval $(-\pi, \pi)$. In the other extreme case, when the noise is negligibly small, a δ-like distribution is obtained.

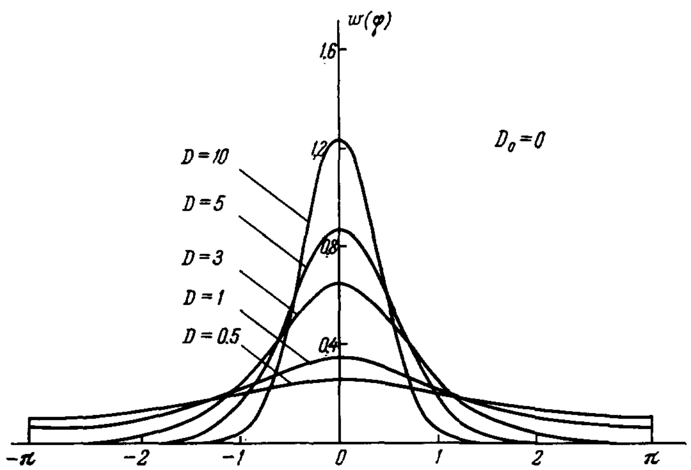

FIG. 4. Probability densities of the phase difference for a first-order synchronization system $\Delta_0 = 0$.

The presence of an initial detuning ($D_0 \neq 0$) leads to asymmetry of the distribution curve. This is seen in Fig. 5, which shows the probability densities $w(\varphi)$ corresponding to values D = 1 and 5, and to several values of the parameter D_0[12].

If we neglect the natural fluctuations of the generators ($\psi = 0$) and regard the external fluctuations $\xi(t)$ as sufficiently broadband quasiharmonic noise with a correlation function

$$k(\tau) = \sigma^2 e^{-\beta|\tau|}\cos\omega_0\tau, \tag{55}$$

we obtain for the phase-type automatic frequency control (AFC) system described by (16) the same results as above, except that now D_0 and D are determined by the relations

$$D_0 = a^2\frac{\Delta_0\beta}{\Delta^2}, \quad D = a^2\frac{\beta}{\Delta}, \tag{56}$$

where $a = A_1/\sigma$—signal/noise ratio.

For the phase-type AFC described by the second-order differential equation (18), we can write the corresponding Fokker-Planck-Kolmogorov equation[25]. If the external noise $\xi(t)$ is assumed to be quasiharmonic with a correlation function (55), then we can obtain an exact solution only in the absence of an

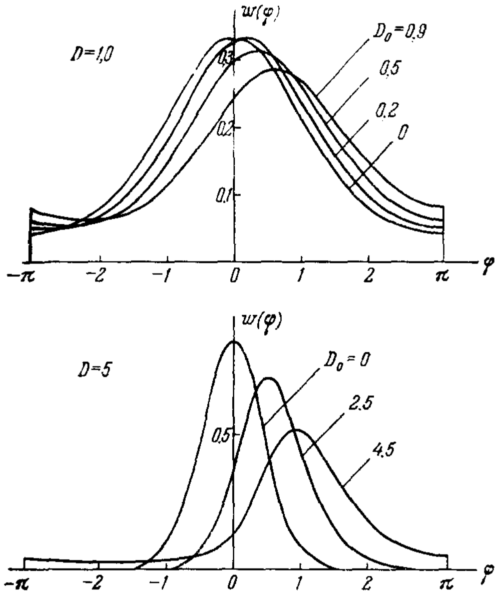

FIG. 5. Probability densities of the phase difference when $\Delta_0 \neq 0$.

initial frequency deviation ($\Delta_0 = 0$). This solution is of the form

$$w_2(\varphi, \dot{\varphi}) = \frac{a}{\Delta} \sqrt{\frac{\beta}{2\pi\alpha}} \frac{1}{2\pi I_0\left(a^2 \frac{\beta}{\Delta}\right)} \times \exp\left(-\frac{a^2\beta}{2\Delta^2}\dot{\varphi}^2 + \frac{a^2\beta}{\Delta}\cos\varphi\right), \qquad (57)$$

where $a = A_1/\sigma$—ratio of the signal amplitude to the mean square value of the noise, and $I_0(z)$—Bessel function of zero order of imaginary argument.

From (57) we obtain the univariate probability density for the phase difference

$$w(\varphi) = \int_{-\infty}^{\infty} w_2(\dot{\varphi}, \varphi)\, d\dot{\varphi} = \frac{1}{2\pi I_0\left(\frac{a^2\beta}{\Delta}\right)} \times \exp\left(\frac{a^2\beta}{\Delta}\cos\varphi\right) \qquad (58)$$

and the univariate probability density for the frequency difference

$$w(\dot{\varphi}) = \int_{-\pi}^{\pi} w_2(\varphi, \dot{\varphi})\, d\varphi = \frac{a}{\Delta}\sqrt{\frac{\beta}{2\pi\alpha}} \times \exp\left(-\frac{a^2\beta}{2\Delta^2\alpha}\dot{\varphi}^2\right). \qquad (59)$$

When $\Delta_0 \neq 0$ it is impossible to obtain an exact solution, and approximate methods were proposed [21,34]. A realization of one of these methods[21] yields the following approximate formulas for the univariate probability densities

$$w(\varphi) = \frac{1}{4\pi^2} e^{\pi D_0} |I_{iD_0}(D)|^{-2} \exp(D_0\varphi + D\cos\varphi) \times \int_{\varphi}^{\varphi+2\pi} \exp(-D_0\gamma - D\cos\gamma)\, d\gamma, \qquad (60)$$

$$w(\dot{\varphi}) = \frac{a}{\alpha\Delta}\left[\sqrt{\frac{\alpha\beta}{2\pi}} - \dot{\varphi}\sqrt{\frac{2\pi\beta}{\alpha}}\, \frac{1-\exp(2\pi D_0)}{4\pi^2\exp(\pi D_0)} \times |I_{iD_0}(D)|^{-2}\right] \exp\left(-\frac{a\beta}{2\alpha\Delta^2}\dot{\varphi}^2\right), \qquad (61)$$

where

$$D_0 = a^2\frac{\Delta_0\beta}{\Delta^2}, \quad D = a^2\frac{\beta}{\Delta}. \qquad (62)$$

Formula (60) coincides with the corresponding formula (51) for the first-order phase-type AFC. Therefore the plots of Fig. 5 can be regarded as constructed on the basis of (60).

