MONOPOLE TRANSITIONS OF ATOMIC NUCLEI

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INTRODUCTION

In the most general case, the transition of a nucleus from an excited state to a lower one can proceed via simultaneous emission of several photons, conversion electrons, and electron-positron pairs.* However, the most probable process accompanying a nuclear transition is usually the emission of a single photon. The emission of one electron or of one pair has a lower probability, but becomes particularly important if the single-photon transition is forbidden. All the remaining processes are much less probable compared with the first three, and play a considerable role only in those cases when the first three processes are simultaneously forbidden.

We can classify nuclear transition by the magnitude of the total angular momentum L carried away by the emitted photons, electrons, and pairs. Thus, for example, if $L = 2$, then the transition is called quadrupole; if $L = 1$, the transition is dipole.† On the other hand, if the total angular momentum of the particles emitted in a nuclear transition is zero, this is a monopole transition.

Since the angular momentum of one photon cannot be equal to zero, single photon emission is absolutely forbidden in monopole transitions.‡

From the angular momentum conservation law we have $|J_i - J_f| \leq L \leq J_i + J_f$, where J_i and J_f are the total angular momenta of the nucleus in the initial and final states. It follows therefore that monopole transitions are possible only if the total angular momentum of the nucleus remains unchanged, that is, when (a) $J_i = J_f = 0$ and (b) $J_i = J_f \neq 0$. In the former case the monopole transition is the only one possible (zero-zero nuclear transition), and in the second case the monopole transition competes with the other non-monopole transitions (J-J nuclear transition).

Monopole transitions can be broken up into two classes, depending on whether the parity of the state of the nucleus changes during the transition or not. Monopole transition without change in parity are called electric or E0 transitions, while monopole transitions with change in parity are called magnetic or M0 transitions (in accordance with the standard classification [1-7]). The main cause of E0 transitions is the Coulomb interaction between the nucleons of the nucleus and the electrons of the atomic shell or the Dirac background.* The remaining interactions, both electromagnetic and non-electromagnetic, are usually negligibly small in E0 transitions. To the contrary, M0 transitions are caused also by non-Coulomb electromagnetic and non-electromagnetic interactions, and for suitable values of the transition energy and of the charge of the nucleus, the latter can predominate over the former.

In E0 transitions, the most probable processes are the emission of one internal-conversion electron or electron-positron pair. These processes are completely forbidden in M0 transitions (in the customarily employed first two orders of perturbation theory). In the latter case, simultaneous emission of two particles is the most probable (for example, the emission of two photons or of one photon and one electron and similar processes).

A characteristic feature of E0 transitions is that the Coulomb interaction between the protons of the nucleon and the electrons of the shell or the Dirac background, which causes these transitions, takes place inside the nucleus (since the monopole moment is constant outside the nuclear volume).† Therefore

*The latter is possible if the energy of the nuclear transition is $\Delta > 2m_0c^2$, where m_0c^2 is the electron rest energy.

†According to the conventional classification of transitions in the theory of multipole radiation.[1-7]

‡In the customarily employed first two orders of perturbation theory (see the end of Sec. 1 concerning the higher approximations).

*Therefore E^0 transitions are also called Coulomb transitions.

†It must be noted that the principal role is played here by the Coulomb interaction between the protons and those electrons which are in states with total angular momentum $j = 1/2$, for they stay inside the nucleus longer than the others.

the probability of electric monopole transitions depends to a considerable degree on the distribution of the charges in the nucleus and on the state of the nucleus. It follows therefore that the investigation of E0 transitions of nuclei is quite useful for the study of the subtle details of nuclear structure or the suitability of some particular nuclear model.

Monopole transitions can be observed both in naturally radioactive elements and in nuclei which are products of various nuclear reactions. It is also possible to excite monopole transitions by inelastic collisions between the electrons and the nuclei.

For a long time (from 1930 through 1948) there were only two known electric monopole transitions, the 0^+-0^+ transitions of RaC′(Po^{214}) and O^{16}. This was followed by discovery of the 0^+-0^+ transition of Ge^{72} (1948), and starting with 1952—of C^{12} and many other nuclei. For a theoretical explanation of 0^+-0^+ transitions various nuclear models are used with lesser or greater success (single-particle, shell, unified, etc.). Since 1955, E0 transitions of the 2^+-2^+ type, occurring in even-even nuclei such as Pt^{196}, U^{232}, Pu^{238}, etc. are also under investigation. In connection with the description of the levels of some of these nuclei on the basis of the non-axial nuclear model of A. S. Davydov and G. F. Filippov, investigations of E0 transitions of the 2^+-2^+ type assume particular significance.

At present there are already more than 20 known observed electric monopole transitions. The data on the experimental observation of magnetic monopole transitions are still doubtful (see page 729).

Sections 1 and 2 of this review are devoted to the general theory and electron excitation of monopole transitions. In the other two sections are given examples of monopole transitions of individual nuclei and theoretical estimates of the nuclear matrix element of the monopole on the basis of various nuclear models. Brief deductions are given in the conclusion.

1. GENERAL THEORY OF MONOPOLE NUCLEAR TRANSITIONS

We consider first electric monopole transitions. The cause of the E0 transitions is the Coulomb interaction of the nucleus with the electron shell of the atom (attempts to attribute one of the causes to non-electromagnetic interactions will be discussed later).

As already mentioned in the introduction, deexcitation of the excited nucleon via an E0 transition cannot be accompanied by emission of a single γ quantum. On the other hand, two-photon emission and other multiple-particle emission in E0 transitions are much less probable than the emission of conversion electrons or the creation of electron-positron pairs.

An important problem of the theory is the calculation of the probability of electron conversion in E0 transitions. The calculations are by perturbation theory, according to which the general formula for the probability of the conversion transition has the form[1-5]

$$W = 2\pi \sum |H'_{if}|^2 \varrho_f, \tag{1.1}$$

where the summation is over all possible initial and final states of the electron spin and ρ_f is the "state density" of the conversion electron, that is, the number of states of the continuous spectrum per unit energy interval*, while H'_{if} is given in the relativistic system of units, assuming an electromagnetic interaction between the nucleus and the electron shell of the atom, by the relation[3,5]

$$H'_{if} = -\alpha^{\frac{1}{2}} \int\limits_{V_{\text{nuc}}} d\mathbf{r}' \int\limits_{V_{\text{nuc}}} d\mathbf{r} \{\psi_f^*(\mathbf{r})\, \boldsymbol{\alpha}\psi_i(\mathbf{r})\, \mathbf{j}(\mathbf{r}') - \psi_f^*(\mathbf{r})\, \psi_i(\mathbf{r})\, \varrho(\mathbf{r}')\} \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}. \tag{1.2}$$

Here V_{nuc} is the region occupied by the nucleus (we shall henceforth not indicate explicitly integration over this region), ψ_i—relativistic wave function of the electron in the initial state†, ψ_f—wave function of the electron in the final state belonging to the continuous energy spectrum, $\rho(\mathbf{r}')$ and $\mathbf{j}(\mathbf{r}')$—density of the charges and currents produced by the nucleons of the nucleus; $\boldsymbol{\alpha}$—Dirac velocity operator, α—fine-structure constant, and k—energy of the transition. Relation (1.2) can be represented in the form

$$H'_{if} = -\alpha^{\frac{1}{2}} \int \{\mathbf{j}_{if}(\mathbf{r})\, \mathbf{A}(\mathbf{r}) - \varrho_{if}(\mathbf{r})\, \varphi(\mathbf{r})\}\, d\mathbf{r}, \tag{1.3}$$

where $\mathbf{A}(\mathbf{r})$ and $\varphi(\mathbf{r})$ are the potentials of the nuclear charges and currents. In view of the fact that the E0 transition is due to the spherically-symmetrical part of the density of these charges and currents, we can put[5,9]

$$\varphi(\mathbf{r}) = \varphi(r), \quad \mathbf{A}(\mathbf{r}) = \Delta\lambda(r), \tag{1.4}$$

where $\varphi(r)$ and $\lambda(r)$ are scalar functions.

Eliminating with the aid of the gauge transformation the vector potential from (1.3), and making the substitution $\rho(\mathbf{r}') \to \rho_{if}(\mathbf{r}') = e\Psi_f^*\Psi_i$, where Ψ_f and Ψ are the wave functions of the initial and final states of the nucleus, and taking into account the interaction between the electron and all the protons of the nucleus, we readily reduce the matrix element H'_{if} to the form

$$H'_{if} = -\alpha \sum_{p=1}^{z} \int\int \Psi_f^*(\mathbf{r}_p)\, \psi_f^*(\mathbf{r})\, \Psi_i(\mathbf{r}_p)\, \psi_i(\mathbf{r}) \frac{d\mathbf{r}\, d\mathbf{r}_p}{|\mathbf{r}-\mathbf{r}_p|}. \tag{1.5}$$

*In the calculation of the probability of pair conversion it is necessary to insert into ρ_f also a factor that depends on the positron momentum and energy.[5]

†In a state belonging to the discrete energy spectrum, if we calculate the probability of electron conversion, or in a state belonging to the continuous spectrum of the negative energy levels, if the probability of pair conversions is calculated.

Inasmuch as integration in (1.5) is over the volume of the nucleus, the quantity H'_{if} and consequently the probability of the E0 transition depends on the values of the electron wave functions inside the nucleus. It follows therefore that the E0 transitions are due principally to the interaction between the nucleons and the electrons whose probability of stay inside the nucleus is maximal. These are the electrons [10] with angular momentum $j = 1/2$.

The formula (1.5) can be also written in the form

$$H'_{if} = \int \varrho_{if}(\mathbf{r}')\,\Phi(\mathbf{r}')\,d\mathbf{r}', \tag{1.6}$$

where Φ satisfies the equation

$$\Delta\Phi = -e\psi_f^*\psi_i. \tag{1.7}$$

Since $\rho(\mathbf{r}')$ is spherically symmetrical, (1.6) will differ from zero only for the symmetrical part of the potential Φ. Because of this, it is first necessary to average the right half of (1.7) over the angles and replace the Laplace operator by its radial part before proceeding to the solution.

By assuming that the electron wave function ψ varies very little inside the nucleus, we can regard it as constant in the zeroth approximation. Then Eq. (1.7) is written in the form

$$\Delta\Phi = -e\varrho_0, \tag{1.7'}$$

where

$$\varrho_0 = \psi_f^*(0)\,\psi_i(0), \tag{1.8}$$

and only the symmetrical solution of (1.7′) gives a non-zero contribution to (1.6). This solution will consist of the general symmetrical solution of the equation $\Delta\Phi = 0$, and the particular solution of the equation $\frac{1}{r}\frac{d^2(r\Phi)}{dr^2} = -e\rho_0$, that is,

$$\Phi = C_1 + \frac{C_2}{r} - \frac{e}{6}\varrho_0 r^2. \tag{1.9}$$

The potential Φ cannot equal C_1 because of the orthogonality of the nuclear wave functions. We cannot put $\Phi = C_2/r$ for Φ must be finite at the center of the nucleus. It follows therefore that

$$\Phi = -\frac{e}{6}\varrho_0 r^2. \tag{1.10}$$

Substituting (1.10) in (1.6) we get

$$H'_{if} = -\frac{2\pi}{3}\psi_f^*(0)\,\psi_i(0)\,Q_0, \tag{1.11}$$

where

$$Q_0 = \frac{e^2}{4\pi}\int \Psi_f^*\Psi_i r^2\,dr \tag{1.12}$$

is the "zeroth moment" of the nucleus[5]. We see from (1.12) that Q_0 is comparable in order of magnitude with the quadrupole moment of the nucleus.

The matrix element H'_{if} can be calculated with greater accuracy by taking into account the variation of the electron wave function ψ inside the nucleus. To this end we expand the radial part of the function ψ in a Taylor series about the point $r = 0$ and confine ourselves to the first two terms of the expansion [11]. If we assume that the nuclear charge density obeys the condition $r^2 q(r) \to 0$ as $r \to 0$, then the expansions of the radial parts of the "large" and "small" components of the Dirac wave functions ψ, for the case when the conversion electron is in states with $j = 1/2$, will have the respective forms

$$(g_{s_{1/2}},\ f_{p_{1/2}}) = C(1 + ar^2 + \ldots), \tag{1.13}$$

$$(f_{s_{1/2}},\ g_{p_{1/2}}) = C(0 + br + \ldots). \tag{1.14}$$

Using (1.13) and (1.14) and calculations similar to those of Church and Weneser [11] we obtain

$$H'_{if} = \frac{1}{6}\,\alpha C_f C_i R^2 \varrho, \tag{1.15}$$

where

$$\varrho = \sum_p \int \Psi_f^*\left[\left(\frac{r_p}{R}\right)^2 - \sigma\left(\frac{r_p}{R}\right)^4 + \ldots\right]\Psi_i\,dr \tag{1.16}$$

(the summation is over all the protons of the nucleus), R is the radius of the nucleus, and

$$\sigma = -\left(\frac{3}{10}\right)(a_i + b_i b_f + a_f^*)\,R^2. \tag{1.17}$$

The dimensionless parameter ρ is called the reduced nuclear matrix element of the electric monopole [11], while the quantity

$$M = \varrho R^2 \tag{1.18}$$

is the nuclear matrix element of the electric monopole.*

Let us focus our attention on the estimate of σ. In particular [11]

$$\sigma_{s_{1/2},\,p_{1/2}} = \frac{R^2}{15}\left[(\varepsilon - V)^2 + (k+1)(\varepsilon - V) + \frac{3}{4}(3k \pm 4)(k \pm 2)\right], \tag{1.19}$$

where ε is the total energy of the bound electron, V the electrostatic potential at the center of the nucleus (of the order of $\alpha Z/R$), and k is the nuclear transition energy (in units of m_0c^2). For $|V| \gg \varepsilon$ the correction σ depends very little on the type of electron shell or the nuclear transition energy. In this case σ is approximately equal to

$$\sigma = \frac{(VR)^2}{15} = \frac{(\alpha Z)^2}{15} \ll 1. \tag{1.20}$$

Figure 1 shows a plot of σ against Z for a definite nuclear transition energy $\Delta = 511$ keV under various assumptions concerning the distribution of the charge in the nucleus [11]. Curve 1 corresponds to uniform distribution of the charge over the surface of the nucleus, curve 2 is obtained for uniform volume distribution of the nuclear charge, and curve 3 corresponds to a nuclear charge with density $q(r) \sim 1/r$. In all cases, as can be seen from the plot, σ lies in the

*We shall henceforth leave out the word "electric."

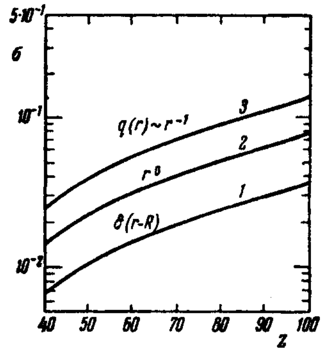

FIG. 1.

approximate range $10^{-2} \le \sigma \le 10^{-1}$ for $50 \le Z \le 90$. Consequently, the second term in the sum under the integral sign in (1.16) is usually neglected, and if it is further recognized that the remaining terms of the sum are also small, the matrix elements M and ρ are determined exclusively by the properties of the nuclear wave functions; they assume the respective forms

$$M = \int \Psi_f^* \Big(\sum_p r_p^2 \Big) \Psi_i \, d\mathbf{r}, \qquad (1.21)$$

$$\varrho = \int \Psi_f^* \Big(\sum_p \frac{r_p}{R} \Big)^2 \Psi_i \, d\mathbf{r}. \qquad (1.22)$$

We have determined the matrix elements M and ρ for E0 transitions evoked by the interaction between the nuclear nucleons and the $j = 1/2$ electron. For the case of interaction between the nucleus and an electron with any j, we obtain the values of M and ρ from (1.21) and (1.22) by making the substitutions $r_p^2 \to (r_p)^{2j+1}$ and $(r_p/R)^2 \to (r_p/R)^{2j+1}$ respectively [12]. In the calculation of the probability of the E0 transition with the aid of the electron Coulomb functions of a point nucleus we obtain for M [13]

$$M = \int \Psi_f^* \Big(\sum_p (r_p)^{2\gamma} \Big) \Psi_i \, d\mathbf{r}, \qquad (1.21')$$

where

$$\gamma = \sqrt{\Big(j + \frac{1}{2}\Big)^2 - \alpha^2 Z^2}.$$

After substituting (1.15) in (1.1) we can write the conversion probability in E0 transition in the form

$$W_e = \varrho^2 \Omega_e, \qquad (1.23)$$

where the factor Ω_e does not depend on the spin states of the nucleus and is completely determined by the electron wave functions. A relatively simple analytic formula for Ω_e, called the reduced probability of the conversion E0 transition, is obtained in the socalled "point nucleus" approximation [1]. Then $a = b = 0$. The constants C_i and C_f are determined from the electron Coulomb functions for a point nucleus, if we put $r = R$. If αZR and $pR \ll 1$ (p is the momentum of the electron), then the reduced probability Ω_e for the K shell will, in accordance with Church and Weneser, be given by the formula

$$\Omega_K = \frac{\alpha^2}{36} \frac{1+\gamma}{\Gamma(2\gamma+1)} \frac{p(\varepsilon+\gamma)}{\alpha Z} (2\alpha Z R)^{2\gamma+2} F(Z, p), \qquad (1.24)$$

where

$$F(Z, p) = \frac{2(1+\gamma)}{[\Gamma(2\gamma+1)]^2} (2pR)^{2\gamma-2} e^{\pi\alpha Z \frac{\varepsilon}{p}} \left| \Gamma\Big(\gamma + \frac{i\alpha Z\varepsilon}{p}\Big) \right|^2 \qquad (1.25)$$

is the Fermi function for the β decay, $\gamma = [1 - (\alpha Z)^2]^{1/2}$ and $\varepsilon = (p^2 + 1)^{1/2}$. The result (1.24) is quite close to the results of other calculations of Ω_K [13-14,7]. In particular, according to Thomas [14] Ω_K is obtained from (1.24) by multiplication by a factor $[1 + (1-\gamma)/\sqrt{3}(1+\gamma)]^2$, which does not differ much from unity.* The result of the approximate calculation of Ω_K made by Blatt and Weisskopf [7] exceeds (1.24), taken in the nonrelativistic approximation [11], by a factor of 4.†

The calculation of the relative reduced probabilities Ω for different shells or subshells, according to Church and Weneser, yields

$$\frac{K}{L} = \frac{2p_K(\varepsilon_K+\gamma) F(Z, p_K)(x+1)\, x^{2\gamma+2}}{p_L(\varepsilon_L+\gamma) F(Z, p_L)(x+2)(2\gamma+1)}, \qquad (1.26)$$

$$\frac{L_I}{L_{II}} = \frac{(2+x)(x-1)(\varepsilon_L+\gamma)}{(2-x)(x+1)(\varepsilon_L-\gamma)}, \quad \frac{L_I}{L_{II}} = \frac{54(1+O(\alpha Z)^2+\ldots)}{(\alpha Z)^2(\varepsilon_L^2-\gamma^2)}, \qquad (1.27)$$

where

$$x = \sqrt{2(1+\gamma)}, \qquad \varepsilon_{K,L} = \sqrt{p_{K,L}^2 + 1}.$$

In the absence of screening $\varepsilon_K = k + \gamma$ and $\varepsilon_L = k + x/2$, where k is the nuclear transition energy (in units of m_0c^2).

Figure 2 shows graphically the dependence of Ω_K on Z and k, established on the basis of (1.24) and (1.25), corrected for (a) the finite nuclear size, (b) the screening effect, and (c) account of the terms of order αZR and pR in the Dirac wave function [11].

