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ELECTRON CAPTURE AND LOSS BY FAST IONS IN ATOMIC COLLISIONS

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THE capture or loss of an electron by a fast ion upon colliding with an atom is one of the most typical phenomena accompanying the passage of fast atomic particles through matter. Interest in these phenomena in the range of ion velocities $v > v_0 = e^2/\hbar$ arises mainly in connection with problems of obtaining fast multiply-charged ions in accelerators and with the topic of the deceleration of these ions in matter. However, owing to experimental difficulties and complexity of the theoretical calculations, the effective cross-sections for loss or capture of electrons in the high-velocity range had until recently been studied mainly for the atoms and ions of hydrogen and helium.[1] For the heavier ions, information existed mainly on the charge composition of certain ion beams after they had passed through a rather thick layer of matter.[2-15] That is to say, we had information on the equilibrium charge distribution established in the ion beam through multiple variations of the charges of the ions. The experimental data on the cross-sections for stripping and charge-exchange of these ions, as one sometimes refers to the processes of loss and capture of electrons by ions, were few in number and disconnected,[3,4,6,7,16-20] while the theoretical calculations [16,21-24] were essentially estimates.

In recent years, mainly owing to the studies of Soviet physicists, extensive experimental material has been obtained on the cross-sections for loss or capture of electrons for the ions of almost all the light elements up to argon, inclusive.[25-33] The information on the equilibrium charge distribution in ion beams[26,31,34-38] has been rounded out, and some theoretical calculations have been made.[39-40] The results of the studies on the stripping and charge-exchange of fast positive ions and atoms in the velocity range $v > v_0 = 2.19 \times 10^8$ cm/sec are systematized and generalized below.

I. MATHEMATICAL INTRODUCTION

1.1. The Fundamental Relations Governing the Charge Composition of an Ion Beam During Passage Through Matter.

The variation of the charge composition of an ion beam while it passes through matter is described by a system of differential equations

$$\frac{d\Phi_k}{dt} = \sum_j \Phi_j \sigma_{jk}, \tag{1.1}$$

where Φ_k is the relative amount of ions of charge k in the ion beam ($\sum_k \Phi_k = 1$), t is the number of atoms of the material in a volume of cross-section 1 cm^2 in the path of the ions, σ_{jk} (where $j \neq k$) is the cross-section of the process whereby an ion of charge j is transformed into an ion of charge k, and $\sigma_{kk} = -\sum_j{}' \sigma_{kj}$ (where the prime on the summation indicates that values j = k are omitted from the summation).

As is shown by experiment, even before the collisions with the atoms of the material have appreciably altered the velocity of the ions, the charge on the particles has undergone repeated change and an equilibrium charge distribution has been established in the ion beam. The latter is independent of the charges on the ions before they enter the material, and is fully determined by the relation between the effective cross-sections for loss and capture of electrons. When an equilibrium state is attained, $d\Phi_k/dt = 0$, so that instead of (1.1) we have

$$\sum_j F_j \sigma_{jk} = 0, \tag{1.2}$$

Here F_j is the relative amount of ions of charge j in the equilibrium distribution, which is equal to the limiting value of Φ_j from (1.1) as $t \to \infty$. Equation (1.2) implies that

$$\sum_{\substack{j \leq i \\ k > i}} F_j \sigma_{jk} = \sum_{\substack{j > i \\ k \leq i}} F_j \sigma_{jk}, \tag{1.3}$$

that is, the decrease in the number of ions having charges $j \leq i$ arising from loss of electrons is equal to the increase in the number of these ions owing to capture of electrons by ions having charges $j > i$. In cases in which we can neglect losses or captures of two or more electrons per collision, Eq. (1.3) is considerably simplified:

$$F_i \sigma_{i,\,i+1} = F_{i+1} \sigma_{i+1,\,i}. \tag{1.4}$$

By solving the system of equations (1.2) or using the approximate relations (1.4), we can determine the values of F_i from the known cross-sections for loss or capture of electrons for the ions of various charges. Thus, we can determine the mean charge $\bar{i} = \sum_i i F_i$ on the ions.

1.2. The Equations Governing the Charge Composition of an Ion Beam in Condensed Media

While Eqs. (1.1) and (1.2) are valid for the passage of ions through any medium, they determine the charge composition of an ion beam only when the quantities σ_{ik} contained in them are fully defined. That is, they refer to ions occurring in previously specified states. This condition is satisfied when the ions pass through rarefied gases. Then the ions collide with the atoms of the medium so seldom that the excited ions formed by the collisions drop to the ground state during the time between collisions. However, when ions pass through solids or liquids, they do not change state during the short time between collisions. Consequently, the quantities σ_{ik} occurring in Eqs. (1.1)–(1.4) are mean values of the cross-sections for ions occurring in various states, depending on the distribution of the ions over these states. In order to describe the variation in the charge composition of an ion beam in a condensed medium, we need equations permitting us to determine the distribution of the electrons contained within an ion over the various states.

Let us limit ourselves in a first approximation to considering ions having only two charges i and i + 1, and assume that excited ions are formed only by electron capture. Then the variation in the total number of electrons $N_i(\nu)$ occurring in the νth state of ions of charge i and the variation in the analogous quantity $N_{i+1}(\nu)$ for ions of charge i + 1, will be described by the following equations:

$$\left.\begin{aligned}\frac{dN_i(\nu)}{dt} &= N_{i+1}(\nu)\,\sigma_{i+1,\,i} + \Phi_{i+1}\sigma_{i+1,\,i}(\nu) - K_i(\nu) - N_i(\nu)\,\sigma_i(\nu),\\ \frac{dN_{i+1}(\nu)}{dt} &= -N_{i+1}(\nu)\,\sigma_{i+1,\,i} + K_i(\nu),\end{aligned}\right\} \tag{1.5}$$

where $\sigma_{i+1,i}(\nu)$ is the electron-capture cross-section for ions of charge i + 1 in the νth state; $\sigma_{i+1,i} = \sum\limits_{\nu}\sigma_{i+1,i}(\nu)$ is the total cross-section for electron-capture by ions of charge i + 1; $\sigma_i(\nu)$ is the cross-section for loss of a particular electron occurring in the νth state of an ion of charge i; and $K_i(\nu)$ is the increase in the value of $N_{i+1}(\nu)$ due to loss of an electron from ions of charge i. The quantities $N_i(\nu)$ are normalized here such that $\sum\limits_{\nu}N_i(\nu) = s_i\Phi_i$, where s_i is the number of electrons in the ion participating in ionization processes.

The quantity $K_i(\nu)$ depends on the cross-sections $\sigma_i(\nu)$ for loss of individual electrons and on the distribution of the electrons over the states in each ion. In general, the latter does not coincide with the distribution of electrons $N_i(\nu)/\Phi_i$ averaged over all the ions of charge i. If the distribution functions over the various states are identical for all the electrons of the ion, as is approximately true when the number s_i of electrons is small, then

$$K_i(\nu) = N_i(\nu)\,\overline{\sigma_{i,\,i+1}}\left(1-\frac{1}{s_i}\right), \tag{1.6}$$

where $\overline{\sigma_{i,i+1}} = \sum\limits_{\nu}N_i(\nu)\sigma_i(\nu)/\Phi_i$ is the mean cross section for loss of an electron by ions of charge i.

If we are interested in the distribution of electrons among several groups of states, then, when each of these groups contains a large number of electrons, the distribution of the electrons over these groups for the individual ions will differ little from the mean distribution. When they are equal,

$$K_i(\nu) = N_i(\nu)\,[\overline{\sigma_{i,\,i+1}} - \sigma_i(\nu)] = N_i(\nu)\,\overline{\sigma_{i,\,i+1}}\left[1-\frac{\sigma_i(\nu)}{s_i\bar{\sigma}_i}\right]. \tag{1.7}$$

If the number of electrons in the group of states under discussion is considerably greater than unity, then the values of $K_i(\nu)$ calculated by (1.6) and (1.7) will turn out to be approximately the same, so that we can almost always use Eq. (1.6).

After an ion beam has passed through a thick enough layer of matter, a definite (equilibrium) distribution of the electrons over the various states has been established in it, whereby $dN_i(\nu)/dt = 0$. Then for the most intense groups of ions having charges $i \approx \bar{i} - \frac{1}{2}$ and $i + 1 \approx \bar{i} + \frac{1}{2}$, we can neglect transitions to other charge states, and use Eqs. (1.5) to determine $N_i(\nu)$ and $N_{i+1}(\nu)$. These equations imply that

$$N_i(\nu) = F_{i+1}\frac{\sigma_{i+1,\,i}(\nu)}{\sigma_i(\nu)}. \tag{1.8}$$

For ions of charges $i < \bar{i} - 1$, which are formed principally from ions of charge i + 1, and vary their charge mainly through loss of an electron, $N_i(\nu)$ is basically determined from $N_{i+1}(\nu)$ by the first equation of the system (1.5). Thence, using (1.6), we derive

$$N_i(\nu) = N_{i+1}(\nu)\,\frac{\sigma_{i+1,\,i}}{\sigma_{i,\,i+1}}\left[1+\frac{\delta_i(\nu)\,N_{i+1}(\nu)}{F_{i+1}}\right] \times\left[1-\frac{1}{s_i}+\frac{\sigma_i(\nu)}{\sigma_{i,\,i+1}}\right]^{-1}, \tag{1.9}$$

where $\delta_i(\nu) = \sigma_{i+1,i}(\nu)/\sigma_{i+1,i}$ is the relative probability of capture of an electron into the νth state.

For ions having $i > \bar{i} + 1$, which are basically produced from ions of charge i − 1, and are then transformed primarily into ions of charge i − 1, the distribution $N_i(\nu)$ of the electrons over the various states depends mainly on $N_{i-1}(\nu)$, and is determined by the second equation of (1.5), whence, using (1.6), we have

$$N_i(\nu) = N_{i-1}(\nu)\,\frac{\overline{\sigma_{i-1,\,i}}}{\sigma_{i,\,i-1}}\left(1-\frac{1}{s_i}\right). \tag{1.10}$$

We can derive relations analogous to (1.4) from both the first and the second equations of (1.5) upon summing over ν with $dN_i(\nu)/dt = 0$:

$$\frac{F_i}{F_{i+1}} = \frac{\sigma_{i+1,\,i}}{\overline{\sigma_{i,\,i+1}}}. \tag{1.11}$$

In addition, for the values of i nearest to $\bar{i} - \frac{1}{2}$, we can derive the following relation from (1.8):

$$\frac{F_i}{F_{i+1}} = \frac{1}{s_i}\,\frac{\sum\limits_{\nu}\sigma_{i+1,\,i}(\nu)}{\sigma_i(\nu)}. \tag{1.12}$$

Thence, if we denote the values of $\delta_i(\nu)$ and $\sigma_i(\nu)$ referring to the lowest energy states as δ_i^0 and σ_i^0, we obtain $F_i/F_{i+1} > \sigma_{i+1,i}\delta_i^0/s_i\sigma_i^0$. In cases in which we can assume the electron-capture cross sections to be the same in condensed and rarefied media, we have

$$\delta_i^0 < \delta_{\max}^0 = \frac{1}{a_i}, \tag{1.13}$$

where

$$a_i = \frac{(F_{i+1}/F_i)_s}{\left(\sum_k k\sigma_{i,\,i+k}/\sigma_{i+1,\,i}\right)_g} = \frac{(\overline{\sigma_{i,\,i+1}})_s}{\left(\sum_k k\sigma_{i,\,i+k}\right)_g};$$

here the subscripts s and g indicate whether the quantities in parentheses refer to condensed (solid) or rarefied (gas) media, respectively. The quantity a_i is the enhancement of the cross section for electron loss in a condensed medium; thus, for ions having $i \approx \bar{i}_s - 1/2$, it also gives an upper limit for the relative probability of capture of electrons into the strongly bound states $\delta^0_{\max}$.

1.3. The Variation in the Equilibrium Charge Distribution in an Ion Beam with Increasing Density of the Medium

In order to describe the variations that take place in the equilibrium charge composition of ion beams as the density of the gaseous medium is increased, we can use Eqs. (1.5) with additional terms to take into account the possibility of spontaneous radiative transition of electrons from the excited state to the ground state. The first equation of (1.5), in which they refer to excited states, is supplemented by the term $-N_i(\nu)/vn\tau_i(\nu)$, and the second equation by the term $-N_{i+1}(\nu)/vn\tau_{i+1}(\nu)$, where v is the ion velocity, n is the number of atoms of the medium per cm^3, and $\tau_i(\nu)$ is the mean lifetime of an electron in the νth state. When $dN_i(\nu)/dt = 0$, we can derive from these equations some expressions for the probability of finding an electron in the νth state:

$$\alpha_i(\nu, n) = \frac{N_i(\nu)}{\sum\limits_{\nu} N_i(\nu)}.$$

In particular, for ions of charge $i \approx \bar{i} - 1/2$, using (1.6), we obtain

$$\alpha_i(\nu, n) = \delta_i(\nu)\, a_i(n)\, [1 + \gamma_i(\nu)]^{-1} B_i(\nu) \tag{1.14}$$

when

$$B_i(\nu) = \left\{1 + \frac{1}{\beta_i(\nu)} + \left(1 - \frac{1}{s_i}\right)\overline{\sigma_{i,\,i+1}}\,[\sigma_i(\nu) + \sigma_{i+1,\,i}\beta_{i+1}(\nu)]^{-1}\right\}^{-1},$$

where

$$\gamma_i(\nu) = \frac{\sigma_i(\nu) - \sigma_i^0}{\sigma_i^0}$$

is the relative increase in the cross section for loss of an electron from the νth state over that for loss of an electron from the ground state, and $\beta_i(\nu) = vn\tau_i(\nu)\sigma_i(\nu)$.

When the quantity $a_i(n)$ equals the ratio of the mean cross section for electron loss in a medium having n atoms/cm^3 to the electron-loss cross section in a rarefied gas, we can derive the following relation from the definition of the mean cross section for electron loss:

$$a_i(n) = 1 + \sum_{\nu} \alpha_i(\nu, n)\,\gamma_i(\nu). \tag{1.15}$$

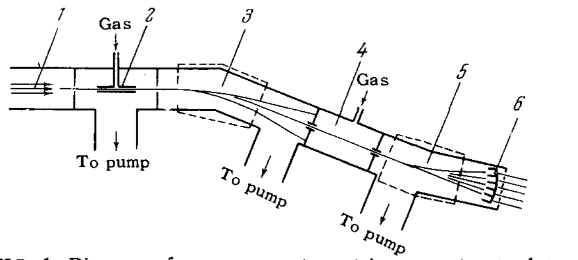

FIG. 1. Diagram of a mass-spectrometric apparatus to determine electron loss and capture cross sections of fast ions. 1 – Ion beam from accelerator; 2 – charge converter; 3 – mass monochromator; 4 – collision chamber; 5 – analyzer; 6 – detectors.

Equations (1.14) and (1.15) imply that

$$a_i(n) = \left\{1 - \sum_{\nu} \delta_i(\nu) B_i(\nu)\left[1 + \frac{1}{\gamma_i(\nu)}\right]^{-1}\right\}^{-1}. \tag{1.16}$$

Hence we see that with increase in n the ratio F_{i+1}/F_i for the two most intense charge groups in an ion beam of equilibrium composition varies mainly within the density range of the medium such that $n \sim 1/v\tau_i^*\sigma_i^*$, where τ_i^* and σ_i^* are the mean lifetime and cross section for electron loss for the most densely populated group of excited states. An analogous result can also be derived for other values of i.

II. EXPERIMENTAL METHOD

2.1. The Mass-spectrometer Method of Determining Effective Cross Sections

Most of the current experimental information on the cross sections for electron loss or capture by fast ions has been obtained by the mass-spectrometer method.[41-45,27] A basic diagram of a suitable mass-spectrometer apparatus is shown in Fig. 1. The fundamental elements of this apparatus are: 1) an accelerator producing an ion beam homogeneous in composition and energy; 2) a charge converter, where ions of differing charges are produced in the beam upon passing through it; 3) a mass monochromator, which isolates particles of a definite charge from the ion beam; 4) a collision chamber, in which the fast ions lose or capture electrons upon colliding with the atoms of a gas; 5) an analyzer for the spatial separation of the fast particles leaving the collision chamber into separate charge groups; and 6) a detection system permitting determination of the relative intensities of the different charge groups.

In the studies of the Moscow group of physicists,[27-30] the fast-particle source was a 72-cm cyclotron with a system for focusing and isolating a monoenergetic ion beam.[46] In the studies of Pivovar and his associates [31-32] the source was a 1.5 MeV electrostatic generator.[47] In the first group of studies, they used a thin celluloid film (of thickness $\sim 2\mu g/cm^2$) as the ion charge converter, while in [32] they used a flowing gas target in the form of a tube of diameter 4 mm and length 40 mm. The mass monochromator was a magnetic analyzer deflecting the ions by 10—15°. The collision chamber was a cylinder several tens of centimeters long, with openings in both ends for passage of the beam.

The particles emerging from the collision chamber were separated into their individual charge components in [27–30] by magnetic analyzer, while in [31–32] they were separated by an electrostatic analyzer. In the first group of studies, they used a system of eight proportional counters to detect the ions. This permitted them to detect simultaneously ions of practically all possible charges, and thus, to determine the relative amount Φ_{ik} of the ions of a given charge k (the subscript i indicates the charge on the ions before passing through the collision chamber). In the experiments of the Khar'kov group,[31–32] the intensity of the flux of charged particles was measured by vacuum-tube electrometers, and the neutral particles by a thermocouple detector. They found from the results of these measurements the ratio of the number of particles of charge k to the number of ions having the original charge i, i.e., the quantity Φ_{ik}/Φ_{ii}.