It can be shown[18,20] that the mean values of the phase derivative $\langle\dot{\varphi}\rangle$ are determined for systems described by (15) and (16) from the formula

$$\langle\dot{\varphi}\rangle = \langle\omega\rangle - \omega_0 = \Delta_0 \frac{\operatorname{sh}\pi D_0}{\pi D_0} |I_{iD_0}(D)|^{-2}. \qquad (63)^*$$

The results of the calculations, made by I. G. Akopyan on the basis of this formula[12], are shown in Fig. 6. From an analysis of (63) and of the plots we see that at very large noise levels ($D \to 0$) the influence of the synchronizing signal is barely noticeable. In this case the mean frequency of the synchronizing generator (SG) turns out to be close to the natural frequency of the generator. In the other extreme case, when the noise is small compared with the signal, a normal synchronous mode is established, of course, if the initial detuning is smaller than the synchronization (holding) bandwidth.

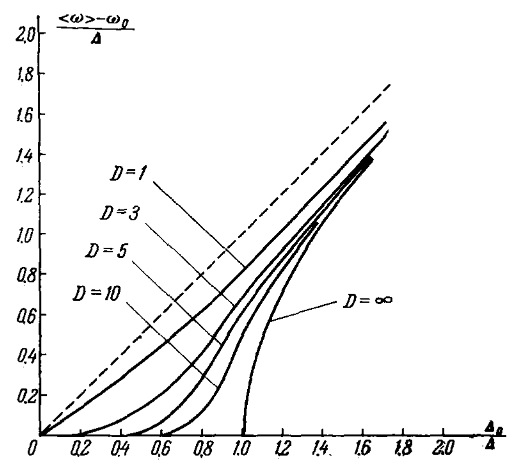

FIG. 6. Dependence of the average frequency shift on the initial deviation for small Δ_0.

If the inequality $\Delta_0 \ll \Delta$ is satisfied, we can determine $\langle\dot{\varphi}\rangle$ from the asymptotic expression of the Bessel function in (63), and obtain the simpler formula

$$\langle\dot{\varphi}\rangle = 2\Delta \operatorname{sh}\left(\pi\frac{\Delta_0}{\Delta}D\right)e^{-2D}. \qquad (64)$$

Figure 7 shows curves plotted in accordance with (64). We see that at zero initial detuning the mean value of the generator frequency difference is zero. This does not mean, however, that a synchronous mode is possible for any noise level in the system.

*sh = sinh

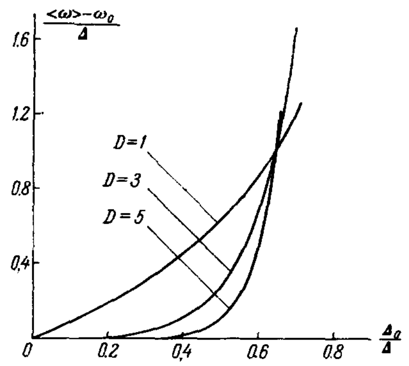

FIG. 7. Plot of the average frequency shift vs. initial deviation for small Δ_0.

For a second-order phase-type AFC we obtain on the basis of (61) the same expression for the mean value of the frequency difference as from (63). In this case, however, formula (63) must be regarded as approximate. In such an approximation, the difference between the first- and second-order systems lies in the values of the variance $\sigma^2_{\dot\varphi}$ of the frequency difference.

For first-order systems with zero initial frequency deviation and $\dot\psi = 0$, we can obtain from (16) the following formula

$$\sigma^2_{\dot\varphi} = \frac{1}{\beta}\left(\Delta\frac{\sigma}{A_1}\right)^2. \qquad (65)$$

The variance of the frequency difference for the second-order system at zero initial detuning is obtained from the expression (59)

$$\sigma^2_{\dot\varphi} = \frac{\alpha}{\beta}\left(\Delta\frac{\sigma}{A_1}\right)^2. \qquad (66)$$

We see that the synchronization system with RC low-pass filter is more immune to interference than the locked-in self-oscillator and phase-type AFC with ideal low-frequency filter. The larger the time constant of the system, the smaller the variance $\sigma^2_{\dot\varphi}$. This result is directly related with the deduction of the linear theory: the deviation of the phase variance accumulated after a certain time decreases with increasing time constant of the LPF.

5. STATISTICAL DYNAMICS

It can be seen from (63) that in the stationary state, in the presence of an initial frequency deviation, the applied noise shifts the mean value of the frequency of the synchronized generator, relative to the mean value of the frequency of the synchronizing signal. The resultant value of the residual frequency deviation is determined by the noise level, by the initial frequency deviation, and by the system parameters.

The physical cause of the frequency deviation can be explained by using automatic phase control as an example, and by considering random-function bursts.[39]

In a closed phase-type AFC, the stable equilibrium states correspond to the values

$$\varphi_k = \frac{\pi}{2} \pm 2k\pi \quad (k = 0, 1, 2, \ldots).$$

The constant $\pi/2$ phase difference between the reference and synchronizing generators is due to the phase detector. Deviations of the phase difference from any specified value φ_k are permissible only within the limits $\pm\pi/2$. In the opposite case it is either impossible to synchronize the system, or else the system will operate in the vicinity of some other stable state, and on going over into the latter state the phase of the synchronized generator will increase by an integral number of cycles ($\pm 2k\pi$).