The calculation of Ω_K with finite nuclear size effects taken into account leads to an increase in the result of (1.24) by a small amount, which increases with Z. In view of the fact that usually $\varepsilon \ll |V|$, the effect of the finite nuclear size is practically the same on the bound and on the free electrons, and depends very little on the nuclear transition energy. The reduced probability Ω_K changes relatively little with the three nuclear charge distributions which were considered above (see Fig. 1) (by 10% for $q(r) \sim \delta(r-R)$, by 30% for $q(r) \sim r^0$, and by 50% for $q(r) \sim r^{-1}$). The curves of Fig. 2 have been obtained assuming a uniform charge distribution over the volume of a nucleus with radius $1.20 \times 10^{-3} A^{1/3}$ cm.

The screening reduces somewhat the result (1.24) and is stronger for L electrons than for K electrons. The effect of screening on the electron functions of

*Thomas obtained his result assuming some definite distribution of nuclear charge. This distribution, however, is not specified in [14].

†Owing to the neglect of one monopole term in the expansion of $1/|\mathbf{r} - \mathbf{r}_p|$ in multipoles.

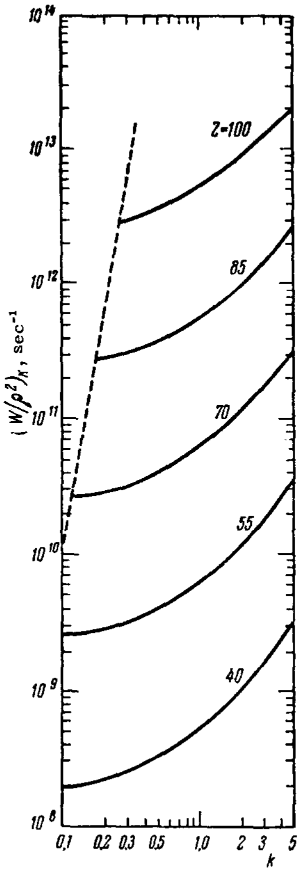

FIG. 2.

the continuous spectrum is appreciable only near the conversion threshold, so that with increasing nuclear transition energy it decreases rather rapidly (the correction to Ω due to this effect is not included in the curves of Fig. 2). The effect of screening decreases with increasing Z.

An account of all three corrections in accordance with Church and Weneser [11] leads to an increase of Ω, for both K and L shells, compared with (1.24), by 25 and 15% respectively for Z = 85 and by smaller values for low Z.

The dependence of Ω on the energy of the nuclear transition is due to the behavior of the continuum electron wave function near the nucleus. Figure 2 shows that Ω increases relatively weakly with k, but depends strongly on Z. The considerable growth of Ω with Z is explained as follows. The E0 transitions, as already mentioned, are due to Coulomb interaction between the nuclear protons and the atomic electrons which penetrate inside the nucleus. E0 conversion will consequently be most probable in atomic shells (or subshells) located as close as possible to the center of the nucleus. It follows therefore that Ω should increase approximately like $(R/a_Z)^3$ or Z^4 (here $a_Z = a_0/Z$ and a_0 is the radius of the first Bohr orbit). Since the shells (or subshells) that are closest to the center of the nucleus do not have an orbital angular momentum different from zero, the E0 conversion takes place predominantly on the $s_{1/2}$ subshells (K, L_I, M_I). However, the K shell is the closest among them, L_I is somewhat farther, and M_I still farther. Therefore Ω will be maximal on the K shell, less on the L_I subshell, and still less on the M_I subshell. In the nonrelativistic case the relative probability of the E0 conversion on these subshells is given by the simple relation [15]

$$K : L_I : M_I = 1 : \frac{1}{8} : \frac{1}{27}.$$

Figure 3 shows the dependence of the relative E0 conversion K/L on Z and k, established under the assumption that the reduced nuclear matrix elements of the monopole ρ is the same for the K and for the L shells. In the calculation of the curves of Fig. 3, no account was taken of the effect of screening on the electron wave functions of the continuous spectrum, which noticeably affects the accuracy of K/L only near the threshold of conversion on the K shell [11] (the latter is noted in Figs. 2 and 3 by dashed lines). As can be seen from Fig. 3, K/L decreases noticeably with increasing Z and increases with k, as is the case also in multipole transitions (see [1-7], at that K/L for the M1 transitions increases with increasing transition energy just as weakly as K/L for the E0 transitions, while K/L increases for E2 transitions much faster with k [11]).

The results of the Church and Weneser calculations for L_I/L_{II} are shown in Fig. 4 for E0 transitions. For comparison, the same figure shows the k-dependence of the ratio L_I/L_{II} for M1 transitions (dashed curves) for Z = 25, 55, and 85. Comparison shows that E0 transitions can be distinguished from M1 transitions by the values of L_I/L_{II}. In addition, from the behavior of the curves of the E0 transitions it can be concluded that E0 conversion on the L_{II} subshell becomes appreciable only for heavy elements and large transition energies.

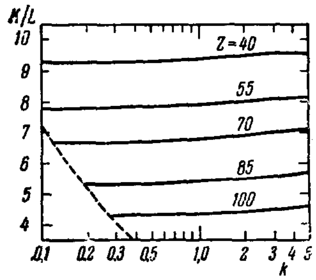

FIG. 3.

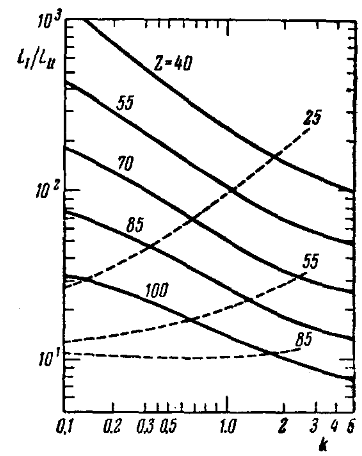

FIG. 4.

Table I

Nucleus	Δ, mc^2	Ω_π, sec^{-1}	Ω_K, sec^{-1}	$\frac{L_I}{L_{II}}$	$\varkappa_K$	K/L_I	$\varkappa_{L_{II}}$	$\varkappa_L$
Ca_{20}^{40}	6.7	$\sim 4\cdot 10^7$	$5.05\cdot 10^8$	342	0.99	7.8	0.965	0.97
Ge_{32}^{72}	1.4	0	$2.56\cdot 10^8$	198	0.96	7.4	0.83	0.945
Zr_{40}^{90}	3.5	$\sim 1.4\cdot 10^6$	$1.16\cdot 10^{10}$	102	0.87	7.6	0.88	0.88
Pd_{46}^{106}	2.27	—	$3.54\cdot 10^{10}$	71	0.88	6.9	0.87	0.87
Po_{84}^{214}	2.85	$\sim 8.7\cdot 10^6$	$8.45\cdot 10^{12}$	19.6	0.68	5.3	0.77	0.75

E_0 conversion on the L_{III} subshell is very small for all values of Z and k, and is weaker than the E_0 conversion on the L_I subshell by a factor 10^8–10^9 [16]. Therefore direct E0 conversion on the L_{III} subshell has very low probability. It is indicated [16] that there is another way of L_{III}-electron conversion, in which the L_{III} electron first goes over into a virtual $s_{1/2}$ state (or $p_{1/2}$ state) with simultaneous emission of an E1-quantum (or M1 and E2 quanta), after which it "converts" in the E0 nuclear transition. The probability of such a process, calculated by Grechukhin [16] with the aid of the Coulomb functions of the electron in the field of the nucleus, without account of screening, turns out to exceed, in the case of the emission of an E1-quantum, the probability of direct E0 conversion on the L_{III} subshell by more than 10^4–10^5 times. On the other hand, the case of emission of M1 and E2 quanta by the L_{III} electron with subsequent E0 conversion has much lower probability.

E0 conversion on the $M_{I,II}$ and other shells, located above the M shells, has so far not been sufficiently well investigated. In [11] the ratio L_I/M_I is estimated to be approximately equal to 3 (for the cases of practical significance).

The theoretical results on electron E0 conversion, obtained by Church and Weneser and given by us in Figs. 2–4, have been qualitatively confirmed by Grechukhin's calculation [12], carried out with the aid of relativistic wave functions of the electron (situated in a field of a nucleus with uniform volume charge) without account of screening.* Grechukhin gives an analytic expression for Ω_K (which we do not present here because of its unwieldiness) and numerical values of Ω_K, K/L_I and L_I/L_{II} for several E0 nuclear transitions (Table I). In the table are indicated also the ratios (κ) of the values of Ω_K, $\Omega_{L_{II}}$, and Ω_L, calculated with the aid of the Coulomb functions of the point nucleus, to the values of the same quantities calculated with account of the finite nuclear size.

The effect of the finite nuclear size on the value of Ω_K was also investigated by Reiner [17–18], who showed that the results of different calculations of the probability of electronic E0 conversion can be represented in the form

$$W_e(E0) = B(Z) F(Z, \varepsilon, R) |M|^2, \tag{1.28}$$

where the factors $F(Z, \varepsilon, R)$ are the same for all the results.

The difference in the calculation methods is manifest only in the value of $B(Z)$. Thus, for $W_K(E0)$ calculated in the "point" nucleus approximation we have in accordance with (1.24) and (1.25)

$$B(Z) = \frac{8}{9}(1+\gamma)^2, \tag{1.29}$$

while a calculation of $W_K(E0)$ under the assumption of a uniform volume charge distribution yields

$$B(Z) = \frac{8}{9}(1+\gamma)^2 \left[\frac{2\gamma}{Z\alpha\chi_1(R) + (\gamma+1)\chi_2(R)}\right]^4, \tag{1.30}$$

where $\chi_1(R)$ and $\chi_2(R)$ are the values of the radial functions which are the solutions of the Dirac equation in the region $r < R$ and on the surface of the nucleus, respectively. $\chi_1(R)$ and $\chi_2(R)$ are normalized to obtain $\lim \chi_2(r) = 1$ as $r \to 0$.

Figure 5 shows plots of $B(Z)$ for different calculations of $W_K(E0)$ [18]. Curves 1–4 have been obtained: (1) assuming uniform distribution of the nuclear charge [18], (2) assuming a "point" nucleus [11] [from Formulas (1.24) and (1.25)], (3) after Thomas [14], and (4) with the aid of the Coulomb functions of the point nucleus [13].

The results of Church and Weneser were than refined somewhat by Listengarten and Band [19]. They have shown that if we take account of the effective screening on the electron wave functions of the continuous spectrum in calculating the reduced E0 conversion probability, then the results of Church and

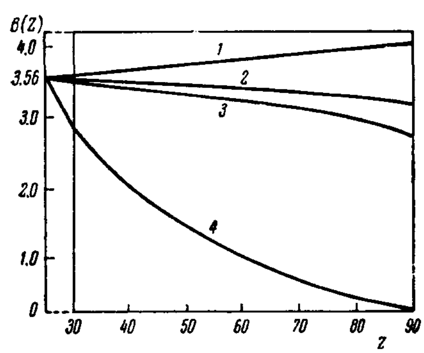

FIG. 5.

*A quantitative comparison of Grechukhin results with those of Church and Weneser is made difficult by the fact that the latter have been obtained with account of the screening and are presented only in graphic form.

Weneser must be reduced by 8% (Z = 98), 6% (Z = 73), and 2% (Z = 49) at transition energies that differ little (by 50—100 keV) from the threshold value of the energy for conversion. Listengarten and Band used in their calculations also the model of a nucleus in the form of a uniformly charged sphere of radius $R = 1.20 \times 10^{-13} A^{1/3}$ cm, and, in addition, the statistical Thomas-Fermi-Dirac atomic model (see [19]). Figures 6 and 7 show their results for $\Omega(Z, k)$ in the case of E0 conversion on the L_I subshell, and the ratio K/L_I.

A monopole electric nuclear transition can be accompanied at transition energies $\Delta > m_0c^2$ by production of electron-positron pairs. The probability of a pair conversion can also be represented in the form (1.23)

$$W_\pi(E0) = \varrho^2 \Omega_\pi(E0), \tag{1.31}$$

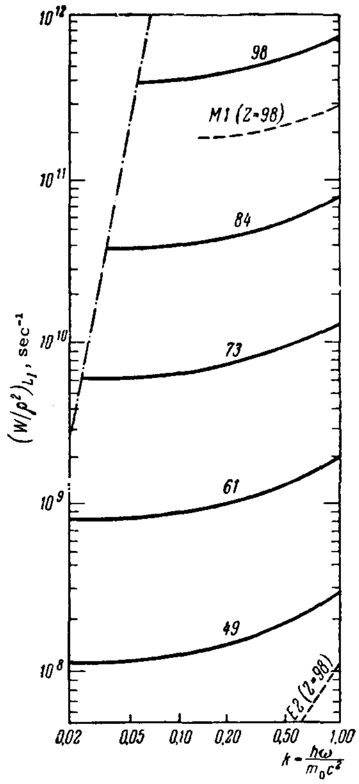

FIG. 6.

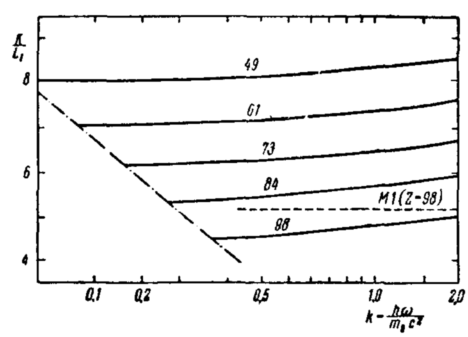

FIG. 7.

where the factor Ω_π depends principally on the form of the wave functions belonging to the continuous spectrum of both the positive and negative electron levels, and ρ is the reduced nuclear matrix element of the monopole. The differential probability for the production of electron-positron pairs, calculated in the Born approximation after Oppenheimer [20], Sakharov [21], and Dalitz [22] in the relativistic unit system is of the form

$$dW_\pi = P(\theta)\, d\varepsilon_+\, d\Omega = |M|^2 \frac{p_- p_+}{9\pi} (\varepsilon_+ \varepsilon_- - 1 + p_+ p_- \cos\theta)\, d\varepsilon_+\, d\Omega, \tag{1.32}$$

where θ is the angle between the directions of motion of the electron and positron, p_+, p_-, ε_+, and ε_- their momenta and energies, and the transition energy is $\Delta = \varepsilon_+ + \varepsilon_-$. Integration of (1.32) with respect to $d\Omega$ or $d\varepsilon_+$ establishes the form of the positron spectrum or the angular distribution of the electrons and positrons.

The total probability W_π was first obtained in the Born approximation for very large transition energies ($\Delta \gg m_0c^2$) by Oppenheimer and Schwinger [23]

$$W_\pi(E\,0) = |M|^2 \frac{1}{135\pi} \left(\frac{\Delta^5}{\hbar^5 c^4}\right) \left(\frac{e^2}{\hbar c}\right)^2. \tag{1.33}$$

A more accurate expression for W_π, calculated in the Born approximation, is given by Dalitz [22] (in the relativistic system of units)

$$W_\pi = |M|^2 \frac{8e^4R^2}{27\pi} \left(\frac{1}{2}k - 1\right)^3 \left(\frac{1}{2}k + 1\right)^2 \times \left\{ \frac{6 + 5\left(s + \frac{1}{s}\right) - 2\left(s + \frac{1}{s}\right)^2}{10s} E(s) - \frac{(1 - s^2)(s^2 + 5s - 2)}{10s^3} K(s) \right\}, \tag{1.34}$$

where $s = (k - 2)/(k + 2)$, k is the transition energy in m_0c^2 units, and $K(s)$ and $E(s)$ are the first and second complete elliptic integrals.

If we take into account the interaction between the pair components, then (1.32) must be multipled by a correction factor, calculated in the Born approximation by Sakharov [24],

$$T = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}. \tag{1.35}$$

Here $\eta = e^2/v$, where e is the electron charge and v is the relative velocity, both in the relativistic system of units. The calculation is based on the assumption that the interaction between the electron and the positron influences appreciably the value of dW_π only at small relative velocities, so that this interaction can be treated in the center of gravity system of the electron and positron as a simple Coulomb interaction $-e^2/r$.

Dalitz [22] investigated the influence of vacuum polarization, radiative corrections, and internal bremsstrahlung, and also the influence of the Coulomb field of the nucleus on the probability of the E0 pair conversion. We shall not write out the rather cumbersome formulas he obtained for dW_π, and only present for illustration the changes which occur in the differential angular distribution of the electrons and positrons, that is, in the function

$$f(\theta, \varepsilon_+, \varepsilon_-) = \varepsilon_+\varepsilon_- - 1 + p_+p_- \cos\theta. \quad (1.36)$$

Calculations with account of vacuum polarization and radiative corrections yield*

$$f(\theta, \varepsilon_+, \varepsilon_-) = \varepsilon_+\varepsilon_- - 1 + p_+p_- \cos\theta + \frac{e^2(\varepsilon_+ - \varepsilon_-)^2 u}{\pi \operatorname{sh} 2u}, \quad (1.37)\dagger$$

where u is connected with the relative velocity v by

$$v = \operatorname{th} 2u. \quad (1.38)$$

The influence of the Coulomb field also changes‡ the function (1.36):

$$f(\theta, \varepsilon_+, \varepsilon_-) = \varepsilon_+\varepsilon_- - 1 + \alpha^2Z^2 + (p_+p_- - \alpha^2Z^2)\cos\theta. \quad (1.39)$$

Investigations have shown [22] that the electron-positron interaction, which is accounted for by the factor (1.35), plays a role only at transition energies Δ that are low, close to the threshold, while the radiative corrections, on the contrary, play a role at large transition energies. However for medium Δ, for example for $\Delta(O^{16}) = 6.05$ MeV, the radiative corrections can reduce W_π only by 0.7%. Numerical calculations for O^{16} lead to the conclusion that the corrections produce the greatest effect at small angles $\theta = 0$ (by 2.3% for the solid angle $\Omega = 10°$, due to the interaction of the pair particles) and for $\theta = 180°$ (by 5%, due to the internal bremsstrahlung). The Coulomb field, as can be seen from Dalitz' formula (1.39), has little influence on the angular distribution in the case of small Z, with the exception of angles close to $\theta = 180°$, for which the number of slow electrons and positrons is largest. Even for $\theta = 180°$, the total number of pairs for O^{16} decreases, according to Dalitz, by less than 1% as a result of this effect [22].

The internal bremsstrahlung accompanying the E0 conversion was investigated also by I. S. Shapiro and Yu. V. Orlov [26,27]. They calculated in the Born approximation on the differential and integral relative probabilities of this radiation, emitted both by the conversion electron [26] and by the pair components [27] in $0^\pm - 0^\pm$ transitions. Figure 8 shows by way of an example the energy spectrum of internal bremsstrahlung photons obtained by Orlov [27], accompanying the pair E0 conversion of the following nuclei: 1) O^{16} with transition energy 6.06 MeV, and 2) C^{12} with transition energy 7.66 MeV. The abscissas are the photon energies ω (in m_0c^2 units), and the ordinates are the values of $\omega N_1(\omega)$ and $\omega N_2(\omega)$, which are the differential relative probabilities of internal bremsstrahlung for O^{16} and C^{12}, respectively, multiplied by ω. The integral relative probability of this process is $N_1 = 3.3 \times 10^{-3}$ for O^{16} and $N_2 = 3.96 \times 10^{-3}$ for C^{12}. These values agree also in order of magnitude with the integral relative probability of internal bremsstrahlung accompanying electron E0 conversion [26,27].

