SOVIET PHYSICS USPEKHI VOLUME 8, NUMBER 3 NOVEMBER, DECEMBER 1965
539.12.01

$K_2 \to 2\pi$ *DECAY AND POSSIBLE CP-NONCONSERVATION*

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Usp. Fiz. Nauk **86**, 231-262 (June, 1965)

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INTRODUCTION

In August, 1964, at the International Conference on High Energy Physics at Dubna, came the first announcement of the sensational results obtained by a group of experimenters at Princeton University in the USA. Christenson, Cronin, Fitch and Turlay had observed the decay into two π mesons of the longlived component of a neutral K beam obtained at the Brookhaven proton accelerator.

The K meson, which is created within a time of 10^{-23} sec in nuclear collisions, is a state with definite strangeness $S = 1$. Its antiparticle $\bar{K}$ has strangeness $S = -1$ and, since the weak interaction can change strangeness, transitions $K \rightleftarrows \bar{K}$ occur. Despite the fact that in such transitions the strangeness changes by two units and that the transitions occur in second order in the weak interaction, there is nevertheless a strong readjustment of the K and $\bar{K}$ states since the corresponding levels are degenerate.

If CP invariance were strict, the states between which there are no transitions and which have a definite mass and lifetime would be the CP-even and CP-odd combinations of the K and $\bar{K}$ mesons. They are called K_1 and K_2 mesons, respectively. These states are respectively the short- and longlived components of the K beam. The lifetimes of the K_1 and K_2 are markedly different. This is related to the fact that the K_1 can decay into two π mesons, while for the K_2 such a decay is forbidden by the conservation of CP parity. The lifetime of the K_1 is approximately 600 times smaller than that of the K_2. Thus the K_1 mesons should rapidly disappear from a K meson beam, and at times much greater than the lifetime for K_1 decay into two π's the K_1's should no longer be observed. Until recently this was strongly supported by the experiments and served as the basic argument in favor of CP conservation.

In the experiment of the Princeton group, [1] they looked for $K \to 2\pi$ decays at a distance of ~ 19 m from the point of creation of the K mesons. For a momentum of $p_K \sim 1.1$ GeV/c (which corresponds to a meson velocity $v \sim 0.91c$), this distance is ~ 300 decay lengths for the K_1. If the experiment were done in vacuum, one could assert that the K_1 mesons should vanish completely from the beam (their residue in the beam $\sim e^{-300}$). Nevertheless decays into two π's were observed in the experiment with a probability 2.6 $\times 10^4$ sec^{-1} (per K_2 meson in the beam). This means that: 1) either there is a contamination of K_1 mesons of order 4×10^{-4}% at large distances from the point of creation (since the probability of $K_1 \to 2\pi$ decay is $\sim 0.7 \times 10^{10}$ sec^{-1}), or 2) the K_2 can decay into two π's.

It is important to point out that the experiment of Christenson, Cronin et al. was not done in vacuum. The K-meson beam passed through air, and at the last stage through a vessel containing helium at atmospheric pressure. In this medium regeneration of the K_1's from K_2 is possible. Regeneration can also occur in the walls of the apparatus. But from various control experiments, and also using existing data and confirmed theoretical notions about regeneration of K_1 mesons in matter, the authors estimated that such an effect, under their experimental conditions, was 10^6 times less than that required to explain the observed effect.

Thus an explanation of this experiment requires a radical change of our notions. All-in-all it appears at present that the most natural conclusion is that CP parity is not conserved.

The invariance of nature with respect to the operation of combined inversion and the related conservation of CP parity—this is the beautiful hypothesis first proposed by Landau [2] to preserve the invariance of empty space (vacuum) under spatial inversion. It arose after it was established in 1956 that spatial parity (P) is not conserved in weak interactions.* The denial of CP conservation (and the associated breakdown of the T invariance of nature) is a difficult step for theoretical physics. Our vaunted "common sense" does not permit us to picture isotropy of space-time together with a preferred direction of the time axis. The connection between conservation of CP and T parity comes from the CPT theorem (cf. Pauli [3]), which has a very profound basis in theory. The principles of special relativity and the connection between spin and statistics (which is itself a consequence of the theory of relativity and the positive character of the energy) automatically lead to conservation of CPT. The connection made by the relativity theory between charge conjugation (particle-antiparticle interchange) and the direction of flow of time extends so far that one can formally interpret an antiparticle as a particle propagating backward in time (cf. Feynman[4]). Generally speaking, it may nevertheless turn out that CPT parity is not conserved, and then the violation of CP invariance observed in the experiment of Christenson, Cronin, et al will not mean nonconservation of time (T) parity. Only future experiments can tell us the correct answer. At the moment it is natural to want to choose the lesser of two evils. In the mathematical structure of our present relativistic physics there is literally no place for a violation of CPT invariance. It is therefore highly probable that the experiment of the Princeton group also indicates the absence in nature of invariance with respect to time inversion $t \to -t$ or, as we say, violation of T invariance.

The transformation $t \to -t$ as a symmetry element in quantum mechanics was first investigated by Wigner.[5] The meaning of time reversal in classical physics has been studied relatively recently (cf. [6]). We shall consider some consequences of time reversal in classical and atomic physics.†

All the interesting consequences are in the form of selection rules: such and such a phenomenon cannot occur if there is T invariance. If our interpretation of the experiment of Christenson, Cronin, et al is correct, this selection rule will not be absolute. Phenomena which contradict T invariance can arise both in atomic and in classical physics via the weak interactions.

a) In the first place, elementary particles can now have an intrinsic dipole moment. Since the dipole moment d is necessarily along the spin σ (there being no other preferred direction), the relation $d \sim \sigma$ contradicts T invariance (as well as P invariance), since d is a time-symmetric vector ($d \to d$ when $t \to -t$), while σ is time-antisymmetric. The absence of electric dipole moments has been checked in many experiments (cf. [8-11]), but the accuracy of these experiments was not sufficient to see the small dipole moment resulting from nonconservation of T parity in weak interactions.

b) T invariance leads to a twofold degeneracy of atomic levels in an arbitrary electric field (Kramers' theorem). Levels with angular momentum projections J_z and $-J_z$ have the same energy. Now such a degeneracy will be lifted. This is related, in particular, to the possible existence of a dipole moment of the electron [cf. point a)].

c) Since the magnetic field H is a time-antisymmetric vector, considerations of T invariance forbid the phenomena of "pyromagnetism" and "piezomagnetism" (the appearance of a magnetic moment in a crystal on heating or compression). Now pyromagnetic and piezomagnetic effects could in principle occur because of the weak interactions.

d) One might find a circular asymmetry of the conductivity in crystals. Usually the absence of circular asymmetry (a difference in conductivity for currents circulating clockwise or counterclockwise in the crystal) is deduced from T invariance.

e) We know the effect of rotation of the plane of polarization of light passing through a medium placed in a magnetic field (Faraday effect). The rotation of the linear polarization vector ϵ' in the transmitted wave compared to the polarization ϵ in the incident wave is proportional to the magnetic field: $\epsilon \times \epsilon' \sim \mathbf{H}$. Breakdown of T and P invariance in the weak interactions would lead in general to a rotation of the polarization in matter because of an externally applied electric field, so that $\epsilon \times \epsilon' \sim \mathbf{E}$.*

We are sure that this is not a complete list of effects which could occur in principle in atomic and macroscopic physics if the K_2 actually decayed into two π's. We should add to this the large number of new phenomena in elementary particle physics, whose consideration is the main content of this survey.

But in order to draw such far-reaching conclusions one must have a confirmation of the Princeton results in many independent experiments. A single experiment is of course not enough for a complete clarification of the situation. We must also remember that the K meson is a very complex object. Its decays through virtual processes involve the whole physics of weak interactions, including the high-energy region. Because of the smallness of the K_1—K_2 mass difference, this system is sensitive to very weak external fields. In other words, the experiment of the Princeton

*It is remarkable that it was precisely the K mesons (charged, in this case) which first exhibited the effect of nonconservation of spatial parity (the famous $\tau - \theta$ problem).

†The reader will find interesting considerations about T invariance in connection with thermodynamics and the law of increase of entropy in the book of Landau and Lifshitz (cf.[7]).

*Aside from this we know that breakdown of P invariance itself can lead to the appearance (because of the weak interactions) of a rotation of the plane of polarization of light in matter containing no optically active molecules (cf. the papers of Zel'dovich and Perelomov[12, 13]).

group takes us into a range of phenomena which no other high-energy experiment has penetrated. Considering how limited our knowledge is, we cannot exclude the possibility that nature has prepared a new surprise for us in this region. In this sense we should consider most seriously the possibility that the Princeton results will be confirmed by other experiments. We shall see later that CP nonconservation is not a unique conclusion to be drawn from the decay of the longlived component of the K beam into two π's. But if we are considering CP parity, it is important to point out that, aside from experiments on spin correlations in the β decay of polarized neutrons [14] (with an accuracy of the order of ten percent) and experiments on asymmetries in nonleptonic decays of hyperons [15] (with even poorer accuracy), there is not a single experiment in which conservation of CP parity in weak interactions has been established to even tens of percent. Up to now, as we have said, it was precisely the absence of $K_2 \to 2\pi$ decays that was regarded as the basic argument in favor of conservation of CP parity.

In this survey we shall discuss the physical consequences of the experiment, accepting its results as fact.

If there is not conservation of CP parity or if the K meson beam passes through some external field (in which $K_2 \rightleftarrows K_1$ transitions can occur), the states K_1 and K_2 no longer coincide with the shortlived K_S and longlived K_L components of the beam. K_S and K_L should be defined as states having a definite mass and lifetime. These are the states that decay according to a simple exponential law. The experiment of Christenson, Cronin, et al measures the probability per unit time of the decay $K_L \to \pi^+\pi^-$.

We present the data on decay rates of the K_S and and K_L mesons into various final states in a Table:

Decay mode	Decay rate, 10^6 sec^{-1}	
	K_S	K_L
$\pi^+\pi^-$	$2/3 \cdot 1,1 . 10^{+4}$	$2.6 \cdot 10^{-2}$
$2\pi^0$	$1/3 \cdot 1,1 \cdot 10^{+4}$	Unknown
Leptons	~ 11	~ 11
$\pi^+\pi^-\pi^0$	$\leqslant 2$	~ 2
$3\pi^0$	$\leqslant 4$	~ 4
	Total $1.1 \cdot 10^4$	~ 18

Section I of the survey gives a general treatment of the phenomena of decay and interference in a K meson beam.*

In Sec. II we study the nature of the information which follows from the data of the Princeton group. Along with the existence of the $K_L \to \pi^+\pi^-$ decay, the smallness of the observed effect requires an explanation. The decay $K_L \to \pi^+\pi^-$ is 4×10^{-6} of the $K_S \to \pi^+\pi^-$ decay.

In Sec. III we discuss attempts to explain the experiment on the basis of a hypothesis of the existence in nature of new fields with a macroscopic range of their forces. Here it is assumed that CP is rigorously conserved.

In Sec. IV we present attempts to explain the experiment on the basis of various mechanisms of breakdown of CP invariance.

It may be that all the models we consider are too speculative to correctly describe the actual situation. But their consideration is important for indicating various specific lines for further experimental study of the problem.

If the longlived component K_L can decay into 2π, in the K meson beam the amplitudes for the $K_S \to 2\pi$ and $K_L \to 2\pi$ decays are coherent, and one can have interference between them. This leads to new features of the time dependence of the two-meson decay. Interference phenomena of this sort are considered in Sec. V.

In the last section, Sec. VI, we enumerate various experiments whose performance is necessary in the future in connection with the $K_L \to 2\pi$ decay and the nonconservation of CP parity.

I. GENERAL PROBLEMS OF THE THEORY OF K MESON DECAY

1. General Theory of the Decay of the K Meson

To study the decay of the K meson we use the Heitler damping theory. An equally convenient method in all respects is that of Weisskopf and Wigner [16] (which was used by Lee, Oehme and Yang [17] in a paper on the symmetry of the weak interactions). But the damping theory is a more general method, which permits us to discuss qualitatively the question of corrections to the exponential decay law.*

Suppose that initially there is a state $|K\rangle$, which corresponds to a K meson created in nuclear collisions practically instantaneously as compared to its lifetime. It is therefore completely consistent to assert that the state $|K\rangle$ is an eigenstate of the energy H, not including the weak interaction (the accuracy of this statement being no worse than 10^{-13}, the ratio of the characteristic times for the strong and weak interactions). The time dependence of the state $|K\rangle$ is given by the total energy operator $H + W$, where W includes both the weak interactions H_W and any macroscopic fields (if such exist).

We want to know the time behavior of the decay amplitude $c_j(t) = \langle j | K; t \rangle$ of the state $|K; t\rangle = e^{-i(H+W)t} |K\rangle$ into any one of the eigenstates j of

*This Section contains the derivation of a rather large number of formulas that are used later on. The reader interested only in the results can cheerfully omit this material. A final summary of the formulas (without derivation) is given in Sec. II.

*The reader will find a simple and physically lucid treatment of the phenomena in a K meson beam when CP parity is conserved in the paper of Zel'dovich. [18]

the Hamiltonian H. We give the following treatment in the rest frame of the K meson.