If the noise causes the fluctuation of the phase difference $\varphi(t)$ to exceed the level $\Phi = \pm\pi/2$, then the system will respond to the random external signal at the instant when Φ goes through the levels $\pm\pi/2$. When $\Delta_0 \neq 0$, the initial frequency deviation will cause the mean system response to be unidirectional with random oscillations determined by the noise.

After the lapse of a certain time, which is determined by the relation between the transient time of the system and the amplitude and duration of the burst $\varphi(t)$ in excess of the level Φ, the operating point returns to the stable state, and a stationary value of φ is established after termination of the transient process.

During the duration of each burst, the phase of the SG increases by 2π, 4π, 6π, etc. This phenomenon can be called a phase jump, although it must be borne in mind that each such jump occurs not instantaneously but after a finite time interval.

At zero initial detuning ($\Delta_0 = 0$) the probability of exceeding the levels $\Phi = \pi/2$ and $\Phi = -\pi/2$ is the same, by virtue of the symmetry of the distribution $w(\varphi)$ (Fig. 4). Therefore the number of phase jumps in both directions is also equal, and consequently $\langle\dot\varphi\rangle = 0$. This explains the absence of an average frequency divergence between the reference and signal generators, although the mode at $\Delta_0 = 0$ cannot be regarded as synchronous.

Strictly speaking, there is a finite probability of phase jumps even under very small random disturbances. For practical purposes, it is convenient to assume that a synchronous mode is established in the system if the probability of the jumps does not exceed some permissible value. This means that the quantity of interest to us is followed-up with practically no frequency error. On the other hand, if the probability of the jumps exceeds the permissible value, then the operating mode of the system must be regarded as asynchronous. Possible quantitative criteria of transition from one mode to another are given in[22].

The presence of initial detuning leads to asymmetry in the distribution $w(\varphi)$ (Fig. 5). The probability that the fluctuations of $\varphi(t)$ will exceed the level

$\Phi = \pi/2$ is not equal to the probability of exceeding the level $\Phi = -\pi/2$. Consequently, the number of phase jumps will not be the same in both directions. The difference in the number of jumps determines the residual frequency detuning, and the sign of the average frequency shift of the SG coincides with the sign of the initial frequency deviation.

The average number of phase jumps per unit time can be calculated if the average number of fluctuation bursts $\varphi(t)$ exceeding the level Φ is known. The average number of bursts at the level Φ per unit time is determined by the formula[39]

$$N(\Phi)=\int_0^\infty \dot{\varphi}\, w_2(\Phi, \dot{\varphi})\, d\dot{\varphi}. \tag{67}$$

We shall consider a case when the fluctuation bursts $\varphi(t)$ are relatively rare, that is, when the average interval $\bar{\theta}$ between the bursts that exceed the level Φ is much larger than the average duration $\bar{\tau}$ of the bursts, $\bar{\theta} \gg \bar{\tau}$.

This is precisely the case of greatest interest, since the opposite case corresponds to very large noise, when operation of the synchronization system is practically impossible.

Let us find the average number of bursts of the function $\varphi(t)$ exceeding the level $\Phi = \pi/2$ at zero initial frequency deviation. To this end we substitute in (67) the expression for $w_2(\pi/2, \dot{\varphi})$ from (57). After simple transformations we obtain

$$N\left(\frac{\pi}{2}\right)=N\left(-\frac{\pi}{2}\right)=\frac{\Delta}{a}\sqrt{\frac{\alpha}{2\pi\beta}}\,\frac{1}{2\pi I_0\left(\frac{a^2\beta}{\Delta}\right)}. \tag{68}$$

We note that not every such burst involves a phase jump. When the burst duration is short, the operating point can return to the same region of stable state from which the fluctuation of $\varphi(t)$ has driven it out. For zero initial detuning, the probability of phase jumps due to the $\varphi(t)$ bursts exceeding the level Φ can be set equal to 0.5. This assumption is in agreement with the physical picture of the effect of noise on a locked-in self-oscillator, given in [25].

Taking the foregoing into account, we obtain for the number N of phase jumps per unit time, referred to the holding band, a value

$$\frac{N}{\Delta}=\frac{1}{a}\sqrt{\frac{\alpha}{2\pi\beta}}\,\frac{1}{4\pi I_0\left(\frac{a^2\beta}{\Delta}\right)}. \tag{69}$$

Formula (69) was used to plot the relative number of phase jumps vs. signal/noise ratio (Fig. 8).

We see that the number of phase jumps depends essentially on the time constant of the filter, and that the number N decreases with increase in $T = RC = 1/\alpha$. This is due to the fact that with increasing filter time constant the fluctuations of $\varphi(t)$ become slower, the number of bursts exceeding the level Φ decreases, and consequently N decreases. Formula (69) demonstrates the relatively poor efficiency of the RC filter. We must therefore find other types of filters that would ensure better interference immunity of synchronization systems at large noise values. Of interest from this point of view is a nonlinear filter[30,34], and also a circuit with an integrator.

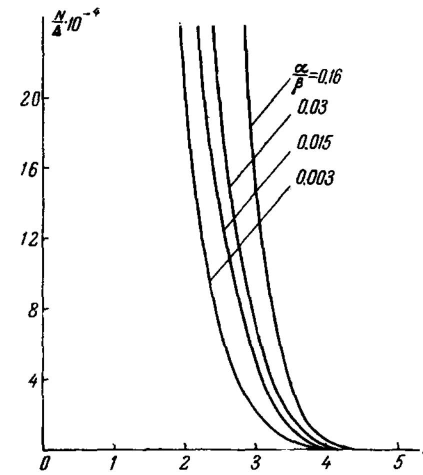

FIG. 8. Dependence of the number of phase jumps on the signal/noise ratio at detunings close to zero.

