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RESONANCE INTERACTIONS OF ELEMENTARY PARTICLES

(Boson Resonances)

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CONTENTS

INTRODUCTION

THE intensive accumulation of experimental data on the interaction between elementary particles brought about by the construction of giant accelerators and large bubble chambers has led to the discovery of a new group of particles—resonances.

A characteristic feature of resonances is their short lifetime ($\tau \sim 10^{-22}$—10^{-23} sec). From the point of view of the experimental possibilities of modern research methods the production and the decay of the new particles occur practically at the same point. Because of this their existence was discovered only with the aid of indirect methods in the study of resonance properties of the products of their decay. This accounts for the name of these particles—resonances.

At the present time the number of resonances discovered already considerably exceeds the number of so-called elementary particles. In this connection a series of questions of principle arises with respect to the nature of all particles. For example, it becomes almost obvious that it makes no more sense to regard all particles as elementary than it would be to consider all atomic nuclei as elementary.

During the last few years the study of resonances has become one of the main problems of high-energy physics. A large number of experimental and theoretical papers has been devoted to this problem.

In the present review we shall discuss in detail the quantum numbers and the properties of boson resonances*.

*Consult also review articles[1-13] on this topic.

The problem of strong and resonance interactions of bosons began to be intensively discussed after the discovery of the maximum in the π^-p cross section curve at a π-meson kinetic energy of $T \approx 900$ MeV [14-16]. In 1955 it was suggested that this maximum is due to a resonance in the $\pi\pi$ system and not in the πN system, since in the latter case the spin of the resonance would be very great ($J \geq {}^{11}/_2$), and this is not very probable[15-16].

In the proposed model the $\pi\pi$ resonance had a mass $M \approx 430$ MeV and a width $\Gamma \approx 100$ MeV. As later investigations of the πN interaction have shown this model does not in fact correspond to reality. However, these suggestions stimulated the development of experiments on the study of the $\pi\pi$ interaction which led to the discovery of a series of resonant π-meson systems.

In 1960 the first experimental results were communicated on the investigation of the $K\pi$ interaction in which the existence of the K* meson was discovered. In 1962—1963 the study of the properties of $K\bar{K}$ pairs and multipion systems began. The more reliably established resonances and their main properties are shown in Table I. Undoubtedly at present it cannot be regarded as complete.

I. ρ MESON

1. Mass and Width

Experimental data on the existence and properties of the ρ meson have been obtained principally in the study of single production of π mesons in the reac-

Table I. Boson resonances*

Designation	Strangeness	$I\,(J^{PG})$ **	CP ***	Mass, MeV	Width (MeV) or τ (sec)	Principal decay modes	Relative probability of decay, %
π	0	$1\,(0^{--})$	$(-)$	π^0—135 $\pi^\pm$—140	π^0—$1\cdot10^{-16}$ sec $\pi^\pm$—$2.5\cdot10^{-8}$ sec	$\pi^0 \to 2\gamma$ $\pi^\pm \to \mu^\pm\nu$	100 100
K	+1	$\frac{1}{2}\,(0^-)$		K^0—498 K^+—494	K_1^0—10^{-10} sec K_2^0—$6\cdot10^{-8}$ sec K^+—$1.22\cdot10^{-8}$ sec	$K_1 \to \pi^+\pi^-$ $K^+ \to \mu^+\nu$	66 64
η	0	$0\,(0^{-+})$	$(-)$	548 ± 1	$\leqslant7$	$\pi^+\pi^-\pi^0$ $\pi^0\pi^0\pi^0$ $\pi^+\pi^-\gamma$ $\gamma\gamma$	30 ± 8 22 ± 9 8 ± 2 40 ± 17
ϱ	0	$1\,(1^{-+})$	$(+)$	750 ± 5	100 ± 10	$\pi^+\pi^-$	100
ω	0	$0\,(1^{--})$	$(+)$	784 ± 1	$9.5\pm2,1$	$\pi^+\pi^-\pi^0$ $\pi^0\gamma$ $\pi^+\pi^-$	85 10 ± 3 $\leqslant0,8$
φ	0	$0\,(1^{--})$	$(+)$	$1018.6\pm0,5$	$3.1\pm1,0$	K_1K_2 $\varrho\pi$	90 $\leqslant10$
K *	+1	$1/2\,(1^-)$		890 ± 1	51 ± 2	$K\pi$	100
f	0	$0\,(\geqslant2^{++})$	$(+)$	1250 ± 25	150 ± 50	$\pi\pi$	100
$\varkappa$ $(\eta2\pi)$ A_1 A_2	1 0 0 0	$1/2$ (?) $0\,(0^{-+})$ $\geqslant1$ (?) $1\,(2^{+-})$	 $(-)$ (?) $(+)$	725 ± 3 $957.5\pm1,5$ 1080 ± 10 1310 ± 15	$\leqslant12$ $\leqslant4$ 100 90	$K\pi$ $\eta2\pi$ $\pi^+\pi^-\gamma$ $\varrho\pi$ $\varrho\pi$ $K\bar{K}$ $\pi\eta$	100 80 20 100 60 20 20
B	0	1 (?)	(?)	1215 ± 18	122 ± 17	$\omega\pi$	100

*In this table the properties of the π and K mesons are given for comparison.
**$I(J^{PG})$ are the isotopic spin, the spin, the parity and the G-parity of the resonance.
***CP is the combined parity.

tions

$$\pi^+ + p \to \pi^+ + \pi^+ + n, \tag{1}$$

$$\pi^- + p \to \pi^- + \pi^+ + n, \tag{2}$$

$$\pi^+ + p \to \pi^+ + \pi^0 + p, \tag{3}$$

$$\pi^- + p \to \pi^- + \pi^0 + p \tag{4}$$

and in the investigation of the processes of antiproton annihilation. The existence of the ρ meson was also discovered in the processes of photoproduction of pions and in proton-proton collisions[17].

A particularly large number of papers has been devoted to the study of the reactions (1)–(4) at $E_\pi \sim 1$ GeV. Already in the earliest work[18-20] carried out with the aid of hydrogen bubble chambers it was noted that there was an indication of the existence of a resonant $\pi\pi$ interaction with M ~ 600 MeV. However, the statistics in these papers were poor [approximately 100 events of type (4)] and, therefore, it was not possible to draw definite conclusions with respect to resonance in the $\pi\pi$ system. A further improvement of statistics to several thousand events of the type (1)–(4) enabled the mass and the width of the ρ meson to be determined.

For example, in [21] the production of mesons in π^+p collisions was investigated at T = 910, 1090 and 1260 MeV. Figure 1 shows the distributions with respect to the effective masses of the $\pi^+\pi^0$ and $\pi^+\pi^+$ meson systems that were obtained. The figure also shows the expected distributions calculated on the basis of statistical considerations.

We shall make a small digression in order to explain briefly what these distributions with respect to effective masses represent, and also on what assumptions the theoretical curves have been obtained, since analogous graphs are widely employed in the study of resonances.

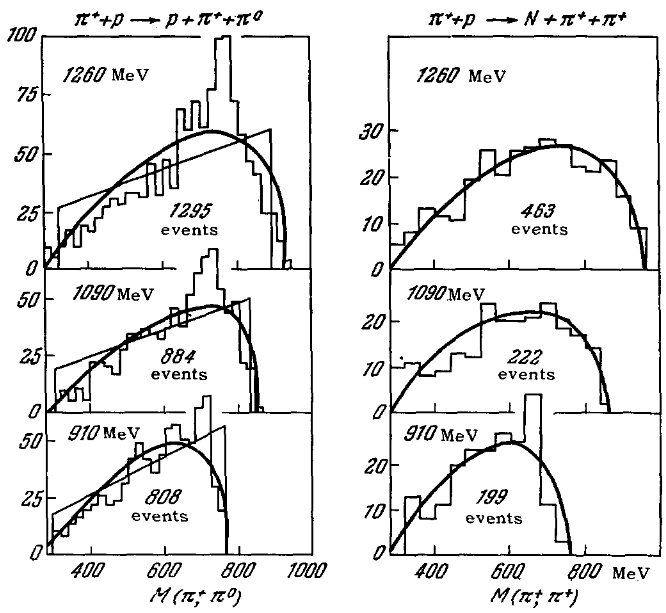

FIG. 1. Distributions with respect to the effective masses of $\pi^+\pi^0$ and $\pi^+\pi^+$ systems obtained in the investigation of reactions (1) and (3).

The effective mass of ν particles is defined by the expression

$$M_\nu = [(\sum_{i=1}^{\nu} E_i)^2 - (\sum_{i=1}^{\nu} \mathbf{p}_i)^2]^{1/2}, \tag{5}$$

where E_i and $\mathbf{p}_i$ are the total energy and the momentum of the corresponding particle, c = 1.* If these particles are products of the decay of another unstable particle-resonance, then from the general results of quantum mechanics it can be shown that the distribution with respect to the effective masses of the ν-particles will have the form[22,23]

$$W(M_\nu) \sim \frac{1}{(M_\nu - M_0)^2 + \left(\frac{\Gamma}{2}\right)^2}, \tag{6}$$

where M_0 is the mass of the unstable particle and Γ is related to its lifetime τ by the energy-time uncertainty relation

$$\Gamma\tau = 1. \tag{7}$$

Thus, by studying the distributions $W(M_\nu)$ one can find a maximum of the type (6), the position and the width of which characterize the mass and the lifetime of the resonance being sought.

We now return to processes (1) and (3) under consideration. In this case if, for example, process (3) occurs only in accordance with

$$\pi^+ + p \to \varrho^+ + p \to \pi^+ + \pi^0 + p, \tag{8}$$

i.e., through the production of a ρ meson, the distribution $W(M(\pi^+\pi^0))$ will have the form (6). However, usually along with reaction (8) reaction (3) is also taking place without the production of a resonance. Then resonance peaks will be observed on a certain background from process (3). Very frequently it is assumed that the distribution $W(M(\pi\pi))$ for a reaction of type (1)—(4) is determined only by the volume of phase space of the states allowed for the given value of the effective mass M_0 of the $\pi\pi N$ system.† Then we have

$$\frac{dW(M_0, M(\pi\pi))}{dM(\pi\pi)} = \int \frac{d^3\mathbf{p}_{\pi_1}}{2E_{\pi_1}} \frac{d^3\mathbf{p}_{\pi_2}}{2E_{\pi_2}} \frac{d^3\mathbf{p}_N}{2E_N} \delta\Big(\sum_{i=3}^{3} \mathbf{p}_i - \mathbf{P}\Big)$$
$$\times \delta\Big(\Big[\Big(\sum_{i=1}^{2} E_i\Big)^2 - \Big(\sum_{i=1}^{2} \mathbf{p}_i\Big)^2\Big]^{1/2} - M(\pi\pi)\Big). \tag{9}$$

Here it is assumed that the matrix element of the πN interaction does not depend on the energies and the momenta of the particles produced and is constant. In subsequent discussion we shall refer to the distribution (9) as the statistical background of the resonance states[24]. Figure 1 shows curves characterizing the statistical background of reactions (1) and (3), and curves in the form of trapezoids characterizing the statistical background from the reactions

$$\pi + N \to N_{33}^* + \pi \to \pi + \pi + N, \tag{10}$$

where N_{33}^* is the isobar with M = 1230 MeV and isotopic spin $I = 3/2$.

As can be seen from the figure, there exists a well defined peak above the statistical background, particularly at T = 1260 MeV with $M \approx 750$ MeV and $\Gamma \approx 100$ MeV in the distribution with respect to the effective masses of the $\pi^+\pi^0$ systems (ρ^+ meson). The absence of a corresponding peak in $\pi^+\pi^+$ systems having I = 2 enables us to conclude that $I(\rho) = 1$, since $I(\pi^+\pi^0)$ can assume only the two values 1 and 2.

Analogous conclusions about the ρ meson were obtained also in other papers on the study of the πN interaction in the energy range for π mesons from 1 to 17 GeV[25-43]. The cross section for the production of ρ mesons has the value of ~ 3.5 mb in reactions (3) and

$$\pi^+ + p \to \pi^+ + p + \pi^+ + \pi^- \tag{11}$$

at $E_\pi^+ \approx 2$ GeV and falls to a few tenths of a millibarn at $E_\pi \approx 10$ GeV[39-41]. It is of interest to note that in the energy range $E_\pi \approx 2$–4 GeV reaction (11) proceeds mainly in accordance with

$$\pi^+ + p \to N_{33}^{*++} + \varrho^0 \to \pi^+ + p + \pi^+ + \pi^- \tag{12}$$

with a cross section of approximately 1 mb[40,41].

The properties and the characteristics of ρ mesons were also studied in annihilation processes[44-48]. In the investigation of the reactions

$$p + \bar{p} \to 2\pi^+ + 2\pi^- + n\pi^0 \qquad (n = 1, 2, \ldots) \tag{13}$$

with the aid of a 72-inch hydrogen bubble chamber it was noted that there is an indication of a doublet structure of the π^0-meson peak ($\pi^+\pi^-$ system)[44].

2. Decay Properties

At present no other possible decays of the ρ meson have been observed apart from the decays $\rho \to 2\pi$. Only rough estimates of the probabilities of these decays are available.

Thus, for example, for the probability of the process

$$\varrho \to 4\pi \tag{14}$$

it has been found[45,46,49] that its upper limit amounts to a few percent of the principal decay of the ρ meson.

It was also established that if the decay

$$\varrho \to \eta + \pi \tag{15}$$

occurs at all, the probability of its occurrence is less than 6×10^{-3} of the probability of the principal decay[45,50].

*Expression (5) is a relativistic scalar, and therefore M_ν can be calculated from the available values of E_i and $\mathbf{p}_i$ in any arbitrary system of coordinates. In future it is assumed in all formulas that $\hbar = c = 1$.

†In some cases the problems of the interference between the background and the resonance channel of the reaction may be significant (cf., for example, Ch. I, Sec. 3).

Some experimental data are also available on the decays

$$\varrho \rightarrow \pi^+\pi^-\gamma \tag{16}$$

and

$$\varrho^0 \rightarrow \text{neutral particles.} \tag{17}$$

In the latter case[51,52]

$$R_1 = \frac{W(\varrho \rightarrow \text{neutral particles})}{W(\varrho \rightarrow \pi^+\pi^-)} \leqslant (6 \pm 40)\%. \tag{17'}$$

Thus, it can be stated that experimental investigation of new modes of the decay of the ρ meson is only beginning.

At the same time there exist a number of theoretical models which enable us to estimate the probabilities of certain modes of decay of ρ mesons[53-58]. For example, the following results were obtained[54-57]

$$R_2 = \frac{W(\varrho^0 \rightarrow \pi^0\gamma)}{W(\varrho^0 \rightarrow \pi^+\pi^-)} \sim 1\%, \tag{18}$$

$$R_3 = \frac{W(\varrho^0 \rightarrow \pi^+\pi^-\gamma)}{W(\varrho^0 \rightarrow \pi^+\pi^-)} \sim 1\%, \tag{19}$$

$$R_4 = \frac{W(\varrho \rightarrow \pi e^+e^-)}{W(\varrho \rightarrow \pi\gamma)} \approx 0.8\% \tag{20}$$

etc. A study of processes (14), (16)—(18) and others can provide essential aid in the determination of the quantum numbers and the properties of the ρ meson.

3. Quantum Numbers

In the first section of this part it was shown that $I(\rho) = 1$. From this it follows that if the observed decay $\rho \rightarrow 2\pi$ is due to the strong interaction (and this is indicated by the large width of the resonance), then as a result of isotopic invariance the spin of the ρ meson must be equal to an odd integer, i.e., its parity is negative*. The G-parity of the ρ meson is positive since it decays according to the strong interaction into two π mesons (cf., Table I)†. However, it is necessary to remember that the conclusion about the isotopic spin of the ρ meson on the basis of which practically all its quantum numbers were determined is of a qualitative character (cf., Sec. 1).

Therefore, other available data on the quantum numbers of the ρ meson are also of particular importance. In this section we shall consider the analysis of the angular distributions of the products of decay of the ρ meson, and in the next section we shall consider data on the cross section of the $\pi\pi$ interaction which largely support the conclusions reached above.

Figure 2 shows the angular distribution of π^- mesons produced in the decay of ρ^- mesons. The distribution is given in the rest system of the ρ mesons with respect to the direction of the incident beam. These data were obtained in the investigation of reaction (4) with the aid of a 50-cm hydrogen bubble chamber irradiated in a beam of π^- mesons of momentum 1.6 GeV/c[49]. Only those cases of production of π mesons were selected in which they emerge in a direction close to the direction of the incident beam of π^- mesons. As can be seen from Fig. 2, in the distribution of π^- mesons resulting from the decay the term $\sim \cos^2 \varphi$ is the dominant one. This fact indicates that

$$J(\varrho) > 1.$$

A more detailed analysis shows that in this distribution there exists also an S-wave (the isotropic part of the distribution) which indicates the presence of $\pi\pi$ interaction with $I = 2$ and $J = 0$.

This circumstance considerably complicates the study of the resonant $\pi\pi$ interaction since problems arise associated with the interference of the interactions of π mesons in states with $I = 1$ and $I = 2$, with isolating the interaction with $I = 1$ in pure form, etc.

A detailed investigation of this problem in [61] has shown that the available experimental data on processes (1)—(4) admit under certain assumptions a large contribution of the $\pi\pi$ interaction with $I = 2$ in the region of the ρ meson. A still more complicated situation occurs in the case of ρ^0 mesons.

In the investigation of peripheral interactions of π^- mesons of momentum 3 GeV/c with protons in reactions (2) it was found that in the distribution of the

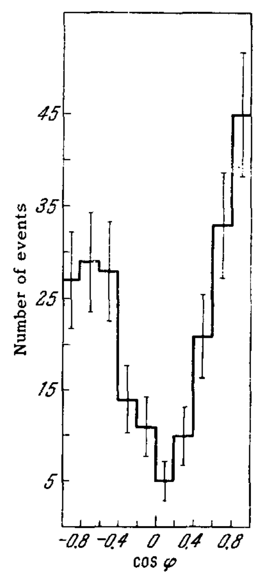

FIG. 2. Angular distribution of π^- mesons produced in the decay of ρ^- mesons.

