538.3

THEORY OF RADIATION FROM A CHARGE PASSING THROUGH AN ELECTRICALLY INHOMOGENEOUS MEDIUM

F. G. BASS and V. M. YAKOVENKO

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CONTENTS

INTRODUCTION

V. L. GINZBURG and I. M. Frank[1] have shown in 1946 that a particle moving uniformly in a straight line across the interface of two media with different dielectric constants emits a unique radiation, called transition radiation. The radiation effects connected with the motion of particles through electrically inhomogeneous media have subsequently attracted the attention of many workers. This group of phenomena is of interest because radiation of the transition type is encountered more frequently, for example, in the study of Cerenkov radiation in organic media, in problems involving the passage of beams of particles through an inhomogeneous plasma such as the solar corona, in accelerators, in counters for the detection of particles, etc. The physical nature of these effects is as follows.

When the charge moves through an electrically inhomogeneous medium, the relation between the phase velocity of the electromagnetic wave at the particle location and the velocity of the particle changes continuously. As a result, the electromagnetic field connected with the particle becomes detached, as it were, from the particle and radiation results.

In this sense, the radiation from a particle moving uniformly in a straight line in a medium with variable dielectric constant is analogous to the radiation from a particle moving non-uniformly in vacuum. In both cases the radiation is connected with the change in the relation between the phase velocity of the electromagnetic waves in the given medium and the particle velocity. The only difference is that in the former case the phase velocity of the wave changes, and in the latter the particle velocity.

It follows from the foregoing that radiation of electromagnetic waves from a charged particle moving uniformly in an electrically inhomogeneous medium is, unlike the Cerenkov effect, a nonrelativistic effect and can be observed at relatively low velocities.

Although the radiation from the particle in an inhomogeneous medium has been the subject of many investigations, there is still no systematic exposition of this very timely question as far as we know. The present review should fill this gap to some degree.

We have purposely disregarded investigations of the radiation produced when a particle passes parallel to a plane interface, since this effect is a modification of Cerenkov radiation. The status of this question as of 1961 was reported in the review of B. M. Bolotovskiĭ[41], and this effect was also the subject of later original papers[33,34,42a,82,85].

In this survey we consider only radiation from a particle in an inhomogeneous medium filling an unbounded space, in which connection we do not concern ourselves with radiation in waveguides[36-38,43,4,68,71,72,127].

1. TRANSITION RADIATION OF A CHARGE PASSING THROUGH THE INTERFACE OF TWO MEDIA

The problem of transition radiation was first solved, as is well known, by Ginzburg and Frank[1] and was later considered by several others[2,5,19,29,74]*.

To solve this problem, we use here the method proposed by Garibyan[4].

Let a charge e, moving uniformly with velocity v, cross the interface between two media with different dielectric constants and permeabilities ϵ and μ. We assume further that the energy lost by the particle per unit path is so small compared with its kinetic energy, and that the velocity of the particle can be regarded as constant. Then the field produced by the particle is described by Maxwell's equations in the form

$$\left.\begin{aligned}&\operatorname{rot}\mathbf{H}=\frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}+\frac{4\pi}{c}e\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t),\quad \operatorname{rot}\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\\&\operatorname{div}\mathbf{B}=0,\quad \operatorname{div}\mathbf{D}=4\pi e\delta(\mathbf{r}-\mathbf{v}t),\\&\mathbf{B}=\mu\mathbf{H},\quad \mathbf{D}=\varepsilon\mathbf{E}.\end{aligned}\right\}\qquad(1.1)\dagger$$

We choose the plane z = 0 as the interface between the media. The particle moves along the positive z direction from the first medium (ϵ_1, μ_1) into the second medium (ϵ_2, μ_2) and crosses the interface at the instant t = 0.

We seek the fields and the currents in the form of expansions in triple Fourier integrals

$$\mathbf{E}(\mathbf{r},\ t)=\int\mathbf{E}(\mathbf{k})e^{i(\mathbf{kr}-\omega t)}d\mathbf{k}\quad\text{etc.}\qquad(1.2)$$

where

$$\omega=\mathbf{kv}=k_z v,\quad \mathbf{D}(\mathbf{k})=\varepsilon(\omega)\mathbf{E}(\mathbf{k}),\quad \mathbf{B}(\mathbf{k})=\mu(\omega)\mathbf{H}(\mathbf{k}).$$

The Fourier components for the fields have the following form

$$\left.\begin{aligned}&\mathbf{E}(\mathbf{k})=\frac{ei}{2\pi^2\varepsilon}\frac{\frac{\omega}{c^2}\varepsilon\mu\mathbf{v}-\mathbf{k}}{k^2-\frac{\omega^2}{c^2}\varepsilon\mu},\\&\mathbf{H}(\mathbf{k})=\frac{\varepsilon}{c}[\mathbf{vE}].\end{aligned}\right\}\qquad(1.3)\ddagger$$

These expressions describe the field of a charge in an unbounded medium and are solutions of the inhomogeneous Maxwell equations.

To find the transition radiation, i.e., the effect connected with the boundary, it is necessary to add to the foregoing solutions also the solutions of the homogeneous Maxwell's equations with arbitrary constants, obtained from the conditions of continuity of the tangential components of the fields.

We denote by ρ and κ the components of the vectors **r** and **k**, lying in the (x, y) plane. The solutions of the homogeneous Maxwell's equations can be written in the form

$$\mathbf{E}_{1,2}(\mathbf{r},\ t)=\int\mathbf{E}_{1,2}(\mathbf{k})e^{i(\varkappa\rho+\lambda_{1,2}z-\omega t)}d\mathbf{k},\qquad(1.4)$$

where

$$\lambda_{1,2}^2=\frac{\omega^2}{c^2}\varepsilon_{1,2}\mu_{1,2}-\varkappa^2.$$

Since the first medium occupies the region of space z < 0, it follows, from the condition that the solution be finite as $z\to-\infty$, that Im $\lambda_1 < 0$.

It is obvious that the radiation fields can propagate in the first medium only in the negative z direction, and therefore Re $\lambda_1 < 0$. From analogous considerations we find that Re $\lambda_2 > 0$ and Im $\lambda_2 > 0$. These signs of Im $\lambda_{1,2}$ pertain to positive ω and should be reversed for negative ω.

From the boundary conditions for the tangential components of the fields $\mathbf{E}_T$ and $\mathbf{H}_T$ we obtain the Fourier components of the radiation field. In the first medium we have

$$E_{1\tau}=\frac{ei}{2\pi^2}\frac{\varkappa\lambda_1}{\zeta}\eta,\quad E_{1z}=\frac{ei}{2\pi^2}\frac{\varkappa^2}{\zeta}\eta,$$

$$\mathbf{H}_1=\frac{ei}{2\pi^2 c}\frac{\omega\varepsilon_1[\boldsymbol{\varkappa}\mathbf{v}]}{v\zeta}\eta,\qquad(1.5)$$

where

$$\eta=\frac{\frac{\varepsilon_2}{\varepsilon_1}-\frac{v}{\omega}\lambda_2}{k^2-\frac{\omega^2}{c^2}\varepsilon_1\mu_1}+\frac{\frac{v}{\omega}\lambda_2-1}{k^2-\frac{\omega^2}{c^2}\varepsilon_2\mu_2},$$

$$\zeta=\varepsilon_2\lambda_1-\varepsilon_1\lambda_2.$$

The radiation fields in the second medium are obtained from (1.5) by interchanging the indices 1 and 2.

We now consider the case when the particle moves from the medium to the vacuum, i.e., $\epsilon_2=\mu_2=1$. We assume furthermore that $\mu_1 = 1$ and $\epsilon_1=\epsilon=\epsilon'+i\epsilon''$.

Let us write out an expression, for example, for the radial component of the electric field of the radiation in vacuum:

$$E_{2\rho}(\varrho,\ z,\ t)=\int_{-\infty}^{\infty}E_\omega e^{-i\omega t}d\omega,$$

$$E_\omega=\frac{ei}{2\pi^2 v}\int\frac{\varkappa\lambda_2\cos\varphi}{\varepsilon\lambda_2-\lambda_1}\eta_2\exp\{i(\varkappa\varrho\cos\varphi+\lambda_2 z)\}\varkappa\,d\varkappa\,d\varphi,$$

$$\eta_2=-\frac{\varepsilon-\frac{v}{\omega}\lambda_1}{k^2-\frac{\omega^2}{c^2}}+\frac{\frac{v}{w}\lambda_1-1}{k^2-\frac{\omega^2}{c^2}\varepsilon},$$

$$\lambda_1^2=\frac{\omega^2}{c^2}\varepsilon-\varkappa^2,\quad \lambda_2^2=\frac{\omega^2}{c^2}-\varkappa^2.\qquad(1.6)$$

The integration with respect to φ is from 0 to 2π, and with respect to κ from 0 to ∞. The integral with respect to φ is expressed in terms of Bessel functions.

*A very illustrative physical interpretation of transition radiation, its peculiarities, and its connection with other phenomena can be found in the paper by I. M. Frank[108], dedicated to the memory of S. I. Vavilov.

† rot ≡ curl.

‡[vE] ≡ v × E.

We introduce the distance R from the point of emergence of the particle from the medium to the point of observation, and the angle θ between the z axis and R, by means of the relations $z = R\cos\theta$ and $\rho = R\sin\theta$. Then, for large R, using the asymptotic expressions for the Bessel functions, we obtain

$$E_\omega(R,\theta)=\int_0^\infty dxW(x)\left\{\exp\left[\frac{i\omega}{c}Rf(x)-\frac{3\pi i}{4}\right]+\exp\left[\frac{i\omega}{c}R\psi(x)+\frac{3\pi i}{4}\right]\right\}, \tag{1.7}$$

where

$$\left.\begin{aligned} x&=\frac{c}{\omega}\varkappa,\quad f(x)=x\sin\theta+\sqrt{1-x^2}\cos\theta,\\ \psi(x)&=-x\sin\theta+\sqrt{1-x^2}\cos\theta,\\ W(x)&=\frac{e}{\pi v}\frac{x\frac{\omega}{c}}{\sqrt{2\pi R\sin\theta}}\frac{\lambda_2}{\varepsilon\lambda_2-\lambda_1}\eta_2(x).\end{aligned}\right\} \tag{1.8}$$

Similar integrals for $\omega R/c \gg 1$ can be readily calculated by the saddle-point method. We consider the first term in the integrand

$$\int_0^\infty dxW(x)\exp\left[\frac{i\omega}{c}Rf(x)-\frac{3\pi i}{4}\right].$$

The saddle point is $x_s = \sin\theta$. We must recognize that the function W(x) has a pole at the point

$$x_p=\left(\varepsilon-\frac{c^2}{v^2}\right)^{\frac{1}{2}},$$

which under the condition $0 \le x_p \le \sin\theta$ makes a contribution to the radiation field, in the form of a cylindrical wave (the other poles are unimportant).

The distance between the saddle point and the pole varies with the frequency; in particular, the pole may coincide with the saddle point (we have in mind the case when $\epsilon'' \to 0$). Depending on this distance, the radiation fields in vacuum differ in structure. Leaving out some intermediate steps, we can represent the radiation field in the following form [54]:

$$E_\omega(R,\theta)=-\left[W(x_s)-\frac{W}{x_s-x_p}\right]\sqrt{\frac{2\pi c}{\omega R}}\cos\theta\exp\left(\frac{i\omega}{c}R\right)+W_0\left\{2\pi i\exp\left[i\frac{\omega}{c}Rf(x_p)-\frac{3\pi i}{4}\right]+\int_{-\infty}^{\infty}\frac{d\xi}{\xi+A}e^{-\xi^2}\exp\left(i\frac{\omega}{c}R-\frac{3\pi i}{4}\right)\right\}, \tag{1.9}$$

where W_0 is the residue of W(x) at the pole $x = x_p$, and

$$\Delta=(x_s-x_p)\sqrt{\frac{\omega R}{2c}}\sec\theta\exp\frac{\pi i}{4}. \tag{1.9'}$$

(It can be shown that the second term in the integrand of (1.7), the exponential of which contains $\psi(x)$ in the argument, makes no contribution to the radiation field in the approximation of large R.)

When the saddle point and the pole are sufficiently far apart ($|\Delta| \ll 1$) we have

$$E_\omega=-W(x_s)\sqrt{\frac{2\pi c}{\omega R}}\cos\theta\exp\left(i\frac{\omega}{c}R\right)+2\pi W_0\exp\left[i\frac{\omega}{c}Rf(x_p)-\frac{\pi i}{4}\right]. \tag{1.10}$$

In the frequency region where $|\Delta| \ll 1$, the pole and the saddle point coincide and $f(x_p) \to 1$, so that

$$E_\omega=-\left[W(x_s)-\frac{W_0}{x_s-x_p}\right]\sqrt{\frac{2\pi c}{\omega R}}\cos\theta\exp\left(\frac{i\omega}{c}R\right)+\pi W_0\exp\left(i\frac{\omega}{c}R-\frac{\pi i}{4}\right). \tag{1.11}$$

Thus, the radiation field, subject to the condition $\epsilon''\sqrt{\omega R/c} \ll 1$, i.e., when the attenuation can be neglected, consists of a spherical and cylindrical wave. The first describes the transition radiation and the second the Cerenkov radiation emerging from the medium to the vacuum.

Indeed, when the particle moves from the medium to the vacuum, the Cerenkov wave is incident on the interface at an angle

$$\cos\theta'=\frac{1}{\beta\sqrt{\varepsilon'(\omega)}},\quad \beta=\frac{v}{c}. \tag{1.11'}$$

If we now use Snell's law, putting $\kappa' = (\omega/c)\sqrt{\epsilon'}\sin\theta'$, and substitute the value of θ' from (1.11'), we obtain $\kappa' = \omega x_p/c$, i.e., $x_p = \sin\nu(\omega)$ precisely determines the angle of refraction of the Cerenkov wave of frequency ω. The left side of the inequality $0 \le x_p < \sin\theta$ is the condition for the excitation of the Cerenkov wave in the medium, while the right side is the condition for the emergence of this wave to the vacuum.

In the limiting case of very large R, when the condition

$$\varepsilon''\sqrt{\frac{\omega}{c}R}\gg 1 \tag{1.12}$$

is satisfied, the situation will be different. Now the attenuation can no longer be regarded as equal to zero, and therefore the condition $|\Delta| \gg 1$ is satisfied in the entire frequency interval. At such distances there is no cylindrical wave, and the radiation field constitutes a spherical wave

$$E_\omega=-W(x_s)\sqrt{\frac{2\pi c}{\omega R}}\cos\theta\exp\left(i\frac{\omega}{c}R\right). \tag{1.13}$$

Expression (1.12) describes both transition and Cerenkov radiation. It is clear that in the presence of attenuation, the contribution to the Cerenkov field emerging to the vacuum is made by a small section of the particle trajectory near the interface. At infinitely large distances from the point of emission of the particle, the waves radiated by a source of finite dimensions are of course spherical.

We note that the remaining components of the electromagnetic field, E_z and $H_\varphi = H$, have a form analogous to that of E_ρ, differing only in inessential multipliers.