More accurate calculations for both the differential and integral probabilities of pair conversion in the E0 transition, carried out with account of the Coulomb field of the nucleus, are contained in [12-14,28]. In [13] Yukawa and Sakata have determined the form of the positron spectrum for Z = 84 and a transition energy 1416 keV. Thomas obtained [14] the following formula for W_π, assuming a certain distribution of the nuclear charge (which, however, is not specifically described)

$$W_\pi = |M|^2 \frac{8\alpha^2}{9\pi(\gamma+1)^4} \int_1^{k-1} (\varepsilon_+\varepsilon_- - \gamma^2) \times p_+p_- F(Z, p_+) F(Z, p_-)\, d\varepsilon_+, \quad (1.40)$$

where ε_-, p_- and ε_+, p_+ are the total energies and momenta of the electron and positron, respectively, k is the transition energy in m_0c^2 units, and the functions F_+ and F_- are given by (1.25). The integration in (1.40) has been carried out by numerical means. Grechukhin [12] gives a rather cumbersome analytic expression for the differential probability dW_π, obtained with the aid of the wave functions ψ_i and ψ_f of an electron with arbitrary j, situated in the field of a uniformly charged spherical nucleus. The calculations have been carried out without account of screening. Owing to the use of the condition $k'R \ll 1$ in the calculations (k' is the electron wave vector), the region of applicability of the foregoing expression is limited to electron energies $\varepsilon_\pm < 15$ MeV. The numerical values of the total probability W_π are given by Grechukhin only for three nuclei and for three values of the transition energy [12] (see Table I).

Zyryanova and Krutov [28] expressed the total probability of pair E0 conversion in $0^\pm - 0^\pm$ transi-

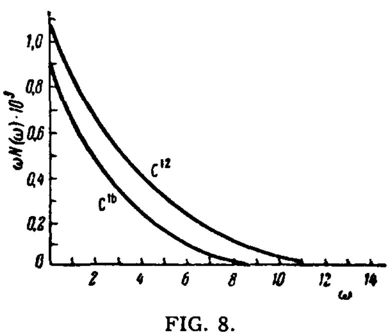

FIG. 8.

*The expression for $f(\theta, \varepsilon_+, \varepsilon_-)$ obtained with account of internal bremsstrahlung, is too cumbersome.[22]

†sh = sinh, th = tanh.

‡Formula (1.39) has been obtained for electrons with j = 1/2. For j = 3/2 it is necessary to replace α^2Z^2 by $\alpha^2A^{2/3}$.

tions in terms of the area of the positron spectrum S, by means of a very simple relation

$$W_\pi(E0) \equiv \varrho^2\Omega_\pi(E0) = \varrho^2 \frac{\pi\alpha^6}{144} A^{\frac{4}{3}} S \frac{m_0c^2}{\hbar}\ \mathrm{sec}^{-1} \qquad (1.41)$$

In the derivation of this relation, the following estimate was used, after Drell and Rose[25], for the nuclear matrix element of the monopole

$$M = \varrho\left(\frac{e}{2m_0c^2} A^{\frac{1}{3}}\right)^2. \qquad (1.42)$$

By comparing (1.42) with the experimental values of M for O^{16}, RaC′, and Ge^{72} it is established[25], that ρ can range for these nuclei between $1/9$ and $1/4$.*

The value of S is calculated from

$$S = \int_1^{k-1} F(\varepsilon_+, Z)\,d\varepsilon_+ = \int_1^{k-1} \{(f_0^- f_0^+ + g_0^- g_0^+)^2 + (f_{-2}^- f_{-2}^+ + g_{-2}^- g_{-2}^+)^2\}_{r=R}\,d\varepsilon_+, \qquad (1.43)$$

where f and g are the radial parts of the wave functions of the ψ_i and ψ_f electron with $j = 1/2$ in the notation of Rose[30], and k is the transition energy in m_0c^2 units. To obtain a more accurate value of S it is necessary to substitute in the integrand values of the functions f and g averaged over the entire volume of the nucleus (and not their values at the point $r = R$, as is done in (1.43)), but the estimates made by Zyryanova and Krutov for $Z \le 84$ and $k \le 5$ show that the more accurate values of S differ from the less accurate ones, calculated by Formula (1.43), by not more than 20%. The function $F(\varepsilon_+, Z)$ determines the form of the positron spectrum. Figures 9-11 show the ratio F/F_{max} as a function of the kinetic energy of the positron T_+ for different values of Z and Δ. In the calculation of the curves, use was made of the tables for relativistic wave functions of the electron in the field of an extended

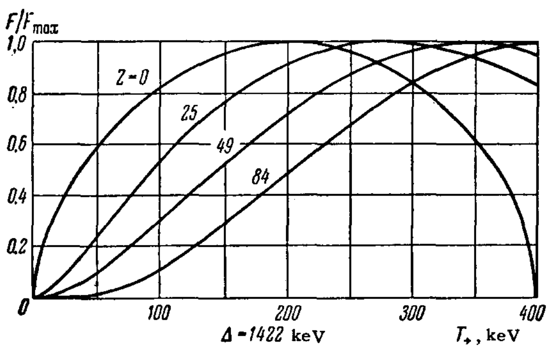

FIG. 9.

*A less accurate estimate for M, suitable only for O^{16} and Ge^{72}, is given in [29]:

$$M = (1.5-2)\left(\frac{e^2}{2m_0c^2}\right)^2. \qquad (1.42')$$

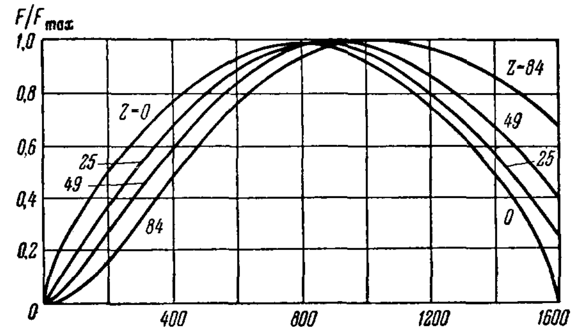

FIG. 10.

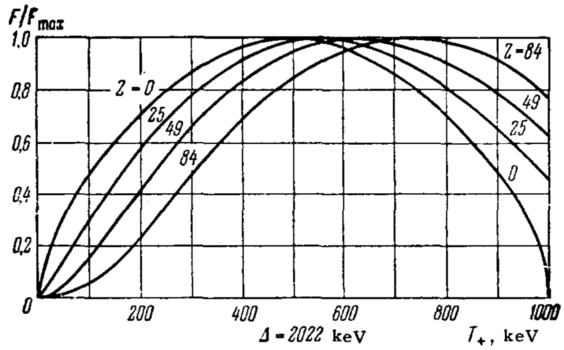

FIG. 11.

nucleus, calculated for β decay by Dzhelepov and Zyryanova[31].

As can be seen from the figures, the form of the positron spectrum depends strongly both on Z and on the energy of the nuclear transition. We can therefore conclude that the frequently used in the investigation of internal conversion of multipole radiation extrapolation of the theoretical data on the form of the positron spectrum, obtained for one value of Z and one transition energy, to other values of the transition energy can lead to considerable errors in case of E0 conversion.

Rather large errors (particularly for large Z) are obtained also when the total probability of pair E0 conversion $W_\pi(E0)$ is calculated in the Born approximation. This is indicated by the rather strong dependence of the integral S (which is proportional to $W_\pi(E0)$) on Z, as established by Zyryanova and Krutov[28] for three values of the transition energy (Table II). These authors have shown that the criterion for the applicability of the Born approximation, $|\alpha Z/\bar{\beta}_\pm|^2 \ll 1$, ($\bar{\beta}_\pm$ are the average electron and positron velocities in the relativistic system of units), which holds true for the calculation of the total probability of pair conversion in the case of multipole transitions, is not suited for E0 conversion. In Fig. 12 are compared the curves of the ratio $W_\pi/W_{\pi(Z=0)}$ against Z in zero-zero nuclear transitions without change of parity, and in E2 transitions, for two values of the transition energy, $k = 2.8$ and $k = 5.2$, satisfying the foregoing criterion

Table II. Area S of positron spectra, in relativistic units

Δ, keV	Z			
	0	25	49	84
1422	0.104	0.132	0.239	0.946
2022	2.41	2.98	5.29	21.8
2622	13.6	16.5	28.3	103

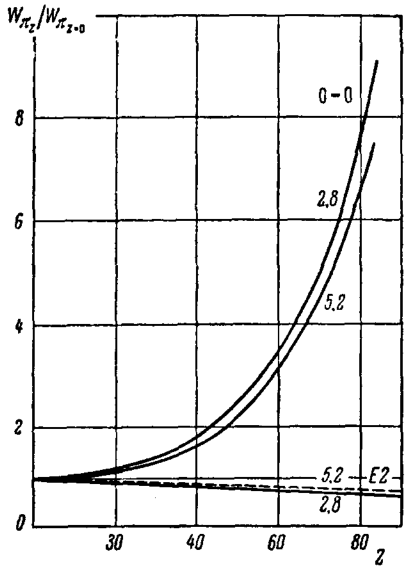

FIG. 12.

(at not too large Z). It is seen from the figure that $W_\pi/W_{\pi(Z=0)}$ for $0^\pm-0^\pm$ transitions increases with Z rapidly beginning with Z > 30, whereas for the E2 transition this ratio differs little from unity even at large values of Z.

In spite of the different behavior of the ratio $W_\pi/W_{\pi(Z=0)}$ for $0^\pm-0^\pm$ and multipole transitions as a function of Z, the shapes of the corresponding positron spectra, obtained by most accurate calculations, are practically the same (at least for not too large values of ε_+). This is seen from a comparison of the curves on Fig. 13. They show the shape of the positron spectra (the dependence of F on ε_+) in the case of Z = 84 and k = 3 for the pair conversion $0^\pm-0^\pm$, E1, and E2 transitions (curves 0—0, D, and

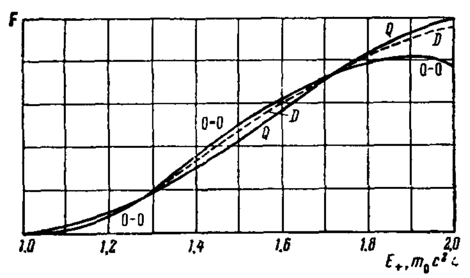

FIG. 13.

Q, respectively). The D and Q curves are based on the exact calculations of Jaeger and Hulm [8], while the 0—0 curve has been obtained by the method described above [28]. Owing to the fact that for large values of ε_+ the D and Q curves have been obtained by extrapolation (the last calculated point is for $\varepsilon_+ = 1.75$), the question of whether the ends of the D and Q curves coincide with the end of the 0—0 curve remains open.

The dependence of the results of the calculation on whether the nucleus is pointlike or extended was also investigated [28]. It was found that the point nucleus approximation leads to an overestimate of $W_\pi(E0)$ (and not to an underestimate, as is the case in the calculations of $W_e(E0)$), compared with the more accurate values obtained with allowance for finite nuclear size, with the maximum deviation reaching 20% for large Z. The shape of the positron spectra remains the same for both methods [28].

The total E0-transition probability, neglecting two-particle and many-particle emission, can be written in the form

$$W(E0) \equiv W_e(E0) + W_\pi(E0) = \frac{1}{\tau}, \qquad (1.44)$$

where τ is the lifetime of the excited nucleus relative to the E0 transition, and by W_e is meant here the probability of total electron conversion, that is, the conversion on all the shells of the atom. If we neglect the interaction between the protons of the nucleus and the atomic-shell or Dirac-background electrons which have an angular momentum $j \neq 1/2$, as is customarily done, then Formula (1.44) can be rewritten

$$\varrho^2(\Omega_e(E0) + \Omega_\pi(E0)) = \frac{1}{\tau}, \qquad (1.45)$$

where

$$\varrho = \int \Psi_f^* \Big(\sum_p \Big(\frac{r_p}{R}\Big)^2\Big) \Psi_i \, d\mathbf{r}.$$

Subsequently, by establishing τ from the experimental data and taking suitable theoretical values of $\Omega_e(E0)$ and $\Omega_\pi(E0)$ from the formulas and plots presented above (pages 718-720), we can determine from (1.45) the values of ρ and $M = \rho R^2$.

It is easiest to measure τ in the case when the E0 transition is the only one possible, that is, in $0^\pm-0^\pm$ nuclear transitions. It is much more difficult to do so for nuclear transitions of the type $J^\pm \to J^\pm$, which we now proceed to consider.

If we assume that the form of the wave functions of the electron depends little in the field of an extended nucleus on whether the total angular momentum of the nucleus is equal to zero or not, then the entire theory developed above applies equally for $0^\pm \to 0^\pm$ and $J^\pm \to J^\pm$ nuclear transitions (it is understood that $J^\pm \neq 0$). All the foregoing formulas remain then in force without change, and the $J^\pm \to J^\pm$ case differs from $0^\pm \to 0^\pm$ only in the different values of the nuclear matrix element ρ.

As indicated above, the E0 transitions of the type $J^{\pm} \rightarrow J^{\pm}$ can compete with the different multipole transitions. In particular, a deexcitation of an excited nucleus, of the type $2^{+}-2^{+}$, can proceed via any of the most probable transitions E0, M1, and E2, the two latter being accompanied either by internal conversion or by emission of γ quanta. There are examples of even-even nuclei ($Pt^{192,196}$ and Hg^{198}), where all these transitions can be observed [32,11]. The lowest levels of such nuclei fit a simple scheme shown in Fig. 14.

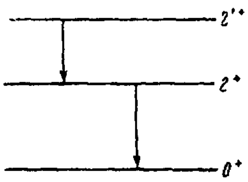

FIG. 14.

Of great importance is a comparison of the theoretical data on the E0, M1, and E2 transitions, since it allows us to predict the cases when observation of E0 transitions in experiment is most promising. Figure 15 shows the probabilities $W_K(E0)$, $W_K(M1)$, $W_\gamma(M1)$, $W_K(E2)$, and $W_\gamma(E2)$ as functions of Z for a transition energy of 511 keV (the E0, M1, M1 (dashed) and the E2 and E2 (dashed) curves, respectively)[11]. The symbols K and γ indicate internal conversion on the K shell and emission of the γ quantum of suitable multipolarity. The E0 curve has been obtained on the basis of the known dependence of the reduced E0 transition probability Ω on Z and k (see Fig. 2) under the assumption that the nuclear matrix element is $\rho = 1$ (The "Weisskopf approximation"[33]). This order of magnitude of ρ corresponds to one-proton transitions with total overlap of the initial and final nuclear wave functions.

The remaining curves are calculated using the data

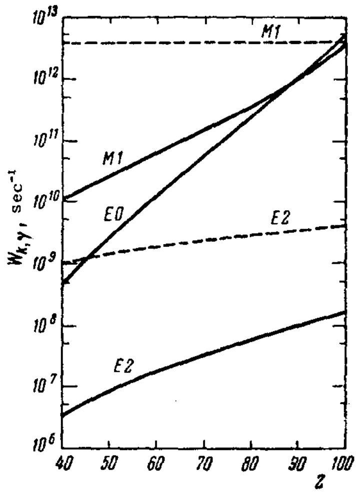

FIG. 15.

of Rose [2], also in the "Weisskopf approximation." The behavior of the curves (Fig. 15) shows that for large Z the "Weisskopf probability" of the E0 transition is much larger than the "Weisskopf probabilities" $W_K(E2)$ and $W_\gamma(E2)$, and becomes almost comparable with the "Weisskopf probabilities" $W_K(M1)$ and $W_\gamma(M1)$ (particularly for the already mentioned nuclei, the 2—2 transitions of which are characterized by a considerable attenuation of the M1-component of the radiation [34]). If we take account of the fact that with increasing transition energy the probabilities $W_\gamma(M1)$ and $W_\gamma(E2)$ increase in proportion to k^3 and k^5, respectively [33], while the probability $W_K(E0)$, as seen from Fig. 2, increases much more slowly with k, we can conclude that the prospects of observing experimentally an E0 transition of the $2^{+}-2^{+}$ type are most favorable for large Z and small k.

The formula from which the probability of an E0 transition of the type $2^{+}-2^{+}$ is determined from the experimental data is derived in the following fashion [11]. If we denote the total coefficient of internal conversion on the K shell in a $2^{+}-2^{+}$ nuclear transition by β^K, and the coefficients of K conversion for the E2 and M1 transitions by α_2^K and β_1^K respectively, then on the basis of the additivity of the probabilities of the E0, M1, and E2 transitions $[W_K = W_K(E0) + W_K(M1) + W_K(E2)]$ there is established between the ratios

$$\frac{W_K(E0)}{W_\gamma(E2)} = \varepsilon_K^2, \quad \frac{W_\gamma(M1)}{W_\gamma(E2)} = \delta^2 \qquad (1.46)$$

a connection

$$\varepsilon_K^2 = (\beta^K - \alpha_2^K) - \delta^2 (\beta_1^K - \beta^K). \qquad (1.47)$$

According to [11], ε_K^2 is a measure of the contribution of E0 conversion to the mixed nuclear transition $2^{+}-2^{+}$. The numerical value of this quantity is usually obtained from the theoretical values of α_2^K and β_1^K, following Rose [181,2], or, more accurately, following Sliv [135]* and the experimental values of β^K and δ^2. The total conversion coefficient β^K is measured in experiment indirectly† (the results of such measurements can be found for some nuclei, for example, in [35,36]). On the other hand, δ^2 can be obtained from the experimental data on the angular correlation between the cascade-emitted γ quanta in mixed $2^{+}-2^{+}$ and subsequent $2^{+}-0^{+}$ nuclear transitions.

According to the theory [37,38], the correlation function has in this case the form

*The most accurate calculations of α_2^K and β_1^K, with account of the so-called intra-nuclear matrix elements, can be made by the method developed in [204-208].

†The presence of an E0 transition is detected from the excess of conversion electrons.

$$W(\gamma\gamma;\ M1+E2)=P_0+\frac{1}{1+\delta^2}\left[A_2^e+2\delta A_2+\delta^2 A_2^m\right]P_2+\frac{1}{1+\delta^2}A_4^e P_4, \qquad (1.48)$$

where $P_i(\cos(\vartheta))$ are Legendre polynomials and the parameters A_2^e, A_2, A_2^m, and A_4^e are tabulated in [38]. The coefficients of P_2 and P_4 are obtained from experiment; knowing these coefficients, we can calculate [11] δ^2 and then ε_K from (1.47). The probability $W_K(E0)$ is obtained from the first relation in (1.46). The probability of emission of an E2 quantum in the nuclear transition 2^+-2^+, which is needed for this purpose, is assumed to be 1.5—2 times larger than the experimentally determined probability $W'_\gamma(E2)$ in the subsequent 2^+-0^+ transition of the nucleus. Such an estimate is presented on the basis of the collective model of the nucleus in [34,39]*. With the aid of the method described above, estimates are made in [32,11] of ρ for several even-even nuclei (see Sec. 3). The latest analysis, however, has shown [43] that this method is inaccurate. The shortcoming of the method lies primarily in the fact that the small uncertainty in the most exact theoretical values of the coefficients α_2^K and β_1^K (an insignificant inaccuracy in the new intranuclear matrix elements) and in the experimental values of β^K (which are on the borderline of the experimental feasibility) leads to considerable differences in the estimates of ρ. This uncertainty has a much smaller effect on the coefficients of $P_i(\cos\vartheta)$ in the angular correlation functions.