When the initial frequency deviation is not equal to zero, the number of phase jumps is not the same in the opposite directions. Calculations by means of (67) give for the relative number of phase jumps in opposite directions at the level Φ, the following formulas:

$$\frac{N_1}{\Delta}=\frac{C}{a}\left[\exp\left(\frac{3}{2}\pi D_0\right)\sqrt{\frac{\alpha}{2\pi\beta}}\times\int_{\pi/2}^{3/2\pi}\exp(-D_0\gamma-D\cos\gamma)\,d\gamma+\frac{\Delta}{2a\beta}\,\mathrm{sh}\,(\pi D_0)\right],$$

$$\frac{N_2}{\Delta}=\frac{C}{a}\left[\exp\left(\frac{\pi}{2}D_0\right)\sqrt{\frac{\alpha}{2\pi\beta}}\times\int_{-\pi/2}^{3/2\pi}\exp(-D_0\gamma-D\cos\gamma)\,d\gamma+\frac{\Delta}{2a\beta}\,\mathrm{sh}\,(\pi D_0)\right],$$

where $C = (1/8)\pi^2 |I_{iD_0}(D)|^2$.

The difference

$$\frac{N_1-N_2}{\Delta}=\frac{C}{a}\sqrt{\frac{\alpha}{2\pi\beta}}\exp\left(\frac{\pi}{2}D_0\right)\times\left[e^{\pi D_0}\int_{\pi/2}^{3\pi/2}e^{-(D_0\gamma+D\cos\gamma)}\,d\gamma-\int_{-\pi/2}^{3\pi/2}e^{-(D_0\gamma+D\cos\gamma)}\,d\gamma\right] \tag{70}$$

gives an idea of the average frequency shift of the synchronized generator due to the noise.

Figure 9 shows the dependence of the relative number of phase jumps in opposite directions on the signal/noise ratio. The shaded area corresponds to the difference $(N_1 - N_2)/\Delta$. We see that with decreasing level of the interacting noise, a decrease takes place not only in the number of probable jumps, but also in the difference $N_1 - N_2$, that is, in the residual frequency deviation.

An analysis of relations (69) and (70) leads to the

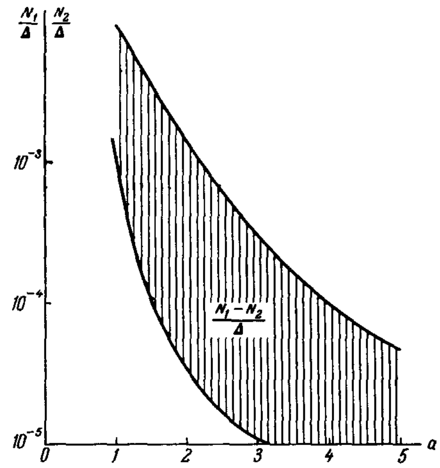

FIG. 9. Number of phase jumps in different directions $\Delta_0 \neq 0$.

following conclusions. First, in the presence of an initial frequency deviation the number of phase jumps in the direction corresponding to the sign of Δ_0 is always larger than the number of jumps in one direction when $\Delta_0 = 0$, provided the remaining parameters of the system and the signal/noise ratio remain the same.

Second, with increasing initial frequency deviation, a decrease takes place in the permissible value of the signal/noise ratio at which phase jumps begin to appear. This means that at detunings close to the synchronization limits phase jumps are possible even in the case of small noise.

Consequently, synchronization devices operate with highest stability at frequency deviations Δ_0 close to zero; when the phase error is likewise at a minimum.

6. METHODS OF EXPERIMENTAL INVESTIGATION OF THE INFLUENCE OF NOISE ON THE OPERATING ACCURACY OF SYNCHRONIZATION DEVICES

A correct understanding of the results of interference-immunity investigations is possible only if a clear-cut idea is gained of the mechanism of the noise. Comprehensive experimental investigations are of help in gaining such an idea. In addition, experiment makes it possible to check the theoretical results and to analyze questions not amenable to a theoretical analysis.

A distinguishing feature of experimental research on the effect of noise on synchronization devices is that it becomes necessary to deal with such a "non-energy" signal characteristic as the phase. This circumstance forces us to seek indirect methods of separating the phase (or the phase difference).

The most successful method was proposed in [12]. Its gist can be understood by examining Fig. 10, which shows the functional diagram of a phase-type AFC with additional elements needed for phase separation.

A flipflop (Ff) transforms the oscillations of the standard generator into a square wave having the

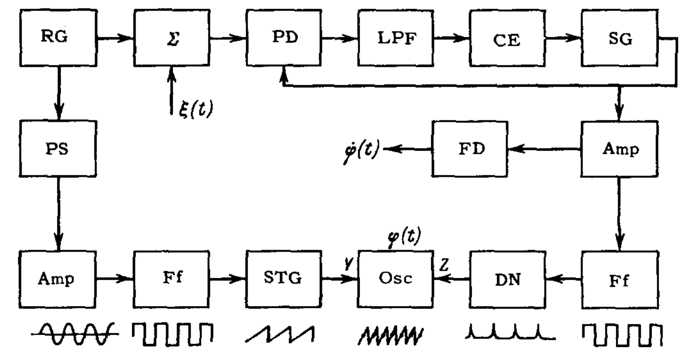

FIG. 10. Block diagram of typical experimental setup.

same repetition period as the reference generator (RG). The leading front of the positive pulse triggers the sawtooth voltage generator (STG). In this case the phase of the sawtooth generator is rigidly connected with the phase of the RG oscillations. This voltage is applied to the Y input of the vertical oscilloscope amplifier.