We now find the spectral density of the radiation. The energy flux of the radiation of electromagnetic waves in a solid-angle element $d\Omega = \sin\theta\, d\theta\, d\varphi$, during the time of flight of the particle, is equal to

$$\frac{dW_{\text{tr}}}{d\Omega} = \frac{c}{4\pi} R^2 \int_{-\infty}^{\infty} E'H\, dt, \tag{1.14}$$

where

$$E' = E_\rho \cos\theta - E_z \sin\theta = H.$$

The spectral density of the transition radiation is written in the form

$$\frac{d^2W_{\text{tr}}}{d\Omega\, d\omega} = cR^2 \mid E'_\omega \mid^2. \tag{1.14a}$$

For the Cerenkov radiation (cylindrical waves) it is convenient to calculate the flux of the Poynting vector through a circular area

$$\varrho,\ \varrho + d\varrho,$$

$$\frac{dW_{\text{c}}}{d\varrho} = \frac{c}{4\pi} \cdot 2\pi\varrho \int_{-\infty}^{\infty} E_\rho H\, dt,$$

hence

$$\frac{d^2W_{\text{c}}}{d\varrho\, d\omega} = 2\pi\varrho c E_{\rho\omega} H^*_\omega. \tag{1.15}$$

We now consider the frequency interval in which $|\Delta| \gg 1$. If we assume zero attenuation and let R become infinite, then the condition can be written in the form

$$|\beta \sin\theta - \sqrt{\beta^2 \varepsilon'(\omega) - 1}| > 0. \tag{1.16}$$

In this frequency region we obtain

$$\frac{d^2W_{\text{tr}}}{d\Omega\, d\omega} = \frac{e^2\beta^2 \sin^2\theta \cos^2\theta}{\pi^2 c (1-\beta^2\cos^2\theta)^2} \times \left| \frac{(\varepsilon - 1)(1 - \beta^2 - \beta\sqrt{\varepsilon - \sin^2\theta})}{(\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta})(1 - \beta\sqrt{\varepsilon - \sin^2\theta})} \right|^2, \tag{1.17}$$

$$\frac{d^2W_{\text{c}}}{d\varrho\, d\omega} = \frac{4e^2}{v^2}\omega \frac{\sqrt{(\beta^2\varepsilon' - 1)[1 + \beta^2(1-\varepsilon')]}}{(1 + \varepsilon'\sqrt{1 + \beta^2(1-\varepsilon')})^2}. \tag{1.18}$$

As already noted, Cerenkov radiation in the form of cylindrical waves is produced not in the entire frequency region $|\Delta| \gg 1$, but only in the part where $0 \le \sqrt{\epsilon'(\omega) - \beta^{-2}} \le \sin\theta$.

In the case when the saddle point and the pole coincide, we obtain the width of the frequency interval from the solution of the equation ($|\Delta| \sim 1$):

$$\delta\omega' \sim \sqrt{\frac{c}{\omega' R}} \left(\frac{d\varepsilon'}{d\omega}\right)^{-1}_{\omega=\omega'},$$

where ω' is determined from the equation

$$\varepsilon'(\omega') - \beta^{-2} = \sin^2\theta.$$

For the transition radiation in this frequency interval $(\omega' \pm \delta\omega')$ we obtain

$$\frac{d^2W_{\text{tr}}}{d\Omega\, d\omega} = \frac{e^2\beta^2 \sin^2\theta\cos^2\theta}{\pi^2 c(1-\beta^2\cos^2\theta)^2} \left| \frac{\varepsilon + 1}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}} \right|^2, \tag{1.19}$$

and the Cerenkov radiation is determined as before by the expression (1.18), but without the factor 4.

Finally, when condition (1.5) is satisfied, the transition and Cerenkov radiations are described by formula (1.17) in the entire frequency interval. The sharp maximum of (1.17), arising when $1 - \beta\sqrt{\epsilon'(\omega) - \sin^2\theta} = 0$, is due to the emergence of the Cerenkov radiation to the vacuum.

For a particle entering into the medium, the formula for the spectral density of the transition radiation (the formula of Ginzburg and Frank[1]) is obtained from (1.17) by replacing β with $-\beta$.

The transition radiation is linearly polarized, and the electric vector lies in the plane passing through the ray in the direction of the charge velocity. For nonrelativistic velocities, the intensity of the radiation is proportional to the square of the particle velocity. On the interface between vacuum and an ideal conductor ($\epsilon = \infty$), the spectral radiation density is[1]

$$\frac{d^2W_{\text{tr}}}{d\Omega\, d\omega} = \frac{e^2\beta^2}{\pi^2 c}\sin^2\theta. \tag{1.17'}$$

The angular distribution of the radiation is in this case the same as for a dipole placed on the interface.

In the ultrarelativistic case, as can be seen from (1.17), the transition radiation has a sharp maximum in the direction $\theta \sim mc^2/E$ (E —kinetic energy of the particle). In this case, as shown by Garibyan[5], the main contribution to the transition radiation forward is made by frequencies larger than optical, for at these frequencies the denominator of (1.17) is small because of the smallness of

$$1 - \beta\sqrt{\varepsilon - \sin^2\theta} \quad (\beta \sim 1,\ \varepsilon \sim 1,\ \theta \ll 1).$$

The spectral density is practically constant up to the limiting frequency $\omega_{\lim} = \omega_0/2\sqrt{[1-\beta^2]}$ (at this frequency the radiation density decreases to one half the radiation density at the lower frequencies), where ω_0 is the plasma frequency of the medium. Substituting in (1.17) the expression for $\epsilon(\omega) = 1 - (\omega_0^2/\omega^2)$ (this expression for $\epsilon(\omega)$ in any medium when $\omega > \omega_0$[110]), and then integrating with respect to the frequencies and the angles, we obtain[5]

$$W_{\text{tr}} = \frac{e^2\omega_0}{3c\sqrt{1-\beta^2}}. \tag{1.20}$$

When the particle enters the medium, the factor $1 - \beta\sqrt{[\epsilon - \sin^2\theta]}$ in the denominator of (1.17) is no longer small, and the transition radiation in vacuum encompasses only the optical part of the spectrum and is determined by a formula that increases logarithmically with the energy[1,5]:

$$W_{\text{tr}} = \frac{e^2}{\pi c}\left(\ln\frac{2}{1-\beta} - 1\right) \int_0^\infty \left(\frac{\sqrt{\varepsilon(\omega)} - 1}{\sqrt{\varepsilon(\omega)} + 1}\right)^2 d\omega. \tag{1.21}$$

It must be pointed out that the expressions presented describe the transition radiation only under the conditions $(\omega/c)R\sin^2\theta \gg 1$. In the region of space $R \sim (c/\omega)\sin^{-2}\theta$, redistribution of the wave field of the transition radiation takes place.* In the ultrarelativistic case, this redistribution region is quite appreciable. In vacuum its order of magnitude is

$$z_v \sim \frac{2c}{\omega(1-\beta^2+\theta^2)},$$

and in the medium (at frequencies larger than optical)

$$z_m \sim \frac{2c}{\omega\left(1-\beta^2+\theta^2+\frac{\omega_0^2}{\omega^2}\right)}.$$

The radiation field of an ultrarelativistic particle, as can be seen from (1.5), is actually redistributed in a region with dimensions $\sim z_v - z_m$. It is obvious that for the formation of the transition radiation it is very important that the particle trajectory be linear over a length on the order of z_m. Multiple scattering can lead to a change in the value of z_m, and consequently also to a change in the spectrum and energy of the transition radiation. The effect of multiple scattering on the transition radiation was considered by Garibyan and Pomeranchuk[15], Garibyan[8], and Pafomov[17,139].

Without dwelling in detail on the contents of these papers, we point out one important circumstance. It turns out that when multiple scattering is taken into account, there exist a certain critical energy

$$E_{cr} = \frac{(mc^2)^3}{E_s^2}\frac{\omega_0 L}{c},$$

(where L is the radiation unit of length, $E_s = 21$ MeV) such that if the particle energy E is lower than critical, multiple scattering does not affect the transition radiation, for in this case the transition radiation field is redistributed essentially along the path in a vacuum, up to the maximum frequency ω_{lim}.

When $E > E_{cr}$, the usual mechanism of transition radiation in the high-frequency region of the spectrum

$$\omega_{lim} \gg \omega > \omega_1 = \left(\frac{\omega_0^2 E}{E_s}\right)^{2/3}\left(\frac{L}{c}\right)^{1/3}$$

begins to break down as a result of multiple scattering, and the multiple scattering itself leads to the appearance of bremsstrahlung. In this case the transition radiation cannot be distinguished from bremsstrahlung and the two must apparently be regarded as a single phenomenon.

The intensity of the transition radiation depends greatly also on the relation between the field redistribution zone and the degree of smearing of the interface. Amatuni and Korkhmazyan have shown[26] that if the smearing is much larger than the redistribution zone the radiation is greatly reduced (see also [140].)

The foregoing formulas are valid, naturally, only for sharp boundaries.

It is also easy to obtain an expression for the spectral density of the transition and Cerenkov radiations in vacuum at the boundary of an isotropic ferrodielectric (Pafomov[19]). Using (1.5), we get

$$\frac{d^2W}{d\Omega\, d\omega} = \frac{e^2\beta^2\sin^2\theta\cos^2\theta}{\pi^2 c(1-\beta^2\cos^2\theta)^2} \times \left|\frac{(\varepsilon-1)(1-\beta\sqrt{\varepsilon\mu-\sin^2\theta})-\beta^2(\varepsilon\mu-1)}{(1-\beta\sqrt{\varepsilon\mu-\sin^2\theta})(\varepsilon\cos\theta+\sqrt{\varepsilon\mu-\sin^2\theta})}\right|^2. \quad (1.18')$$

It must be noted that in a ferrodielectric the group velocity can in principle be negative, i.e., the directions of the phase and group velocities can be opposite.* This can be readily shown by substituting in Maxwell's equation a solution in the form of plane waves ($E, H \sim \exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$). We then obtain

$$\mathbf{k}E^2 = \frac{\omega\mu}{c}[\mathbf{EH}] = \frac{4\pi}{c^2}\omega\mu\mathbf{S}.$$

Undamped electromagnetic waves exist both when $\epsilon > 0$, $\mu > 0$ and when $\epsilon < 0$, $\mu < 0$, since the refractive index is real in both cases. From the latter equality it is seen that in the first case the Poynting vector coincides in direction with the wave vector $\mathbf{k}$, while in the second case $\mathbf{S}$ and $\mathbf{k}$ are directed opposite each other, i.e., the group velocity is negative. In this case the solutions are advanced potentials. If the particle velocity exceeds the phase velocity of the light ($n\beta > 1$), then all the elementary waves of the excitation propagate, with equal phase, in a direction θ satisfying the equality $n\beta\cos\theta = 1$. The Cerenkov waves produced in this manner have an equal-phase surface in the form of a cone with vertex directed backward (in distinction from retarded potentials, when the vertex of the cone is directly forward). The phase velocity of the Cerenkov waves makes an acute angle θ with the direction of motion of the particle, while the energy propagates in the opposite direction. Accordingly, a sharp peak in the intensity of the transition radiation, due to the Cerenkov radiation, should take place when a particle enters the medium. This can be readily verified by introducing a small damping

$$\varepsilon = \varepsilon' + i\varepsilon'', \qquad \mu = \mu' + i\mu'' \qquad (\varepsilon'' > 0, \quad \mu'' > 0).$$

Indeed, in the region of frequencies with negative group velocity ($\epsilon' < 0$, $\mu' < 0$), we should get, from the conditions that the solution be finite as $z \to \infty$,

$$\mathrm{Re}\,\lambda_1 = \frac{\omega}{c}\sqrt{\varepsilon'\mu' - \sin^2\theta} < 0.$$

Taking this circumstance into account and replacing β by $-\beta$, we verify that the spectral density (1.18)

*More accurately, the zone of redistribution of the radiation is determined from the condition that there be no interference between the radiation field and the particle field, i.e.,

$$z_{1,2} \sim \left|\frac{\omega}{v} - \lambda_{1,2}\right|^{-1}.$$

*Particle-radiation singularities connected with negative group velocity in an anisotropic medium were investigated in the dissertation of Pafomov[18].

has a sharp maximum in the direction corresponding to the angle of emission of the Cerenkov radiation θ_0, ($\sin^2\theta_0 = \epsilon'\mu' - \beta^{-2}$).

This singularity of transition radiation, connected with the negative group velocity, appears also in anisotropic media[18,19,55,56] and in media with spatial dispersion[85].

In conclusion it must be noted that by using perfectly analogous arguments we can calculate the transition radiation not only for a point charge, but also for an arbitrary current, for example for dipole moments[24], a charged filament[11], etc.[25,46,65-68]

The quantum theory of transition radiation was developed in papers by Ternovskiĭ[126] and Garibyan[9]. It follows from their papers, however, that the quantum corrections to the classical formulas for transition radiation are of no particular interest in the case of the ordinary electron densities. They can become appreciable only when the charge enters from vacuum in very dense matter, for example into an atomic nucleus.

2. RADIATION OF A CHARGE PASSING THROUGH A MOVING INTERFACE

The question of the transition radiation in moving media is of definite interest in the study of the interaction between individual particles and electron clusters, for example, in the generation of radio waves, for astrophysical applications, etc.

In the present section we develop the results obtained by Barsukov and Bolotovskiĭ[39] and Barsukov and Naryshkina[35].

Let us assume that in a reference frame x, y, z, ict the interface between two media with dielectric constants ϵ_1 and ϵ_2, measured in the reference frame at rest, is the plane $z = -ut$, where u is the velocity of the interface. The charge e moves along the z axis with velocity v. We change over to a reference frame connected with the interface. The velocity of the charge in this system is written in the form

$$w=\frac{u+v}{1+\frac{uv}{c^2}}$$

and the electromagnetic fields of the radiation are determined by expressions (1.4) and (1.5).

To obtain the radiation fields in the stationary reference frame, it is necessary to use the Lorentz transformation for the field.

We shall henceforth confine ourselves to the case when the moving medium is a plasma with $\epsilon_2' = 1 - (\omega_0'^2/\omega'^2)$, and $\epsilon_1' = 1$ (the primes denote quantities pertaining to the frame affixed to the boundary).

As a result of a calculation similar to that given in the preceding section, we obtain for the total radiation energy when the relative particle velocity is $w \approx c$

$$W_r^+=\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}\,W_0^+(w) \qquad (2.1)$$

for radiation directed forward and

$$W_r^-=\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}\,W_0^-(w) \qquad (2.2)$$

for radiation directed backward.

Here $W_0^+(w)$ and $W_0^-(w)$ denote the total energy radiated in the corresponding direction in the stationary medium, and are determined by formulas (1.20) and (1.21) in which v is replaced by w. Substituting the expression for $W_0^-(w)$ and introducing the Lorentz-transformed frequency

$$\omega=\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}\,\omega',$$

we obtain

$$W_r^-=\frac{e^2}{\pi c}\int_0^\infty\left(\frac{\sqrt{\varepsilon}-1}{\sqrt{\varepsilon}+1}\right)^2\left(\ln\frac{2}{1-\frac{w}{c}}-1\right)d\omega, \qquad (2.3)$$

where

$$\varepsilon=1-\frac{\omega_0''^2}{\omega^2},\quad \omega_0''=\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}\,\omega_0'.$$

The expression for the radiation energy has formally the same form as in the case of a stationary interface, but with a different material plasma frequency. When $u \approx c$, ω_0'' becomes much smaller than ω_0'. This effect is equivalent to increasing the plasma density by a factor $2(1-u/c)^{-1}$.

Integrating (3.3) with respect to the frequency, we get

$$W_r^-=\frac{e^2\omega_0''}{15\pi c}\left\{\ln\left[\frac{1}{\left(1-\frac{u}{c}\right)\left(1-\frac{v}{c}\right)}\right]-1\right\}. \qquad (2.4)$$

As can be seen from (2.4), W_r^- becomes quite appreciable when $u \sim c$.

When $w \approx c$, the total energy radiated forward (when the particle is emitted from the plasma)

$$W_r^+=\frac{e^2\omega_0'}{3c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \qquad (2.5)$$

i.e., in this case the radiation energy is independent of the velocity of the wall, and coincides exactly with the expression for the transition radiation in the case of the stationary interface.

3. TRANSITION RADIATION IN OBLIQUE INCIDENCE OF THE CHARGE ON THE INTERFACE

The radiation produced in oblique incidence of the charge on the interface has a very important property: unlike in normal incidence, the radiation is generally speaking polarized in two mutually perpendicular directions.* This circumstance is very important for

*A similar polarization is possessed also by transition radiation in the case of normal incidence of a charge on the boundary of a gyrotropic medium[56, 142, 143].

the detection of transition radiation, since the polarization of the transition radiation is one of the most important factors which make it possible to distinguish it from all possible extraneous effects.

Transition radiation in the case of oblique incidence of the charge was considered in papers by Korkhmazyan,[29,32], Garibyan[6,7], and Pafomov[22].

We present below an exposition of the results of these papers.

Assume that a charge is incident with velocity v on the plane z = 0 which separates the media ϵ_1 and ϵ_2. The x axis is chosen such that the vector v is in the (x, z) plane, while the z axis is directed from the first medium (ϵ_1) into the second medium (ϵ_2).