It is established in [43] that if account is taken of the latter circumstance with respect to the angular correlation of the cascade emission of the K-conversion electron in the 2^+-2^+ transition and the γ quantum in the subsequent 2^+-0^+ transition, then experiments on the determination of this correlation (and on the determination of the γ-γ correlation) will be perfectly sufficient for a more accurate estimate of ρ. The angular correlation function of the type $e_K-\gamma$ for the mixed transition E0 + M1 + E2 has the form [43]

$$W(e_K\gamma;\ E0+M1+E2)=\frac{1+p^2}{1+p^2+q^2}W(e_K\gamma;\ E2+M1)+\frac{q^2}{1+p^2+q^2}P_0+\frac{q}{1+p^2+q^2}b_0P_2, \qquad (1.49)$$

where

$$W(e_K\gamma;\ M1+E2)=P_0+\frac{1}{p^2+1}(b_2^eA_2^e+2pb_2A_2+p^2b_2^mA_2^m)P_2+\frac{1}{1+p^2}(b_4^eA_4^e)P_4, \qquad (1.50)$$

$$p^2=\frac{W_K(M1)}{W_K(E2)},\quad q^2=\frac{W_K(E0)}{W_K(E2)}. \qquad (1.51)$$

*Theoretical as well as experimental investigations of E2 transitions of the type 2^+-0^+ in even-even nuclei are also treated in [40-42].

p has here the same sign as δ. The sign of δ, on the other hand, is determined from the γ-γ angular correlation. The parameters $A_2^{e,m}$ and A_2 are the same as in (1.48). The values of the parameters b_2^e, b_2, b_2^m, and b_4^e are given in [38,43] for both a point and an extended nucleus. The parameter b_0 is defined as depending only on the interference between the conversion E0 and E2 electrons, and is calculated theoretically. The effect of the new conversion intranuclear matrix elements for the E2 and M1 transitions on the angular correlation e_K-γ is neglected (at least for the type of nuclei under consideration). Figure 16 shows the parameter b_0 plotted against the energy of the nuclear transition, k, obtained in [43] for the particular case Z = 78 and the cascade $2^{+\prime}\to 2^+\to 0^+$ [where we know $W'_\gamma(E2)$ for the $2^{+\prime}\to 2^+$ transition]. (This plot can be readily generalized to cover any other nuclear spin sequence, provided only that Z remains unchanged.) By measuring experimentally the coefficients of the polynomials P_2 and P_4, we can determine p and q, and then use the formulas

$$W_K(E0)=\alpha_2^K W_\gamma(E2)\ \text{and}\ W_K(E0)=\Omega_K\rho^2 \qquad (1.51')$$

to estimate ρ. Since measurement of the coefficient of P_4 entails certain difficulties [43], we can confine ourselves to measurement of only one coefficient, that of P_2, using the formula $p^2=\beta_1^K\delta^2/\alpha_2^K$ to determine p (the signs of δ and p coincide), and determining δ^2 from the γ-γ angular correlation. The conversion coefficients β_1^K and α_2^K are taken from the theory (Rose [181,2] or Sliv [135]).

As noted above, the E0 transition is absolutely forbidden only with respect to the emission of a single photon. As to the simultaneous emission of two or more photons, for example, or the emission of one photon and one conversion electron, such processes (as well as other processes of second and higher order) can occur in E0 transitions, although

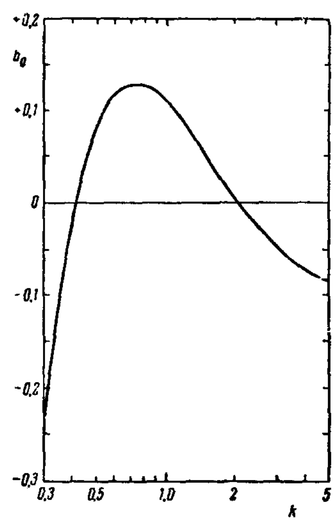

FIG. 16.

their probability is much lower than the probability of pure internal electron and pair conversion. A characteristic feature of all the foregoing processes of simultaneous emission of many particles (both photons and electrons) is that these particles should have a continuous energy spectrum.

Let us concentrate our attention first on photon emission. Since the probability of simultaneous emission of more than two quanta is a quantity of higher order of smallness compared with the probability of a two-quantum emission, we shall confine the discussion to the latter. The differential probability of emitting two electric multipolarity γ quanta in a nuclear 0—0 transition without a change in parity is calculated by perturbation theory and is expressed in a relativistic system of units by the formula [44-46,5]

$$dW_{\omega_1\omega_2}=\frac{8}{9\pi}\sum_{M_1M_2}\left\{\sum_{s}\left[\frac{(Q_{LM_2})_{fs}(Q_{LM_1})_{si}}{E_i+E_s-\omega_1}+\frac{(Q_{LM_1})_{fs}(Q_{LM_2}^{*})_{si}}{E_i-E_s-\omega_2}\right]^2\right\}(\omega_1\omega_2)^3\,d\omega_1, \tag{1.52}$$

where Q_{LMi}—electric multiple moments, and ω_1 and and ω_2 are the frequencies of the γ quanta. Summation is over all possible (virtual) states of the nucleus with total angular momentum $J' = L$* and with parity opposite to the parity of the 0 states. Formula (1.52) is suitable also for simultaneous emission of two magnetic quanta, if we replace Q_{LMi} in (1.52) by the magnetic multipole moments M_{LMi} (see [3,5,7]) and sum over the virtual states $J' = L$ with the same parity as the 0 states.

The first to calculate the total probability of monopole emission of two dipole electric quanta were Oppenheimer and Schwinger in 1939 [23] for 0^+—0^+ transition of the O^{16} nucleus; account was taken there of only one virtual state with $J' = 1$ and negative parity, the energy of which was $\Delta' \approx 20$ MeV. On the other hand, the energy of the 0^+—0^+ transition was $\Delta = 6.06$ MeV. The calculations yielded the formula †

$$W_{\omega_1\omega_2}=\frac{2}{945\pi}\frac{\Delta^5}{\hbar^5c^4}\left(\frac{e^2}{\hbar c}\right)^2\left(\frac{\Delta}{\Delta'}\right)^2|M|^2. \tag{1.53}$$

The total probability $W_{\omega_1\omega_2}$ of two-photon emission in E0 transition was calculated by Grechukhin[12] for the case when the nucleus has so-called "dipole levels" of high density ‡ . Then there are grounds for assuming that the calculation of $W_{\omega_1\omega_2}$ can be restricted to summation over those virtual states which belong to these levels. Recognizing that the energy of the dipole level Δ' is much larger than the energy of the E0 transition Δ, and using in rough approximation the connection between the following matrix elements:

$$\int\Psi_f^*\Big(\sum_{p>p'}r_pr_{p'}\Big)\Psi_i\,dr\approx-\frac{1}{2}\int\Psi_f^*\Big(\sum_p r_p^2\Big)\Psi_i\,dr, \tag{1.54}$$

which has been established on the basis of [48], Grechukhin calculated [12] the following simple formula for the probability of simultaneous emission of two electric dipole quanta, in the relativistic system of units,

$$W_{\omega_1\omega_2}\approx e^4\frac{\Delta^5}{1890\pi}\left(\frac{\Delta}{\Delta'}\right)^2S'\varrho^2R^4, \tag{1.55}$$

where S' satisfies the inequality

$$\left\{1+\frac{a}{2-a}-\frac{a^2}{2(2-a)^2}\right\}\leqslant S'\leqslant\left\{1+\frac{a}{1-a}-\frac{a^2}{(1-a^2)}\right\},\quad a=\frac{\Delta}{\Delta'}. \tag{1.56}$$

If we put $S' = 1$, then Formula (1.55) coincides, apart from a constant factor, with (1.53).

The dependence of the differential probability on the photon energy (that is, the form of the γ spectrum) in the case of simultaneous emission of one photon and one conversion electron, was investigated theoretically by means of a formula [46] analogous to (1.52), in 1948, by Goldberger [49] in connection with the proposed presence of 0—0 transitions in the Ir^{192} nucleus[50]. The curves for this dependence, both for the $0^+ \to 1^- \to 0^+$ transition (1^- virtual state, Fig. 17, curves Ia) and for the $0^+ \to 1^+ \to 0^+$ transition (1^+ virtual state, Fig. 18, curves Ib) were obtained only with account of one virtual state with $J' = 1$, the energy of which is 20 times larger (Fig. 17—18, curves I with B = 20) or smaller (Fig. 17—18, curves I with B = 0)* then the transition energy $\Delta = 58$ keV. † It is seen from Figs. 17 and 18 that the energy distribution of the γ quanta (the abscissas

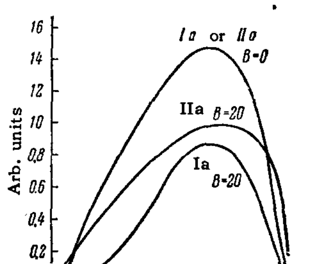

FIG. 17.

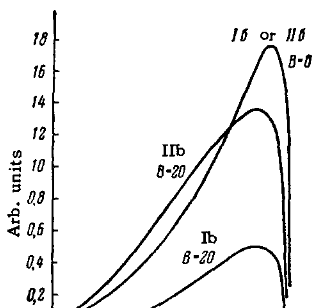

FIG. 18.

*If the two-quantum E0 transition is of the $J^\pm \to J^\pm$ type, then the summation should be carried out over the virtual states with

$$J'=|J-L|,\ \ |J-L|+1,\ldots,J+L-1,\ \ J+L.$$

†This formula would be the same if the transition were 0^-—0^-, and the virtual state were to have $J' = 1$ and positive parity.

‡By "dipole" is meant here a nuclear level to which the nucleus can go from the ground level by absorbing a dipole quantum. The sharp increase in the density of such levels in the vicinity of energies $\Delta' = 40\,A^{-0.2}$ MeV can apparently be attributed to the fact that some nuclei have a giant resonance in the absorption of photons with frequencies close to $\omega_{res} = \Delta'/\hbar$.[12,47]

*Curves IIa and IIb will be discussed later.

†This energy is not sufficient for conversion on the K shell. Therefore an L electron is emitted together with the photon.

represent the γ-quantum energy in units of Δ) has such a sharply pronounced maximum that in the experimental observation of such a spectrum it can be taken as a single γ line. Table III lists the results of the calculations of the average lifetime of the excited nucleus with respect to 0—0 transitions of different types.*

Table III

Transition	B	τ, sec
Ia $0^+ \to 1^- \to 0^+$	≈ 0	$5.4 \cdot 10^{-2}$
»	≈ 20	$1.18 \cdot 10^{-2}$
$I6$ $0^+ \to 1^+ \to 0^+$	≈ 0	$2.40 \cdot 10^{2}$
»	≈ 20	$7.45 \cdot 10^{5}$
IIa $0^- \to 1^+ \to 0^+$	≈ 0	$3.22 \cdot 10^{-3}$
$0^- \to 1^- \to 0^+$	≈ 20	$8.70 \cdot 10^{-3}$
$0^+ \to 0^+$	—	$4.26 \cdot 10^{-8}$

Both the processes considered above, as well as other processes of second and higher order, are the only ones possible for electromagnetic interaction between a nucleus and the electron shell of the atom, if we deal with 0—0 transitions with change of parity, that is, with M0 transitions. These $0^{\pm}$—$0^{\mp}$ transitions were also investigated in the cited paper by Goldberger [49]. Assuming that the deexcitation of the excited nucleus (Ir^{192}) is accompanied in the $0^{\pm}$—$0^{\mp}$ transition also by emission of one photon and one conversion electron, the author has calculated, for the same initial data as in the $0^{\pm}$—$0^{\pm}$ transition, the form of the γ spectra (see Fig. 17—18, curves IIa and IIb), and the average lifetime of the nucleus in the excited $0^{\pm}$ state. The forms of the curves for the E0 and M0 transitions turned out to be similar, and for the case when only an intermediate state which is very close to the ground state ($B \approx 0$) is considered in the calculations, they coincide (see Figs. 17—18). The average lifetime τ of the nucleus in the excited state, relative to simultaneous emission of one photon and one conversion electron, is even smaller in the M0 transition than in the E0 transition, but it is approximately 10^5 times larger compared with pure electron E0 conversion. On the other hand, compared with the lifetime of isomeric states, τ is 10^9—10^{12} times smaller.

Second-order processes different from the one considered but also occurring in M0 transitions were investigated even earlier by Sachs [46], whose work was closely related with the question that arose at the end of the thirties, whether the $0^{\pm}$—$0^{\mp}$ transitions can be regarded as isomer transitions. He considers in that paper two-photon M0 transitions with simultaneous emission of two conversion electrons.

*The process of simultaneous emission of one photon and one conversion electron in an E0 transition can be accompanied by internal bremsstrahlung, but the probability of the latter is 10^7 times smaller than that of the former[27].

The simultaneous emission of two electron-positron pairs in M0 transitions is considered to be impossible for the following reasons. One of the states of the nuclei between which the $0^{\pm}$—$0^{\mp}$ transition occurs, should have a sufficiently large lifetime, in view of the fact that this transition is strongly forbidden. As is well-known, the lifetime of a nucleus is the longer, the smaller the difference between the excited and normal energy levels. Inasmuch as a very large energy, corresponding to a very large difference in the indicated energy levels, is necessary for the production of two pairs, the lifetime of the corresponding excited states should be very short, and this contradicts the strong forbiddenness of the $0^{\pm}$—$0^{\pm}$ transition.

The formula for the differential probability of two-quantum emission in M0 transitions is obtained from (1.52) by simply replacing one of the electric moments by a magnetic moment, so that in this case one of the emitted quanta should belong to the electric radiation and the other to the magnetic one. An estimate of the total probability $W_{\omega_1\omega_2}$ is made in [46] by taking into account one of the virtual states with $J' = 1$, the energy of which $\Delta' = 1$ MeV is much higher than the energy of the transition Δ (by a factor of 100 and more). The dependence of $W_{\omega_1\omega_2}$ for $0^{\pm}$—$0^{\mp}$ transition on ε and Δ turns out to be the same as for the $0^{\pm}$—$0^{\pm}$ transition. Table IV lists estimates of the average lifetime of the excited $0^{\pm}$ state $\tau_{\omega_1\omega_2}$ relative to two-photon emission, as a function of the transition energy Δ.

Table IV

Δ, eV	$\tau_{\omega_1, \omega_2}$, sec	τ_{K_1, K_2}, sec	τ_{L_1, L_2}, sec	$\tau\,(\Delta J=5)$, sec
10^5	$4 \cdot 10^2$	$2 \cdot 10^6$	$2 \cdot 10^7$	$3 \cdot 10^6$
$5 \cdot 10^4$	$5 \cdot 10^4$	$5 \cdot 10^7$	$2 \cdot 10^8$	$3 \cdot 10^8$
$2.5 \cdot 10^4$	$6 \cdot 10^6$	10^{10}	$9 \cdot 10^9$	$3 \cdot 10^{11}$
10^4	$4 \cdot 10^9$	—	$4 \cdot 10^{10}$	—
$5 \cdot 10^3$	$5 \cdot 10^{11}$	—	$7 \cdot 10^{11}$	—

The process of internal two-electron conversion on the K and L shells is treated analogously. For example, the starting formula for the probability of simultaneous emission of two K conversion electrons is of the form [46]

$$W_{K_1K_2} = 2\pi \int\limits_0^{\Delta - 2\varepsilon_K} \varrho(\varepsilon')\,\varrho(\varepsilon) \left| \frac{H_{is} H_{fs}}{E_i - E_s - \varepsilon_K - \varepsilon'} \right|^2 d\varepsilon, \quad (1.57)$$

where H_{is} and H_{sf} are matrix elements of the interaction operator of the shell electron with the electric and magnetic moments of the nucleus, E_i and E_s are the energies of the nucleus in the initial and intermediate states, ε_K is the ionization energy of the K electron, ε and ε' are the kinetic energies of the conversion electrons, and $\rho(\varepsilon)$ and $\rho(\varepsilon')$ are the densities of the electron states. In order to apply Formula (1.57) to the conversion of L electrons, it is necessary to replace ε_K by ε_L, where ε_L is the L electron ionization energy. Estimates for $W_{K_1K_2}$ and $W_{L_1L_2}$ are given under the same initial assumptions as for $W_{\omega_1\omega_2}$. As a result of the calculations, the following relationship was established for $W_{K_1K_2}$:

$$W_{K_1K_2} \backsim (\Delta - 2\varepsilon_K)^3 (\Delta' - \varepsilon_K)^{-2} \tag{1.58}$$

with an analogous relation for $W_{L_1L_2}$. The dependence on Δ of the lifetimes of the nucleus, $\tau_{K_1K_2}$ and $\tau_{L_1L_2}$, relative to the transition $0^{\pm}-0^{\mp}$, for the case of two-electron K and L conversion, is also given in Table IV [46] (with $\varepsilon_K = 10^4$ eV and $\varepsilon_K = 10^3$ eV). For comparison, the lifetimes of an excited nucleus relative to a transition in which the total angular momentum of the nucleus changes by 5, taken from [51], are also shown. It is seen from Table IV that only for small energies of the $0^{\pm}-0^{\mp}$ transition does its probability drop sufficiently to become comparable with the probability of a transition with $\Delta J = 5$. In this case the transitions $0^{\pm}-0^{\mp}$ can be distinguished from the transitions with large ΔJ only by the shape of the γ spectra or of the conversion spectra. It was indeed by these features that it was then established [52-53] that the isomer transitions are characterized by a considerable change in the total angular momentum J of the nucleus (and are not 0—0 transitions with change in parity).

The theory of monopole transitions developed above is based on the assumption that they are caused by electromagnetic interaction between the nucleons of the nuclei and the electrons of the atomic shell or of the Dirac background. The question of non-electromagnetic interaction between these particles, which could also lead to the occurrence of monopole transitions, was considered only as applied to 0—0 transitions of O^{16}. A theory of non-electromagnetic interaction is developed in [20,29,54-56] in analogy with the β-decay theory. It is assumed that the nucleons of O^{16} interact directly with the electron-positron field. The probability of the 0^+-0^+ transition is calculated from Formula (1.1). The perturbation operator H′ is taken [29] in the form customary for β decay

$$H' = g \sum \int (\Psi^+ \hat{O} \Psi)(\psi^+ \hat{O} \psi)\, dv, \tag{1.59}$$

where g is the constant of the electron-nucleon interaction, Ψ and ψ are the wave functions of the nucleon and electron in the occupation-number representation, and $\hat{O}$ is an operator which determines the type of the interaction. Analysis has shown [29,54] that if the transition under consideration is once forbidden with pseudo-vector coupling, then the spectrum shape of the positrons and the angular distribution of the electron-positron pairs will agree with the experimental results. However, in view of the once-forbidden nature of the transition, the parities of the initial and final 0-states of the O^{16} nucleus should be different, but in fact they are the same [1,57]. Shapiro [56] indicates that if the operator $\hat{O}$ is chosen such as to make the expressions $\Psi^+\hat{O}\Psi$ and $\psi^+\hat{O}\psi$ four-vectors, then by taking H′ in the form of a product of the time components of these vectors it is possible to obtain an angular distribution of electron-positron pairs agreeing with the experimental data, with the parities of both zero-states now the same.

In spite of these qualitative successes of the theory of direct non-electromagnetic interaction in explaining the 0—0 transition of the O^{16} nucleus, it nevertheless is not confirmed by experiment quantitatively. Calculating the probability of the 0—0 transition of O^{16} by means of formula (1.1) with account of (1.59), and comparing it with the experimental data, we can determine the constant g, which was found to be [29,58]

$$g \approx 5m_0c^2\left(\frac{e^2}{m_0c^2}\right)^3, \tag{1.60}$$

where m_0 is the electron mass. The value of this constant obtained, on the other hand, from experiments on electron-neutron scattering [59] is incomparably smaller:

$$g \approx 0.2m_0c^2\left(\frac{e^2}{m_0c^2}\right)^3. \tag{1.61}$$

If we now take account of the fact that the nonelectromagnetic electron-proton and electron-neutron interactions should be the same in magnitude, then the inconsistency of the explanation of the transition under consideration by attributing it to direct non-electromagnetic electron-nucleon interaction becomes obvious. At best one can merely state that the probability of the 0^+-0^+ transition of O^{16}, calculated under the assumption of a direct non-electromagnetic interaction, is only a very small correction (on the order of 10^{-3}) to the probability of this transition due to the electromagnetic electron-nucleon interaction.