*The complete wave function for a system of two mesons must be symmetric with respect to a simultaneous interchange of isotopic and space variables in view of the identity of the particles (bosons). The isotopic part of the wave function of two π mesons with $I = 1$ is antisymmetric with respect to an interchange of the isotopic variables. Therefore, the space part must also be antisymmetric with respect to an interchange of the spatial variables, i.e., the parity of the system is negative:

$$P = (-1)^J = -1.$$

†The G-parity of the π meson is negative, the G-parity of a system of π mesons is equal to the product of the G-parities of all the π mesons composing the system[59,60].

type shown in Fig. 2 there exists a considerable "forward-backward" asymmetry[72,62]. Interpretation of this distribution in terms of $\pi\pi$ scattering has shown that there exists a strong, or even a resonance type, interaction of π mesons with I = 0 and J = 0. Thus, in the region of the ρ^0 meson *one can expect the existence also of another resonance with* I = 0. In this connection it appears to be of interest to study processes in which π mesons are produced in a singlet isotopic state. As an example we can quote the reaction

$$d + d \rightarrow He_2^4 + \pi^+ + \pi^-. \tag{21}$$

Thus, on the basis of these data, the conclusion that $J(\rho) = 1$ cannot be regarded as final, although it is the most probable one.

4. Cross Section for the $\pi\pi$ Interaction and the ρ Meson

The existence of a resonance decaying into two π mesons (ρ meson), must lead to the appearance of a maximum in the energy dependence of the cross section of the $\pi\pi$ interaction. An investigation of this maximum would allow one to draw more definite conclusions with respect to the quantum numbers of ρ mesons. At the present time there exist no colliding beams of π mesons, and, therefore, only indirect conclusions about the $\pi\pi$ interaction are possible.

The greater part of the known data on the $\pi\pi$ interaction was obtained from experiments on the single production of pions in πN collisions with the aid of a method proposed by Chew and Low[63]. They noted that one-meson diagrams (Fig. 3 and 4) have a pole when the square of the momentum transferred to the nucleon is $\Delta^2 = -m_\pi^2$. In this case the cross section for $\pi\pi$ scattering is given by the expression

$$\sigma_{\pi\pi}(\omega) = -4\pi f^{-2} F(\omega^2, -m_\pi^2)\, p^2 [\omega^2(\omega^2 - 4m_\pi^2)]^{1/2}, \tag{22}$$

where f is the constant of the πN interaction ($f^2 \approx 0.08$), p is the momentum of the incident π mesons in the laboratory coordinate system, ω is the total energy of the secondary mesons in their center-of-mass system and the value of the quantity

$$F(\omega^2, \Delta^2) = \frac{d^2\sigma(\pi N \rightarrow \pi\pi N)}{d\Delta^2\, d\omega^2} (\Delta^2 + m_\pi^2)^2 \tag{23}$$

is taken for $\Delta^2 = -m_\pi^2$. In this case we need not take into account contributions from processes described by

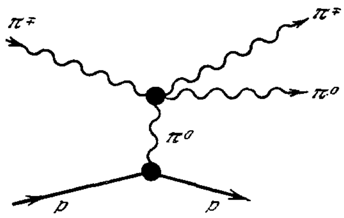

FIG. 3

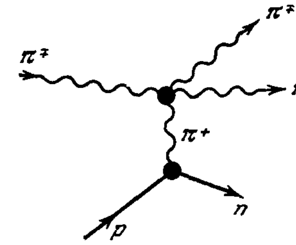

FIG. 4

other diagrams since these diagrams have no poles. However, all these assertions are valid for the unphysical domain of the transferred momenta, and, therefore, the cross section $\sigma_{\pi\pi}(\omega)$ can be obtained only by means of an extrapolation of the experimental values of

$$\frac{d^2\sigma(\pi N \rightarrow \pi\pi N)}{d\Delta^2\, d\omega^2}$$

in the domain $\Delta^2 \geq 0$ to the point $\Delta^2 = -m_\pi^2$. This extrapolation will turn out to be most reliable if one utilizes data in the domain $\Delta^2 \leq m_\pi^2$, i.e., those closest to the pole. However, selection of πN interactions in the domain $\Delta^2 \lesssim m_\pi^2$ corresponds to measurement of processes with a cross section of the order of several tenths of a microbarn superimposed on a background of substantially more probable processes, and at the present time this is a complicated experimental problem. Therefore, in those cases when the statistical accuracy of the material being analyzed turns out to be insufficient for dividing it up according to two parameters ω^2 and Δ^2, a somewhat different method of treating the experimental data is utilized. In this method it is assumed that all cases of reactions (1)—(4) for $\Delta^2 \leq (nm_\pi)^2$, where n = 1, 2, 3 ..., are described by one-meson diagrams (cf., Figs. 3 and 4) and the cross section for the interaction of the virtual π meson with the primary pion depends only on ω (in the general case $\sigma = f(\omega^2, \Delta^2)$).* We then have

$$\frac{d^2\sigma(\pi N \rightarrow \pi\pi N)}{d\Delta^2\, d\omega^2} (\Delta^2 + m_\pi^2)^2 = \frac{f^2 \Delta^2}{4\pi p^2 m_\pi^2} \sqrt{\omega^2(\omega^2 - 4m_\pi^2)}\, \sigma_{\pi\pi}(\omega). \tag{24}$$

After expression (24) has been integrated over Δ^2 one can compare with experiment the theoretically calculated cross section $d\sigma/d\omega^2$. This circumstance enables us to utilize less accurate data on processes (1)—(4) than in the case of the extrapolation procedure. This method has been given the name "physical-region-plot method."

The two methods considered above yield substantially the same result, if in the physical domain of

*In reference[64] a method is proposed for calculating the cross section of processes described by one-meson diagrams (cf., Figs. 3 and 4) in the physical domain of the transferred momenta, without assuming that the cross section for the interaction of a virtual π meson with an incident pion depends only on ω. In this method data are utilized on the pion form-factors of the nucleon obtained from an analysis of single production of mesons in nucleon-nucleon collisions.

transferred momenta reactions (1)–(4) are described primarily by one-meson diagrams. In the opposite case it is hard to say as to which method gives a result closer to reality. To reach such a conclusion additional investigations are necessary.

We now proceed to analyze the available experimental data*. In the energy range of primary π mesons that has been studied the most ($E \approx 1$ GeV) data are available which show that the contribution of processes (1)–(4) described by diagrams of Figs. 3 and 4 is not the dominant one.

Thus, for example, a comparison of the experimental and the theoretical distributions of the secondary π mesons with respect to the angle between them in the πN system at $E_\pi \approx 1$ GeV has shown that the contribution to the cross section of reaction (2) made by the process described by the diagram of Fig. 4 does not exceed 30%. Theoretical calculations were carried out utilizing formula (24), and also utilizing the statistical theory taking into account the production of the N_{33}^*-isobar [65].

In 1962 Treiman and Yang proposed general criteria which must be satisfied by all processes described by the diagrams of Figs. 3 and 4 [66]. The point of their proposal consists of the following. The structure of the one-meson diagrams of Figs. 3 and 4 leads to an absence of correlations (apart from the kinematic ones) between the particles in the nucleon and pion vertices. This is associated with the fact that the virtual particle (π meson) has a spin equal to zero. In this case, for example, the differential cross sections for the reactions (1)–(4) should not depend on the relative orientation of the vectors $[\mathbf{K}_1 \times \mathbf{K}_2]$ and $[\mathbf{p}_1 \times \mathbf{p}_2]$. Here $\mathbf{K}_1$ and $\mathbf{K}_2$ are the momenta of the secondary mesons, $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the nucleons in the πN system†. Moreover, in virtue of isotopic invariance all the characteristics of reactions (3) and (4) described by one-meson diagrams must be identical [67].

A detailed check of the applicability of the one-meson approximation for T = 1.25 GeV was made in [68]. The observed events of type (2) with $\Delta^2 \lesssim 16\ m_\pi^2$ were divided into two groups with respect to the angle α (α is the angle between the directions $[\mathbf{K}_1 \times \mathbf{K}_2]$ and $[\mathbf{p}_1 \times \mathbf{p}_2]$). The coefficients of the "forward-backward" asymmetry in the distribution of the π^- mesons in the rest system of the secondary mesons turned out to be equal to 0.40 ± 0.09 for the cases with $\alpha \leq 90°$ and 0.08 ± 0.09 for $\alpha > 90°$.‡

*A detailed summary of the data on the cross section of the elastic $\pi\pi$ interaction is available in reference [13].

†It should be emphasized that the fact that the Treiman-Yang criterion is satisfied is a necessary condition for the possibility of the description of the processes by means of one-meson diagrams, but it is not sufficient.

‡The asymmetry coefficients are equal to the ratio of the difference between the number of π^- mesons emerging into the forward and the backward hemispheres to the total number of π^- mesons.

Thus, there exists a strong dependence of the asymmetry coefficient on the angle α, i.e., the selected experimental material does not satisfy the Treiman-Yang criterion.

Results of the investigations of reactions (3) and (4) with incident π mesons of momentum 1.25 GeV/c lead to a similar conclusion [69]. It has turned out that the ratio of the cross sections of these processes for $\Delta^2 \lesssim 9\ m_\pi^2$ is equal to

$$\frac{\sigma(\pi^+ p \to \pi^+ p \pi^0)}{\sigma(\pi^- p \to \pi^- p \pi^0)} = 4.6 \pm 0.3. \tag{25}$$

As has been noted above, in the case when the dominant contribution comes from processes described by one-meson diagrams the ratio (25) must be close to unity.

Thus, analysis of experimental data shows that at an energy of incident mesons in the neighborhood of 1 GeV reactions (1)–(4) cannot be described by one-meson diagrams only and one must take into account contributions of other possible diagrams. For illustration we reproduce data on the cross sections of reactions

$$\pi^+ + \pi^0 \to \pi^+ + \pi^0, \tag{26}$$
$$\pi^- + \pi^0 \to \pi^- + \pi^0, \tag{27}$$

obtained in [69] by the "physical-region-plot method" (Fig. 5). As can be seen from the diagram, there is a broad maximum in the curve for the cross section of reaction (26) with $M_{res} = 725 \pm 25$ MeV which can be identified with the ρ^+ meson. On the other hand in the $\pi^-\pi^0$ cross-section curve there is no indication at all of the existence of a ρ^0 meson. The extrapolation procedure applied to the same experimental data gives

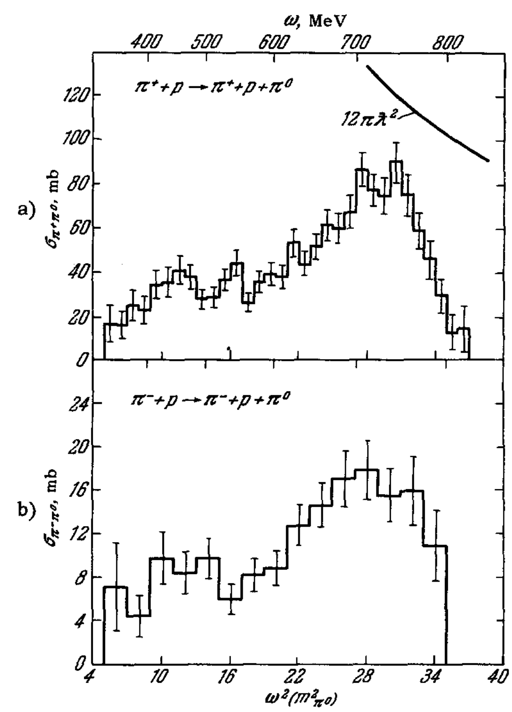

FIG. 5. Total cross sections for $\pi\pi$ scattering ($\omega^2(\pi_0^2)$ is the square of the total energy of the π mesons in units of $m^2_{\pi^0}$).

the same results for the cross sections of reactions (26) and (27) in the domain of energies corresponding to the ρ meson, and different results outside this region. The authors conclude that there is a dominant contribution from the process described by a one-meson diagram only in reaction (3). But in the case of reaction (4) a significant role is played by processes described by other diagrams, for example, by a diagram in which the interaction in the final state is taken into account (Fig. 6). Therefore, the results obtained in this paper from an analysis of events of type (3) can be regarded as a confirmation of the existence of the ρ^+ meson. As the energy of the primary π mesons is increased the situation becomes somewhat more clear. For example, reactions (1) and (2) with $\Delta^2 \lesssim 5m_\pi^2$ were studied at pc = 1.75 GeV with the aid of the scintillation counter method[70].

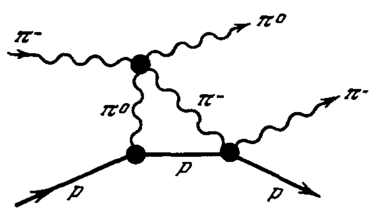

FIG. 6.

In this case the use of the "physical-region-plot method" made possible the discovery of the ρ^0 meson with M ≈ 750 MeV and Γ = 190 MeV.

On the other hand, in [49,71], in which reactions (2) and (4) were studied at pc = 1.59 GeV, results were obtained indicating that the dominant contribution is made by processes described by the diagrams of Figs. 3 and 4, for

$$\Delta^2 \leqslant 8m_\pi^2. \tag{28}$$

In these papers there exists a clear indication of the existence of ρ^0 and ρ^+ mesons obtained by means of the "physical region plot method" taking into account the nucleon form-factor[64] (Fig. 7).

The maximum value of the cross section for $\pi^+\pi^-$ scattering is in this case equal to

$$\sigma_{\max}(\pi^+\pi^- \to \pi^+\pi^-) = (108 \pm 16)\ \mathrm{mb}. \tag{29}$$

As is well known, the maximum cross section for the resonance scattering of π mesons is given by the expression

$$\sigma_{\max} \leqslant 4\pi(2J+1)\lambda^2 \tag{30}$$

and for the case $J(\rho) = 1$

$$\sigma_{\max} \leqslant 12\pi\lambda^2 = 120\ \mathrm{mb} \tag{31}$$

(here $\lambda\!\!\!^{-}$ is the de Broglie wavelength of the π meson). Thus, the good agreement between the values (29) and (31) of the cross section is evidence in favor of $J(\rho) = 1$.* If the nucleon form-factor is not taken into account we obtain

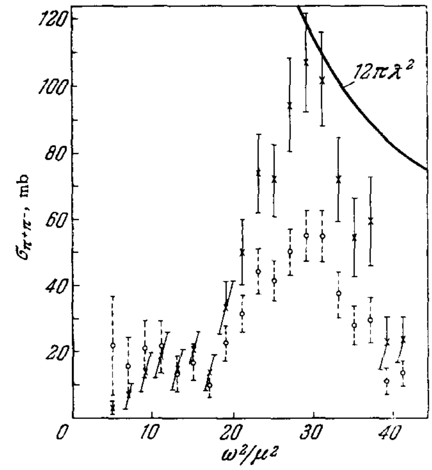

FIG. 7. Cross section for $\pi^+\pi^-$ scattering. ω/μ is the total energy of the dipion in units of m_π. The theoretical curve represents the cross section for the resonance $\pi\pi^-$ scattering with J = 1. ○ – Chew and Low formula; × – Selleri formula.

$$\sigma_{\max}(\pi^+\pi^- \to \pi^+\pi^-) = (56 \pm 8)\ \mathrm{mb} \tag{32}$$

(cf., Fig. 7), and this does not agree with the expected value (31).

An investigation of single production of π mesons in π^-p collisions at pc ≈ 3 GeV has shown that events with $\Delta^2 \lesssim 10\ m^2$ satisfy the Treiman-Yang criterion[62,72]. The results of the analysis of these events are given in Sec. 3 of this chapter. In the main they confirm the existence of the ρ meson.

Thus, the data on the cross section of $\pi\pi$ scattering obtained by the method of Chew and Low are an additional argument in favor of the existence of the ρ meson with I = J = 1.

In summary of the discussion of the problem of the quantum numbers of the ρ meson it should be emphasized that for its final solution new experiments are needed investigating the production and the decay of ρ mesons (cf., Ch. IV). In particular, a discovery of a decay of the type

$$\rho^0 \to \pi^0 + \gamma \tag{18'}$$

would provide evidence that $J(\rho) \neq 0$.

II. f-MESON

A resonance in the $\pi\pi$ interaction with M = 1250 MeV and I = 0 was found almost simultaneously by two different groups in the investigation of the

*It should be noted here that $\sigma_{\max} \approx 120$ mb for $\pi^+\pi^-$ scattering in the domain of energies corresponding to the ρ meson. This is related to the circumstance that the inelastic channels for the decay of the ρ meson ($\rho \to 4\pi$, $\rho \to \eta\pi$ etc.) constitute a small fraction of the decay $\rho \to 2\pi$ (cf., Sec. 2 of this chapter), and, therefore, the phase of $\pi\pi$ scattering is mainly real and the inequality (31) can be approximately regarded as an equality.

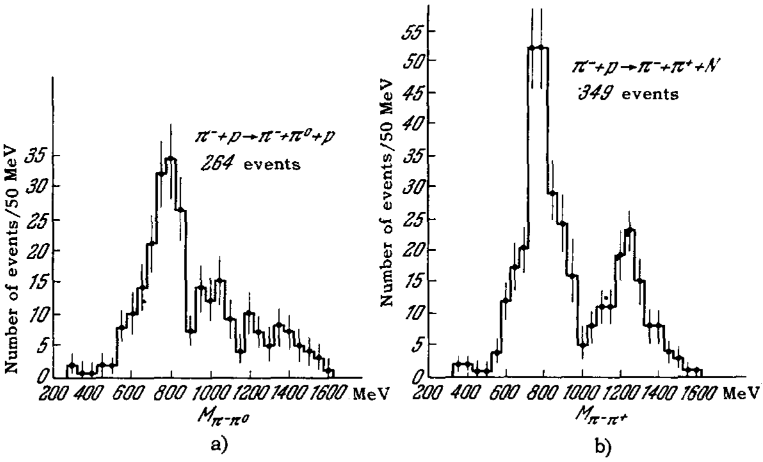

FIG. 8. Distribution with respect to the effective masses of the $\pi^-\pi^0$ and $\pi^-\pi^+$ systems.

interaction of π^- mesons with protons (the f meson) [73,74]. The great interest in this resonance and an extensive search for it are due to the fact that the recently developed new direction in the theory describing processes of interaction between particles at high energies (so called Reggistics) predicts the existence of a particle with $J = 2$, $I = 0$ and a mass in the range 1.0–1.4 GeV[75]. As will be seen later, the properties of an f-meson are close to those of this hypothetical particle. This problem is discussed in greater detail in [76].