The amplitudes $c_j(t)$ satisfy the Schrödinger equation

$$i\frac{\partial c_j(t)}{\partial t}=\varepsilon_j c_j(t)+\sum_{j'}W_{j,j'}c_{j'}(t) \tag{1.1}$$

with the initial condition $c_j(0)=\delta_{jK}$. Here ϵ_j are the eigenvalues of H: $H|j\rangle=\epsilon_j|j\rangle$, $W_{j,j'}\equiv\langle j|W|j'\rangle$. We write Eq. (1.1) with its boundary condition as an integral equation

$$c_j(t)=c_j(0)e^{-i\varepsilon_j t}-i\sum_{j'}\int_0^t W_{j,j'}e^{-i\varepsilon_j(t-t')}c_{j'}(t')\,dt', \tag{1.2}$$

whose solution we assume in the form of a Fourier integral

$$c_j(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F_j(E)e^{-iEt}\,dE. \tag{1.3}$$

Then

$$F_j(E)=i\frac{c_j(0)}{E-\varepsilon_j+i\delta}+\sum_{j'}W_{j,j'}\frac{F_{j'}(E)}{E-\varepsilon_j+i\delta}. \tag{1.4}$$

Among the states $|j\rangle$ there is a state $|\bar{K}\rangle$ corresponding to the $\bar{K}$ meson, which in the absence of the weak interactions has a mass m identical with that of the K meson. The degeneracy of the states $|K\rangle$ and $|\bar{K}\rangle$ permits a separate treatment of the decay amplitudes $c_{\bar{K}}(t)$ and $c_K(t)$. It is therefore convenient to separate out the Fourier components of these amplitudes in (1.4). We look for a solution of the form

$$F_j(E)=\frac{1}{E-\varepsilon_j+i\delta}[T_{j,K}(E)F_K(E)+T_{j,\bar{K}}(E)F_{\bar{K}}(E)], \tag{1.5}$$

with $j\neq K,\bar{K}$, where by definition $T_{j,K}(E)$ and $T_{j,\bar{K}}(E)$ are zero for $j=K$ or $\bar{K}$. In the following we shall agree to assign the index j to states not including K or $\bar{K}$.

We then get from (1.5) and (1.4) the system of two equations

$$\sum_{b=K,\bar{K}}\left[(E-m)\delta_{a,b}+\frac{i}{2}\lambda_{a,b}(E)\right]F_b(E)=c_a(0) \tag{1.6}$$

(where a is either K or $\bar{K}$), and the matrix $\lambda(E)$ is

$$\lambda_{a,b}(E)=2i\left[W_{a,b}+\sum_{j\neq K,\bar{K}}\frac{W_{a,j}T_{j,b}(E)}{E-\varepsilon_j+i\delta}\right], \tag{1.7}$$

where $T_{j,b}(E)$ is the solution of the following system:

$$T_{j,b}(E)=W_{j,b}+\sum_{j'\neq K,\bar{K}}\frac{W_{j,j'}T_{j',b}(E)}{E-\varepsilon_{j'}+i\delta}\quad(j\neq K,\bar{K}). \tag{1.8}$$

Now it is convenient to go over to the other representation. In place of the states $|K\rangle$ and $|\bar{K}\rangle$ we consider their linear combinations $|K_S\rangle$ and $|K_L\rangle$:

$$|\alpha\rangle=\sum_{b=K,\bar{K}}U_{b\alpha}|b\rangle\quad(\alpha=K_S,\,K_L). \tag{1.9}$$

We write the corresponding covectors by using the inverse matrix

$$\langle\alpha|=\sum_{b=K,\bar{K}}\langle b|U^{-1}_{\alpha b}\quad(\alpha=K_S,\,K_L) \tag{1.10}$$

so that the conditions

$$\langle\alpha|\alpha'\rangle=\delta_{\alpha,\alpha'},$$
$$\sum_{\alpha=K_S,K_L}|\alpha\rangle\langle\alpha|=\sum_{b=K,\bar{K}}|b\rangle\langle b|$$

are satisfied.

In accordance with these definitions, the amplitudes and matrix elements in the new representation have the form

$$c_\alpha(t)=\sum_{b=K,\bar{K}}U^{-1}_{\alpha b}c_b(t),$$
$$\langle j|W|\alpha\rangle=\sum_{b=K,\bar{K}}\langle j|W|b\rangle U_{b\alpha}\quad(\alpha=K_S,\,K_L). \tag{1.11}$$

Equation (1.6) in the new representation is

$$\sum_{\beta=K_S,K_L}\left[(E-m)\delta_{\alpha,\beta}+\frac{i}{2}\lambda'_{\alpha,\beta}(E)\right]F_\beta(E)=c_\alpha(0)$$
$$(\alpha=K_S,\,K_L). \tag{1.12}$$

It is now convenient to choose the matrix U in (1.9) and (1.10) so that the matrix $\lambda'(E)=U^{-1}\lambda(E)U$ in (1.12) is diagonal. We then get

$$F_\alpha(E)=\frac{c_\alpha(0)}{E-m+\frac{i}{2}\lambda_\alpha(E)}, \tag{1.13}$$

where $\lambda_\alpha(E)$ are the eigenvalues of $\lambda(E)$. From now on we limit ourselves to the lowest order in the interaction W in the matrix $T_{j,\alpha}(E)$. We then get from (1.5)

$$F_j(E)=\sum_{\alpha=K_S,K_L}W_{j,\alpha}\frac{F_\alpha(E)}{E-\varepsilon_j+i\delta}\quad(j\neq K,\bar{K}), \tag{1.14}$$

and the final expression for the amplitudes

$$c_\alpha(t)=\frac{c_\alpha(0)}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-iEt}\,dE}{E-m+\frac{i}{2}\lambda_\alpha(E)}\quad(\alpha=K_S,\,K_L), \tag{1.15}$$

$$c_j(t)=\sum_{\alpha=K_S,K_L}\frac{W_{j,\alpha}c_\alpha(0)}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-iEt}\,dE}{(E-\varepsilon_j+i\delta)\left(E-m+\frac{i}{2}\lambda_\alpha(E)\right)}\quad(j\neq K,\bar{K}). \tag{1.16}$$

The properties of the eigenvalues $\lambda_\alpha(E)$ can be investigated by writing the matrix $\lambda(E)$ as the sum of two terms:

$$\lambda(E)=\Gamma(E)+2iM(E), \tag{1.17}$$

where Γ and M are hermitian 2×2 matrices with the respective matrix elements

$$\Gamma_{ab}(E)=2\pi\sum_{j\neq K,\bar{K}}{}'\int W_{a,j}W_{j,b}\delta(E-\varepsilon_j)n_j(\varepsilon_j)\,d\varepsilon_j, \tag{1.18}$$

$$M_{a,b}(E)=W_{a,b}+P\sum_{j\neq K,\bar{K}}{}'\int\frac{W_{a,j}W_{j,b}}{E-\varepsilon_j}n_j(\varepsilon_j)\,d\varepsilon_j, \tag{1.19}$$

where $n_j(\epsilon_j)$ is the density of states with energy ϵ_j.

The integral in (1.19) is taken in the sense of the principal value. This integral diverges formally and must be cut off at some energy in a theory that correctly takes account of the contribution of the high energy region. We shall never calculate explicitly the integral in (1.19). All we need is the possibility of writing $\lambda(E)$ in the form (1.17).

Then it follows simply from the positive definiteness of $\Gamma(E)$ and the hermiticity of $\Gamma(E)$ and $M(E)$ that the real parts of the eigenvalues $\gamma_S(E) = \mathrm{Re}\,\lambda_S(E)$ and $\gamma_L(E) = \mathrm{Re}\,\lambda_L(E)$ of the matrix $\lambda(E)$ are positive. The absolute value of $\lambda_\alpha(E)$ is very small. We shall show later that $\lambda_\alpha(E) \sim 10^{-5}$ eV. On the other hand, the scale δ over which $\lambda_\alpha(E)$ varies significantly as a function of E is determined by the masses of the elementary particles (more precisely, by the energies ϵ_j). Thus, to very good accuracy $\lambda_\alpha(E)/\delta \sim 10^{-11}$–$10^{-13}$, and we can neglect the dependence on E in the argument $\lambda_\alpha(E)$ in (1.15) and (1.16), i.e., we can assume that $\lambda_\alpha(E) \approx \lambda_\alpha(m)$. Then the only singularities of the integrands in (1.15) and (1.16) will be simple poles in the lower half of the complex E plane. The integration is now elementary. We get

$$c_\alpha(t) = c_\alpha(0)\, e^{-im_\alpha t - \frac{1}{2}\gamma_\alpha t} \quad (\alpha = K_S, K_L), \tag{1.20}$$

$$c_j(t) = \sum_{\alpha = K_S, K_L} \frac{W_{j,\alpha} c_\alpha(0)}{\varepsilon_j - m_\alpha + \frac{i}{2}\gamma_\alpha}\left(1 - e^{i(\varepsilon_j - m_\alpha) - \frac{1}{2}\gamma_\alpha t)}\right) \quad (j \neq K, \bar{K}). \tag{1.21}$$

We have also introduced the new notation. If we separate out the real and imaginary parts of $\lambda_\alpha(m)$:

$$\lambda_\alpha(m) = \gamma_\alpha + 2i\Delta_\alpha, \tag{1.22}$$

then $\gamma_\alpha > 0$ will determine the lifetime of the state $|\alpha\rangle$, while the quantity $m_\alpha = m + \Delta_\alpha$ gives the mass of the state $|\alpha\rangle$.

The structure of expressions (1.20) and (1.21) is in no way related to the various simplifying assumptions we made in deriving them. In particular the restriction to the lowest approximation in the interaction W is in no way fundamental. Including higher approximations gives only small corrections to m_α and γ_α, but we still find these parameters from experiment. The form of the interaction W was not fixed by us. The exponential character of the decay of the state $|\alpha\rangle$ is, however, related to our assumption that the quantity $\lambda_\alpha(E)$ is a constant.

The probability per unit time for decay of the K meson into channel j is given by the expression

$$\frac{\partial}{\partial t}|c_j(t)|^2 = \Gamma_j(t).$$

This probability is obtained from (1.21) in the form

$$\Gamma_j(t)\, d\varepsilon_j = 2\pi \left|\sum_\alpha W_{j,\alpha} c_\alpha(t)\right|^2 \delta(\varepsilon_j - m)\, n_j(m)\, d\varepsilon_j, \tag{1.23}$$

if we neglect the change with energy ϵ_j of the matrix elements $W_{j,\alpha}$ and the density of states $n_j(\epsilon_j)$ on scales of order $\gamma_\alpha, \Delta_\alpha$. If the interaction W also contains contributions from any external fields, the decay $\alpha \to j$ still proceeds through the weak interaction, so that $W_{j,\alpha} = (H_W)_{j,\alpha}$ in formula (1.23). In the sequel we shall say that the amplitude for the decay $\alpha \to j$ is the quantity

$$A(\alpha \to j) = \langle j | H_W | \alpha \rangle\, c_\alpha(0). \tag{1.24}$$

In accordance with the definition (1.11) and formula (1.20), the time dependence of the amplitude for the K and $\bar{K}$ states is given by the expression

$$c_{\hat{a}}(t) = \sum_{\alpha = K_S, K_L} \sum_{b = K, \bar{K}} U_{\hat{a}\alpha} U^{-1}_{\alpha b} c_b(0)\, e^{-im_\alpha t - \frac{1}{2}\gamma_\alpha t} \quad (a = K, \bar{K}). \tag{1.25}$$

2. Nonexponential Corrections to the Decay Law

The matrix $\lambda_{a,b}(E)$ has branch points corresponding to the masses of physical states to which transitions are possible from the $|K\rangle$ and $|\bar{K}\rangle$ states. In our approximation in the interaction W, it is obvious from formulas (1.17) or (1.18) and (1.19) that all the singularities of this type lie on the real axis of the complex E plane. Consider, for example, the threshold for creation of π mesons. The corresponding energy is $E_{thr} = 2m_\pi$. Deforming the contour of integration in (1.15) into the lower halfplane, we get an integral of the jump in the function $(E - m + (i/2)\gamma_\alpha(E))^{-1}$ over the cut:

$$c_\alpha(t) = \frac{c_\alpha(0)}{2\pi} \int_{E_{thr}}^{\infty} \frac{\gamma_\alpha(E)}{(E - m_\alpha)^2 + \frac{1}{4}\gamma_\alpha^2(E)}\, e^{-iEt}(-i\, dE). \tag{1.26}$$

Again deforming the contour into the lower halfplane and computing separately the contribution from the pole at $E - m_\alpha \approx -(i/2)\gamma_\alpha$, we get

$$c_\alpha(t) = c_\alpha(0)\, e^{-im_\alpha t - \frac{1}{2}\gamma_\alpha t}$$

$$+ \frac{c_\alpha(0)}{2\pi} e^{-iE_{thr} t} \int_0^{-i\infty} \frac{\gamma_\alpha(E_{thr} + z)}{(z + E_{thr} - m_\alpha)^2 + \frac{1}{4}\gamma_\alpha^2}\, e^{-izt}(-i\, dz).$$

The asymptotic behavior is easily calculated. If $\gamma_\alpha(E_{thr} + z) \sim (z/E_{thr})^{n/2}\gamma_\alpha$ when $z \to 0$, since $E_{thr} - m_\alpha \gg \gamma_\alpha$, the correction term to the exponential decay will be

$$\sim c_\alpha(0)\, e^{-iE_{thr} t} \frac{\gamma_\alpha}{t(E_{thr} - m_\alpha)^2 (E_{thr} t)^{n/2}}. \tag{1.27}$$

The relative contribution of this term when $\gamma_\alpha t \sim 1$ is $\sim 10^{26}(10^{-13})^{n/2}$. The presence of other branch points, which are related to the possibility of decay of the K and $\bar{K}$ into other channels, leads to the appearance of additional corrections to the basic decay law $e^{-1/2\gamma_\alpha t}$. The general structure of these corrections is again of the form of (1.27), where E_{thr} is the energy of the corresponding threshold, and n determines the nature of the singularity (branch point) at the point $E = E_{thr}$.