Since the experimental apparatus always contains residual gas, one uses the appropriate solutions of Eqs. (1.1) generalized to the case of passage of ions through a mixture of gases in calculating the cross sections σ_{ik} from the experimental values of Φ_{ik} or Φ_{ik}/Φ_{ii}. When the pressures of the residual and the admitted gases are small enough, one can calculate the cross sections by the simpler formulas

$$\Phi_{ik}-\Phi'_{ik}=\sigma_{ik}t \tag{2.1}$$

and

$$\frac{\Phi_{ik}}{\Phi_{ii}}-\frac{\Phi'_{ik}}{\Phi'_{ii}}=\sigma_{ik}t, \tag{2.2}$$

where Φ'_{ik} is the relative amount of ions of charge k in the beam of particles of original charge i after it has passed through the residual gas.

Let us consider the broader range of gas pressures limited by the condition that $0.6 \leq \Phi_{ii} \leq 1$ for any initial ion charge. Here, taking into account the possibility that the ions can alter their charge in the regions on the beam path before and after the collision chamber, the values of Φ_{ik} are expressed with an accuracy up to 1—2% as follows:[27]

$$\Phi_{ik}=\delta_{ik}+g_{ik}+\frac{1}{2}\sum_p(g_{ip}g_{pk}+\gamma g'_{ip}g_{pk}-\gamma g_{ip}g'_{pk})$$
$$+\frac{1}{6}\sum_{pq}(g_{ip}g_{pq}g_{qk}+2\gamma g'_{ip}g_{pq}g_{qk}-\gamma g_{ip}g'_{pq}g_{qk}-\gamma g_{ip}g_{pq}g'_{qk}). \tag{2.3}$$

Here $\delta_{ik}=0$ when $i \neq k$, and $\delta_{ii}=1$, $g_{js}=\sigma_{js}t+\sigma'_{js}t'$, $g'_{js}=\sigma'_{js}t'$, $\gamma=(\beta'_1-\beta'_2)-(\beta_1-\beta_2)$, and β_1 and β_2 are the relative amounts of gas molecules occurring on the path of the ions before and after the collision chamber, respectively. The quantities σ, t, and β refer to the admitted gas, and the quantities σ', t', and β' to the residual gas.

Hence as $t \to 0$, we have

$$\Phi_{ik}-\Phi'_{ik}=\left\{\sigma_{ik}+\frac{1}{2}\sum_p[(1+\gamma)g'_{ip}\sigma_{pk}+(1-\gamma)\sigma_{ip}g'_{pk}]\right\}t \tag{2.4}$$

and

$$\frac{\Phi_{ik}}{\Phi_{ii}}-\frac{\Phi'_{ik}}{\Phi'_{ii}}=a_{ik}t, \tag{2.5}$$

where

$$a_{ik}=\sigma_{ik}+\frac{1}{2}\sum_p[(1+\gamma)g'_{ip}\sigma_{pk}+(1-\gamma)\sigma_{ip}g'_{pk}]-g'_{ii}\sigma_{ik}-\sigma_{ii}g'_{ik}.$$

We see from Eqs. (2.3)—(2.5) that the relations (2.1) and (2.2), which constitute the basis of the mass-spectrometer method, are valid only near t = 0 and at a small enough residual-gas pressure.* Hence, a necessary element of the mass-spectrometer method is to check for the absence of distortions in the values of the cross-sections arising from the presence of residual gas or a possible non-linearity in the relation of Φ_{ik} or of Φ_{ik}/Φ_{ii} to t.

The coefficient a_{ik} in Eq. (2.5) is found from the results of measuring Φ_{ik}/Φ_{ii} as a function of t, and approximating it when necessary by a second-degree polynomial:

$$\frac{\Phi_{ik}}{\Phi_{ii}}=\frac{\Phi'_{ik}}{\Phi'_{ii}}+a_{ik}t+b_{ik}t^2.$$

If $\Phi_{ik}/\Phi_{ii} \gg \Phi'_{ik}/\Phi'_{ii}$ at t values where the contribution of the quadratic terms to the quantity Φ_{ik}/Φ_{ii} is small, then the coefficient a_{ik} is close to σ_{ik}. If this condition is not satisfied, then we must study the dependence of the coefficient a_{ik} on the pressure of the residual gas to get the correct value of σ_{ik}.

In cases where one is determining experimentally the cross sections of all the electron loss and capture processes with varying initial charges on the ions, one can find the deviation of the difference $\Phi_{ik}-\Phi'_{ik}$ from $\sigma_{ik}t$ by using the obtained values of Φ_{ik} and Φ'_{ik} with the aid of Eq. (2.3), without studying the relation of Φ_{ik} to t and t'. In a first approximation, the values of σ_{ik} and σ'_{ik} necessary for calculating these corrections can be obtained from the approximate relations (2.1) and from $g'_{ik}=\Phi'_{ik}$. One can find accurate values of σ_{ik} by the method of successive approximations. This method of calculating the cross sections has been applied by Dmitriev, Nikolaev, et al.[27–30] They supplemented Eq. (2.3) with certain terms proportional to t^4 and t^5 in order to ensure an accuracy of calculation of 0.2%, and, when determining the cross sections for simultaneous loss of four

*The condition for applicability of Eqs. (2.1) and (2.2) is often identified with the single-collision condition, according to which each ion must undergo no more than one collision resulting in charge alteration in the collision chamber. However, in actuality, the single-collision condition is not sufficient for applicability of these formulas. For example, in determining the electron-loss cross-sections σ_{01} of hydrogen atoms at high energies, where the inverse cross section σ_{10} is much smaller than σ_{01}, the single-collision condition will be satisfied up to gas pressures such that $\sigma_{01}t$ will be of the order of 1 or even 2, whereas Eqs. (2.1) and (2.2) will no longer be valid here.

or five electrons, to avoid exclusion of the possibility of loss of the same number of electrons through successive collisions with only one electron lost each time. They determined the complete set of corresponding values of Φ_{ik} from the set of Φ'_{ik} values and the values of Φ_{ik} obtained at any single pressure in the collision chamber. The values of Φ_{ik} obtained at different pressures gave mutually independent sets of σ_{ik} values. The agreement of the σ_{ik} values corresponding to different gas pressures indicated not only the validity of the corrections made, but also the internal consistency of the obtained experimental data.

Another source of errors in determining the cross sections for loss or capture of electrons by the mass-spectrometer method is the scattering of the fast particles when they collide with the atoms of the gas. As a result, a fraction of the ions scattered at large angles does not reach the detector. The size of this error depends on the angular distribution of the ions having the initial and altered charges, and on the geometrical conditions of the experiment, which are characterized by the function $S(\theta)$.[48,28] This function is equal to the ratio of the fraction of the particles detected when scattered at the angle θ to the fraction of the particles detected when scattered at an angle $\theta = 0$. It can be calculated for any experimental setup if one knows the distribution of velocity directions of the primary ions, or, roughly speaking, the divergence of the ion beam.* More crudely, the geometrical conditions of experiment can be characterized by the maximum scattering angle θ_m of the particles detected, which is defined by the relation $S(\theta_m) \approx 0.5$. When the dimensions of the entrance aperture of the detector are large, the value of θ_m is approximately equal to the ratio of the width or diameter of the exit channel of the collision chamber to its length. In[27-30], $\theta_m \approx 0.005$, and in [31-32], $\theta_m \approx 0.015$.

In order to prevent scattering from distorting appreciably the values of the measured cross-sections, the number of ions of a given charge scattered at angles $\theta \gtrsim \theta_m$ must be small in comparison with the number of ions of the same charge scattered at angles at which $S(\theta)$ differs little from unity. One usually tests for the absence of these distortions [31,32,45] by making measurements with varying diameters of the exit aperture of the collision chamber. If the values of Φ_{ik}/Φ_{ii} do not vary thereby, one assumes the corresponding cross section to be correct. However, constancy of the values of Φ_{ik}/Φ_{ii} as θ_m varies over a certain limited range of angles cannot serve in itself as a guarantee of the constancy of this quantity when θ_m is increased further. We

*In[27-30], e.g., $S(\theta) = 1 - 50\,\theta$ when $0 < \theta < 0.0036$, and $S(\theta) \approx \exp(0.9 - 300\theta)$ when $\theta > 0.0036$.

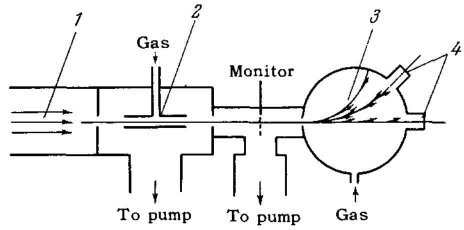

FIG. 2. Diagram of an experimental apparatus to determine overall cross sections for stripping and charge exchange of ions by the method of attenuation of the beam in a magnetic field. 1 – Ion beam from accelerator; 2 – charge converter; 3 – collision chamber in magnetic field; 4 – detectors.

can guarantee it to remain constant only in those cases in which the number of particles scattered at angles $\theta \gtrsim \theta_m$ is negligibly small. Thus, the test for the absence of distortions in the values of the measured cross-sections requires that we estimate the relative number of particles scattered at angles $\theta \gtrsim \theta_m$.

We can take the maximum error of the measured cross section to be the value of the overall cross section $\sigma_p(\theta_m)$ for scattering of ions at angles $\theta > \theta_m$, or according to more realistic estimates,[27] the quantity $0.5\,\sigma_p(\theta_m)$. For $\theta \gtrsim 0.001$, these quantities can be calculated relatively well.[49-50]

This method of estimating the maximum error in the values of the measured cross sections arising from scattering of the fast ions was applied in [27-30]. According to these estimates, in most of the studied cases of loss or capture of a single electron, the error due to incomplete detection of the scattered particles was considerably less than the random errors. As usual, the latter amounted to $\sim 10\%$ of the obtained cross sections.

2.2 The Determination of Cross Sections by the Method of Attenuation of the Ion Beam in a Magnetic Field. Allison and his associates[51-53,26] used the method of attenuation of the ion beam in a transverse magnetic field in their studies to determine the electron loss and capture cross sections of helium and lithium ions of energies up to 450 keV. The collision chamber in this method is placed in a magnetic field, so that the beam of fast particles, which usually contains ions of differing charges, is separated there into individual charge components, one of which is directed into the detector (Fig. 2). The alteration of the charge of the ions when they collide with the gas atoms results in a considerable change in their trajectories. Consequently, they leave the original beam and do not enter the detector. Measurement of the extent of attenuation of the beam as the pressure in the collision chamber is raised makes it possible to determine the sum $\sum_k{}' \sigma_{ik}$ of the cross sections for loss and capture of electrons.

The values of $\sum_k{}' \sigma_{ik}$ for ions of charge i were found by measuring the beam intensity R_i of these ions at various gas pressures in the collision chamber by the formula

$$R_i(t)/R_i(0) = \exp\left(-\sum_k{}' \sigma_{ik} t\right), \qquad (2.6)$$

where t is the number of gas atoms per volume of 1 cm^2 cross section lying along the path of an ion in the magnetic field. Equation (2.6) does not take into account the attenuation of the beam owing to scattering of the ions without alteration of charge. Control measurements of the attenuation of the undeviated beam in the absence of a magnetic field showed that scattering played a substantial role only in the attenuation of a beam of singly-charged lithium ions, for which the quantity $\sum_k{}' \sigma_{ik}$ is small. Hence, the values of $\sum_k{}' \sigma_{ik}$ were not determined for these ions.

Errors in the obtained values of the cross sections can also arise from variation in the charge composition of the primary beam as the gas pressure increases in the portion of the apparatus through which the beam passes before it enters the magnetic field. In order to ensure constancy of the charge composition of the primary beam, an equilibrium charge distribution was established in it with a charge converter 2 (Fig. 2) filled with the same gas as the collision chamber. However, they did not take into account the dependence of the charge distribution on the gas density.

Since the value of the sum $\sum_k{}' \sigma_{ik}$ is fundamentally determined by one or two terms, the values of $\sum_k{}' \sigma_{ik}$ in many cases are equal to the cross sections of the dominating process. For example, when $v \sim v_0$, the values of $\sum_k{}' \sigma_{ik}$ for lithium ions having i = 0, 2, and 3 are equal within the limits of experimental error to σ_{01}, σ_{21}, and σ_{32}, respectively. The cross sections obtained, especially at the highest ion energies, are somewhat smaller as a rule than those found by the mass-spectrometer method.[27,29,31,32] For helium ions, this difference does not exceed the limits of experimental error. The overall trend in the velocity dependence of the cross sections can be established from the total of the existing results. It indicates that at ion energies ~400 keV, in most cases where discrepancies occur, the cross sections found by Allison are too low.

By combining the beam-attenuation method with the mass-spectrometer method, Allison determined the values of σ_{02} and σ_{20} for helium atoms and ions. First he found the quantities $\sigma_{01} + \sigma_{02}$ and $\sigma_{21} + \sigma_{20}$ by the beam-attenuation method, and then found the ratios $\sigma_{02}/(\sigma_{01} + \sigma_{02})$ and $\sigma_{20}/(\sigma_{21} + \sigma_{20})$ by the mass-spectrometer method from the experimental values of Φ_{02}/Φ_{01} and Φ_{20}/Φ_{21} obtained at low pressures in the collision chamber. In the latter experiments, they used as the collision chamber a flowing gas target 2* (see Fig. 2), while the singly-charged ions were converted into neutral and doubly-charged particles in an additional gas target. In calculating the cross-sections, they took into account the possibility of formation of neutral and doubly-charged particles through two collisions with capture or loss of only one electron per collision, and they used the known values of σ_{10} and σ_{12}. The obtained cross sections σ_{02} and σ_{20} agree well with the results of mass-spectrometer measurements.

2.3 The method of determining the equilibrium charge composition of ion beams. The equilibrium charge distribution in an ion beam passing through matter is determined by mass-spectrometric analysis of the charge composition of the beam after it has passed through a film of a solid material or a flowing gas target of suitable thickness. When the ions pass through a rarefied gas, the charge distribution practically does not differ from the equilibrium distribution, provided that $\sigma_{i+1,i}t \gtrsim 4$ for $i \sim \bar{i}$. When $v \sim v_0$, this condition is satisfied when $t \gtrsim 10^{16}$ atoms/cm^2 (for gases of atomic number $Z_{med} \sim 15$, this corresponds to a thickness of $\gtrsim 0.3\ \mu g/cm^2$). At $v \sim 10^9$ cm/sec, it is satisfied when $t \gtrsim 10^{17}$—10^{18} atoms/cm^2. In a collision chamber several tens of centimeters long, a gas target of this thickness is produced at $p \sim 10^{-2}$—10^{-1} mm Hg. This is the pressure at which almost all of the measurements of the equilibrium charge composition of ions beams in gases have been performed.

In an ion beam passing through a solid, establishment of an equilibrium charge distribution requires a greater target thickness. This is because an equilibrium distribution of the electrons over the various states must be established for all electrons including the relatively strongly bound ones, whose cross sections for removal are small. In the experiments performed up to now, the solid targets used have been thin films of various materials of thicknesses from 2—5 to 30 $\mu g/cm^2$, and in some cases up to 150 $\mu g/cm^2$.

One usually tests for attainment of the equilibrium state by measuring the charge distribution with targets of varying thicknesses. In the experiments of Nikolaev, Dmitriev, et al.[34,35] the attainment of the equilibrium state was also tested with varying initial charges on the ions. This seems preferable for gaseous media, since one increases the thickness of a gas target by increasing the pressure of the gas, whereas the equilibrium charge distribution generally does not remain constant upon increase of the gas density.

Errors can arise in studying the equilibrium

*The flowing gas target 2 was produced in a tube about 20 cm long, having apertures of diameter about 0.9 mm for passage of the beam.

charge composition of ion beams in a gas, owing to retention of part of the scattered ions by the walls of the collision chamber. The reason for this is that one has to make the exit aperture of the collision chamber small to achieve a considerable pressure drop between the collision chamber and the adjoining vacuum space where the beam is separated into its individual charge components. In the experiments conducted up to now, the ratio of the diameter or width of the exit aperture to the length of the collision chamber has amounted to 0.002—0.006. In fact, the scattering angle of the particles being measured exceeded this value, since the region of the ion path in which the equilibrium distribution was established was usually considerably shorter than the length of the collision chamber. As the gas pressure is raised, the length of this region is shortened. Hence, errors due to limitation of the angle of the particles being measured is tested by measurements at different gas pressures, which are performed in all experiments. In the experiments of Pivovar and his associates,[31] they also tested for the absence of distortions due to ion scattering by measurements with an increased diameter of the exit aperture of the collision chamber.

The values of F_i are related by the normalization condition $\sum_i F_i = 1$, and are not mutually independent. Hence, in comparing the results of different measurements, it is convenient to compare the ratios F_{i+1}/F_i, which are more simply related to the electron loss and capture cross sections than the F_i values are, and which vary monotonically with varying ion velocity. This makes it possible to make a reliable interpolation between the experimental values of F_{i+1}/F_i for different velocities and to obtain continuous curves for F_i as a function of v.[14] The values of $F_i \gtrsim 0.1$ are usually determined with an accuracy of 1—5%.