An investigation of processes (2) and (4) at pc = 3 GeV carried out by means of a 20-inch hydrogen bubble chamber has demonstrated the existence of two peaks in the distribution with respect to the effective masses of the $\pi^+\pi^-$ systems corresponding to ρ^0 and f mesons (Fig. 8)[73]. The absence of a peak in the $M(\pi^-\pi^0)$ distribution at $M \approx 1250$ MeV provides a basis for the assumption that $I(f) = 0$ (cf. also [77]). An analysis of the distribution with respect to $M(\pi^+\pi^-)$ shows that $M(f) = (1250 \pm 25)$ MeV and $\Gamma(f) = 100 \pm 50$ MeV; the cross section for the production of f mesons in reaction (2) has a value in the neighborhood of 1 mb at pc = 3 GeV. A search for possible decays

$$f \to 4\pi \tag{33}$$

gave negative results corresponding to probabilities comparable to the probabilities of the principal decay

$$f \to 2\pi. \tag{34}$$

A further study of the reaction (2) and (4) using the same experimental arrangement made it possible to show that $J(f) \neq 0$, since the angular distribution of the π mesons in process (34) has a strongly pronounced anisotropic character (cf., Ch. I, Sec. 3)[62]. From this one can draw the conclusion that $J(f) \geq 2$. Indeed, since the isospin of the f meson is equal to zero, then in virtue of isotopic invariance $J(f)$ can be equal only to an even integer (cf., Ch. I, Sec. 3).

The results of the work on the study of the f-meson carried out by means of a 300 liter bubble chamber filled with a mixture of freon (CF_3Br) and propane (C_3H_8) agree with the previously quoted results[74]. The chamber was placed in a magnetic field of 17.1 kG and was irradiated by π^- mesons of momentum 6.1 GeV/c. Events of type (2) were selected. The developed method of measurement and analysis of these events enable this to be carried out with good accuracy. In the selected experimental material the admixture of other events does not exceed 15% and is of no significance in the study of the properties of the f-meson. It was found that $M(f) = 1260 \pm 35$ MeV and $\Gamma(f) \leq 200$ MeV. An analysis of the angular distribution of the π mesons has shown that with a probability of 500:1 the spin of the f meson is different from zero.

A study of the peripheral interactions of π^- mesons with protons ($\Delta^2 \leq 15\ m_\pi^2$) at pc = 4 GeV also confirmed the existence of the f meson[78] (cf., also [72,32,78]). The cross section for the reaction

$$\pi^- + p \to n + f \to n + \pi^+ + \pi^- \tag{35}$$

turned out to be equal to 0.42 + 0.06 mb, while the ratio of the probabilities of the decays was given by

$$\frac{W(f \to \pi^+\pi^-\pi^+\pi^-)}{W(f \to \pi^+\pi^-)} \leqslant 0.08 \pm 0.06. \tag{36}$$

The angular distribution of the π^--mesons produced in reaction (35) in the rest system of the f-meson is shown in Fig. 9. In the same figure are also shown theoretical curves for the case $J = 2$ and 4. Comparison with experimental data shows that the value $J = 2$ turns out to be preferable over $J = 4$.

An analysis of the data obtained by the Chew-Low method taking the nucleon form-factor into account (cf., Ch. I, Sec. 4) has made it possible to determine the cross section for $\pi^+\pi^-$ scattering in the region of f and ρ mesons (Fig. 10). The same figure also shows theoretical curves for $J = 0$, 2 and 4 taking into account another possible decay channel

$$f^0 \to 2\pi^0. \tag{37}$$

As can be seen from Fig. 10 the best agreement with experiment corresponds to the value $J(f) = 2$. Thus, the combined available experimental data show that

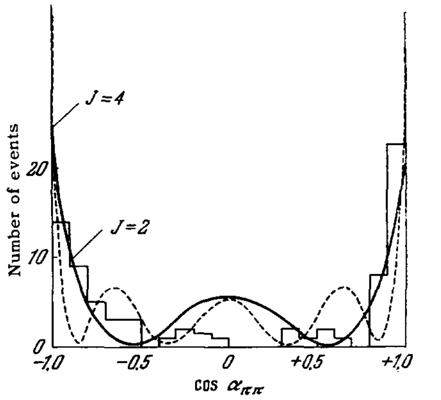

FIG. 9. Angular distribution of the π^- mesons produced in the decay of the f^0 meson. The theoretical curves have been calculated for J = 2 and J = 4.

J(f) ≥ 2, and that the preferred value is J(f) = 2. However, a further increase is necessary in the statistics of cases with the production of an f meson in order to make more definite conclusions with respect to its spin. The G-parity of the f meson is positive, since it decays into two π mesons according to the strong interaction. The spatial parity is also positive and is determined by the fact that I(f) = 0 (cf., Ch. I, Sec. 3).

In conclusion we note that a search for the f meson in the reaction

$$\pi^+ + p \to \pi^+ + p + f \to \pi^+ + p + \pi^+ + \pi^- \quad (38)$$

at pc ≈ 3.5 GeV gave a negative result[41]. The cross section for the production of the f meson turned out to be less than 0.1 mb, while the cross section for the production of the ρ^0 meson in the analogous process (11) at the same energy of incident π mesons amounts to 1.5 mb. Such a difference in the cross sections for

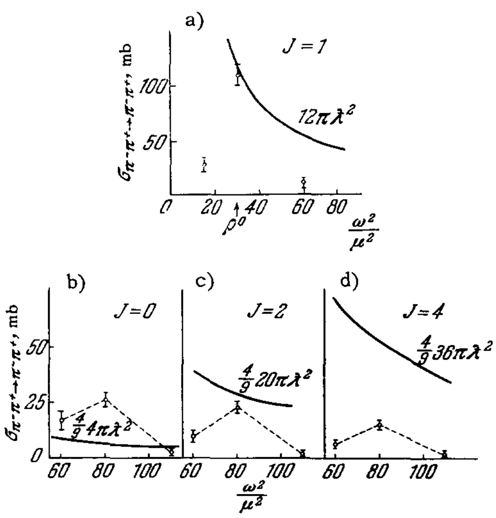

FIG. 10. Cross section for $\pi\pi$ scattering obtained from an analysis of the reactions $\pi^- p \to \pi^+\pi^- n$: a) in the region of the ρ resonance; b) in the region of the f resonance for J = 0; c) in the region of the f resonance for J = 2; d) in the region of the f resonance for J = 4. ω/μ is the total energy of the dipion in units of m_π.

the production of f and ρ^0 mesons can be explained, for example, in the case when the processes (11) and (38) can be described[79] by corresponding one-meson diagrams and

$$\Gamma(\rho^0) = 2.5\,\Gamma(f).$$

Some problems related to the associated production of f mesons and nucleon isobars are discussed in [80].

III. INVESTIGATION OF THE $\pi\pi$ INTERACTION AT LOW ENERGIES (ABC ANOMALY)

A study of the production of π mesons in proton-deuteron collisions

$$p + d \to He_2^3 + \pi^+ + \pi^- \quad (39)$$

has shown that there exists a narrow peak in the momentum spectrum of the recoil nuclei (Fig. 11)[81]. At first this anomaly was interpreted as a proof of the existence of a new resonance with M = 310 MeV and Γ = 10 ± 6 MeV. From the names of the authors of this paper[81] it has received the name of the ABC resonance. Later it was reported that it had been observed in processes of annihilation in πN collisions and in meson photoproduction reactions[44,82,83].

The problem of the existence and the properties of ABC mesons has also been discussed in theoretical papers, particularly in connection with Regge poles [84-89]. It was noted that the experimental data on the decay of K mesons are in contradiction with the existence of the ABC resonance[86,88].

An analysis of the observed anomaly in pd collisions given in [90,91] has shown that it does not have a resonance character and can be explained by the interaction of π mesons in the final state with I($\pi\pi$) = 0. A more detailed experimental investigation of this problem confirms this conclusion. Thus, in [92-99] where the $\pi\pi$ interaction was investigated at low energies, the ABC meson was not found. As an illus-

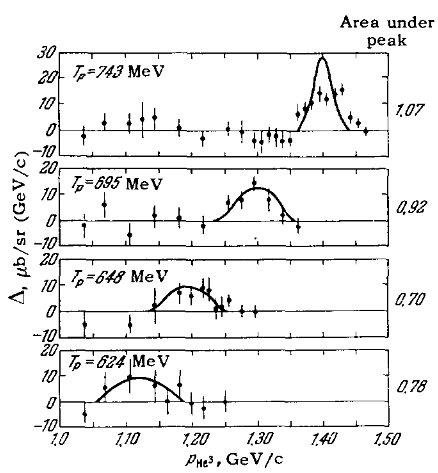

FIG. 11. The momentum spectrum of the He_2^3 nuclei produced in the reaction (39) obtained after subtraction of the statistical background (cf., reference[81]).

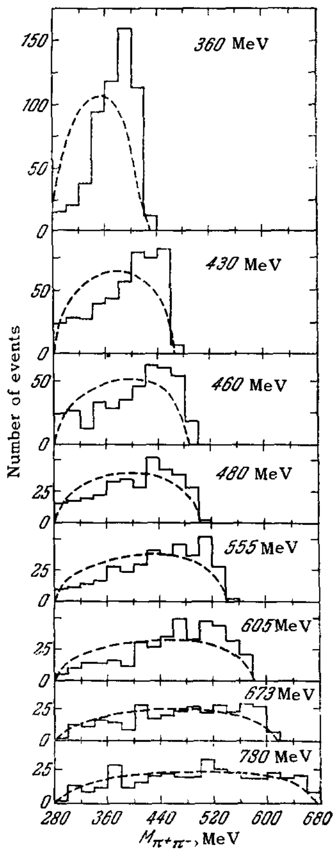

FIG. 12. Distributions with respect to the effective masses of the $\pi^+\pi^-$ systems produced in reaction (2).

tration we quote the results of an investigation of reaction (2) carried out with the aid of a 72-inch hydrogen bubble chamber at kinetic energies of incident π mesons of 360—780 MeV [99]. The distributions obtained for $M(\pi^+\pi^-)$ are given in Fig. 12. From this diagram it can be seen that even though there is an excess of observed points above the phase curve for high values of the effective mass of the $\pi^+\pi^-$ system at T = 360—605 MeV, nevertheless, there is no basis for concluding that there are resonances with masses in the range from 280 to 680 MeV.

A more detailed investigation of the interaction between protons and deuterons carried out by the group which had discovered the anomaly under discussion has shown that a nonresonant interaction of π mesons in the final state with I = 0 and a scattering length of $(2 \pm 1)/m_\pi$ satisfactorily explains the experimental data [100-103].

A similar explanation of the ABC anomaly was proposed in [104]. In this case it was noted that if the reaction (39) proceeds according to the scheme

$$p+d \to \mathrm{He}_2^{3*}+\pi_1, \tag{40}$$

$$\mathrm{He}_2^{3*} \to \pi_2+\mathrm{He}_2^3, \tag{41}$$

$$\pi_1+\pi_2 \to \pi_1+\pi_2, \tag{42}$$

which is described by a triangular diagram (Fig. 13),

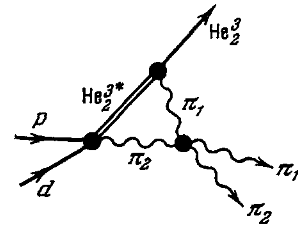

FIG 13

then in the distribution with respect to $M(\pi_1\pi_2)$ there will appear a peak with $M \approx 310$ MeV at a proton kinetic energy of 740 MeV (here He_2^{3*} is a helium nucleus in which one of the nucleons is replaced by an isobar). The appearance of a peak in the distribution with respect to $M(\pi_1\pi_2)$ is associated with the presence of a logarithmic singularity in the diagram of Fig. 13. Calculations show that for a range for $\pi\pi$ scattering $\sim 1/m_\pi$ the ABC anomaly is well described by a triangular diagram.

Thus, at present there is no convincing evidence to support the existence of the ABC resonance.

IV. SEARCH FOR NEW RESONANT $\pi\pi$ SYSTEMS

At present there exist approximately another twenty observed peaks in the distributions with respect to the effective masses of two π mesons (Table II). But, as a rule, these peaks occur in only one or two papers and are statistically poorly supported. Therefore, even the very fact of the existence of an anomaly is not certain (let alone its resonant nature). Moreover, in similar work of other groups carried out with high accuracy these peaks are absent (cf., for example, [99]).

Of particular interest are indications of the possible existence of resonances with I = 0 and 2 in the neighborhood of the ρ^0 meson (cf., Table II). Results of the study of the angular distribution of π mesons produced in the decay of the ρ^0 meson also indicate the existence of a resonance with $M = M(\rho^0)$ and I = 0 (cf., Ch. I, Sec. 3). Therefore, it appears to be of great interest to investigate the $\pi\pi$ interaction in the neighborhood of the ρ-peak in states with definite values of isotopic spin, for example, in reactions (1), (21) and

$$p+p \to d+\pi^0+\pi^+. \tag{43}$$

A simultaneous investigation of such processes will make it possible to answer questions regarding the properties of the ρ meson and regarding other resonances in this range of effective masses of two π mesons (cf., also [108-111]).

V. ω MESON

1. Mass and Width

In 1957 in connection with the analysis of experimental data on electron-proton scattering a suggestion was made of the existence of a heavy neutral meson with I = 0 and $J^P = 1^-$ (ω meson) [116].

Table II

M, MeV*	Γ, MeV	I	References	M, MeV*	Γ, MeV	I	References
310–320	10	0	81, 83, 44	650		1 or 2	109
330		0	82	760		0	82, 106
330		1	82	760		2	82, 106
330		2	82	880		—	38
400	50		36, 112	920	150	—	115
520	70	0 or 1	36	990		0	82
570		1	105	990		1	82
580		0	82	990		2	82, 106
580		2	82, 106	1200		1	106
600		1	112, 113	1200		2	106
600	75	2	114, 112	1400			27

*In the table are given approximate values of the masses of the resonances.

Experimentally the ω meson was observed for the first time in 1961 in the investigation of the annihilation processes

$$p+\bar{p}\rightarrow\pi^{+}+\pi^{+}+\pi^{-}+\pi^{-}+\pi^{0} \quad (44)$$

by means of a 72-inch hydrogen bubble chamber at pc = 1.61 GeV[117,118]. A narrow peak was found in the distribution with respect to effective masses of the $\pi^+\pi^-\pi^0$ system with $M(\omega) = 787$ MeV and $\Gamma \leq \Gamma_{res} = 24$ MeV (Γ_{res} is the experimental resolving power of the apparatus). The absence of a corresponding peak in the distributions with respect to $M(\pi^{\pm}\pi^{\pm}\pi^0)$ and $M(\pi^{\pm}\pi^{\pm}\pi^{\mp})$ provides grounds for assuming that $I(\omega) = 0$.

By now the ω meson has been observed in many reactions, for example[119-133,40,41,71]

$$p+\bar{p}\rightarrow 3\pi^{+}+3\pi^{-}+\pi^{0}, \quad (45)$$

$$\pi^{+}+d\rightarrow p+p+\omega, \quad (46)$$

$$\pi^{\pm}+p\rightarrow\pi^{\pm}+p+\omega, \quad (47)$$

$$K^{-}+p\rightarrow\Lambda+\omega, \quad (48)$$

$$p+p\rightarrow p+p+\omega, \quad (49)$$

$$p+\bar{p}\rightarrow K+\bar{K}+\omega \quad (50)$$

etc.

Figure 14 shows a characteristic peak corresponding to the ω meson obtained in reaction (46); here also one can see a small peak with $M \sim 550$ MeV (η meson).

The width of the ω meson was determined with the aid of a 30-inch hydrogen bubble chamber in the investigation of the annihilation of stopped antiprotons in the reaction[34]

$$p+\bar{p}\rightarrow K^{+}+K^{-}+\pi^{+}+\pi^{-}+\pi^{0}. \quad (51)$$

In this experiment it was possible to obtain $\Gamma_{res} \approx 2$ MeV, which is by an order of magnitude better than in other experiments. Such a considerable improvement in the resolving power of the apparatus was achieved by a special selection of events of type (51). In particular, only those events were studied in which both K mesons were stopped in the chamber. This circumstance made it possible to measure the energies and the momenta of the K mesons from their range in hydrogen considerably more accurately than from the curvature of their track in the magnetic field, as has been done in other experiments.

The reaction (51) has turned out to be very convenient from this point of view, since with a high degree of probability it proceeds along channel (50), and in this case the total kinetic energy of the particles produced amounts to only approximately 100 MeV and the K mesons are generally stopped in the chamber. The range-energy relation was checked in the same experiment for protons and pions of known energies.

By measuring the angle between the K mesons it is possible to determine $M(\pi^+\pi^-\pi^0)$ in accordance with the formula

$$M(\pi^{+}\pi^{-}\pi^{0})=[(M-E_{-}-E_{+})^{2}+(\mathbf{p}_{+}+\mathbf{p}_{-})^{2}]^{1/2}, \quad (52)$$

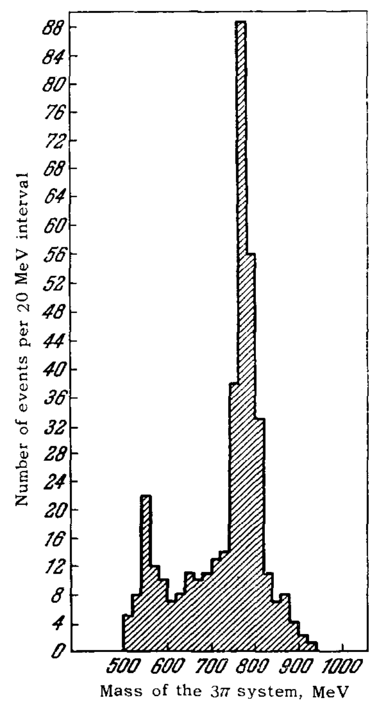

FIG. 14. The spectrum of the effective masses of the $\pi^+\pi^-\pi^0$ systems produced in the reaction (46).

where $M = M(p) + M(\bar{p})$ and the subscripts on E and p denote the signs of the charge of the K mesons [cf., formula (5)]*.

Altogether 119 events (51) were found when both K mesons were stopped in the chamber. The magnitude of the error in the determination of $M(\pi^+\pi^-\pi^0)$ from formula (52) for these events lies in the range from 0.6 to 1.2 MeV.

Figure 15 shows the distribution of the cases obtained with respect to $M(\pi^+\pi^-\pi^0)$. The solid curve which gives the best agreement with the experiment was calculated by means of a formula of type (6) taking into account the efficiency of recording the K mesons in the chamber; the dotted curve gives the corresponding statistical background in reaction (51).