But the problem of finding such corrections may not arise because of their smallness. In fact the amplitude (1.27) is determined by the contribution of states whose energies differ markedly from the energy of the initial state, since the Fourier integral of (1.27) contains only the narrow range $E \sim E_{thr} = 2m_\pi$. But in any experiment on K meson decay one can only include events in which the energy of the decaying state lies in the region of the peak corresponding to the pole at the point $E \approx m_\alpha$ in (1.26). The theoretical width of the peak is γ_α, while experimentally it is determined by the resolving power of the apparatus $\delta \gg \gamma_\alpha$ (in present day experiments $\delta \sim 1$ MeV). In principle it is impossible to distinguish from the background any events that do not coincide with the peak. In fact just what is the sense of our initial assumption that at time $t = 0$ only the amplitude $c_K(0)$ for the K meson state was different from zero? It is valid only so long as we are interested only in a very narrow region around the peak, of order γ_α, since the position and shape of the peak are essentially determined by $c_K(0)$. The region far away from the peak depends essentially on details of the wave packet describing the initial state. Thus the inclusion of correction terms of the form of (1.27) is completely inconsistent with our choice for the initial state. It is beyond the accuracy set here.

There still remains the important question of exactly what is the actual contribution of nonexponential corrections to the decay law. From the experimental conditions we can get rid of all events not falling within an interval $\sim \delta$ (the experimental resolution) around the peak at $E \approx m_\alpha$. It seems reasonable to cut off the integral in (1.26) by using some function $\rho(E)$ such that $\rho(E) = 0$ when $|E - m_\alpha| > \delta$, $\rho(m_\alpha) = 1$. We choose the law of falloff of $\rho(E)$ at the ends of the interval δ so as to simplify the further calculations:

$$\varrho(E) \to \left(\frac{E - m_\alpha \pm \delta}{m_\alpha}\right)^k \text{ for } E \to m_\alpha \mp \delta.$$

This corresponds to choosing the initial state to be a wave packet with a "width" $\sim \delta$ (in this connection cf. the papers of Schwinger [19] and of Jacob and Sachs, [20] where one can also find references to earlier work). The time dependence of the resulting corrections to the exponential law is essentially determined by the nature of the falloff of $\rho(E)$ when $E \to m_\alpha - \delta$. We find (for $\gamma_\alpha \ll \delta \ll m_\alpha$)

$$c_\alpha(t) = c_\alpha(0)\, e^{-im_\alpha t - \frac{1}{2}\gamma_\alpha t} + o\left(c_\alpha(0)\, e^{-im_\alpha t}\, \frac{\gamma_\alpha}{\delta^2 t}\, \frac{1}{(m_\alpha t)^k}\right).$$

When $\gamma_\alpha t \sim 1$ the correction terms are completely negligible for any values $k > 0$, and become important at values of t so large that $c_\alpha(t)$ becomes less than $10^{-22}(10^{-13})^k$. Thus it appears impossible to explain the experiment of Christenson, Cronin et al on the basis of deviations from the exponential law.

II. $K_L \to \pi^+\pi^-$ DECAY AND CONSERVATION OF T, CP AND CPT.

1. Formulas for States and Decay Amplitudes.

Since the physical amplitudes (1.23) and (1.25) contain products of matrix elements of the matrix U and its inverse (introduced in (1.9) and (1.10)), it is meaningful to look for U within a factor, which corresponds to an arbitrary normalization of the states $|K_S\rangle$ and $|K_L\rangle$. The matrix U brings the operator $\Gamma + 2iM$, which we shall call the "mass operator," to diagonal form [cf. (1.17)]. The matrix U can be chosen as

$$U = \frac{1}{\sqrt{1+s^2}}\begin{pmatrix} s^2 & 1 \\ rs & -rs \end{pmatrix}, \quad U^{-1} = \frac{1}{\sqrt{1+s^2}}\begin{pmatrix} 1 & 1/rs \\ 1 & -s/r \end{pmatrix}. \quad (2.1)$$

Accordingly the states $|K_S\rangle$ and $|K_L\rangle$, which diagonalize the mass operator and which in this sense have a definite mass and lifetime, will be the following combinations of $|K\rangle$ and $|\bar{K}\rangle$ (cf., for example, Sachs [2]):

$$|K_S\rangle = \frac{1}{\sqrt{1+s^2}}\,(s^2|K\rangle + rs\,|\bar{K}\rangle),$$
$$\langle K_S| = \frac{1}{\sqrt{1+s^2}}\left(\langle K| + \frac{1}{rs}\langle\bar{K}|\right),$$
$$|K_L\rangle = \frac{1}{\sqrt{1+s^2}}\,(|K\rangle - rs\,|\bar{K}\rangle),$$
$$\langle K_L| = \frac{1}{\sqrt{1+s^2}}\left(\langle K| - \frac{s}{r}\langle\bar{K}|\right). \quad (2.2)$$

The parameters r and s appearing in these formulas are

$$r = \left(\frac{\Gamma_{\bar{K},K} + 2iM_{\bar{K}K}}{\Gamma_{K\bar{K}} + 2iM_{K\bar{K}}}\right)^{1/2}, \quad (2.3)$$

$$s = \sqrt{1+\eta^2} - \eta,$$
$$\eta = \frac{\Gamma_{\bar{K},\bar{K}} - \Gamma_{K,K} + 2i(M_{\bar{K},\bar{K}} - M_{K,K})}{2[(\Gamma_{K,\bar{K}} + 2iM_{K,\bar{K}})(\Gamma_{\bar{K},K} + 2iM_{\bar{K},K})]^{1/2}}. \quad (2.3')$$

The matrix elements of Γ and M were defined in (1.18) and (1.19). In the future we shall write the amplitudes $c_{K_S}(t)$ and $c_{K_L}(t)$ simply as $K_S(t)$ and $K_L(t)$ [similarly, $c_K(t) \equiv K(t)$, $c_{\bar{K}}(t) \equiv \bar{K}(t)$]. The quantities $K_S(t)$ and $K_L(t)$ change with time according to the simple exponential law

$$K_\alpha(t) = K_\alpha(0)\, e^{-im_\alpha t - \frac{1}{2}\gamma_\alpha t} = K_\alpha(0)\, e^{-imt} e^{-\frac{1}{2}(\gamma_\alpha + 2i\Delta_\alpha)t}, \quad (2.4)$$

where the parameters γ_α and Δ_α are the real and imaginary parts of the eigenvalues of the matrix $\Gamma + 2iM$:

$$\gamma_\alpha + 2i\Delta_\alpha = \Gamma_{K,K} + 2iM_{K,K} + [(\Gamma_{\bar{K},K} + 2iM_{\bar{K},K})(\Gamma_{K,\bar{K}} + 2iM_{K\bar{K}})]^{1/2}\begin{cases} 1/s, & \alpha = S, \\ -s, & \alpha = L. \end{cases} \quad (2.5)$$

We note that the states $|K_S\rangle$ and $|K_L\rangle$ are neither orthogonal nor normalized, so that they are not "states" in the usual quantum mechanical sense. The time dependence of the "normal" K and $\bar{K}$ meson states is given from (1.25) in the form

$$K(t)=\frac{1}{\sqrt{1+s^2}}[s^2K_S(0)e^{-im_St-\frac{1}{2}\gamma_St}+K_L(0)e^{-im_Lt-\frac{1}{2}\gamma_Lt}],$$

$$\bar{K}(t)=\frac{sr}{\sqrt{1+s^2}}[K_S(0)e^{-im_St-\frac{1}{2}\gamma_St}-K_L(0)e^{-im_Lt-\frac{1}{2}\gamma_Lt}], \quad (2.6)$$

where

$$K_S(0)=\frac{1}{\sqrt{1+s^2}}\left[K(0)+\frac{1}{sr}\bar{K}(0)\right],$$

$$K_L(0)=\frac{1}{\sqrt{1+s^2}}\left[K(0)-\frac{s}{r}\bar{K}(0)\right] \quad (2.7)$$

(where we are particularly interested in the case where $\bar{K}(0)=0$).

The time dependence of the probability for decay of the K meson into any state j is given according to (1.23) in the form

$$\Gamma_j(t)=2\pi n_j(m)|\langle j|H_W|K\rangle K(t)+\langle j|H_W|\bar{K}\rangle\bar{K}(t)|^2$$
$$=2\pi n_j(m)|\langle j|H_W|K_S\rangle K_S(t)+\langle j|H_W|K_L\rangle K_L(t)|^2. \quad (2.8)$$

The decay rate w is usually defined as the probability for decay during a time interval t small compared to the lifetime of the decaying particle, i.e., for $\gamma_\alpha t \ll 1$:

$$w(K_\alpha\to j)=2\pi n_j(m)|\langle j|H_W|K_\alpha\rangle K_\alpha(0)|^2. \quad (2.9)$$

Here K_α can be either the K_S or K_L, or the K or $\bar{K}$-meson. It is the decay rate w that is most frequently determined in experiments. It is determined by the decay amplitude $A(K_\alpha\to j)$ [cf. formula (1.24)]:

$$w=2\pi n_j(m)|A(K_\alpha\to j)|^2,$$
$$A(K_\alpha\to j)=\langle j|H_W|K_\alpha\rangle K_\alpha(0).$$

2. Limitations Following from T, CP, and CPT Invariance.

Invariance of the interaction W under time reflection (T), combined inversion (CP), and the operation CPT lead to various relations between matrix elements. To obtain these, we first determine the result of applying these operations to the K meson state. We adopt the conventions

$$\left.\begin{array}{r}CP|K\rangle=|\bar{K}\rangle,\\ T|K\rangle=\langle K|,\\ CPT|K\rangle=\langle\bar{K}|,\end{array}\right\} \quad (2.10)$$

assuming that the arbitrary phase factor that can appear in such transformations is equal to unity. For the class of problems we shall treat, this arbitrary choice of phase is completely unimportant. The only important point is that the K meson is assumed to transform according to the usual laws for a scalar particle under the operations CP and T.

The eigenstates $|j\rangle$ of the Hamiltonian H in general transform differently under CP and T. Since we are assuming that $[H, CP]=0$, we could choose as the states $|j\rangle$ eigenstates of CP. But this is not always convenient. The only important thing is that the law of transformation of the states $|j\rangle$ under CP is the same as for the eigenstates $|j_0\rangle$ (corresponding eigenstates of the free Hamiltonian H_0). Let $|j'\rangle$ be the state obtained from $|j\rangle$ by the operation CP.

If among the states $|j\rangle$ we consider states of zero angular momentum, definite parity and isospin, then for such states the standard representation of the T operation is*

$$T|j\rangle=e^{2i\delta_j}\langle j|. \quad (2.11)$$

Here the phase δ_j depends on the nature of the interaction in the state $|j\rangle$ (cf., for example, Gell-Mann and Watson[22]). If this interaction is small (for example, in the state $|\pi^+e^-\tilde{\nu}\rangle$, where only electromagnetic forces are important), the state $|j\rangle$ can be regarded as a product of wave functions for free particles, and the phase $\delta_j=0$.

Thus we obtain the following relations between matrix elements for the respective cases of T, CP and CPT invariance:

$$\left.\begin{array}{ll}\langle K|W|j\rangle=e^{2i\delta_j}\langle j'|W|\bar{K}\rangle & \text{(from } CPT),\\ \langle K|W|j\rangle=\langle\bar{K}|W|j'\rangle & \text{(from } CP),\\ \langle K|W|j\rangle=e^{2i\delta_j}\langle j|W|K\rangle & \text{(from } T).\end{array}\right\} \quad (2.12)$$

We have made use of the antiunitary character of the T operation, the hermiticity of the Hamiltonian and the invariance condition $W=\hat{O}^{-1}W\hat{O}$, where $\hat{O}$ is one of the three operations CPT, CP, or T.

Using expressions (1.18) and (1.19) for the matrices Γ and M, we get from (2.12)

$$\Gamma_{K,K}+2iM_{K,K}=\Gamma_{\bar{K},\bar{K}}+2iM_{\bar{K},\bar{K}} \quad \text{(from } CPT), \quad (2.13')$$

$$\left.\begin{array}{l}\Gamma_{K,K}+2iM_{K,K}=\Gamma_{\bar{K},\bar{K}}+2iM_{\bar{K},\bar{K}},\\ \Gamma_{K,\bar{K}}+2iM_{K,\bar{K}}=\Gamma_{\bar{K},K}+2iM_{\bar{K},K}\end{array}\right\} \quad \text{(from } CP). \quad (2.13'')$$

Because of the hermiticity of Γ and M, the relations (2.13) mean that the matrix elements $\Gamma_{K,\bar{K}}$ and $M_{K,\bar{K}}$ are real.