The values of F_{i+1}/F_i obtained in different experiments agree within the limits of error in the overwhelming majority of cases. Only in [10,35], where nitrogen ions were passed through a celluloid film, did they obtain somewhat lower values of F_{i+1}/F_i than in [8,12] (see Fig. 22). In the former experiments, where they used somewhat thinner targets, the thickness might not have been sufficient for the relatively strongly bound electrons to make the transition to the excited states. In addition, the possibility of error in determining the ion velocity was apparently not ruled out in [10], since the latter was calculated from the parameters of the cyclotron, whereas the mean velocity of multiply charged ions obtained in a cyclotron, as is known,[54] can be lower than that of protons or helium ions.

The mean charge $\bar{i} = \sum_i iF_i$ of the ions can be determined also without studying the charge distribution. Thus, the mean charge of nitrogen ions that had passed through a nickel foil was found [5] by comparing the electric charge carried by the ion beam before and after passage through the foil, while the mean charge of uranium-fission fragments in various gases was found [3,4,15] from the value of the mean deflection of the ions in a magnetic field; here, owing to collisions with the atoms of the gas, they varied in charge repeatedly in the region of the magnetic field, and the deflection depended on the value of $\bar{i}$. Some information on the charge of fast particles at $v > 10^9$ cm/sec has also been obtained [36,55–58] by measuring the mean energy losses $-(dE/dx)$ of the ions. For ions of $Z = 1$ at such velocities, $\bar{i} = 1$, and the ratio of the quantity $-(dE/dx)_Z$ for ions of nuclear charge Z to the value $-(dE/dx)_1$ for protons is equal to the square of the effective charge i^{*2} of the ions. At high enough velocities, at which the decelerating ability of the material ceases to depend on the charge of the ions, the effective charge i^* must coincide with the root-mean-square value $(\overline{i^2})^{1/2} = (\sum_i i^2F_i)^{1/2}$. On the other hand, when $\overline{i^2} \gg 1$, it must agree with the mean charge $\bar{i}$. The experimental data obtained in[36,38,55–57] indicate that the values of i^* in a solid differ from $\bar{i}$ by no more than 2% for ions with $Z = 5$—10 at $v = (2-4) \times 10^9$ cm/sec, while at $v \approx 1.5 \times 10^9$ cm/sec, they differ by $\sim 5\%$. However, one will apparently observe values of i^* this close to $\bar{i}$ at considerably higher velocities for ions of the heavier elements. In particular, this is indicated by the following fact: [59] the values of i^* differ from $(\overline{i^2})^{1/2}$ by 10—30% at $v \sim 3 \times 10^8$ cm/sec for ions with $Z < 7$, but for ions with $Z \sim 15$—18, they differ by a factor of 1.5—2.

III. ELECTRON-LOSS BY FAST IONS

3.1. Effective Cross Sections for Electron Loss. Loss of electrons by fast ions when they collide with the atoms of a medium is in essence a process of further ionization of the atomic particles. According to general theoretical conceptions,[21] if the relative velocity v of the colliding particles is high enough, the ionization cross sections must diminish with increasing velocity, in line with the decrease in the interaction time. In the velocity range below the orbital velocity v_e of the electron being removed, the cross sections must increase with increasing velocity, owing to the adiabatic nature of the collisions. We should expect a maximum cross section at $v \sim v_e$. Experiment corroborates these ideas (Fig. 3). According to the existing experimental data, the velocity v_m at which the cross section reaches a maximum is determined mainly by the binding energy I of the electron being removed: [29]

$$v_m = \gamma u, \tag{3.1}$$

where $u = (2I/\mu)^{1/2}$, and μ is the mass of an elec-

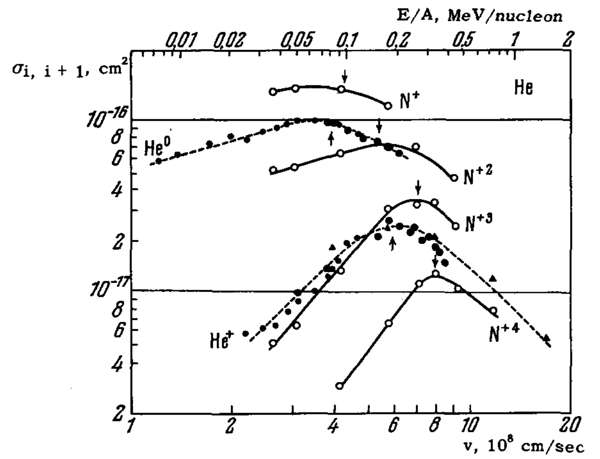

FIG. 3. Cross sections $\sigma_{i,\ i+1}$ for loss of a single electron for helium and nitrogen ions in helium as functions of the ion velocity v. The experimental data for nitrogen ions (o) are from[29], and for helium atoms and ions are: • from[1], ● from[31], and ▲ from[29]. The arrows indicate the values v = 1.35 u_i.

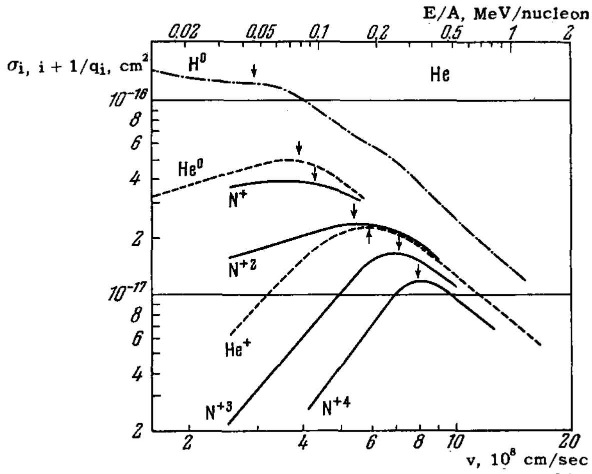

FIG. 4. Values of $\sigma_{i,\ i+1}/q_i$ for atoms and ions of hydrogen (–.–), helium (– –), and nitrogen (–) in helium. The arrows correspond to velocities v = 1.35u_i.

tron.* In helium $\gamma \approx 1.3$, in nitrogen $\gamma \approx 1.5$ and in krypton $\gamma \approx 2$.

Eq. (3.1) can also be written in the form usual for Massey's adiabatic criterion:[60]

$$\frac{\Delta E a}{\hbar v_m} \sim 1, \tag{3.2}$$

where ΔE is the difference in the binding energy of the electron before and after collision, and the quantity a characterizes the distance at which interaction takes place. However, in distinction from the processes of electron capture by singly-charged ions and neutral atoms, for which a depends on the initial charge of the ion and the number of captured electrons,[61, 62, 45] the quantity a depends on I for electron-loss processes, and proves to be of the order of magnitude of the dimensions of the ion: if we assume that $\Delta E = I$, we obtain from Eq. (3.1) that $a = 2\gamma\hbar/\mu u$, or $a = 2\gamma r/n$. Here $r = n\hbar/\mu u$, is a quantity approximating the mean radius of the shell from which the electron is being removed, and n is the principal quantum number. Hence we see that the quantity a is determined by the amount of momentum μu that must be imparted to the electron to remove it from the ion, rather than by the dimensions of the ion. The ratio $\Delta E/v$ is the change in momentum Δp of the ion in a long-range inelastic collision. Hence, we can interpret[63] Eq. (3.2) as being the equation determining the value of the momentum change Δp_m of the ion for which the probability of the process under discussion is a maximum. In the present case, $\Delta p_m = \mu u/2\gamma$. Thus, condition (3.1) implies that the electron-loss cross sections attain a maximum when the change in the momentum of the ion in long-range ionizing collisions amounts to a definite fraction ($\sim 1/3$) of the momentum μu imparted to the electron.

The values of the cross sections depend substantially on the number of electrons q_i in the outer shell of the ion: for equal ionization potentials, the cross sections $\sigma_{i,i+1}$ for loss of a single electron for various ions are approximately proportional to q_i,[29] while the cross sections for simultaneous loss of two electrons are proportional to $q_i(q_i - 1)$. Hence, if we divide the values of $\sigma_{i,i+1}$ shown in Fig. 3 by q_i, then the quantities $\sigma_{i,i+1}/q_i$ form a family of nonintersecting curves (Fig. 4). We see from Fig. 4 that as

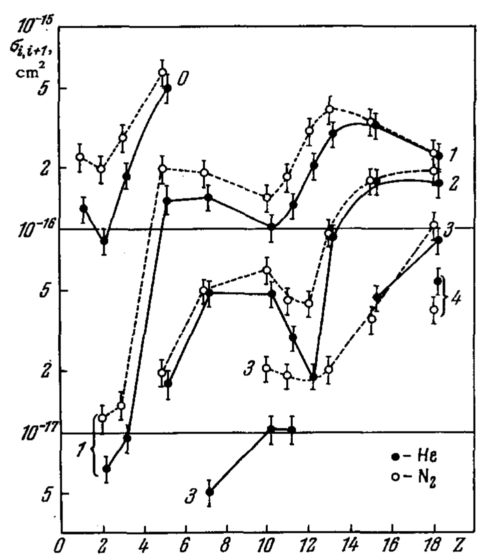

FIG. 5. The relation of the cross sections $\sigma_{i,\ i+1}$ for loss of a single electron to the nuclear charge Z of the ions at v = 2.6×10^8 cm/sec in helium (●) and nitrogen (O) according to the experimental data of[1, 26, 29, 31].

*For hydrogen atoms, for which the cross sections σ_{01} in helium hardly vary in the region v < u (see Fig. 4), the quantity γu denotes the velocity above which the cross sections begin to diminish.

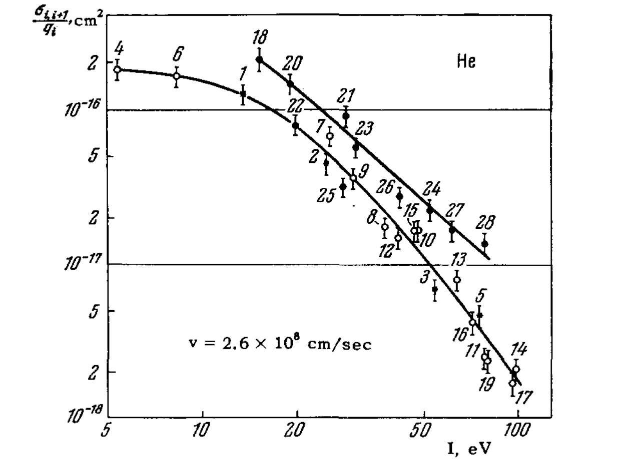

FIG. 6. Values of $\sigma_{i,\,i+1}/q_i$ in helium at $v = 2.6 \times 10^8$ cm/sec as a function of the binding energy I_i of the electron. The asterisks, open, and solid circles refer to K, L, and M electrons, respectively. 1 – H; 2, 3 – He^0, He$^+$; 4, 5 – Li^0, Li$^+$; 6 – 8 – B^0 – B^{+2}; 9 – 11 – N$^+$ – N^{+3}; 12 – 14 – Ne$^+$ – Ne^{+3}; 15 – 17 – Na$^+$ – Na^{+3}; 18, 19 – Mg$^+$, Mg^{+2}; 20, 21 – Al$^+$, Al^{+2}; 22 – 24 – P$^+$ – P^{+3}; 25 – 28 – Ar$^+$ – Ar^{+4}.

the ionization potential I_i decreases, the cross section per electron increases monotonically. Here $\sigma_{i,i+1}/q_i$ proves to depend on I_i much more strongly in the low-velocity region where $v < u_i$ than at higher velocities.

Since the quantities I_i and q_i do not vary monotonically upon increase of the nuclear charge Z of the ions, the relation of $\sigma_{i,i+1}$ to Z also proves to be non-monotonic (Fig. 5). The values of $\sigma_{i,i+1}/q_i$ determined from the cross sections given in Fig. 5 are shown in Fig. 6 as a function of I_i. We see from Fig. 6 that the values of $\sigma_{i,i+1}/q_i$ for ions whose outer electrons belong to K or L shells lie on a single general curve within an accuracy of 20%. The values of $\sigma_{i,i+1}/q_i$ for M electrons fit another curve, except for individual cases. One observes a similar pattern at other velocities and in other gases.

The values of $\sigma_{i,i+1}/q_i$ deviate from the overall curve only for ions having large numbers q_i. For these, the quantity $\sigma_{i,i+1}/q_i$, especially in heavy gases, is only part of the cross section σ_i for loss of a particular electron, as defined by the relation

$$\sigma_i = \frac{1}{q_i}(\sigma_{i,\,i+1} + 2\sigma_{i,\,i+2} + 3\sigma_{i,\,i+3} + \ldots). \qquad (3.3)$$

If we take into account the simultaneous loss of several electrons, the scatter in the experimental points on the curve of the relation of σ_i to I_i turns out to be considerably less than for $\sigma_{i,i+1}/q_i$. As a rule, it does not exceed the experimental limits of error (Fig. 7). Thus, the effective cross section for removal of a particular electron is determined by its binding energy, and within an accuracy of 20%, it does not depend on the number of electrons in the shell. This means that the removal of each of the electrons is independent of that of the others.

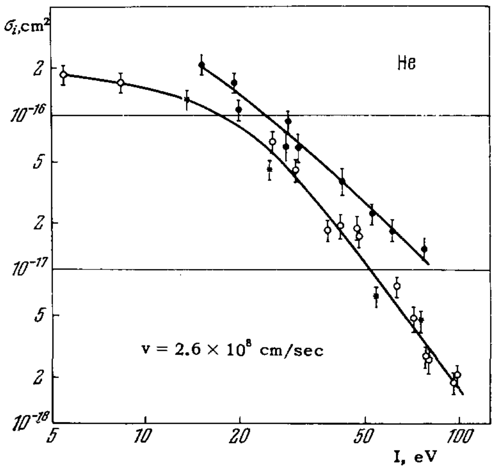

FIG. 7. The relation of the cross section σ_i for loss of a particular electron to I_i at $v = 2.6 \times 10^8$ cm/sec in helium. The symbols are the same as in Fig. 6.

This conclusion also follows from the experimental data on the cross sections for simultaneous loss of several electrons.[30] The cross sections for loss of a pair of electrons found from the experimental cross sections $\sigma_{i,i+s}$ for loss of varying numbers of electrons

$$\sigma_i^{(2)} = \left(\frac{1}{C_q^2}\right) \sum_{s \geqslant 2} C_s^2 \sigma_{i,\,i+s} = \frac{2}{q_i(q_i-1)} \times (\sigma_{i,\,i+2} + 3\sigma_{i,\,i+3} + 6\sigma_{i,\,i+4} + \ldots) \qquad (3.4)$$

are independent of q_i.

Figure 8 shows some typical relations between electron-loss cross sections in various media. As

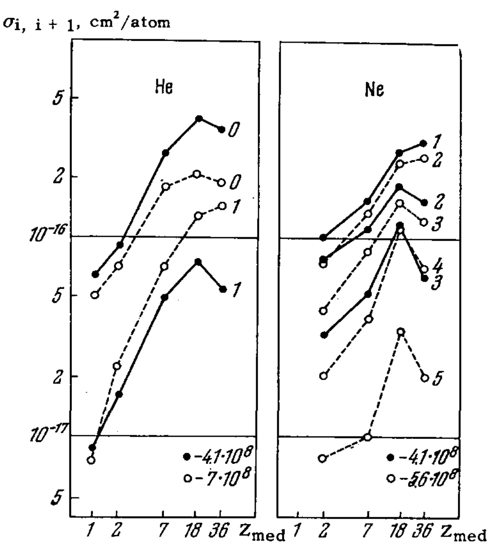

FIG. 8. The relation of the cross sections $\sigma_{i,\,i+1}$ for loss of one electron to the atomic number Z_{med} of the medium for atoms and ions of helium and neon according to the results of[29, 31]. The charge of the ions is indicated next to the curves. The solid circles correspond to a velocity $v = 4.1 \times 10^8$ cm/sec, and the open circles to $v = 7 \times 10^8$ cm/sec for helium ions and $v = 5.6 \times 10^8$ cm/sec for neon ions.

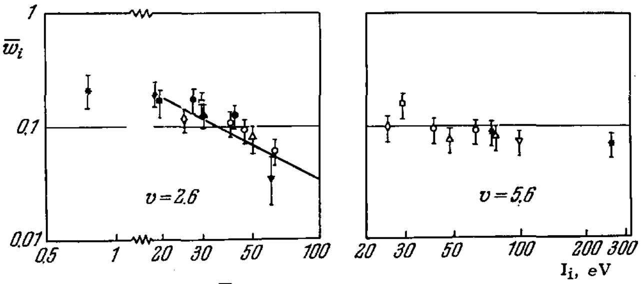

FIG. 9. Values of $\overline{w}_i = (2/q_i)(\sigma_{i,\,i+2}/\sigma_{i+1,\,i+2})$ in helium at $v = 2.6 \times 10^8$ and 5.6×10^8 cm/sec as functions of the binding energy I_i of the electron, according to the experimental data of[1, 29, 30, 64, 65]. The asterisks, open, and solid symbols refer to K, L, and M electrons, respectively. The inclined straight line corresponds to $\overline{w}_i = AI_i^{-1}$ with $A = 3.5$ eV; ◇, ✳ – $q_i = 2$; △ – 3; □ – 4; ▽ – 5; o – 6–8.

the atomic number Z_{med} of the medium increases from 1 to 18, the cross sections increase. However, if we go from $Z_{med} = 18$ (argon) to $Z_{med} = 36$ (krypton) at $v \lesssim 5 \times 10^8$ cm/sec, the cross-sections for most ions decrease. This indicates that the deformation of the atoms of the medium affects the values of the cross sections considerably.[29] With decreasing ion charge i and increasing v, the extent of the decrease of the cross sections in heavy media declines, and at $v > 10^9$ cm/sec, the cross sections in krypton are comparable with those in argon or exceed them.