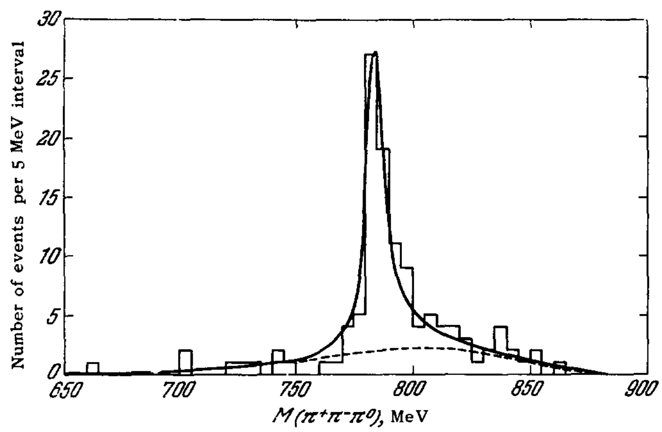

FIG. 15. Spectrum of the effective masses of the $\pi^+\pi^-\pi^0$ systems produced in the reaction (51).

Figure 16 gives a picture of this distribution. From an analysis of these data the following results were obtained: $M(\omega) = 784.0 \pm 0.9$ MeV and $\Gamma = 9.5 \pm 2.1$ MeV, which corresponds to $\tau(\omega) = (0.69 \pm 0.15) \times 10^{-22}$ sec. Interesting proposals for the measurement of $\Gamma(\omega)$ by other methods are discussed in [135-140].

The cross section for the production of ω mesons in πN and KN interactions has a value $\sim 1-2$ mb at $E = 2-4$ GeV and does not exceed a few tenths of a millibarn in annihilation processes at $E(\bar{p}) = 1-3$ GeV.

2. Dalitz Plots

Before going on to a discussion of the quantum numbers of the ω meson we shall consider a general method of analysis of three-particle states. This method was proposed by Dalitz in 1953 as applied to the decay

$$K^{\pm} \rightarrow \pi^{\pm} + \pi^{\pm} + \pi^{\mp} \tag{53}$$

and is widely used at present [141-144].

*The effective mass of a group of particles calculated from the characteristics of the other particles participating in the reaction (in our case p, $\bar{p}$, K^+ and K^- particles) is usually called the residual or "missing" mass.

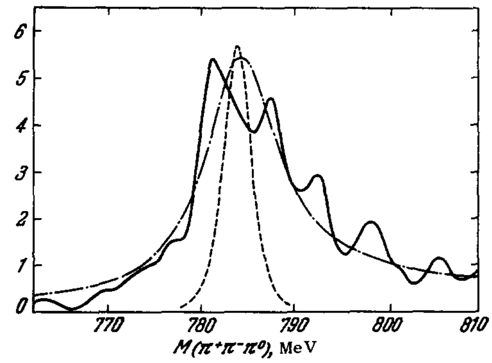

FIG. 16. Distribution of cases represented in Fig. 15. ——— experiment; — . — curve of best fit — — resolution.

Its essence consists of the following. The general expression for the probability of decay of any particle of mass M_0 into three other particles has the form [145]

$$W(M_0 \rightarrow m_1, m_2, m_3) = \frac{1}{(2\pi)^5} \int \frac{|M|^2}{2M_0} \delta(\Sigma p_i)\, \delta(M_0 - \Sigma E_i) \frac{d^3p_1 d^3p_2 d^3p_3}{2E_1 2E_2 2E_3}. \tag{54}$$

Here $|M|^2$ is the square of the modulus of the matrix element. If the decaying particle has no spin or is unpolarized, then the matrix element depends only on two variables. Indeed, of the nine possible variables, p_{1i}, p_{2i}, p_{3i} $(i = 1, 2, 3)$ the energy-momentum conservation law leaves only five independent. Further, if the particle M_0 is unpolarized, i.e., if in its rest system there are no privileged directions in space, then the probability of its decay is independent of the orientation of the plane of decay.* Thus, of the five variables only two remain on which the matrix element depends.

If for these two variables one chooses the total energies of the two particles E_1 and E_2 in the center-of-mass system for the three particles, then appropriate transformations bring formula (54) into the form

$$W(M_0 \rightarrow m_1 m_2 m_3) = \frac{\pi^2}{2M_0 (2\pi)^5} \int\int |M(E_1, E_2)|^2 \, dE_1\, dE_2. \tag{55}$$

From this expression it can be seen that if the available experimental material on the decay $M_0 \rightarrow m_1, m_2, m_3$ is plotted on a diagram in which E_1 and E_2 are chosen as the variables, then the density of points in this diagram will be proportional to the square of the modulus of the matrix element. Thus, with the aid of such a diagram we immediately have a visual impression of the behavior of the matrix element, and this considerably simplifies the determination of the quantum numbers of the decaying particle.

It is evident that this property of the Dalitz plot remains unchanged if instead of the variables E_1 and E_2 we choose the variables T_1 and T_2 or T_1, T_2-T_3.†

*In the case that the particles are initially polarized we must replace $|M(E_1E_2)|^2$ in expression (55) by $\overline{|M(E_1E_2)|^2}$ where the horizontal bar denotes avaraging over the initial polarization.

†Indeed, $E = T + m$ and $dE_1 dE_2 = dT_1 dT_2$.

Sometimes Dalitz plots are constructed also in terms of the variables $M^2_{1,2}$ and $M^2_{2,3}$ (the squares of the effective masses of two particles). We shall show that in this case also the density of points will be proportional to the square of the modulus of the matrix element. By definition we have

$$M^2_{1,\,2} = (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2. \qquad (5')$$

The conservation laws give the following equations in the rest system of the particle M_0:

$$E_1 + E_2 = M_0 - E_3, \qquad (56)$$
$$|\mathbf{p}_1 + \mathbf{p}_2| = p_3. \qquad (57)$$

From this we have

$$M^2_{1,\,2} = (M_0 - E_3)^2 - p_3^2 \qquad (58)$$

and

$$dM^2_{1,\,2} = -2\,(M_0 - E_3)\,dE_3 - 2E_3\,dE_3 = -2M_0\,dE_3, \qquad (59)$$

i.e., $dM^2_{1,2}$ depends linearly on dE_3. Therefore we have

$$dM^2_{1,\,2}\,dM^2_{2,\,3} \sim dE_3\,dE_1. \qquad (60)$$

The use of the variables M^2_{ik} has certain advantages since they are relativistically invariant and, therefore, the Dalitz plot does not depend on the coordinate system in which the experimental data were obtained. A practical application of these diagrams will be shown below in the discussion of the quantum numbers of the ω and η mesons.

3. The Quantum Numbers of the ω Meson

The quantum numbers of the ω meson were determined from the study of the decays

$$\omega \rightarrow \pi^+ + \pi^- + \pi^0. \qquad (61)$$

We assume that this process is due to the strong interaction; then $G(\omega) = -1$ *[59,60]. If we restrict ourselves to the value of the spin $J \le 1$, then there exist four possible combinations for the spin and the spatial parity of the ω meson: 0^+, 0^-, 1^+ and 1^-. The quantum numbers 0^+ are forbidden by the law of conservation of parity.

*In the case of boson systems of strangeness equal to zero it is very convenient to utilize the quantum number $\hat{G}$, since the G-parity of a system is equal to the product of the G-parities of the particles of which it is composed. The operation G is defined as a product of two known operations: charge conjugation and rotation in isotopic space about the I_2 axis by 180°, i.e.,

$$\hat{G} = \hat{C}e^{i\pi I_2}. \qquad (62)$$

From this it is clear that the parity of the system remains unchanged in the case of strong interaction and can be altered in the case of electromagnetic interactions. For neutral systems the following equation holds

$$G = C(-1)^I. \qquad (63)$$

Here C is the charge parity of the system.

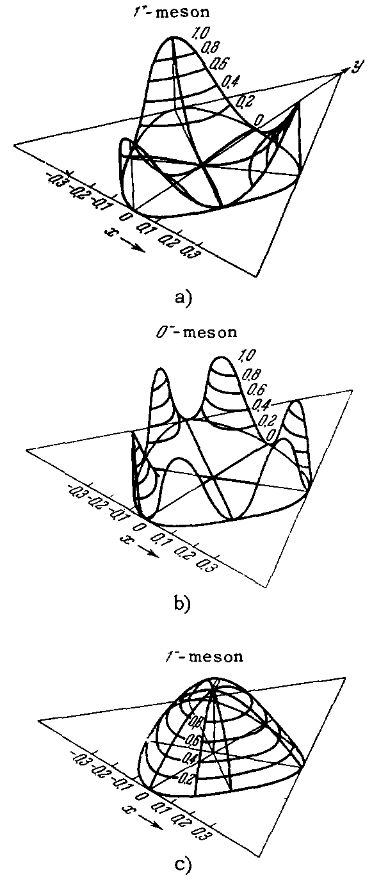

FIG. 17. Simplest matrix elements for 1^+, 0^- and 1^- mesons. $x = \frac{T_- - T_+}{\sqrt{3}Q}$ and $y = \frac{T_0}{Q}$.

The fact that the latter combination is forbidden can be easily seen if one considers the products of the decay $\omega \rightarrow 3\pi$ in terms of a single π meson and a dipion (for example, a π^0 meson and a $\pi^+\pi^-$ dipion). We denote by **L** the orbital angular momentum of the π mesons of the dipion in their c.m.s., and by **l** the angular momentum of the third π meson with respect to the dipion in the c.m.s. of the ω meson. If the spin of the ω meson is zero (we are speaking of the possible combination 0^+), it is necessary to have $l = L$, i.e., the spatial parity of the 3π-system*

$$P = (-1)^{l+L+3} = (-1)^{2L+3} \qquad (64)$$

will be negative ($P = -1$). Thus, the combination 0^+ and $I = 0$ is impossible for an ω-meson.

For the remaining three combinations of the quantum numbers one can write down the matrix elements for the decay (61) for minimum values of L and l (the so-called simplest matrix elements). They are shown in Table III[118].

Figure 17 shows the dependence on the Dalitz vari-

*The number three in the exponent appears as a result of taking into account the intrinsic parity of π mesons which is negative.

Table III. Simplest matrix elements for the decay $\omega \to 3\pi$

J^P	l	L	Simplest matrix elements*	Matrix element equal to zero
1^-	1	1	$(\mathbf{p}_0 \times \mathbf{p}_+) + (\mathbf{p}_+ \times \mathbf{p}_-) + (\mathbf{p}_- \times \mathbf{p}_0)$	At the boundary of the Dalitz plot.
0^-	1 and 3	1 and 3	$(E_- - E_0)(E_0 - E_+)(E_+ - E_-)$	Along straight lines when $E_- = E_0$, $E_0 = E_+$, $E_+ = E_-$
1^+	0 and 2	1	$E_-(\mathbf{p}_0 - \mathbf{p}_+) + E_0(\mathbf{p}_+ - \mathbf{p}_-) + E_+(\mathbf{p}_- - \mathbf{p}_0)$	When $\mathbf{p}_0 = \mathbf{p}_+$, $\mathbf{p}_+ = \mathbf{p}_-$, $\mathbf{p}_- = \mathbf{p}_0$

*Here the subscripts on E and p denote the sign of the charge of the π mesons.

ables of the simplest matrix elements in arbitrary units

$$x = \frac{T_- - T_+}{\sqrt{3}Q} \text{ and } y = \frac{T_0}{Q} \quad (Q = T_+ + T_- + T_0).$$

In Fig. 18 the Dalitz plot shows the experimental data on the decay $\omega \to 3\pi$ obtained in [131]. A visual comparison of the theoretical and the experimental distributions (Fig. 18) shows that the combinations 1^+ and 0^- are in contradiction with experiments since they give a density of points equal to zero at the center of the plot and increasing towards the boundary.

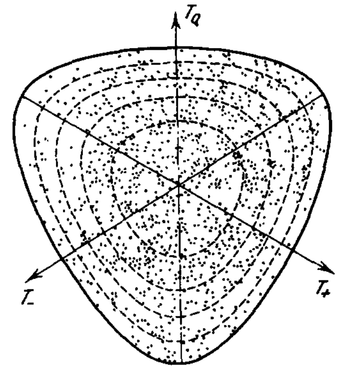

FIG. 18. Dalitz plot for the decays $\omega \to \pi^+\pi^-\pi^0$ (1100 events). T_0, T_-, T_+ are the kinetic energies of the π mesons.

The combination 1^- gives a maximum density of points at the center of the diagram which then falls off to zero at the boundary, and this is in agreement with the experimental data.

Figure 19 shows the theoretical and the experimental dependence of the density of points on the distance from the center of the diagram, which support this conclusion[118].

It is of interest to note that although the analysis for $J > 1$ was not given here, the simplest matrix elements for the quantum numbers 2^+ and 2^- give a density of points equal to zero at the center of the diagram.

Thus, an analysis of experimental data carried out with the aid of the simplest matrix elements on the assumption that $G(\omega) = -1$ and $I \leq 1$ shows that the ω meson is a vector particle with negative parity ($J^{PG} = 1^{--}$). However, in this case there is no certainty that in a matrix element of general form those properties of the simplest matrix element are preserved which were utilized for the determination of the spin and parity of the resonant state.

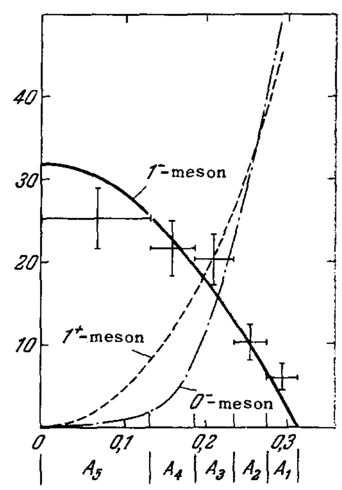

FIG. 19. Theoretical and experimental dependence of the density of points on the distance to the centre of the Dalitz plot[118].

A discussion of the most general properties of the matrix elements of general form which do not depend on the dynamics of the process has shown that the fact that they vanish at definite values of the momenta coincides in the case of three-pion systems with selection rules which follow from the form of the simplest matrix elements (cf., Table III*)[146]. Thus, the absence of experimental points on the boundaries of a Dalitz plot and their presence in the rest of the diagram for the decay $\omega \to 3\pi$ is a strong argument in favor of $J^P(\omega) = 1^-$, with this assertion not being restricted to a specific form of the matrix element. A search for the forbidden configurations of the momenta of the particles also has the advantage that in this case the problem of the interference of resonance processes with the background is solved very simply.

*A number of general properties of multipion systems have been considered in references[147-152].

Indeed, since in these domains the matrix element of the resonance interaction is equal to zero there will be no interference.

The analysis of the decay (61) given above was made on the assumption that this decay proceeds in accordance with the strong interaction ($G(\omega) = -1$). In connection with the small width of the resonance $\Gamma \approx 9$ MeV it was suggested that the decay $\omega \rightarrow 3\pi$ is an electromagnetic process, i.e., $G(\omega) = +1$[148]. However, subsequent investigations have excluded this version. Thus, the study of the decay[45]

$$\omega \rightarrow 4\pi, \tag{65}$$

which must proceed in the case $G(\omega) = +1$ in accordance with the strong interaction and with greater probability than the decay (61), has shown that

$$\frac{W(\omega \rightarrow \pi^+\pi^-\pi^0\pi^0)}{W(\omega \rightarrow \pi^+\pi^-\pi^0)} \leqslant 12\% \tag{66}$$

and

$$\frac{W(\omega \rightarrow \pi^+\pi^-\pi^+\pi^-)}{W(\omega \rightarrow \pi^+\pi^-\pi^0)} \leqslant 5\%. \tag{67}$$

On the other hand, in the case $G(\omega) = -1$ the ratio

$$\frac{W(\omega \rightarrow \text{neutral particles})}{W(\omega \rightarrow \pi^+\pi^-\pi^0)} \sim \alpha \tag{68}$$

($\alpha = 1/137$ is the fine structure constant).

Indeed, the decay

$$\omega \rightarrow \pi^0 + \pi^0 + \pi^0 \tag{69}$$

is forbidden both according to the strong and the electromagnetic interactions, since the charge parity of the ω-meson is negative, while that of three π^0 mesons is positive.* Therefore, the process (69) can occur only as a result of the weak interaction. The probability of this decay is considerably lower than that of electromagnetic processes of the first order

$$\omega \rightarrow \pi^0 + \gamma, \tag{70}$$
$$\omega \rightarrow \pi^0 + \pi^0 + \gamma, \tag{71}$$

which are the ones determining the order of magnitude of the value of the ratio (68)[130,131,125].

For all the combinations of quantum numbers with $G(\omega) = +1$ the probability of the decay of ω into neutral particles must be greater than or comparable to the probability of the decay (61), since this process is an electromagnetic process of the second order. Experimental study of the relation (68) has shown that

$$\frac{W(\omega \rightarrow \text{neutral particles})}{W(\omega \rightarrow \pi^+\pi^-\pi^0)} = 0.10 \pm 0.03. \tag{72}$$

The estimates of the magnitude of this ratio made by other groups agree with (72)[131,125,130]. Thus, the low probability of the radioactive decay of the ω-meson also confirms the correctness of the assumption $G(\omega) = -1$.

*If the bosons have $I = 0$, then $G = C(-1)^I = C$, i.e., $C(\omega) = -1$. The C-parity of a π^0 meson is positive.

Table IV. Data on the possible quantum numbers of the ω-meson ($I(\omega) = 0$)

Possible values of $J^{PG}(J \leq 1)$	Excluded by the following arguments
0^{--}	Dalitz plot; the decay $\omega \rightarrow \pi^0\gamma$
0^{+-}	Conservation of parity; the decay $\omega \rightarrow \pi^0\gamma$
1^{--}	
1^{+-}	Dalitz plot
0^{-+}	Dalitz plot; the small value of the ratio $\omega \rightarrow$ neutral particles/$\omega \rightarrow \pi^+\pi^-\pi^0$; the decay $\omega \rightarrow \pi^0\gamma$
0^{++}	Conservation of parity; the decay $\omega \rightarrow \pi^0\gamma$
1^{-+}	The small value of the ratio $\frac{\omega \rightarrow 4\pi}{\omega \rightarrow 3\pi}$; the decay $\omega \rightarrow \pi^0\gamma$
1^{++}	Dalitz plot; the small value of the ratios $\frac{\omega \rightarrow 4\pi}{\omega \rightarrow 3\pi}$ and $\frac{\omega \rightarrow \text{neutral particles}}{\omega \rightarrow \pi^+\pi^-\pi^0}$; the decay $\omega \rightarrow \pi^0\gamma$

*A more detailed justification for this table may be found in references[45,148].

Finally, the observation of the decay (70) is a direct proof of the fact that $G(\omega) = C(\omega) = -1$, and that its spin cannot be equal to zero[153].*

In conclusion of this section we reproduce Table IV in which a summary is given of the arguments helping to exclude various sets of quantum numbers with the exception of $J^{PG}(\omega) = 1^{--}$.