From (2.13) and (2.3) we find that the parameters r and s, giving the diagonal combinations of the K and $\bar{K}$ mesons, are

$$s=1 \quad \text{(from } CPT), \quad (2.14')$$
$$r=1,\ s=1 \quad \text{(from } CP). \quad (2.14'')$$

It is immediately obvious from (2.2) that, in accor-

*One chooses either in or out states as the states $|j\rangle$. Then for example,

$$T|j;\ \text{in}\rangle=\langle j;\ \text{out}|=\sum_{j'}\langle j;\ \text{out}|j';\ \text{in}\rangle\langle j';\ \text{in}|=\sum_{j'}S_{j,j'}\langle j';\ \text{in}|.$$

Here $S_{j,j'}$ is a matrix element of the S matrix. If the states $|j\rangle$ are classified according to angular momentum, parity and isospin (if we neglect electromagnetic corrections), $S_{j,j'}=e^{2i\delta_j}\delta_{j,j'}$, and we get the result (2.11).

dance with general principles, when we have CP invariance,

$$|K_S\rangle=\frac{1}{\sqrt{2}}\{|K\rangle+|\bar{K}\rangle\}\equiv|K_1\rangle,$$
$$|K_L\rangle=\frac{1}{\sqrt{2}}\{|K\rangle-|\bar{K}\rangle\}\equiv|K_2\rangle.$$

These states are the CP-even and CP-odd combinations of $|K\rangle$ and $|\bar{K}\rangle$. They correspond to the K_1 and K_2 mesons.

3. The $K_L\to\pi^+\pi^-$ Decay.

The ratio of the amplitudes for the $K_L\to\pi^+\pi^-$ and $K_S\to\pi^+\pi^-$ decays can be obtained from (2.2), (2.7) and (2.9):

$$\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}=\frac{\langle\pi^+\pi^-|H_W|K_L\rangle}{\langle\pi^+\pi^-|H_W|K_S\rangle}=\frac{1-\xi sr}{s^2+\xi sr},\quad \xi=\frac{\langle\pi^+\pi^-|H_W|\bar{K}\rangle}{\langle\pi^+\pi^-|H_W|K\rangle}. \tag{2.15}$$

Here CPT invariance leads to the condition $s = 1$ [cf. (2.14)].

Two π mesons in an arbitrary state, being subject to Bose statistics, are a CP-even system:

$$CP|\pi^+\pi^-\rangle=(-1)^{2L}|\pi^+\pi^-\rangle=|\pi^+\pi^-\rangle,$$

where L is the orbital angular momentum. In our case only the state $L = 0$ enters. Thus conservation of CP parity means

$$s=1,\quad r=1,\quad \xi=1. \tag{2.16}$$

In this case

$$\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}=0.$$

Thus the longlived component of the K meson beam cannot in this case decay into two π mesons. On the other hand the experiment of Christenson, Cronin et al indicates that

$$\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}\approx 2\cdot 10^{-3} \tag{2.17}$$

(where we use absolute values).

Nonconservation of CPT (together with CP conservation) cannot explain the results of the Princeton group. As a rule, in other phenomena in a K meson beam also, conservation of CP parity masks any possible nonconservation of CPT. On the other hand CPT invariance has a very profound basis in theoretical physics, and there is no reason at present to doubt the existence of such a symmetry. We shall assume that the weak interactions conserve CPT.

But it is possible that in the experiment of the Princeton group the K mesons interacted with some external field such that

$$W=V+H_W, \tag{2.18}$$

where H_W is the usual weak interaction, V the interaction with the external field; we shall refer to this field by this same letter V. We shall not consider in detail the nuclear interactions of the mesons with the medium in which the beam is propagating. Such an effect is closely related to the experimental conditions, and if these are set correctly the absence of interactions with the medium can be checked in control experiments.

Nevertheless the propagation of the K mesons occurs in the physical vacuum, where one cannot exclude the presence of fields produced by distant objects. In this case even if the weak interaction H_W is CP and CPT invariant, we cannot speak of conservation of CP and CPT in decays, since the K mesons do not form a closed system.

It is hard to imagine an external field that changes the strangeness, so we set $V_{K,\bar{K}} = 0$, and in addition $V_{K,j} = V_{\bar{K},j} = 0$. But the matrix elements $V_{K,K}$ and $V_{\bar{K},\bar{K}}$ may be different. This leads to an apparent nonconservation of CP and CPT. (The parameter s in (2.3″) will differ from unity). The important point is that the effects of known types of fields can be neglected, because they are either too small (if we are dealing with electromagnetic interactions, where $V_{K,K}$ and $V_{\bar{K},\bar{K}}$ may differ because of the difference in the radii of the charge distributions for the K and $\bar{K}$), or have the same effect for a particle and its antiparticle (if we are dealing with gravitational forces).

III. $K_L\to\pi^+\pi^-$ DECAY AND POSSIBLE EXISTENCE OF NEW FIELDS.

1. $K\to\pi^+\pi^-$ Decay in an External Field. General Considerations.

Imagine a K meson in its rest system interacting with an external field. The nature of this interaction does not matter as yet; the only important point is the assumption that the K and $\bar{K}$ have different potential energies in this field. We are assuming that the weak interaction H_W conserves CP. This means that $r = 1$ and $\xi = 1$ in (2.15), but that s can differ slightly from unity so that the decay $K_L\to\pi^+\pi^-$ can occur. Using (2.3′) and (1.19), we obtain in lowest approximation in the external field

$$s=1-i\frac{V_{\bar{K},\bar{K}}-V_{K,K}}{[(\Gamma_{K,\bar{K}}+2iM_{K,\bar{K}})(\Gamma_{\bar{K},K}+2iM_{\bar{K},K})]^{1/2}}. \tag{3.1}$$

Using (2.5), it is easy to express s in terms of the differences in width and mass for the K_S and K_L mesons:

$$s=1-i\frac{2(V_{\bar{K},\bar{K}}-V_{K,K})}{\gamma_S-\gamma_L+2i(m_S-m_L)}. \tag{3.2}$$

Now (using (2.15) or $r = \xi = 1$) we get the ratio of the amplitudes:

$$\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}=\frac{V_{\bar{K},\bar{K}}-V_{K,K}}{2\left(m_S-m_L-\frac{i}{2}(\gamma_S-\gamma_L)\right)}. \tag{3.3}$$

This ratio was found in the rest system of the K meson. Its form in the laboratory system is determined by the laws of transformation of the matrix elements $V_{\bar{K},\bar{K}}$ and $V_{K,K}$. Thus, in addition to a violation of CP invariance one will in general also observe a dependence of the ratio of the amplitudes on the reference system, which manifests itself as an apparent violation of relativistic invariance.

2. Possible Nature of the New Fields.[23,24]

Imagine that the field V is produced by macroscopic bodies. As sources of this field we may imagine either the hypercharge Y or the third component of the isospin T_3. Then K and $\bar{K}$ would have opposite charges with respect to the new interactions. Suppose that

$$V_{\bar{K},\bar{K}} = \frac{1}{2} V_0, \qquad V_{K,K} = -\frac{1}{2} V_0.$$

It is convenient to express the result in terms of the potential V in the laboratory system, where the K mesons move with velocity v (in the experiment of Christenson, Cronin et al, $\gamma = (1 - v^2)^{1/2} \approx 2.5$). This can be done if we know the tensorial character of the new field. If it is a vector field, the static potential V is the fourth component of a vector, and $V_0 = \gamma V$. (This assumption is quite natural, since in a local theory scalar and tensor fields have the same static potential for a particle and its antiparticle.) We find

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \frac{\gamma}{2} \frac{V}{m_S - m_L - \frac{i}{2}(\gamma_S - \gamma_L)} \approx 2 \cdot 10^{-3} \quad (3.4)$$

(where we mean equality of the absolute values).

In this case we get a very interesting effect of dependence of the ratio of the amplitudes on the energy of the K-meson beam. If such an apparent violation of relativistic invariance were observed experimentally, this would be a serious argument in favor of the existence of the V-field. This dependence of the ratio (3.4) on velocity is a fundamental result. Even if the field V is scalar we should still get the factor γ in (3.4). In fact, in a local theory (without derivative coupling) a scalar field has the same interaction with particle and antiparticle (the K and $\bar{K}$ mesons in our case), and this is not what we want. So we must take the scalar field with gradient coupling, and this again leads to a linear dependence on energy as in (3.4).

Since the mass difference and difference of widths of the K_S and K_L mesons are approximately the same and of order 10^{-5} eV, $V \sim 10^{-8}$ eV. Thus, because of the smallness of the mass difference, the experiment of the Princeton group is sensitive to very weak potentials. There is not another experiment on elementary particles at present that has a comparable sensitivity to low energies.

If the V-field is a vector field, its quanta must have a mass. This is related to the fact that the source of the field is a nonconserved current, since the "charges" Y or T_3, unlike the electric charge, are not rigorously conserved. Furthermore, if the V-field is massless, gauge invariance implies that we cannot measure the absolute value of the potential V, but it is just the absolute value of V that enters in the amplitude ratio (3.4).

If the mass of the quanta of the V-field were $\mu \sim R_g^{-1}$, where R_g is the radius of the Galaxy, then

$$V \sim f_g^2 \frac{M_g}{m_p} R_g^{-1}, \quad (3.5)$$

where f_g is the coupling constant, M_g is the mass of the Galaxy, m_p is the proton mass (M_g/m_p is the hypercharge of the Galaxy, or except for a factor of two, the third component of its isospin). Taking $R_g \sim 10^{22}$ cm, $M_g \sim 10^{68}$ m_p, we get $f_g^2 \sim 10^{-49}$.

If the range of the new forces is determined by the radius of the Solar system, then $\mu \sim R_S^{-1}$, where R_S is the distance from the Earth to the Sun ($R_S \sim 1.5 \times 10^{13}$ cm). For the corresponding constant we find $f_S^2 \sim 10^{-47}$.

Finally, if $\mu \sim R_e^{-1}$, where R_e is the Earth radius ($R_e = 6 \times 10^8$ cm), then $f_e^2 \sim 10^{-46}$.

As pointed out by Weinberg,[25] there is a strong limitation on the mass of the quanta of the V-field which is imposed by considering effects of real radiation of soft quanta. The amplitude for emission of a real quantum with polarization ϵ_μ and momentum q in the $K \to 2\pi$ decay is

$$\frac{fA(K \to 2\pi)\, 2p\varepsilon}{(2\pi)^{3/2}\sqrt{2q_0}\,[(p-q)^2 - m^2]}, \quad (3.6)$$

where p is the momentum of the K meson, $A(K \to 2\pi)$ is the amplitude for $K \to 2\pi$ decay without emission of soft quanta, f is one of the constants f_g, f_S, or f_e. (Since we are assuming that the quanta carry off a small momentum, we can neglect emission from the immediate region in which the $K \to 2\pi$ decay occurs.) Then the relative probability for emission of a quantum with energy $\omega \leqslant E$ is

$$R = \frac{f^2}{4\pi^2\mu^2} \int_\mu^E \frac{(\omega^2 - \mu^2)^{3/2}\, d\omega}{\left(\omega - \frac{\mu^2}{2m}\right)^2} \approx \frac{f^2E^2}{8\pi^2\mu^2}. \quad (3.7)$$

If $E \sim 100$ MeV and $\mu \sim R^{-1}$, then $f^2/\mu^2 \sim 10^{17}\ (\text{MeV})^{-2}$ and $R \sim 10^{13}$. If $\mu \sim R^{-1}$, then we would get $f_S^2/\mu^2 \sim 10\ (\text{MeV})^{-2}$ and $R \sim 10^3$, i.e., in both cases the K meson would be completely unstable with respect to decay with emission of quanta. Finally, in the last case $\mu \sim R_e^{-1}$, $f_e^2/\mu^2 \sim 10^{-6}\ (\text{MeV})^{-2}$, and we get $R \sim 10^{-4}$. Thus this is the most realistic case. Here one can still increase the mass somewhat, but the range of the forces should still be enough so that the K meson "feels" the macroscopic objects around it. We conclude that the mass μ should lie within the limits

$$10^3\,\text{cm} \leqslant \frac{1}{\mu} \leqslant 10^9\,\text{cm}. \quad (3.8)$$

It seems improbable that the mass of the quanta of the V-field would fall precisely within the interval (3.8). This is one of the difficulties with this theory.

One could however look for a way out here. In fact the arguments that led to (3.8) are to a large extent internally contradictory. The condition $f^2E^2/\mu^2 \sim 1$ in the theory of the vector field means going into the region of effectively strong coupling (cf., for example, [26]). When $f^2E^2/\mu^2 \gg 1$, the diagram we are considering, with radiation of a single quantum, gives an effect whose magnitude is incompatible with unitarity. It is necessary to consider other processes, which should lead (if the theory of interaction of a vector field with a nonconserved current makes any sense at all) to the appearance of an effective form factor that falls off rapidly with increasing E. Thus, R in (3.7) cannot be greater than unity. But exactly what the value of R is when $f^2E^2/\mu^2 \gg 1$ we cannot say, since we cannot make any calculations in this region.

The theory with a macroscopic V-field is also unusual in that the potential V has an absolute significance (it is measured in the experiment of Christenson, Cronin, et al.). Here we have a profound difference from electrodynamics, where only gradients of the potential are measurable. There are arguments (cf., for example, [27]) that it is difficult to make such a theory compatible with the principles of the theory of relativity. But it seems to us that these arguments do not represent a proof. To settle the question one would have to measure (3.4) as a function of the energy of the K meson.