The procedure for finding the electromagnetic fields of the radiation is in this case precisely the same as in Sec. 1. We therefore present directly expressions for the Fourier components of the radiation[7]:

$$\left.\begin{aligned} \mathbf{H}_{1,2}(\mathbf{k}) &= \frac{c}{\omega}[(\boldsymbol{\varkappa}+\mathbf{n}\lambda_{1,2})\mathbf{E}_{1,2}], \\ \varkappa E_{1,2\tau}+\lambda_{1,2}E_{1,2n}(\mathbf{k}) &= 0, \\ \mathbf{E}_{1,2\tau}(\mathbf{k}) &= \boldsymbol{\varkappa}E_{1,2\varkappa}(\mathbf{k})+\mathbf{v}_xE_{1,2x}(\mathbf{k}), \end{aligned}\right\} \tag{3.1}$$

$$E_{1,2\varkappa}=\frac{ei}{2\pi^2}\frac{1}{\lambda_{1,2}\varepsilon_{2,1}-\lambda_{2,1}\varepsilon_{2,1}}\left[\lambda_{1,2}\left(\frac{\frac{\varepsilon_{2,1}}{\varepsilon_{1,2}}-\frac{v_z}{\omega}\lambda_{2,1}}{k^2-\frac{\omega^2}{c^2}\varepsilon_{1,2}}+\frac{\frac{v_z}{\omega}\lambda_{2,1}-1}{k^2-\frac{\omega^2}{c^2}\varepsilon_{2,1}}\right)\right.$$
$$\left.+\frac{k_xv_x}{\omega}(k_z-\lambda_{1,2}-\lambda_{2,1})\left(\frac{1}{k^2-\frac{\omega^2}{c^2}\varepsilon_{1,2}}-\frac{1}{k^2-\frac{\omega^2}{c^2}\varepsilon_{2,1}}\right)\right], \tag{3.2}$$

$$E_{1,2x}=\frac{ei}{2\pi^2}\frac{\omega(\lambda_{2,1}-k_z)}{c^2(\lambda_{1,2}-\lambda_{2,1})}\left(\frac{1}{k^2-\frac{\omega^2}{c^2}\varepsilon_{1,2}}-\frac{1}{k^2-\frac{\omega^2}{c^2}\varepsilon_{2,1}}\right), \tag{3.3}$$

where $\omega = \mathbf{k}\cdot\mathbf{v} = k_Xv_X + k_Zv_Z$, $\mathbf{n}$ is a unit vector along the z axis, and $E_\kappa(\mathbf{k})$ and $E_X(\mathbf{k})$ are the components of the tangential components E_τ along κ and along v_X.

To find the flux of the differently polarized radiation we resolve the vectors $\mathbf{E}$ and $\mathbf{H}$ into components.

One of the components—the longitudinally polarized one—lies in the plane containing the wave vector and the normal drawn to the boundary at the point of entry of the particle, while the other—transversely polarized—is perpendicular to the first. Denoting further the longitudinally polarized components by the symbol $\parallel$ and the transversely polarized by $\perp$, we obtain

$$\begin{aligned} E_x^{\parallel} &= \cos\varphi(\varkappa E_\varkappa+\cos\varphi v_xE_x), \\ E_x^{\perp} &= \sin^2\varphi v_xE_x \end{aligned} \tag{3.4}$$

etc., where φ is the angle between κ and the x axis.

The electromagnetic radiation flux during the entire time of flight of the particle is best obtained by means of the formula

$$W=\frac{c}{4\pi}\int[\mathbf{EH}]_z\,dx\,dy\,dt. \tag{3.5}$$

In integrating with respect to t, x, and y we obtain δ-functions; taking further account of the fact that

$$dk_z=\frac{d\omega}{v_z},\quad k_x=\frac{\omega}{c}\sqrt{\varepsilon}\sin\theta\cos\varphi,\quad k_y=\frac{\omega}{c}\sqrt{\varepsilon}\sin\theta\sin\varphi,$$
$$\varkappa=\frac{\omega}{c}\sqrt{\varepsilon}\sin\theta,$$

we obtain the spectral density of the radiation in a solid-angle element*.

Thus, for the radiation in medium 2 (forward) we have[32]

$$\frac{d^2W^{\parallel}}{d\Omega\,d\omega}=\frac{4\pi^2\omega^2}{cv_z^2}\sqrt{\varepsilon_2}\left[\frac{\omega}{c}\sqrt{\varepsilon_2}\sin\theta E_{2x}(\omega,\ \theta,\ \varphi)\right.$$
$$\left.+v_xE_{2x}(\omega,\ \theta,\ \varphi)\cos\varphi\right]^2, \tag{3.6}$$

$$\frac{d^2W^{\perp}}{d\Omega\,d\omega}=\frac{4\pi^2\omega^2}{cv_z^2}\varepsilon_2^{3/2}v_x^2E_{2x}^2(\omega,\ \theta,\ \varphi)\cos\theta\sin^2\varphi. \tag{3.7}$$

The total radiation density is determined by the sum of (3.6) and (3.7).

As can be seen from (3.7), in the case of normal incidence of the charge on the boundary ($v_X = 0$) the radiation is longitudinally polarized.

Let us consider some particular cases of our results.

As $v_Z \to 0$, the transition radiation vanishes, as it should, since the particle does not cross the interface. When $\theta = \pi/2$, the radiation is also equal to zero if $\epsilon_1 \neq \infty$. This result is due to the fact that the wave propagating along the interface either cannot satisfy the boundary condition ($\epsilon_2 > \epsilon_1$) or else is "drawn into the medium" 1 ($\epsilon_2 < \epsilon_1$).

If we put in (3.6) and (3.7) $\epsilon_2 = 1$ and $\epsilon_1 = \infty$, then we obtain the angular distribution on the interface between vacuum and an ideal conductor:

$$\frac{d^2W^{\parallel}}{d\Omega\,d\omega}=\frac{e^2\beta_z^2}{\pi^2c}\left[\frac{\sin\theta-\beta_x\cos\varphi}{(1-\beta_x\sin\theta\cos\varphi)^2-\beta_z^2\cos^2\theta}\right]^2, \tag{3.8}$$

$$\frac{d^2W^{\perp}}{d\Omega\,d\omega}=\frac{e^2\beta_z^2}{\pi^2c}\left[\frac{\beta_x\cos\theta\sin\varphi}{(1-\beta_x\sin\theta\cos\varphi)^2-\beta_z^2\cos^2\theta}\right]^2. \tag{3.9}$$

As can be seen from (3.8), when $\sin\theta = \beta_X\cos\varphi$ the radiation polarization is purely transverse. Let us consider the transition radiation of a nonrelativistic particle. Neglecting in (3.2) and (3.3) all the terms that contain β, we get[22]

$$\frac{d^2W^{\parallel}}{d\Omega\,d\omega}\simeq\frac{d^2W}{d\Omega\,d\omega}=\frac{e^2\beta_z^2}{\pi^2c}\frac{\varepsilon_2^{1/2}|\varepsilon_2-\varepsilon_1|^2\sin^2\theta\cos^2\theta}{|\varepsilon_1\cos\theta+\sqrt{\varepsilon_2(\varepsilon_1-\varepsilon_2\sin^2\theta)}|^2}. \tag{3.10}$$

Thus, the transition radiation is polarized in this approximation in the same manner as for normal incidence, and the radiation density differs only by the factor $(\beta_Z/\beta)^2$.

For relativistic velocities, the main contribution to the transition radiation is made, as before, by the frequencies $\omega_0 < \omega < \omega_0/2\sqrt{[1-\beta^2]}$, and the radiation is

*We confine ourselves here to the case of large distances, when the cylindrical wave is completely transformed into a spherical one.

concentrated around the trajectory of the particle in a cone with an apex angle $\sim\sqrt{[1-\beta^2]}$. The transversely polarized radiation component is smaller in order of magnitude by a factor $(1-\beta^2)$ than the longitudinal component, i.e., this radiation is practically completely longitudinally polarized.

4. TRANSITION RADIATION WITH ACCOUNT OF THE SPATIAL DISPERSION OF THE DIELECTRIC CONSTANT

The radiation from sources moving in a medium depends not only on the nature of the source and on the character of its motion, but also on the electromagnetic properties of the medium. From this point of view, it is of definite interest to determine the radiation from charged particles in media with spatial dispersion. We have referred so far to media in which the connection between the induction and field-intensity vectors was local. This means that we considered media in which the value of the induction at a given point is determined by the field at the same point. Yet it is sometimes necessary to take into account the influence of the field in all of space on the induction at a given point, and this leads to the appearance of several new effects, particularly to the appearance of additional waves [57,69,70]. Mathematically this circumstance is described by an integral relation between the field and the induction, namely

$$D_i(\mathbf{r},\ t)=\int\limits_{-\infty}^{t} dt' \int d\mathbf{r}'\hat{\varepsilon}_{ik}(t,\ t',\ \mathbf{r},\ \mathbf{r}')E_k(\mathbf{r}',\ t'). \qquad (4.1)$$

For processes which are homogeneous in space and in time, the operator $\hat{\epsilon}$ depends only on the differences $\mathbf{r}-\mathbf{r}'$ and $t-t'$. In this case the nonlocal connection between the field and the induction can be replaced by a local connection between the Fourier components, i.e.,

$$D_i(\omega,\ \mathbf{k})=\varepsilon_{ik}(\omega,\ \mathbf{k})E_k(\omega,\ \mathbf{k}). \qquad (4.2)$$

The dependence of the dielectric tensor $\epsilon_{ik}(\omega,\mathbf{k})$ on the wave vector $\mathbf{k}$ is called spatial dispersion.

The effect of spatial dispersion on the energy lost to radiation by a particle passing through an interface between vacuum and a medium was first considered by Zhelnov [52] and later by many others [53-56,58,59,104]. The operator $\hat{\epsilon}_{ik}(t,t',\mathbf{r},\mathbf{r}')$ can then be obtained by starting from a concrete model of the medium and making some physical assumptions concerning the properties of its surface. This is the situation, for example, in problems involving a bounded hot plasma [57].

It is possible, however, to approach the problem phenomenologically, by postulating the form of the operator $\hat{\epsilon}_{ik}(\omega,\mathbf{k})$ [70]. It then becomes necessary to introduce additional boundary conditions for Maxwell's equations, since their order, generally speaking, increases for media with spatial dispersion. Some arbitrariness enters in this case in the choice of boundary conditions. However, as follows from [52,58,59,86], the intensity of the radiation depends very little on these conditions.

We investigate the influence of spatial dispersion and transition radiation, using an electronic plasma as an example. To describe the properties of the plasma we use a linearized kinetic equation with self-consistent field in the form [57]

$$\frac{\partial f}{\partial t}+\nu f+\mathbf{u}\nabla f+e\mathbf{u}\mathbf{E}\frac{\partial f_0}{\partial\varepsilon}=0. \qquad (4.3)$$

If f_0 is the plasma electron distribution function, f the nonequilibrium addition, ϵ, u, and ν the energy, velocity, and effective collision frequency of the electrons, and e the electron charge. On the interface $t = 0$ the electrons satisfy the condition for specular reflection*

$$f(z=0,\ u_z)=f(z=0,\ -u_z). \qquad (4.4)$$

The plasma occupies the half space $z > 0$. The field equations can be represented in the form

$$\Delta\mathbf{E}-\frac{\partial^2 E}{c^2\,dt^2}-\operatorname{grad}\operatorname{div}\mathbf{E}-\frac{4\pi}{c^2}\frac{\partial}{\partial t}\left\{e\int uf\,d\mathbf{p}+q\mathbf{v}\,\delta(z-vt)\,\delta(\varrho)\right\}=0,$$
$$\mathbf{v}=(0,\ 0,\ v), \qquad (4.5)$$

where q is the charge of the particle.

To solve the problem, all the quantities entering in (4.3) and (4.5) are best expanded in Fourier integrals of the type

$$\mathbf{A}(\mathbf{r},\ t)=\int \mathbf{A}(\omega,\ \varkappa,\ z)\,e^{i(\varkappa\rho-\omega t)}\,d\varkappa\,d\omega \qquad (4.6)$$

[compare with (1.2)].

Solving (4.3) with boundary condition (4.4), and continuing the electric field into the region $z < 0$ (the tangential components E_T in even fashion and the normal component E_z in odd fashion [57]), we obtain from (4.5)

$$E_k(\omega,\ \mathbf{k})=L_{ki}^{-1}(\omega,\ \mathbf{k})N_i(\omega,\ \mathbf{k}) \qquad (4.6')$$

(summation from 1 to 3 is carried out over repeated indices),

$$L_{ik}=\frac{\omega^2}{c^2}\varepsilon_{ik}(\omega,\ \mathbf{k})-k^2\delta_{ik}+k_ik_k,$$
$$N_\alpha=E'_\alpha(0,\ \varkappa,\ \omega)-i\varkappa_\alpha E_z(0,\ \varkappa,\ \omega)$$

($\alpha = x,y$ and the prime denotes the derivative with respect to z),

$$N_z=\frac{iq\omega}{2\pi c^2}\left[\delta_+\left(\frac{\omega}{v}+k_z\right)-\delta_+\left(\frac{\omega}{v}-k_z\right)\right],$$
$$\delta_+(x)=\frac{1}{\pi}\int\limits_0^\infty e^{i\alpha x}\,d\alpha=\delta(x)+\frac{i}{\pi}P\frac{1}{x},\quad k^2=\varkappa^2+k_z^2. \qquad (4.7)$$

*The case of diffuse reflection of electrons from the boundary was considered by A. Ts. Amatuni.[141]

The symbol P denotes that the singularity at $x = 0$ must be understood in the sense of the principal value. The Fourier components $\mathbf{E}(\omega, \mathbf{k})$ are related with $\mathbf{E}(\omega, \kappa, z)$ by the equation

$$E_i(\omega, \varkappa, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} E_i(\omega, \mathbf{k}) e^{ik_z z} dk_z. \tag{4.8}$$

The dielectric tensor $\epsilon_{ik}(\omega, \mathbf{k})$ in the plasma is equal to

$$\varepsilon_{ik}(\omega, \mathbf{k}) = \delta_{ik} - \frac{4\pi}{i\omega} e^2 \int \frac{u_i u_k \frac{\partial f_0}{\partial \varepsilon}}{i(\omega - \mathbf{k}\mathbf{u}) - \nu} d\mathbf{p}. \tag{4.9}$$

For an isotropic medium this tensor can be represented in the form [57]

$$\varepsilon_{ik} = \left(\delta_{ik} - \frac{k_i k_k}{k^2}\right) \varepsilon^{\mathrm{tr}}(\omega, \mathbf{k}) + \frac{k_i k_k}{k^2} \varepsilon^{\mathrm{l}}(\omega, \mathbf{k}), \tag{4.10}$$

where $\epsilon^l(\omega, \mathbf{k})$ and $\epsilon^{\mathrm{tr}}(\omega, \mathbf{k})$ are the longitudinal and transverse dielectric constants; for a plasma

$$\varepsilon^{\mathrm{l}}(\omega, \mathbf{k}) = 1 + \frac{4\pi e^2}{\omega k^2} \int d\mathbf{p} \frac{(\mathbf{k}\mathbf{u})^2}{\omega - \mathbf{k}\mathbf{u} + i\nu} \frac{\partial f_0}{\partial \varepsilon}, \tag{4.11}$$

$$\varepsilon^{\mathrm{tr}}(\omega, \mathbf{k}) = 1 + \frac{2\pi e^2}{\omega k^2} \int d\mathbf{p} \frac{[\mathbf{k}\mathbf{u}]^2}{\omega - \mathbf{k}\mathbf{u} + i\nu} \frac{\partial f_0}{\partial \varepsilon}. \tag{4.12}$$

It is convenient to represent the general solution of Maxwell's equations in vacuum, as in Sec. 1, in the form of a sum of solutions of the homogeneous and inhomogeneous equations [see (1.3) and (1.5)]. Omitting the calculation procedure, which is perfectly analogous to that given in Sec. 1, we obtain for the spectral density of the transition radiation in vacuum [54]

$$\frac{d^2W}{d\Omega\, d\omega} = \frac{q^2\beta^2 \sin^2\theta \cos^2\theta}{\pi^2 c (1 - \beta^2\cos^2\theta)^2} |\eta(\omega, \theta)|^2, \tag{4.13}$$

$$\eta(\omega, \theta) = \frac{1 - \beta\zeta - \beta\gamma(1 - \beta^2\cos^2\theta)}{\zeta + \cos\theta}, \tag{4.14}$$

$$\zeta = \frac{i\omega}{\pi c} \int_{-\infty}^{\infty} \frac{dk_z}{k^2} \left\{ \frac{k_z^2}{\frac{\omega^2}{c^2}\varepsilon^{\mathrm{tr}}(\omega, \mathbf{k}) - k^2} + \frac{\sin^2\theta}{\varepsilon^{\mathrm{l}}(\omega, \mathbf{k})} \right\}, \tag{4.15}$$

$$\gamma = \frac{\omega^3}{v^2 c} \int_{-\infty}^{\infty} \frac{k_z\, dk_z}{k^2} \delta_+\left(\frac{\omega}{v} - k_z\right) \left[\frac{1}{\frac{\omega^2}{c^2}\varepsilon^{\mathrm{tr}} - k^2} - \frac{1}{\frac{\omega^2}{c^2}\varepsilon^{\mathrm{l}}(\omega, \mathbf{k})} \right]. \tag{4.16}$$

The foregoing expressions solve completely the formal aspect of the problem of transition radiation in our case. Setting $\epsilon^{\mathrm{tr}}(\omega, \mathbf{k})$ and $\epsilon^l(\omega, \mathbf{k})$ in (4.13) and (4.14) equal to $\epsilon(\omega)$, we obtain the known Ginzburg and Frank expression [1] for the transition radiation in a medium without account of spatial dispersion.