4. Electromagnetic Decays of ω Particles

As was shown in the preceding section the decay of the ω meson ($J^{PG} = 1^{--}$) into neutral particles is an electromagnetic process. At present a study of these decays has begun with the aid of bubble chambers filled with heavy liquids (xenon, freon, propane, mixture of propane with freon etc.)

With the aid of a 17-liter bubble chamber filled with a mixture of propane (C_3H_8) and xenon the decay (70) in the reaction[153]

$$\pi^- + p \rightarrow n + \omega \rightarrow n + \pi^0 + \gamma \tag{72'}$$

was studied. Experiments were carried out for π^--meson momenta of 1.25, 1.55 and 2.8 GeV/c. Those cases were measured when three or more e^+e^- pairs produced by γ quanta in the chamber were directed towards the point at which the π^- meson was stopped, on the condition that the stoppage of the meson was not accompanied by any traces of nuclear interaction (prongless stars). The background is primarily due to the multiple production of π^0 mesons. For the selection of the cases of decay $\omega \rightarrow \pi^0\gamma \rightarrow 3\gamma$ a kinematic method was used since the energy of the conversion pairs was not measured and, therefore, it was

*The C-parity of a γ quantum is negative.

not possible to obtain the distribution with respect to $M(\gamma\gamma\gamma)$. The essence of this method consists of the following. In the case of the decay $\omega \rightarrow 3\gamma$ one can draw a circular cone through the directions of the three γ quanta. The aperture of this cone has a minimum angle (β_{min}) which depends on the mass of the ω meson and on its energy in the (πp) system:

$$\sin\frac{\beta_{min}}{2}=\frac{M(\omega)}{E}. \tag{73}$$

Figure 20 shows the distributions with respect to the angle β of events corresponding to the reaction

$$\pi^-+p\rightarrow n+3\gamma, \tag{74}$$

after the background has been subtracted. Arrows indicate the values of the angles of aperture (β_{min}) of the decay cone for the ω meson with M = 782 MeV. As can be seen from Fig. 20 the majority of the events lies in the range of angles greater than β_{min}, as is expected for the decay (70). A small number of cases with $\beta < \beta_{min}$ can be explained by statistical fluctuations, by the background from the reaction

$$\pi^-+p\rightarrow K^0+\Sigma^0+\pi^0 \tag{75}$$

or by systematic errors not taken into account. Undoubtedly an improvement in the statistics of events

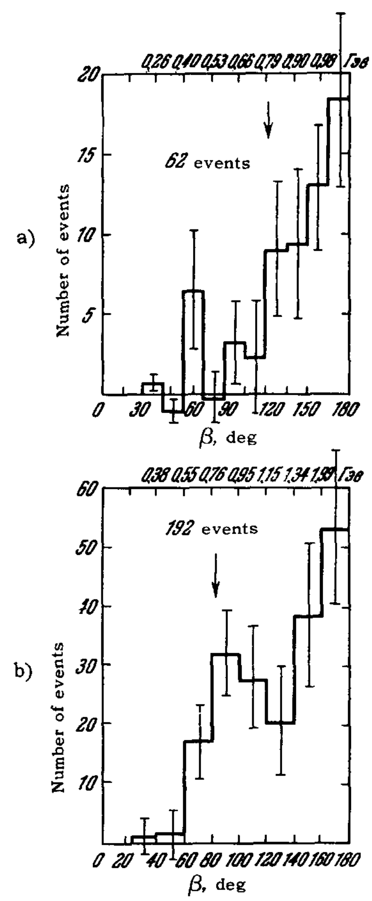

FIG. 20. Distribution of cases corresponding to the reaction $\pi^-p\rightarrow n+3\gamma$ with respect to the angle β. a) pc = 1.55 Gev; b) pc = 2.8 Gev.

(74) and a study of the decays (70) by other methods are required for a final conclusion about its existence.

A comparison of the probabilities of the decays (70) and the decays

$$\omega\rightarrow \text{neutral particles} \tag{76}$$

shows that the decay (70) is the principal one among the neutral types of decay.

In the same article an estimate was given of the value of the ratio

$$\frac{W(\omega\rightarrow 2\pi^0\gamma)}{W(\omega\rightarrow\pi^0\gamma)}\leqslant 0.1. \tag{77}$$

This result is in agreement with the theoretical estimate which was obtained on the assumption that the radiative decays of the ω meson (71) proceed in accordance with

$$\omega\rightarrow\rho^0+\pi^0\rightarrow\pi^0+\pi^0+\gamma. \tag{78}$$

A study of the distribution with respect to the effective masses of $\pi^+\pi^-\gamma$ systems[51] formed in πN interactions at E ~ 7 GeV has shown that there exists a peak at M ~ 760 MeV which can be associated with the decay

$$\omega\rightarrow\pi^++\pi^-+\gamma. \tag{79}$$

In [124,155] an estimate was obtained of the value of the ratio

$$\frac{W(\omega\rightarrow e^+e^-)}{W(\omega\rightarrow\pi^+\pi^-\pi^0)}\leqslant 0.01, \tag{80}$$

which agrees with the corresponding theoretical calculation[156-158].

Thus, the investigation of the electromagnetic decays of the ω-meson has only begun. There exists a number of theoretical models which predict the probabilities of these decays[154,156-160]. A comparison of these models with experimental results presents a possibility of determining the "strength" of the interaction of the ω meson with the ρ meson and with other particles (cf., for example,[154]).

5. ρ-ω Transitions

The quantum numbers for the ρ and the ω mesons are the same with the exception of the isotopic spin, and, therefore, as the result of electromagnetic interactions each of them will contain an admixture of the state with the other isotopic spin. The wave functions for these mixed states can be written in the form[161]

$$\tilde{\rho}^0=\rho^0+\frac{\delta}{\Delta m}\omega \tag{81}$$

and

$$\tilde{\omega}=\omega-\frac{\delta}{\Delta m}\rho^0. \tag{82}$$

Here δ is the matrix element for the electromagnetic transition $\omega \rightleftarrows \rho$, Re ($\Delta$m) ≈ 35 MeV (the difference in the masses of the ρ and ω mesons), Im (Δm)

$= \Gamma(\rho) - \Gamma(\omega) = 100$ MeV, ρ, ω are the wave functions of the states with a definite isotopic spin ($I(\rho) = 1$ and $I(\omega) = 0$).

As can be seen from formulas (81) and (82) the admixture of the state with the other isotopic spin due to the electromagnetic interaction ($\delta \sim 5$ MeV) will be considerable if the difference in the masses of the mesons is small.

Specific calculations show that in the case of ω and ρ mesons the decay

$$\omega \longrightarrow \rho^0 \longrightarrow \pi^+ + \pi^- \qquad (83)$$

will amount to several percent of the principal decay (61)[160,162,164]. The process

$$\rho^0 \longrightarrow \omega \longrightarrow 3\pi \qquad (84)$$

will not be significant due to the short lifetime of the ρ^0 meson.

The decay $\omega \to 2\pi$ was discovered in the investigation of the π^-p and K^-p interactions[124,165,166]. In [124] the value of the ratio

$$\frac{W(\omega \to 2\pi)}{W(\omega \to 3\pi)} = 0.045 \pm 0.016 = 4.5 \pm 1.6\% \qquad (85)$$

was determined.

Reaction (2) was studied at a momentum of 1.7 GeV/c in a hydrogen bubble chamber. Approximately 2137 events were recorded with $M(\pi^+\pi^-)$ in the neighborhood of the ρ-peak. Figure 21a shows the distribution of the effective masses. The distribution has a broad peak in the neighborhood of 650–850 MeV which is asymmetric with respect to the value $M(\pi^+\pi^-) \approx M(\rho^0) = 750$ MeV (393 events between 750 and 800 MeV and 298 events between 750 and 700 MeV). If we assume that the ρ^0 meson has the same mass and a symmetrically shaped peak, as do the ρ^+ mesons, the asymmetry may be brought about by the concentration of events in the neighborhood of the ω peak (780 MeV) as a result of the decays $\omega \to 2\pi$. For a more clearcut separation of the ω peak a distribution of events was constructed which have a value of Δ^2 in the range 0.25–0.70 (GeV/c)^2 (Fig. 21b). In this case a sharp peak is observed corresponding to the ω meson. The authors explain this fact by the circumstance that in events involving small momentum transfer those processes predominate which are described by one-meson diagrams in which the production of the ω meson is forbidden by G-parity and ρ mesons are produced with high intensity (cf., Ch. I, Sec. 4). At large momentum transfers (0.25–0.70 (GeV/c)^2) these processes are not significant and, therefore, the decay $\omega \to 2\pi$ is observed more clearly. The value of the ratio (85) is ~ 0.05. A more careful analysis of all the data on the decay $\omega \to 2\pi$ has shown that the value of the ratio (85) does not exceed 0.8%.[166].

Undoubtedly new experiments are needed for a more exact determination of the probability of the decays $\omega \to 2\pi$.

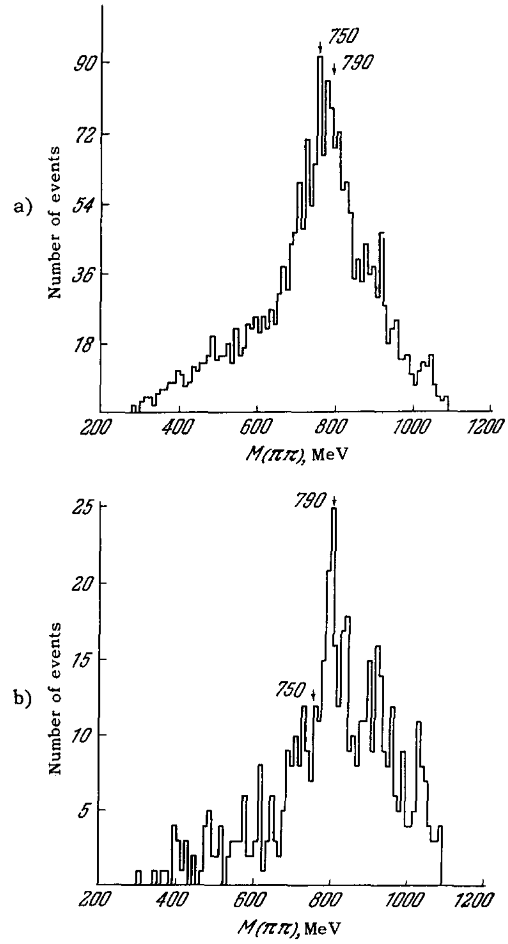

FIG. 21. a) Distribution with respect to the effective masses of the $\pi^+\pi^-$ systems produced in the reaction (2); b) distribution with respect to the effective masses of the $\pi^+\pi^-$ systems produced in the reaction (2). $0.25\ (\mathrm{GeV/c})^2 \leq \Delta^2 \leq 0.70\ (\mathrm{GeV/c})^2$.

VI. η MESON

An investigation of the resonance interactions of elementary particles has led to the discovery of η mesons which on the basis of their properties must be placed among elementary particles and not among resonances.* Indeed, the lifetime of η mesons exceeds by several orders of magnitude the lifetime of the resonances ($\tau(\eta) \sim 10^{-17}$–10^{-18} sec); they are produced in strong interaction processes and decay in accordance with the electromagnetic interaction, as do the π^0 mesons. However, η mesons are usually discussed among the group of resonances due to the common methods used in detecting them.

1. Quantum Numbers

The production of η mesons was observed in many reactions, for example[40,41,121,122,124,125,131,168–174]

*The properties of the η meson are discussed in detail in the review article[167].

$$\pi^+ + d \to p + p + \eta, \quad (86)$$

$$K^- + p \to \Lambda + \eta, \quad (87)$$

$$p + p \to p + p + \eta, \quad (88)$$

$$\pi^+ + p \to \pi^+ + p + \eta, \quad (89)$$

$$\pi^- + p \to \pi^- + p + \eta \quad (90)$$

with the subsequent decay

$$\eta \to \pi^+ + \pi^- + \pi^0. \quad (91)$$

It has also been reliably established that there exist no charged analogues of the η^0 meson, i.e., η^+ and η^- decaying in accordance with

$$\eta^\pm \to \pi^\pm + \pi^+ + \pi^-. \quad (92)$$

From this it follows that the isotopic spin is $I(\eta) = 0$. The mass of the η meson is equal to 548 ± 1 MeV, the width of the corresponding resonance does not exceed 7 MeV and can be wholly ascribed to experimental error.

The cross section for the production of η mesons in πN collisions is approximately 1 mb at $E_\pi = (1-3)$ GeV and falls to several tenths of a millibarn at still higher energies.

In processes of the type (86)–(90) the spectrum of the effective masses of neutral particles (the "missing mass" spectrum) has a sharp peak at the mass of the η meson (cf., for example, Fig. 22). From this it was obtained[175] that

$$\frac{W(\eta \to \text{neutral particles})}{W(\eta \to \pi^+\pi^-\pi^0 + \eta \to \pi^+\pi^-\gamma)} = 2.7 \pm 0.6. \quad (93)$$

Since η mesons are produced in strong interactions, it is natural to suppose at first that the decay (91) also occurs as a result of strong interaction. Then the G- and C-parities of the η meson are negative (cf. ω meson) and decays into an even number of π mesons are forbidden. Decay into five mesons is energetically impossible ($5m_\pi < m_\eta$), and the only possible remaining decay is $\eta \to 3\pi$. The last conclusion also refers to neutral decays and this ruins the whole scheme since the decay

$$\eta \to 3\pi^0 \quad (94)$$

is impossible both in accordance with the strong and the electromagnetic interactions due to the law of conservation of C-parity (cf., Ch. V, Sec. 3). Experiment gives the opposite results. Therefore, it is necessary to conclude that electromagnetic and not strong interactions are responsible for decays of the type

$$\eta \to \text{neutral particles}. \quad (95)$$

A further essential step in the solution of the problem of the quantum numbers of the η meson is associated with the analysis of the corresponding Dalitz plot.

If we assume that the decay (91) is brought about by strong interactions ($G(\eta) = C(\eta) = -1$), then for $J(\eta) \leq 2$ the following sets of quantum numbers are possible: 0^+, 0^-, 1^+, 1^-, 2^+ and 2^-. The set 0^+ is forbidden by the law of conservation of parity (cf. Ch. V, Sec. 3). For each of the other sets there exist regions in the Dalitz plot for which the density of phase points is equal to zero[146]. Figure 23 shows the experimental distribution[131]. It can be seen that the phase points are distributed quite uniformly, i.e., the assumption $G(\eta) = -1$ does not agree with experiment and should be replaced by $G(\eta) = +1$. This means that all the decays of the η mesons are electromagnetic decays.

In particular, the decay (91) is associated with an electromagnetic process of the second order, which involves the virtual emission and absorption of a γ quantum[148]. In this case the G-parity changes sign, while the isotopic spin is altered by unity, i.e., $I(3\pi) = 1$. We now prove this assertion. When a γ quantum is emitted the isotopic spin is either not altered, or changes by unity; the same applies in the case of ab-

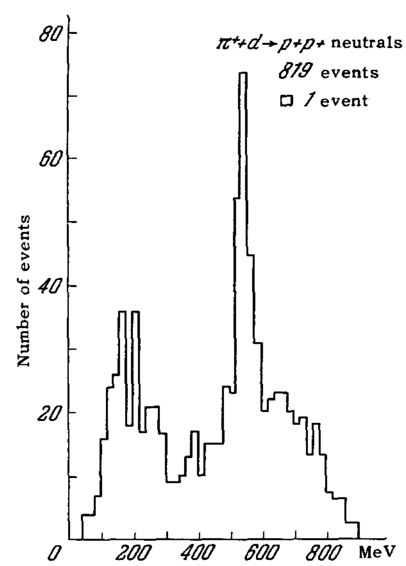

FIG. 22. Distribution with respect to the effective masses of the neutral particles produced in the reaction $\pi^+ d \to pp +$ neutral particles.

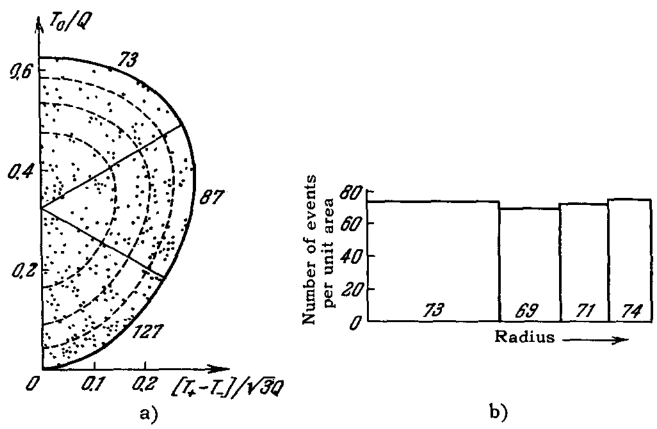

FIG. 23. a) Dalitz plot for the decays $\eta \to \pi^+\pi^-\pi^0$; b) radial density of the phase points for the decays $\eta \to \pi^+\pi^-\pi^0$.

sorption. Therefore, as a result of an electromagnetic process of the second order associated with the emission and absorption of a virtual γ quantum the isotopic spin may change by 0, 1, and 2. On the other hand, the relation

$$G = (-1)^I C \tag{63'}$$

establishes a connection between the changes of G-parity and of isotopic spin, since the charge parity is conserved in electromagnetic processes. Since after the decays (91) and (94) the G parity is altered, the isotopic spin will also change, but only by unity. The initial isospin is $I(\eta) = 0$, and, therefore, the final isospin is $I(3\pi) = 1$. This point is discussed in greater detail in [176].

We now return to Dalitz plots. For the case $I(3\pi) = 1$ they already have a different form, and for the sets 1^+ and 1^-, as before, there exist regions with zero density of points, while for the set 0^- there are no such regions. (Here we have restricted ourselves to the case $J(\eta) \leq 1$[146].)

As a result it appears reasonable to assume that $J^{PG} = 0^{-+}$. Here a degree of caution is called for since the statistics so far are not very good, and ambiguity is possible in the subtraction of the background. It should also be emphasized that in the case of strong interaction the analysis has not been carried out for $J \geq 3$, and for the electromagnetic interactions even for $J = 2$.

A confirmation of the correctness of the choice of the quantum numbers for the η meson (0^{-+}) is given by the results of the experiments on the observation of radiative decays.