3.2. The Mean Probability of Removal of Electrons. Since in the cases under discussion the motion of an ion with respect to the atoms of the medium can be considered classically,

$$\sigma_{i,\,i+s} = 2\pi \int W_{i,\,i+s}\varrho\, d\varrho, \tag{3.5}$$

where $W_{i,i+s}$ is the probability of simultaneous loss of s electrons, and ρ is the impact parameter. The quantities $W_{i,i+s}$ can be expressed in terms of the mean probability w_i of loss of an individual electron, the mean probability $w_i^{(2)}$ of loss of a pair of electrons, etc. In most cases we can limit ourselves to considering the electrons of the outer shell, and then[30]

$$W_{i,\,i+s} = C_q^s \sum_{t=0}^{q-s} C_{q-s}^t (-1)^t w_i^{(s+t)} = C_q^s w_i^{(s)} - \sum_{t=1}^{q-s} C_{t+s}^t W_{i,\,i+s+t}, \tag{3.6}$$

where $C_q^s = q!/s!(q-s)!$ is the number of combinations of q electrons taken s at a time.

We can derive the following from Eqs. (3.5) and using the relation $w_i^{(1+t)}\, w_i = w_i^{(t)}$, which is valid for the loss of electrons from a single shell:

$$\overline{w}_i = \frac{\int w_i W_{i+1,\,i+2}\varrho\, d\varrho}{\int W_{i+1,\,i+2}\varrho\, d\varrho} = \frac{2}{q_i}\frac{\sigma_{i,\,i+2}}{\sigma_{i+1,\,i+2}}. \tag{3.7}$$

The quantity $\overline{w}_i$ defined thus is the probability of loss of an individual electron from an ion of charge i averaged over the region of impact parameters making the major contribution to the cross section for loss of the next electron.

According to the existing experimental data,[1,30] the mean probability of loss of a particular electron $\overline{w}_i$ does not depend on the number of electrons in the shell, and in most cases it turns out to be relatively small (Fig. 9). At $v \approx 2.5 \times 10^8$ cm/sec for ions with $I_i = 20-70$ eV, we can assume that $\overline{w}_i \approx AI_i^{-1}$, where $A \approx 4$ eV in helium, but $A \approx 8$ eV in heavier gases. For negative hydrogen ions having $I_i \approx 0.8$ eV, the value of $\overline{w}_i$ varies from ~ 0.2 (in helium) to ~ 0.8 (in nitrogen and oxygen), as is indicated by the studies of Fogel' and his associates.[64,65] At $v = (6-12) \times 10^8$ cm/sec, the values of $\overline{w}_i$ are about the same for all ions having $I_i = 25-500$ eV: in helium $\overline{w}_i \sim 0.1$, and in the heavier gases ~ 0.2. The over-all mean probability of loss of electrons $q_i\overline{w}_i = 2\sigma_{i,i+2}/\sigma_{i+1,i+2}$ proves to be near unity for ions of large q_i.

The values of σ_i and $\overline{w}_i$ permit us to estimate the mean value of the impact parameters ρ_i for the collisions making the major contribution to the cross section, since we may assume that

$$\sigma_i = \pi\overline{\varrho_i^2}\,\overline{w}_i. \tag{3.8}$$

If we limit ourselves to the first term in the expression (3.3) for σ_i, then we obtain from Eqs. (3.8), (3.7), and (3.3):

$$\pi\overline{\varrho_i^2} \approx \sigma_{i,\,i+1}\,\sigma_{i+1,\,i+2}/2\sigma_{i,\,i+2}. \tag{3.9}$$

For ions having I_i from 20 to 150 eV with $v \sim (2.5-10) \times 10^8$ cm/sec, the values of $\overline{\rho}_i$ lie between 0.5×10^{-8} and 3×10^{-8} cm. Since $\overline{w}_i$ depends weakly on v, the values of $\overline{\rho}_i^2$ vary in about the same way with varying velocity as the cross sections $\sigma_{i,i+1}$ do. The maximum values of $\overline{\rho}_i$ are approximately equal to $(1-2) \times 10^{-8}$ cm. That is, they are close to the sum of radii of the electron shells of the colliding particles.

The experimental data on the cross sections for simultaneous loss of three electrons make it possible also to draw some conclusions on the form of the relation of $\overline{w}_i$ to ρ in the range $\rho \sim \rho_i$. Similarly to $\overline{w}_i$, the quantity

$$\overline{w}_i(2) = \frac{\int w_i W_{i+1,\,i+3}\varrho\, d\varrho}{\int W_{i+1,\,i+3}\varrho\, d\varrho} = \frac{3}{q_i}\frac{\sigma_{i,\,i+3}}{\sigma_{i+1,\,i+3}} \tag{3.10}$$

is the probability of loss of a particular electron averaged over the range of impact parameters making the major contribution to the cross section for loss of the next pair of electrons. The quantity $\overline{w}_i(2)$ coincides with $\overline{w}_i$ only when the probability $w_i(\rho)$ is equal either to some constant or to zero for all values of ρ. According to the existing experimental data, $\overline{w}_i(2) \approx 1.3\,\overline{w}_i$ in helium, while in nitrogen, argon, and krypton $\overline{w}_i(2) \approx 2\overline{w}_i$. This indicates that the probability of loss of an electron is approximately proportional to $1/\rho$, i.e., $w \approx w'\rho'/\rho$, over the rather broad range of impact parameters ρ extending from ρ' to ρ''. The experimental values $\bar{w}_i(2)/\bar{w}_i = 1.35$ and 2 correspond to ratios $\rho''/\rho' \sim 5$ and 14. Here in helium, $w' \approx 2\bar{w}_i \sim 0.2-0.4$, and $\rho'' = 1.15\bar{\rho}_i$, while in the other gases $w' \sim 4\bar{w}_i \sim 1$, and $\rho'' \approx 1.3\bar{\rho}_i$. Hence, the previously-defined impact parameter $\bar{\rho}_i$ is close to the value ρ'' below which the probability of loss of an individual electron w is approximately inversely proportional to ρ, while the mean probability $\bar{w}_i$ is equal to the probability $w(\rho)$ at $\rho = (0.4-0.5)\bar{\rho}_i$.

Thus, in the velocity range where the electron-loss cross-section is near a maximum, collisions having relatively large impact parameters, for which the ionization probability is small, make the major contribution to the cross section. In line with this, the appreciable influence of deformation of the atoms of the medium noted in Sec. 3.1 on the value of the electron-loss cross section in heavy gases becomes understandable, together with the large effect of the screening of the Coulomb field of the nuclei of the atoms of the medium by the atomic electrons, as will be discussed in the next section.

3.3 Fundamental Results of the Theoretical Studies. Comparison of Experiment with Theory. According to well-known criteria,[66,68,21] the Born approximation must give correct results for $v \gg Z_{med}v_0$, and also for ions having $i \gg Z_{med}$ in the region $v < Z_{med}v_0$. Up to now, cross sections have been calculated in the Born approximation for loss of K electrons for ions having $i = Z - 1$ in hydrogen and helium.[69,71,40] In helium, the calculated cross sections in the region where the Born approximation can be applied are in good agreement with the experimental values (Fig. 10). However, there are some discrepancies in hydrogen,[39] which can be due either to experimental errors or to the fact that the calculation was performed for atomic hydrogen, while the experimental cross sections refer to the molecular gas.

For ions having $Z > 2Z^*_{med}$, where Z^*_{med} is the effective charge of the nuclei of the atoms of the medium (in hydrogen, $Z^*_{med} = 1$, but in helium $Z^*_{med} = 1.69$), when $v > 3Zv_0$, the values of $\sigma_{Z-1,Z}$ are given by the following approximate formula:[40]

$$\sigma_{Z-1,Z} = 4\pi a_0^2\left(\frac{v_0}{Zv}\right)^2 \times [Z^2_{med}(1+0.55\ln A)+Z_{med}(1+0.55\ln B)]. \quad (3.11)$$

Here A is equal to the lesser of the quantities $1.6v/Zv_0$ and $Z/2Z^*_{med}$, and B is equal to the lesser of the quantities $1.6v/Zv_0(1 + 1.6I_{med}/Z^2\mu v_0^2)$ and Z/Z^*_{med}, $a_0 = \hbar/\mu e^2 = 0.53 \times 10^{-8}$ cm is the atomic unit of length, and I_{med} is the binding energy of an electron in an atom of the medium. In the range $v < Zv_0$, the values of $\sigma_{Z-1,Z}$ for the same ions coincide with the cross-sections calculated in [67,69,73] for loss of a K electron upon collision of an ion with

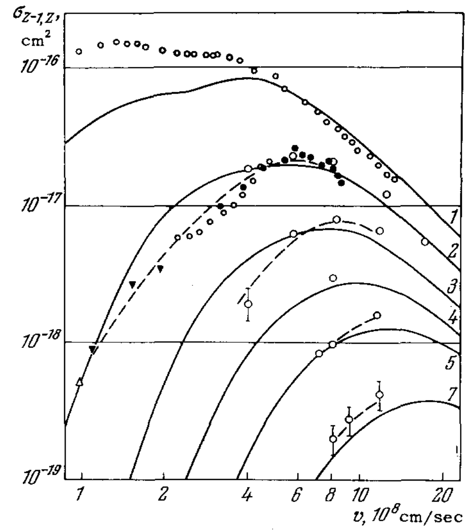

FIG. 10. Cross sections $\sigma_{Z-1,Z}$ for loss of a K electron in helium. The curves give the results of the calculations in the Born approximation, and the points are experimental values: o – from[1], ● – from[31], ○ – from[29], △ – from[41], and ▼ – from[72]. The numbers next to the curves indicate the nuclear charge Z of the ions.

atomic nuclei:

$$\sigma_{Z-1,Z} = \pi a_0^2 Z^2_{med} Z^4 F_K\left(\frac{v}{Zv_0}\right). \quad (3.12)$$

According to [69,73], when $v \ll Zv_0$, we have here

$$F_K\left(\frac{v}{Zv_0}\right) = \frac{2^{19}}{45}\left(\frac{v}{Zv_0}\right)^8.$$

For ions having $Z \lesssim Z_{med}$, the increasing degree of shielding of the Coulombic field of the nuclei of the atoms of the medium by the atomic electrons has the consequence that a much smaller role is played by collisions in which the momentum of the atoms of the medium varies by an amount $q < \mu Zv_0$. Hence, when $v \gtrsim 3Zv_0$, the Born approximation for these ions leads to the same results as does the free-collision approximation, in which the electron-loss cross section is considered to be equal to the cross section for scattering of a free electron moving with a velocity v by an atom of the medium, whereby the momentum of the electron changes by an amount $q > \mu u$. Values of σ_i have been calculated in this approximation for ions passing through hydrogen and helium.[39] In particular, for ions having $u = (0.4-2)Z_{med}v_0$ in the high-velocity range,

$$\sigma_i = \pi a_0^2 \frac{v_0^3}{Z^*_c uv^2}(1.2Z^2_{med} + 3.4Z_{med}). \quad (3.13)$$

For all ions having $u < 2Z_{med}v_0$ at $v \geq 2Z_{med}v_0$, the calculated σ_i values in helium agreed with the experimental values.[39] Owing to the shielding of the Coulomb field of the nuclei of the atoms of the medium by the atomic electrons, the cross sections σ_i for ions having $u \leq Z_{med}v_0$ for any $v > u$ prove to be considerably smaller than those given by Bohr's [21] well-known formula derived in the free-collision approximation without taking shielding into account:

$$\sigma_i = 4\pi a_0^2 \left(\frac{v_0^2}{uv}\right)^2 (Z_{\text{med}}^2 + Z_{\text{med}}). \quad (3.14)$$

For ions having $u < Z_{\text{med}} v_0$ in the region of v from $\sim u$ to $Z_{\text{med}} v_0$, i.e., when not too fast ions are passing through heavy materials, the σ_i values can be estimated from the generalized Bohr formula[21,29] derived in the free-collision approximation through a classical treatment of the scattering of an electron in the strongly shielded field of an atom:

$$\sigma_i = \pi a_0^2 Z_{\text{med}}^{2/3} \frac{v_0^2}{uv}. \quad (3.15)$$

In accord with experiment, the relation of σ_i to u in heavy gases according to this formula turns out to be the same as from (3.13), while σ_i depends more weakly on v. The rather complex dependence of the cross sections on Z_{med} observed experimentally is not reflected in this formula. Since classical mechanics is inapplicable for small scattering angles, Eq. (3.15) should give a correct result for $u \gg Z_{\text{med}}^{1/3} v_0$, i.e., in nitrogen at $I \gg 50$ eV, and in krypton at $I \gg 150$ eV. In actuality, the values of σ_i calculated from Eq. (3.15) for $Z_{\text{med}} \geq 2$ in the region $u < v$ differ from the experimental values by a factor of no more than two, even at $I \sim 5$—20 eV.

In estimating the electron-loss cross sections of uranium fission fragments in various gases at $v \sim u$, Bohr and Lindhard [23] used an expression for the cross section for loss of a particular electron derived in the free-collision approximation for cases of collision of ions with nuclei:

$$\sigma_i = 4\pi a_0^2 Z_n^2 \left(\frac{v_0^2}{uv}\right)^2 \left[1 - \left(\frac{u}{2v}\right)^2\right]. \quad (3.16)$$

In light media, the nuclear charge Z_n was assumed equal to Z_{med}, and Eq. (3.16) was reduced to the portion of the expression for σ_i in the free-electron approximation for $u \gg Z_{\text{med}} v_0$. This corresponds to collisions of the electron being removed with the nuclei of the atoms of the medium, so that the effects arising from the presence of the atomic electrons were neglected. In discussing cross-sections in heavy media, when $v < Z_{\text{med}} v_0$, they assumed that $Z_n = Z_{\text{med}}^{1/3} v/v_0$ in taking shielding into account. Here σ_i proves to depend more strongly on u than according to Eqs. (3.13) and (3.15), while σ_i depends more weakly on v when $v \gg u$. We can obtain Eq. (3.15), which is close to actuality, from (3.16) by assuming $Z_n = Z_{\text{med}}^{1/3} (uv)^{1/2}/2$.

Bell [22] has also made some calculations of electron-loss cross-sections in the velocity range in which the cross section is near the maximum for uranium fission fragments in oxygen, and Gluckstern [16] has calculated them for ions of oxygen, neon, phosphorus, and argon in argon. The calculations were performed classically, with account taken of the orbital velocity of the electrons being removed.

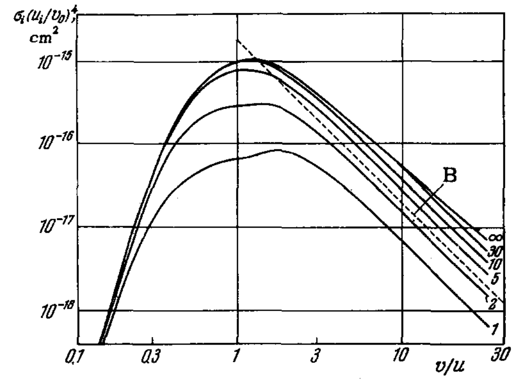

FIG. 11. Values of $\sigma_i u_i^4/v_0^4$ for ions having $i = Z - 1$ in helium as a function of v/u_i. The solid lines are for the Born approximation, and the dotted line according to Bohr's formula (3.14); the numbers next to the curves denote Z.

The fundamental qualitative regularities in the values of the cross sections that they could establish from these calculations agree with the experimental values. The electron-loss cross sections calculated by Gluckstern for oxygen ions at $v \sim 10^9$ cm/sec differ from the experimental values by no more than 20%.

The conditions for applicability of Firsov's [74] approximate calculations are satisfied in the region $v \ll v_0$ for many cases of electron loss by neutral atoms in heavy gases. However, even at $v = 1.2\, v_0$, the experimental values of $\sum_k{}' \sigma_{0k}$ for atoms of $Z \geq 5$ in nitrogen, argon, and krypton differ from Firsov's value for σ_{01} by a factor of no more than 2.5.[29]

The results of the most recent experimental and theoretical studies confirm the conclusion drawn by Fogel' et al.[64] that it is impossible to reduce the values of $\sigma_{i,i+1}/q_i$ for different ions in a given medium to a common curve depending only on the quantity v/u, as Krasner [75] has tried to do. For a number of reasons, in particular the differing effect of the shielding of the Coulomb field of the nuclei of the atoms of the medium on the cross section, the shape of the $\sigma_i(v/u)$ curve also proves to differ for different ions (Fig. 11). Nevertheless, the general regularities in the values of the cross sections established by the experimental and theoretical studies, and their concrete values for a number of ions permit us to make reliable estimates even for ions for which there are no experimental data. Thus Dmitriev [76] has calculated the cross sections $\sigma_{i,i+1}$ for loss of a single electron for all atoms and positive ions of the light elements having Z from 3 to 10 in helium and for ions having Z = 3, 5, 7, and 10 in nitrogen in the velocity range from 10^8 to 3×10^9 cm/sec.