The existence of the new interaction would lead to a violation of the principle of equivalence of gravitational and inertial mass, since in addition to the ordinary Newtonian attraction between bodies there would now be a force depending on the number of nucleons. A modification of the Eötvos experiment (cf. Dicke [28]) gives a limit for the coupling constant for such an interaction:

$$f^2 < 2\cdot 10^{-45}. \tag{3.9}$$

The values of f_g^2, f_S^2 and f_e^2 do not contradict this relation. It is interesting that if the range of the new force field is determined by the size of the earth R_e, Dicke's experiment does not give as strong a restriction on f^2 as that from the original experiments of Eötvos, which have an accuracy which is two orders of magnitude poorer.*

The nature of the new interaction (attraction or repulsion) could in principle be established by experiments on regeneration of K mesons in matter. In these experiments one should study the interference between the amplitude for $K_2 \to K$ regeneration in matter and the amplitude for the $K_2 \to K_1$ transformation in the V-field. In this case (cf., for example, [23])

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \frac{1}{2}\,\frac{\gamma V - \frac{2\pi n}{m}(f_K - f_{\bar{K}})}{m_S - m_L - \frac{i}{2}(\gamma_S - \gamma_L)}, \tag{3.10}$$

where n is the density of matter, f_K and $f_{\bar{K}}$ are the amplitudes for forward scattering of K and $\bar{K}$ mesons in matter.* The quantity $2\pi n/m\ (f_K - f_{\bar{K}})$ is the difference of the effective interaction energies of K and $\bar{K}$ with matter.

If the decay $K_L \to \pi^+\pi^-$ is actually explained by the existence of some new long-range forces, then nowhere except in the decay of the neutral K mesons should one detect an apparent violation of CP invariance. The degree of violation of CP invariance in the decays of neutral mesons will in all cases be determined by the small parameter $1 - s$ [cf. (3.3), (3.2)]. Thus, for example, the charge asymmetry in the leptonic decays of the long- and shortlived components of the K meson beam are given by the formulas

$$\left.\begin{aligned} \frac{w(K_L \to \pi^+e^-\tilde{\nu})}{w(K_L \to \pi^-e^+\nu)} &= 1 - 2\,\mathrm{Re}\,(1-s),\\ \frac{w(K_S \to \pi^+e^-\tilde{\nu})}{w(K_S \to \pi^-e^+\nu)} &= 1 + 2\,\mathrm{Re}\,(1-s). \end{aligned}\right\} \tag{3.11}$$

These formulas are gotten assuming the $\Delta S = \Delta Q$ rule. To get (3.11) we must use (2.9), (2.2) and (2.7), with $r = 1$ and $(1 - s) \ll 1$. In calculating the matrix elements $\langle \pi e\nu |\, H_W \,| K_{L,S}\rangle$ we must consider that $\langle \pi^+e^-\tilde{\nu}|\, H_W \,|K\rangle = 0$ and $\langle \pi^-e^+\nu|\, H_W\,|\bar{K}\rangle = 0$ because of the $\Delta S = \Delta Q$ rule and, in addition, $\langle \pi^+e^-\tilde{\nu}|\, H_W\,|\bar{K}\rangle = \langle \pi^-e^+\nu|\, H_W\,|K\rangle$ because of the conservation of CP parity in the decays.

We also mention the formulas

$$\left.\begin{aligned} \frac{w(K_S \to \pi^-e^+\nu)}{w(K_L \to \pi^-e^+\nu)} &= 1 - 4\,\mathrm{Re}\,(1-s),\\ \frac{w(K_S \to \pi^+e^-\tilde{\nu})}{w(K_L \to \pi^+e^-\tilde{\nu})} &= 1. \end{aligned}\right\} \tag{3.11'}$$

It is interesting that while the charge asymmetry (3.11) also appears when $s = 1$, because of violation of CP invariance in weak interactions (for the case $r \neq 1$, cf. later), the difference in lifetimes of the K_S and K_L relative to decay into $\pi^-e^+\nu$ (in a K beam satisfying the $\Delta S = \Delta Q$ rule) occurs only when $s \neq 1$ and thus precisely characterizes the degree of nonconservation of CPT.

*Recently Lee[29] proposed a theory in which the V-field is scalar and massless. In his theory the potential V is constant over time and space. One cannot simply subtract such a background and associate its energy with the vacuum, since V has opposite signs for particles with opposite hypercharges. Here there will be no observable violations of the equivalence principle. But the velocity dependence of the ratio (3.4) remains.

*One can make an estimate of the effect of the matter on the experiment of Christenson et al. Using data on the cross section for interaction of K mesons with light nuclei, it is reasonable to assume for helium that $\sigma_K - \sigma_{\bar{K}} \sim 10$ mb (where σ_K and $\sigma_{\bar{K}}$ are the total cross section for K and $\bar{K}$ respectively). On the other hand, from the optical theorem $\mathrm{Im}\,(f_K - f_{\bar{K}}) = p/4\pi\,(\sigma_K - \sigma_{\bar{K}})$. Since $n \approx 3 \times 10^{19}$ atoms per cc and $p/m \approx 2$, $\mathrm{Im}\, f_K \sim |f_K|$, we get $2\pi n/m\,(f_K - f_{\bar{K}}) \sim 6 \times 10^{-12}$ eV. This means that in the experiment of Christenson et al the interaction with the medium gives for the amplitude ratio $A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)$ a value that is $10^3 - 10^4$ times smaller than that observed.

IV. $K_L \to \pi^+\pi^-$ DECAY AND NONCONSERVATION OF CP.

1. Semiphenomenological Treatment.

We shall consider the data on the $K_L \to \pi^+\pi^-$ decay as evidence for CP nonconservation. We shall again assume that CP is conserved in strong and electromagnetic interactions, and that CPT is rigorously conserved. We shall also assume that there are no external fields of the type considered in Sec. III. We then have from (2.15) and (2.14)

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \frac{1-\xi r}{1+\xi r} \approx 2\cdot 10^{-3}. \tag{4.1}$$

This equation is to be understood to apply to the absolute values, and now holds in an arbitrary coordinate system. From (4.1) we see that the violation of CP invariance observed experimentally is characterized by the two parameters ξ and r. The first is related to the nonconservation of CP directly in the $K \to \pi^+\pi^-$ decay, while the second characterizes the CP nonconservation in virtual processes.*

A very important point is that the parameters ξ and r are completely independent. But it is difficult to imagine a situation in which ξ and r differ markedly from unity and still have $\xi r \approx 1$ to within 0.4%, as we must have [from (4.1)]. To understand this point it is convenient to introduce the amplitudes for K decay into states of definite isospin T:

$$\langle 2\pi,\ T \mid H_W \mid K\rangle = A_T e^{i\delta_T},$$
$$\langle 2\pi,\ T \mid H_W \mid \bar{K}\rangle = A_T^* e^{i\delta_T}. \tag{4.2}$$

Here δ_T is the phase for scattering of the two π mesons in a state with isospin T. The fact that it is just A_T^* that appears in the second equation of (4.2) is a consequence of CPT invariance. CP conservation would mean that A_T is real, but now $A_T = a_T e^{i\varphi_T}$, where

$$a_T = |A_T|, \qquad \varphi_T \neq 0. \tag{4.3}$$

The ratio $\langle \pi^+\pi^- \mid H_W \mid \bar{K}\rangle / \langle \pi^+\pi^- \mid H_W \mid K\rangle$, expressed in terms of the amplitudes a_T, is equal to

$$\xi = \frac{\sqrt{2}\, a_0 e^{i(\delta_0-\varphi_0)} + a_2 e^{i(\delta_2-\varphi_2)}}{\sqrt{2}\, a_0 e^{i(\delta_0+\varphi_0)} + a_2 e^{i(\delta_2+\varphi_2)}} \equiv e^{-2i\varphi_0}(1+x). \tag{4.4}$$

Since the $\Delta T = 1/2$ rule is well confirmed by experiment (in particular for K decays) we should have $a_2/a_0 \ll 1$. This means that the parameter x in (4.4) is

$$x \approx -i\sqrt{2}\,\frac{a_2}{a_0}\, e^{i(\delta_2-\delta_0)} \sin(\varphi_2-\varphi_0) \ll 1. \tag{4.5}$$

*Using the definition of ξ in (2.15) and the formula $|K_2\rangle = \frac{1}{\sqrt{2}}\{|K\rangle - |\bar{K}\rangle\}$, we get $\langle \pi^+\pi^- \mid H_W \mid K_2\rangle = \frac{1}{\sqrt{2}}(1-\xi) \times \langle \pi^+\pi^- \mid H_W \mid K\rangle$. Thus we may say that $\xi \neq 1$ characterizes the nonconservation of CP arising from $K_2 \to \pi^+\pi^-$ decay itself, while $r \neq 1$ is related to the CP nonconservation which results from the mixing of the K_2 and K_1 states.

On the other hand the quantity r [cf. (2.3)] can be written in the form

$$r^2 = \frac{a_0^2 e^{2i\varphi_0} + \Gamma' + 2i(M^{(0)} + M')}{a_0^2 e^{-2i\varphi_0} + \overset{*}{\Gamma'} + 2i(\overset{*}{M}{}^{(0)} + \overset{*}{M'})}, \tag{4.6}$$

where Γ' and M' are determined by the contributions to the mass operator from all possible states except that of two π mesons with T = 0. Since the $K \to 2\pi$ decay is most important and the amplitude for T = 0 predominates in it, $a_0^2 \gg |\Gamma'|$. But we cannot claim that $M^{(0)} \gg M'$, since transitions off the energy shell are important here, and in particular three π mesons in the intermediate state can make M' comparable with $M^{(0)}$. On the other hand, a_0^2 and $M^{(0)}$ should be approximately equal, since a_0^2 determines the "width" of the K meson, while $M^{(0)}$ and M' determine the mass difference of K_S and K_L (cf. (2.5)). Thus a "strong" nonconservation of CP in the $K \to 2\pi$ decay (in other words, a large value of the phase φ_0 and a large deviation of the parameter ξ from unity) is contradicted by the experimental data, since the value of r would not be proportional to $e^{2i\varphi_0}$, and consequently the quantity $1-\xi r$ in (4.1) would not be small compared to unity.

Strictly, this may not be the case if $M^{(0)}$ and M' have the same phase as $a_0^2 e^{2i\varphi_0}$. Then $r \sim e^{2i\varphi_0}$, and the $K_L \to \pi^+\pi^-$ decay will again be suppressed.* But equality to within tenths of a percent of the phases of $a_0^2 e^{2i\varphi_0}$, $M^{(0)}$ and M', in the presence of a "strong" violation of CP invariance, may be regarded as nothing but a surprising accident.

Thus aside from the very fact that there is a possible violation of CP invariance, the new thing in our situation is also that the violation of CP is a very weak effect. The violation of CP invariance in weak interactions, leading to nonleptonic decays with strangeness change, should amount to tenths of a percent in the amplitude. Such small effects of CP nonconservation also cannot as yet be excluded from the ordinary β decay of the neutron.

The measurement of effects of CP nonconservation in weak interactions is done as a rule by observing correlations of the type $\sigma \times (p_1 \times p_2)$, $p_1 \times (p_2 \times p_3)$, $\sigma_1 \times (\sigma_2 \times \sigma_3)$. (An exception is the decay of the neutral K mesons, where there are other ways of testing

*This phenomenon (examples of which we shall meet repeatedly in the sequel) is related to the fact that CP nonconservation is manifested in the differences in phase of the corresponding amplitudes. The phase of an individual amplitude is as much a matter of convention as the phase of a single state. The assertion that CP is not conserved because some amplitude is complex, when it should be real if we start from the rules (2.12), is meaningless because, together with the amplitudes, the rules (2.12) themselves can be changed by including an additional phase factor. This permits us to shift the reference phase for all phases by setting $\varphi_0 = 0$. As we see from (4.2), this is equivalent to a redefinition of the phase of the K meson, $|K\rangle \to e^{i\varphi_0}|K\rangle$. We shall not make use of this possibility since we must then change the rules (2.10) and (2.12), which is *inconvenient pedagogically.*

CP conservation.) These quantities change sign when $t \to -t$ (when CPT invariance holds, tests of T and CP conservation are the same thing). But we cannot claim that observation of such correlations implies nonconservation of CP. (In fact, polarization of protons perpendicular to the plane of scattering is observed in pp collisions, but this does not mean that CP is not conserved in strong interactions.) Here there is a difference in principle from the situation where spatial parity (P) is not conserved. There it is sufficient to observe some pseudoscalar quantity. The difference is connected with the presence of final state interaction. As a rule, the amplitude for a real decay is a superposition of the amplitudes for decays into states with definite angular momentum, parity and (or) isospin. Each of these amplitudes has "its own" scattering phase $e^{i\delta}$ in the final state. (We saw, in the example of the $K \to \pi^+\pi^-$ decay, that

$$A(K \to \pi^+\pi^-) = \sqrt{2} A_0 e^{i\delta_0} + A_2 e^{i\delta_2},$$

where $A_0 = |A_0| e^{i\varphi_0}$, $A_2 = |A_2| e^{i\varphi_2}$, $\varphi_0 = \varphi_2 = 0$ if CP is conserved.) The interference of the different amplitudes results in a correlation of the type $\sin(\delta - \delta')\sigma \times (\mathbf{p}_1 \times \mathbf{p}_2)$, which thus occurs even if CP is conserved.* (We note that the scattering phase $\delta \to -\delta$ for $t \to -t$.) Only in decays of the type $\pi \to \mu + \nu$, $\mu \to e + \nu + \tilde{\nu}$, is there no interaction in the final state. In decays of the type $n \to p + e^- + \tilde{\nu}$ there is electromagnetic interaction, and $\sin(\delta - \delta') \sim \alpha$ (or $\sim Z\alpha$ if we are dealing with nuclear β decay). Thus, effects of CP nonconservation at the 0.1% level in β decay (not to speak of the nonleptonic decays of hyperons) are practically impossible to distinguish on the background of final state interactions.