The first experiments on the detection of the possible decays

$$\eta \to \gamma + \gamma \qquad (J \neq 1,\ C = +1), \tag{95'}$$

$$\eta \to \pi^0 + \gamma \qquad (J \neq 0,\ C = -1), \tag{96}$$

carried out with the aid of a bubble chamber which records γ-quanta by their electron-positron conversion pairs, demonstrated the radiative decay of the η meson[177]. However, on the basis of the results obtained it was not possible to choose between the reactions (95) and (96)[178]. Similar results were also obtained in the investigation of the photoproduction of η mesons[177-182]

$$\gamma + p \to p + \eta. \tag{97}$$

In [183] the reaction

$$\pi^- + p \to n + \eta \tag{98}$$

was investigated at pc = 1.15 GeV with the aid of a bubble chamber filled with a mixture of C_3H_8 and CF_3Br (the radiation length is 22 cm). The chamber was placed in a magnetic field H = 17500 Gauss which enabled the energy of the e^+e^- pairs to be measured by the magnetic deflection in spite of the large role

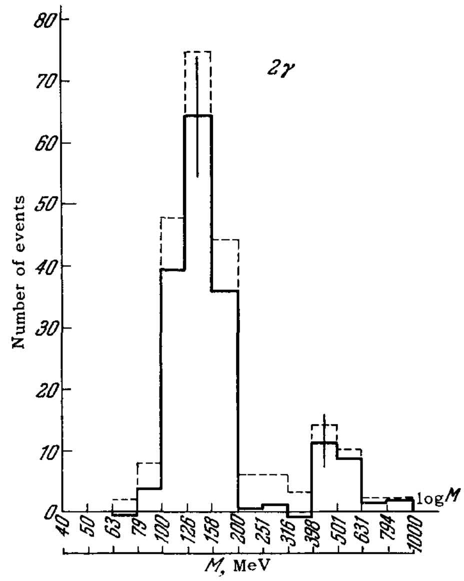

FIG. 24. Distribution with respect to the effective masses $M(\gamma\gamma)$ obtained in the study of the reaction (98).

played by multiple Coulomb scattering. The accuracy of the energy measurements was not great and amounted to 30%. The authors selected photographs with two e^+e^- pairs and, assuming that both γ quanta were produced in the decay of the same particle, evaluated its mass. Figure 24 shows the distribution of the effective masses $M(\gamma\gamma)$. The first peak corresponds to the π^0 meson (the average value of the mass is 138.5 ± 3.7 MeV), the second peak corresponds to the η meson (the average value of the mass is 573 ± 26 MeV). After subtracting the background associated with the reaction

$$\pi^- + p \to \pi^0 + \pi^0 + n, \tag{99}$$

there remain 21 ± 6 events in the neighborhood of the second maximum (the background consists of seven events). The decay (95) was also observed in [175,184].

Thus, the totality of available data apparently points to the existence of the decay (95). From this it follows first of all that the spin of the η meson cannot be equal to unity (the situation is completely analogous to the case of the decay $\pi^0 \to 2\gamma$). It is generally assumed that $J(\eta) = 0$, although, as has been noted above, at present there are as yet no sufficiently convincing grounds for making such a choice*. In this connection [185] is of interest, in which it is proposed to investigate reaction (86) near its energy threshold. It can be shown that in this case it is forbidden for a pseudoscalar η meson and is allowed for any value of its spin different from zero. Experimentally this problem has not been investigated as yet. Another possibility for

*In reference[172] it is shown that the angular distribution of the π^0 mesons in the η-meson system is isotropic, and this agrees with the assumption $J(\eta) = 0$.

determining $J(\eta)$ consists of the application of standard methods associated with angular distributions of η mesons, and of π mesons resulting from the decay.

The existence of the decay (95) shows that $G(\eta) = C(\eta) = +1$, since the γ quantum has negative C-parity. The decay

$$\eta \to \pi^+ + \pi^- \tag{100}$$

is an allowed one from the point of view of spin, G-parity and charge parity. Under these conditions the fact that it does not occur can only denote that the spatial parity of the η meson is negative ($P(2\pi) = +1$, if $I(2\pi) = 0$).

Some other possibilities of determining the quantum numbers are discussed in [186,187].

Thus, the totality of available data shows that the most probable set of quantum numbers of the η meson is 0^{-+} and $I(\eta) = 0$.* In conclusion we display Table V which summarizes the arguments for the possible choices of the quantum numbers of the η meson.

Table V. Data on the possible quantum numbers of the η-meson ($I(\eta) = 0$)

Possible values of $J^{PG}(J \le 1)$	Excluded by the following arguments
0^{--}	Dalitz plot, the decays $\eta \to \gamma\gamma$, $\eta \to 3\pi^0$
0^{+-}	Conservation of parity, the decays $\eta \to \gamma\gamma$, $\eta \to 3\pi^0$
1^{--}	Dalitz plot, the decays $\eta \to \gamma\gamma$, $\eta \to 3\pi^0$
1^{+-}	Dalitz plot, the decays $\eta \to \gamma\gamma$, $\eta \to 3\pi^0$
0^{++}	Conservation of parity, absence of the decay $\eta \to \pi^+\pi^-$
0^{-+}	
1^{-+}	Dalitz plot, the decay $\eta \to \gamma\gamma$
1^{++}	Dalitz plot, the decay $\eta \to \gamma\gamma$

2. The Decay Properties of η Mesons

The pseudoscalar η meson with $I = 0$ can decay only as a result of electromagnetic or weak interactions (ch. VI, Sec. 1). In particular, possible modes of decay of the first and of the second order with respect to the electromagnetic interaction are given by

$$\eta \to \pi^+ + \pi^- + \gamma, \tag{101}$$
$$\eta \to \pi^+ + \pi^- + \pi^0 + \gamma, \tag{102}$$
$$\eta \to \pi^+ + \pi^- + \pi^0, \tag{91'}$$
$$\eta \to \pi^0 + \pi^0 + \pi^0, \tag{94'}$$
$$\eta \to \pi^+ + \pi^- + \gamma + \gamma, \tag{103}$$
$$\eta \to \pi^0 + \gamma + \gamma, \tag{104}$$
$$\eta \to \pi^0 + \pi^0 + \gamma + \gamma, \tag{105}$$
$$\eta \to \pi^0 + \pi^0 + \pi^0 + \gamma + \gamma \tag{106}$$

and

$$\eta \to \gamma + \gamma. \tag{95''}$$

Experimental investigation of these processes encounters essential methodological difficulties due to the necessity for recording neutral particles or of measuring the energy of the charged particles very accurately. However, at present the first results on the radiative decays of the η meson are already available.

Of particular interest are the data on the relative partial widths of processes of the first order in the electromagnetic interaction (101) and (102).

In [190] in which a 72-inch hydrogen bubble chamber was used at pc = 1170 MeV the reaction

$$\pi^+ + p \to \pi^+ + p + \pi^- + \pi^+ + X^0, \tag{107}$$

was studied in great detail, where the symbol X^0 represents some neutral particles (one or several π^0 mesons, γ quanta etc.). 76 such cases were selected, and they all turned out to be associated with the production of an η meson. Calculations of the effective mass of the neutral particles (X^0) showed that all the events satisfied the assumption of either $X^0 \equiv \pi^0$, or $X^0 \equiv \gamma$. No other types of decay of the η meson were found (including the decay $\eta \to \pi^+\pi^-\pi^0\gamma$). The ratio of the probabilities for decay is

$$\frac{W(\eta \to \pi^+\pi^-\gamma)}{W(\eta \to \pi^+\pi^-\pi^0)} = 0.26 \pm 0.08. \tag{108}$$

In this case very accurate measurements of the energy of the charged particles (~ 1%) in reaction (107) made it possible to determine the ratio (108) without recording the γ quanta. The small value of the ratio that was obtained is unexpected. Indeed, the decay (101) as an electromagnetic process of the first order in α must have a higher probability than the decay (91).* A discussion of this problem will be given below.

Estimates of the probabilities for other channels for the decay of the η meson were obtained[175] using the same experimental arrangement as in [190]. In this case the reactions

$$\pi^+ + p \to \pi^+ + p + \eta \to \pi^+ + p + e^+ + e^- + X^0, \tag{109}$$
$$\pi^+ + p \to \pi^+ + p + \eta \to \pi^+ + p + \gamma + X^0 \tag{110}$$

were investigated. In reaction (110) only those events were selected in which the γ quantum was converted

*It is of interest to note that long before the discovery of the η meson properties of a particle with the same quantum numbers were discussed in references[188,189]. The same references also proposed a method for discovering such a particle which was utilized later, and which is based on finding the value of the so called "missing mass."

*An analogous situation also exists in the case of the ratio of the probabilities of the decays $\eta \to 3\pi\gamma$ and $\eta \to 3\pi$ which is even smaller than (108)[190]. However, in this case the decay $\eta \to 3\pi\gamma$ corresponds to a smaller volume in phase space. Both an experimental and a theoretical investigation of this problem are of considerable interest.

into an e^+e^- pair in the hydrogen bubble chamber:

$$\gamma + H \rightarrow e^+ + e^- + H \qquad (111)$$

(the conversion probability is $\sim 2.5\%$).

Calculations of the effective mass of X^0 made it possible to pick out the cases of the decay $\eta \rightarrow 2\gamma$ ($M(X^0) = 0$) and $\eta \rightarrow 3\pi^0$ ($M(X^0)$ lies between appropriate kinematic limits). It turned out in this case that in the domain of $M(X^0)$ where kinematics allows only decays into two π^0 mesons not a single case is observed. From this the authors conclude that the background from the reaction

$$\pi^+ + p \rightarrow \pi^+ + p + \pi^0 + \pi^0 \qquad (112)$$

has a very small value in the neighborhood of the η peak.

As a result the following estimates were obtained:

$$\frac{W(\eta \rightarrow 3\pi^0)}{W(\eta \rightarrow \pi^+\pi^-\pi^0)} = 0.83 \pm 0.32 \qquad (113)$$

and

$$\frac{W(\eta \rightarrow 2\gamma)}{W(\eta \rightarrow \pi^+\pi^-\pi^0)} = 1.24 \pm 0.56. \qquad (114)$$

For the ratio (113) there exist definite theoretical predictions. As has been noted above (cf., ch. VI, Sec. 1) in the decay $\eta \rightarrow 3\pi$ the isotopic spin of the three π mesons is $I = 1$. In this case, starting with the isotopic structure of the wave function for three π mesons, it can be shown[191] that

$$\frac{W(\eta \rightarrow 3\pi^0)}{W(\eta \rightarrow \pi^+\pi^-\pi^0)} \leqslant \frac{3}{2}, \qquad (115)$$

which agrees with (113).

In the course of the investigation of the photoproduction of η mesons the value of the ratio

$$\frac{W(\eta \rightarrow 2\gamma)}{W(\eta \rightarrow 3\pi^0 + \eta \rightarrow \pi^0\gamma\gamma)} = 0.80 \pm 0.25 \qquad (116)$$

was determined[181].

In these results the fact is noteworthy that the probabilities of the decays $\eta \rightarrow 2\gamma$ and $\eta \rightarrow 3\pi$ are of the same order of magnitude. According to theoretical estimates which basically take into account only the difference in the volumes in phase space for these processes the probability for the decay $\eta \rightarrow 2\gamma$ is many times greater than the probability for the decay $\eta \rightarrow 3\pi$ (the value of the ratio (114) is approximately 100).

One of the possible explanations of the anomalously small values of the ratios (108) and (114) consists of assuming the existence of a strong $\pi\pi$ interaction (or resonance) with $I = 0$ which is responsible for increasing the probability of the decay $\eta \rightarrow 3\pi$. Indeed, in the decay $\eta \rightarrow 3\pi$ such a state can be realized ($I(3\pi) = 1$ and $I(\pi^+\pi^-) = 0$ or 2), while in the decay $\eta \rightarrow \pi^+\pi^-\gamma$ the isotopic spin of the two π mesons cannot be equal to zero, $I(\pi^+\pi^-) = 1$.*

*In the decay $\eta \rightarrow \pi^+\pi^-\gamma$ the C-parity is conserved, i.e., $C(\pi^+\pi^-\gamma) = +1$. On the other hand $C(\pi^+\pi^-\gamma) = C(\pi^+\pi^-)C(\gamma)$ and $C(\gamma) = -1$, i.e., $C(\pi^+\pi^-) = -1$. Since $C(\pi^+\pi^-) = (-1)^l = (-1)^I$, then $I(\pi^+\pi^-) = 1$.

Calculations made on the assumption that there exists a resonance with $M = 370$ MeV and $\Gamma = 50$ MeV have shown[192] that

$$\frac{W(\eta \rightarrow 2\gamma)}{W(\eta \rightarrow 3\pi)} = 3.3. \qquad (117)$$

On the other hand if the decay $\eta \rightarrow \pi^+\pi^-\gamma$ proceeds according to the scheme[193]

$$\eta \rightarrow \rho^0 + \gamma \rightarrow \pi^+ + \pi^- + \gamma, \qquad (118)$$

then

$$\frac{W(\eta \rightarrow \pi^+\pi^-\gamma)}{W(\eta \rightarrow 3\pi)} \approx 0.4. \qquad (119)$$

As can be seen, these estimates agree with the experimental data. An investigation of the energy spectrum of π^0 mesons produced in the decay $\eta \rightarrow \pi^+\pi^-\pi^0$, also points to the possible existence of a strong $\pi\pi$ interaction (Fig. 25)[194].

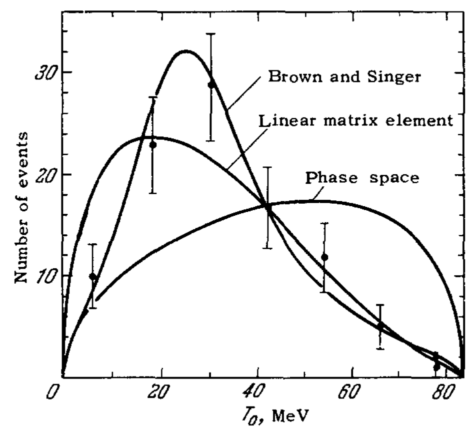

FIG. 25. Energy spectrum of π^0 mesons produced in the decay $\eta \rightarrow \pi^+\pi^-\pi^0$. For the meaning of the theoretical curves cf. reference[194] and Ch. VI, Sec. 2.

The theoretical curve (Brown and Singer) which gives the best agreement with the experimental data was calculated on the assumption of the existence of a $\pi\pi$ resonance with $M = 381 \pm 5$ MeV and $\Gamma = 48 \pm 8$ MeV[192].* A further investigation of this problem is of considerable interest.

Some information on the nature of the decays $\eta \rightarrow 3\pi$ can be obtained from the analogy of the processes[195-204,206]

$$K^{\pm} \rightarrow \pi^{\pm} + \pi^+ + \pi^-, \qquad (120)$$

$$K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0. \qquad (121)$$

It turns out that the three π mesons produced in the decays $K_2^0 \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ can have the same quantum numbers. Indeed, in the case of the decay $\eta \rightarrow 3\pi$ $J^P(3\pi) = 0^-$ and $I(3\pi) = 1$ (Ch. VI, Sec. 1). In the decay $K_2^0 \rightarrow 3\pi$, since $J(K) = 0$, then $L(\pi^+\pi^-) = l(\pi^0)$ (in terms of a dipion and a single π^0 meson for the

*An indication of the existence of a strong $\pi\pi$ interaction with $I(\pi\pi) = 0$ and $M \sim 3\, m_\pi$ was also obtained in the study of the reactions $\pi N \rightarrow \pi\pi N$[99].

reaction (121)). From this we have

$$P(3\pi)=(-1)^{l+L+3}=(-1)^{2l+3}=-1. \qquad (122)$$

In weak interaction processes the combined parity (CP) is conserved, and, therefore,

$$CP(3\pi)=(-1)(-1)^{I(\pi^+\pi^-)}=-1, \qquad (123)$$

since $CP(K_2^0) = -1$. Consequently, the isotopic spin $I(\pi^+\pi^-) = 0$ or 2. The total spin of the three π mesons can be equal to 1, 2 and 3. If one uses the rule $\Delta I = 1/2$, then only one possibility remains: $I(3\pi) = 1$ ($I(K) = 1/2$).

Thus, the parity, isospin and spin of 3π-systems produced in the processes $K_2^0 \to 3\pi$ and $\eta \to 3\pi$ coincide. It is true that these systems may differ with respect to other quantum numbers [l, L and $I(\pi^+\pi^-)$]. However, the fact that there is not much difference in the masses of the K and the η mesons gives grounds for assuming that in these quantum numbers there will also not be any great difference. In this connection, if one assumes that the matrix element for the processes $K^0 \to 3\pi$ and $\eta \to 3\pi$ depends only on the interaction of the π mesons in the final state, the meson spectra in these decays should be similar[196]. The experimental data does not contradict this assumption[196–198]. A simultaneous study of these processes will enable one to obtain new information on the mechanism of the decay of the η meson.

In conclusion of this section we note that the decay $\eta \to 3\pi$ occurs as a result of electromagnetic processes of the second order. This means that in strong interactions the isotopic spin is conserved with an accuracy up to terms of order $\alpha^2 \sim 10^{-4}$. Naturally, we are here not so much making a final assertion, but rather indicating the direction of future investigations[205]. Some conclusions drawn from the non-occurrence of the decay $\eta \to \pi^+\pi^-$ associated with the conservation of P- and CP-parities in strong interactions are discussed in [206].

3. Lifetime of the η Meson

The electromagnetic nature of the decays of the η meson determines its relatively long lifetime. According to various theoretical estimates $\tau(\eta) \sim 10^{-17}$–10^{-18} sec, i.e., the expected width of the η peak is $\sim 10^{-4}$–10^{-3} MeV. With the present experimental accuracy when the resolving power of the apparatus is no better than several MeV the determination of $\Gamma(\eta)$ by direct methods is not possible. Observation of gaps between the points of production and of decay of η mesons [determination of $\tau(\eta)$] requires very high spatial resolution, since the magnitude of these gaps is ~ 0.01–$0.001\,\mu$ (this situation is analogous to the determination of the lifetime of the π^0 meson)[205,209,210].

Therefore, indirect methods of the determination of $\Gamma(\eta)$ acquire great significance. For example, investigating the photoproduction of η mesons in the Coulomb field of a heavy nucleus, i.e., utilizing a process inverse to the decay $\eta \to 2\gamma$, it is possible to make an estimate of the magnitude of $\Gamma(\eta)$. In this case the value of the effective cross section is proportional to the width of the η meson and at $Z \sim 100$ and $E_\gamma \sim 4$ GeV attains a value of 3×10^{-28} cm^2/sr if $\Gamma(\eta \to \gamma\gamma) \sim 150$ eV[211,212]. In order to separate this process from the nuclear production of η mesons one can utilize the very narrow angular distribution (the most probable angle is 0.5°) and the rapid increase of the cross section with $Z(\sigma \sim Z^2)$.

