WHAT IS VERIFIED BY MEASUREMENTS OF THE GRAVITATIONAL FREQUENCY SHIFT?

V. L. GINZBURG

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It was only three years ago that it became possible to reliably measure [1] the gravitational frequency shift (to weigh photons), although the effect had been predicted by Einstein already in 1907. [2] The measurement of the gravitational frequency shift (one talks more commonly of the red shift, having in mind the shift of the lines in the spectrum of the sun and of stars) is usually viewed as one means of testing the general theory of relativity. In connection with the experiments of Pound et al [1], however, there have appeared in the literature statements to the effect that these experiments do not properly introduce anything new and do not constitute a test of the general theory of relativity. Precisely such a point of view has been clearly expressed, for example, in the article by Ya. A. Smorodinskiĭ, [3] published recently in UFN (it is indicated in this article that the experiments [1] "test nothing beyond the law of energy conservation").

It seems to me that this conclusion is incorrect and that the experiments of Pound et al do solve a problem, which is in fact the problem posed by Einstein in 1907, and thereafter in a more complete form in 1911;* moreover, these experiments have a direct bearing on the general theory of relativity.

The problem was formulated by Einstein with characteristic precision and clarity: [4]

"The relativity theory leads to the conclusion that the inertial mass of a body increases with increasing energy content; if the energy increment is E then the increment in the inertial mass is E/c^2, where c is the velocity of light. Is there an increase in the gravitational mass corresponding to this increase in the inertial mass? If there is not, then a body will fall with different accelerations in the same gravitational field, depending on its energy content. The very satisfactory result of the theory of relativity according to which the law of conservation of mass is contained in the law of energy conservation would be false, since in such a case the law of mass conservation in its old form would have to be discarded for inertial mass, whereas it would remain valid for gravitational mass. This conclusion should be considered most improbable. On the other hand the conventional theory of relativity contains no arguments from which one could conclude that the weight of a body depends on its energy content. But we will show that it follows as a necessary conclusion, from our hypothesis on the equivalence of the systems K and K′, that energy must have weight."

The referred-to system K, which is at rest relative to an inertial system, contains a uniform gravitational field (acceleration of the gravitational force g), while the system K′ is uniformly accelerated with respect to inertial systems without gravitational fields (the acceleration of the K′ system is equal to $-g$). The equivalence hypothesis, i.e., the hypothesis of complete physical equality of the systems K and K′, is indeed the essence of "the principle of equivalence" (it is known from the general theory of relativity that this principle is of local character: it applies only to a sufficiently small region of space-time; see in this connection [5,6]).

It follows from the principle of equivalence, if use is also made of the special theory of relativity and the law of energy conservation in the system K′, that the increment in the gravitational mass is

$$\Delta m_g = \Delta m_i = \frac{\Delta E}{c^2},$$

where Δm_i and ΔE are respectively the increments in the inertial mass and the energy of the body. This is precisely what had been shown by Einstein, who obtained in the same paper [4] the formula for the gravitational frequency shift

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\varphi}{c^2}$$

by an independent approach (in the sense that no energy considerations were used). But we are considering in fact the same effect. If light (say, a wave packet) of energy E has a gravitational mass E/c^2, then the change in its energy ΔE on propagation from the point with gravitational potential φ_1 to the point with gravitational potential $\varphi_2 = \varphi_1 - \Delta\varphi$ is equal to

$$\Delta E = \frac{E}{c^2}\Delta\varphi.$$

It only remains to relate the energy E with the frequency ν of the light. If quantum considerations are resorted to, then $E = h\nu$ and the indicated formula for $\Delta\nu$ follows immediately. In this connection the use of the quantum picture, as in many other analogous cases (see, for example, [7]), is convenient also in solving classical problems (the Planck constant h drops out from the answer). One may, however, use just as well

*The Einstein 1911 paper[4] is not only more detailed and discusses the question more fully—it is also more accessible (it is available in Russian translation). We shall therefore cite this paper and not the earlier one.[2]

the laws of energy and momentum conservation in their classical form, but in addition relate E with ν by taking into account that for a slow variation of the parameters the ratio $E/\nu = \text{const}$ (adiabatic invariant).

Thus it is sufficient to adopt the principle of equivalence or the relation

$$\Delta m_g = \frac{\Delta E}{c^2},$$

in order to arrive at the formula for the gravitational frequency shift in the first order approximation in φ/c^2. The converse of this assertion is false: the gravitational mass of light could be equal to zero and a gravitational frequency shift would appear if, for example, the inertial mass m_i depended on the gravitational potential φ. This is precisely what is assumed in the Lorentz-invariant (and in that sense fully consistent) gravitational theory of Nordström.[8,9] In that theory, however, light rays are not deflected in the gravitational field of the sun, in disagreement with the general theory of relativity and with observations.

It is clear from the foregoing that it is impossible to arrive at the correct expression for the gravitational frequency shift on the basis of the energy conservation law only, without any assumptions about the relation between m_g and E or the effect of a gravitational field on m_i. This follows also, of course, from Smorodinskiĭ's paper[3] in which the difference in the energies of two states of a nucleus located at a height H above the earth is taken to be equal to

$$\Delta mc^2\left(1+\frac{gH}{c^2}\right).$$

But in this case one has already assumed precisely the thing that is to be proved by experiments on measurement of the gravitational frequency shift.