We now proceed to an analysis of the results in several particular cases.

In the case of weak spatial dispersion, the transverse and longitudinal electric constants can be represented in the form [57]

$$\varepsilon^{\mathrm{tr}}(\omega, \mathbf{k}) = \varepsilon(\omega) - \alpha^{\mathrm{tr}} \frac{c^2 k^2}{\omega^2},$$
$$\varepsilon^{\mathrm{l}}(\omega, \mathbf{k}) = \varepsilon(\omega) - \alpha^{\mathrm{l}} \frac{c^2 k^2}{\omega^2}. \tag{4.17}$$

Substituting these values of the dielectric constant in (4.15) and (4.16), we get

$$\zeta = \frac{1}{\varepsilon(\omega)} \left\{ \sqrt{\frac{\varepsilon(\omega)}{1 + \alpha^{\mathrm{tr}}} - \sin^2\theta} + \frac{\sin^2\theta}{\sqrt{\frac{\varepsilon(\omega)}{\alpha^{\mathrm{l}}} - \sin^2\theta}} \right\}, \tag{4.18}$$

$$\gamma^{-1} = \left(1 + \beta\sqrt{\frac{\varepsilon(\omega)}{1 + \alpha^{\mathrm{tr}}} - \sin^2\theta}\right)\left(1 + \beta\sqrt{\frac{\varepsilon(\omega)}{\alpha^{\mathrm{l}}} - \sin^2\theta}\right)$$
$$\times \left[\sqrt{\frac{\varepsilon(\omega)}{1 + \alpha^{\mathrm{tr}}} - \sin^2\theta} + \sqrt{\frac{\varepsilon(\omega)}{\alpha^{\mathrm{tr}}} - \sin^2\theta}\right] \alpha^{\mathrm{l}}. \tag{4.19}$$

For a nonrelativistic electron plasma with

$$f_0 = \frac{n_0}{(2mT)^{3/2}} \exp\left(-\frac{p^2}{2mT}\right)$$

(where T is the temperature in energy units), calculation of the integrals in (4.11) and (4.12) under conditions of spatial dispersion ($\omega \gg k\sqrt{[3T/m]}$) yields*

$$\varepsilon(\omega) = 1 - \frac{\omega_0^2}{\omega^2}\left(1 - \frac{i\nu}{\omega}\right), \quad \omega_0^2 = \frac{4\pi e^2 n_0}{m},$$
$$\alpha^{\mathrm{tr}} = \frac{\omega_0^2 T}{\omega^2 mc^2}, \quad \alpha^{\mathrm{l}} = 3\frac{\omega_0^2 T}{\omega^2 mc^2}. \tag{4.20}$$

In this case the contribution due to α^{tr} is negligibly small. For a completely degenerate electron Fermi gas with

$$f_0(p) = \begin{cases} \frac{3n_0}{3\pi p_0}, & p < p_0, \\ 0, & p > p_0 \end{cases}$$

(where $p_0 = (3\pi^2)^{1/3}\hbar n^{1/3}$ is the limiting Fermi momentum) we have

$$\alpha^{\mathrm{l}} = \frac{3}{5}\frac{\omega_0^2 v_0^2}{\omega^2 c^2}, \quad v_0 = \frac{p_0}{m}.$$

When the charge is emitted from the plasma, the radiated Cerenkov longitudinal waves contribute to the field in the vacuum. Indeed, the phase velocity of the longitudinal waves, as follows from the dispersion equation

$$\varepsilon(\omega) - \alpha^{\mathrm{l}} \frac{c^2 k^2}{\omega^2} = 0,$$

can be much smaller than the velocity of the charge. Then when the charge moves to the boundary of the plasma, the Cerenkov longitudinal wave is incident on the boundary at an angle

$$\cos\theta = \frac{\omega}{\mathbf{k}\mathbf{v}} = \frac{c}{v}\sqrt{\frac{\alpha^{\mathrm{l}}}{\varepsilon'}}, \tag{4.21}$$

where $\epsilon' = \mathrm{Re}\,\epsilon$. In addition to the reflected longitudinal and transverse waves, a transverse wave is transformed from the longitudinal one on the boundary. The angle at which this wave propagates in vacuum is determined from Snell's law and is equal to

$$\sin\nu = \sqrt{\frac{\varepsilon'(\omega)}{\alpha^{\mathrm{l}}} - \beta^{-2}}. \tag{4.22}$$

Using a derivation similar to that in Sec. 1, we can show that in our case the Cerenkov wave going into the vacuum is cylindrical near the boundary. As $R \to \infty$ it becomes transformed into the spherical wave. The spectral density of the Cerenkov and transition radiation is then described by expressions (4.13), (4.14), (4.18), and (4.19), in which v is replaced by $-v$. It

*We neglect the Landau damping.

follows from (4.19) that the maximum radiation due to the emergence of the Cerenkov waves into vacuum occurs when condition (4.22) is satisfied, and that the region of the frequencies in which the Cerenkov generation takes place is determined by the inequality $0 \leq \sin\nu \leq 1$. We now proceed to investigate the singularities of the radiation in the region where the conditions of strong spatial dispersion are satisfied, namely

$$\frac{\omega}{k} \ll \sqrt{\frac{3T}{m}}.$$

In this approximation the backward transition radiation is equal to [54]

$$\frac{d^2W}{d\Omega\, d\omega} = \frac{q^2\beta^2 \sin^2\theta \cos^2\theta}{\pi^2 c\,(1-\beta^2\cos^2\theta)^2} \left| \frac{1-\beta\zeta}{\zeta+\cos\theta} \right|^2, \tag{4.23}$$

where ζ is the surface impedance.

In the case of a Maxwellian plasma, the impedance is equal to

$$\zeta = \left(\frac{2}{27\pi}\right)^{1/2} \left(\frac{\omega^2\sqrt{mT}}{n_0 e^2 c}\right)^{1/3} (1 - i\sqrt{3}). \tag{4.24}$$

Since the impedance is small for a Maxwellian plasma (because $\omega^2/\omega_0^2 \ll T/mc^2 \ll 1$), the transition radiation in the entire region of angles $\cos\theta \gg |\zeta|$ is the same as for an ideally conducting medium [see (1.17′)]. If the medium has sufficiently large magnetic permeability, the conditions for strong spatial dispersion are satisfied because of the large value of μ, and the impedance ζ, which is proportional to $\sqrt{\mu}$, may not be small compared with unity.

To conclude this section, we consider the role of weak spatial dispersion of the dielectric constant near the exciton absorption band [69,57,70], when the dielectric constant is large. As is well known, allowance for spatial dispersion leads in this case to the appearance of additional transverse waves (exciton waves).

The question of transition and Cerenkov radiation of a charge near the exciton light-absorption frequencies in a dielectric was considered in [86], where the influence of boundary conditions for the polarization vector on the radiation intensity is examined, and where it is shown that the intensity of the radiation depends very little on these conditions. We confine ourselves to the case when the condition $P_n|_{z=1} = 0$ is satisfied for the polarization vector. This condition is equivalent to continuity of the normal component of the electric field $E_{1n}|_{z=0} = E_{2n}|_{z=0}$. It is easy to show that the condition of specular reflection from the surface of the plasma also leads to this equation. Therefore to estimate the spectral density of radiation, with allowance for spatial dispersion near the exciton absorption bands in the dielectric, we can make use of formulas (4.13)–(4.16). We represent $\epsilon^{tr}(\omega, \mathbf{k})$ in the form [69]

$$\frac{1}{\varepsilon^{tr}(\omega, \mathbf{k})} = \frac{1}{\varepsilon(\omega)} + \beta_0 \frac{c^2k^2}{\omega^2}, \tag{4.25}$$

and we replace $\epsilon^l(\omega, \mathbf{k})$ by $\epsilon(\omega)$ (we disregard longitudinal waves), where $|\epsilon(\omega)| \gg 1$ and β_0 is a parameter characterizing the spatial dispersion. Calculating the integrals (4.15)–(4.16) for a particle entering the medium, we obtain

$$\frac{d^2W}{d\Omega d\omega} \simeq \frac{q^2\beta^2}{\pi^2 c} \left| \frac{\sqrt{\varepsilon}\left(1+\beta\sqrt{-\frac{1}{\varepsilon\beta_0}-\sin^2\theta}\right) - \beta\sqrt{-\frac{1}{\varepsilon\beta_0}}}{(\sqrt{\varepsilon}\cos\theta+1)\left(1+\beta\sqrt{-\frac{1}{\varepsilon\beta_0}-\sin^2\theta}\right)} \right|^2 \times \sin^2\theta\cos^2\theta. \tag{4.26}$$

This formula has been obtained under the assumption that $\beta \ll 1$, $\epsilon' = \mathrm{Re}\,\epsilon \gg 1$, and $\epsilon'^2\beta_0 \ll 1$. The dispersion relation for the exciton wave takes the form

$$k^2 = -\frac{\omega^2}{c^2}\frac{1}{\varepsilon\beta_0}.$$

In the region of frequencies where $\epsilon' > 0$ the exciton wave has negative group velocity when $\beta_0 < 0$. The real part of the radical $[-1/\epsilon\beta_0 - \sin^2\theta]^{1/2}$ is negative in this case and the sharp radiation maximum due to emergence of Cerenkov waves of exciton origin into the vacuum occurs when the particle enters the medium at an angle θ_0, where

$$\sin^2\theta_0 = \frac{1}{\varepsilon'|\beta_0|} - \beta^{-2} \tag{4.27}$$

[compare with (1.18)].

5. RADIATION FROM A POINT CHARGE PASSING THROUGH A PLATE*

In this section we obtain the radiation produced when a point charge passes through an isotropic plate of thickness a (Garibyan and Chalikyan [13,14], Pafomov [17,18,20]).

The Fourier component of the radiation fields in vacuum have in this case the form [14]:

$$E_{1\tau} = \frac{ei\varkappa}{2\pi^2F}\left[\left(\frac{\varepsilon}{\lambda_1}+\frac{1}{\lambda_2}\right)\alpha_1 e^{-i\lambda_1 a} + \left(\frac{\varepsilon}{\lambda_1}-\frac{1}{\lambda_2}\right)\alpha_2 e^{i\lambda_1 a} + \frac{2\varepsilon}{\lambda_1}\beta_1 e^{i\frac{\omega}{v}a}\right],$$

$$E_{2\tau} = -\frac{ei\varkappa}{2\pi^2F}\left[\left(\frac{\varepsilon}{\lambda_1}-\frac{1}{\lambda_2}\right)\alpha_1 e^{i\lambda_1 a} + \left(\frac{\varepsilon}{\lambda_1}+\frac{1}{\lambda_2}\right)\alpha_2 e^{-i\lambda_1 a} \tag{5.1}$$

$$+ \frac{2\varepsilon}{\lambda_1}\beta_2 e^{-i\frac{\omega}{v}a}\right] e^{-i\left(\lambda_2-\frac{\omega}{v}\right)a}, \tag{5.2}$$

where

$$F = \left(\frac{\varepsilon}{\lambda_1}+\frac{1}{\lambda_2}\right)^2 e^{-i\lambda_1 a} - \left(\frac{\varepsilon}{\lambda_1}-\frac{1}{\lambda_2}\right)^2 e^{i\lambda_1 a}, \tag{5.3}$$

$$\left.\begin{matrix}\alpha_1\\ \alpha_2\end{matrix}\right\} = \frac{\pm\frac{\varepsilon}{\lambda_1}-\frac{v}{\omega}}{k^2-\frac{\omega^2}{c^2}} + \frac{\mp\frac{1}{\lambda_1}+\frac{v}{\omega}}{k^2-\frac{\omega^2}{c^2}\varepsilon}, \qquad \left.\begin{matrix}\beta_1\\ \beta_2\end{matrix}\right\} = \frac{\mp\frac{1}{\lambda_2}+\frac{v}{\omega}}{k^2-\frac{\omega^2}{c^2}} + \frac{\pm\frac{1}{\lambda_2\varepsilon}-\frac{v}{\omega}}{k^2-\frac{\omega^2}{c^2}\varepsilon}. \tag{5.4}$$

*We do not consider here the ionization energy loss of the particle in the plate [5, 20].

The normal components of the electric field and the magnetic field are connected with E_T by the equations

$$E_{1n}(k)=\frac{\varkappa}{\lambda_2}E_{1\tau},\quad E_{2n}=-\frac{\varkappa}{\lambda_2}E_{2\tau},$$

$$\mathbf{H}=\frac{c}{\omega}[(\varkappa-\mathbf{n}\lambda_1)\mathbf{E}_{1\tau}],\quad \mathbf{H}_2(\varkappa,\omega)=\frac{c}{\omega}[(\varkappa+\mathbf{n}\lambda_2)\mathbf{E}_{2\tau}] \tag{5.5}$$

($\mathbf{n}$ is a unit vector along the normal).

The expressions with index 1 describe the radiation field ahead of the plate, while those with index 2—behind the plate in the direction of particle motion. We recall that

$$k^2=\frac{\omega^2}{v^2}+\varkappa^2,\quad \lambda_{1,2}^2=\frac{\omega^2}{c^2}\varepsilon_{1,2}-\varkappa^2,\quad \varepsilon_1=\varepsilon,\quad \varepsilon_2=1,$$

and that the real and imaginary parts of $\lambda_{1,2}$ are positive.*

We consider first the case when R is much larger than the thickness of the plate a ($R \gg a$). Then the saddle point, as in a semi-infinite medium, is $\kappa_s = (\omega/c)\sin\theta$. Following the usual procedure, we can easily obtain a formula for the spectral density of the forward radiation [18]:

$$\frac{d^2W}{d\Omega\, d\omega}=\frac{e^2v^2}{\pi^2c^3}\frac{\sin^2\theta\cos^2\theta}{(1-\beta^2\cos^2\theta)^2}\,|A(\omega,\theta)|^2,$$

$$A=\frac{(\varepsilon-1)\left[(x-y)(1-\beta x)(1-\beta^2+\beta x)e^{i\frac{\omega}{c}xa}+(x+y)(1+\beta x)(1-\beta^2-\beta x)e^{-i\frac{\omega}{c}xa}\right]}{\left[(x+y)^2e^{-i\frac{\omega}{c}xa}-(x-y)^2e^{i\frac{\omega}{c}xa}\right](1-\beta^2x^2)}-\frac{(\varepsilon-1)\left[2x(1-\beta^2-\beta^3y-\beta^2x^2)e^{-i\frac{\omega}{v}a}\right]}{\left[(x+y)^2e^{-i\frac{\omega}{c}xa}-(x-y)^2e^{i\frac{\omega}{c}xa}\right](1-\beta^2x^2)},$$

$$x,\ y=\frac{c}{\omega}\lambda_{1,2},\quad \beta=\frac{v}{c}. \tag{5.6}$$

The energy radiated in the region in front of the plate can be obtained from (5.6) by replacing v with $-v$.