There are, however, a whole variety of mechanisms of violation of CP invariance for which the observed small relative probability of $K_L \to \pi^+\pi^-$ decay should still give rise to interesting possibilities of observation of other processes.

2. CP Nonconservation and Transitions with $\Delta T \neq 1/2$. [30,31]

There are very interesting consequences if CP is not conserved in decays in which the $\Delta T = 1/2$ rule is violated. In the $K \to 2\pi$ decay this "promotes" the decay to the state with isospin $T = 2$.

In addition to the parameter ξ introduced earlier, it is convenient to use

$$\xi' = \frac{\langle 2\pi^0 | H_W | \overline{K} \rangle}{\langle 2\pi^0 | H_W | K \rangle}. \qquad (4.7)$$

Expressing this ratio in terms of the amplitudes for transition to states with definite isospin, we have

$$\xi' = \frac{a_0 e^{i(\delta_0 - \varphi_0)} - \sqrt{2} a_2 e^{i(\delta_2 - \varphi_2)}}{a_0 e^{i(\delta_0 + \varphi_0)} - \sqrt{2} a_2 e^{i(\delta_2 + \varphi_2)}} = e^{-2i\varphi_0}(1 + x'), \qquad (4.8)$$

where $x' \approx i 2\sqrt{2} a_2/a_0 e^{i(\delta_2 - \delta_0)} \sin(\varphi_2 - \varphi_0)$. Writing r in the form $r = e^{2i\varphi_0}(1 - \epsilon)$, we find in lowest approximation in ϵ, x, and x', from (4.1) and the analogous formula for the $K_L \to 2\pi^0$ decay,

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \approx \frac{1}{2}(\varepsilon - x) \approx 2 \cdot 10^{-3}, \qquad (4.9')$$

$$\frac{A(K_L \to 2\pi^0)}{A(K_S \to 2\pi^0)} \approx \frac{1}{2}(\varepsilon - x') = ? \text{ (no experimental data).} \qquad (4.9'')$$

On the other hand,

$$\frac{A(K_S \to \pi^+\pi^-)}{A(K_S \to 2\pi^0)} = \frac{1 + \xi r}{1 + \xi' r} \frac{A(K \to \pi^+\pi^-)}{A(K \to 2\pi^0)} \approx \sqrt{2} + o(\varepsilon, x, x'). \qquad (4.10)$$

The last step in (4.10) is a consequence of the $\Delta T = 1/2$ rule in the $K \to 2\pi$ decay. But the analogous ratio of amplitudes for the decay of the longlived component K_L may be completely different. In fact, from (4.9'), (4.9'') and (4.10) we get

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_L \to 2\pi^0)} = \sqrt{2}\, \frac{\varepsilon - x}{\varepsilon - x'}. \qquad (4.11)$$

Taking account of (4.10) and the equation $x' = -2x$ [cf. (4.8) and (4.5)], we easily get the two limiting cases:

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_L \to 2\pi^0)} = \sqrt{2} \quad \text{for } \varepsilon \gg x, \qquad (4.12)$$

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_L \to 2\pi^0)} = -\frac{1}{2} \quad \text{for } \varepsilon \ll x. \qquad (4.13)$$

The first case [formula (4.12)] corresponds to CP nonconservation mainly because of virtual processes. Here $\epsilon \approx 4 \times 10^{-3}$ [this follows from (4.9')] and

$$\left| \sqrt{2} \frac{a_2}{a_0} \sin(\varphi_2 - \varphi_0) \right| \ll 4 \cdot 10^{-3}.$$

The ratio a_2/a_0 can be estimated as 4×10^{-2} from data on the ratio of probabilities for $K^+ \to \pi^+\pi^0$ and $K \to \pi^+\pi^-$ decays; then $|\varphi_2 - \varphi_0| \ll 0.7 \times 10^{-1}$.

The second case [formula (4.13)] is possible if the CP nonconservation is caused mainly by interference of the $\Delta T = 1/2$ and $\Delta T = 3/2$ (or $\Delta T = 5/2$) amplitudes, and in this sense is an indication of CP nonconservation in nonleptonic decays with change of strangeness. One might imagine (though this is not necessary in the variant we are considering) that it is just the interactions that give rise to transitions with $\Delta T = 3/2$ (or $\Delta T = 5/2$) that are the "carriers" of the CP nonconservation (i.e., only the phase $\varphi_2 \neq 0$). We then get from (4.9')

$$\left| \sqrt{2} \frac{a_2}{a_0} \sin\varphi_2 \right| \approx 4 \cdot 10^{-3} \text{ and } \varphi_2 \approx 0.7 \cdot 10^{-1}.$$

It is easily shown that $\epsilon = (1 - r) \sim a_2^2/a_0^2 \cdot \varphi_2$, and in fact is much less than the parameter $x \sim a_2/a_0 \cdot \varphi_2$.

Thus, measurement of the ratio (4.11) is of very great interest. In addition to our discussion, measurement of this ratio is important in connection with the

*It is important, however, that correlations of the form $\mathbf{p}_1 \times (\mathbf{p}_2 \times \mathbf{p}_3)$ and $\boldsymbol{\sigma}_1 \times (\boldsymbol{\sigma}_2 \times \boldsymbol{\sigma}_3)$ can occur when CP is conserved only if the final state interaction does not conserve spatial parity.

remark of Greenberg and Messiah[32] that there are no experiments in which it has actually been verified that the π meson is a boson. If this is not the case, we cannot assert that the $\pi^+\pi^-$ state has a definite CP parity. In this case CP may be conserved, $K_L \equiv K_2$, but the decay $K_2 \to \pi^+\pi^-$ will not be forbidden. But the state $|2\pi^0\rangle$ with total angular momentum $l = 0$ has positive CP parity independent of the statistics. Thus the $K_2 \to 2\pi^0$ decay is forbidden by CP conservation independent of the statistics.

3. CP Nonconservation and Leptonic Decays

The analogy suggested by the nonconservation of spatial parity is to look for the class of interactions that would be the "carriers" of CP nonconservation. The violation of CP invariance in processes caused by such interactions would be maximal. This sort of picture is by no means necessary. But it is still important to consider various models of this type, since predictions following from them may stimulate experimental investigations.

Let us suppose that CP is conserved in nonleptonic decays of strange particles and in all decays with conservation of strangeness. Nonconservation of CP will occur in the $K \to 2\pi$ decay through virtual leptonic decays that change the mass operator $\Gamma + 2iM$ (cf. [33]).

In this model the parameter ξ in (4.1) is equal to unity because of CP conservation in nonleptonic decays, while the parameter r is

$$r^2 = \frac{\Gamma^l_{\bar{K},K} + \Gamma^N_{\bar{K},K} + 2i\left(M^l_{\bar{K},K} + M^N_{\bar{K},K}\right)}{\overset{*}{\Gamma}{}^l_{\bar{K},K} + \Gamma^N_{\bar{K},K} + 2i\left(\overset{*}{M}{}^l_{\bar{K},K} + M^N_{\bar{K},K}\right)}. \tag{4.14}$$

Here Γ^l and M^l contain only leptonic contributions to the mass operator. They make possible virtual transitions of the type $K \xrightarrow{\Delta S=\Delta Q} \pi^- e^+ \nu \xrightarrow{\Delta S=-\Delta Q} \bar{K}$. It is important that the nondiagonal matrix elements $\Gamma^l_{\bar{K},K}$ and $M^l_{\bar{K},K}$ are different from zero only if the $\Delta S = \Delta Q$ rule is violated. It is clear that this is not connected with the specific choice of intermediate states, but is a reflection of the fact that in the transition $K \to j \to \bar{K}$ the strangeness changes by two units while the charge does not change at all.

It is known that leptonic decays of the K meson are 600 times less probable than the $K \to 2\pi$ decay. Thus $\Gamma^l_{\bar{K},K}/\Gamma^N_{\bar{K},K} \ll 1$. If an analogous inequality holds for the matrix elements $M^l_{\bar{K},K}$ and $M^N_{\bar{K},K}$*, we get for the parameter $1 - r$, which in accordance with (4.1) determines the amplitude ratio $A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-)$,

$$1 - r \approx -\frac{\mathrm{Im}\,\Gamma^l_{\bar{K},K} + 2i\,\mathrm{Im}\,M^l_{\bar{K},K}}{\Gamma^N_{\bar{K},K} + 2iM^N_{\bar{K},K}}. \tag{4.15}$$

We shall assume that the matrix elements for transitions with $\Delta S = \Delta Q$ and with $\Delta S = -\Delta Q$ are of the same order, and differ in phase by $\sim \pi/2$ (then $\mathrm{Im}\,\Gamma^l_{\bar{K},K} \sim \Gamma^l_{\bar{K},K}$). If we take $\Gamma^l_{\bar{K},K}/\Gamma^N_{\bar{K},K} \approx M^l_{\bar{K},K}/M^N_{\bar{K},K} \approx 1/600$, then

$$\frac{1-r}{1+r} \approx \frac{1}{2}\,\frac{1}{600}. \tag{4.16}$$

This is somewhat less than the value 2×10^{-3} given by experiment. One cannot of course attribute serious importance to such a small difference. The only important thing is that from the point of view of violation of CP invariance in leptonic decays the observed effect is large.

In this model, among others, the most important point is apparently the assumption of the existence of decays with $\Delta S = -\Delta Q$. Such decays have as yet not been observed. It is interesting, however, that the latest data supporting the $\Delta S = \Delta Q$ rule must be interpreted differently if there is a large CP violation in such decays.

Let us consider the situation in more detail for the example of the $K \to \pi e \nu (K_{e3})$ decay. To simplify the formulas we introduce a new notation for the matrix elements:

$$\left.\begin{aligned} f &= \langle \pi^- e^+ \nu \,|\, H_W \,|\, K\rangle, \\ f^* &= \langle \pi^+ e^- \tilde{\nu} \,|\, H_W \,|\, \bar{K}\rangle \end{aligned}\right\} \quad \Delta S = \Delta Q,$$

$$\left.\begin{aligned} g &= \langle \pi^+ e^- \tilde{\nu} \,|\, H_W \,|\, K\rangle, \\ g^* &= \langle \pi^- e^+ \nu \,|\, H_W \,|\, \bar{K}\rangle \end{aligned}\right\} \Delta S = -\Delta Q. \tag{4.17}$$

In (4.17) we have already used the CPT invariance of the weak interactions. The additional requirement imposed by the CP invariance is the reality of the amplitudes f and g.* The time dependence of the decay probabilities $K \to \pi^+ e^- \tilde{\nu}$ and $K \to \pi^- e^+ \nu$ is given by the formulas

$$\Gamma_{\pi^+ e^- \bar{\nu}}(t) \sim |(g + rf^*)\,e^{-\frac{1}{2}\gamma_S t} + (g - rf^*)\,e^{-\frac{1}{2}\gamma_L t} e^{i\Delta t}|^2, \tag{4.18'}$$

*This assumption is actually quite arbitrary. The assumption of a predominant role for the 2π intermediate state is justified if we are dealing with transitions on the energy shell (factor $\Gamma_{\bar{K},K}$), but the states including lepton pairs may give a large contribution to $M_{\bar{K},K}$ because the domain of integration over energy of the intermediate states in $M_{\bar{K},K}$ will in this case no longer be "cut off" by the strong interactions, and should in general be considerably larger than for two π mesons (cf.[34]).

*The amplitude f, for example, is equal to $[F_1(q^2)\,p_\mu + F_2(q^2)q_\mu] \times \bar{u}_e \gamma_\mu (1 + \gamma_5)\, v_\nu$ where u_e and v_ν are the Dirac spinors for the electron and antineutrino, p_μ are the components of the K meson momentum transferred to the leptons. There are similar expressions for the amplitudes for $K_{\mu 3}$, $K^{\pm}_{e3}$ and $K^{\pm}_{\mu 3}$ decays. The form factors F_1 and F_2 are real if CP is conserved. From the fact that the decays into an electron and in a μ meson are comparable, it follows that $F_2 < F_1$, since the same form factors can be used for the electron and the μ mesonic decays. Since the factor $q_\mu \gamma_\mu$ can be reduced to the electron mass by using the Dirac equation, the contribution of F_2 to the decay probability will be very small for the electron decay.

$$\Gamma_{\pi^- e^+ \nu}(t) \sim |(f+rg^*)e^{-\frac{1}{2}\gamma_S t} + (f-rg^*)e^{-\frac{1}{2}\gamma_L t}e^{i\Delta t}|^2 \quad (4.18'')$$

($\Delta = m_S - m_L$ is the mass difference of the mesons).

These formulas are obtained from (2.8) taking account of (2.4) and (2.2) (where we note that s = 1 from the invariance of CPT).