The oscillations of the synchronized generator are also transformed into a square wave, which is transformed in the differentiator (DN) into very narrow pulses corresponding to the leading fronts of the positive pulses. These are fed to the Z input of the oscilloscope. Intensity modulation superimposes clear and bright points onto the sawtooth pattern on the oscilloscope screen. If the phase difference between the reference and signal generators is zero, then the points are located at the very start of the sawtooth. If the oscillations are not in phase the brightness markers are located in the linear portion of the sawtooth pattern. The phase shift can be measured, since the entire swing of the sawtooth corresponds to 2π.

By reducing the sweep rate of the oscilloscope beam in the horizontal direction, it is possible to merge the sawtooth voltage traces into a continuous raster, on the background of which one can see the line formed by the individual intensity pips. The position of the line is determined by the phase difference between the reference and signal generators. To compensate for the phase shift in the circuit elements, and also to locate the initial phase line at the center of the raster, a phase shifting network is connected in one of the branches of the circuit controlling the oscilloscope beam. The brightness control makes it possible to remove the raster image, leaving only the phase position line visible on the screen.

If a phase-type AFC circuit is subjected to both a signal and a noise $\xi(t)$, phase fluctuations are observed on the oscilloscope screen. Large noise causes phase jumps amounting to an integral number of periods. On the screen, each phase jump is displayed by a drift of the representative point from the raster and its reappearance on the other side.

Figure 11 shows by way of an example oscillograms of the phase of the synchronized generator in a phase-type AFC system with RC filters. Oscillo-

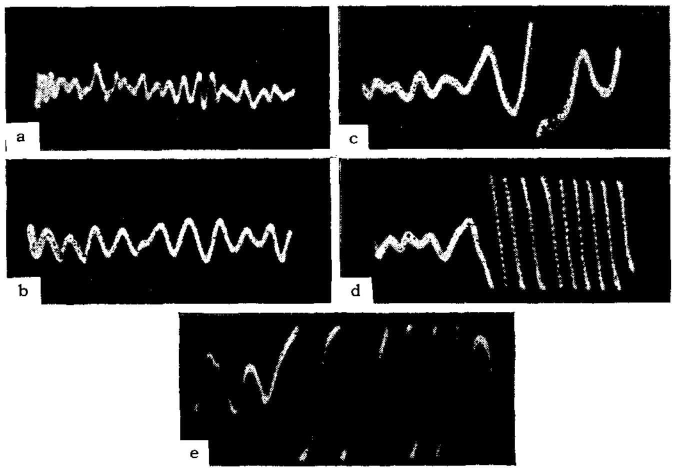

FIG. 11. Phase oscillograms in a system of synchronization under the influence of fluctuation noise.

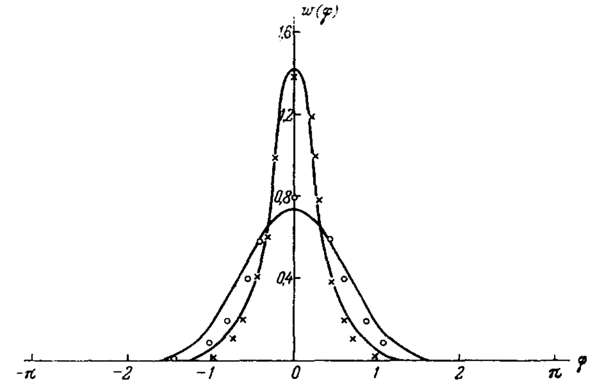

FIG. 12. Experimentally plotted probability densities of the phase difference, for $\Delta_0 \approx 0$.

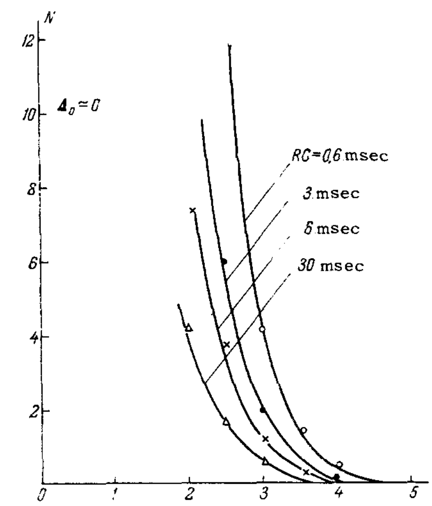

FIG. 13. Experimental dependence of the number of phase jumps on the signal/noise ratio for several time constants of the low pass filter.

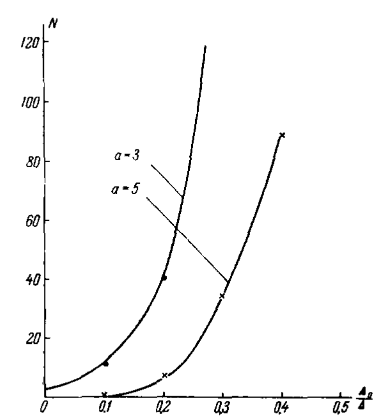

FIG. 14. Dependence of the number of phase jumps on the initial detuning, obtained by experimental means.

grams a and b correspond to phase fluctuations due to normal external noise, with oscillogram b corresponding to a large time constant of the RC filter (the sweep rate is the same in both cases). Photograph c shows a single phase jump of 2π. The presence of a large initial frequency deviation causes several phase jumps in one direction, as can be seen on photographs d and e, taken for detunings with opposite signs.

The procedure described, in addition to having great clarity, also affords the possibility of experimentally plotting univariate probability densities (for example, photometrically), to count the number of phase jumps, and to investigate, if further modified, nonstationary processes in synchronization systems. It is possible to investigate analogously the effect of modulated signals, harmonic noise, and other types of noise on synchronization devices.