Let us investigate the results (5.6) for a thin plate ($|\sqrt{\varepsilon}|\omega a/c \ll 1$) in the case of a nonrelativistic particle. Expanding $\exp(i\omega xa/c)$ and $\exp(-i\omega xa/c)$ in a series and retaining the first terms of the expansion, we obtain the angular distribution of the transition-radiation spectral density in the form:

$$\frac{d^2W}{d\Omega\, d\omega}=\frac{e^2\beta^2}{\pi^2c}\left|1-\frac{1}{\varepsilon}\right|^2\sin^2\theta\sin^2\left(\frac{\omega a}{2v}\right). \tag{5.7}$$

The radiation in front of the plate is described by the same formula, since the latter does not depend on the sign of the velocity. When $\sin(\omega a/2v)=1$, the radiation energy exceeds the corresponding value in the case of a half-space, owing to interference of the electromagnetic waves radiated by the particle on going through the boundary of the plate.

It must be noted that the foregoing formula interprets correctly the transition radiation experimentally observed in thin films (see [58-61]).

For a relativistic particle ($\omega a/v \ll 1$) the spectral radiation density is written in the form

$$\frac{d^2W}{d\Omega\, d\omega}=\frac{e^2\omega^2a^2}{4\pi^2c^3}\left|\varepsilon-1\right|^2\frac{\sin^2\theta\cos^2\theta}{(1-\beta^2\cos^2\theta)^2}. \tag{5.8}$$

Integrating over the angles, we get

$$\frac{dW}{d\omega}=\frac{e^2\omega^2a^2}{4\pi c^3}|\varepsilon-1|^2\left(\ln\frac{4}{1-\beta^2}-3\right). \tag{5.9}$$

For a relativistic particle, the radiation is also independent of the sign of the velocity and increases with increasing particle energy.

We now consider the case of a "thick" plate. We assume that the inequalities under which Cerenkov radiation is possible are satisfied, namely

$$R \geqslant a \gg \frac{c}{\omega\sqrt{\varepsilon}}.$$

When finding the asymptotic values of the radiation field, it is necessary to take into account the effect of the exponentials in the expressions for E_{1T} and E_{2T} on the position of the saddle point. Expanding F^{-1} in powers of the small quantity

$$\left(\frac{\varepsilon}{\lambda_1}-\frac{1}{\lambda_2}\right)^2\Big/\left(\frac{\varepsilon}{\lambda_1}+\frac{1}{\lambda_2}\right)^2,$$

we obtain for the radial component of the Cerenkov-radiation field [14]

$$E_{1\rho}(R,\theta,t)=2\pi i\,\mathrm{Res}\left(-\frac{e}{2\pi^2v}\right)\int_{-\infty}^{\infty}d\omega\varkappa^{3/2}\exp\left(-i\omega t-\frac{3\pi i}{4}\right),$$

$$\frac{\left[\left(\frac{\varepsilon}{\lambda_1}-\frac{1}{\lambda_2}\right)\alpha_2e^{i\lambda_1a}+\frac{2\varepsilon}{\lambda_1}\beta_1e^{i\frac{\omega}{v}a}\right]}{\left(\frac{\varepsilon}{\lambda_1}+\frac{1}{\lambda_2}\right)^2}$$

$$\times\sum_{n=0}^{\infty}\left(\frac{\frac{\varepsilon}{\lambda_1}-\frac{1}{\lambda_2}}{\frac{\varepsilon}{\lambda_1}+\frac{1}{\lambda_2}}\right)^{2n}\exp\left\{iR\left[\varkappa\sin\theta+\lambda_2\cos\theta+\frac{a}{R}\lambda_1(2n+1)\right]\right\}, \tag{5.10}$$

where Res denotes the residue at the pole

$$\varkappa_p=\frac{\omega}{c}\sqrt{\varepsilon-\beta^{-2}}.$$

This residue must be taken into account if the pole is to the left of the line of steepest descent.

It is seen from (5.10) that the terms containing α_2

*Here, unlike in (1.4), we seek the solution in the region $-a < z < 0$ in the form $A\exp(i\lambda_1 z)+B\exp(-i\lambda_1 z)$, and in the region $z > 0$ and $z < -a$ in the form $\exp(i\lambda_2 z)$ and $\exp(-i\lambda_2 z)$, respectively.

and β_1 have different saddle points. If the pole is to the left of the saddle points corresponding to the terms with α_2 and β_1, then the residues must be taken at the poles α_2 and β_1.

It is easy to show that in this case the residues cancel each other and the field is equal to zero. Therefore the residue makes a contribution to the radiation field only for those frequencies at which the pole κ_p is to the left of the line of steepest descent corresponding to the term with β_1, and to the right of the line of steepest descent of the term with α_2.

As a result we obtain the following conditions for the Cerenkov-radiation frequencies [14]:

$$\beta \sin\theta \left(1 - \frac{as}{R} \frac{\cos^2\theta}{\sqrt{\varepsilon(\omega) - \sin^2\theta}}\right) \leqslant \sqrt{\beta^2 \varepsilon(\omega) - 1}$$

$$\leqslant \beta \sin\theta \left(1 - \frac{ap}{R} \frac{\cos^2\theta}{\sqrt{\varepsilon(\omega) - \sin^2\theta}}\right), \tag{5.11}$$

where $s = 2n + 2$ and $p = 2n + 1$ (ϵ real). For the spectral density of the Cerenkov radiation through a circular area ρ, $r + d\rho$ into the space ahead of the plate we obtain [14]

$$\frac{d^2 W_{1c}}{d\varrho\, d\omega} = \frac{4e^2}{v^2} \omega \sum_{n=0}^{\infty} \frac{[1 - \varepsilon\sqrt{1 + \beta^2(1-\varepsilon)}]^{4n+2}}{[1 + \varepsilon\sqrt{1 + \beta^2(1-\varepsilon)}]^{4n+4}}$$

$$\times \sqrt{(\beta^2\varepsilon - 1)[1 + \beta^2(1-\varepsilon)]}. \tag{5.12}$$

Similar calculations for the radiation in the region of space behind the plate yield the following expression:

$$\frac{d^2 W_{2c}}{d\varrho\, d\omega} = \frac{4e^2}{v^2} \omega \sum_{n=0}^{\infty} \frac{[1 - \varepsilon\sqrt{1 + \beta^2(1-\varepsilon)}]^{4n}}{[1 + \varepsilon\sqrt{1 + \beta^2(1-\varepsilon)}]^{4n+2}}$$

$$\times \sqrt{(\beta^2\varepsilon - 1)[1 + \beta^2(1-\varepsilon)]}, \tag{5.13}$$

where the bands of the Cerenkov frequencies are determined from the condition (5.11) with $s = 2n + 1$, $p = 2n$. It is obvious that the series expansion in (5.12) and (5.13) corresponds to Cerenkov radiation arriving at the given point as a result of multiple reflection of the waves from the walls of the plate.

From a comparison of (5.12) and (5.13) we see that forward Cerenkov radiation exceeds the backward radiation by a factor

$$\frac{[1 + \varepsilon\sqrt{1 + \beta^2(1-\varepsilon)}]^2}{[1 - \varepsilon\sqrt{1 + \beta^2(1-\varepsilon)}]^2}$$

Putting $a \ll R$ we find from (5.11) that at a given point there will move at an angle θ a packet of waves with a narrow frequency spectrum

$$\Delta\omega \sim \frac{a}{R} \left(\frac{d\varepsilon}{d\omega}\right)^{-1}$$

and with a fundamental frequency determined from the condition $\sin\theta = [\epsilon - \beta^{-2}]^{1/2}$. In this case there is no cylindrical Cerenkov wave (just as for the case of a half-space, when $R \to \infty$), and the intensity of the Cerenkov radiation through the circular area ρ, $\rho + d\rho$ vanishes, naturally, as $R \to \infty$. The intensity of the Cerenkov radiation in a solid-angle element $dW_C/d\Omega$, on the other hand, is finite (it is independent of R) and is proportional to the square of the thickness of the plate [18].

It is interesting to note that the Cerenkov radiation in the backward direction vanishes if the angle of incidence of the generated Cerenkov ray is equal to the Brewster angle.

Indeed, as is known from optics, in the case of incidence at the Brewster angle the reflected ray is at a right angle to the refracted ray. In the case of radiation at the Brewster angle, the condition $\beta^2\epsilon^2 = 1 + \epsilon$ is satisfied, and the statement made above follows from (5.12).

Radiation from a charge passing through the plates of a ferrodielectric, and also through a plate cut from a uniaxial crystal, is of interest from the point of view of the Cerenkov-radiation singularities referred to in Sec. 1. An investigation of this problem was made by Pafomov [18,20].

Silin and Fetisov [58,59] took into account spatial dispersion of the dielectric constant to determine the transition radiation in a plate. In solving this problem they assumed that the reflection of the conduction electrons from the surface is specular.

6. RADIATION OF A PARTICLE MOVING IN A MEDIUM WITH PERIODICALLY VARYING PROPERTIES

Direct estimates show that the intensity of transition radiation is low. The photon emission probability is of the order of $1/137$ per particle. In his Nobel prize lecture [109], I. M. Frank noted the possibility of intensifying transition radiation by adding up the transition radiation from a particle passing through a periodically complicated structure. In this connection, an analysis of the passage of particles through a layered medium is of a special interest.

The earliest of the rather extensive group of papers devoted to this question is that of Ya. B. Faĭnberg and N. A. Khizhnyak [78]*, who considered a simple model of a layered medium, such that the solution of the problem can be completed without any approximations. This paper deals with a periodically stratified dielectric with period $a + b$, consisting of two plates with material constants ϵ_1, μ_1 and ϵ_2, μ_2, and with thicknesses a and b. The particle moves along the z axis perpendicular to the interfaces between the plates.

Using the symmetry of the problem, we can reduce Maxwell's equations to a single equation for the z-component of the induction vector $D_z = D = \epsilon(\omega, z)E_z$, which has the following form:

*See also[138].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial D}{\partial r}\right)+\varepsilon\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial D}{\partial z}\right)+k^2\varepsilon\mu D$$
$$=-\frac{ik\varepsilon\mu}{\pi c}e^{i\frac{\omega}{v}z}\frac{\delta(r)}{r}+\frac{e\varepsilon}{\pi v}\frac{\delta(r)}{r}\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}e^{i\frac{\omega}{v}z}\right)\left(k=\frac{\omega}{c}\right). \quad (6.1)$$

Equation (6.1) holds true for any dependence of ϵ on z.

We seek D(r, z) in the form of a Fourier-Bessel integral

$$D(r,\ z)=\int_0^\infty X(r)J_0(\varkappa r)\varkappa\, d\varkappa, \quad (6.2)$$

where J_0 is a Bessel function of zero order.

For X we have the following expression:

$$\varepsilon\frac{d}{dz}\left(\frac{1}{\varepsilon}\frac{dX}{dz}\right)+(k^2\varepsilon\mu-\varkappa^2)X=-\frac{ik\varepsilon\mu}{\pi c}e^{i\frac{\omega}{v}z}$$
$$+\frac{\varepsilon e}{\pi v}\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}e^{i\frac{\omega}{v}z}\right). \quad (6.3)$$

In our case (6.3) is an equation with constant coefficients. The boundary conditions for (6.3) are the usual boundary conditions of electrodynamics on the interface between dielectrics of different kinds, and the condition that the solution be periodic with period L = a + b.

The solution of (6.3) is obtained by usual means. However, greatest interest attaches not to the solution itself, but to its value averaged over the period L at the instant t = z/v:

$$\bar{E}(\omega,\ \varkappa)=\frac{1}{L}\int_0^L\frac{1}{\varepsilon(\omega,\ z)}X(z)\,e^{-i\frac{\omega}{v}z}dz. \quad (6.4)$$

The superior bar denotes in this section averaging over the period.

The thermal energy lost by a particle per unit length, averaged over the period, can be readily determined in terms of this quantity using the known formula[110]

$$\frac{d\bar{I}}{dz}=e\int\bar{E}_z(\omega,\ \varkappa)\varkappa\, d\varkappa\, d\omega. \quad (6.5)$$

The expression for $\bar{E}_z(\omega,\kappa)$, is too cumbersome to present here; we proceed directly to an analysis of the results.

The value of the integral in (6.5) is determined by the integrand poles with respect to ω, which in turn are the roots of the following equations:

$$\left.\begin{array}{c}\text{a) } \varepsilon_1(\omega)=0,\quad \text{b) } \varepsilon_2(\omega)=0,\\ \text{c) } \cos\frac{\omega L}{v}=\cos p_1a\cos p_1b-\frac{1}{2}\left(\frac{p_1\varepsilon_2}{p_2\varepsilon_1}+\frac{p_2\varepsilon_1}{p_1\varepsilon_2}\right)\sin p_1a\sin p_2b,\\ \text{d) } p_1^2-\frac{\omega^2}{v^2}=0,\quad \text{e) } p_2^2-\frac{\omega^2}{v^2}=0,\end{array}\right\} \quad (6.6)$$

where

$$p_{1,2}^2=k^2\varepsilon_{1,2}\mu_{1,2}-\varkappa^2.$$

Equations (6.6a) and (6.6c) determine the polarization losses in an infinite medium with constants ϵ_1, μ_1, and ϵ_2, μ_2. The roots of (6.6c) correspond to the radiation losses that are specific for a periodically inhomogeneous medium. Equation (6.6c) is a dispersion equation. Finally, the roots of (6.6d) and (6.6e) describe Cerenkov radiation in unbounded media.

Let us consider the polarization losses in a layered dielectric. Let $\epsilon_2(\omega) = 0$ when $\omega = \omega_0$. (We assume that $\mu_1 = \mu_2 = 1$.) The corresponding formula for the losses is of the form

$$-\frac{d\bar{I}}{dz}=\frac{e^2}{v^2}\frac{\omega_0}{\left(\frac{d\varepsilon_2}{d\omega}\right)_{\omega=\omega_0}}\frac{b}{L}\ln\left[\frac{\varkappa_{\max}^2b^2+\left(\frac{\omega_0b}{v}\right)^2}{1+\left(\frac{\omega_0b}{v}\right)^2}\right]$$
$$-\frac{2e^2}{v^2}\frac{\omega_0}{\left(\frac{d\varepsilon_2}{d\omega}\right)_{\omega=\omega_0}}\frac{b}{L}\left\{\frac{1+\frac{\omega_0^2b^2}{2v^2}-\cos\frac{\omega_0b}{2v}}{\left(\frac{\omega_0b}{v}\right)^2\left[1+\left(\frac{\omega_0b}{v}\right)^2\right]}\right.$$
$$\left.+\frac{v}{b\omega_0}\operatorname{arctg}\frac{b\omega_0}{v}\right\}; \quad (6.7)^*$$

κ_{max} is the maximum value of the wave vector, at which the macroscopic theory is still applicable.[110]

The first term in (6.7) describes the polarization losses in a layer of thickness b, while the second term is connected with the presence of boundaries. From (6.7) it follows that the boundaries decrease the polarization losses. A more detailed investigation shows that polarization losses reach a maximum in the middle of the layer and drop off towards the edges[78].

To be able to analyze the radiation losses, we must know the frequency dependences of $\epsilon(\omega)$ and $\mu(\omega)$.

Such an analysis is presented in [78] for several particular cases. Without dwelling on these cases, we point out one general deduction. Assume that we have a layered medium with a dielectric constant periodic in the coordinate z with period L. If in some region of frequencies the wavelength in the medium is much larger than the change in the dielectric constant, then in this frequency region the periodic medium is equivalent, from the point of view of its electrodynamic properties, to a uniaxial crystal with a dielectric tensor having components

$$\varepsilon_{zz}=\left(\frac{1}{L}\int_0^L\frac{dz}{\varepsilon(z)}\right)^{-1},\quad \varepsilon_{rr}=\frac{1}{L}\int_0^L\varepsilon(z)\,dz. \quad (6.8)$$

Therefore the energy lost by the particle to Cerenkov radiation can be determined from the formula[111]

$$-\frac{d\bar{I}}{dz}=\frac{e^2}{c^2}\int\left(1-\frac{c^2}{v^2\varepsilon_{rr}}\right)\omega\, d\omega. \quad (6.9)$$

This result is generalized to the three-dimensional case if we impose on the "amplitudes" of the oscillations of the dielectric and magnetic constants some additional limitations.

