A LIQUID-FILLED PENDULUM FOR DEMONSTRATION OF THE DIURNAL ROTATION OF THE EARTH

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PROOFS of the rotation of the earth are based on the fact that a Coriolis force acts on every body moving relative to the earth. This force is regarded as the effect of the rotating earth on the relative motion of the body. The action of the Coriolis force reduces to the effect that a body moving on the rotating earth will either be deflected in a direction perpendicular to the instantaneous value of its relative velocity, or will exert a corresponding pressure on its support. This effect is most graphically evident in the behavior of moving liquids, which produce a pressure on the walls of the solid body containing it in the direction of action of the Coriolis force (the so-called Beer law). Therefore, it is natural to use a moving liquid in the demonstration apparatus. However, in the review of Grammel,[1] among the numerous examples of experimental proof of the diurnal rotation of the earth, there is only a single example of a hydraulic experiment (Sec. 10, p. 356), that of Perot and Comb, based on the law of areas as applied to liquid flow.

A pendulum moves under the force of gravity due to the mass of the earth. The Coriolis force manifests itself in this motion, and is brought about by the daily rotation of the earth. The latter effect is relatively small, as a consequence of the small angular velocity of the earth's rotation. The motion of the pendulum is the resultant of the action of two forces, which do not depend on the experimenter: the earth's gravitational attraction and the Coriolis force. For a demonstration of the daily rotation of the earth, the problem lies in the preparation of pendulums which are sensitive to the Coriolis effect.

One can suggest a pendulum with a solid bob, as proposed by Foucault and Poshekhonov.[2] The visible shift of the pendulum relative to the earth is observed as the rotation of the plane of vibration with a constant angular velocity (Foucault pendulum) or with a variable one (Poshekhonov pendulum). Foucault pendulums of large dimensions are required for successful observation of the effect. Thus the Parisian Foucault pendulum has a length of 67 m and a mass of 28 kg, while the Leningrad one is 96 m in length and 54 kg in mass. The preparation of special Cardan suspensions for the demonstrations pendulums made it possible to shorten their length to several meters (for a mass of about 30 kg). Thus the Foucault pendulum of the Moscow planetarium has a length of 4.6 m while those of the Volgograd and Leningrad planetariums are about 7 m in length.

The Poshekhonov pendulum is a rod with a bob and a frame which can rotate about an azimuthal axis in the direction of rotation of the earth.

Pendulums with solid bobs, proposed at various times by Foucault and Poshekhonov, have not had widespread application because of constructional and experimental difficulties.

One of the ways of increasing the quality of pendulums designed for the demonstration of the rotation of the earth is to use non-wetting liquids with a free surface. Experiment shows that the "sensitivity" of the Foucault pendulum to the rotation of the earth (for fixed mass of the bob and length of the suspension) is sharply increased by replacing the solid bob with a composite one: a rigid sphere, partially filled (up to its middle) with a heavy non-wetting liquid. The observed increase in the demonstration qualities of such a pendulum is explained by the fact that its bob consists of two masses in different states (liquid and solid), which behave differently under the action of the Coriolis force. This difference amounts to the following: in any displacement of the center of mass of the solid, the location of its atoms (or molecules) remains fixed relative to this center of mass. In the liquid case, it can change. Therefore, use of the combined liquid-solid bob in a pendulum used for the demonstration of the diurnal rotation of the earth makes it possible to produce a parametric liquid-filled pendulum with greater availability (in comparison with the Foucault pendulum) and simplicity (in comparison with the Poshekhonov pendulum).

Under the action of the Coriolis component of the inertial force, the filler periodically changes its shape inside the container, with a frequency equal to the frequency of oscillation of the pendulum. The volume of the liquid is left unchanged. The shape and volume of the container also remain unchanged in this case. The filler, following the motion of the pendulum, moves inside the container, striving to align its free surface perpendicular to the instantaneous value of the direction of the geometric sum of the gravitational and Coriolis forces, and the compound bob of the pendulum acquires an acceleration in the horizontal direction, perpendicular to the plane of oscillation, i.e., in the azimuthal plane. In other words, the pendulum exhibits rapidly progressing deformation of the initial swing plane with simultaneous rotation of this plane, as the result of the periodic change in the distribution of its mass.