Attempts were made [55] to relate the 0^+-0^+ transitions in O^{16} with non-electromagnetic electron-nucleon interaction relized via a meson field. According to this theory, the process of electron-positron pair production in the 0^+-0^+ transition can be described as follows. At first the nucleus goes from the excited state to the normal state, emitting a virtual meson. Then the electron, which is in a state with negative energy, absorbs this meson and goes over into a state with positive energy. It is shown in [55] that under a suitable choice of the type of meson and the form of the operator for the energy of interaction between the meson field and the light particles we can reconcile the theory with experiment, both with respect to the shape of the positron spectrum, and with respect to the angular distribution of the pairs, but, contrary to the experimental data, the parities of the initial and final zero-states of O^{16} must then be different.

Thus, we have verified with the 0^+-0^+ transition of the O^{16} nucleus as an example that the non-electromagnetic electron-nucleon interactions can be neglected in E0 transitions. It is easy to note, however, that in M0 transitions, the role of these interactions can become appreciable. The point is that the electromagnetic interactions in M0 transitions lead only to the occurrence of multi-particle emission processes, the probabilities of which may turn out to be much smaller (by approximately 10^2 times, for suitable values of Z and k) than the probabilities for the emission of one conversion electron or pair, induced by non-electromagnetic electron-nucleon interactions.

In this connection, attempts were made in [60] to detect electron positron pairs with total energy 10.98 MeV, emitted in the M0 transition $0^- \rightarrow 0^+$ of O^{16}. A determination of the probability of such a transition would make it possible to check whether this M0 transition is due to non-electromagnetic interactions. The experiment has shown that the number of pairs with energy close to 11 MeV is so small that there are not sufficient grounds for attributing these pairs to the $0^- \rightarrow 0^+$ transition. They are more likely to belong to the cosmic radiation background. Nonetheless, the following estimates are given in [60]: (1) the ratios of the number of 10.98-MeV pairs to the number of pairs and to the number of γ quanta with energy 3.86 MeV (0^+-1^- transition) turn out to be respectively $< 2 \times 10^{-2}$ and $< 2 \times 10^{-5}$, and (2) the lifetimes are $\tau_\pi(0^- \rightarrow 0^+) > 2 \times 10^{-8}$ sec.

In addition to the multiple emission processes in E0 transitions, which we have considered above, studies were made recently of other higher-approximation effects, particularly the so-called "electron and electron-nuclear bridges" [209-213]. By way of an example of an "electron bridge" we can mention the following process. As a result of interaction with the nucleus, the atomic-shell electron absorbs the virtual photon emitted by the nucleus and goes over into a state in the continuum, after which it returns to the initial state, emitting a γ quantum. The calculation of the probability of the "electron bridge" process has shown [212-213] that the theoretical value of $W_e(E0)$ is not only made more exact, but in the case of an unfilled atomic shell and in a preferred direction the E0 nuclear transitions can be accompanied by single-photon emission, which to be sure has very low intensity and cannot be measured by modern experimental techniques ($W\gamma(E0)/W_e(E0) = \pm 10^{-5}$ for an electron in an unfilled K shell).

It has also been established [212-213] that for M0 transitions the forbiddenness of the single-electron conversion (or single-photon) deexcitation of the nucleus (or deexcitation of the nucleus by emission of one conversion pair) is lifted by the "electron-nuclear bridge" process. In the simplest case this process consists of a double exchange of virtual photons between the nucleus and the shell electron, with the transition of the latter first to intermediate and then from intermediate into final states. An estimate of the probability of single-conversion deexcitation of the nucleus in an M0 transition through an "electron-nuclear bridge" is made in [213] on the basis of a single-particle nuclear model and yields $W_K(M0) \approx 2 \times 10^4$ sec^{-1}, that is, a value which is already experimentally observable. It must be noted that the contribution from the higher subshells (L_{II}, L_{III}, $M_{II}, \ldots$) to the single-electron M0 conversion is much larger than in the case of E0 conversion.

2. EXCITATION OF ELECTRIC MONOPOLE TRANSITIONS BY ELECTRONS

The investigation of both elastic and inelastic scattering of different particles by an atomic nucleus is one of the most important methods of studying nuclear properties. If the particles are charged and have energy much lower than the Coulomb barrier of the nucleus (precisely such particles will be dealt with in this section), then the scattering of the particles will follow the well-known laws of electromagnetic phenomena. The results of the investigations of particle scattering can be then interpreted more rigorously and more accurately than in the case when the interactions between the particles and the nucleus are of non-electromagnetic character.

As a result of the inelastic collision between a particle and a nucleus the latter goes over from the ground state into an excited state. Theoretical and experimental investigations have shown that the behavior of the inelastically scattered particles can yield information on the energy and probability of this transition and also on its multipolarity and type.

Electrons exhibit characteristic features in inelastic collisions with nuclei. Electrons have that advantage over heavy charged particles moving with subbarrier velocities, that they penetrate freely inside the nucleus and can pass through it*, so that a study of inelastic electron scattering can give more accurate information on the details of the structure and the wave functions of the nucleus (particularly in monopole transitions) than does the investigation of inelastically scattered heavy particles.

The theory of monopole excitation of nuclei by electrons is based on the general theory of inelastic electron scattering. This raises the following question: find the effective cross section for inelastic scattering of electrons if we know 1) the initial and final states of the nucleus plus electron system, described by wave functions Ψ_i, Ψ_f and ψ_i, ψ_f, respec-

*We assume here, of course, that the conditions under which non-electromagnetic interactions between the electron and the nucleons in the nucleus can be neglected are satisfied.

tively; 2) the energy Δ transferred by the electron to the nucleus, and 3) the interaction between the electron and the nucleons of the nucleus, described by the formula

$$V=\int\left(\varrho\varphi-\frac{1}{c}\mathbf{jA}\right)d\mathbf{r}, \qquad (2.1)$$

where φ and A are the field potentials, and ρ and j are the charge and current densities produced by all the system particles (that is, by the nucleons and the electron). Exact quantum mechanical calculations have shown [61–64], that the excitation cross sections do not change if the interaction V is replaced by the simpler expression [65]*

$$V'=-e^2\sum_p\frac{e^{k|\mathbf{r}-\mathbf{r}_p|}}{|\mathbf{r}-\mathbf{r}_p|}-\frac{e\mathbf{v}}{c^2}\sum_n\int\frac{\mathfrak{j}_n e^{ik|\mathbf{r}-\mathbf{r}_n|}}{|\mathbf{r}-\mathbf{r}_n|}=V_1+V_2, \qquad (2.2)$$

where $k=\Delta/\hbar c$, v and r are the velocity and radius vector of the electron, and p and n are indices pertaining to the proton and nucleon, respectively. The presence of the factors $\exp[ik\mid \mathbf{r}-\mathbf{r}_n\mid]$ in (22) takes account of retardation effects.

Let us consider the case when $k\mid \mathbf{r}-\mathbf{r}_n\mid$ is small, that is, the delay effect can be neglected. Then we should have

$$|\mathbf{r}-\mathbf{r}_n|\ll\lambda, \qquad (2.3)$$

where λ is the wavelength corresponding to the transition energy Δ. The order of magnitude of λ differs for the known monopole transitions of different nuclei, but is in no case less than 10^{-11} cm. If we now recognize that the electron must penetrate inside the nucleus in order to excite the monopole transition (see Sec. 1), condition (2.3) is satisfied.

We now expand the interaction V' in a multipole series. However, in view of the fact that in the expansion of the second term of V_2 there is no monopole term [64–66,9], we confine ourselves to an expansion of the Coulomb part V_1 of the interaction (2.2) [with account of condition (2.3)]. This expansion can be carried out in two ways [65]:

$$V_1=-\sum_p\frac{e^2}{|\mathbf{r}-\mathbf{r}_p|}=-4\pi e^2\sum_{p,L,M}\frac{1}{2L+1}\frac{r_p^L}{r^{L+1}}Y_L^M(\theta,\Phi)Y_L^{M*}(\theta_p,\Phi_p) \qquad (2.4)$$

for $r_p<r_e$ and

$$V_1=-\sum_p\frac{e^2}{|\mathbf{r}-\mathbf{r}_p|}=-4\pi e^2\sum_{p,L,M}\frac{1}{2L+1}\frac{r^L}{r_p^{L+1}}Y_L^M(\theta,\Phi)Y_L^{M*}(\theta_p,\Phi_p) \qquad (2.5)$$

for $r_p>r_e$.

The first series will not yield monopole transitions, since the term with $L=0$ does not depend on r_p and its matrix element will vanish because of the orthogonality of the nuclear wave functions. A nonzero monopole matrix element is contained only in the expansion (2.5). Since the latter applies only to electrons, it is they which can excite the monopole transitions directly.

The problem of the electron excitation of monopole transitions is simplest to deal with in the Born approximation. In this case both the initial and final states of the electron, ψ_i and ψ_f, are plane waves; this is possible under the following conditions:

1. The kinetic energy of the electron is much larger than the transition energy

$$T\gg\Delta. \qquad (2.6)$$

2. The inequality

$$\frac{Ze^2}{\hbar v}\ll 1, \qquad (2.7)$$

holds, where v is the electron velocity. Inequality (2.7) signifies essentially that the de Broglie wavelength of the electron exceeds greatly the classical minimum distance from the electron to the center of the nucleus [65]. For extremely relativistic electron velocities, (2.7) turns into the inequality*

$$\frac{Ze^2}{\hbar c}\ll 1,\ \text{i.e.}\ \frac{Z}{137}\ll 1. \qquad (2.8)$$

The theory of inelastic scattering of charged particles by a nucleus yields in the Born approximation the following formula for the differential effective cross section [63–65]

$$d\sigma(\theta,\Phi)=\frac{k_f^2}{\hbar^2 v_i v_f}\left|(\mathbf{k}_f,\Psi_f\left|\sum_p\frac{e^2}{|\mathbf{r}-\mathbf{r}_p|}\right|\mathbf{k}_i,\Psi_i)\right|^2 d\Omega, \qquad (2.9)$$

where k_i and k_f are the wave vectors of the electron in the initial and final states. After integrating in (2.9) over the electron wave functions we obtain for the differential cross section of monopole excitation [63,64]

$$d\sigma_{E0}(\theta)=\left(\frac{Ze^2}{\hbar c}\right)^2\frac{1}{\mathbf{k}_i^2}\left|F_{E0}(J\to J,K)\right|^2\times\frac{\frac{4m_0c^2}{\hbar^2k_ik_f}+\frac{k_i}{k_f}+\frac{k_f}{k_i}-\frac{k^2}{k_i\cdot k_f}+2\cos\theta}{\left(\frac{k_i}{k_f}+\frac{k_f}{k_i}-2\cos\theta\right)^2}d\Omega. \qquad (2.10)$$

*It is indicated in [67,68] that for a sufficiently large magnitude or range of the scattering potential, condition (2.8) must be replaced by the more rigorous condition

$$\frac{Ze^2}{\hbar c}\ln\frac{a_0}{r_0'}\ll 1, \qquad (2.8')$$

where a_0 is the Bohr radius and r_0' is the smallest of the following quantities: the de Broglie wavelength of the electron, the radius of the nucleus, and the ratio a_0/Z. This condition is so stringent that even C^{12} is at the borderline of its applicability. On the other hand, experiments on the inelastic scattering of fast electrons by C^{12} give for the effective cross section σ values that are comparable with the results of the Born-approximation calculation.[69]

*In the relativistic case we must take in place of v the Dirac velocity operator $c\boldsymbol{\alpha}$.

Here

$$F_{E0}(J\to J, K)\equiv F_{E0}(K)=\frac{1}{Z}\Big(f\Big|\sum_p j_0(Kr_p)\Big|i\Big) \quad (2.10')$$

is the so-called form factor for the E0 transition, $j_0(Kr_p)$ is the Bessel spherical function $j_L(Kr_p)$ with $L = 0$. In the extreme relativistic case, taking into account (2.6) and the equation*

$$\hbar K=\hbar\,[k_i^2+k_f^2-2k_ik_f\cos\theta]^{\frac{1}{2}}\approx 2\hbar k_i\sin\frac{\theta}{2} \quad (2.11)$$

we obtain for the differential cross section of monopole excitation †

$$d\sigma_{E0}(\theta)=\left(\frac{Ze^2}{\hbar c}\right)^2\frac{\cos^2\frac{\theta}{2}}{4k_i^2\sin^4\frac{\theta}{2}}\left\{\frac{1}{Z^2}\Big|\Big(f\Big|\sum_p j_0(Kr_p)\Big|i\Big)\Big|^2\right\}d\Omega. \quad (2.12)$$

The first factor in (2.12) (in front of the curly brackets) is the differential cross section for elastic scattering of electrons by a point charge in the relativistic Born approximation.

We assume now that $Kr_p \ll 1$ (or, which is the same, $KR \ll 1$). Then the form factor in (2.12) can be expressed in terms of the nuclear matrix element of the monopole M, using the expansion

$$j_0(Kr_p)=1-\frac{(Kr_p)^2}{2\cdot 6}+\cdots \quad (2.15)$$

Confining ourselves to the first two terms of the series (2.15) and recognizing that the first yields zero after substitution in the form factor (in view of the orthogonality of the nuclear wave functions), we obtain for the square of the form factor ‡

$$|F_{E0}(J\to J, K)|^2\approx\frac{K^4}{36Z^2}\Big|\Big(f\Big|\sum_p r_p^2\Big|i\Big)\Big|^2=\frac{K^4|M|^2}{36Z^2}. \quad (2.16)$$

If we recognize that when $f = i$ and $KR \ll 1$, the square of the form factor (2.10′) is almost equal to unity and (2.12) yields in this case the differential cross section for elastic scattering of electrons, $d\sigma$, then the ratio of $d\sigma_{E0}$ to $d\sigma$ will be given approximately by (2.16), that is, we have

$$\sqrt{\frac{d\sigma_{E0}}{d\sigma}}\approx K^2\frac{|M|}{6Z}. \quad (2.17)$$

The smaller K, the more accurate (2.17). This formula is used to determine the nuclear matrix element of the monopole from the experimental data on elastic and inelastic electron scattering (see below).

A theoretical analysis of the cross sections calculated in the Born approximation for both elastic and inelastic electron scattering [formulas (2.12)—(2.14)], as a function of the values of K, has shown[65] that when $KR \ll 1$ the elastic scattering predominates. With increasing KR, the excitation cross section of transitions with low multipolarity L (in particular, monopole transitions) increases first, and when $KR \gg 1$ the transitions with high multipolarity begin to be most intensely excited.

The case of excitation of monopole transitions by electrons having threshold energies (that is, kinetic energies close to the energies of the monopole transitions), was first considered theoretically by K. Ter-Martirosyan [9]. He obtained the following formula for the differential effective cross section of electron excitation of electric monopole transitions (in the relativistic system of units)

$$d\sigma_{E0}=\frac{(\alpha Z)^2}{18}N_{p_i}N_{p_f}\frac{p_f}{p_i}\{\varepsilon_i\varepsilon_f+1-(\alpha Z)^2$$
$$+[p_ip_f-(\alpha Z)^2]\cos\theta\}S_0\,d\Omega, \quad (2.18)$$

where α—fine structure constant, N_{pi} and N_{pf} are coefficients of the type

$$N_p=(2p)^{2(\gamma-1)}e^{\frac{\pi\alpha Z\varepsilon}{p}}\Big|\Gamma\Big(\gamma+i\frac{\alpha Z\varepsilon}{p}\Big)\Big|^2, \quad (2.19)$$

$\gamma=\sqrt{1-(\alpha Z)^2}$, p_i, p_f, ε_i, and ε_f are the initial and final momenta and energies of the electron. On the other hand, S_0 is given by

$$S_0=\frac{1}{\lambda_0^2}\left|\frac{4Q_{if}^{(0)}}{[\Gamma(2\gamma+1)]^2}\right|^2, \quad (2.20)$$

where $\lambda_0 = \hbar/m_0c$ is the Compton wavelength, introduced to facilitate the transition to the ordinary units, and Q_{if}^0 is connected with the nuclear matrix element of the monopole by the equation

$$(-1)^{M_i}C^{00}_{J_i,-M_i,J_f,M_f}Q_{if}^{(0)}=\frac{3}{\gamma(2\gamma+1)}\int\Psi_i^*r^{2\gamma}\Psi_f\,dr, \quad (2.21)$$

$C^{00}_{J_i,-M_i,J_f,M_f}$ are the Clebsch-Gordan coefficients, and Ψ_i and Ψ_f are the nuclear wave functions. In 0-0 transitions and for negligibly small αZ, the quantity $Q_{if}^{(0)}$ is the usual nuclear matrix element of the monopole for single-particle excitation of the nucleus.

Formula (2.8) has been obtained with the aid of the

*It is easy to see that (2.11) and all the formulas derived on its basis will not be applicable for small scattering angles.

†In the EL transition, the square of the form factor is[65]

$$|F_{EL}(K)|^2\equiv|F_{EL}(J_i\to J_f, K)|^2=\frac{4\pi^2}{Z^2}\frac{K^{2L}}{[(2L+1)!!]^2}B(CL, K), \quad (2.13)$$

where B(CL, K) is a quantity obtained from the reduced probability of radiative transition

$$B(EL)=\frac{1}{2J_i+1}\sum_{M_iM_f}\Big|\Big(J_f, M_f\Big|\sum_p r_p^LY_L^M(\theta_p,\Phi_p)\Big|J_i, M_i\Big)\Big|^2 \quad (2.14)$$

by the substitution $r_p^L\to\frac{(2L+1)!!}{K^2}j_L(Kr_p)$. The effective cross section for the EL transition is also expressed by (2.12), except that the form factor is given by (2.13).

‡It is easy to see from (2.13)–(2.14) that the same K-dependence will hold for the electric quadrupole form factor when $KR \ll 1$. If we estimate the form factors for the monopole and quadrupole excitations on the basis of the single-particle model of the nucleus, we find that the cross sections of these excitations will be almost equal in magnitude for identical experimental conditions.[65]

wave functions of an electron in a nuclear Coulomb field, without account of the finite nuclear size and the screening effect. It is assumed that the excitation energy Δ is not very large, $\Delta \ll 100\,A^{1/3}$ MeV (A—atomic weight of the nucleus being excited), so that $p_i R \ll 1$, where p_i is the initial momentum of the electron in the relativistic unit system and R is the nuclear radius. The latter condition is equivalent to including in the calculations only those scattered-electron states which are characterized by the quantum numbers $j = 1/2$.

The electron wave functions used in the calculations are taken in the form of four-component relativistic functions ψ_{p_i, ν_i} and ψ_{p_f, ν_f} where ν_i and ν_f are the quantum numbers determining the spin states of the electron before and after scattering, while p_i and p_f are the initial and final momenta of the electron at infinity. As $r \to \infty$ ψ_{p_i, ν_i} is represented asymptotically in the form of the sum of a plane and an outgoing spherical wave, while ψ_{p_f, ν_f} is the sum of a plane and an incoming spherical wave.