To measure the statistical characteristics of the phase derivative, it is sufficient to connect a frequency discriminator (FD) to the synchronized generator.

Measurement of the locking and holding bands, and also an exact determination of the mean value of the synchronized-generator frequency under the influence of noise, are best made with electronic frequency meters operating on the principle of counting the number of "zeroes."

Figure 12 shows the experimentally plotted probability densities $w(\varphi)$ for a second-order phase-type AFC system. The points shown on the figure correspond to the theoretical deviation of the curves. We see that the agreement between the theoretical and experimental results is perfectly satisfactory.

The experimentally plotted dependence of the number of phase jumps in one direction against the signal/noise ratio, for different time constants of the RC filter, is shown in Fig. 13. The initial frequency deviation was in this case close to zero. The qualitative agreement between the experimental dependence $N(a)$ and the theoretical one (Fig. 8) is further evidence of the expediency of the described experimental procedure.

At a fixed noise level (a = const), the number of phase jumps depends on the magnitude of the initial detuning, as can be seen from Fig. 14.

7. EFFECT OF FLUCTUATIONS ON THE OPERATION OF RELAXATION OSCILLATORS

Let us consider qualitatively the influence of internal fluctuations on the operation of relaxation oscillators, which are widely used in pulsed radars, oscilloscopes, and other types of apparatus. By way of an example we shall consider a symmetrical multivibrator (Fig. 15).

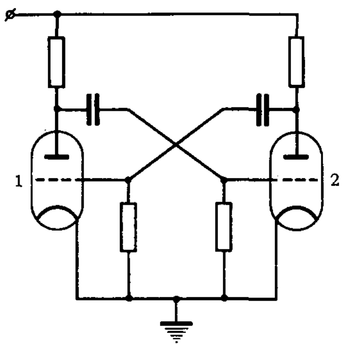

FIG. 15. Diagram of symmetrical multivibrator.

In the absence of fluctuations, the operation of the multivibrator is characterized by the fact that tubes 1 and 2 alternately conduct and are cut-off at equal fixed time intervals T/2, determined by the circuit parameters. The output of the multivibrator is then a discontinuous oscillation, constituting a periodic sequence of almost rectangular pulses with a repetition period T. The spectrum of the oscillations is discrete.

If we take into account the fluctuations of the circuit elements themselves (in particular, the tube shot noise), then the tube will not start and stop at rigidly fixed instants of time, but at instants that are subject to a random "flicker." As a result, the durations of the pulses generated by the multivibrator as well as the pulse repetition periods will be subject to fluctuations[43,44].

A characteristic feature of these fluctuations is that they accumulate[45]. If, for example, the intrinsic fluctuations have caused a "premature" conduction of tube 1, then this does not of necessity give rise to a likewise premature cutoff of the tube; its cutoff instant will be random, not connected statistically with the conduction instant. Therefore the variance σ_N^2 of the instant of appearance of the N-th pulse, being the variance of a sum of N independent random quantities, increases in proportion to N:

$$\sigma_N^2 = N\sigma_1^2.$$

In other words, we are observing here an effect analogous to the time growth of the variance of the phase in sinusoidal self-oscillating systems. As a result, the self oscillations of the multivibrator become aperiodic and the oscillation spectrum becomes continuous[46].

If we define σ_1^2 as the variance of the duration of a single pulse, then quantitative estimates show[43] that the relative value of $2\sigma_1/T$ is of the order of 10^{-5}.

8. INTERFERENCE IMMUNITY OF PULSED METHODS OF SYNCHRONIZATION

The need for pulsed synchronization of radio devices arises in radio communication, in radiotelemetry, and in other fields. In the simplest variant, the purpose and the operation of a pulse-synchronized radio line consist in the following.

Assume that some device transmits not only information pulse signals (which carry useful information), but also one or several synchronization pulses. It is assumed that the synchronization pulses precede the information pulses in time, and are rigidly connected with the latter. The synchronization pulses serve to "prepare" the information receiver for the reception of information pulses only. They can be received by the same receiver or else by a special synchronization receiver. The latter variant is preferable, since it provides increased interference immunity.

The presence of noise results in different time relations between the information pulses and the synchronization pulses on the transmitting and receiving ends. Any such mismatch is always undesirable.

Without stopping to analyze in detail different technical methods of pulsed synchronization and the corresponding systems, let us consider one simple concrete example, which enables us to explain the nature of the problem.

Let the synchronization signal $s_1(t)$ be a video pulse of rectangular form with known amplitude A and duration τ_u. It is received against a white noise background $\xi_1(t)$ (with spectral intensity N_0) by a special receiver, which is optimal with respect to interference immunity[40,41]. The synchronization pulse from the output of this receiver acts on an electronic relay, which should "turn on" the information receiver, via a special device, only during the time interval when the information pulses should appear.

By electronic relay is meant here a device that operates whenever the voltage applied to it exceeds a certain threshold value H.

As is well known[42], the optimal synchronization receiver for this case is a linear filter matched to the signal. The signal $s_2(t)$ at the output of the matched filter coincides in form with the autocorrelation function of the input signal, while the correlation function of the output of the normal noise $\xi_2(t)$ has the form of the autocorrelation function of the input signal.

The maximum peak value of the output pulse $s_2(t)$ is equal to

$$s_{2m} = kE, \quad E = \int_{-\infty}^{\infty} s_1^2(t)\,dt = A^2\tau_u, \tag{71}$$

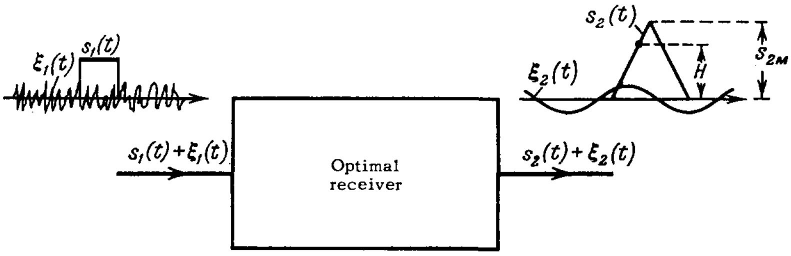

FIG. 16. Operating principle of optimal receiver.

where E—"energy" of the signal and k—some amplification coefficient.