For a pendulum with a solid bob, we introduce the concept of reduced length l_0, defined as

$$l_0=\frac{J}{Ml}, \tag{1}$$

where l is the distance from the point of support to the center of mass, M is the mass of the pendulum, and J is the moment of inertia of the pendulum relative to the axis of rotation, defined according to the relation

$$J=\sum_{i=1}^{n} m_i r_i^2, \tag{2}$$

where n is the number of particles, m_i is the mass of the i-th particle, and r_i is its distance from the point of support of the pendulum. The quantities s, l_0 and J do not change during the vibration if the pendulum has a solid bob. For a pendulum with a composite bob, they change during vibration and are expressed by the relations

$$l_0=l_{0\text{s}}+l_{0\text{c}}=\frac{J_\text{s}}{Ml}+\frac{J_\text{v.c}}{Ml} \tag{3}$$

and

$$J=J_\text{s}+J_\text{v.c}, \tag{4}$$

where l_{0S} and J_S are their steady components, which pertain to the container, l_{0C} and J_{VC} are their variable components, pertaining to the filler. If we denote (for the pendulum with composite bob) by l_{01} the reduced length of the pendulum with the composite bob under the force of gravity, by l_{02} the reduced length under the combined action of gravity and the Coriolis force, and by l_{0VC_1} and l_{0VC_2} the corresponding lengths for the liquid filler, we then get the result that for action of the force of gravity alone,

$$l_{01}=l_{0\text{s}}+l_{0\text{v.c1}} \tag{5}$$

and for action of the force of gravity and of the Coriolis force,

$$l_{02}=l_{0\text{s}}+l_{0\text{v.c2}}. \tag{6}$$

As a criterion which characterizes the demonstration qualities of the pendulum with a composite bob, it is convenient to select the difference $\Delta l_0 = l_{01} - l_{02}$ which, according to relations (5) and (6), with account of (3), is equal to

$$\Delta l_0=\frac{J_{0\text{v.c1}}-J_{0\text{v.c2}}}{Ml}, \tag{7}$$

where J_{0VC_1} is the static moment of inertia of the filter in the position of equilibrium, and J_{0VC_2} is the moment of inertia of the filler whose form changes under the action of the Coriolis forces.

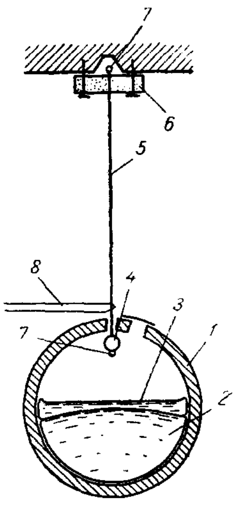
Taking it into account that, according to (7), the change in the reduced length of a pendulum with a compound bob depends on a change in the reduced length of the filler (the active factor) for an unchanged reduced length of the container (the passive factor), we come to the conclusion that the mass of the filler should be as large as possible in comparison with the mass of the container (the mass of the suspension can be neglected).

Thus, as a result of the change in the distribution of the masses of the compound bob during the period of motion of the pendulum, the latter performs parametric vibrations under the action of the Coriolis force. The parameter is the reduced mass of the pendulum, which changes periodically under the action of the Coriolis force (which changes the shape of the moving filler with the free surface and consequently changes the moment of inertia of the pendulum). It is just the presence of the parametric vibrations which distinguishes the proposed pendulum from those of Foucault and Poshekhonov, which execute forced vibrations.