VII. MULTIPION RESONANT SYSTEMS

In connection with the discovery of two- and three-pion resonant systems (ρ, f, ω, and η mesons) the question naturally arises of the existence of such resonances which might decay primarily into four, five and greater number of π mesons. In the case that a considerable number of such resonances were to be discovered the situation concerning them would, possibly, remind one to a certain extent of the situation existing in the formation of nuclei from nucleons, and one might hope to find common regularities in their properties. In the opposite case there can exist several resonances whose properties can be quite different. The solution of this problem is of considerable interest.

At the present time a search has been started for resonances decaying into four π mesons[123,214,215] (cf. the Appendix).

A study of processes of the type

$$\pi^- + p \to \Lambda + K + n\pi \qquad (n = 1, 2, 3, 4, \ldots) \qquad (124)$$

has shown that in the distribution with respect to the effective masses of the $\pi^+\pi^-\pi^+\pi^-$ system there exists a peak with M = 1340 GeV and $\Gamma \approx 140$ MeV[123]. An indication of the existence of a resonance in the 4π system with M = 1.4 GeV was obtained in [214,215]. However, further investigations are needed in order to elucidate the nature of these anomalies.

1. The B Meson ($\pi\omega$ Resonance)

A study of the reactions[41,137]

$$\pi^\pm + p \to \pi^\pm + p + \omega, \qquad (125)$$

carried out by means of hydrogen bubble chambers in the range of momenta from 3.24 to 4 GeV/c has shown that there exists a peak in the distribution with respect to $M(\pi^\mp\pi^+\pi^-\pi^0)$ (B meson, Fig. 26). The mass of the B meson is equal to 1.22 GeV and $\Gamma = 0.100 \pm 0.020$ GeV.

It was also established that

$$\frac{W(B^+ \to \pi^+\pi^+\pi^-\pi^0)}{W(B^+ \to \pi^+\omega \to \pi^+\pi^+\pi^-\pi^0)} \leqslant 0.5. \qquad (126)$$

Evidently $I(B) = 1$. It is of interest to note that the B meson is the first resonance which decays in accordance with

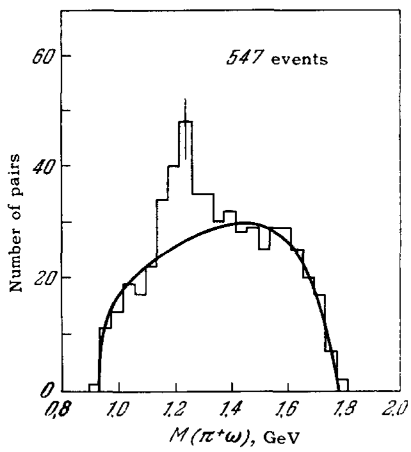

FIG. 26. Distribution of effective masses of $\pi^+\omega$ systems produced in reaction (127).

$$B^{\mp} \to \omega + \pi^{\mp} \tag{127}$$

involving an already known resonance.

The possible quantum numbers and the properties of the new resonance are discussed in [216,217]. However, an improvement in the statistics and further study of the nature of the anomaly under discussion are needed before a conclusion that the B meson exists can be drawn.

VIII. RESONANT K-MESON SYSTEMS

At present experimental data are gradually being accumulated on the production and decay of pairs of K and $\bar{K}$ mesons and a study of the corresponding resonances is beginning. In this connection we shall first mention a very interesting property of the $K^0\bar{K}^0$ systems which has turned out to be useful for the determination of the quantum numbers of the resonances, and we shall then go on to the available experimental results.

1. $K^0\bar{K}^0$ Decay Mode as a Detector of the Parity of the System

We shall consider some decay properties of pairs of K^0 and $\bar{K}^0$ mesons[218,221].

The $K^0\bar{K}^0$ system, just as any other boson-antiboson system, has a very important particular feature: its combined parity is always positive (CP = +1). Indeed, for any arbitrary orbital angular momentum l of the system charge conjugation yields a phase factor $(-1)^l$; spatial reflection P yields exactly the same factor, so that after a CP-transformation the combined phase factor is equal to $(-1)^{2l} = +1$.

On the other hand, decays of the type

$$K^0\overline{K^0} \to K_1^0K_1^0, \tag{128}$$

$$K^0\overline{K^0} \to K_2^0K_2^0, \tag{129}$$

$$K^0\overline{K^0} \to K_1^0K_2^0, \tag{130}$$

are observed experimentally where K_1^0 are shortlived mesons having CP = +1 and K_2^0 are longlived mesons with CP = −1.

The pairs $K_1^0K_1^0$ and $K_2^0K_2^0$, since the particles of which they are composed are identical, are always in states with even angular momenta and have a combined parity CP = +1.

The combined parity of the pairs $K_1^0K_2^0$ depends on the orbital angular momentum of the system and is equal to $(-1)^{l+1}$. Therefore, if this pair is produced from a $K^0\bar{K}^0$ pair which, as has been shown above, has CP = +1, then it can have only odd angular momenta. Thus, for even orbital angular momenta the $K^0\bar{K}^0$ pair can decay only in accordance with (128) and (129), while for an odd angular momentum it can decay only in accordance with (130).

Thus, the decay scheme is in this case a detector of the parity of the orbital angular momentum of the system. Since the parity of the system is defined by the factor $(-1)^l$, the decay scheme is also a detector of the parity of the system.

If there exists a resonance interaction between K and $\bar{K}$ mesons in a state with a definite orbital angular momentum, then the corresponding neutral system $K^0\bar{K}^0$ will decay only in accordance with (128) and (129) or (130) depending on the parity of the resonance, with the decays in accordance with (128) and (129) being of equal probability[218,220]. This important property of pairs of K^0 and $\bar{K}^0$ mesons was utilized for the determination of the quantum numbers of the φ meson.

2. The φ Meson

Investigations of the interactions of K^- mesons in the momentum range 1.8–2.2 GeV/c with protons carried out with the aid of hydrogen bubble chambers led to the discovery of a resonance decaying into $K\bar{K}$ pairs, which has been called the φ meson[222–224].

In these papers the reactions studied were

$$K^- + p \to \Lambda + K^+ + K^-, \tag{131}$$

$$K^- + p \to \Lambda + K^0 + \overline{K^0}. \tag{132}$$

Altogether 46 events of type (131) and 52 events of type (132) were found. In the distribution with respect to the effective masses of the $K\bar{K}$ systems there exists a sharp resonance peak with $M(\bar{K}K) \approx 1019$ MeV and $\Gamma \leq 3-5$ MeV (Fig. 27 of [223]). The cross section for the production of the φ meson in these reactions is equal to $50 \pm 6\,\mu$b.

The width of the resonance peak of the φ meson was determined in the study of the annihilation of antiprotons stopped in a 30-inch hydrogen bubble chamber (cf., Ch. V, Sec. 1).[225] The reaction investigated was

$$p + \bar{p} \to K^+ + K^- + \pi^+ + \pi^-. \tag{133}$$

Only those events were selected in which both K mesons were stopped in the chamber. The error in the determination of $M(K^+K^-)$ was equal to ±0.6 MeV. Figure 28 gives a picture of the observed events with

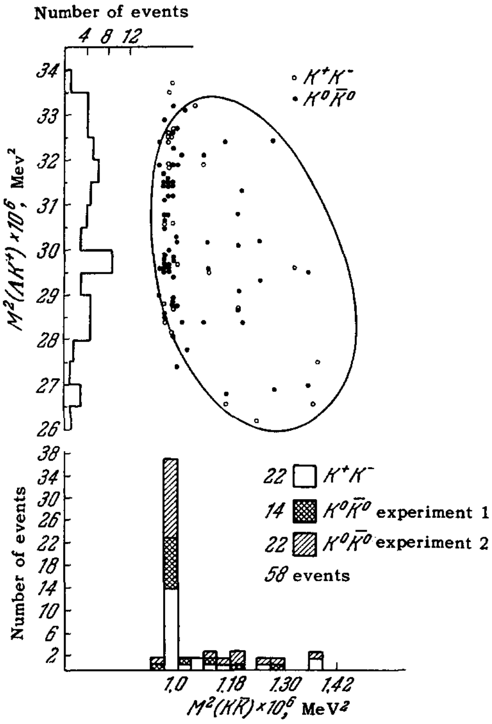

FIG. 27. Dalitz plot for the reaction $K^- p \to \Lambda K\bar{K}$.

respect to $M(K^+K^-)$ and the corresponding resolving and best agreement curves[225].* It was found that

$$M(\varphi) = 1018.6 \pm 0.5 \text{ MeV and } \Gamma = 3.1 \pm 1.0 \text{ MeV}. \quad (134)$$

An essential role in the determination of the quantum numbers of the φ meson was played by its neutral modes of decay:

$$\varphi \to K^0\overline{K^0}. \quad (135)$$

In particular, it was established that the φ meson decays in accordance with

$$\varphi \to K^0\overline{K^0} \to K_1^0K_2^0, \quad (136)$$

i.e., that its parity is negative, and the value of its spin is equal to an odd integer (cf. Ch. VIII, Sec. 1).

This fact was established in the study of the reaction

$$K^- + p \to \Lambda + \varphi \to \Lambda + K^0 + \overline{K^0}, \quad (137)$$

$$K^0\overline{K^0} \to K_1^0K_2^0, \quad (130')$$

where only the decays of the Λ and K_1^0 particles were recorded in the chamber. The corresponding kinematic program enabled one to separate out events of the type (137). In [223] there were found 23 such events, while not a single event of type (137) was observed with the decays

$$K^0\overline{K^0} \to K_1^0K_1^0, \quad (128')$$

*Some questions associated with the representation of experimental distributions in such a form are discussed in references[226-228].

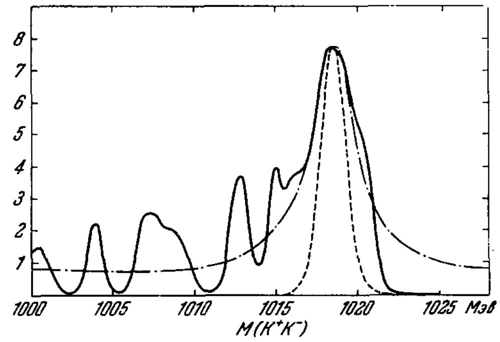

FIG. 28. Distribution with respect to the effective masses of K^+K^- systems produced in the reaction $p\bar{p} \to K^+K^- \pi^+\pi^-$ [225]. ——— experiment; – – resolution; – . – curve of best fit, $\Gamma = 3.1$

in spite of the high efficiency of recording K_1^0 decays. Appropriate calculations show that the hypothesis that the φ-meson decays in accordance with (128) and (129) yields results which differ by 12 standard deviations from the experimental results[223].

Some conclusions regarding the value of the spin of the φ-meson can also be drawn from the value of the ratio

$$\alpha(J) = \frac{W(\varphi \to K_1^0K_2^0)}{W(\varphi \to K_1^0K_2^0 + K^+K^-)}. \quad (138)$$

An essential dependence $\alpha(J)$ on the spin of the resonance appears in connection with the small release of energy in the decay $\varphi \to K\bar{K}$ (Q= 20—30 MeV). In this case the difference in the masses of the $K^\pm$ and K^0 mesons and the Coulomb interaction strongly affect the value of $\alpha(J)$ in the presence of centrifugal barriers ($J \neq 0$).*

Appropriate calculations[223,229,230] show that

$$\alpha(J=1) = 0.39 \text{ and } \alpha(J=3) = 0.26. \quad (139)$$

In [223] the value $\alpha = 0.45 \pm 0.10$ was obtained, i.e., the value $J(\varphi) = 1$ is favored.

The angular distributions of the decay K-mesons ($\varphi \to K\bar{K}$) also agree better with $J(\varphi) = 1$[222]. However, the statistics of the events is insufficient for final conclusions.

A search for the decays[223]

$$\varphi \to \pi^+ + \pi^- \quad (140)$$

has shown that

$$\frac{W(\varphi \to 2\pi)}{W(\varphi \to K\bar{K})} \leqslant 0.2. \quad (141)$$

Theoretical estimates of the value of this ratio obtained primarily from a comparison of the volumes in phase space for the corresponding decays of the φ meson give values ~ 100[229]. Such a strong suppression of the decay (140) can be explained by the difference in the G-parity of the φ meson and the system

*If these effects are neglected then the value of $\alpha(J)$ does not depend on the spin of the resonance and is equal to 0.5 in virtue of isotopic invariance.

composed of two π mesons (we recall that $G(\pi\pi) = +1$). Consequently, $G(\varphi) = -1$.

From this it follows that since the G-parity of the $K\bar{K}$ system is determined by the expression*

$$G(K\bar{K}) = (-1)^{l+I}, \tag{142}$$

and the spin of the φ meson is equal to an odd integer, then $I(\varphi) = 0$. Thus, the φ meson has the following quantum numbers: $I(J^{PG}) = 0\,(1^{--})$, i.e., the same ones as for the ω meson. In this connection a "mixing" of the φ and ω mesons is possible as a result of strong interactions (cf. also Ch. V, Sec. 5)[231-232].

The results of the investigations of the production of φ mesons in πp collisions and of the search for the decay

$$\varphi \to \varrho + \pi \tag{143}$$

have turned out to be unexpected. It was found[233,223,41] that

$$\frac{\sigma(\pi^- p \to \pi^- p\varphi)}{\sigma(\pi^- p \to \pi^- p\omega)} \leqslant 0.012 \tag{144}$$

at pc = 3.7 GeV and

$$\frac{W(\varphi \to \varrho\pi)}{W(\varphi \to K\bar{K})} \approx 0.35 \pm 0.2. \tag{145}$$

Theoretical estimates of the value of the ratio (145) obtained from considerations associated with volumes in phase space and with a range of interaction $r \approx 1/2m_\pi$, yield values greater by an order of magnitude than (145). On the other hand, the small value of the cross section for the production of φ mesons in πp collisions, compared with the cross section for the production of ω mesons which have the same quantum numbers, seems very strange.

Both these experimental results can be explained if one assumes that

$$\frac{g^2_{\rho\varphi\pi}}{g^2_{\rho\omega\pi}} \leqslant (1-5)\% \tag{146}$$

(here g are the corresponding coupling constants) [233,230]. The weak coupling of the φ meson to the ρ and π mesons can be explained to a certain extent within the framework of the unitary symmetry model (nonconservation of unitary parity in the decays (143)[234]).

3. $K\bar{K}$ Interaction at Low Energies. $K\bar{K}\pi$ Resonance

A study of the interaction of K mesons produced in π-collisions has demonstrated the existence of a broad maximum of resonance type with $M(K\bar{K}) \approx 1.02$ GeV and $\Gamma \approx 100$ MeV[235-239]. In this case a study was made of the reaction

$$\pi^- + p \to K^0 + \bar{K}^0 + n + m\pi \qquad (m = 0, 1, 2, \ldots) \tag{147}$$

with the subsequent decays

$$K^0\bar{K}^0 \to K_1^0 K_1^0. \tag{128'}$$

Thus, the system $K^0\bar{K}^0$ has an even orbital angular momentum and positive parity (Ch. VIII, Sec. 1). The angular distribution of the K_1^0 mesons has an anisotropic character, i.e., $J(K^0\bar{K}^0) \geq 2$[235]. However, the statistics of the obtained events (147) is insufficient for a final conclusion (37 events). Other possible quantum numbers of the $K^0\bar{K}^0$ resonances are discussed in [240-243].

The anomaly in the distribution $M(K^0\bar{K}^0)$ can also be explained by the interaction of the K mesons in the final state. In this case the range for $K^0\bar{K}^0$ scattering does not have to be very great, $\sim 1/m_\pi$[104].

A further investigation of this phenomenon is of considerable interest.

Data on a possible resonance in the $K\bar{K}\pi$ system were obtained in study of the reaction[244]

$$p + \bar{p} \to K^0 + K^\mp + \pi^\pm + \pi^+ + \pi^-. \tag{148}$$

It has turned out that $M_{res}(K^0K^\pm\pi^\mp) \approx 1410$ MeV and $\Gamma \sim 60$ MeV. In [244,245] an indication was obtained of the existence of a resonance in $K_1^0K^\pm$ systems with $M \approx 1.02$ GeV.

New experiments are needed in order to elucidate the nature of these anomalies[246].

IX. THE K* MESON

The existence of the K* meson was first discovered in the study of the K^-p interaction in the reaction[247]

$$K^- + p \to \bar{K^0} + \pi^- + p. \tag{149}$$

By now the K* meson has been observed in the processes of interaction of $K^\pm$ mesons[126,247-257] and of π^- mesons[258-267] with nucleons, and also in annihilation processes[268].

A characteristic distribution of the effective masses of the $\bar{K}^0\pi^-$ system in reaction (149) is shown in Fig. 29 ($M(K^*) = 890.4$ MeV and $\Gamma = 47$ MeV). The cross section for the production of K* mesons in this reaction at pc = 1.15 GeV is equal to 1.3 ± 0.3 mb.

The isotopic spin of the K* meson was determined

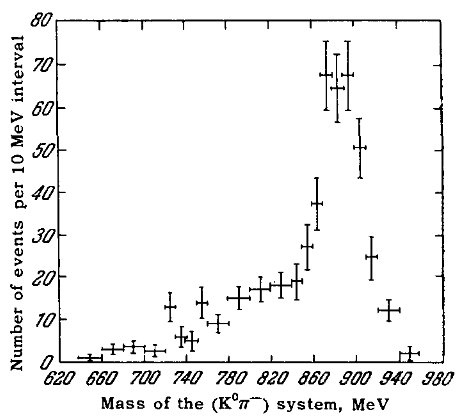

FIG. 29. Distribution of effective masses of $K^0\pi^-$ systems produced in the reaction (149).