The variance of the normal noise $\xi_2(t)$ at the output of the matched filter is

$$\sigma_2^2 = k^2 \frac{N_0 E}{2}. \tag{72}$$

Consequently, the ratio of the peak value of the signal to the mean square value of the noise is

$$\frac{s_{2m}}{\sigma_2} = \sqrt{\frac{2E}{N_0}}. \tag{73}$$

Figure 16 shows the input synchronization signal $s_1(t)$ and the input white noise $\xi_1(t)$, and also the signal $s_2(t)$ and the noise $\xi_2(t)$ at the output of the synchronization receiver.

If we disregard the signal distortion during propagation and the distortion due to various technical instabilities of the apparatus, then, in the absence of noise, for we can always choose for a specified signal energy E a relay operation threshold

$$H < s_{2m} = kE, \tag{74}$$

which determines exactly ("rigidly") the instant of relay operation.

In the presence of noise, such a rigid fixation of the instant of operation of the relay is practically impossible.

Depending on the ratio of the threshold voltage H to the noise level σ_2, two cases should be distinguished:

1. If the noise level is small compared with the threshold voltage ($\sigma_2 \ll H$), we can disregard the highly improbable false operations of the relay. The noise will cause a "flickering" of the instant of relay operation, and by the same token will disturb the rigid time relation between the instant of relay operation and the information pulses.

It can be shown[43] that the mean-square value σ_0 of the random instant of operation of the relay is determined by the formula

$$\sigma_0 = \frac{\sigma_2}{S_H}, \quad S_H = \left.\frac{ds_2(t)}{dt}\right|_{t=t_0}, \tag{75}$$

where the instant of time t_0 is determined from the equation $s_2(t_0) = H$. For a rectangular synchronizing pulse, formula (75) takes the form

$$\sigma_0 = \tau_u \frac{\sigma_2}{s_{2m}} = \tau_u \sqrt{\frac{N_0}{2E}}. \tag{76}$$

It is seen from this formula that to reduce the "flicker" of the instant of operation of the relay, at a fixed energy E of the rectangular synchronizing pulse, it is necessary to reduce the duration of the pulse, that is, to increase the slope S_H of the output pulse.

2. When the level of the noise is comparable with or larger than the threshold value ($\sigma_2 \geq H$), false operations of the relay will occur for the most part in the interval between the synchronization pulses, particularly if their off-duty factor is large[47]. If we denote the interval between neighboring pulses by T, then the average number of false operations within a time T is determined by a well-known formula[39], which can be written in the form

$$n_T(H) = \mu T \exp\left(-\frac{H^2}{2\sigma_2^2}\right), \tag{77}$$

where the coefficient μ is defined in terms of the noise correlation function. (We do not consider here the feasibility of calculating this coefficient mathematically.)

It is seen from (77) that to reduce the number of false operations it is necessary to increase to the utmost the threshold H. In accordance with (74), this calls for an increase in the energy of the synchronizing pulse. The energy can be increased by increasing the radiated peak power or by lengthening the pulse. The increase in the peak power is limited by the technical capabilities (so-called "peak" limitations), and an increase in the pulse duration is undesirable from the point of view of the instability of the instant of relay operation, caused by the noise (76).

From the foregoing very simple example we can now see clearly the requirements that must be imposed on the wave form of signals used in pulsed-synchronization lines. With the peak radiation power limited, the synchronization signals must have the necessary energy E and as "narrow" an autocorrelation function as possible. Such requirements are usually imposed on radar signals when it is necessary to obtain high target-range measurement accuracy and high resolution in range.

These requirements are satisfied to a known degree by special signals with intra-pulse modulation (in particular, phase manipulation). These include signals constructed on the basis of the codes of Barker[48,49], Huffman[50], etc.

The principle of construction of such signals consists roughly speaking in the following. Starting from "peak" limitations, one determines the overall duration of the pulse τ_u at which the required signal energy E is attained. The time interval τ_u is broken up into n elementary sub-intervals of length $\Delta = \tau_u/n$. For example, the phase of the high-frequency oscillation at each of the elementary sub-intervals can assume two values: φ and $\varphi \pm \pi$. By choosing the number n and the phase alternation sequence in each of the elementary sub-intervals, we can obtain a signal with a narrow correlation function. Thus, for the Barker code with n = 13, the signal autocorrelation function (hence the signal at the output of the optimal receiver) has a fundamental narrow peak in the form of an isosceles triangle with a base 2Δ, and 12 identical "lobes" of the same form, but the height of each lobe is $1/13$ of the height of the main peak.

Detailed information on methods of formation and reception of such signals can be found in the periodical literature.

Note added in proof. Recently the field of application of synchronization devices (in particular, phase-type automatic frequency control), has broadened further. It turns out that certain modifications of phase-type automatic frequency control are optimal for interference-immune reception of continuous stochastic signals against a background of fluctuating noise[52].

[1] V. V. Migulin, UFN **33**, 353 (1947).

[2] Antonov, Korshunov, Meleshko, and Panasyuk, PTÉ no. **6**, 20 (1959).

[3] Zarubezhnaya radioélektronika (Foreign Electronics) **9**, no. 2 (1959) [Probably: K. Rohrich, Z. Flugwiss. **6**, 266 (1958)].

[4] P. V. Shmakov, Televideniye (Television), Svyaz'izdat, 1960.