IV. ELECTRON CAPTURE BY FAST IONS

4.1. Fundamental Results of the Theoretical Studies. Quantum-mechanical calculations of electron-capture cross sections have been made only for the simplest cases of electron capture by protons and helium ions upon passing through hydrogen and helium. The results of these calculations have been discussed in the reviews of Bates and McCarroll [77] and Gerjuoy.[79] The cross section for simultaneous capture of two electrons has also been calculated for helium nuclei passing through helium.[79] Cross sections agreeing with the experimental values are obtained by calculation with rather cumbersome formulas. However, the general characteristics of the electron-capture process are also reflected in the very simple approximate formula first derived by Brinkman and Kramers[80] and then in a somewhat more general form by Schiff [81] for the cross-section for capture of an electron by an atomic nucleus of charge Z into states of principal quantum number n from the ground state of a hydrogen-like ion:

$$\sigma = \frac{2^{18}}{5}\pi a_0^2 Z^2\left(\frac{Z}{n}\right)^3 Z_{\text{med}}^5\left(\frac{v}{v_0}\right)^8\left[\left(\frac{v}{v_0}\right)^4 + 2\left(\frac{v}{v_0}\right)^2\left(Z_{\text{med}}^2+\frac{Z^2}{n^2}\right)+\left(Z_{\text{med}}^2-\frac{Z^2}{n^2}\right)^2\right]^{-5}. \quad (4.1)$$

Equation (4.1) implies that the probability of electron capture is highest when the electrons have a mean orbital velocity $v_{e,\text{med}} = Z_{\text{med}}v_0 \sim v$, and are captured into states of orbital velocity $v_e = Zv_0/n \sim v$, while the capture cross sections into more highly excited states are proportional to $(Z/n)^3$.

In more complex cases of electron capture, we can only estimate the cross-sections by the approximate methods of Bohr,[21] Bell,[22] and Bohr and Lindhard.[23] In spite of the difference in the methods of calculation and their differing approach to the electron-capture phenomenon, these methods generally lead to approximately the same results, in qualitative agreement with the conclusions drawn from (4.1).

In estimating electron-capture cross sections $\sigma_{i,i-1}$ by the Bohr method, the value of $\sigma_{i,i-1}$ is assumed proportional to the cross section σ' for collision of the ion with an electron, imparting to the latter the momentum $\sim\mu v$ necessary to bring about capture. The necessary momentum transfer $\sim\mu v$ between the electron and the atom of the medium is most probable for electrons having $v_{e,\text{med}} \sim v$. Hence we assume that $\sigma_{i,i-1} = \sigma' f k$, where k is the number of electrons in the atom of the medium having $v_{e,\text{med}} \sim v$, and f is their probability of capture. In cases in which the ion can be assumed to be a point charge, $\sigma' \sim 4\pi a_0^2 i^2 (v_0/v)^4$. When electrons are captured from states having $v_e < v$, according to Bohr,[21] $f \sim (v_e/v)^3$. Hence, for cases when one of the atomic electrons is captured by a nucleus into states having $n > Zv_0/v$, we have

$$f \sim \left(\frac{Zv_0}{nv}\right)^3, \quad (4.2)$$

while we obtain an expression for the cross sections agreeing with (4.1) within the accuracy of the coefficient for $Z/n \ll v/v_0$ and $Z_{\text{med}} \approx v/v_0$:

$$\sigma \sim 4\pi a_0^2 Z^2\left(\frac{Z}{n}\right)^3\left(\frac{v_0}{v}\right)^7. \quad (4.3)$$

Bohr [21] assumed on the basis of a statistical model of the atom that

$$k \sim Z_{\text{med}}^{1/3}\frac{v}{v_0} \quad (4.4)$$

for nuclei of $Z < v/v_0$ passing through heavy gases, and obtained

$$\sigma_{Z-1,Z} \sim 4\pi a_0^2 Z^5 Z_{\text{med}}^{1/3}\left(\frac{v_0}{v}\right)^6. \quad (4.5)$$

Bohr's method has been used to estimate the electron-capture cross sections of the various ions of nitrogen.[24] Account was taken here of the fact that the effective charge exerted by the ion on the electron being captured can exceed the charge i of the ion. Hence, e.g., for ions having $i \sim 2-4$ and $v \sim (5-10) \times 10^8$ cm/sec in heavy gases, the following was obtained:

$$\sigma_{i,i-1} \sim 4\pi a_0^2 i^3 Z_{\text{med}}^{1/3}\left(\frac{v_0}{v}\right)^5. \quad (4.6)$$

In calculating the values of $\sigma_{i,i-1}$ by Bell's method [22], we assume the electron being captured to be a particle moving according to the laws of classical mechanics. As long as the force exerted on it by the ion does not exceed that exerted by the atom, we neglect the effect of the ion on the motion of the electron. After this, we neglect the interaction of the electron with the atom. If here the total energy of the electron with respect to the ion turns out to be negative when calculated with its orbital velocity taken into account, we assume the electron to have been captured.

Bohr and Lindhard [23] applied a somewhat more simplified method, and derived simple approximate formulas for the electron-capture cross sections of highly-charged ions. The cross sections were estimated by comparing the electron-release distance R_0, at which the force exerted on the electron by the ion and by the atom of the medium become equal, with the maximum capture radius R_c at which an electron of velocity v with respect to the ion proves to be bound. They also took into account the relation between the collision time R_c/v and the time $a_{e,\text{med}}/v_{e,\text{med}}$ necessary for completion of the process of release of the electron from its binding to the atom of the medium ($a_{e,\text{med}}$ is the mean radius of the orbit of the electron in the atom of the medium). Just as in the result of Bell's calculations, it turned out here that electrons of orbital velocities $v_{e,\text{med}} \sim v/2$, for which $R_0 \approx R_c$, make the major contribution to the cross-section. The capture cross-section of each of these electrons is determined by the capture radius $R_c = ia_0(v_0/v)^2$, i.e.,

$$\sigma \sim \pi a_0^2 i^2\left(\frac{v_0}{v}\right)^4. \quad (4.7)$$

Hence, taking (4.4) into account, we obtain

$$\sigma_{i,i-1} \sim \pi a_0^2 i^2 Z_{\text{med}}^{1/3}\left(\frac{v_0}{v}\right)^3. \quad (4.8)$$

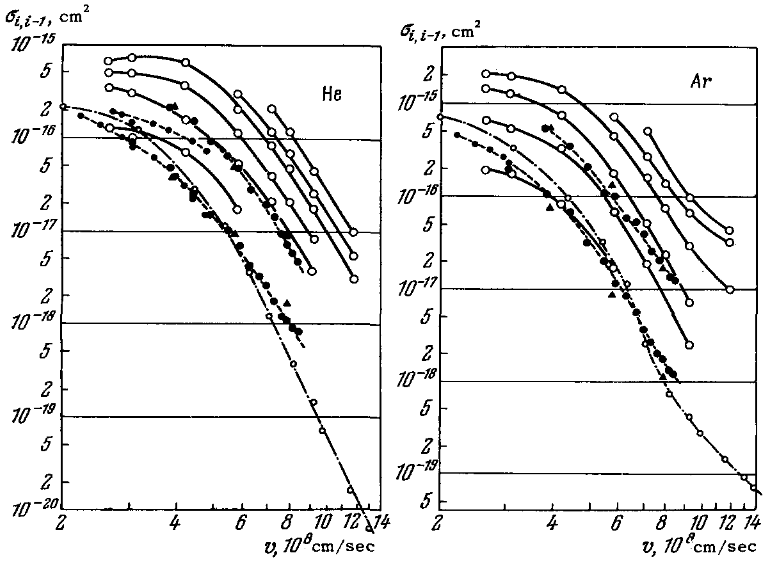

FIG. 12. Cross sections $\sigma_{i,\,i\,-\,1}$ for capture of a single electron by protons, and helium and nitrogen ions in helium and argon. o – For H^+, from[1]; •, ▲, and ●, for He^+, He^{+2}, from[1, 27, 31, 32], respectively; ○, for $N^+ - N^{+6}$, from[27].

Equation (4.8) agrees qualitatively with the results of calculations by Bell's method.[22,16] The values of $\sigma_{i,i-1}$ given by Eq. (4.8) differ from those calculated by Bell's method by a factor of no more than three. Since the quantity πR_c^2 coincides with the cross-section for imparting a momentum of $\mu v/2$ to the electron, Eqs. (4.7) and (4.8) can be derived by Bohr's method by assuming $f = 1/4$. Eq. (4.7) agrees with (4.1) to within the accuracy of a constant factor when $i = Z$ and $Z/n \approx Z_{med} \approx v/v_0$.

Since the electron is taken to be a classical particle in estimating cross-sections by the Bell and Bohr-Lindhard methods, the results obtained are valid only for ions of high enough charges. For example, when $i < v/v_0$, an electron localized in a region of dimensions R_c will have a mean velocity $\sim v_{cr} = v_0 a_0/R_c = v^2/iv_0 > v$, in agreement with the uncertainty relations. However, it was assumed in deriving (4.7) that the velocity of the electron with respect to the ion amounted to $\sim v$. If we estimate the probability that the electron under discussion should have the velocity $\sim v$ necessary for capture to be $(v/v_{cr})^3$, then the value $(iv_0/v)^3$ obtained for the probability coincides with the capture probability f introduced by Bohr. The fact that the effective charge of the ion can exceed the value i places a further limitation on the region of applicability of (4.7) and (4.8). Consequently, we can consider them valid only for $i \gtrsim Z^{1/3} v/v_0$.[24]

When fast ions pass through hydrogen or helium, $v_{e,med} \ll v$ and $a_{e,med}/v_{e,med} \gg R_c/v$ for all the atomic electrons. Then the release of an electron in the time R_c/v has a probability of the order of $(R_c/v)/(a_{e,med}/v_{e,med}) \sim iZ_{med}^2 (v_0/v)^3$, as Bohr and Lindhard had assumed, and hence,

$$\sigma_{i,\,i-1} \sim \pi a_0^2 i^3 Z_{med}^3 \left(\frac{v_0}{v}\right)^7. \tag{4.9}$$

The relation of $\sigma_{i,i-1}$ to i, Z_{med}, and v given by (4.9) proves to differ from that given by (4.1) for $i = Z$, $Z/n \sim v/v_0$, and $Z_{med} \ll v/v_0$. This is because, in distinction from (4.9), Eq. (4.1) corresponds to the capture of electrons of orbital velocities $\sim v/2$.[82] The number of such electrons in hydrogen and helium atoms can be estimated to be

$$k \sim Z_{med} \left(4Z_{med}^{*} \frac{v_0}{v}\right)^5 \left(1 + 4Z_{med}^{*2} \frac{v_0^2}{v^2}\right)^{-4}. \tag{4.10}$$

Now, we may assume, in agreement with the results obtained in the first Born approximation, that mainly electrons of orbital velocities $\sim v/2$ are captured in light media. Then for highly-charged ions, instead of (4.9), we have from (4.7) and (4.10):

$$\sigma_{i,\,i-1} \sim \pi a_0^2 i^2 Z_{med} Z_{med}^{*5} 2^{10} \left(\frac{v_0}{v}\right)^9 \left(1 + 4Z_{med}^{*2} \frac{v_0^2}{v^2}\right)^{-4}, \tag{4.11}$$

while for nuclei of low charges

$$\sigma_{Z,\,Z-1} \sim \pi a_0^2 Z^5 Z_{med} Z_{med}^{*5} \left(\frac{2v_0}{v}\right)^{12} \left(1 + 4Z_{med}^{*2} \frac{v_0^2}{v^2}\right)^{-4}. \tag{4.12}$$

Equations (4.11) and (4.12) agree qualitatively with (4.1). For uranium fission fragments, the values of $\sigma_{i,i-1}$ in hydrogen and helium calculated by Eqs. (4.9) and (4.11) differ by a factor of no more than 1.6. However, the relation of $\sigma_{i,i-1}$ to Z_{med} and v given by (4.11) is closer to experiment.[4]

The value of the electron-capture probability for varying impact parameters has been calculated for the case of electron capture by protons in hydrogen.[80,81,83,84] The calculations indicate that for $v = (1-2)v_0$, the major contribution to the cross section comes from collisions with impact parameters $\rho \sim (1-3)a_0$, while with increasing v, the relative importance of closer collisions gradually increases. In the range of impact parameters making the major contribution to the cross section, the probability of electron capture amounts to ~ 0.5 when $v \sim v_0$, but rapidly declines with increasing v.

4.2. <u>Results of Experimental Studies of Single-electron Capture. Comparison with Theory</u>. For $v \gtrsim 4 \times 10^8$ cm/sec, the values of $\sigma_{i,i-1}$ rapidly decrease with increasing v for all ions (Fig. 12).

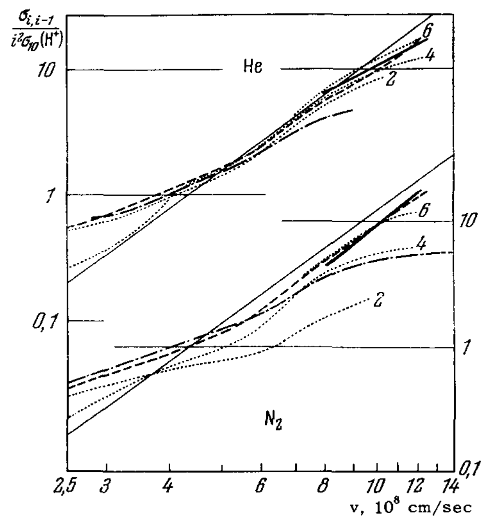

FIG. 13. The ratio of the values of $\sigma_{i,\,i-1}/i^2$ for various ions to the electron-capture cross section of protons at the same velocity in helium and in nitrogen: —.— helium nuclei, — — lithium nuclei, —— boron nuclei, . . . nitrogen ions of charges i = 2, 4 and 6 (indicated next to the curves), from the experimental data of[1, 26, 27, 32]. The inclined straight line corresponds to $\sigma_{i,\,i-1} = i^2 (v/2v_0)^3 \sigma_{10}(H^+)$.

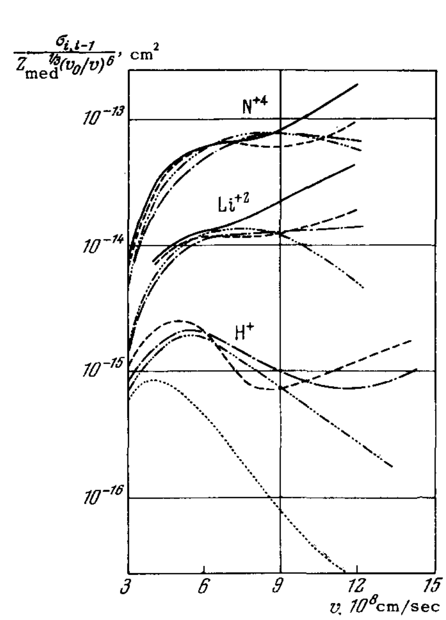

FIG. 14. The relation of the values of $\sigma_{i,\,i-1}/[Z_{med}^{1/3}(v_0/v)^6]$ to v for the ions H^+, Li^{+2}, and N^{+4} in helium (—. . .—), nitrogen (—.—), argon (— —), krypton (—), and hydrogen (. . .), from the results of measurements of $\sigma_{i,\,i-1}$ given in[1, 26, 27].

Highly-charged ions show the weakest dependence of $\sigma_{i,i-1}$ on v, while protons show the strongest. The experimental values of σ_{10} for protons in hydrogen and helium agree with the best theoretical values.[77] For hydrogen and helium nuclei, the relation of $\sigma_{Z,Z-1}$ to v in hydrogen differs little from that given by (4.1). However, the absolute magnitudes of $\sigma_{Z,Z-1}$ are 3—5 times smaller than those given by (4.1).* The cross sections $\sigma_{Z,Z-1}$ for nuclei of $Z > 1$ are related to the values of σ_{10} for protons in the same medium by the following approximate relations:

$$\left.\begin{array}{ll}\sigma_{Z,\,Z-1} \approx Z^2 (v/2v_0)^3 \sigma_{10} & Z > v/2v_0,\\ \sigma_{Z,\,Z-1} \approx Z^5 \sigma_{10} & Z < v/2v_0.\end{array}\right\} \quad (4.13)$$

In agreement with the conclusions of Sec. 4.1, the first of these relations holds not only for nuclei (replacing Z by i), but also for any ions of sufficiently high charge (Fig. 13). This means that the ratio, introduced by Bohr, of the electron-capture probability for nuclei of low charge to the maximum probability shown by highly-charged ions is approximately equal to $(2Zv_0/v)^3$. As the charge of the ions decreases, the velocity dependence of the cross sections generally becomes stronger. However, there is no unequivocal relation between this dependence and the charge of the ion or the ionization potential. For example, for ions having unfilled K shells, the values of $\sigma_{i,i-1}$ decrease more rapidly with increasing v than for other (heavier) ions having the same charge or ionization potential. This evidently involves the more rapid increase in the effective charge of the latter.*

The dependence of the cross sections on the velocity and the medium is more complex than Eqs. (4.5), (4.6), and (4.8) would indicate. This is because the shell structure of the atoms of the medium, which is not reflected in these formulas, has a considerable influence on $\sigma_{i,i-1}$. The values of $\sigma_{i,i-1}$ calculated by either these formulas or by Eqs. (4.9), (4.11), and (4.12) differ from the experimental values by factors up to five on either the high or the low side. However, the idea underlying these formulas, that electrons having orbital velocities $v_{e,med} \sim v$ are preferentially captured, has been confirmed experimentally.

The outer shells of the atoms He, N, Ar, and Kr contain 2, 5, 8, and 8 electrons, respectively, having mean orbital velocities $v_{e,med} \sim 4 \times 10^8$ cm/sec. The next shells contain 0, 2, 8, and 18 electrons, having $v_{e,med} \sim (1-2) \times 10^9$ cm/sec.

Correspondingly, the values of $\sigma_{i,i-1}$ in these gases in the v range from 3×10^8 to 9×10^8 cm/sec vary in approximately the same way (Fig. 14). Here,

*The more accurate values of σ_{21} calculated by Schiff[81] for helium nuclei in hydrogen for $v = (3-7) \times 10^8$ cm/sec differ from the experimental values[22] by a factor of no more than two.