As is well known, the parametric vibrations are executed more easily the greater the change in the parameter and the smaller the energy loss in the system to friction or resistance. Consequently, to shorten the time of increase of the effect to a visually observable value, it is necessary to attempt to decrease the energy loss to friction and resistance of the suspension to the azimuthal shift of the compound bob, and also to friction of the liquid against the container. The presence of friction between the liquid and the container, and the lag of its shift relative to the force can also distort the results of the experiment, and the imperfection of the suspension can lead to an additional deformation during the swing of the pendulum. In this case the effect observed experimentally can depend on the Coriolis force only in a small degree. Therefore, the filler should possess as much free surface as possible, and should be non-wetting relative to the material of the container, while the suspension should be elastically-symmetric relative to the vertical suspension.

A favorable combination is the use of mercury, which has a large specific gravity, and ping pong balls. The latter have a small mass (≈ 2.5 g) for a large volume (≈ 15 cm^3 of liquid when half-filled) and high strength (when filled with water they did not break when dropped from 2.4 m). They are easily machined, they do not contaminate the filler, and they are cheap and can be obtained in any quantity.

To prepare the pendulum, two holes of different diameters are drilled or burned through the ping pong ball at a small distance from one another. The ball is filled halfway (to the middle) by a non-wetting liquid, preferably mercury, through the larger of these holes. Then a little water is added (to prevent possible evaporation of the mercury). A very small rubber ball is attached to one end of the suspension (string or thin metal wire of length 0.25—0.5 m). The diameter of this ball corresponds to the diameter of the larger hole in the ping pong ball. Then the free end of the suspension is passed successively through the



large and small holes of the ping pong ball, and the larger hole is then sealed. The free end of the suspension is passed (with the help of a needle) through a rubber slab about 1 cm thick (for example, an eraser) and, after the needle has been removed, the suspension is terminated by a knot. If the rubber slab is then mounted horizontally at a height greater than the length of the pendulum, then the thread of the suspension, which is fastened by knots at the lower part of the rubber ball and at the upper side of the slab, will connect the bob with the point of the suspension not rigidly, but through elastic (rubber) symmetric shock absorbers, which reduce to a minimum the additional deformations of the suspension, thereby increasing the relative effect of the Coriolis force.

The pendulum with a compound bob is illustrated in the drawing, in which the following notation is used: 1—container, 2—liquid filler (mercury), 3—water, 4—shock absorber in the form of a rubber ball, 5—suspension thread, 6—rubber shock absorber, 7—knot in suspension thread. To start the pendulum, the thread 8 is burned through; this thread holds the pendulum in an initially deflected position. Such a pendulum, with an initial angle of inclination of about 10°, guarantees direct observation of the action the earth's rotation within 3—5 minutes after starting at a latitude of about 60° (at Leningrad). Its plane of oscillation is deformed and turns in one direction, tending to become established in the plane of the geographical parallel (in the east-west direction) and in this sense it becomes a sort of compass. The effect is most clearly seen if the initial plane of vibration of the pendulum is to the north-south direction, for which the shift of the liquid by the Coriolis force is a maximum.

Such a pendulum, consisting of a ping pong ball half filled with mercury with the shock-absorber suspension described above, makes it possible to observe the effect with a suspension 0.25-0.5 m long.

It is advantageous to make the observation on the shadow produced by the bob on a sheet of paper with a grid of radial lines by means of an electric light placed at the position of support of the pendulum.

Thus the pendulum with liquid filler possesses the property of accumulating the effects of small Coriolis accelerations, displaying the action of the Coriolis force on the liquid-solid bob of the pendulum, in contrast with the pendulums of Foucault and Poshekhonov, which exhibit its effect on a solid bob. It is a unique compass, indicating the direction perpendicular to the true (geographic) meridian.

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[1] R. Grammel', Usp. Fiz. Nauk **3**, 335 (1923).

[2] G. L. Poshekhonov, Author's certificate No. 94733, 1950.

Translated by R. T. Beyer