As shown in [80], in the case when both the dielec-

*arctg ≡ $\tan^{-1}$

tric constant and the permeability are arbitrary periodic functions of the coordinate, and the wavelength is much shorter than the period, the electrodynamic properties of the medium are described by dielectric and magnetic tensors of the form

$$\varepsilon_{ik}=\bar{\varepsilon}\delta_{ik}-\frac{1}{\bar{\varepsilon}}\sum_n|\alpha_n|^2\frac{n_in_k}{n^2},$$
$$\mu_{ik}=\bar{\mu}\delta_{ik}-\frac{1}{\bar{\mu}}\sum_n|\beta_n|\frac{n_in_k}{n^2}, \tag{6.10}$$

where α_n and β_n are the Fourier components of the dielectric constant and the permeability, respectively. The summation in (6.10) is over all the reciprocal-lattice vectors.

As shown by Gertsenshtein and Tatarskii [112], formulas of the type (6.10) hold true if, in addition to limitations formulated above, the following requirement is satisfied

$$\left(\frac{1}{\bar{\varepsilon}}\sum|\alpha_n|^2\right)\left(\frac{2\pi L}{\lambda}\right)^7\ll 1,\ \left(\frac{1}{\bar{\mu}}\sum|\beta_n|^2\right)\left(\frac{2\pi L}{\lambda}\right)^7\ll 1. \tag{6.11}$$

In this case the quantities $\frac{1}{\bar{\epsilon}}\sum|\alpha_n|^2$ and $\frac{1}{\bar{\mu}}\sum|\beta_n|^2$ may be far from small. Knowing ϵ_{ik} and μ_{ik}, we can calculate the particle losses from the known formula [87].

The loss of an ultrarelativistic particle in a structure consisting of sharply bounded layers of a dielectric was considered in [10,16,17,76]. We shall not stop to discuss these results, since all the qualitative deductions are the same as in the case of a periodically inhomogeneous medium whose properties vary smoothly with the coordinate. We now proceed to an analysis of such a case.

The medium considered in several papers [16,75-77,79] had a dielectric constant that depended on the coordinate like

$$\varepsilon(\omega,\ z)=\varepsilon_0(\omega)+\Delta\cos\frac{2\pi z}{l}. \tag{6.12}$$

The most detailed study of this case is given in the paper by Ter-Mikaelyan [76], which we shall follow below. General deductions concerning the character of the spectrum radiated by a particle moving in a periodically inhomogeneous medium can be obtained by starting from the laws of particle energy and momentum conservation

$$\delta E=\hbar\omega,$$
$$\frac{\mathbf{v}\delta\mathbf{p}}{v}-\hbar k\cos\theta=\hbar\frac{2\pi r}{l}\cos\psi. \tag{6.13}$$

Here δE —change in particle energy due to the emission of a quantum with frequency ω, δp —change in the momentum of the particle, $\mathbf{k}$ —wave vector, θ —angle between the direction of the emitted quantum and the vector $\mathbf{v}$, ψ —angle between the Oz axis and the vector $\mathbf{k}$, and r is an integer.

The second formula of (6.12) describes the conservation of the projection of the momentum of the particle plus emitted quantum system on the direction of motion of the particle.

If we assume that the particle energy loss is small compared with the energy of the particle itself, then $\delta E = \mathbf{v}\cdot\delta\mathbf{p}$ and both formulas in (6.13) can be combined in one

$$\frac{\omega}{v}-k\cos\theta=\frac{2\pi r}{l}\cos\psi. \tag{6.14}$$

The modulus of the wave vector is connected with the frequency ω by the dispersion equation of the medium $k = k(\omega)$. From (6.14) we obtain for $\cos\theta$ the equation

$$\cos\theta=\left(\frac{\omega}{v}-\frac{2\pi r}{l}\right)k^{-1}(\omega). \tag{6.15}$$

From the requirement that $|\cos\theta| < 1$ follow the inequalities

$$-1\leqslant\left(\frac{\omega}{v}-\frac{2\pi r}{l}\right)k^{-1}(\omega)\leqslant 1. \tag{6.16}$$

From the inequalities (6.16) we can find for each r the region of the frequencies radiated by the particle. Formulas of the type (6.13)–(6.16) can be easily obtained for an arbitrary medium whose dielectric constant is periodic in the three coordinates. The deductions made with respect to the character of the radiated spectrum remain in force here, too.

As noted by Ter-Mikaelyan, Eq. (6.13) is equivalent to Bragg's law in the theory of diffraction of x rays.

It follows from (6.13) that the radiation of electromagnetic waves by a particle takes place for a definite relation between the wavelength $\lambda = 2\pi/k$ and the lattice parameter, i.e., there occurs a peculiar parametric type of spatial resonance.

For the special case of a layered medium, considered by Faĭnberg and Khizhnyak, the frequencies corresponding to the parametric resonance are determined by Eq. (6.6c).

If $|\bar{\epsilon}-\epsilon|/\bar{\epsilon}\ll 1$, then the influence of the inhomogeneous medium on the properties of the emitted quantum can be neglected and we can put in (6.13)–(6.16) that $k=\frac{\omega}{c}\sqrt{\bar{\epsilon}(\omega)}$.

We note that as $r\to\infty$ Eq. (6.16) goes over into the usual conditions for Cerenkov radiation.

We now consider the ultrarelativistic case, when we can put $v\approx c$ and we can define $\epsilon(\omega, z)$ by means of a formula such as

$$\varepsilon=1-\frac{4\pi e^2\bar{N}}{m\omega^2}-\frac{4\pi e^2N'}{m\omega^2}\cos\frac{2\pi z}{l}. \tag{6.17}$$

Substituting

$$k=\frac{\omega}{c}\sqrt{1-\frac{4\pi e^2\bar{N}}{m\omega^2}}$$

in (6.16c) and recognizing that when $v\approx c$

$$2\left(1-\frac{v}{c}\right)=\left(\frac{mc^2}{E}\right)^2,$$

we obtain the following inequalities, which define the region of the radiated frequencies:

$$\frac{E}{\hbar} \gg \omega_{\max} = \frac{2e^2\bar{N}l}{mcr}\left[1-\sqrt{1-\frac{e^2\bar{N}l^2}{\pi mc^2r^2}\left(\frac{mc^2}{E}\right)^2}\right]^{-1} \geqslant \omega \geqslant \frac{2e^2\bar{N}l}{mcr}\left[1+\sqrt{1-\frac{e^2\bar{N}l^2}{\pi mc^2r^2}\left(\frac{mc^2}{E}\right)^2}\right]^{-1} = \omega_{\min}. \quad (6.18)$$

From the fact that the radicand is positive it follows that the energy of the radiated particle should satisfy the inequality

$$\left(\frac{E}{mc^2}\right) \geqslant \left(\frac{E}{mc^2}\right)_{\text{thr}} = \frac{l}{2\pi cr}\sqrt{\frac{4\pi e^2\bar{N}}{m}}. \quad (6.19)$$

Thus, the radiation of electromagnetic waves by an ultrarelativistic particle is a threshold effect, i.e., waves will be radiated by the particle if its energy is not lower than that given by inequality (6.19).

The conditions (6.18) can be greatly simplified if we stipulate satisfaction of the inequality

$$\frac{e^2\bar{N}l^2}{\pi mc^2r^2}\left(\frac{mc^2}{E}\right)^2 \ll 1. \quad (6.20)$$

Then

$$\frac{E}{\hbar} \gg \omega_{\max} \approx \frac{4\pi c}{l} r\left(\frac{E}{mc^2}\right)^2 \geqslant \omega \geqslant \frac{\bar{N}e^2}{m}\frac{l}{cr}. \quad (6.21)$$

We note that from (6.19) and (6.20) we get the following limitation on the order of the spectrum:

$$r \ll r_{\max} = \frac{l}{2\pi}\left(\frac{4\pi e^2\bar{N}}{mc^2}\right)^{1/2}. \quad (6.22)$$

From inequality (6.15) and the relations that follow it we get

$$\theta \leqslant \frac{r}{l\left(\frac{\bar{N}e^2}{\pi mc^2}\right)^{1/2}} \ll 1, \quad (6.23)$$

i.e., the radiation from an ultrarelativistic particle lies in a narrow cone in a predominantly forward direction, as already mentioned several times above.

It is interesting to note that transverse electromagnetic waves can be radiated in a layered medium having a dielectric constant described by (6.17), but not in a non-layered medium.

We shall not discuss here spectra with $r \gg r_{\max}$, since, as shown in [76], the radiation intensity corresponding to these spectra is quite small.

The expression for the intensity of radiation produced by a particle moving in a medium with a dielectric constant described by a formula of the type (6.17) was derived by the WKB method by Ter-Mikaelyan in the following form:

$$\frac{d^2W}{dz\,d\omega} = \frac{\omega e^2}{c^2}\sum_r J_r^2(B)\left[1-\frac{1}{\varepsilon_0\beta^2\omega^2}\left(\omega-\frac{2\pi rc}{l}\right)^2\right], \quad (6.24)$$

where $d^2W/dzd\omega$ is the spectral density of the particle radiation per unit path, J_r is a Bessel function of order r, $B = l\omega\Delta/4\pi\sqrt{\epsilon_0}\, c\cos\theta$, and $\beta = v/c$.

In the derivation of (6.24) it was also assumed that $l \gg c/\omega\sqrt{\epsilon_0}$ (the condition for the applicability of the WKB method).

Each of the terms in the sum in (6.24) corresponds to the radiation intensity of the r-th order spectrum. It is easy to see [see (6.15) and below] that the expression in the square brackets in (6.24) is equal to $\sin^2\theta$.

The spectrum with r = 0 corresponds to Cerenkov radiation modified by the presence of the inhomogeneity.

We note that the radiation with $r > 0$ can occur when the Cerenkov condition $(v/c)\sqrt{\epsilon} > 1$ is not satisfied.

The intensities of the spectra with r = 0 and ± 1 were calculated in [26,79] under the assumption that $B \ll 1$ and $\Delta/\epsilon_0 \ll 1$, but for an arbitrary ratio of λ to l. Éĭdman[106] considered the calculation of the radiation from a particle traveling at an angle to the Oz axis in a medium with a dielectric constant described by formula (6.12). The radiation of an oscillator in such a medium was calculated by Khachatryan[89]. The same question is dealt with in the paper by V. V. Tamoĭkin[136], in which it was pointed out that resonance between the transition radiation and the oscillator vibrations is possible, so that the latter can build up.

K. A. Barsukov and B. M. Bolotovskiĭ[40,130] investigated the energy lost by a particle in an inhomogeneous nonstationary medium with a dielectric constant that varies like

$$\varepsilon(z, \omega t) = \varepsilon_0(\omega) + \Delta\cos\left(\frac{2\pi}{l}z - \omega_0 t\right). \quad (6.25)$$

The calculation, made in the WKB approximation, leads to the following expression for the spectral density of the field radiated by the particle per unit path:

$$\frac{d^2W}{dz\,d\omega} = \frac{e^2\,\text{sign}\,(v-u)}{c^2}\sum_r J_r(B')\left[1-\frac{(\omega-r\Omega)^2}{\varepsilon_0\beta^2\omega^2}\right]\omega. \quad (6.26)$$

Here

$$u = \frac{\omega_0 l}{2\pi}, \quad \Omega = \frac{2\pi}{l}|v-u|, \quad B' = \frac{\omega l\Delta}{4\pi c\sqrt{\varepsilon_0}\left|\cos\theta - \frac{u}{c\sqrt{\varepsilon_0}}\right|}.$$

The conditions that determine the radiated frequencies are obtained from the requirement that the expression in the square bracket of (6.26) must be positive. These conditions can also be derived from conservation laws, in analogy with the procedure used for a medium whose properties are time-independent. B. V. Khachatryan[133] studied the radiation from a system of cylindrical plasmoids in a medium having a dielectric constant given by (6.12), and investigated the influence of the fluctuations of the plasmoid dimensions and of the distances between plasmoids on the spectrum of the field.

Barsukov investigated the radiation from a charge moving in a magnetoactive plasma situated in an external magnetic field that varies periodically with the coordinate[131]. His results are close to those of Ter-Mikaelyan (cf. [76]).

So far we have considered a particle moving through an infinite periodic medium. Let us now discuss the radiation of a particle passing through a stack consisting of a finite number of plates. This question is the subject of papers by Garibyan [10] and Pafomov [17]. We confine ourselves to a report of the results of [17].

In Sec. 5 we gave a formula for the radiation from a particle passing through a thin plate. Summing the radiation from m plates, we obtain

$$\frac{d^2W_m}{d\omega\, d\Omega} = \frac{\sin^2 \frac{\omega m}{2c}\,[a + b(1 - \beta\cos\theta)]}{\sin^2 \frac{\omega}{2c}\,[a + b(1-\beta\cos\theta)]}\,\frac{d^2W}{d\omega\, d\Omega}, \tag{6.27}$$

where $d^2W_m/d\omega d\Omega$ is the radiation from a particle passing through m plates (per unit solid angle and per unit frequency interval), $d^2W/d\omega d\Omega$ is the radiation from a particle passing through one plate, a is the thickness of the plate, and b the distance between plates. The factor preceding $d^2W/d\omega d\Omega$ in (6.27) describes the interference between radiation from different plates.

If

$$\frac{\omega}{2c}\, m\,[a + b(1-\beta\cos\theta)] \ll 1,$$

then

$$\frac{d^2W_m}{d\omega\, d\Omega} = m^2 \frac{d^2W}{d\omega\, d\Omega},$$

i.e., the radiation is summed coherently. If

$$\frac{\omega}{2c}\, m\,[a + b(1-\beta\cos\theta)] \gg 1,$$

then the summation of the radiation is incoherent and

$$\frac{d^2W}{d\omega\, d\Omega} = m\,\frac{d^2W}{d\omega\, d\Omega}.$$

For an arbitrary value of

$$\frac{\omega}{2c}\, m\,[a + b(1-\beta\cos\theta)]$$

the interference factor has maxima proportional to m^2, and widths of order m when

$$\frac{\omega}{2c}\,[a + b(1-\beta\cos\theta)] = \pi\nu, \tag{6.28}$$

where ν is an integer.

7. RADIATION OF A PARTICLE IN A MEDIUM HAVING A FLUCTUATING DIELECTRIC CONSTANT

In many cases it is of interest to study the radiation of a particle in a medium whose dielectric constant fluctuates. Phenomena of this kind can occur when corpuscular streams pass through the solar corona, ionosphere, or interstellar space. The energy lost by the particle to radiation should experience a sharp increase when the particle passes through matter, when it is in a phase-transition state, etc.

The radiation loss of a particle can be calculated by two methods—by an averaging method and by perturbation theory. Naturally, when these methods are applied to the same problem they lead to the same results. However, the averaging method seems preferable to us, since it makes it possible to reduce the problem of particle energy loss in a fluctuating medium to the known problem of particle energy loss in a medium with a specified dielectric tensor.

Maxwell's equation in a medium with dielectric constant $\hat{\varepsilon}$ is of the form

$$\operatorname{rot}\operatorname{rot}\mathcal{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{\varepsilon}(\mathbf{r},\, t)\,\mathcal{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}. \tag{7.1}$$

In the general case, in the presence of temporal and spatial dispersion, the dielectric constant $\hat{\varepsilon}(\mathbf{r},t)$ is an integral tensor operator which is a random function of its arguments. Naturally, the electric field $\mathcal{E}$ is also a random function. We write $\hat{\varepsilon}$ and $\mathcal{E}$ in the form

$$\hat{\varepsilon} = \overline{\hat{\varepsilon}} + \delta\hat{\varepsilon},\quad \mathcal{E} = \mathbf{E} + \xi,\quad \mathbf{E} = \overline{\mathcal{E}}. \tag{7.2}$$

The bar denotes here statistical averaging, $\delta\hat{\varepsilon}$ is the fluctuation of the dielectric tensor, and ξ is the fluctuation of the electric field.

Let us derive an equation for the average field $\mathbf{E}$. To this end we average (7.1) and subtract the averaged equation from (7.1). As a result we obtain the following system for the determination of $\mathbf{E}$ and ξ:

$$\begin{gathered}\operatorname{rot}\operatorname{rot}\mathbf{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\overline{\hat{\varepsilon}}\mathbf{E} + \overline{\delta\hat{\varepsilon}\xi}) = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t},\\ \operatorname{rot}\operatorname{rot}\xi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\overline{\hat{\varepsilon}}\xi + \delta\hat{\varepsilon}\mathbf{E} + \delta\hat{\varepsilon}\xi - \overline{\delta\hat{\varepsilon}\xi}) = 0.\end{gathered} \tag{7.3}$$

To obtain an equation for $\mathbf{E}$ we must express the second equation of (7.3) in terms of $\mathbf{E}$, and then substitute the resultant expression into the first equation of (7.3). This procedure can be carried out in closed form if we neglect the last two terms in the parentheses of the second equation of (6.3). We shall assume that this neglect is permissible; the conditions under which this can be done will be formulated later.