*We should note in this regard the values of σ_{10} calculated by Schiff[81] for helium ions in helium without taking into account the increase in the effective charge of the ion in close collisions. They decline with increasing v more rapidly than the experimental values: they coincide at $v = (3-4) \times 10^8$ cm/sec, but at $v = 6 \times 10^8$ cm/sec, they are 1.7 times smaller than the experimental values.

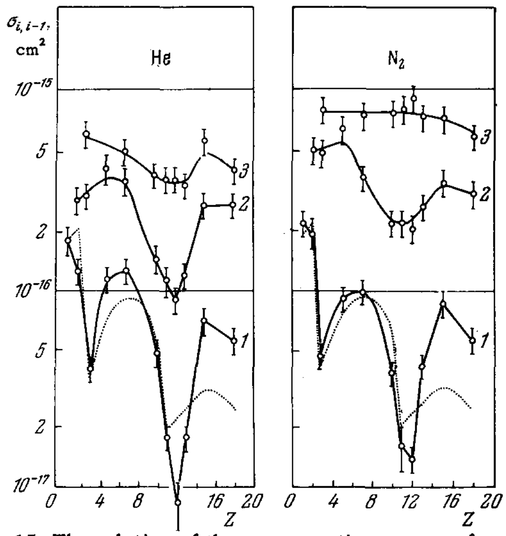

FIG. 15. The relation of the cross section $\sigma_{i,\,i-1}$ for capture of a single electron to the nuclear charge Z of the ions at $v = 2.6 \times 10^8$ cm/sec in helium and nitrogen, according to the experimental data of[1, 26, 27]. The charge i of the ions is indicated next to the curves. The dotted line represents the calculated relations of σ_{10} to Z.

on the average, the ratios of cross sections in these gases for various ions are approximately equal to 4 : 5 : 8 : 10. However, in the velocity range from 9×10^8 to $\sim 13 \times 10^8$ cm/sec, the relation of $\sigma_{i,i-1}$ to v in these media begins to differ considerably, and the ratios of cross sections at $v \sim 12 \times 10^8$ cm/sec prove to be close to 1 : 4 : 8 : 20. The fact that these ratios are close to the ratios of the number of electrons in the inner shells of these atoms indicates that a transition takes place in the region $v = (9-13) \times 10^8$ cm/sec to capture of electrons from the inner shells.

As the existing experimental data on $\sigma_{i,i-1}$ indicate, electrons are captured with greatest probability into states of binding energy $I \sim \mu v^2/2$. A proof of this is the typical pattern of the relation of $\sigma_{i,i-1}$ to Z shown in Fig. 15: For the triply-charged ions, for which the binding energy of the electron upon capture into the ground state is $I_{i-1} > \mu v^2$, the values of $\sigma_{i,i-1}$ depend weakly on Z. However, for the singly-charged ions, for which $I_{i-1} < \mu v^2/4$, the cross sections undergo considerable periodic fluctuations with varying Z. We observe an especially large increase in the cross sections upon going from Li^+ ions to He^+, and from Na^+ to Ne^+. Now, the systems of energy levels into which an electron captured by Li^+ and He^+ ions can enter differ practically solely in the absence of a vacant site in the K shell of the Li^+ ion. Thus, we should ascribe practically the entire difference in the values of σ_{10} for these ions to the capture of the electron into the ground state of the helium atom. Similarly, the increase in σ_{10} on going from Na^+ to Ne^+ indicates that the Ne^+ ions capture the electron into the ground state in 60% of the cases. As the number of vacant sites in the L shell increases with decreasing Z from 10 to 7, the cross sections increase further. The fact that the cross sections are not enhanced as we go from Al^{+3} to Mg^{+3}, Na^{+3}, and Ne^{+3}, i.e., as vacant sites appear in the L shell, indicates that the fraction of electron captures into the L shell for these ions does not exceed the limits of experimental error, i.e., 20%. For the doubly-charged ions, the increase in cross sections as Z decreases from 12 to 7–5 does not begin when vacant sites appear in the L shell, but when the binding energy of the L electrons has declined to a value $I \sim 1.5\,\mu v^2/2$.

Let us assume, in accord with (4.1), that for ions of low charge the relative probability of electron capture into a completely empty shell is proportional to $I^{3/2}$, while for a partly-filled shell it is also proportional to the number of vacant sites in the shell. Then we obtain the curve drawn in Fig. 15 for the relation of σ_{10} to Z. We see from the diagram that at $v \sim 3 \times 10^8$ cm/sec the experimental relation of σ_{10} to Z in the range of Z from 1 to 11 is close to the calculated relation, while in the range $Z > 12$, the experimental cross-sections increase more than the calculated ones do. As will be shown below, this discrepancy does not involve an increase in the probability of capture for $Z > 12$, and is apparently due to an increase in the effective charge of the ions. In the range of higher velocities, the lighter ions also show an analogous increase in the cross sections.

4.3. The Mean Probability of Electron Capture. Similarly to (3.5), we can assume that

$$\sigma_{i,\,i-s} = 2\pi \int W_{i,\,i-s}\varrho\, d\varrho, \qquad (4.14)$$

where the quantities $W_{i,i-s}$ are related to the number k of effectively capturable electrons in an atom of the medium, and to the mean capture probabilities f_i of an individual electron, $f_i^{(2)}$ for an individual pair, $f_i^{(3)}$ for a triplet of electrons, etc., by an equation analogous to (3.6):

$$W_{i,\,i-s} = C_k^s \sum_{t=0}^{k-s} C_{k-s}^t (-1)^t f_i^{(s+t)} = C_k^s f_i^{(s)} - \sum_{t=1}^{k-s} C_{t+s}^t W_{i,\,i-s-t}. \qquad (4.15)$$

The quantity $W_i(2) = W_{i,i-2}/W_{i,i-1}$ is the contingent probability for capture of the second electron. Its mean value in the range of impact parameters making the major contribution to the cross section is

$$\overline{W}_i(2) = \frac{\int W_i(2)\, W_{i,\,i-1}\varrho\, d\varrho}{\int W_{i,\,i-1}\varrho\, d\varrho} = \frac{\sigma_{i,\,i-2}}{\sigma_{i,\,i-1}}. \qquad (4.16)$$

If the probability of electron capture is small, so that the quantities $W_{i,i-1}$ and $W_{i,i-2}$ are mainly determined by the first term of (4.15), then $W_i^{(2)} \approx (k-1) f_i^{(2)}/2f_i$. Now, as is shown by experiment, the ionic charge i exerts no substantial effect on the mean capture probability. Hence, whenever ions of charges i and i − 1 capture electrons primarily into the same states, we can assume that $f_i^{(2)} \approx f_i f_{i-1}$. Then $W_i(2) \approx 0.5(1 - k^{-1}) W_{i-1,i-2}$, and

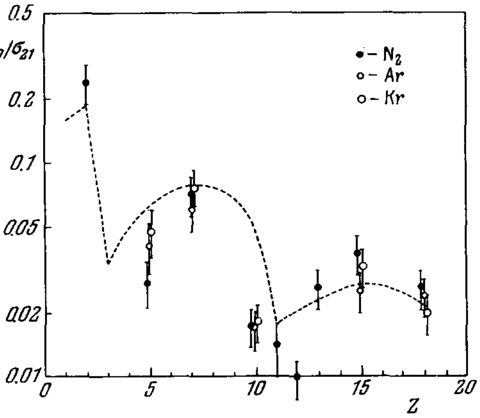

FIG. 16. The ratio σ_{20}/σ_{21} for doubly-charged ions of different elements at $v = 2.6 \times 10^8$ cm/sec in nitrogen, argon, and krypton, according to the results of[27, 28, 52].

$\overline{W}_{i,i-1} \approx 2(1-k^{-1})^{-1}\sigma_{i+1,i-1}/\sigma_{i+1,i}$. If we use a relation analogous to (3.8), $\sigma_{i,i-1} = \pi\bar{\rho}_i^2\overline{W}_{i,i-1}$, we can estimate from the found values of $\overline{W}_{i,i-1}$ and $\sigma_{i,i-1}$ the value of the impact parameters of the collisions making the major contribution to the cross-section:

$$\pi\bar{\varrho}_i^2 \approx (1-k^{-1})\frac{\sigma_{i+1,\,i}\,\sigma_{i,\,i-1}}{2\sigma_{i+1,\,i-1}}. \qquad (4.17)$$

Figure 16 gives typical values of $\sigma_{i,i-2}/\sigma_{i,i-1}$ for ions of low charges. Here we see that the relation of σ_{20}/σ_{21} to Z at $v = 2.6 \times 10^8$ cm/sec generally agrees with the calculated curve shown above in Fig. 14. The ratio σ_{20}/σ_{21} at Z = 12 is diminished to the same extent as the cross section σ_{10}, owing to the competing effect of the loss of the weakly-bound electron, which will be discussed somewhat further on. On the other hand, the relatively small values of σ_{20}/σ_{21} for Z = 10 involve a decrease in the probability of electron capture into the L shell, owing to the exceedingly large binding energy of L electrons (the experimental values of σ_{20}/σ_{21} for Z = 10 agree with those calculated with capture into the L shell neglected).

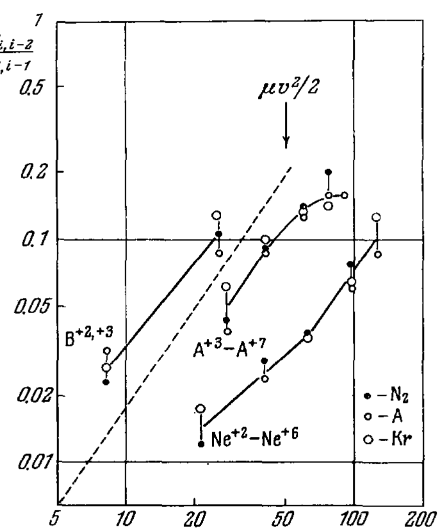

FIG. 17. The ratios $\sigma_{i,\,i-2}/\sigma_{i,\,i-1}$ for boron, argon, and neon ions in nitrogen, argon, and krypton at $v \approx 4.1 \times 10^8$ cm/sec as functions of the electronic binding energy I_{i-2} in the ground state of the produced ion of charge i – 2 (from[28]). The dotted line corresponds to a relation of the type $I^{3/2}$.

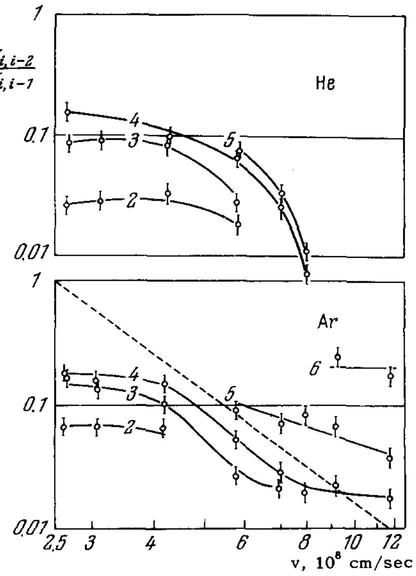

FIG. 18. The relation to v of the ratios $\sigma_{i,\,i-2}/\sigma_{i,\,i-1}$ for nitrogen ions in helium and argon, according to the results of[27, 28]. The numbers next to the curves indicate the charge i of the ions. The dotted line corresponds to a relation of the type v^{-3}.

As the ion charge i increases, the ratios $\sigma_{i,i-2}/\sigma_{i,i-1}$ increase (Fig. 17). However, as the values of $\sigma_{i,i-2}/\sigma_{i,i-1}$ approach 0.2, their dependence on i weakens, and they become practically constant. When $\sigma_{i,i-2}/\sigma_{i,i-1} < 0.1$ for the ions of a given element, these ratios are approximately proportional to $I_{i-2}^{3/2}$. That is, the increase in the mean probability of electron capture as the charge on the ion increases is no greater than the increase in the capture probability arising from the increase in the binding energy of the captured electron. Thus, we observe no appreciable direct effect of the ion charge on the mean probability of electron capture. Again, this corresponds to the ideas on electron-capture probability presented in Sec. 4.1.

In accord with the results of theoretical calculations of the electron-capture probabilities of protons in hydrogen and the cross-section σ_{20} for helium nuclei in helium, the values of $\sigma_{i,i-2}/\sigma_{i,i-1}$ for all ions in helium rapidly decline with increasing v (Fig. 18) in the range $v > 5 \times 10^8$ cm/sec. Consequently, at $v \geq 8 \times 10^8$ cm/sec, the values of $\sigma_{i,i-2}/\sigma_{i-1}$ do not exceed 0.01.* The variation in the

*The smallest of the measured ratios $\sigma_{i,\,i-2}/\sigma_{i,\,i-1} \sim 0.0003$ is known from the results of Afrosimov et al.[25] for protons in helium at $v \approx 5.5 \times 10^8$ cm/sec.

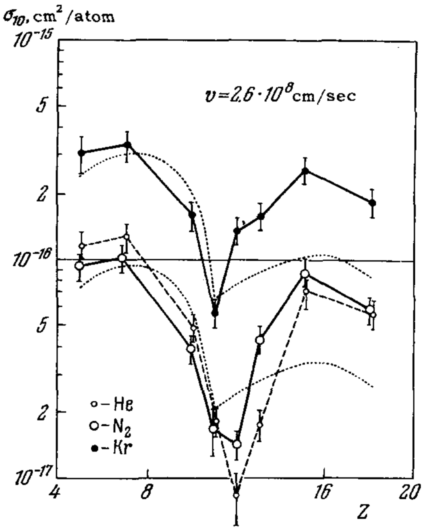

FIG. 19. The relation of σ_{10} to Z in helium, nitrogen, and krypton at v = 2.6 × 10^8 cm/sec, from the data of[27]. The dotted curves give the calculated relation of σ_{10} to Z.

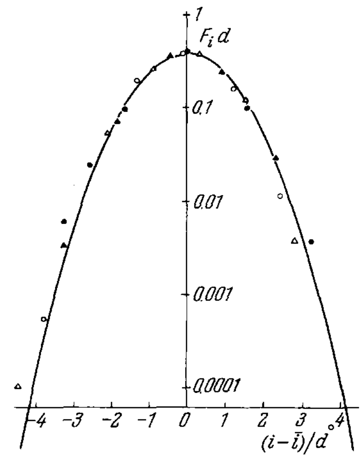

FIG. 20. Values of $F_i d$ for an equilibrium charge distribution in a beam of nitrogen ions upon passing through nitrogen (●, ○) or a celluloid film (▲, △), from the results of[12, 35]. The open symbols pertain to an ion velocity v = 6 × 10^8 cm/sec, and the solid symbols to v = 12 × 10^8 cm/sec. The solid curve represents a Gaussian distribution.

mean capture probability also basically determines the velocity dependence of the cross sections, since the quantity $\pi\bar{\rho}_i^2$ depends much more weakly on the velocity. However, when ions pass through nitrogen, argon, and krypton, whose atoms contain electrons of mean orbital velocities $\sim 10^9$ cm/sec, the values of $\sigma_{i,i-2}/\sigma_{i,i-1}$ for highly-charged ions at v ~ 10^9 cm/sec remain about the same as at v ~ 3 × 10^8 cm/sec (see Fig. 18). Thus, the decline in the cross sections with increasing velocity for these ions in these media involves a decrease in the quantity $\pi\bar{\rho}_i^2$. At v ~ 10^9 cm/sec, for which the electrons are captured from the innermost shell, the values of $\pi\bar{\rho}_i^2$ are 1½ orders of magnitude smaller than at v ~ 3 × 10^8 cm/sec. The values of $\bar{\rho}_i$ agree in order of magnitude with the dimensions of the region occupied by the electrons subject to capture in the atoms of the medium. For ions of low charges, in accord with the conclusions of Sec. 4.1, the values of $\sigma_{i,i-2}/\sigma_{i,i-1}$ generally decrease with increasing v, although not according to such a simple law as is given by Eq. (4.2): while at

$$v \sim (5 - 8)\cdot 10^8 \text{ cm/sec}$$

the ratios $\sigma_{i,i-2}/\sigma_{i,i-1}$ are proportional to v^{-3}, in the range

$$v = (8 - 12)\cdot 10^8 \text{ cm/sec}$$

where a transition occurs to electron capture from the inner shells of the atoms of the medium, the values of $\sigma_{i,i-2}/\sigma_{i,i-1}$ vary only slightly (see Fig. 18).

4.4. Competition Between Electron Capture and Loss. The processes of electron loss and capture are opposites: the former increases the charge on the ions, but the latter decreases it. Hence, if the probability of electron loss happens to be near unity in the collisions making the major contribution to the capture cross section, the latter will be appreciably diminished. For example, as the pertinent estimates show, we should expect such a decrease in the value of $\sigma_{i,i-1}$ in nitrogen at v = 2.6 × 10^8 cm/sec for Mg^+ ions, and for Al^+ ions to a somewhat lesser extent. The experimental relations of σ_{10} and of σ_{20}/σ_{21} to Z (see Figs. 15 and 16) confirm this conclusion. However, in krypton the value of σ_{12} at the same velocity is twice as great as in nitrogen, and the value of σ_{10} is almost four times as great. Consequently, there is a greater increase in the value of the mean impact parameter for capture in krypton than in the value of the impact parameter at which the probability of electron loss approaches unity, and we observe no decrease in the capture cross section (Fig. 19).

We should expect a decrease in the electron-loss cross sections for highly-charged ions, for which $\sigma_{i,i-1} \gg \sigma_{i,i+1}$. However, no clearcut manifestations of an effect of electron capture on the electron-loss cross sections have been observed.