From (2.18) we obtain by integrating with respect to θ

$$\frac{\sigma_{E0}}{S_0} = \frac{2\pi (\alpha Z)^2}{9} N_{p_i} N_{p_f} \frac{p_f}{p_i} [\varepsilon_i \varepsilon_f + 1 - (\alpha Z)^2]. \quad (2.22)$$

At the threshold value of the incoming-electron energy (that is, for $p_f \to 0$ or $\varepsilon_i - 1 \to k$), the quantity σ_0/S_0 has a non-zero limit equal to

$$\lim_{p_f \to 0} \frac{\sigma_{E0}}{S_0} = \frac{2\pi^2 (\alpha Z)^2}{9} (2\alpha Z)^{2\gamma-1} \left\{ \frac{N_{p_i}}{N_{p_f}} \right\}_{\varepsilon_i = k+1} (k + 2 - \alpha^2 Z^2) \quad (2.23)$$

(because $\lim_{p_f \to 0} p_f N_{p_f} = \pi (2\alpha Z)^{2\gamma - 1}$). Starting from this limit, σ_{E0}/S_0 increases monotonically with p_f. Figure 19 shows curves of σ_{E0}/S_0 against $\varepsilon_f - 1$, calculated [9] for the particular case $\Delta = 1$ MeV $\approx 2m_0c^2$, for a nucleus with Z = 50 and Z = 80. An approximate numerical estimate [9] yields for S_0 a value 10^{-28}—10^{-26} cm^2. For comparison we present the results obtained in the Born approximation in accordance with the formula [9]

$$\lim_{\alpha Z \to 0} \frac{d\sigma_{E0}}{S_0} = \frac{(\alpha Z)^2}{18} \frac{p_f}{p_i} (\varepsilon_f \varepsilon_i + 1 + p_i p_f \cos\theta)\, d\Omega, \quad (2.24)$$

$$\lim_{\alpha Z \to 0} \frac{\sigma_{E0}}{S_0} = \frac{2\pi (\alpha Z)^2}{9} \frac{p_f}{p_i} (\varepsilon_f \varepsilon_i + 1). \quad (2.25)$$

The calculation of σ_{E0} with account of the finite nuclear size was made by Grechukhin [12], but likewise for a scattered electron with $j = 1/2$ and with not too high an energy ($\varepsilon_i \ll 15$ MeV for heavy nuclei). The calculation yielded a rather cumbersome formula for σ_{E0} (see [12]).

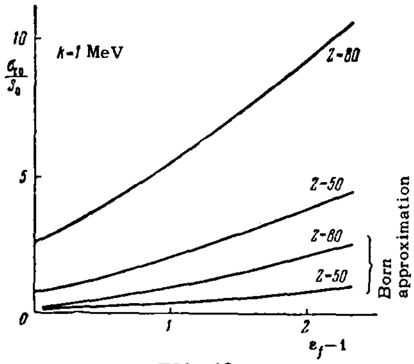

FIG. 19.

The dependence of the differential effective cross section of the monopole excitation by electrons on the angle between the momenta p_i and p_f can be represented by the function [12] (after integrating over the energies ε_i and ε_f)

$$\frac{d\sigma_{E0}}{d\Omega} = \frac{\sigma_0}{4\pi} (1 + b_0 \cos\theta)\, \varrho^2 \cdot 10^{-30} \left[\frac{\text{cm}^2}{\text{sr}} \right]. \quad (2.26)$$

Table V [12] gives the numerical values of σ_0 and b_0 for the excitation of electric monopole transitions of Ca^{40}, Ge^{72}, Zr^{90}, Pd^{106} and RaC′ (Po^{214}) for incident-electron energy values 10 m_0c^2 and 20 m_0c^2 (when $\varepsilon_i = 20\ m_0c^2$ the phase shift due to the finite size of the nuclei must be taken into account; this was not done in [12]).

The influence of the finite nuclear size on the results of the calculations of σ_{E0} is illustrated in Table VI [12], which lists the ratio of σ_{E0} calculated for the foregoing monopole transitions with the aid of the Coulomb functions of the point nucleus to the values calculated with finite nuclear size taken into account. Table VI shows that the finite size of the nuclei leads to a considerable change (a reduction to almost one-half) of the effective cross section of monopole excitation by electrons only in the case of large Z.

Experiments aimed at observing the excitation of either monopole or multipole nuclear transitions by fast electrons, so as to confirm the values of σ calculated in the Born approximation, were carried out

Table V

Nucleus	Energy 10 m_0c^2		Energy 20 m_0c^2		Nucleus	Energy 10 m_0c^2		Energy 20 m_0c^2	
	σ_0	b	σ_0	b		σ_0	b	σ_0	b
Ca^{40}_{20}	0.054	0.93	0.34	1	Pd^{106}_{46}	2.52	0.99	7.4	1
Ge^{72}_{32}	1.08	0.97	3.2	1					
Zr^{90}_{40}	0.98	0.97	3.7	1	Po^{214}_{84}	67	0.98	190	1

Table VI

Nucleus	Ca^{40}_{20}	Ge^{72}_{32}	Zr^{90}_{40}	Pd^{106}_{46}	Po^{214}_{84}
$\varkappa$	0.98	0.97	1.0	1.16	1.87

principally at Stanford [69-73]. To obtain fast electrons (with energy 190 MeV and more), a linear accelerator was used. The electrons were scattered by thin targets and then magnetically analyzed and detected with Cerenkov counters so as to obtain the angular distributions of different groups (a detailed description of the experiments is found in Hofstadter's review [73]).

Monopole excitation has been established for the time being only in the scattering of electrons by C^{12} nuclei. Inelastic and elastic scattering on these nuclei were observed simultaneously. Figure 20 shows the energy distribution of the scattered electrons for a definite scattering angle [69,73]. The initial electron energy is 187 MeV and the scattering angle 80°. We see that the curve is characterized by maxima of the elastic (first peak on the right) and inelastic scattering (the remaining peaks are shifted relative to the first by amounts equal to the excitation energies of the corresponding nuclear levels), the second of the inelastic peaks (counting from the right), for an approximate energy of 177 MeV, corresponds to electric monopole excitation of C^{12} with transition energy 7.6 MeV.* The width of the maxima deviates from Gaussian on the low-energy side, owing to the presence of the so-called "tail" due to the radiation processes in the target. The strongest such "tail" is possessed by the elastic-scattering maximum.

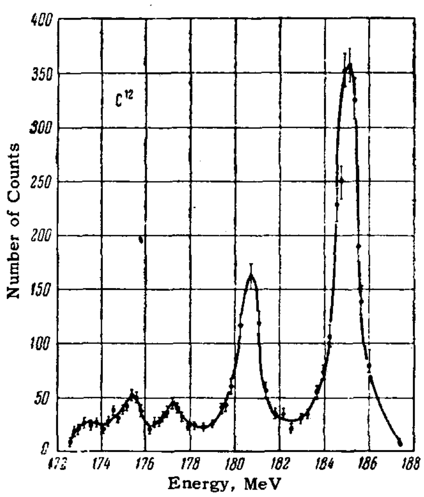

FIG. 20.

*Experimental results on the observation of a 7.6-MeV 0^+-0^+ E0 transition from excited C^{12} produced in various nuclear reactions, are found, for example, in [74-77] (for more details see Sec. 3, page 310-311).

This "tail" decreases approximately in inverse proportion to the difference between the given energy and the energy corresponding to the maximum of elastic scattering.

Investigations of the inelastic scattering of an electron with initial energies of 150 and 80 MeV have shown [69,71] that at higher initial electron energies, the inelastic peaks are better separated, while the elastic peak grows relatively more slowly with increase in these energies (this essentially confirms the theory; see page 732). In addition, it turns out that at higher initial electron energies, the "tails" indicated above will be smaller so that it will be much easier to separate the inelastic peaks against a lower background. It must be noted, however, that at very high initial electron energies (for example, for $KR \gg 1$), the nuclear transitions with large L (and not with $L = 0$) will most probably be excited.

In comparisons of the experimental values of the effective excitation cross section with the theoretical values account must be taken of the radiative corrections and the bremsstrahlung effect. According to Schwinger[78,79] the radiative corrections reduce the observed excitation intensity by approximately 8% at small angles and almost 20% at large scattering angles.* The bremsstrahlung effect is somewhat more appreciable: it causes the excitation intensity to decrease by 20% at small angles and by almost 40% at large scattering angles[78,79]. Figure 21 shows [71,73] curves, plotted with account of these corrections, for the effective cross sections as functions of the scattering angle for both elastic (curve 1) and inelastic scattering (curves 2, 3, and 4 for the excitation of the levels 4.43, 7.66 (0^+ level) and 9.61 MeV, respectively). It is seen from the figure that curves 2, 3 and 4 for inelastic scattering are much less steep than the curve for the elastic scattering (the height of the elastic peak varies by approximately 2×10^6 times in the angle interval from 35 to 138°). In the case of sufficiently large scattering angles, therefore, the cross section for the excitation of any of the levels becomes larger than the cross-section for elastic scattering. In addition, inasmuch as curves 2 and 4 are almost parallel, we can conclude that the 4.43 and 9.61 MeV levels correspond to excited nuclear states with identical total angular momenta and parities (states 2^+).

*It must be noted that the radiative corrections vary slowly with the angle and therefore influence principally the absolute effective excitation cross section.

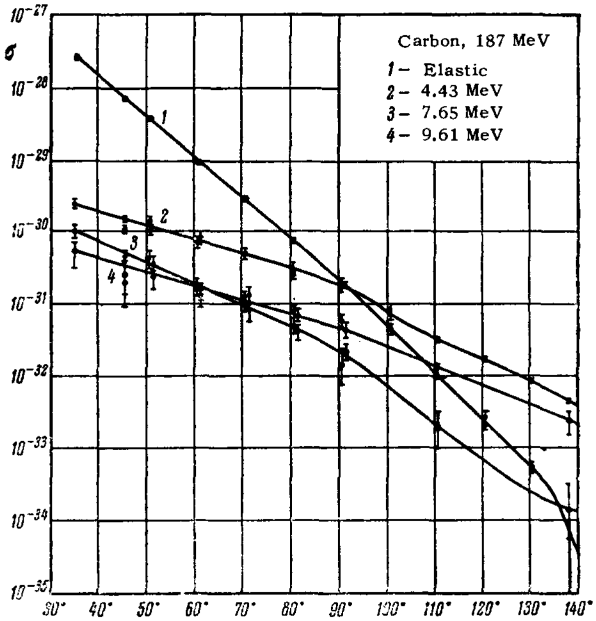

FIG. 21.

In the interpretation of the experimental results it is frequently convenient to deal not with the differential effective excitation cross section but with the "measured" square of the form factor $|F(K)|^2$, defined as the ratio of the measured differential effective excitation cross section to the calculated differential effective cross section for elastic scattering by a point nucleus. In particular, in the case of monopole excitation of C^{12} ($\Delta = 7.66$ MeV), this "measured" form factor was used successfully by Schiff [80] to determine the nuclear matrix element $M(C^{12})$ of the monopole in accordance with Formula (2.17) (by extrapolating the experimental values of $(d\sigma_{E0}/d\sigma)^{1/2}$ to the limiting values obtained for $K \to 0$). He obtained $M(C^{12}) = 3.8 \times 10^{-26}$ cm^2.* It is possible to obtain M also from the absolute value of the cross section of monopole excitation as a function of the scattering angle, but this method of determining the nuclear matrix element of the monopole is considered to be less reliable.[80]

The experimental data on the excitation of nuclear transitions by electrons with threshold energies are so far skimpy. The dependence of the cross section σ for the excitation of Cd^{114} on the kinetic energy $\mathcal{E}_i - 1$ of the incoming electrons was investigated in [81]. It was found that this cross section is practically zero until the kinetic energy of the electrons reaches the excitation energy, after which it increases abruptly, and then σ decreases monotonically to the next excitation threshold, then increases suddenly, and the process repeats. Owing to the lack of enough experimental data, it is difficult to compare them with the theory of monopole excitations of nuclei by electrons with threshold energies [9].

*A somewhat different Schiff value for M, approximately 5×10^{-26} cm^2, is given in [65].

3. EXAMPLES OF MONOPOLE TRANSITIONS OF NUCLEI

We consider now the observed monopole transitions of individual nuclei. All turned out to be E0 transitions (one attempt of observing the M0 transition was discussed at the end of Sec. 1). Most of the reliably established E0 transitions are of the 0^+—0^+ type. Transitions of this type were identified principally by observing the concomitant emission of the internal conversion electrons and electron-positron pairs, with complete absence of γ quanta of energy equal to the transition energy. The monopole nature of the investigated transition is further confirmed by the agreement between the experimental values of K/L, L_I/L_{II}, W_K/W_π and other quantities with the theoretical values (see Sec. 1).

The presence of E0 transition admixtures in nuclear transitions of the "$J^+ \to J^+$" type can be detected by observing some excess of conversion electrons or electron-positron pairs over the number that should be emitted in the conversion processes that compete with the corresponding γ radiation in the absence of the E0 transition. For a quantitative determination of the E0 admixture, the angular correlations of various cascade processes are investigated and the results compared with theory (see Sec. 1).

O^{16}. One of the most thoroughly investigated monopole transitions[20-23,28,54-58,82-86] is the 0^+—0^+ transition of the O^{16} nucleus. This transition is produced by proton bombardment of F^{19} via the reaction

$$_9F^{19} + {}_1H^1 \to {}_{10}Ne^{20*} \to {}_2He^4 + {}_8O^{16}. \quad (3.1)$$

The O^{16} nucleus can occur in either the ground state or in four excited states with energies that differ relatively little from one another (Fig. 22) [1]. Three of these states (which we denote by ${}^\gamma O^{16}$) are characterized by the fact that the transition between them and the ground state are accompanied by emission of photons. On the other hand, the transition from the fourth and lowest excited state, ${}^\pi O^{16}$, to the ground state gives rise to electron-positron pairs. The latter cannot be attributed to internal conversion of the photons, for in this case the number of pairs should be much smaller (1%) than the number of photons, whereas actually these numbers are comparable [1]. In addition, photons with energy equal to the difference of the levels ${}^\pi O^{16}$ and O^{16} (6.051—0.010 MeV)[102] are missing from the radiative spectrum of O^{16}, whereas photons with energies 7.09, 6.99, and 6.14 MeV, values equal to the differences of the ${}^\gamma O^{16}$ and O^{16} levels, are present there. If it is also taken into account that the spin of O^{16} in the ground state vanishes, then it follows on the basis of

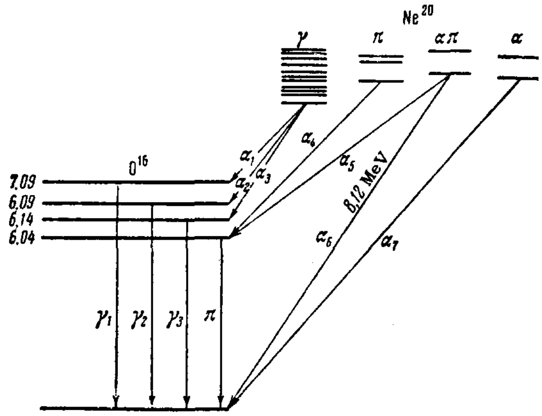

FIG. 22.

the general theory of monopole transitions (Sec. 1) that the ${}^{\pi}O^{16} \rightarrow O^{16}$ transition is 0—0.

It has been demonstrated experimentally [1] that the O^{16} levels responsible for the emission of photons on the one hand, and of electron-positron pairs on the other, appear at different resonant proton energies. This means that there are several groups of levels of the compound nucleus Ne^{20}, and α decay from one group is accompanied by a transition to the levels ${}^{\gamma}O^{16*}$, while decay from the other group causes transitions to the level ${}^{\pi}O^{16}$ (See Fig. 22). If we assume that the data on the level groups of Ne^{20} are perfectly reliable (this pertains particularly to the $\alpha\pi$ group), then it can be shown [1] that the transition ${}^{\pi}O^{16} \rightarrow O^{16}$ occurs without a change in parity. Indeed, the transition from the $\alpha\pi$ group can be both to the ${}^{\pi}O^{16}$ level and to the normal level. We denote the parity of the $\alpha\pi$ state by $d_{\alpha\pi}$. Then the parities of the states ${}^{\pi}O^{16}$ and O^{16} will respectively be $d_{\pi} = d_{\alpha\pi}(-1)^{L_{\pi}}$ and $d_0 = d_{\pi\alpha}(-1)^{L_0}$, where L_{π} and L_0 are the angular momenta carried away by the α particle in the transitions ${}^{\alpha\pi}Ne^{20} \xrightarrow{\alpha_5} {}^{\pi}O^{16}$ and ${}^{\alpha\pi}Ne^{20} \xrightarrow{\alpha_5} O^{16}$ (see Fig. 22). Since the angular momenta in the states ${}^{\pi}O^{16}$ and O^{16} are equal to zero, L_{π} and L_0 are equal to each other. Consequently also $d_0 = d_{\pi}$, that is, the parities of the nuclear wave functions describing the states ${}^{\pi}O^{16}$ and O^{16} are identical, and since A and Z of O^{16} are both even, they are also positive.

According to general theory of monopole transitions (Sec. 1), the transition ${}^{\pi}O^{16*} \rightarrow O^{16}$ should be accompanied not only by pair but also by electron internal conversion. However, as shown by calculations [21], the probability of the latter is smaller than the probability of the former by a factor 28,000. So far, no E0 conversion electrons from O^{16} have been observed.

As already noted (see page 721), the energy distribution of the positrons can be obtained by starting with Formula (1.32), which expresses the differential probability for the production of an electron-positron pair in 0—0 transitions.* It is then possible to compare the result with the experimental data. The results of this comparison turned out to be fully satisfactory [1,83].

The angular distribution of the electron positron pairs obtained on the basis of (1.32) is also in fair agreement with experiment [84,85].

This distribution was investigated in [84] by the coincidence method with two different setups ("close" and "far") of counter-telescopes relative to the source of the electron-positron pairs (that is, relative to the location of the proton-bombarded CaF_2 target). After integration of (1.32) with respect to the energy and account of the counter efficiency, the theoretical formula for the angular distribution assumes the form

$$P(\theta)_{\text{theor}} = A[1 + (0.9937 - \delta')\cos\theta], \qquad (3.2)$$

where δ' depends on the dimensions of the useful area and the location of the counters (for infinitesimally small counters $\delta' = 0$). For the two specific mentioned counter installations we obtain from (3.2) [84]

$$1)\ P(\theta)_{\text{theor}} \approx A[1 + (0.955 \pm 0.003)\cos\theta], \qquad (3.3)$$

$$2)\ P(\theta)_{\text{theor}} \approx A[1 + (0.974 \pm 0.002)\cos\theta]. \qquad (3.4)$$

On the other hand, the experimental dependence of the function $P(\theta)$ on θ was found to be for these two cases

$$1)\ P(\theta)_{\text{exp}} = A[1 + (0.948 \pm 0.012)\cos\theta], \qquad (3.5)$$

$$2)\ P(\theta)_{\text{exp}} = A[1 + (0.980 \pm 0.009)\cos\theta]. \qquad (3.6)$$

Comparing (3.3), (3.4) with (3.5), (3.6) we see that the theoretical and experimental coefficients of $\cos\theta$ differ from each other in the mean by 0.002 ± 0.008, that is, by an insignificant amount. Some contradiction between the theoretical and experimental angular distributions of the pairs, observed in [57,86], must be attributed to insufficient experimental accuracy [84]†.

Present day experiment is incapable of detecting the different corrections to the angular distribution of the pairs (radiative) and also the corrections that take into account the internal bremsstrahlung and the Coulomb interaction between the pair components, as well as other corrections (see page 722) although it is predicted [84] that the presence of such corrections can be established in the future.