*Indeed, since $C(K\bar{K})=(-1)^l$, then $G(K\bar{K})=C(-1)^I=(-1)^{l+I}$ (cf. Ch. VIII, Sec. 1).

in the simultaneous study of processes (149) and

$$K^- + p \rightarrow K^{*-} + p \rightarrow K^- + \pi^0 + p. \qquad (150)$$

It was found[247, 248] that

$$\frac{W(K^{*-} \rightarrow K^-\pi^0)}{W(K^{*-} \rightarrow \bar{K}^0\pi^-)} = 0.75 \pm 0.35. \qquad (151)$$

From the isotopic invariance it follows that the value of this ratio for $I(K^*) = 1/2$ is equal to 0.5, while for $I(K^*) = 3/2$ it is equal to 2. From this one can assume that $I(K^*) = 1/2$. This conclusion is also supported by a special investigation of the possible resonance states of the $\bar{K}\pi^-$ system with $I = 3/2$ at pc = 1.51 GeV in the reaction[250]

$$K^- + n \rightarrow K^- + \pi^- + p. \qquad (152)$$

It has been shown that a resonance in the $K^-\pi^-$ system in the mass range 0.6—1.0 GeV is not produced with a cross section greater than 0.01 mb. At the same time and at the same momenta of the primary K^- mesons the cross section for the production of a K^* meson in reaction (149) is equal to 1.6 mb[251]. A similar result was also obtained in the study of the $K^0\pi^-$ interaction in πN collisions[158].

The first conclusions with respect to the possible value of the spin of the K^* meson were obtained in [247]. In this case a study was made of the production of new mesons near the threshold of the reaction

$$K^- + p \rightarrow K^{*-} + p. \qquad (153)$$

Experiments were carried out at a K^--meson momentum of 1150 MeV/c; the threshold momentum is 1080 MeV/c. The angular distribution of the K^* mesons in a K^-p system is of isotropic character. In this connection it is possible to assume that the K^* mesons are born in an S-state and the analysis according to Adair can be carried out for arbitrary angles of emission of the meson[269-270]. If $J(K^*) > 1$, then the angular distribution of the decay mesons in the rest system of the K^* meson must be anisotropic. For $J(K^*) = 0$ or 1 isotropic distributions are also possible. It was shown that the values $J(K^*) > 1$ disagree with experimental data.

A study of the reaction

$$K^{*+} + p \rightarrow K^{*0} + N_{33}^{*++} \rightarrow K^{*+} + \pi^- + p + \pi^+ \qquad (154)$$

has made it possible to obtain additional information on the spin of the K^* meson[255-257]. The reaction (154) was studied with the aid of a hydrogen bubble chamber irradiated by a beam of K^+ mesons with pc = 1.96 GeV.

The cross section for this process turned out to be equal to 1.1 ± 0.2 mb[257]. Figure 30 shows distributions with respect to the effective masses of $K^+\pi^-$ and $p\pi^+$ systems.

For an analysis utilizing Adair's method 69 events were selected in which the cosine of the angle of emission of the K^* meson in the (K^+P) c.m.s. was in the range

$$1.0 \geqslant \cos\vartheta_{K^*} \geqslant 0.8. \qquad (155)$$

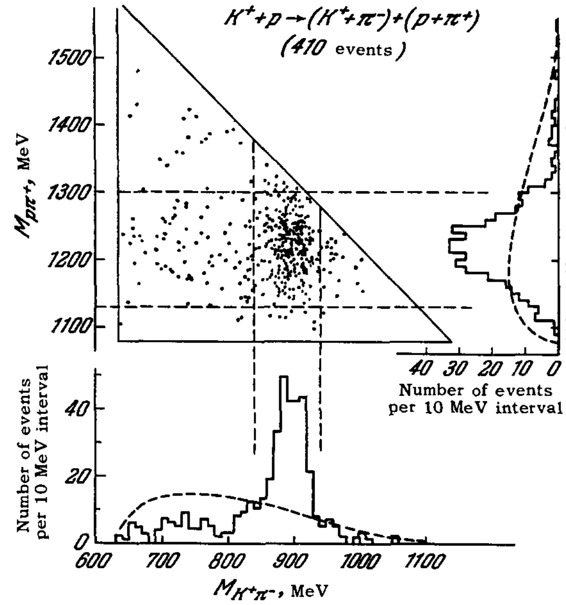

FIG. 30. Dalitz plot for the reaction $K^+p \rightarrow (K^+\pi^-) + (p\pi^+)$.

Figure 31 gives the obtained angular distributions of the K^+ mesons. The sharp anisotropy of this distribution immediately excludes the value $J(K^*) = 0$. Comparing this result with the one obtained above ($J(K^*)' \leq 1$) we conclude that $J(K^*) = 1$.

On the other hand, conclusions regarding the value of the spin of the K^* meson can also be obtained from an analysis of the angular distributions of the K^+ mesons. In Table VI the corresponding angular distributions are given for the decays $K^*(I_{K^*}(\alpha))$ and N_{33}^{*++} $(I_{N_{33}^{*++}}(\beta))$ for the allowed components of spin of these particles along the direction of the primary beam of the K^+ mesons.*

The distributions obtained agree well with $I(\alpha) = \cos^2\alpha$ for the decays of K^* mesons and do not contradict $I(\beta) = 1 + 3\cos^2\beta$ for the decays of the N_{33}^{*++}-isobar. This circumstance is an additional argument in favor of $J(K^*) = 1$.

An investigation of the reaction[267]

$$\pi^- + p \rightarrow \Sigma^0 + K^{*0} \qquad (157)$$

for momenta of incident π^- mesons equal to 2.17 and 2.25 GeV/c has also shown the existence of a significant anisotropy in the distribution of the decay K mesons with respect to the normal to the plane of production of the Σ hyperon and the K^* meson. This fact excludes the value $J(K^*) = 0$.

*The law of conservation of the component of the total angular momentum of colliding particles in this case has the form

$$m_J(K^*) + m_J(N_{33}^{*++}) = \pm 1/2,$$

since the component of the orbital angular momentum along the direction of the primary K^+ meson is equal to zero.

Table VI

$m_J(K^*)$	$m_J(N_{33}^{*++})$	$I_{K^*}(\alpha)$	$I_{N_{33}^{*++}}(\beta)$
+1	$-1/2$	$1/2\sin^2\alpha$	$1+3\cos^2\beta$
0	$-1/2$	$\cos^2\alpha$	$1+3\cos^2\beta$
-1	$+3/2$	$1/2\sin^2\alpha$	$3\sin^2\beta$

A very interesting method of determining the spin of the K* meson was proposed in [271]. It is based on the properties of pairs of K^0 and $\bar{K}^0$ mesons produced in the reaction

$$p+\bar{p}\to K^0+\bar{K}^0+\pi^0. \tag{158}$$

Experimental data on the annihilation of antiprotons stopped in hydrogen show that it occurs in the S-state [272]. In this case the processes

$$p+\bar{p}\to K^0+\overline{K^{*0}}, \tag{159}$$

$$p+\bar{p}\to \bar{K}^0+K^{*0} \tag{160}$$

with the subsequent decays

$$K^0\overline{K^{*0}} \text{ or } \bar{K}^0K^{*0}\to K_1^0K_2^0\pi^0 \tag{161}$$

are forbidden if J(K*) = 0, i.e., again, as in the case of the φ meson, the type of decay of the system is a detector of its spin and parity.

An experimental investigation of this process was carried out with the aid of an 81-cm hydrogen bubble chamber [269]. The reaction studied was

$$p+\bar{p}\to K_1^0+\text{neutral particles}. \tag{162}$$

The momentum distribution of the K_1^0-mesons is shown in Fig. 32. The maximum occurring in the distribution at pc = 610 MeV corresponds to reactions (159) and (160). An analysis of events associated with the decays of the KK* system along the channel (161) has shown that the number of such cases is equal to

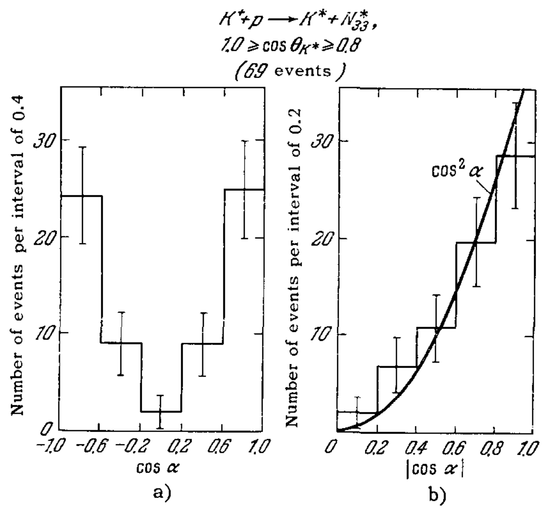

FIG. 31. Angular distributions of K^+ mesons produced in the decays $K^{*0}\to K^+\pi^-$ (in the rest system of the K* meson).

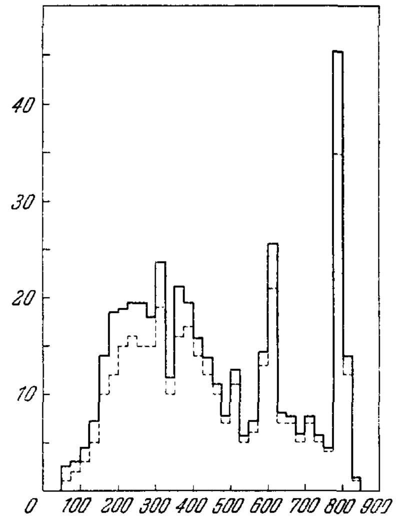

FIG. 32. Momentum distribution of K_1^0 mesons in the reaction $p\bar{p}\to K_1^0+$ neutral particles. ——— corrected for the probability of decay – – measured distribution.

36.5 ± 15. Thus, in this case also there is an indication that J(K*) = 1.

Thus, the totality of data on the spin of the K* meson provides grounds for assuming that J(K*) ≠ 0 and that most probably J(K*) = 1. This means that the Kπ system is in a P-state and the K* meson has positive parity with respect to the K-meson. Therefore, the parity of the K* meson with respect to the Λ-particle will be the same as in the case of the K meson, i.e., negative (cf., Table I).

We also note that other possible experiments on the determination of J(K*) [273-278]. For example, if J(K*) = 0, then the reactions

$$K+\mathrm{He}_2^4\to \mathrm{He}_2^4+K^*, \tag{163}$$

$$K^*\to K+\gamma, \tag{164}$$

$$K^*\to K+\pi+\pi \tag{165}$$

are forbidden. However, in the case of the vector meson the reactions (164) and (165) are of low probability. Theoretical estimates have shown [277-278] that

$$\frac{W(K^*\to K\gamma)}{W(K^*\to K\pi)}\approx 0.8-0.15\% \tag{166}$$

and

$$\frac{W(K^*\to K\pi\pi)}{W(K^*\to K\pi)}\approx 0.3-3\%. \tag{167}$$

In [279-285] dynamic models of the K* meson and the role it plays in the processes of interaction of elementary particles are discussed.

X. κ MESON (K*(725))

The extensive experimental material on the interaction of elementary particles obtained up to the present time enables one to begin a study of those resonances the cross section for the production of which amounts to several microbarns. The κ meson ($K^*(725)$) belongs to just such resonances.

The first results on the κ meson were obtained in the investigation of the reactions[258,259,266]

$$\pi^- + p \rightarrow \Sigma^- + \pi^0 + K^+, \qquad (168)$$

$$\pi^- + p \rightarrow \Sigma^- + \pi^+ + K^0 \qquad (169)$$

in the momentum range from 1.51 to 2.36 GeV/c. In the distribution with respect to $M(K^+\pi^0)$ and $M(K^0\pi^+)$ there are resonance peaks with $M \approx 8.85$ MeV (K^* meson) and $M \approx 725$ MeV (κ meson, Fig. 33). The width of the κ peak is $\Gamma \leq 20$ MeV. The authors estimate the probability of the appearance of the peak at $M \approx 725$ MeV as a result of statistical fluctuations as being $\sim 0.2\%$. The cross section for the production of the κ meson amounts to $\sim 4\,\mu$b, i.e., almost by an order of magnitude lower than the cross section for the production of the K^* meson in the same reaction ($\sigma \approx 30\,\mu$b).

Some indirect data on the isotopic spin of the κ meson were obtained in the study of the process[266]

$$\pi^- + p \rightarrow \Sigma^+ + \pi^- + K^0. \qquad (170)$$

In this case no resonance peaks are observed in the region of the κ meson ($I(\pi^-K^0) = 3/2$). An analogous conclusion is also obtained in [250]. Therefore, one can suppose that $I(\kappa) = 1/2$.

An indication of the possible existence of the κ meson was also obtained in the study of the reaction

$$K^- + p \rightarrow \overline{K^0} + \pi^- + p \qquad (171)$$

in the momentum range for K^- mesons from 1.0 to 1.7 GeV/c[249]. The cross section for the production of κ mesons in this reaction is $\sim 40\,\mu$b, while the cross section for the production of the K^* meson is

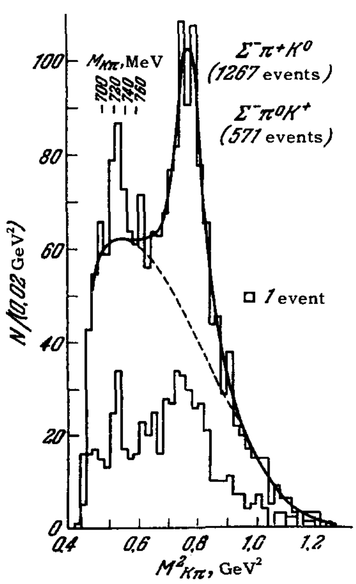

FIG. 33. Distribution with respect to $M^2(K\pi)$ obtained in the study of the reactions (168) and (169).

$\sim 1000\,\mu$b. It was found that $M(\kappa) = 723 \pm 3$ MeV and $\Gamma \leq 12$ MeV. Due to the poor statistics (~ 30 κ mesons) one cannot draw any definite conclusions about the other quantum numbers of this particle (cf., Appendix and [286-289]).

CONCLUSION

The discovery of a large group of new particles-resonances shows that apparently it makes no more sense to regard all particles as elementary than it does to regard atomic nuclei as elementary. At the present time there exist several attempts of establishing regularities (symmetries) in the properties of the particles enabling one to unify a large number of different particles into a small number of groups (multiplets).

The most popular models are those of Sakata, Gell-Mann and Ne'eman[290-293]. For example, in the models of Gell-Mann and Ne'eman[292-291] the bosons and hyperons (N, Λ, Σ, Ξ) are combined into multiplets of 8 particles, i.e., 8 mesons with $J^P = 0^-$ (π, K, η), 8 mesons with $J^P = 1^-$ (ρ, K^*, φ or ω) etc.

All the particles belonging to the same multiplet have the same values of spin and of parity. The formula for the masses of the particles obtained for this model gives a good description of the experimental data[293].

We also note the important role played by vector mesons. Their existence on the basis of the generalized gauge invariance was discussed by Sakurai[294] et al. On the other hand, Ogievetskiĭ and Polubarinov have shown that if vector mesons have a spin equal to unity in an interaction, then in this case isotopic invariance and the conservation of baryon and hyperon charges in strong interactions[295] follows from their existence. In this case the vector mesons must have negative parity. As can be seen from Table I all the vector mesons indeed do have $P = -1$.

A review of the basic phenomena associated with resonances shows that they touch upon such fundamental problems as the problem of the elementary nature of particles, problems of the theory of interactions etc. Therefore, a further detailed study of the properties of the known resonances and a search for new ones are of great interest.

APPENDIX

New Data

In this section are presented the main results of the investigations of boson resonances which were published in physics journals up to April 1965 or reported at the XII International Conference on High Energy Physics (Dubna, 1964).

These results refer primarily to new boson resonances discovered recently. As regards resonances which have been discussed in detail in the present review, in this case two additions can be made concerning f^0 and κ mesons.

I. The f^0 Meson

An investigation of the angular characteristics of the decays

$$f^0 \to \pi^+ + \pi^- \tag{172}$$

has shown that the assumption that $J(f^0) = 2$ agrees well with the experimental data, while the assumption that $J(f^0) = 0$ or 1 is in disagreement with them[296]. In [297-298] the decay

$$f^0 \to \pi^0 + \pi^0 \tag{173}$$

was discovered.

The value of the ratio[299,298]

$$\frac{f \to \pi^0 + \pi^0}{f \to \pi^+ + \pi^-} \approx 0.5, \tag{174}$$

is what is expected in the case $I(f^0) = 0$. Thus, the totality of new data on the f^0 meson confirms the conclusions with respect to its quantum numbers made earlier (cf. Table I).

II. The κ Meson

In [300] a study was made of the reaction

$$K^+ + p \to p + K^{\overset{0}{+}} + \pi^+ + \pi^- + \pi^{\overset{+}{0}} \tag{175}$$

In the distribution with respect to the effective masses of $K\pi$ systems there are well defined resonance pions at $M \approx 888$ MeV (K^* meson) and $M \approx 723$ MeV (κ meson). Thus, the existence of the κ meson can be taken as proven. In the same reference it was found that

$$\frac{K^* \to \varkappa + \pi}{K \to K + \pi} \leqslant 0.01. \tag{176}$$

III. New Mesons

1. $\eta 2\pi$ resonance. In the study of the reactions[301,302] of the type

$$K^- + p \to \Lambda + X^0 \tag{177}$$

in the momentum range from 2.3 to 2.7 GeV/c a new meson with $M = 975.5$ was found which decays in accordance with the channels

$$X^0 \to \eta + \pi^+ + \pi^-, \tag{178}$$

$$X^0 \to \pi^- + \pi^+ + \gamma. \tag{179}$$

An analysis of the experimental data has shown that the most probable quantum numbers of the $\eta 2\pi$ meson are $0(0^{-+})$, i.e., it is a "heavy" analog of the η meson.

2. A_1 and A_2 mesons. In [303-304] A_1 and A_2 resonances were observed. In this case processes of the type

$$\pi^+ + p \to p + \pi^+ + \pi^+ + \pi^- \tag{180}$$

where investigated.

In the distribution with respect to $M(\pi^+\pi^+\pi^-)$ when the combination of π^- and one of the π^+ mesons gave $M(\pi^+\pi^-) = M(\rho^0)$, resonance pions with $M(A_1) = 1080$ MeV and $M(A_2) = 1310$ MeV were discovered.

Thus, the principal mode of decay of the A-mesons is the decay

$$A_{1,2} \to \rho + \pi. \tag{181}$$

For the preferred quantum numbers of the A_2 meson given in work reported at the XII Conference on High Energy Physics refer to Table I.

3. $K\pi\pi$ resonances. A study of the distributions with respect to the effective masses of the $K\pi\pi$ systems produced in πN and $\bar{p}p$ collisions has demonstrated the existence of peaks of resonance type with $M = 1175$ MeV, 1215 MeV, and 1270 MeV[305]. Also a resonance peak was observed in the $K\pi\pi\pi$ system with $M = 1630$ MeV[305]. However, in all these cases a further study of the nature of these anomalies is necessary.

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