V. EQUILIBRIUM CHARGE DISTRIBUTION IN ION BEAMS

5.1. The Fundamental Laws of Equilibrium Charge Distributions. In ion beams passing through a solid or gaseous material, the distribution of the ions among the most intense charge groups is nearly Gaussian:

$$F_i \approx (2\pi d^2)^{-1/2} \exp\left[-\frac{(i-\bar{i})^2}{2d^2}\right], \qquad (5.1)$$

and is basically characterized by two parameters: the mean charge $\bar{i} = \sum_i iF_i$, and the half-width of the dis-

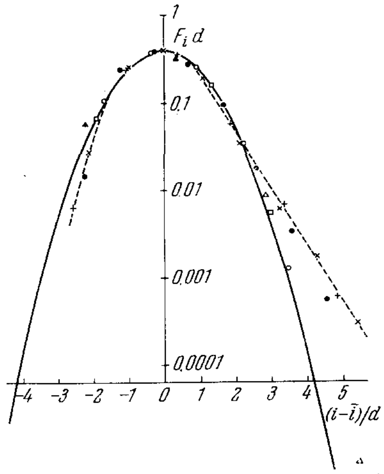

FIG. 21. Values of $F_i d$ for an equilibrium charge distribution in beams of ions of lithium (△), boron (□), nitrogen (o), sodium (+), phosphorus (×), and argon (●) upon passing through nitrogen at $v = 2.6 \times 10^8$ cm/sec, from the results of[35]. The solid curve represents a Gaussian distribution, while the dotted curve is the mean distribution for the ions of $Z > 10$; △ – Li; □ – B; o – N; + – Na; × – P; ● – Ar.

tribution $d = [\sum_i (i - \bar{i})^2 F_i]^{1/2}$ (Figs. 20 and 21). The only cases excluded are those for $\bar{i} < 1$ or $\bar{i} > Z - 1$, for which the distribution turns out to be highly asymmetric with respect to $\bar{i}$. One observes systematic deviations from Gaussian form also for ions of $Z > 10$ when passing through a gas (Fig. 21).

The closeness of the equilibrium charge distribution to Gaussian form involves the approximately linear relation of $\ln(F_{i+1}/F_i)$ to i (Figs. 22 and 23). To explain, note that if the difference $\Delta_i = \ln(F_{i+1}/F_i) - \ln(F_i/F_{i-1})$ is independent of i, then

$$F_i = A \exp\left[-\frac{(i-i_0)^2}{2d^2}\right],$$

where $d^2 = 1/\Delta_i$. Here, when $1 < \bar{i} < Z - 1$, the coefficient A is close to $(2\pi d^2)^{1/2}$, and i_0 is close to $\bar{i}$.[14] The systematic enhancement of the F_i values over the Gaussian values for ions of $Z > 10$ when $i > \bar{i} + 3d$ is due to the fact that F_{i+1}/F_i abruptly ceases to decrease with increasing i when $i \gtrsim \bar{i} + 2d$.

The mean charge on the ions of each element depends on their velocity and on the medium through which they are passing (Figs. 24 and 25). The difference between the maximum and minimum values of $\bar{i}$ in rarefied gases amounts to ~20% in most cases. The value of $\bar{i}$ in the lightest solid media is greater than in heavy solids by 8–10% for uranium fission fragments, and by about 2% for nitrogen ions. At $v \sim (2.5–5) \times 10^8$ cm/sec for ions having $Z = 3–7$, the mean charge in solid materials exceeds the

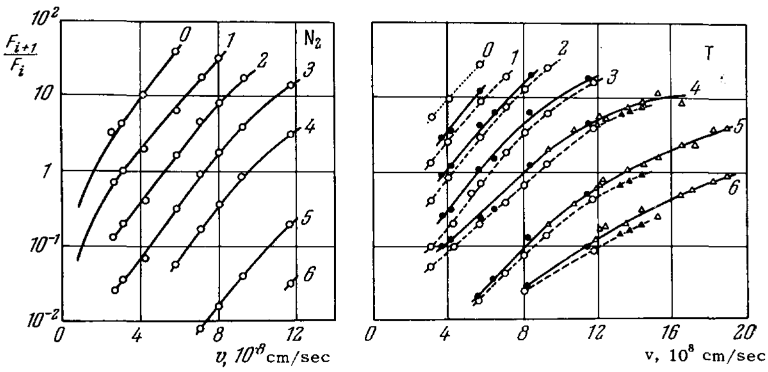

FIG. 22. The ratios F_{i+1}/F_i for a beam of nitrogen ions upon passing through nitrogen or an organic film as a function of the ion velocity. o – Data from[35]; ● – from[12]; △ – from[8]; △ – from[10]; the lines at $v < 2 \times 10^8$ are from[85]. The numbers next to the curves give the values of i.

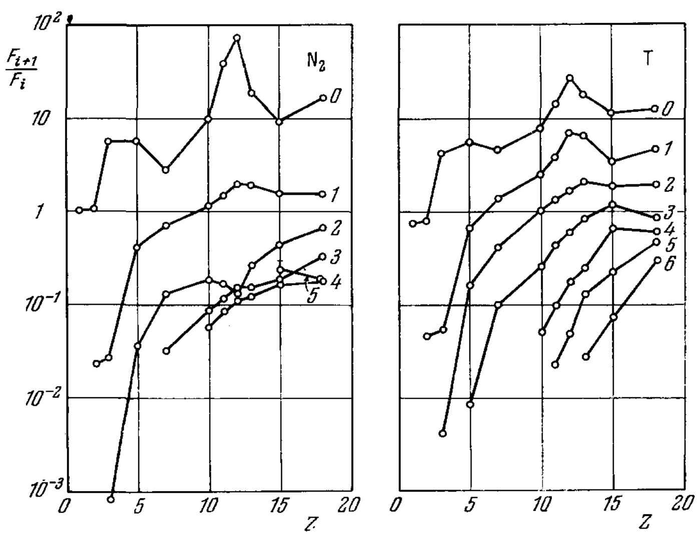

FIG. 23. Values of F_{i+1}/F_i for the ions of different elements upon passing through nitrogen or a celluloid film at $v = 2.6 \times 10^8$ cm/sec, from the experimental data of[1, 12, 13, 26, 35]. The values of i are indicated next to the curves.

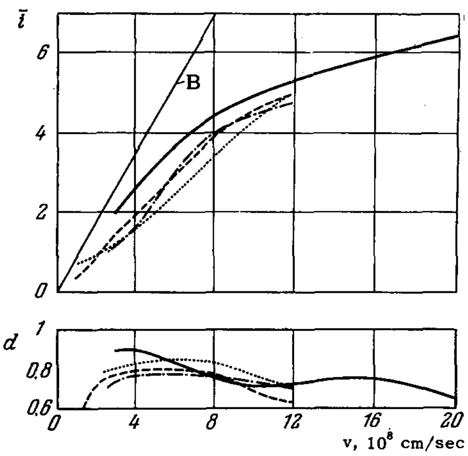

FIG. 24. The relation of the mean charge $\bar{i}$ and the half-width d of the distribution to the velocity v for nitrogen ions upon passing through helium, nitrogen, krypton, and a solid, from the results of[8, 12, 35, 85]. B – according to Bohr's formula (5.3); ... He; – – N_2; –·– Kr; —— solid.

maximum value of $\bar{i}$ in gaseous media by 15—20%, or by 50—80% for ions having Z = 10—18. This difference decreases with increasing velocity. The mean charge of uranium fission fragments passing through a gas increases by $\sim$10—15% with increase of the gas pressure up to 10—50 mm Hg, but it increases much more slowly with further increase in the pressure.[3] An analogous increase in the mean charge of nitrogen ions takes place as the gas pressure is increased from $\sim 10^{-3}$ to $\sim 10^{-2}$ mm Hg.[86]

At a fixed velocity, the values of $\bar{i}$ for the ions of different elements vary on the average approximately as $Z^{3/4}$ in solids, and as $Z^{3/5}$ in gases (Fig. 25). The velocity at which the mean degree of ionization $\bar{i}/Z$ of the ions in a given medium attains an assigned value is proportional on the average to Z^{α}; here $\alpha \approx 0.5$ for all gases when $\bar{i}/Z \lesssim 0.6$, but for solids, the exponent α varies from 0.2 to 0.5 as $\bar{i}/Z$ increases from 0.2 to 0.6.

In line with this, we can represent the values of $\bar{i}/Z$ in gases as a function of the parameter $vZ^{-\alpha}$. In particular, we can assume for $\bar{i}/Z < 0.6$ that

$$\frac{\bar{i}}{Z} \approx \frac{kv}{Z^{1/2} v_0}, \tag{5.2}$$

where k = 0.4 in nitrogen and argon, 0.35 in helium, and 0.38 in krypton. When $v \gtrsim 3 \times 10^8$ cm/sec, the experimental values of $\bar{i}/Z$ for ions having Z > 2 generally differ from the values given by Eq. (5.2) by no more than 5—10%.

In krypton, in which the function $\bar{i}(v)$ differs further from linearity, these deviations amount to $\sim$10—20%.

The values of d (see Figs. 24 and 25) for ion beams passing through different gases differ as a rule by no more than 10%. When the ions pass through solids, the d values are usually somewhat larger than in gases at the same value of $\bar{i}$. In the velocity range in which $\bar{i} \sim (0.3—0.8)$ Z, the values of d depend weakly on v, while they decline as we move away from this range. On the average, the values of d increase with increasing Z approximately as $Z^{1/2}$. Since the quantities

$$\Delta_i = \ln\frac{F_{i+1}}{F_i} - \ln\frac{F_i}{F_{i-1}}$$

increase as we go from the ionization of one electron shell to that of another, the relation of d to v and Z shows minima (see Figs. 24 and 25).

5.2. The Reasons for the Regularities Observed in the Equilibrium Charge Composition of Ion Beams Passing Through Rarefied Gases. Owing to the relative smallness of the cross sections for simultaneous loss or capture of several electrons, the distribution of the ions among the most intense charge groups is largely determined by the cross sections for loss or capture of a single electron. These groups obey Eq. (1.4). Since the ratios $\sigma_{i,i+1}/\sigma_{i+1,i}$ are approximately proportional to exp (−mi), the equilibrium charge distribution over the most probable charge states proves to be nearly Gaussian, with a half-width $d \approx m^{-1/2} = (m_{capture} + m_{loss})^{-1/2}$, where $m_{capture}$ and m_{loss} are the mean values of $\ln(\sigma_{i+1,i}/\sigma_{i,i-1})$ and $\ln(\sigma_{i-1,i}/\sigma_{i,i+1})$ in the region $i \sim \bar{i}$. Processes of simultaneous loss or capture of several electrons

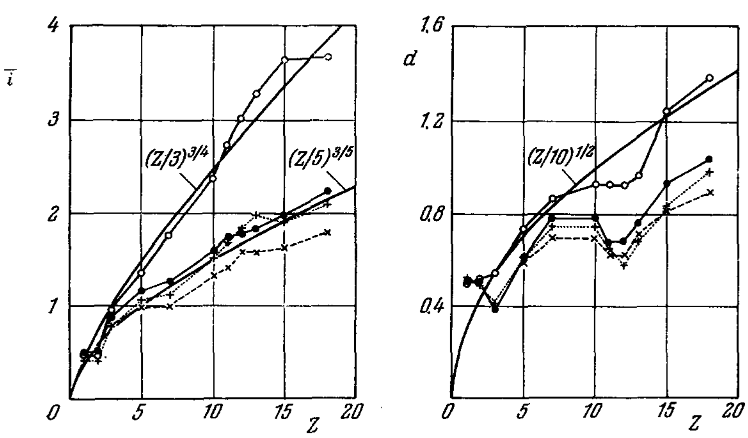

FIG. 25. Values of $\bar{i}$ and d for the ions of different light elements at v = 2.6 × 10^8 cm/sec in helium (+), nitrogen (●), krypton (×), and celluloid (○), from the experimental data of[1, 12, 13, 26, 35].

exert a determining influence on the number of ions occurring in charge states of low probability.[28,30]

The results of studying the cross-sections for electron loss and capture indicate that ions with charges $i < \bar{i} - d^2a - 2$ [where a is the value of $\ln(\sigma_{i+s,i}/\sigma_{i+s+1,i})$ averaged over s] are largely formed from ions having charges $i \sim \bar{i} - d^2a$, through simultaneous capture of several electrons. On the other hand, ions having $i > \bar{i} + d^2b + 2$ [where b is the value of $\ln(\sigma_{i-s,i}/\sigma_{i-s-1,i})$ averaged over s] are largely formed from ions of charge $i \sim \bar{i} + d^2b$ through simultaneous loss of several electrons. Here the values of F_i prove to be larger than the Gaussian ones, while the ratios F_{i+1}/F_i depend weakly on i.* Since $b < a$, the transition point where F_{i+1}/F_i becomes constant occurs at a smaller difference between i and $\bar{i}$ in the region $i > \bar{i}$. For ions having $Z > 10$, for which $d^2 \sim 0.5$ and $b \sim 0.2-0.4$, the ratios F_{i+1}/F_i become constant even at $i \gtrsim \bar{i} + 1$ (see Fig. 23). Hence, the cross sections for loss of several electrons exert an appreciable influence on the values of $\bar{i}$ and d as well. For example, if at $v = 2.6 \times 10^8$ cm/sec all the cross-sections for loss of two or more electrons became zero, while the rest of the cross-sections remained unchanged, the mean charge of magnesium ions in nitrogen would decrease by 5%, while for nitrogen, neon, and argon ions it would decrease by 25–30%. Here the value of d for nitrogen, neon, and magnesium ions would decrease by 5–10%, while it would decrease by more than 25% for argon ions. Nevertheless, simultaneous loss of several electrons in itself leads practically only to a broadening of the distribution. This is because the mean charge of the ions would be practically invariant if the sum of the cross-sections for loss of individual electrons remained constant as the cross-sections for loss of several electrons declined to zero (i.e., the cross section for loss of a single electron became equal to $\Sigma_s s\sigma_{i,i+s}$).

Processes of loss of several electrons generally play a greater role as Z increases; the degree of broadening of the charge distribution due to simultaneous loss of electrons is also increased. For example, at $v \sim 3 \times 10^8$ cm/sec, half of the increase in d as Z increases from 7–10 to 18 is due to an increase in the fraction of the electrons that are removed in a single collision.

Although an appreciable fraction of the quantities $\bar{i}$ and d involves the cross sections for loss of two or more electrons, the fundamental laws governing these quantities are due to the corresponding properties of the cross sections for loss or capture of a single electron. In particular, the relatively small value of the mean charge of ions in helium at $v \sim (2-10) \times 10^8$ cm/sec is explained by the relatively large cross section for electron capture, while the small value of $\bar{i}$ in krypton at $v \lesssim 4 \times 10^8$ cm/sec is explained by the anomalously small cross section for loss of an electron. Since the ratios $\sigma_{i+1,i}/\sigma_{i,i-1}$ and $\sigma_{i-1,i}/\sigma_{i,i+1}$ vary much less in going from one medium to another than the cross sections $\sigma_{i,i-1}$ and $\sigma_{i,i+1}$ do, the exponents m_{capture} and m_{loss}, and hence also the values of d, turn out to be rather close for different gases. Now, the cross sections for loss or capture of an electron depend on the ionization potential I_i and the number q_i of electrons in the outer shell of the ion. Hence, as Z increases, the exponents m_{capture} and m_{loss} vary non-monotonically, and go through maxima at $Z = 3$ and $Z = 11$-12. Correspondingly, the relation of d to Z shows minima at the same values of Z (see Fig. 25).

Since the quantities I_i and q_i exert a strong effect on the cross-sections for loss or capture of an electron, we should expect also a minimum on the curve of $\bar{i}$ plotted against Z in the region $Z = 10-11$ (at $v \sim 3 \times 10^8$ cm/sec). The absence of this minimum (see Fig. 25) is mainly due to the increase in the value of F_2/F_1 for the ions from nitrogen to sodium, and to the disappearance of the minimum in the dependence of F_2/F_1 on Z (such a minimum exists for F_3/F_2; see Fig. 23). The increase in the F_2/F_1 values is due to the decrease in σ_{21} owing to a decrease in the probability of capture of an electron into the ground state and states lying close to it for ions having Z from 7 to 11. This is due to the extremely large binding energy in these states.

5.3. Interpretation of the Features of the Equilibrium Charge Composition of Ion Beams in Condensed Media. Bohr and Lindhard[23] explained the higher value of $\bar{i}$ in a beam of fast particles that had passed through a condensed medium by the increase in the cross sections for loss of electrons that were unable to drop from the excited state to the ground state within the time between two successive collisions of the ion with the atoms of the media. This explanation has now been confirmed by the experimentally-obtained[87] increase in electron-loss cross sections as the time between collisions is shortened, and by the experimental proofs given in Sec. 4.2 of preferential capture of electrons into highly-excited states.

Neufeld[88] has pointed out that electron-loss cross sections can be enhanced in condensed media by the polarization of the medium by the fast ion. According to his calculations, for helium ions at $v = 2v_0$, the ratio F_2/F_1 in liquid argon should be 40% greater than in gaseous argon, while it should be only ~10% greater if one neglects polarization. Since Neufeld assumed that the ion passing through the material moves in a homogeneous electric field which is responsible for all of its deceleration in the material, his calculation gives the maximum imaginable in-

*Eq. (1.4) implies that when $i < \bar{i} - d^2a - 2$, we have $F_{i+1}/F_i \approx (\sigma_{i_a,i+1}/\sigma_{i_a,i})/(\sigma_{i+1,i+2}/\sigma_{i,i+1})$, where $i_a = \bar{i} - d^2a$. However, when $i > \bar{i} + d^2b + 2$, we have $F_{i+1}/F_i \approx (\sigma_{i_b,i+1}/\sigma_{i_b,i})/(\sigma_{i+1,i}/\sigma_{i,i-1})$, where $i_b = \bar{i} + d^2b$.