Usually one measures experimentally the probabilities of the decays (4.18) after summation over the spin and energy variables of the leptons. We indicate such a summation by the brackets $\langle\ \rangle$. Recent experiments [35] have measured the ratio of the coefficients in front of the exponential factors $e^{-\gamma_S t}$ and $e^{-\gamma_L t}$ in formulas (4.18′) and (4.18″). This ratio is

$$\alpha = \frac{w(K_S \to \pi e \nu)}{w(K_L \to \pi e \nu)} = \frac{\langle |g+f^*|^2\rangle}{\langle |g-f^*|^2\rangle}. \quad (4.19)$$

The parameter r is equal to unity quite accurately, as can be seen from (4.19) and (4.16).

The value $\alpha = 0.85^{+1.8}_{-0.85}$ was found. If CP were conserved, a value of unity for α would indicate the absence of decays with $\Delta S = -\Delta Q$ (i.e., g = 0). If, as is assumed in our model, CP is not conserved and $g \sim if$, then the existence of $\Delta S = -\Delta Q$ decays is compatible with the condition $\alpha = 1$.

To get information about the amplitude g, one must measure the ratio

$$\frac{w(K \to \pi^+ e^- \tilde{\nu})}{w(K \to \pi^- e^+ \nu)} = \frac{\langle |g|^2\rangle}{\langle |f|^2\rangle} \text{ when } \gamma_S t \ll 1, \quad (4.20)$$

but the experimental time resolution so far attained is insufficient for investigating this ratio.

There exist, however, data favoring the $\Delta S = \Delta Q$ rule from other processes (cf., Okun' [36]). In order to explain the suppression of the $\Delta S = -\Delta Q$ amplitudes in these processes, one needs various additional assumptions. One can therefore not deny that our just-hatched model is already on the verge of contradiction by experiments. Still we shall consider the consequences following from the model.

In the K_{e3} and $K_{\mu 3}$ decays there should be a large time-dependent charge asymmetry. CP violation in itself leads to a charge asymmetry, but if the $\Delta S = \Delta Q$ rule holds, the charge asymmetry

$$\frac{\Gamma(K \to \pi^+ e^- \tilde{\nu})}{\Gamma(K \to \pi^- e^+ \nu)} = |r|^2 \approx 1 - 2\mathrm{Re}(1-r) \quad (4.21)$$

is small and independent of the time. This follows directly from (4.18′) and (4.18″) with g = 0.

The charge asymmetry in K_L decays

$$\frac{\Gamma(K_L \to \pi^+ e^- \tilde{\nu})}{\Gamma(K_L \to \pi^- e^+ \nu)} = \frac{|g-rf^*|^2}{|rg^*-f|^2}$$

is a small effect of order (1 − r) in all cases.

In the model we are considering, the maximal effect of CP nonconservation should occur when there is interference between $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ amplitudes, since they have a large relative phase. Together with the CP violation, there should be a violation of T invariance. Thus, for example, in the $K_L \to \pi e \nu$ and $K_L \to \pi \mu \nu$ decays, the transverse polarization $\sigma_\perp \sim \mathbf{k} \times \mathbf{k}_\pi$ of the electron or μ meson should be large.

If CP nonconservation reduces simply to a shift in phase by $\pi/2$ of the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ interactions, there will be no observable effects in the decays of $K^\pm$ mesons or hyperons, because of the lack of the necessary interference. Thus the form factors F_1 and F_2, which determine the amplitude for $K^\pm_{\mu 3}$ decay (cf., the last footnote) can have only a common phase. Only in the decay of neutral K's does such a mechanism for CP violation manifest itself.

4. The Possible Existence of New, Very Weak, Interactions.

Another model that we shall consider (cf. [37]) starts from the usual form of the weak interaction as a product of currents

$$H_W = \frac{G}{\sqrt{2}} J_\mu J^+_\mu, \quad (4.22)$$

where $J_\mu = l_\mu + g_\mu + s_\mu - i\alpha t_\mu$. Here l_μ, g_μ, and s_μ are the usual leptonic, hadronic strangeness-conserving, and hadronic strangeness-changing currents (in accordance with the $\Delta S = \Delta Q$ rule); t_μ is a new current, which changes strangeness according to the $\Delta S = -\Delta Q$ rule; it is defined so that the factor i guarantees that the following terms in H_W are CP-odd:*

$$\left.\begin{aligned} H^l_W &= i\alpha \frac{G}{\sqrt{2}}(l_\mu t^+_\mu - t_\mu l^+_\mu),\\ H^{N1}_W &= i\alpha \frac{G}{\sqrt{2}}(g_\mu t^+_\mu - t_\mu g^+_\mu),\\ H^{N2}_W &= i\alpha \frac{G}{\sqrt{2}}(s_\mu t^+_\mu - t_\mu s^+_\mu). \end{aligned}\right\} \quad (4.23)$$

The interaction $H^{N_2}_W$ gives transitions with $\Delta S = 2$. A CP violating contribution to the mass operator already appears in first order in $H^{N_2}_W$. The small parameter (1 − r) is of order

$$1 - r \sim \frac{\langle \bar{K}|H^{N2}_W|K\rangle}{m_S - m_L - i/2(\gamma_S - \gamma_L)} \sim \eta\frac{Gm}{\Delta} \quad (4.24)$$

(where Δ, the mass difference $m_S - m_L$, is $\sim 10^{-5}$ eV).

Data on the $K_L \to \pi^+\pi^-$ decay (cf. (4.1)) give the estimate $\eta \sim 10^{-11}$ (we can assume $\xi = 1$ in (4.1.) Deviation of ξ from unity arises only from the inter-

*We note that in first order in the weak interaction one cannot obtain CP nonconservation using the usual expression for the current J_μ in the V-A theory of the weak interactions. The current s_μ can always be multiplied by an arbitrary factor exp $(i\varphi_S)$. Such a transformation corresponds to conservation of strangeness in the absence of weak interactions. The current l_μ can always be multiplied by an arbitrary factor exp $(i\varphi_\nu)$. This corresponds to conservation of neutrino number in the absence of weak interactions. We can thus always arrange it that the three currents l_μ, s_μ and g_μ are added with a common phase $e^{i\varphi}(l_\mu + g_\mu + s_\mu)$ (and this phase, too, can be eliminated by a transformation corresponding to charge conservation). Thus CP invariance imposes no limitations on the form of the current.

action $H_W^{N_1}$, which gives corrections $\sim \eta$ (but not $\eta\sigma m/\Delta$) to the ordinary weak interaction which produces transitions with $\Delta S = 1$).

In such a model the only really observable effects of CP nonconservation are connected with the condition $r \neq 1$. These are (except for the fact of $K_L \to \pi^+\pi^-$ decay) the charge asymmetry in the leptonic decays of neutral K mesons [cf. (4.21)].

The scale of all such effects is determined by the very small parameter η. Such effects are, in particular, the violation of T invariance in leptonic decays of neutral K mesons (because of interference between the $\Delta S = -\Delta Q$ interaction with H_W^l and the usual weak interaction, in the same way as in Sec. 4), the time dependence of the charge asymmetry (cf. Sec. 4), CP violation in nonleptonic decays with $\Delta S = 1$ (because of interference between $H_W^{N_1}$ and the ordinary weak interaction).

5. CP Nonconservation and Currents of the Second Kind.

One interesting possibility is to relate the CP nonconservation to currents of the second kind, which have so far been assumed not to occur. The model considered below (cf. [38]) is closely related to the assumption of SU_3 symmetry of the strong interactions. It arose in connection with experiments on the $K_L \to \pi^+\pi^-$ decay, but does not explain the size of the effect observed experimentally. However, even if in the future the $K_L \to \pi^+\pi^-$ decay is explained in some way not involving CP noninvariance, there is no basis a priori for excluding a large CP violation in other processes. In this sense the various predictions of this model deserve serious consideration.

The hadronic part of the vector J_μ^V and axial J_μ^A currents appearing in the weak interactions is a definite combination of components of the octet of hermitian currents. The quantities $J_\mu^{(i)} = J_\mu^{V(i)} + J_\mu^{A(i)}$ transform in a definite way under the CP operation:

$$CPJ_\mu^{(i)}(CP)^{-1} = \eta_i J_\mu^{(i)} \text{ (minus sign for } \mu = 4),$$
$$\eta_i = 1 \quad (i = 2, 3, 5, 6, 8), \qquad \eta_i = -1 \quad (i = 1, 4, 7). \tag{4.25}$$

The weak interaction describing, say, β or μ decays of baryons, has the form of a product of charge currents

$$\frac{G}{\sqrt{2}}(J_\mu l_\mu + \text{c.c.}). \tag{4.26}$$

For decays without change of strangeness, $J_\mu = \cos\theta\,(J_\mu^{(1)} + iJ_\mu^{(2)})$, and increases the charge of the hadron by unity ($\Delta Q = 1$), while for decays with change of strangeness $J_\mu = \sin\theta\,(J_\mu^{(4)} + iJ_\mu^{(5)})$ and gives transitions with $\Delta S = \Delta Q = 1$.

We note that for the CP operation,

$$CPJ_\mu(CP)^{-1} = J_\mu^+,$$
$$CPl_\mu(CP)^{-1} = l_\mu^+, \tag{4.27}$$

and the interaction (4.26) conserves CP. The nonleptonic interactions are obtained as a product of hadronic currents $J_\mu^{(i)}$ and also conserve CP under the transformation rules (4.25).

Now we suppose that the currents $J_\mu^{(i)}$ have a mixed behavior under CP. Namely, let $J_\mu^{(i)} = J_\mu^{(i),R} + J_\mu^{(i),I}$, where $J_\mu^{(i),R}$ has the same spatial and unitary properties as $J_\mu^{(i)}$, while $J_\mu^{(i),I}$ differs in the sign of its transformation by CP:*

$$CPJ_\mu^{(i),I}(CP)^{-1} = -\eta_i J_\mu^{(i),I} \text{ (plus sign for } \mu = 4). \tag{4.28}$$

It is obvious that the charge current J_μ appearing in the weak interaction (4.26) will also have the structure $J_\mu = J_\mu^R + J_\mu^I$, where

$$CPJ_\mu^R(CP)^{-1} = (J_\mu^R)^+, \quad CPJ_\mu^I(CP)^{-1} = -(J_\mu^I)^+. \tag{4.29}$$

The current J_μ^I (like the currents $J_\mu^{(i),I}$) is said to be a current of the second kind. The interaction (4.26) will now be invariant under CP reflection. The behavior of J_μ^R and J_μ^I under CP transformation has interesting consequences for the properties of the matrix elements for leptonic decays. These matrix elements, for the vector and axial parts of the current, respectively, have the form (for baryon decays)

$$V_\mu = f_1\gamma_\mu + f_2\sigma_{\mu\nu}q_\nu + f_3q_\mu,$$
$$A_\mu = g_1\gamma_\mu\gamma_5 + g_2q_\mu\gamma_5 + g_3\sigma_{\mu\nu}q_\nu\gamma_5. \tag{4.30}$$

It is important that the terms f_3q_μ and $g_3\sigma_{\mu\nu}q_\nu\gamma_5$ "arise" only from the currents of the second kind J_μ^I, and the others only from J_μ^R. This will always be the case if the strong interactions are SU_3 invariant and conserve CP.

In fact the group SU_3 always contains a reflection operation O_R, which changes either the sign of ΔQ or the signs of both ΔQ and ΔS. In the first case this is the transformation that changes a neutron into a proton ($n \rightleftarrows p$). In the second case it is the transformation $n \rightleftarrows \Sigma^-$. These operations give the transformations

$$J_\mu^R \xrightarrow{O_R} (J_\mu^R)^+,$$
$$J_\mu^I \xrightarrow{O_R} (J_\mu^I)^+. \tag{4.31}$$

J_μ^R and J_μ^I transform in the same way (with the same phase), since their properties under SU_3 are the same. Consequently the currents J_μ^R and J_μ^I have definite and opposite parities with respect to the prod-

*The two-component nature of the neutrino imposes strong restrictions on the choice of the analogous device for violation of CP through weak currents. A CP violation could be introduced by adding to the usual lepton current the expression

$$\bar{u}_e(k_e + k_\nu)_\mu(1 + \gamma_5)v_\nu.$$

But to good accuracy such terms are forbidden by the data concerning $\pi^+ \to e^+\nu/\pi^+ \to \mu^+\nu$.

uct of the two operations: CP and O_R-reflection. Their matrix elements must also have this property if the strong interactions are CP and SU_3 invariant. The matrix elements of V_μ and A_μ transform in the same way under the transformation O_R, since they are similar components of an SU_3 vector. But we know that $f_3 q_\mu$ changes sign relative to the first two terms in the matrix element V_μ under the CP transformation (cf., for example,[39]). Similarly $g_3 \sigma_{\mu\nu} q_\nu \gamma_5$ changes sign relative to the first two terms in the matrix element A_μ under the CP transformation. Consequently the terms $f_3 q_\mu$ and $g_3 \sigma_{\mu\nu} q_\nu \gamma_5$ have different parities under the reflection $CP \times O_R$. Thus $f_3 q_\mu$ and $g_3 \sigma_{\mu\nu} q_\nu \gamma_5$ are the matrix elements, respectively, of the vector and axial parts of the current of the second kind, J_μ^I. Using the property (4.29) we can in the usual way show that f_1, f_2, g_1, g_2 are real, while f_3 and g_3 are imaginary.