[5] Yu. P. Bakaev, Radiotekhnika i élektronika **3**, 227 (1958).

[6] N. L. Teplov, Élektrosvyaz' (Elect. Communication) **1**, 28 (1959).

[7] Berzunov, Lobanov, and Semenov, Odnopolosnaya modulyatsiya (Single-band Modulation), Svyaz'izdat, 1962.

[8] I. R. Costas, IRE Trans. CS-7 (1), (1957).

[9] B. Van der Pol, Phil. Mag. Ser. **7**, 3 (13), 65 (1927).

[10] A. Andronow and A. Witt, Arch. f. Elektr. **24**, 99 (1930).

[11] A Andronov and A. Vitt, Zh. prikl. fiz. (J. of Appl. Phys.) **6** (4), 3 (1930).

[12] I. G. Akopyan, Candidate's Dissertation, Moscow State Univ. 1959.

[13] Andronov, Pontryagin, and Vitt, JETP **3** (3), 165 (1933).

[14] I. L. Bershteĭn, DAN SSSR **20** (1), 11 (1938).

[15] I. L. Bershteĭn, ZhTF **11** (4), 305 (1941).

[16] I. L. Bershteĭn, Izv. AN SSSR, ser. fiz. **14** (2), 145 (1950).

[17] S. M. Rytov, JETP **29**, 304 (1955), Soviet Phys. JETP **2**, 217 (1956).

[18] R. L. Stratonovich, Radiotekhnika i élektronika **3**, 497 (1958).

[19] I. G. Akopyan and P. S. Landa, ibid. **7**, 1285 (1962).

[20] V. I. Tikhonov, Avtomatika i telemekhanika **20**, 1188 (1959).

[21] V. I. Tikhonov, ibid. **21**, 301 (1960).

[22] V. I. Tikhonov and K. B. Chelyshev, Radiotekhnika i élektronika **8**, 331 (1963).

[23] G. S. Gorelik, Izv. AN SSSR ser. fiz. **14** (2), 187 (1950).

[24] Kuznetsov, Stratonovich, and Tikhonov, DAN SSSR **97** (4), 639 (1954).

[25] R. L. Stratonovich, Izbrannye voprosy fluktuatsiĭ v radiotekhnike (Selected Problems in Fluctuations in Radio), Soviet Radio, 1961.

[26] V. I. Tikhonov, Radiotekhnika i élektronika **2**, 502 (1957).

[27] N. N. Bogolyubov and Yu. A. Mitropol'skiĭ, Asimptoticheskie metody v teorii nelineĭnykh kolebaniĭ (Asymptotic Methods in the Theory of Nonlinear Oscillations), Gostekhizdat, 1955.

[28] V. I. Tikhonov, Radiotekhnika i élektronika **6**, 1082 (1961).

[29] M. R. Kaplanov and V. A. Levin, Avotomaticheskaya postroĭka chastoty (Automatic Frequency Control), Gosenergoizdat, 1962.

[30] M. V. Kapranov, Nauchn. dokl. vyssh. shkoly (Science Reports of the Universities, Radio and Electronics), **5** (11), 1774 (1960).

[31] V. V. Shakhgil'dyan, Élektrosvyaz' **9**, 22 (1961).

[32] V. V. Shakhgil'dyan, Radiotekhnika **16** (19), 28 (1961).

[33] Kapranov, Ivanov, and Ivanova, Radiotekhnika i élektronika **5**, 1774 (1960).

[34] Yu. N. Bakaev, Avtomatika i telemekhanika **23**, 1179 (1962).

[35] G. S. Gorelik and G. A. Elkin, Radiotekhnika i élektronika **2**, 28 (1957).

[36] V. I. Tikhonov, Fluktuatsionnye protsessy (Fluctuation Processes), Press of Zhukovskiĭ Aviation Institute, 1961.

[37] Kuznetsov, Stratonovich, and Tikhonov, JETP **26** (2), 189 (1954).

[38] S. P. Morgan, Tables of Bessel Functions of Imaginary Order and Imaginary Argument, Inst. of Techn., California, 1947.

[39] V. I. Tikhonov, UFN **77**, 449 (1962), Soviet Phys. Uspekhi **5**, 594 (1963).

[40] V. A. Kotel'nikov, Teoria potentsial'noĭ pomekhustoĭchivosti (Theory of Maximum Interference Immunity), M. Gosenergoizdat 1956.

[41] S. E. Fal'kovich, Priem radiolokatsionnykh signalov na fone fluktuatsionnykh pomekh (Reception of Radar Signals Against Fluctuating Noise), Soviet Radio, 1961.
[42] J. L. Turin, Matched Filters (Russ. Transl.) in: Zarubezhnaya elektronika (Foreign Electronics) 3, 30 (1961).
[43] V. I. Tikhonov, Vestnik, Moscow State University 5, 31 (1956).
[44] I. M. Kogan and I. B. Pogozhev, Radiotekhnika **14** (10), 57 (1959).
[45] I. N. Amiantov and V. I. Tikhonov, Radioetekhnika **14** (4), 9 (1959).
[46] B. S. Tsybakov and V. P. Yakovlev, Radiotekhnika i élektronika **4**, 543 (1959).
[47] V. I. Tikhonov, ibid. **2**, 31 (1956).
[48] R. H. Barker, Communication Theory, Butters-worth, 1953.
[49] H. Sherman, IRE Trans. IT-2 (1), 24 (1956).
[50] D. A. Huffman, Synthesis of Linear Sequential Coding Networks, MIT, Cambridge, 1955.
[51] Weatherby, Proc. IEEE 51 (12), 1737 (1963).
[52] N. K. Kul'man and R. L. Stratonovich, Radiotekhnika i élektronika **9**, 67 (1964).

Translated by J. G. Adashko