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*We recall that this formula has been derived assuming a Coulomb interaction between the protons of the nucleus and the electrons of the Dirac background.

†According to Groshev and Shapiro[1] difficulties still remain in explaining why the transition from the level ${}^{\pi}Ne^{20}$ (see Fig. 22) is possible to the level ${}^{\pi}O^{16*}$ (with emission of an α particle), and is impossible to the level O^{16}, although the levels ${}^{\pi}O^{16*}$ and O^{16} differ only in energy. It is supposed[1] that a way out of the difficulty can be found by introducing additional selection rules with respect to some imprecise quantum number.

The experimental value of the total pair-conversion probability W_π for the 0^+-0^+ of the O^{16} nucleus is equal to $\sim 1.4 \times 10^{10}$ sec^{-1} according to the latest data[84]. From a comparison with the reduced probability Ω_π, calculated from Formula (1.34), a value 3.8×10^{-26} cm^2 has been obtained for the nuclear matrix element of the monopole[84]; this is presently regarded as the most accurate value. The corresponding reduced monopole nuclear matrix element is $\rho(O^{16}) \approx 2/5$ (for $R = 1.2 \times 10^{-13} A^{1/3}$)*. Theoretical estimates of the nuclear monopole matrix element of O^{16} based on different nuclear models are discussed in Sec. 4.

RaC′(Po^{214}). As long ago as in 1930 it was suggested[88,15] that the conversion line corresponding to the transition energy 1416 keV, observed in the RaC (Po^{214}) spectrum, belongs to the 0^+-0^+ transition†, since no corresponding line was found in the γ spectrum. Many years, however, have passed before the assumed E0 transition in RaC′ was fully confirmed by various quantitative data.

A comparison of the relative conversion coefficients K/L, L_I/L_{II}, and L_{III}/L_{II} (which pertain to the transition from the 1416 keV level to the ground level) with their theoretical values for different multipoles has shown[88-90] that the considered transition cannot belong to any of the electric or magnetic multipole transitions, meaning that it is a 0—0 transition. On the other hand, comparison of say the experimental values of K/L with the theoretical ones for E0 transitions (Church and Weneser[11]) is satisfactory.

However, the de-excitation of the excited nucleus in the E0 transition can proceed also via production of electron-positron pairs. The theoretical ratio of the probability of E0 conversion on the K shell to a probability of pair conversion, is equal for RaC′ to 170 (after Thomas[14]) or 420 (after Sakharov[21]). Experiment yields for W_K/W_π values in the range $440 \le W_K/W_\pi \le 625$[82,91,92]. These values are seen to be closer to the second of the given theoretical results. It is nevertheless assumed that the experimental values of W_π are insufficiently accurate (since their determination entabils non-unique operations[93]).

At the present time the following sequence has been established for the low-lying excited levels of RaC′ (Fig. 23[93,94]). The excited 0^+ state (1416 keV) is in this case the sixth and not the first excited state, as was the case with the O^{16} nucleus.

Various types of de-excitation of the nucleus from the 1416-keV level were investigated, such as emission of long-range α particles (group α_3), conversion transition to the ground level, single-photon or conversion transitions to the 609-keV level. The remaining possible processes accompanying this de-excitation of the nucleus have low probability and were not observed[93].

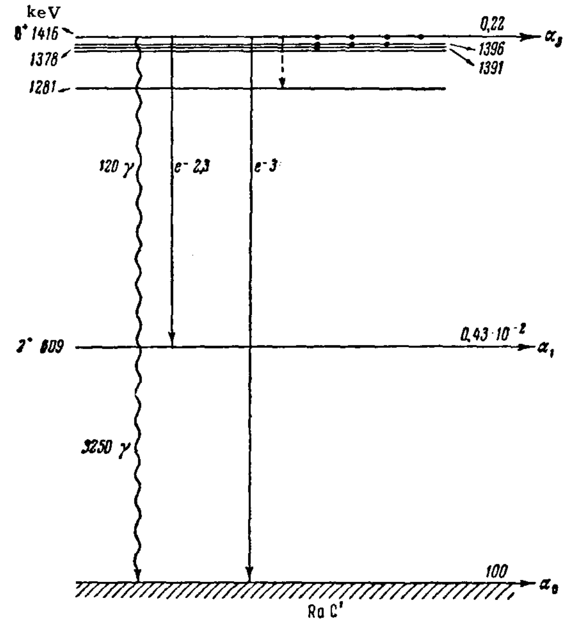

FIG. 23. The transition $\alpha_3 - \alpha_0$: e^- 3.5).

The most important problem is to determine the absolute probability of the E0 transition or (which is the same) the partial lifetime τ_e of the 1416 keV level relative to the conversion transition to the ground state. Unfortunately, the value of τ_{e^-} has not yet been accurately determined. One of the methods for estimating it is based on the relation

$$\frac{\lambda_{e^-}}{\lambda_{\alpha_3}} = \frac{N_{e^-}}{N_{\alpha_3}}, \tag{3.7}$$

where λ_{e^-} and λ_{α_3} are the probabilities of the considered conversion transition and emission of long-range α particles, and N_{e^-} and N_{α_3} are the numbers of internal-conversion electrons and α particles of group α_3 per decay, which are known from experiment; λ_{α_3} is obtained from the experimental value of the probability for the emission of α particles from the ground state of RaC′ (group α_0). By extrapolating the known formula from α-decay theory to the case of the excited levels[93,95] we obtain

$$\lambda = De^{-4\alpha \frac{c}{v} Z(2u_0 - \sin 2u_0)}, \tag{3.8}$$

where v —velocity with which the α particle and the nucleus move apart, $u_0 = \cos^{-1}(ER/2Ze^2)$, E is the total energy released in ergs, and α is the fine-structure constant. Dzhelepov and Shestopalova[93] obtained in this manner a value $\tau_e = 3.0 \times 10^{-10}$ sec for the partial lifetime of the 1416 keV state (for $R = 1.2 \times 10^{-13} A^{1/3}$ cm), which differs noticeably from the earlier estimates of Bethe[96] ($\tau'_{e^-} = 8 \times 10^{-11}$ sec) and Drell and Rose[25] ($\tau''_{e^-} = 2.5 \times 10^{-11}$ sec).

The accuracy of the result (τ_{e^-}) depends on the extent to which Formula (3.8), derived for a spherical

*A value $\rho(O^{16}) = 1/2$ is given in [11-87].

†The states are positive because RaC′ is even-even.

nucleus, is applicable to the somewhat deformed RaC′ nucleus ($\Delta R/R = 0.2$ [97]). In a rougher approximation it is possible to calculate τ_{e^-} by comparing the sought probability with the probability of γ-quantum emission in the transition from the 1416 keV to the 609-keV level, using the Weisskopf approximation (with corrections by the Sunyar method[97]) to obtain the theoretical value of $\lambda\gamma$. In this case τ_{e^-} was found to equal 3.5×10^{-12} sec[93].

In view of the lack of sufficiently accurate data on the absolute probability of the E0 transition of RaC′, comparison of the experimental and theoretical data cannot yield (see Sec. 1) exact values of the reduced nuclear monopole matrix element ρ. An estimate of ρ for RaC′ was first made in 1930 by Fowler, who used the single-particle model of the nucleus to calculate the E0-transition probability.* He obtained for ρ a value close to $1/20$. In later work[25,98], comparison of the reduced E0-transition probability calculated with account of the finite nuclear size with the experimental partial lifetime of the 0^+ state of RaC′, namely $\tau_{e^-} = 2.5 \times 10^{-11}$ sec, yielded for ρ values in the range $1/9 \le \rho \le 1/4$ and a value of $1/5$[98]. If we use in this comparison $\tau_{e^-} = 3.0 \times 10^{-10}$ sec, ρ is decreased by a factor of approximately 3.5.

$\underline{Ge^{72,70}}$. The 0^+-0^+ transition of Ge^{72} with energy 0.69 MeV is the third observed monopole nuclear transition. As was established in[99-101], the excited Ge^{72} nucleus with energy 0.69 MeV is produced by bombardment of Ga^{72} with slow neutrons and subsequent β decay. The transition of Ge^{72} from the first excited (0.69-MeV) state to the ground state is accompanied by emission of an intense conversion line with energy close to 0.69 MeV, without a corresponding γ quantum. The lifetime of this excited state τ_{e^-} was found to be 3×10^{-7} sec[102]. The presence of strongly converted high-energy radiation together with the short lifetime of the excited state indicate that the 0.69-MeV transition of Ge^{72} cannot be classified as isomeric[103] but must be a monopole transition of the 0^+-0^+ type [101,104].

Comparing the experimental value of $\tau_{e^-}(0^{+*})$ of Ge^{72} with theory[11] (allowing for $W_\pi = 0$) we obtain [102] $\rho(Ge^{72}) = 0.11$. An earlier estimate[98] of $\rho(Ge^{72})$ was twice as large.

A type 0—0 E0 transition was also observed in Ge^{70}, which is the β-decay product of Ga^{70}; the 0^{+*} state is in this case a second-excited state with energy 1.215 MeV, so that the nucleus can become de-excited both by electron E0-conversion (no pair E0 conversion was observed) and by cascade emission of two γ quanta with energies (0.173 ± 0.002) and (1.042 ± 0.005) MeV. The partial lifetime of the 0^{+*} state of Ge^{70} relative to the emission of the E0-conversion K electrons was found to be $\tau_{e^-} = (24 \pm 1.2) \times 10^{-7}$ sec [102]. Comparison of τ_{e^-} with theory yields $\rho(Ge^{70}) = 0.09$, that is, a value differing little from $\rho(Ge^{72})$.

$\underline{C^{12}}$. The opinion was expressed in 1954[105] that the small number of electron-positron pairs with maximum energy 7.66 MeV, observed in the $Be^8(\alpha\gamma)C^{12}$ reaction, offers evidence of the presence of an electric monopole transition of the 0^+-0^+ type in C^{12}, inasmuch as no corresponding γ emission (that is, emission with energy 7.66 MeV)* was observed. On the other hand, observation of γ quanta with energies 3.16 and 4.43 MeV[76] indicates a cascade. The cascade type, $0^+ \to 2^+ \to 0^+$, was then established on the basis of an investigation of the angular correlation of these quanta in [77]. The existence of excited levels 0^+ and 2^+ of C^{12} was also confirmed by experiment on inelastic scattering of electrons[69-73], which were already mentioned in Sec. 2. From the data of these experiments a value of 3.8×10^{-26} cm^2 was obtained[80] for the C^{12} nuclear monopole matrix element (its theoretical estimates are given in Sec. 4).

Pursuant to this, particular attention was paid to the study of the excited 0^+ state of C^{12}, in connection with the question of the sources of stellar energy and the theory of the origin of elements. It was predicted for the first time in [107] that the carbon nucleus produced in the assumed cascade of nuclear reactions $He^4 + He^4 \to Be^8$ and $Be^8(\alpha\gamma)C^{12}$, which occurs in the later stages of stellar evolution† —red giants—should be in an excited state with energy 7.6—7.7 MeV. It is precisely the presence of this resonance level in C^{12} which causes the probability of the $Be^8(\alpha\gamma)C^{12}$ reaction to become comparable with the probability of a reaction of the inverse type at sufficiently high temperatures.

The probability of two successive nuclear reactions $2He^4 \to Be^8$ and $Be^8(\alpha\gamma) \to C^{12}$ ‡ is given by the formula[109]

$$W = 3^{\frac{5}{2}} 8\pi^3 \frac{\hbar^5}{m_\alpha^{\frac{5}{2}} (kT)^3} (\varrho\chi_\alpha)^2 \frac{\Gamma_\gamma \Gamma_\alpha}{\Gamma_\gamma + \Gamma_\alpha} e^{-\frac{Q}{kT}} \ \mathrm{sec}^{-1} \qquad (3.9)$$

where m_α is the α-particle mass, T the absolute temperature, ρ the density, χ_α the concentration of helium (by weight), and Q an energy equal to the difference between the energy of the excited 0^+ state and the binding energy of C^{12} (in mass units $Q = m(C^{12}) - 3m_\alpha$). The quantity Γ_γ is the partial width of the 7.66 MeV level of C^{12} for transition of this nucleus to the ground level, and Γ_α is the partial width of the

*This model consisted of an α particle moving in a nuclear field with a potential in the form of a spherically-symmetrical box with walls of 1) infinite and 2) finite height.

*The 7.66 MeV level of C^{12} was observed as long ago as in 1952[106].

†When almost the entire hydrogen in the core of the star is converted into helium and gravitational contraction causes the temperature of this core to reach 10^{8}°K and the density to reach 10^5 g/cm^3.[108]

‡Much attention is paid presently to the theory of these reactions, since reactions similar to $Be^8(\alpha\gamma)C^{12}$ can explain the formation of the nuclei O^{16} Ne^{20} and others (for example, $C^{12}(\alpha\gamma)O^{16}$).[110]

7.66 MeV level for α decay. Inasmuch as $\Gamma_\alpha \gg \Gamma_\gamma$ [110], we have $\Gamma_\gamma\Gamma_\alpha/(\Gamma_\gamma + \Gamma_\alpha) \approx \Gamma_\gamma$ and W is proportional to Γ_γ. On the other hand, the partial width Γ_γ consists of the partial width of the 0^{+*} level for monopole transition and the partial width for the above-indicated γ cascade, that is, $\Gamma_\gamma = \Gamma(E0) + \Gamma(E2)$. We see therefore that for an exact determination of the probability of carbon production in a stellar medium it is necessary to have as accurate a value of the probability of the C^{12} electric transition as possible (along with $\Gamma(E2)$).

Using the experimental value given above for the C^{12} nuclear monopole matrix element, Salpeter [110] obtained $\Gamma(E0) \approx \Gamma_{e\pm} = 4 \times 10^{-5}$ eV, where $\Gamma_{e\pm}$ is the partial width of the 0^+ level for emission of monopole electron-positron pairs (the remaining processes accompanying the E0 transition are neglected). The upper limit for $\Gamma_{e\pm}$ was established in [111] by using the experimentally measured ratio of the number of monopole pairs to the number of γ quanta with 4.43 MeV energy and known relative populations of the 7.66 and 4.43 MeV levels [112]. The result obtained was $\Gamma_{e\pm} < 7 \times 10^{-5}$ eV which does not contradict Salpeter's estimate. On the other hand, the ratio $\Gamma_{e\pm}/\Gamma_\alpha$ is approximately 7×10^{-6} according to [113] and 6.6×10^{-6} according to [114], and much smaller than the value $\Gamma_{3.23\gamma}/\Gamma_\alpha \approx (3.3 \pm 0.9) \times 10^{-4}$ [115], where $\Gamma_{3.23\gamma}$ is the partial width of the 7.66 MeV level for the emission of a 3.23-MeV γ quantum.

Because the energy Q in (3.9) depends greatly on the exact value of the excitation energy of the 0^+ state, more and more accurate measurements were made of the 7.66-MeV level. By way of an example we present some values of this level: (7.66 ± 0.02) MeV [116] (1955), (7.658 ± 0.027) MeV [117] (1956), (7.653 ± 0.008) MeV [109] (1957), and (7.654 ± 0.009) MeV [110], and finally 7.656 MeV [118].

Zr^{90}. The presence of a 0^+-0^+ for Zr^{90} was predicted theoretically by Ford [119]. According to a shell model, the protons in nuclei beyond Z = 38 are first placed in the $p_{1/2}$ levels (the first at Z = 39 and second at Z = 40), and then in the $g_{9/2}$ levels. Inasmuch as the 0-order scheme gives a sufficiently large difference between the particle levels $g_{9/2}$ and $p_{1/2}$, the proton shell of Zr^{90} can be regarded as almost filled. The discovery of the isomer transition $5^- \to 0^+$ with energy 2.3 MeV [120] confirms the quasi-filling of the proton shell of Zr^{90} and simultaneously points to the existence of a configuration $p_{1/2}g_{9/2}$ ("5^-" state of Zr^{90}). In the 0-order scheme, the levels determined by the nucleon configurations $p_{1/2}$, $p_{1/2}g_{9/2}$, and $g_{9/2}^2$ are at equal distances from one another (Fig. 24, left half of the level diagram). If on the other hand we take account of the residual interaction between the nucleons, then these levels split and drop in such a way that the downward shift of the levels of the $(g_{9/2})^2$ and $(p_{1/2})^2$ configurations is much larger than that of the

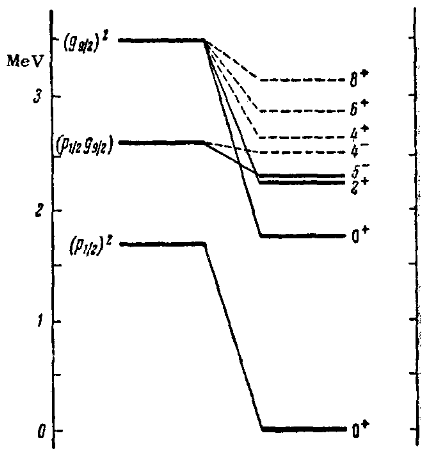

FIG. 24.

levels of the $(p_{1/2}g_{9/2})$ configurations, and consequently the 0^{+*} level is located below the 5^- level [119,121]* (Fig. 24). Thus, the first excited state of Zr^{90}, as can be seen from Fig. 24, is a 0^+ state.

The transition from this state to the normal state was observed then experimentally [123]. In the β radiation of Y^{90}($_{39}Y^{90} \to {}_{40}Zr^{90}$) there were observed conversion electrons belonging to Zr^{90} with energy 1.75 MeV (1.734 ± 0.005 MeV according to [124]). Inasmuch as no photons were found with such energy, we must assume that we are dealing here with internal conversion of a monopole transition of the $0^+ \to 0^+$ type. This deduction is confirmed also by the observation of the positron spectrum of Zr^{90} with maximum positron energy 0.8 MeV.

Electron and pair conversion of the 0^+-0^+ transitions of Zr^{90} were subsequently investigated in detail, with the ratios of the number of conversion electrons and the number of pairs to the number of β electrons† emitted by Y^{90} per second determined with greater accuracy, and with measurement of the lifetime of the excited 0^+ state of Zr^{90} and other quantities characterizing to a greater or lesser degree the considered 0^+-0^+ transitions [124-132,102].

In [124] the experimental and theoretical data concerning the values of $K/(L_I + M_I)$ and W_e/W_π for the E0 transition of Zr^{90} are compared and ρ is determined from Formula (1.45), using the experimental value of the lifetime τ of the excited 0^+ state of Zr^{90}. The best agreement between theory and experiment was found for W_e/W_π. The theoretical value of W_e/W_π is 2.4 (as given by Zyryanova and Krutov [28] and Church and Weneser [11]) or 2.8 (as given by Thomas [14] and Church and Weneser [11]), while the experimental value [124] is 3.0. The result of the com-

*More accurate calculations of all these levels can be found in [122].

†On the basis of these ratios we obtain the values of $K/(L_I + M_I)$, W_K/W_π, and W_e/W_π.

parison is somewhat worse for $K/(L_I + M_I)$, the experimental value of which is 5 [124] or 4 [131], while theoretically $K/(L_I + M_I) = 7.1$ (see [11,132]).

The measured lifetimes $\tau(0^{+*})$ vary. For example in [129] the value given is $\tau = (8.5 \pm 3) \times 10^{-9}$ sec, whereas [132] gives $(90 + 6) \times 10^{-9}$ sec*, that is, one order of magnitude larger. In this connection, two values are obtained for the reduced nuclear monopole matrix element, $\rho = 0.18$ in the first case [124] and $\rho = 0.56$ in the second [132].