Let $\hat{\varphi}$ be the Green's operator of the equation

$$\operatorname{rot}\operatorname{rot}\xi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\overline{\hat{\varepsilon}}\xi = 0.$$

From the second equation of the system (7.3) we obtain for ξ the following expression:

$$\xi(\mathbf{r},\, t) = \int \overline{\hat{\varphi}(\mathbf{r},\, \mathbf{r}';\, t,\, t')(\delta\hat{\varepsilon}\mathbf{E})_{\mathbf{r}',\, t'}}\,d\mathbf{r}'\,dt'. \tag{7.4}$$

The indices $\mathbf{r}'$ and t' denote that the expression $(\delta\hat{\varepsilon}\mathbf{E})$ is taken at the points $\mathbf{r}'$ and t'. Substituting (7.4) in the first equation of (7.3) we ultimately obtain for $\mathbf{E}$ the equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\hat{\varepsilon}^{(e)}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \tag{7.5}$$

where the effective dielectric constant is defined by

$$\hat{\varepsilon}^{(e)}\mathbf{E} = \overline{\hat{\varepsilon}}\mathbf{E} + \int \overline{\delta\hat{\varepsilon}(\mathbf{r},\, t)\,\varphi(\mathbf{r},\, t;\, \mathbf{r}',\, t')(\delta\hat{\varepsilon}\mathbf{E})_{\mathbf{r}'t'}}\,d\mathbf{r}'\,dt'. \tag{7.6}$$

This method was first applied to the problem of propagation of electromagnetic waves in a fluctuating medium by Lifshitz, Kaganov, and Tsukernik [113], and developed by Kaner[114] and by one of the authors[81].

We shall henceforth consider only homogeneous and stationary media, i.e., media in which $\bar{\epsilon}$ does not depend explicitly on r or on t, and the correlation relations between $\delta\epsilon_{ik}(\mathbf{r}, t)$ and $\delta\epsilon_{ik}(\mathbf{r}', t')$ have the following form:

$$\overline{\delta\varepsilon_{ik}(\mathbf{r}, t)\,\delta\varepsilon_{i'k'}(\mathbf{r}', t')} = W_{iki'k'}(\mathbf{r}-\mathbf{r}', t-t'). \quad (7.7)$$

In this case the expression for $\hat{\epsilon}^{(e)}\mathrm{E}$ can be written:

$$(\hat{\varepsilon}^{(e)}\mathbf{E})_i = \int \bar{\varepsilon}_{ik}(\mathbf{r}-\mathbf{r}', t-t')\,E_k(\mathbf{r}', t')\,d\mathbf{r}'\,dt' + \int \varphi_{em}(\mathbf{r}-\mathbf{r}', t-t')\,W_{ilmk}(\mathbf{r}-\mathbf{r}', t-t')\,E_k(\mathbf{r}', t)\,d\mathbf{r}'\,dt'. \quad (7.8)$$

Going over to Fourier transforms with respect to the coordinates and the time, we can rewrite (7.8) in the form

$$(\hat{\varepsilon}^{(e)}\mathbf{E})_i = \hat{\varepsilon}^{(e)}_{ik}(\omega, \mathbf{k})\,E_k(\omega, \mathbf{k}), \quad (7.9)$$

where

$$\varepsilon^{(e)}_{ik}(\omega, \mathbf{k}) = \bar{\varepsilon}_{ik}(\omega, \mathbf{k}) + \int \varphi_{em}(\boldsymbol{\varrho}, \tau)\,W_{ilmk}(\boldsymbol{\varrho}, \tau)\,e^{-i(\omega\tau - k\rho)}\,d\boldsymbol{\varrho}\,d\tau. \quad (7.10)$$

It is interesting to note that the presence of fluctuations leads to the appearance of spatial and temporal dispersion even in the case when there is no such dispersion for the average value of the dielectric constant. Knowing the effective dielectric tensor, we can easily obtain the particle energy loss by means of the formula[57]

$$\frac{d^2I}{d\omega\,dz} = -\mathrm{Re}\,\frac{ie^2}{2\pi c^2 v}\int (\mathbf{k}\mathbf{v}) \times \frac{|(\mathbf{v}, \hat{b}^{-1}\mathbf{k})|^2 - (\mathbf{v}, \hat{b}^{-1}\mathbf{v})(\mathbf{k}, \hat{b}^{-1}\mathbf{k}) + (\mathbf{v}\hat{b}^{-1}\mathbf{v})}{1-(\mathbf{k}\hat{b}^{-1}\mathbf{k})}\,dk, \quad (7.11)$$

where

$$\hat{b} = k^2 - \frac{(\mathbf{k}, \mathbf{v})^2}{c^2}\,\hat{\varepsilon}^{-1}.$$

Let us consider an isotropic medium for which $\hat{\epsilon}$ and W_{iklm} are isotropic tensors characterized by the scalars $\bar{\epsilon}(\omega)$ and $W(\rho)$. We shall also assume that W does not depend on τ, nor does $\bar{\epsilon}$ on k. Under these assumptions, the effective dielectric tensor takes the following form[81,113-115]:

$$\varepsilon^{(e)}_{ik}(\omega, \mathbf{k}) = \bar{\varepsilon}(\omega)\,\delta_{ik} + \frac{\omega^2}{c^2}\int W(\varrho)\left[\delta_{ik} + \frac{c^2}{\omega^2\bar{\varepsilon}(\omega)}\frac{\partial^2}{\partial\varrho_i\,\partial\varrho_k}\right] \times \frac{\exp\left[-i\frac{\omega}{c}\sqrt{\bar{\varepsilon}(\omega)}\,\varrho + i\mathbf{k}\boldsymbol{\varrho}\right]}{\varrho}\,d\boldsymbol{\varrho}. \quad (7.12)$$

Thus, the formulas presented provide a complete solution of the problem of the energy lost by a particle moving in a medium having random inhomogeneities.

Methods similar to the foregoing were used to determine the particle loss in a statistically inhomogeneous medium by Tamoĭkin and Biragov[103] and by Kalashnikov and Ryazanov[92]. Kapitza[88], Ter-Mikaelyan[77], and Tamoĭkin[102] based their analysis on perturbation theory, i.e., they assumed that E in formula (7.4) for ξ is the same as it would be for a medium with $\delta\hat{\epsilon} = 0$, and then, knowing $\hat{\xi}$, they determined the intensity of the radiated field by means of the usual formulas.

We proceed now to a direct exposition of the results. If the inequality $\frac{\omega}{c}\sqrt{\bar{\epsilon}}\,l \ll 1$ is satisfied (l is the characteristic correlation radius), and if W depends only on the absolute value of the vector ρ, then the dielectric constant of the fluctuating medium is given by the following formula:

$$\varepsilon^{(e)} = \bar{\varepsilon} - \frac{1}{3}\frac{\overline{\delta\varepsilon^2}}{\bar{\varepsilon}}\left(1 - 2\frac{\omega^3}{c^3}\,\bar{l}^3\,\bar{\varepsilon}^{3/2}\right). \quad (7.13)$$

Here

$$\overline{\delta\varepsilon^2} = W(0), \quad \bar{l}^3 = \frac{1}{\overline{\delta\varepsilon^2}}\int_0^\infty \varrho^2 W(\varrho)\,d\varrho.$$

Apart from the inequalities indicated above, the limits of applicability of this formula are determined also by the relation $(2\pi l/\lambda)^7\overline{\delta\epsilon^2} \ll 1$[112] [cf. (6.11)].

Substitution of (7.13) and (7.11) leads to the following formula for the particle energy loss[103]:

$$\frac{d^2I}{d\omega\,dz} = -\frac{e^2\omega^4\overline{\delta\varepsilon^2}\,l^3}{6\pi^2c^3v^2\bar{\varepsilon}^{3/2}}\left\{2\ln\left|\frac{v\varkappa_{\max}}{\omega\sqrt{1-\beta^2\bar{\varepsilon}}}\right| - \beta^2\bar{\varepsilon}\right\}. \quad (7.14)$$

$\kappa_{\max}$ is the maximum value of the wave vector at which macroscopic electrodynamics is still applicable. In the derivation of this formula it was assumed that $\beta^2\bar{\epsilon} < 1$, i.e., the conditions for Cerenkov radiation are not satisfied for the average value of the dielectric constant. In this case, which is of most interest, the only reason for the radiation is the presence of fluctuations. In addition, it was assumed in the calculation of the energy loss that $\omega l/v \ll 1$.

If the medium is one-dimensional, i.e., if $\hat{\epsilon}$ depends only on one coordinate, say on z, then the effective dielectric constant is a tensor with components

$$\varepsilon^{(l)}_{rr} = \varepsilon^{(l)}_{xx} = \varepsilon^{(l)}_{yy} = \bar{\varepsilon}(z), \quad \varepsilon^{(l)}_{zz} = [\overline{\varepsilon^{-1}(z)}]^{-1}, \quad (7.15)$$

i.e., such a medium is equivalent in its electrodynamic properties to a uniaxial crystal[81]. We have already encountered a similar situation in the analysis of the loss in a periodically inhomogeneous medium (see the preceding section). In this case the energy loss is determined by formula (6.11).

Tamoĭkin[102] considered the energy lost by a particle in a plasma, taking into account weak spatial dispersion and fluctuations of the electron density.

The action of the operator $\hat{\epsilon}(\omega)\mathrm{E}$ was specified in the form

$$\hat{\varepsilon}(\omega)\,\mathbf{E} = \bar{\varepsilon}(\omega)\,\mathbf{E} + \frac{3T}{m}\frac{4\pi e^2N}{m}\,\nabla(\nabla\mathbf{E}), \quad (7.16)$$

where

$$\bar{\varepsilon}(\omega) = 1 - \frac{4\pi e^2 N}{m};$$

T is the temperature of the electron gas in energy units, $N = \bar{N} + \delta N$, $\bar{N}$ is the average value of the electron density, and δN is the fluctuation of the electron density.

It is assumed that $\delta N \ll \bar{N}$. It is easy to see that the fluctuation of the dielectric constant is determined by the relation

$$\delta\varepsilon = -4\pi e^2 \delta N/\omega^2 m.$$

As is well known, in the absence of a magnetic field there is no Cerenkov radiation of transverse waves. Cerenkov radiation of longitudinal waves is possible under the condition

$$v^2 > 3v_T^2/\bar{\varepsilon} \quad (v_T^2 = T/m).$$

If $v^2 < 3v_T^2/\bar{\epsilon}$, both longitudinal and transverse waves are radiated, owing to the presence of fluctuations.

If $v^2 > 3v_T^2/\bar{\epsilon}$, then an additional process takes place, consisting in the transformation of the longitudinal Cerenkov waves into transverse ones, owing to scattering by the fluctuations.

We consider first the case $v^2 < 3v_T^2/\bar{\epsilon}$. We obtain for the intensity of the radiation loss of transverse waves the expression

$$\frac{d^2 I_\perp}{d\omega\, dz} = -\frac{2\omega^4 e^2 \overline{\delta\varepsilon^2} l^3}{3\pi c^3 v^2 \bar{\varepsilon}^{3/2}} \left[\ln \left| \frac{1 - \dfrac{\bar{\varepsilon} v^2}{3v_T^2}}{1 - \beta^2 \bar{\varepsilon}} \right| - \beta^2 \bar{\varepsilon} - \frac{\bar{\varepsilon} v^2}{3v_T^2 \left(1 - \dfrac{\bar{\varepsilon} v^2}{3v_T^2}\right)} \right]. \tag{7.17}$$

Formula (7.17) is similar in appearance to (7.14). There is no need, however, for introducing here the limiting momentum κ_{max}, for cut-off takes place automatically at distances on the order of the Debye radius when spatial dispersion is taken into account.

The radiation intensity increases abruptly when v tends to $\sqrt{3}\, v_T/\sqrt{\bar{\epsilon}}$, corresponding to an approach to the Cerenkov threshold of plasma wave emission.

The expression for the loss due to radiation of longitudinal waves differs from (7.17) by a factor $\frac{1}{3}\sqrt{3} \times (c^3/v_T^3)$. Since this factor is much larger than unity, it follows from the foregoing that the intensity of radiation of longitudinal waves is many times larger than the intensity of radiation of transverse waves.

The corresponding formula is applicable under the conditions

$$\frac{\omega}{v} l \ll 1, \quad \frac{\omega \sqrt{\bar{\varepsilon}}\, l}{\sqrt{3}\, v_T} \ll 1.$$

We shall not stop to discuss the case $v^2 > 3v_T^2/\bar{\epsilon}$, or the angular distribution of the radiated field. The corresponding results and their discussion can be found in the cited papers.

8. RADIATION OF A PARTICLE IN THE PRESENCE OF OBSTACLES

The inhomogeneity of the medium in which the particle moves causes a change in the phase velocity of the electromagnetic waves, and this in turn causes radiation of an electromagnetic field even at subluminal particle velocity.

The same result is obtained when the particle moves over an uneven surface. The physical nature of the phenomenon is that the charge induced by the particle does not move on the surface uniformly even when the particle itself moves uniformly.

If the particle velocity is much smaller than the velocity of light, then the problem is electrostatic and can be solved by the image method. Such an analysis was carried out by Askar'yan, Gorodinskiĭ, and Éĭdman[103], using as an example a particle incident on an ideally conducting sphere.*

The gist of the method of images is to replace the system comprising the sphere and the moving charge, in accordance with the known rules, with some specified distribution in vacuum of charges moving generally speaking in non-uniform fashion, such as to satisfy the boundary conditions on the surface of the sphere. Knowing the motion and the distribution of the charges, we can easily obtain their radiation loss.

The calculation given in [105] gives for the spectral density of the field radiated per unit solid angle the following expression

$$\frac{d^2 W}{d\Omega\, d\omega} = \frac{e^2 \omega^2 \sin^2\theta}{4\pi^2 c^3} \left[\frac{v^2}{\omega^2} \frac{1}{(1+\beta\cos\theta)^2} + a^2 |J|^2 + \frac{2v}{\omega} a \frac{\operatorname{Im} J}{1+\beta\cos\theta} \right], \tag{8.1}$$

where

$$J = \frac{1}{2} \left\{ -1 + i\frac{\omega}{v} a - \frac{\omega^2}{v^2} a^2 e^{i\frac{\omega}{v} a} E_i\left(-\frac{i\omega a}{v}\right) \right\},$$

θ is the angle between the particle trajectory and the direction to the point of observation, a is the radius of the sphere, and E_i is the integral exponential function.

As $a \to \infty$, (8.1) goes over into the well known expression for transition radiation for normal incidence of the particle on a perfectly conducting half-space. As $a \to 0$ (collision with neutral particle), (8.1) takes the form:

$$\frac{d^2 W}{d\Omega\, d\omega} = \frac{e^2 v^2 \sin^2\theta}{4\pi^2 c^3 (1+\beta\cos\theta)^2}. \tag{8.2}$$

Formulas (8.1) and (8.2) were obtained under the assumption that $a \ll \lambda$ and $v^2/c^2 \ll 1$. However, formula (8.1) gives a correct answer also when $a \to \infty$.

*We note that a remark was made in[105] to the effect that Askar'yan's results in[97] are incorrect. As shown in[137], the results are actually correct.

This circumstance is connected with the special symmetry of the problem in this case.

It is interesting to note that the image of a subluminal particle can acquire superluminal velocity if the surface on which the charge is incident is chosen in suitable manner. Noteworthy from among problems of this type is the one dealing with radiation incident on a sphere from a current ring[116,117].

Dnestrovskiĭ and Kostomarov considered the energy loss of a nonrelativistic particle moving near a surface possessing axial symmetry[99,100]. The particle radiation intensity is expressed in terms of the regular part of the Green's function of the electrostatic problem for the given surface. It must be mentioned that the determination of the Green's function itself is quite difficult and, generally speaking, cannot be carried out in general form. The authors have succeeded in solving approximately, with the aid of a method they developed, the problem of radiation from a particle entering into a round waveguide with an infinite flange.