Values of a_i, δ_i^0, and q_i^* for ions having $i \approx i_s - 1/2$ at $v = 2.6 \times 10^8$ cm/sec

Ion	$Z-i$	a_i	δ^0_{max}	δ^0_σ	q_i^*	$\frac{q_i^*}{Z-i}$
Li^+	2	—	—	—	0.15	0.07
B^+	4	1.4	0.7	0.6	1.8	0.4
N^+	6	1.9	0.5	0.4	2.5	0.4
Ne^{+2}	8	7.9	0.13	$\leqslant 0.1$	3.7	0.5
Al^{+3}	10	21	0.05	$\leqslant 0.1$	4	0.4
P^{+3}	12	24	0.04	$\leqslant 0.1$	6	0.5
Ar^{+3}	15	4.6	0.2	$\leqslant 0.1$	4	0.3

crease in F_2/F_1 in a condensed medium. Actually, the values of F_2/F_1 for helium ions passing through celluloid, aluminum, and silver exceed the values of F_2/F_1 in nitrogen and argon by ~10%. Thus, there is apparently no reason to list the polarization of the medium among the fundamental factors determining the enhancement in the mean charge of ions in condensed media.

Excited ions in a beam of fast particles can be formed either by excitation of the electrons occurring in the ion, or by capture of electrons into excited states. The main source of the enhancement of the mean ion charge in a condensed medium is apparently electron capture into highly-excited states. For example, at $v \sim 3 \times 10^8$ cm/sec, a considerable increase in $\bar{i}$ begins at $Z \gtrsim 7$, i.e., at the point where ions having $i \approx \bar{i}_s + 1/2$ (where $\bar{i}_s$ is the mean charge of the ions in a solid medium) begin to capture electrons preferentially into highly-excited states. The excitation of ions in collisions with atoms of the medium should play a relatively small role in these cases, since electron capture into strongly bound states occurs with a lower probability, whereas the excitation cross section amounts to something of the order of $\sigma_{i,i+1}$ for a rarefied gas, which is considerably smaller than the electron-capture cross section $\sigma_{i,i-1}$. Among the experimental values of $(F_{i+1}/F_i)_s$ for a solid medium and $(\sum_k k\sigma_{i,i+k}/\sigma_{i+1,i})_g$ for a rarefied gas, we can find by Eq. (1.13) the maximum possible values δ^0_{max} of the relative probability of electron capture into strongly-bound states, corresponding to the assumption that all of the excited ions are formed through electron capture. These quantities agree with the values of δ^0_σ obtained from the experimental relation of $\sigma_{i,i-1}$ to Z (see the table). Thus Eq. (1.13) does not require us to take into account ion-excitation processes in order to explain the increase in the mean charge of the ions in condensed media.

Owing to the low probability of electron capture into strongly bound states, an appreciable fraction of the electrons occurring in the ions having charges $i \sim \bar{i}_s$ happen to be in excited states of binding energies $I \sim \mu v^2/2$. We can estimate the number q_i^* of these electrons by using the results of studying the cross sections for electron loss and capture, by means of Eq. (1.11). Here we assume that $\overline{\sigma_{i,i+1}} = q_i^*\sigma_i^* + \sigma^0_{i,i+1}$, where σ_i^* is the cross section for loss of an individual electron from a state of binding energy $\sim \mu v^2/2$, and $\sigma^0_{i,i+1}$ is the cross section for loss of the electrons remaining in the lower energy states. [We can also estimate q_i^* for ions having $i < i_s - 1$ and $i > \bar{i}_s$, from the value of q_i^* for $i \approx \bar{i} - 1/2$, using Eqs. (1.9) and (1.10).] As the results of these estimates show, from one-third to one-half of all of the electrons occur in highly-excited states for ions of $Z \gtrsim 5$ (see the table). The cross section for loss of these electrons then determines the quantity $\overline{\sigma_{i,i+1}}$, i.e., $\overline{\sigma_{i,i+1}} \approx q_i^*\sigma_i^*$.

Owing to the decrease in the binding energy of the electrons that determine $\overline{\sigma_{i,i+1}}$, the dependence of $\overline{\sigma_{i,i+1}}$ and F_{i+1}/F_i on i becomes weaker, and d increases in solids. The marked weakening of the effect of the ionization potential of the ion on $\overline{\sigma_{i,i+1}}$ causes the minimum in the relation of d to Z to vanish (see Fig. 25). In line with the increase in the values of d and $b = \overline{\ln(\sigma_{i-s,i}/\sigma_{i-s+1,i})}$, the transition to constant values of F_{i+1}/F_i in the region $i > \bar{i}$ in solids should occur at a point where i differs more greatly from $\bar{i}$. Owing to the transition of part of the electrons to excited states, electron capture into the more strongly bound states characterized by higher capture probabilities becomes possible for ions having $I_i < \mu v^2/2$. Consequently, the electron-capture cross section is increased, and the values of F_{i+1}/F_i are somewhat diminished in solid media for ions of low charge (see, e.g., the values of F_1/F_0 in Fig. 23).

As Bohr and Lindhard[23] proposed, the decrease in the values of $\bar{i}_s$ with increasing Z_{med} involves the increase in the mean binding energy of the electrons captured by the ion. In such a case, a certain narrowing of the equilibrium charge distribution should accompany the decrease in $\bar{i}_s$. Such a narrowing actually occurred in the experiments with nitrogen ions,[12] but it was observed most strongly in going from beryllium to celluloid, rather than in the region $Z_{med} \sim 7-80$, where the major variation in the value of $\bar{i}_s$ occurs. Apparently, just as in gaseous media, one can explain the lowering of the mean charge of the ions as Z_{med} in increased from ~10 to ~80 by the decrease in the cross-sections for electron loss owing to polarization of the atoms of the medium.

According to (1.16), the increase in the electron-loss cross sections as the density of the medium is raised should occur largely at an atom concentration $n \sim 1/v\tau\sigma_i^*$ in the medium. Since the lifetime τ of the excited states for ions of light elements is much greater than for uranium fission fragments, they should show *an increase in mean charge at considerably lower densities of the medium.* If the increase in the mean charge of the fission fragments is observed largely at gas pressures ~10–50 mm Hg, then for ions of the light elements with $v \sim 10^9$ cm/sec, $\sigma_i^* \sim 10^{-16}$ cm^2, and $\tau \sim 10^{-7}$ sec, the increase in $\bar{i}$ should occur at $n \sim 10^{14}$ atoms/cm^3, i.e., at a gas pressure of 10^{-3}–10^{-2} mm Hg, as is confirmed by experiment.*[86]

Although the mean charge of the ions increases with increasing gas density, it still does not reach the values of $\bar{i}_s$ obtained when the ions are passed through solid materials. In the opinion of Bohr and Lindhard,[23] the smaller value of $\bar{i}$ in a dense gas is to be explained by the smaller electron-loss cross sections. The latter are due to the redistribution of the excitation energy of the electrons among the various electrons of the ion that occurs during the time between collisions of the ion with the atoms of the gas. Such an energy redistribution will lead to a decrease in the cross sections only when the value of σ_i decreases more rapidly than I^{-1} with increasing binding energy I of the electron. Now, this relation between σ_i and I holds when $I \geq \mu v^2/2$ [29,30] whereas electrons show the greatest probability of capture into states having $I \sim \mu v^2/2$. Thus, the cited decline in the electron-loss cross section should actually take place.

5.4. Methods of Calculating the Mean Charge of Ions. The value of the mean charge $\bar{i}$ of the ions is determined by the cross sections for electron loss and capture, just as the overall equilibrium charge distribution in the ion beam is. Hence, a systematic theoretical calculation of the mean charge of the ions requires the calculation of these cross sections. Bell,[22] Bohr and Lindhard,[23] and Gluckstern[16] have performed such a calculation of $\bar{i}$ for ions of $Z > 2$. Owing to the approximate nature of the calculations of the cross sections, the values of $\bar{i}$ obtained are not very accurate, and differ from the experimental values by 20–30% in many cases. Apparently, in a number of cases, one can get more precise values of $\bar{i}$ in this way by taking into account the results of the most recent studies and using semiempirical methods to calculate the cross sections.

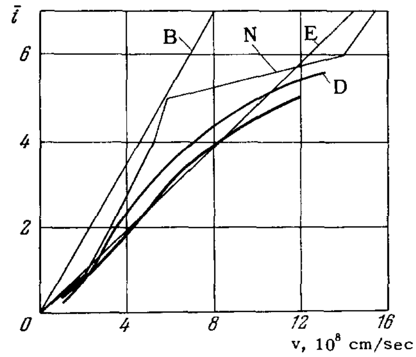

FIG. 26. The mean charge of nitrogen ions in nitrogen as a function of the ion velocity v. B – according to Bohr's formula (5.3), N – as calculated by Neufeld[91], D – as calculated by Dmitriev[93], E – according to the empirical formula (5.2). The heavy line shows the experimental results.[35,85]

In view of the difficulties of calculating the cross sections, methods of determining the mean charge of ions that do not require a preliminary calculation of the cross sections are of considerable interest. Above all, these include the methods based on the criteria of Bohr and Lamb. According to Bohr,[89] an ion moving in a material retains only those electrons whose orbital velocities $v_e > v$. According to Lamb,[90] the mean charge of the ions is determined by the condition that the binding energy I of the retained electrons is greater than $\mu v^2/2$. If we apply these criteria to real ions, as Neufeld[91] has done, they imply that the dependence of $\bar{i}$ on v must become greatly weakened (Fig. 26) as we go from the ionization of one electron shell to the ionization of another, while minima should appear on the curve of $\bar{i}$ versus Z. Actually, this is not observed, since the values of σ_{ik} and hence also $\bar{i}$ are greatly affected, not only by the values of I_i or v_e, but also by the number of electrons in the outer shell of the ion, the extent to which this shell is filled, and the effective charge of the ion. Each of these factors acts in the direction of smoothing the relation of $\bar{i}$ to v and Z. Hence, these criteria lead to better agreement with experiment if we use a statistical model to describe the ions.

Using the simplest approximate expression for the orbital velocity of an electron in the statistical model, $v_e = iZ^{-1/3}v_0$, Bohr found that

$$\bar{i} = Z^{1/3}\frac{v}{v_0}. \tag{5.3}$$

Bohr's formula gives for various gases a correct mean dependence of i on v when $\bar{i}/Z \lesssim 0.6$. It gives

*This implies that the equilibrium charge distribution at pressures $\sim 10^{-2}$ mm Hg, at which one usually measures the equilibrium charge state of a beam, can differ from the equilibrium distribution in a more rarefied gas. Such a discrepancy is indicated by the fact that when one substitutes the experimental values of F_i and σ_{ik} into (1.4) the right-hand side of this relation usually turns out to be larger than the left-hand side. For ions having Z from 2 to 18, this excess amounts on the average to a value $h \sim$ 10–15%. Thus, when $d \sim 0.5-1$, the mean charge in a rarefied gas should be less than the measured charge by an amount $\Delta\bar{i} \approx d^2h \sim 0.03-0.15$.

a somewhat weaker dependence of $\bar{i}$ on Z than the actual one: for ions having $Z \sim 40-50$, the values of $\bar{i}$ calculated by (5.3) exceed the experimental values by 20—30%, while for ions having $Z = 7-20$, they exceed the latter by a factor of 1.5—2.

In order to obtain $\bar{i}$ values closer to the experimental values, Brunings, Knipp, and Teller [92] suggested that the ion retains electrons having $v_e > v\gamma$, where γ is a slowly-varying coefficient of the order of unity, and performed some more thorough calculations of the mean orbital velocity of the electrons. Either the most weakly bound, or the outermost electron was being removed in an ion described by a statistical model. In the first variant (with γ_1 = const.), the quantity $\bar{i}/Z$ for ions of $Z \gtrsim 12$ proved to be a function of $vZ^{-2/3}$, while for ions of $Z \sim 6-10$ in the range $\bar{i}/Z \lesssim 0.6$, it was a function of $vZ^{-\alpha}$ with $\alpha \approx 0.55$. In the second variant (with γ_2 = const.), the exponent $\alpha \approx 1/3$ for $\bar{i}/Z \leq 0.6$. For gaseous media the experimental value $\alpha = 0.5$ lies between these extreme values, but for solid media $\alpha < 1/3$ when $\bar{i}/Z < 0.4$. The coefficient γ must depend on v to give agreement with experiment. This implies that we can obtain from Bohr's criterion, as generalized by introducing the coefficient γ, the correct mean relation of the values of $\bar{i}$ in gases to v and Z, and get values of $\bar{i}$ near the actual ones by using a statistical model to describe the ion and assuming that in the process of stripping the ion an electron is removed that has an orbital velocity proportional to $Z^{1/2}$ at the given value of $\bar{i}/Z$. The simplest function $\bar{i}(Z, v)$ that can be obtained here for $\bar{i} < 0.6Z$ agrees with the approximate empirical formula (5.2).

Dmitriev [93] and Livesey [94] have used unique methods of calculating the mean charge of the ions, based on the actual values of the ionization potentials. In Dmitriev's calculations, the mean charge of the ions was assumed to be equal to the sum of the probabilities of removal of each of the Z electrons of the atom. The probability P of removal of an electron was assumed to depend only on v/u. Here a definite value of u was ascribed to each electron: the ith electron in sequence corresponded to the value of u found from the ith successive ionization potential of the atom. The value of $P(v/u)$ was taken equal to the probability of removing an electron from a hydrogen atom, i.e., the value of the mean charge of hydrogen ions at the same value of v/u in the same medium. Owing to the simultaneous use of all the potentials for successive ionization of the atom, the relation of $\bar{i}$ to v proves to be smoother and the values of $\bar{i}$ closer to the experimental than for the usual application of Lamb's criterion to an actual atom. The values of $\bar{i}$ obtained by Dmitriev's method for ions of $Z \leq 20$ in nitrogen generally differ from the experimental values by no more than 20%. For uranium fission fragments, they differ by no more than 10%. However, the relation of $\bar{i}$ to Z for $v < 4 \times 10^8$ cm/sec remains non-monotonic, while the width of the calculated equilibrium charge distribution is too high by $\sim 30\%$.

In calculating the mean charge of ions having Z from 3 to 10 in nitrogen and in a nuclear photoemulsion, Livesey used the mean charge of hydrogen and helium ions in these media. He assumed that at any assigned value of v/u_Z, where $u_Z = Zv_0$, the degree of ionization of the K shell for ions of all elements having $Z \geq 2$ is the same. At a definite value of v/u_{Z-3}, where $u_{Z-3} = (2I_{Z-3}/\mu)^{1/2}$ is the orbital velocity of the first L electron, he assumed the degree of ionization of the L shell for lithium ions to coincide with the degree of ionization of hydrogen atoms. For neon ions, he assumed the degree of ionization to coincide with that of the K shell of helium at the same values of v/u_Z. He calculated the degree of ionization of the L shell of ions having Z from 4 to 9 from assumed intermediate curves of the relation of the degree of ionization to v/u. The values that Livesey used for the mean degree of ionization of the K and L shells differ little from those obtained when calculated by Dmitriev's method. Hence, the $\bar{i}$ values given by Livesey for ions of $Z = 7-10$ in nitrogen are close to the $\bar{i}$ values obtained by Dmitriev; like the latter, they differ from the experimental values by no more than 20%. Since the $\bar{i}$ values for hydrogen and helium ions in a gas and in a solid are close together, the $\bar{i}$ values obtained by Dmitriev and Livesey for the ions of different elements in solid media differ little from the $\bar{i}$ values in a gas. However, actually for ions with $Z \gtrsim 10$, the mean charge in a solid is considerably greater than in a gas.

The semi-empirical methods based on established regularities in the equilibrium charge distributions and on the use of concrete experimental data on $\bar{i}$ should occupy an important place among the approximate methods of calculating the mean charge of ions not requiring a preliminary calculation of the cross sections for electron loss and capture. Unfortunately, these methods have not been widely enough developed.

For example, one can base the development of one of these methods on the concept of the quantity $\bar{i}/Z$ as a function of $vZ^{-\alpha}$. Papineau [95] has used this idea in its simplest form to calculate the mean charge of ions having Z from 3 to 10 in a nuclear photoemulsion. He assumed that the values of $\bar{i}/Z$ are functions of $vZ^{-2/3}$ for all these ions. To find this function $f(vZ^{-2/3})$, he used the experimental data existing at that time on the values of $\bar{i}/Z$ in different media. In a more correct application of this method for various media, evidently one should adopt a function $f(vZ^{-\alpha})$, and in addition, take into account a certain dependence of α on $\bar{i}/Z$, Z, and the medium. In calculating values of $\bar{i}$ in gases for which one lacks the experimental data necessary to construct the function $f(vZ^{-\alpha})$ over the required interval of $\bar{i}/Z$ values, it is evidently expedient to adopt as this function a curve of $\bar{i}/Z = f(vZ^{-\alpha})$ averaged over several gases. In particular, when $\bar{i}/Z = 0.2-0.6$, we can use Eq. (5.2).

Note added in proof. A semi-empirical method of calculating equilibrium charge compositions of ion beams with account taken of the hypotheses advanced here has been described in a recent paper.[96]

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