Similar arguments in connection with the $K_{\mu 3}$ decay permit the conclusion that in such a model the form factor F_2 in the amplitude for $K_{\mu 3}$ decay [cf. the footnote following Eq. (4.17)] "came" from a current of the second kind, and is therefore pure imaginary. It is interesting that a large value for the imaginary part of F_2 is not in contradiction with existing data on the $K_{\mu 3}$ decay.

Observable effects of CP nonconservation are associated with interference of currents of the first and second kinds. It is important that f_3 and g_3 participate only in forbidden transitions. Thus the violation of CP and T invariance should be small in β decays with a low energy release Q, even if the absolute values of f_3 and g_3 are comparable with f_1 and g_1.

But a large value of currents of the second kind should show itself clearly in virtual transitions and nonleptonic decays of particles. In this sense the small effect seen in the $K \to 2\pi$ decay cannot be explained.

In effects where CP violation cannot be observed (spectra, decay probabilities, longitudinal polarizations), currents of the second kind will appear only on the background of second-forbidden transitions, since, because of the 90° phase shift, interference terms of the type $\mathrm{Re}\, f_1 f_3^*$ will be absent.

To observe effects of violation of T invariance, one must look for correlations of the type $\boldsymbol{\sigma} \times (\mathbf{p}_1 \times \mathbf{p}_2)$ or $\boldsymbol{\sigma}_1 \times (\boldsymbol{\sigma}_2 \times \mathbf{p})$ in decays with large energy release, in particular in decays of hyperons, $K_{\mu 3}$ decay, experiments on μ capture, or in neutrino experiments.* The scale of the observable effects should be of order QR, where R is the size of the decaying system ($R \sim m^{-1}$ or m_p^{-1} for decays of K mesons and baryons).

*It is especially convenient to study the correlation $\boldsymbol{\sigma} \times [\mathbf{p}_\Lambda \times \mathbf{p}_e]$ in the $\Sigma^\pm \to \Lambda e^\pm \nu$-decay. Here we have a relatively large energy release (compared to β decay), and the polarization of the Λ can be measured directly from its decay.

6. $K_L \to \pi^+\pi^-$ Decay and CP Nonconservation in Strong Interactions.

So far all the models considered dealt with CP nonconservation in weak interactions. We have assumed that CP is strictly conserved in strong and electromagnetic interactions. But it has been pointed out by L. B. Okun'[40] that the accuracy with which CP conservation has been established in strong interactions is not sufficient to exclude the possibility of $K_L \to \pi^+\pi^-$ decay because of a small violation of CP invariance in the strong interactions. In fact, let us imagine that the strong interactions are not CP invariant. Then the state $|\pi^+\pi^-\rangle$ of two strongly interacting π mesons no longer need have the same CP parity as do two free π mesons. Let $|\pi^+\pi^-\rangle = |\pi^+\pi^-; CP = 1\rangle + |\pi^+\pi^-; CP = -1\rangle$. Then, in the fundamental formula (4.1), the parameter ξ is equal (if the weak interaction conserves CP) to:

$$\xi = \frac{\langle \pi^+\pi^- | H_W | \overline{K} \rangle}{\langle \pi^+\pi^- | H_W | K \rangle}$$

$$= \frac{\langle \pi^+\pi^-; CP = 1 | H_W | K \rangle - \langle \pi^+\pi^-; CP = -1 | H_W | K \rangle}{\langle \pi^+\pi^-; CP = 1 | H_W | K \rangle + \langle \pi^+\pi^-; CP = -1 | H_W | K \rangle}. \quad (4.32)$$

If the CP-odd admixture in the state $|\pi^+\pi^-\rangle$ is small (of order 0.1%), then $1 - \xi \sim 10^{-3}$. The same order of violation of CP appears in the mass operator, so that $1 - r \sim 10^{-3}$. In accordance with (4.1), this all leads to an experimentally observable probability for $K_L \to \pi^+\pi^-$ decay.

If we assume that the CP violating correction of 0.1% in the strong interaction at the same time conserves spatial (P) parity, we can understand the absence of dipole moments of nucleons (cf. [8,9]). Data supporting T invariance (according to the CPT theorem this is equivalent to CP conservation) of the strong interactions, obtained by measuring polarizations in pp-scattering at low energies,[41] and also from a comparison of cross sections for direct and inverse reactions,[42] have an accuracy of 2–3%.

Thus, in order to exclude this possibility we require an increase in accuracy of an order of magnitude in experiments to test CP conservation in strong interactions.

V. INTERFERENCE PHENOMENA IN A K-MESON BEAM.

We now consider various interesting new effects, which occur in a K-meson beam if the $K_L \to 2\pi$ decay exists. These phenomena are related to the possibility of interference of the $K_L \to 2\pi$ and $K_S \to 2\pi$ decays (cf., for example, [43]). Here, as a rule, the precise mechanism of the CP violation is unimportant. The interference is determined by the amplitude ratio

$$\frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \frac{1 - \xi sr}{s^2 + \xi sr} \equiv \beta, \quad \beta = |\beta| e^{i\chi}. \quad (5.1)$$

Thus, for example, the probability for decay into two π mesons as a function of time is obtained directly from (2.8):

$$\Gamma_{2\pi}(t) = w(K_S \to 2\pi)\left| e^{-\frac{1}{2}\gamma_S t} + \beta\frac{K_L(0)}{K_S(0)} e^{-\frac{1}{2}\gamma_L t + i\Delta t}\right|^2, \quad (5.2)$$

where $\Delta = m_S - m_L$. (In the laboratory coordinate system we must substitute $t \to t_{lab}/\gamma$, where t_{lab} is the time in the lab system, and γ is the Lorentz factor).

If we are considering a K meson beam ($K(0) = 1$, $\bar{K}(0) = 0$) then $K_L(0)/K_S(0) = 1$ (cf. (2.7)). Then

$$\Gamma_{2\pi}(t) \sim e^{-\gamma_S t} + |\beta|^2 e^{-\gamma_L t} + 2|\beta| e^{-\frac{1}{2}(\gamma_S+\gamma_L)t}\cos(\Delta t + \chi). \quad (5.3)$$

When $e^{-\gamma_S t} \sim |\beta|^2$, $e^{-\gamma_L t} \sim 1$, the interference term has a marked effect (cf. [43]). In the experiment of Christenson, Cronin, et al, they measured the absolute value of the ratio (5.1). A measurement of the function $\Gamma_{2\pi}(t)$ allows us to determine the phase χ.

If we are considering a K beam ($K(0) = \bar{K}(0) = 1$), then $K_L(0)/K_S(0) = -s^2$ [cf. (2.7)]. In formula (5.2), $s = 1$ if CPT is conserved. Thus in expression (5.3), for a $\bar{K}$ beam only the sign of the interference term is changed. If CPT $\neq 1$, we get additional small corrections, whose form will depend on the specific form of the CPT violation. Thus, for example, in the model of Bernstein et al.[23,24] (macroscopic field) these corrections are easily calculated using formulas (3.2) and (3.3).

Another possible way of observing interference is to place a piece of material (regenerator) in the beam of K_L mesons (i.e., at a sufficient distance from the point of creation of K and $\bar{K}$). The amplitudes for 2π decay of the K_L and K_S mesons (produced in the matter by regeneration) are coherent, and again we can have interference. We shall not give a detailed investigation of such phenomena here. We restrict ourselves to some rather general remarks.

The admixture of K_S mesons appearing after passage of the beam through a layer of matter of thickness d (when $t_0 = d/v$ is the time in the lab system for passage through the layer), is given by the regeneration coefficient α (cf., for example, [44]):

$$K_S(t_0) = \alpha K_L(t_0) = \alpha K_L(0)\, e^{-\lambda_L' t_0},$$
$$\alpha = R\,(1 - \exp(-(\lambda_S' - \lambda_L')\, t_0)). \quad (5.4)$$

Here

$$\lambda_{L;S}' = i\,(\varepsilon_{L;S} + pv) + \frac{1}{2\gamma\tau_{L;S}},$$

where $\epsilon_{S;L}$ are the respective energies of the K_S and K_L mesons, p is their momentum and $\tau_{S;L}$ their lifetimes. The value of R depends on the properties of the material and is expressed linearly in terms of its density and the difference of the amplitudes for forward scattering of the K and $\bar{K}$ mesons in the material. Even for dense materials, R is small (thus, for copper, $R \lesssim 10^{-1}$).

On emerging from the material, each of the amplitudes K_S and K_L develops with time according to the factors $e^{-\lambda_S' t}$ and $e^{-\lambda_L' t}$, respectively. Thus the probability for decay into two π mesons at a time t after emergence from the material is [again from formula (2.8)]

$$\Gamma_{2\pi}(t) = w(K_S \to 2\pi)\left|\frac{K_L(t_0)}{K_S(0)}\right|^2 |\alpha e^{-\lambda_S' t} + \beta e^{-\lambda_L' t}|^2 \quad (5.5)$$
$$= w(K_S \to 2\pi)\left|\frac{K_L(t_0)}{K_S(0)}\right|^2 \left|\alpha e^{-\frac{t}{2\gamma\tau_S}} + \beta e^{-\frac{t}{2\gamma\tau_L}} e^{i\frac{\Delta}{\gamma}t}\right|^2.$$

But if the decay is observed inside the medium (for example, in a gas or liquid) at time t_0, then*

$$\Gamma_{2\pi}(t_0) = w(K_S \to 2\pi)\left|\frac{K_L(t_0)}{K_S(0)}\right|^2 |\alpha + \beta|^2. \quad (5.6)$$

By changing the thickness of the layer of material, its density and the substance (in other words, by changing the modulus and phase of α), one can change the interference picture markedly. In principle one could achieve vanishing of the probability $\Gamma_{2\pi}$ in (5.6) because of completely destructive interference.

VI. EXPERIMENTS NEEDED FOR THE PROBLEM OF THE $K_L \to \pi^+\pi^-$ DECAY AND CP NONCON-

We now enumerate various experiments whose performance in the future is particularly important. The need for each of the experiments on the list has already been noted in connection with the various models we have treated. We have also indicated the expected size of the effect. (In the summary, we refer to the appropriate place in this survey.) But these experiments are primarily important for their own sake, independent of the models in which they were proposed.

1. Measurement of the ratio $w(K_L \to \pi^+\pi^-)/w(K_L \to 2\pi^0)$ (cf. IV, Sec. 2).
2. Measurement of the time dependence of the probability for $K \to \pi^+\pi^-$ decay at times with $e^{-\gamma_S t} \sim 10^{-6}$ (cf. V).
3. Measurement of the dependence of the ratio $w(K_L \to \pi^+\pi^-)/w(K_S \to \pi^+\pi^-)$ on the velocity of the K mesons (cf. III, Secs. 1, 2).
4. Measurement of the charge asymmetry in leptonic decays of the long-lived component $w(K_L \to \pi^+ e^- \tilde{\nu})/w(K_L \to \pi^- e^+ \nu)$ (cf. III, Sec. 2, IV, Sec. 3).
5. Measurement of the ratio $w(K_S \to \pi e \nu)/w(K_L \to \pi e \nu)$ (cf. IV, Sec. 3, III, Sec. 2).

*We note that

$$|\lambda_S' - \lambda_L'| \approx \frac{1}{\gamma}\left|i\Delta + \frac{1}{2\tau_S}\right| \approx \frac{1}{2\gamma\tau_S}.$$

If $t_0/\gamma\tau_S \gg 1$, then $\alpha \approx R$ and does not depend on t_0. If, on the other hand, $t_0/\gamma\tau_L \ll 1$, then $K_L(t_0) \approx K_L(0)$ and also is independent of t_0. Thus when $\gamma\tau_S \ll t_0 \ll \gamma\tau_L$, the quantity $\Gamma_{2\pi}(t_0)$ in (5.6) is practically independent of t_0.

6. Measurement of the charge asymmetry $\Gamma(K\to\pi^+e^-\tilde{\nu})/\Gamma(K\to\pi^-e^+\nu)$ as a function of time (cf. IV, Sec. 3).

7. Measurement of the transverse (perpendicular to the production plane) polarization of μ mesons and electrons in $K^{\pm}_{\mu 3}$, $K^{\pm}_{e3}$, $K_{\mu 3}$, and K_{e3} decays, and observation of correlations of the type $\sigma_\mu \times (p \times p_\pi)$ (cf. IV, Secs. 3, 5).

8. Observation of correlations of the type $\sigma\times(p_1\times p_2)$ and $\sigma_1\times(p\times\sigma_2)$ in β decay, leptonic decays of hyperons, μ capture, and neutrino experiments (cf. IV, Sec. 5).

9. Test of T invariance in nonleptonic decays of hyperons. Regarding the nature of these correlations, cf. [15,39].

10. Increase in accuracy in experiments testing CP invariance in strong interactions (cf. IV, Sec. 6).

11. Observation of interference between $K_L\to\pi^+\pi^-$ and $K_S\to\pi^+\pi^-$ amplitudes in regeneration experiments (cf. V).

Note added in proof. A confirmation of the results of[1] has recently appeared in two experiments: De Bouard et al., Phys. Letters 15,58 (1965); W. Galbraith et al., Phys. Rev. Letters 14,383 (1965). No energy dependence of the ratio $K_L\to\pi^+\pi^-/K_S\to\pi^+\pi^-$ was seen. Thus the effect of an external field (cf. III) appears to be excluded. There has been little clarification of the theoretical aspects of the problem, even though more than ten papers have appeared which are not included in this summary.

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