Increased radiation intensity can naturally be obtained if the particle moves over a system of spheres or some other bodies in such a way that the individual fields radiated as the particle moves over each sphere are summed coherently. It is easy to see that the best conditions for coherent addition are obtained if the bodies over which the particle moves are periodically arranged. A corresponding analysis was presented by Amatuni and Oganesyan[28].

Let us calculate the radiation from a particle traveling over a sphere of radius a at an impact distance b. In analogy with the case considered in the beginning of this section, we assume that the inequalities $\beta \ll 1$ and $a/\lambda \ll 1$ are satisfied, so that the radiation can be regarded as dipole, and we can use the image method to determine the dipole moment.

Simple calculations yield for the components of the dipole moment d the following expressions:

$$d_x=0,\ d_y=-\frac{ea^3vt}{(b^2+v^2t^2)^{3/2}},\quad d_z=-\frac{ea^3b}{(b^2+v^2t^2)^{3/2}}. \tag{8.3}$$

Knowing the dependence of the dipole moment on the time, we can use known formulas[118] to find the spectral density of radiation in a unit solid angle. The corresponding formula will be of the form

$$\frac{d^2W}{d\omega\,d\Omega}=\frac{e^2\omega^6a^6}{\pi^2c^3v^4}\left[K_1^2\left(\frac{b\omega}{v}\right)+K_0^2\left(\frac{b\omega}{v}\right)(1-\sin^2\theta\sin^2\varphi)\right], \tag{8.4}$$

where K_0 and K_1 are modified Hankel functions, and θ and φ are the angles in the spherical coordinate system.

We write out also the values of the spectral energy density radiated in all the directions $(dW/d\omega)$, the energy $dW/d\Omega$ radiated at all frequencies in a unit solid angle, and the total radiated energy W:

$$\frac{dW}{d\omega}=\int\limits_{\Omega}\frac{d^2W}{d\omega\,d\Omega}d\Omega=\frac{e^2\omega^6a^6}{\pi c^3v^4}\left\{K_0^2\left(\frac{b\omega}{v}\right)+K_1^2\left(\frac{b\omega}{v}\right)\right\}, \tag{8.5}$$

$$\frac{dW}{d\Omega}=\int\limits_{\omega}\frac{d^2W}{d\omega\,d\Omega}d\omega=\frac{225e^2v^3}{409c^3b}[7\sin^2\theta+5(1-\sin^2\theta\sin^2\varphi)], \tag{8.6}$$

$$W=\int\limits_{\omega,\,\Omega}\frac{d^2W}{d\omega\,d\Omega}d\omega\,d\Omega=1.76\frac{e^2v^3}{bc^3}\left(\frac{a}{b}\right)^6. \tag{8.7}$$

The maximum radiation corresponds to a frequency $\omega_{max} = 2.2\,v/b$:

$$\frac{dW}{d\omega}=\begin{cases}\dfrac{8e^2a^6\omega^4}{3\pi c^3v^2b^2}, & \omega\ll\omega_{max},\\[2mm] \dfrac{8e^2a^6\omega^5}{3\pi c^3v^3b}e^{-\frac{2b\omega}{v}}, & \omega\gg\omega_{max}.\end{cases} \tag{8.8}$$

An analogous calculation leads to the following expressions for a charged filament moving over a cylinder:

$$\frac{d^2W}{d\omega\,d\varphi}=\frac{8\tau^2a^4\omega^3}{\pi^3c^2v^2}e^{-\frac{2b\omega}{v}},\quad W=\frac{16\tau^2v^2b^2}{\pi^2c^2l^2}. \tag{8.9}$$

Here φ is the azimuthal angle, $(0\le\varphi\le\pi)$, and τ is the charge per unit filament length. The first formula of (8.9) has a maximum when $\omega = 3v/2b$.

We now consider the radiation of electromagnetic waves when some charge moves over a system of m spheres or cylinders which are separated from one another. We assume the charge to move parallel to the line joining the centers. We also assume that $(a/r)^3 \ll 1$; in this case the resultant field is the sum of the fields due to the radiation from the image in each of the spheres or cylinders.

The magnetic field **H** radiated by such a system has at the instant t the following form[28]:

$$\mathbf{H}=\frac{1}{c^2R_0}\sum_{\nu=1}^{m}\left[\mathbf{d}\left(t_\nu-\frac{R_0^{(\nu)}}{c}\right)\mathbf{n}\right],$$

where R_0 is the distance from the center of the system to the point of observation,

$$t_\nu=t-\frac{r(\nu-1)}{c},\quad R_0^{(\nu)}\approx R_0-r(\nu-1)\cos\theta.$$

The spectral density of the radiation per unit angle is given by the formula

$$\frac{d^2W_m}{d\omega\,d\Omega}=\frac{\sin^2\frac{m\omega r}{2v}(1-\beta\cos\theta)}{\sin^2\frac{\omega r}{2v}(1-\beta\cos\theta)}\frac{d^2W}{d\omega\,d\Omega}. \tag{8.10}$$

The first factor describes the radiation of an individual body, and the second the interference of radiation from different bodies. This factor is typical of diffraction-grating theory. Naturally, formula (8.10) can be used to describe the radiation from a system of arbitrary independent radiating bodies.

If the system length is small $\left(\frac{m\omega r}{2v}(1-\beta\cos\theta)\ll 1\right)$, the second factor is equal to m^2, i.e., coherent addition of the radiation from the individual bodies takes place.

For an arbitrary value of $\frac{m\omega r}{2v}(1-\beta\cos\theta)$, the second factor has maxima which are proportional to m^2 and have a width of the order of $1/m$ when

$$\frac{\omega r}{2v}(1-\beta\cos\theta)=\pi l, \tag{8.11}$$

where l is an integer.

Relation (8.11) is the Bragg-Wulff condition for a plane grating. From the condition $\cos\theta \leq 1$ at a specified value of the wavelength we can obtain the following limitations on the order of l:

$$\left|\frac{1}{\beta}-\frac{l\lambda}{r}\right| \ll 1. \tag{8.12}$$

Finally, when $\frac{\omega r}{2v}(1-\beta\cos\theta) \gg 1$, the interference factor will be of the order of m at all points except in the direction of the principal maxima, i.e., incoherent summation of the fields radiated by the individual bodies takes place in this case. (We note that we are dealing here with a situation analogous to that produced when a particle travels through a stack of plates; see Sec. 6.)

Radiation from a particle moving along a continuous periodic surface was investigated by Bass and Khankina[82], and also by Parygin[90].

The method used in [82] was first proposed by Lysanov[119] and by Meechem[120], who showed that if the surface $z=\zeta(x,y)$ satisfies the inequalities

$$|\nabla\zeta|_{\max} \ll 1, \quad \frac{\omega}{c}|\zeta|_{\max}|\nabla\zeta|_{\max} \ll 1$$

(the subscript max denotes the maximum value), then the investigation of the propagation of the waves over the periodically uneven surface reduces to a solution of an integral equation with a difference kernel.

For the time-averaged particle energy-loss, the following expression was obtained in [82]:

$$\frac{d^2I}{dx\,d\omega}=-\frac{e^2(1-\beta^2)\,\omega}{2\pi v^2}\,\mathrm{Re}\int_{-\infty}^{\infty}d\varkappa\sum_{l,\,r,\,s=-\infty}^{\infty}B_{-s,\,-l}(\mu_{00}\zeta_{\max}),$$

$$B_{s,\,r}(\mu_{0e}\zeta_{\max})\exp\left[i(\mu_{00}+\mu_{0e})\,b\right]\frac{\mu_{-s,\,l-r}}{\mu_{00}\mu_{0e}}. \tag{8.13}$$

We have introduced here the following notation: $B_{pq}(\mu_{00}\zeta_{\max})$ are the coefficients of the Fourier expansion of $\exp\left[i\mu_{00}\zeta_{\max}\frac{\zeta(x,y)}{\zeta_{\max}}\right]$, and

$$\mu_{\gamma\delta}^2=\frac{\omega^2}{c^2}-\left(\frac{\omega}{v}+\frac{2\pi}{L_x}\gamma\right)^2-\left(\varkappa_y+\frac{2\pi}{L_y}\delta\right)^2,$$

$$\mu_{00}^2=-\frac{\omega^2}{v^2}(1-\beta^2);$$

L_x and L_y are the periods of the function $\zeta(x,y)$.

We consider the particular case when $\zeta = A\cos[(2\pi/L)x]$ and $(\omega b/v)(1-\beta^2)^{1/2} \gg 1$. If these assumptions are satisfied, then (8.13) takes the form

$$\frac{d^2I}{dx\,d\omega}=-\frac{e^2(1-\beta^2)^{1/4}}{2\sqrt{\pi b v}}\,\mathrm{Re}\sum_{s=-\infty}^{\infty}J_s^2\left(\frac{\omega}{v}A(1-\beta^2)^{1/2}\right)$$

$$\times\sqrt{\frac{\omega^2}{c^2}-\left(\frac{\omega}{v}+\frac{2\pi}{L}s\right)^2}\;e^{-2b\frac{\omega}{v}\sqrt{1-\beta^2}}, \tag{8.14}$$

where J_s is the modified Bessel function of order s.*

It follows from (8.14) that only harmonics with $s<0$ can be radiated.

From the condition that the radicand must be real follow the following limitations on the number of the radiated harmonic at fixed frequency:

$$-\frac{\omega L}{2\pi v}(1+\beta)<s<-\frac{\omega L}{2\pi v}(1-\beta). \tag{8.15}$$

From the same condition we can obtain the limitation on the radiated frequency at a fixed number of the harmonic:

$$\omega<-\frac{2\pi s v}{L(1-\beta)}. \tag{8.16}$$

In the paper referred to, the radiation of a particle moving over a periodic surface was investigated in a perturbation approximation, in the form developed in [121]. The limits of applicability of this method are determined for the given case by the inequalities

$$|\nabla\zeta|_{\max} \ll 1, \quad \frac{\omega}{v}(1-\beta^2)^{1/2}|\zeta|_{\max} \ll 1.$$

In the case when $\zeta = A\cos(2\pi x/L)$ the formula for the particle loss is written in the form

$$\frac{d^2I}{dx\,d\omega}=-\frac{e^2A^2(1-\beta^2)^{1/4}\,\omega^{3/2}\left[\frac{2\pi}{L}-\frac{\omega}{v}(1-\beta^2)\right]^2}{2\sqrt{\pi b}\,v^{5/2}\sqrt{\frac{\omega^2}{c^2}-\left(\frac{\omega}{v}-\frac{2\pi}{L}\right)^2}}\,e^{-2b\frac{\omega}{v}(1-\beta^2)^{1/2}} \tag{8.17}$$

The limitation on the frequency in (8.17) is obtained from (8.16) by putting $s=-1$. It is interesting to note that in the perturbation-theory approximation, when $\omega/v-2\pi/L$ is close to ω/c, the loss increases. This is connected with the known effect of resonant rise in the field amplitude when the field glides over a periodically uneven surface. When $\omega/v-2\pi/L \approx \omega/c$, perturbation theory in the form used by us cannot be employed. A more correct calculation, similar to that given in [125], should lead to a finite value of the losses in this case.

Parygin[90] considered, using a somewhat different form of perturbation theory[122], the disturbance of a modulated beam moving over a periodically uneven surface.

Along with the radiation from a particle moving over a periodically uneven surface, it is of interest to consider a particle that moves over a statistically uneven surface. A problem of this type was solved by Bass and Khankina[83]. We shall not repeat the calculation method, and present only the final result for one particular case:

$$\frac{d^2I}{dx\,d\omega}=-\frac{4e^2\omega^{9/2}\overline{\zeta^2}\,\overline{L}^2(1-\beta^2)^{1/4}(1+\beta^2)}{3\sqrt{\pi b}\,v^{5/2}c^3}\,e^{-2b\frac{\omega}{v}(1-\beta^2)^{1/2}}, \tag{8.18}$$

*Some errors have crept into[82], as a result of which formulas (10) and (11) of that paper, which correspond to our formulas (8.13) and (8.14), are incorrect.

where

$$L^{-2}=\frac{1}{\overline{\zeta^2}}\int_0^\infty \varrho K(\varrho)\,d\varrho,\quad K(\varrho)=\overline{\zeta(\mathbf{r}+\varrho)\zeta(\mathbf{r})}$$

is the correlation function of the uneven surface, the bar denotes statistical averaging, and $\overline{\zeta^2}=K(0)$ the mean square of the height of the roughness. Formula (8.18) is valid if the following inequalities are satisfied:

$$\frac{\omega}{v}|\zeta|_{\max}\ll 1,\quad \frac{\omega}{v}(\overline{L}^2)^{1/2}\ll 1,\quad |\nabla\zeta|_{\max}\ll 1,$$

$$\frac{\omega}{v}b(1-\beta^2)^{1/2}\gg 1.$$

Formulas (8.13), (8.14), (8.17), and (8.18) can be rewritten for moving radiators of different types by multiplying them by a certain factor. Thus, for a dipole this factor is of the form

$$\frac{\omega^2}{e^2}[(1-\beta^2)P_z^2+P_x^2]$$

(P_x and P_z—are the corresponding components of the dipole moment); for a filament

$$\frac{2\tau^2}{e^2}\sqrt{\frac{\pi a v}{\omega(1-\beta^2)^{1/2}}},$$

etc. It is interesting to note that the formulas obtained in this manner hold true for a filament without any limitations on $\omega b/v$.

In conclusion let us consider the radiation from various types of charges moving past a semi-infinite ideally conducting screen. The field connected with the motion of the charge can be represented as a superposition of plane damped electromagnetic waves. The radiation from the charge is connected with the diffraction of these waves by the edge of the screen, since diffraction of a damped wave can give rise to undamped diffraction waves.

The diffraction of plane electromagnetic waves by a semi-infinite screen has been thoroughly investigated. This problem can be reduced to a system of paired integral equations, which are solved by the Wiener-Hopf method[123]. The superposition of the diffracted waves yields the radiation field. Without dwelling in detail on the methodological aspect of the problem, we present the results.

The radiation from a charged particle moving past a screen was calculated by Kazantsev and Surdutovich[93], that of a particle moving past a semi-infinite screen by Sedrakyan[124], and that of a charged filament by Sedrakyan[91] and by Bolotovskiĭ and Voskresenskiĭ[42]. We present the results of the last papers. It is assumed in the calculation that the filament is parallel to the edge of the screen and moves at a distance a from it. The angle between the filament velocity and the plane of the screen is $\pi-\vartheta$. The radiation intensity in the frequency intervals $d\omega$ and angle intervals $d\varphi$ is determined by the relation

$$\frac{d^2W}{d\omega\,d\varphi}=\frac{\tau^2}{\pi\omega}\,\frac{1-\beta\cos\vartheta}{\beta}\,\frac{\sin^2\frac{\varphi}{2}}{(\cos\varphi+\beta^{-1}\cos\vartheta)^2+\beta^{-2}-1}\,e^{-2a\frac{\omega}{v}(1-\beta^2)^{1/2}}. \tag{8.19}$$

Integration of (8.19) over the angles leads to the following expression

$$\frac{dW}{d\omega}=\frac{\tau^2\beta^2}{\omega(1-\beta^2)^{1/2}}\,e^{-2a\frac{\omega}{v}(1-\beta^2)^{1/2}}. \tag{8.20}$$

This result was obtained with the aid of asymptotic formulas which are not always valid. Thus, when $\vartheta=0$ and $\beta=1$ we have $dW/d\omega=0$, something which does not follow from (8.20). The divergence of the intensity in regions of small ω is connected with the slow decrease of the field of the filament with distance, compared with the field of a finite source. This divergence can be eliminated by taking into account the finite thickness or the finite conductivity of the screen.

Sedrakyan[134] considered radiation from a filament moving past a screen at a distance b from the edge of the screen. The presence of the edge of the screen introduces into the formula for the transition-radiation fields corrections which decrease like $(kd)^{-1/2}$ with increasing d. There are also several papers dealing with the radiation from a linear source moving over two planes[132] or over an infinite system of planes[128].

The radiation of a charge moving through an opening was investigated essentially by numerical means[95-97]. We shall not stop to discuss the results here, referring the reader to the original papers.

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