If it is assumed that the difference in the energies (inertial masses) of states of the system (atom, nucleus, etc) is independent of position and, consequently, of the values of the potential at the given point, as follows from the existing theory and as is assumed in [3], then the experiments of Pound et al prove the existence of a corresponding gravitational mass

$$\Delta m_g = \frac{\Delta E}{c^2}$$

for excited nuclei and photons. This conclusion, it is true, has to a large extent been clear[10] already from the Eötvös experiments with masses of different chemical composition (these experiments have been recently repeated with the result:[11] $(m_g - m_i)/m_i < 10^{-10}$). In addition, the very fact that it has not been possible to reliably measure (say accurate at least to $\lesssim 10\%$) the gravitational frequency shift for as long as 53 years after Einstein's work,[2] leaves one without any doubts as to the usefulness of the experiments of Pound et al.

In one way or another the meaning of similar experiments has been completely clear even before the general theory of relativity has been consistently formulated.[12] It was therefore never proposed that the measurement of the gravitational frequency shift could test the equations of the gravitational field in some approximation lying beyond the limits of direct application of the equivalence principle (in particular, it is not possible to establish the fact that space is curved by measuring the frequency). Since this principle lies at the foundation of the theory it is with full justification that its verification may be considered at the same time as verification of the general theory of relativity itself. On the other hand, the gravitational frequency shift is "noncritical" in the sense that it can be obtained (in first approximation in φ/c^2) in various gravitational theories, or better said speculations,[13,14] which differ from the general theory of relativity, and sometimes are not even in agreement with the principle of equivalence.[8,9] Here, however, we are simply encountering the well known "asymmetry" between overthrowing and confirming of a theory in the natural sciences (see, in particular, [14,15]). Concretely, the violation of the formula

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\varphi}{c^2},$$

would unconditionally disprove the general theory of relativity, and so already for this reason it is useful to have it tested. The verification of this formula, however, as that of other conclusions of the theory considered by themselves, does not yet prove the validity of the theory since these conclusions could follow from other theories.*

Let us discuss two more questions touched upon in the article[3]. It is indicated in the article that "in the general theory of relativity it is necessary to make use of quantum clocks—a circumstance that points to the deep connection between geometry and quanta (cf. Wigner[9])". It seems to me, however, that exactly the opposite conclusion follows from Wigner's[17] paper: as a result of analyzing the possibilities of measurements in the general theory of relativity Wigner arrives at the conclusion that "clocks are essentially nonmicroscopic objects" and that "the essentially nonmicroscopic nature of the basic representations of the general theory of relativity seems to us unavoidable." The extension of the representations of the general theory of relativity to the microscopic (quantum) domain indeed presents difficulties and unclear moments, but this is nothing but again a confirmation of the macroscopic (nonquantum) character of this the-

*According to P. Bergmann[16] Einstein himself thought that the measurement of the three so called "critical effects" (gravitational frequency shift, bending of light rays in the field of the sun and the precession of the perihelion of planets) was not as important as establishing to a higher precision the equality $m_g = m_i$. In this connection any improvement on the results of Eötvös (see[11]) is of interest, and the corresponding experiments may be considered with full justification as means of testing the general theory of relativity.

ory. In the nonquantum region, on the other hand, there are no difficulties of principle on the question of the measurement of an interval or some other quantity, as far as is known.* It is only this last circumstance that makes the general theory of relativity consistent. We do not even mention the fact that the Planck constant h does not appear in the general theory of relativity, which gives one a formal basis for considering the theory as nonquantum.

The last remark that seems to me appropriate here has to do with the deviation of light rays in the field of the sun. The corresponding deviation is given by†

$$\alpha = \frac{4\kappa M_{\odot}}{c^2 R}$$

(κ is the gravitation constant, R is the distance of closest approach of the ray to the center of the sun), and is an effect of order $1/c^2$, testifying to the curvature of space. The point is that the indicated result does not follow from the principle of equivalence combined with the special theory of relativity, but is due to an integral effect independent of the choice of the frame of reference (the metric is assumed to be Galilean at infinity). This circumstance has been emphasized by Einstein himself (see [19], p. 85), but recently there appeared in the literature the opposite assertion, carelessly repeated also in my paper [14]. Aside from the already mentioned general argument (the integral character of the effect) the impossibility of explaining the deviation of the rays by the angle α on the basis of the principle of equivalence is expounded in detail in the papers [6,20].

*In particular the gravitational frequency shift may be measured with the help of a classical oscillating circuit. If the gravitational frequency shift to be measured is sufficiently large, which is in principle quite possible, then no difficulties arise in the use of the circuit connected with the establishing of approximate identity and calibration in general of circuits lying at different points and serving to measure the frequency.

In the cited 1911 paper Einstein obtained a value smaller by a factor of two, which can be arrived at (as was already done by Soldner in 1801) on the basis of classical mechanics and the relation $m_g = m_i$ as applied to light corpuscles (the calculation is given, for example, in[18]); calculations based on classical mechanics (where in the end the velocity v of the corpuscles is set equal to c) agree in this case with relativistic calculations since one is dealing here with a small deviation in a direction perpendicular to the velocity of the particle (photon).

The agreement between the predictions of the theory and observations as applied to all three "critical effects," as well as a number of other circumstances (for details see [14,15]), present additional reasons for believing that the general theory of relativity rests on exceptionally firm foundations.

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