Pt^{196}. The first two excited and the ground level of Pt^{196} form the sequence $2^{+\prime} \rightarrow 2^+ \rightarrow 0^+$ with transition energies 334.0 ($2^{+\prime} \rightarrow 2^+$) and 356.5 keV ($2^+ \rightarrow 0^+$). The nuclear transition $2^{+\prime} \rightarrow 2^+$ is a mixture of the E0, M1, and E2 transitions. From the theory of such transitions, developed in pages 725-727, it follows that in order to determine the E0 admixture in the $2^{+\prime} \rightarrow 2^+$ transition of Pt^{196} it is possible to use the following four independent experiments: experiments on the establishment of the angular correlations $\gamma-\gamma$ and $e_K-\gamma$† in the cascade $2^{+\prime} \rightarrow 2^+ \rightarrow 0^+$, on the measurement of the absolute probability $W'_\gamma(E2)$ of the E2 transition in the second link of the cascade ($2^+ \rightarrow 0^+$)‡, and the determination of the total coefficient of internal conversion in the first link of the cascade ($2^{+\prime} \rightarrow 2^+$). Gerholm and Petterson [133] used the results of the first three experiments to calculate $q^2(Pt^{196}) = W_K(E0)/W_K(E2)$ and the nuclear matrix element of the monopole $\rho(Pt^{196})$, namely the results of investigations [133] of the $e_K-\gamma$ angular correlation and experimental values of the probability $W'_\gamma(E2)$ and of $\delta^2 = W_\gamma(M1)/W_\gamma(E2)$ [36,134]. Since the coefficients of $P_i(\cos\theta)$ in the function (1.49), which describes the $e_K-\gamma$ angular correlation were investigated in [133] with allowance for the possible time dependence of the perturbation due to the magnetic interaction between the electron shell excited by the preceding K capture or the K conversion and the nuclear magnetic moment, the value obtained for q lies in the range $0.24 \le q \le (0.56 \pm 0.01)$ (the lower limit for q has been obtained with account of the indicated perturbation, and the upper one without it).

The uncertainty in ρ is somewhat larger. It is due not only to the uncertainty in the number q, but also in the uncertainty of the quantity $W'_\gamma(E2)$ [see (1.51′)]. On the other hand, the uncertainty of $W_\gamma(E2)$ is explained by the fact that this quantity is not determined by experiment, but is usually obtained from the known experimental value of $W'_\gamma(E2)$ by using the ratio of the reduced probabilities $B(E2; 2^{+\prime} \rightarrow 2^+)/B(E2; 2^+ \rightarrow 0^+) = b$, established on the basis of various model-dependent considerations, and which therefore assumes various values.* Thus, for example, according to the "free oscillation" model [34,138] we have $b = 2$. Then ρ lies in the range [133] $0.017 \le \rho \le (0.039 \pm 0.007)$. On the other hand, if b is calculated with the aid of the Davydov and Filippov theory of non-axial nuclei [139], its value is $^{10}/_7$ and we obtain $0.013 \le \rho \le 0.04$. [140]

Gerholm and Petterson [133] give one more estimate, with a wider range $0.009 \le \rho \le 0.05$, obtained under the assumption that $\frac{1}{2} \le B(E2; 2^{+\prime} \rightarrow 2)/B(E2; 2^+ \rightarrow 0) < 2$.†

It must be noted that the first estimate of $\rho(Pt^{196})$ was made by Church and Weneser [11,32], who used the experimental data on the $\gamma-\gamma$ angular correlation, $W'_\gamma(E2)$, and the total conversion coefficient β^K (this method of determining ρ is described in pages 725-726). They obtained $\rho < \frac{1}{34}$, which does not contradict the estimates indicated above for the nuclear monopole matrix element.

Rare earths (Ce^{140}, Sm^{152}, and Gd^{152}). It was established in [142-143] that in β decay of La^{140}, and also in electron capture or β decay of Pr^{140}, the Ce^{140} nucleus can occur in an excited 0^+ state with energy 1902 keV. Two conversion K and L lines, corresponding to an averaged transition energy 1902 keV, were observed in the Ce^{140} spectrum. No corresponding γ line was found (4×10^{-4} quantum per decay). Measurement of the relative conversion yielded $K/L = 6.33$. An estimate of the lower boundary for the conversion coefficient yields $\beta^K > 0.38$. One cannot attribute such a high value of β^K to a high-multipolarity transition ($L > 10$), for in this case the 1902-keV state would be an isomer state with a lifetime $> 10^{10}$ years, whereas in fact the lifetime is approximately equal to 38 hours. Consequently, it can be concluded that the transition under consideration is a 0^+-0^+ transition.

Investigations have shown [142,143] that the 1902 keV level of Ce^{140}, which is the second excited level, is much more frequently excited in the decay of Pr^{140} than in the decay of La^{140}. The number of excitations of this level amounts in the second case to 0.013% of the number of excitations in the first case.

E0 transitions were also observed in Sm^{152} and

*Approximately the same values of τ were obtained also in [102,130].

†It should be noted that investigations of the angular correlation in the case of Pt^{196} are preferred in the sense that the correlation function is practically independent of the theoretical uncertainty in the K-conversion coefficient for Ml-radiation. [43,134]

‡Measurement of $W'_\gamma(E2)$ is essential, since the absolute probability of emission of an E2 quantum, $W'_\gamma(E2)$, in the first link of the cascade ($2^{+\prime} \rightarrow 2^+$) has not yet been determined experimentally.

*The quantity $B(E2; 2^{+\prime} \rightarrow 2^+)/B(E2; 2^+ \rightarrow 0^+) = b$ was also determined recently experimentally, principally from data on the $\gamma-\gamma$ angular correlation and measurements of the relative conversion coefficients K/L. [136] The most promising with respect to the determination of b are experiments on the Coulomb excitation of the $2^{+\prime}$ level. [137]

†The value $b = B(E2; 2^{+\prime} \rightarrow 2^+)/B(E2; 2^+ \rightarrow 0) = 1/2$ is taken from [140], in which it was established for Pt^{194}. It is assumed in [133] that the values of b for Pt^{194} and Pt^{196} should be the same. After correcting the mistake noted by Listengarten and Band [19] we obtain finally $0.013 \le \rho \le 0.070$.

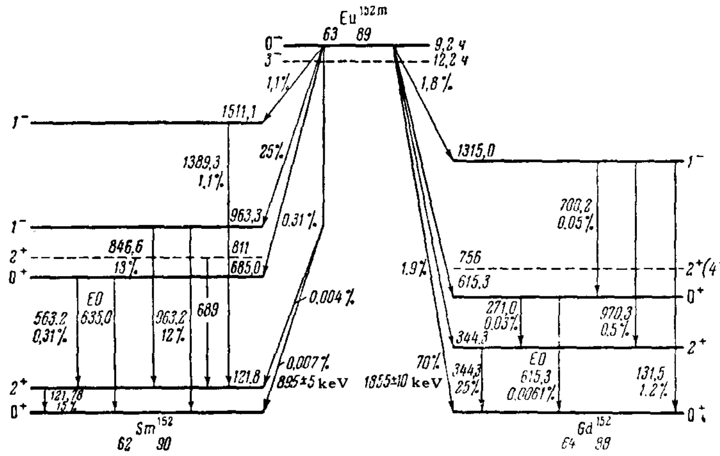

FIG. 25. (Transition on the left $1^- - 2^+$: 831.6 keV).

Gd^{152}, which are the decay products of Eu^{152}. Figure 25 shows a diagram of the low-lying collective purely rotational (two lower) and β-vibrational (two upper) levels of Sm^{152} and Gd^{152}, with indication of the E0 transitions [144].* The levels shown dashed in the figure arise in the decay of the isomer Eu^{152} with half life 12.2 years. The diagram shows also the percentage ratios of the numbers of the observed transitions to the number of Eu^{152} decay events.

Sm^{152}. Two types of E0 transitions have been established: 0^+-0^+ with energy 685 keV [145] and 2^+-2^+ with energy 689 keV [146] (see Fig. 25). Using the theoretical values [147] of $B(E2;\ 2^{+\prime} \to 2^+)/B(E2;\ 2^+ \to 0^+) \simeq {}^{10}/_{7}$ and of the conversion coefficient $\alpha_2^K(2^{+\prime} \to 0^+)$ for Sm^{152}, and also the experimental data on the electron conversion of the 689 and 811 keV transitions [148], an estimate was made in [146] of the ratio of the intensity of the E0 conversion and radiation of E2 quanta in the $2^{+\prime} \to 2^+$ transition. This ratio was found to equal approximately 0.07.

Subsequently the investigations of the γ-ray and internal conversion electron spectra produced in the $(n\gamma)$ reaction on Ga^{155} and Ga^{157} have shown [214] that some of the transitions of Ga^{156} and Ga^{158} can be classified as monopole (as 0^+-0^+ and 0^+-2^+ transitions with energies 1010 and 1041 keV respectively in the case of Ga^{156} and as 2^+-2^+ transitions with energies 1436, 1373, 1405, and 1454 keV in the case of Ga^{158}).

$U^{234,232}$. The radioactive isotope U^{234} is usually obtained as a result of α decay of Pu^{238}, β decay of the isomer Pa^{234}, or electron capture by Np^{234}. Investigations of both the γ and conversion spectrum of U^{234} have established [149-152] two E0 transitions of the U^{234} nucleus, one of 0^+-0^+ type with energy 812 keV†, and the second 2^+-2^+ with energy 810 keV.

Several groups of conversion electrons (K, L, M_I, and N_I electrons) accompanying the foregoing E0 transitions, were observed in [152]. The ratio of the intensities of the K-electron lines for the 810 and 812 keV transitions was found to be $1:(4 \pm 1)$, from which it follows that the probability of the second of these transitions is much larger than that of the first. The identification of the 810 and 812 keV E0 transitions was confirmed by comparing the experimental value [152] of $K:L_I:M_I$ and of $K:(L_I+M_I)$ with the theoretical ones [11,19]. Figure 26 shows the scheme of several low-lying levels* of U^{234}, which is the decay product of Np^{234}, with indication of the E0 transitions and their relative intensities [152].

Electric monopole transitions of the 0^+-0^+ type with energy 816.4 keV and the 2^+-2^+ with energy 817.5 keV have been observed [155] also for the U^{232} nucleus, produced as a result of β^- decay of Pa^{232}.

In addition to the foregoing examples of E0 transitions, it must be noted that E0 transitions were observed also for $Ca^{40,42}$ (see [203,216,132]), Pd^{106} (see [124]), Cd^{114} (see [154]), $Th^{230,232}$ (see [155-156]),

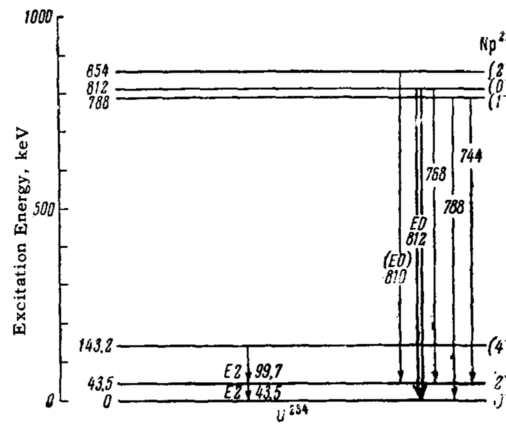

FIG. 26.

*The transition 2^+-2^+ is indicated on the basis of [146].

†A somewhat different value for the energy of this transition, 805 keV, is also given in the literature.[153]

*The state 1^-, corresponding to the 787 keV level, has not yet been sufficiently accurately established in [152].

$U^{236,238,240}$ (see [156-157]), Pu^{238} (see [153,158]) and Bi^{212} (see [159]). The E0 transition of Bi^{212} is the only type $0^-{-}0^-$ transition observed to date ($\Delta = 176$ keV, with 0^- and 0^- the fourth and second excited states[159]). The existence of electric monopole transitions of the type J—J was also predicted and an estimate of the parameter ρ was made by the method described in pages 288-289 on the basis of the experimental data for Hg^{198} (see [11]), Pt^{192} (see [11]) and Au^{197} (see [87]). A summary of the most important experimental data on E0 transitions is given in Table VII.

Of particular interest is the transition of Au^{197} since, unlike all the other nuclei given above, it can have an angular momentum with half integer J only. The scheme of the Au^{197} levels produced as a result of β decay of Pt^{197} or electron capture of Hg^{197} is given in Fig. 27[87]. The values of the number J corresponding to the levels shown in the scheme have been accurately determined, with the exception of the 268-keV level.

There are grounds for assuming[87] that the 191-keV transition of Au^{197}, which corresponds to an appreciable experimental internal conversion coefficient $\beta^e = 2.5$[160], should be regarded as an (E0 + M1) transition of the $\frac{1}{2}{-}\frac{1}{2}$ type (to this end it is sufficient to prove that $J = \frac{1}{2}$ for the 268-keV level).

The 268 keV level cannot be assigned $J = \frac{3}{2}$, since β^e would then be ≤ 1.[87] Values of J larger than $\frac{3}{2}$ are excluded, for they would contradict the character of the β decay[87]. Consequently the 191-keV transition is actually $\frac{1}{2}{-}\frac{1}{2}$.

Since electric quadrupole radiation is forbidden in the $\frac{1}{2} \rightarrow \frac{1}{2}$ transition, formula (1.47) for the E0 admixture simplifies greatly. We have

$$\beta^e = \frac{W_e(E0) + W_e(M1)}{W_\gamma(M1)} = \frac{W_e(E0)}{W_\gamma(M1)} + \beta_1^e. \qquad (3.10)$$

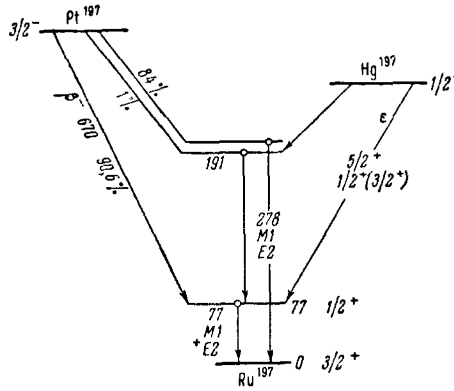

FIG. 27.

Substituting in (3.10) the experimental value $\beta^e = 2.5$ [160], the theoretical value $\beta_1^e = 1.0$, and the value of $W_\gamma(M1)$ calculated by the Moszkowski formula[2], we get $W_e(E0) \simeq 4 \times 10^{11}$ sec^{-1}, and then in the usual manner $\rho \approx 0.5$[87]. After correcting the error noted by Listengarten and Band, we have $\rho = 0.70$.

4. THEORETICAL ESTIMATES OF THE NUCLEAR MONOPOLE MATRIX ELEMENT

In Sec. 1 we developed a general theory for E0 transitions, according to which the most important characteristics are the quantities M and ρ—the nuclear and reduced nuclear monopole matrix elements. It was also shown (Secs. 1—3) how to obtain the experimental values of M and ρ by comparing the results of the theory with the experimental data. A very important problem is the comparison of these experimental values with the theoretical estimates of M and ρ made on the basis of different nuclear models, for it is possible to establish in this way the degree

Table VII. The subscript in the designation for the spins of the excited nuclei denotes the serial number of the excited state, $e^\pm$—conversion with production of electron-positron pairs, e^-—electron conversion (the remaining notation is explained in the text)

Nucleus	Transition	Δ, Mev	Conversion	$\tau(E0)$, sec	ρ_{exp}
1. C^{12}	$0_2^+ \to 0^+$	7.656	$e^\pm$	$\approx 1.5 \cdot 10^{-11}$	$\frac{1}{2}$
2. O^{16}	$0_1^+ \to 0^+$	6.051	$e^\pm$	$7.2 \cdot 10^{-11}$	0.42
3. Ca^{40}	$0_1^+ \to 0^+$	3.348	$e^\pm$, e^-	$3.4 \cdot 10^{-9}$	0.15
4. Ca^{42}	$0_2^+ \to 0^+$	1.836	$e^\pm$, e^-		0.41
5. Ge^{70}	$0_2^+ \to 0^+$	1.215	e^-	$24 \cdot 10^{-7}$	0.09
6. Ge^{72}	$0_1^+ \to 0^+$	0.69	e^-	$3 \cdot 10^{-7}$	0.11
7. Zr^{90}	$0_1^+ \to 0^+$	1.734	e^-, $e^\pm$	$90 \cdot 10^{-9}$	0.056
8. Cd^{114}	$0_4^+ \to 0^+$	1.308	e^-	$5 \cdot 10^{-10}$	0.63
9. Pd^{106}	$0_2^+ \to 0^+$	1.137	e^-	$\gg 10^{-8}$	
10. Pt^{196}	$2_2^+ \to 2_1^+$	0.334	e^-		0.013÷0.070
11. Au^{197}	$\left(\frac{1}{2}\right)_2^+ \to \left(\frac{1}{2}\right)_1^+$	0.191	e^-	$2.5 \cdot 10^{-12}$	0.70
12. RaC′(Po^{214})	$0_6^+ \to 0^+$	1.416	e^-, $e^\pm$	$2.5 \cdot 10^{-11}$	0.17

to which any particular model is applicable to any specific nucleus whose monopole transition is observed.

The theoretical estimates of ρ are based on different model representations of the nucleus. It is simplest to estimate ρ by starting with the single-particle model. Since in this case the states of the nucleus are distinguished by the states of the "external" nucleon moving in the field of a fixed nuclear residue ("core"), the value of ρ can be readily expressed in the form

$$\varrho \equiv \sum_p \int \Psi_f^* \left(\frac{r_p}{R_i}\right)^2 \Psi_i \, d\mathbf{r} = \left(\frac{Z}{A^2} + \delta_p\right) \int u_f^* \frac{r^2}{R^2} u_i \, d\mathbf{r} \equiv \left(\frac{Z}{A^2} + \delta_p\right) \varrho', \quad (4.1)$$

where u_i and u_f are the wave functions of the initial and final states of the nucleon. In the first term the Z/A^2 in the sum preceding the integral takes into account the core recoil [7], whereas the second term δ_p has a value 0 or 1, depending on whether the external nucleon is a neutron or a proton. The numerical value of the integral ρ' can be obtained only by making some specific assumption with respect to the form of the averaged potential of the nucleons and the solution of the corresponding Schrodinger equation. In very rough approximation, putting $u_f = u_i = \text{const}$ for $r \le R$ and $u_f = u_i = 0$ for $r > R$, we obtain $\rho' = 0.6$. In all other cases we should expect $\rho' < 0.6$ [7]. As can be seen from (4.1), for single-proton transitions in medium and heavy nuclei $\rho \approx \rho'$, meaning that $\rho < 0.6$. Even in the lightest nuclei for which 0^+-0^+ transitions can still be observed, this estimate varies little (for example $\rho \le 0.625$ for C^{12}).

Estimates of M were made for C^{12} and O^{16} on the basis of the α-particle model [80]. According to this model, the nuclei C^{12} and O^{16} consist of three and four α particles, respectively. In the equilibrium position, the α particles are located at the vertices of an equilateral triangle (C^{12}) or tetrahedron (O^{16}). The excited 0^+ state of each of these nuclei is characterized by radial oscillations with equal phase, executed by the α particles about the equilibrium position ("pulsating") model.

Schiff [80] gives a general formula with which to calculate the nuclear matrix element M in this case:

$$M = ZR_\alpha h \sqrt{\frac{2}{Am_n\Delta}}, \quad (4.2)$$

where m_n is the mass of the nucleon, Δ the excitation energy, and R_α the distance from the center of the α particle (in the equilibrium position) to the center of the nucleus. Assuming that the nuclear radius R is larger than the radius R_α by 1.0×10^{-13} cm [80], Formula (4.2) yields $M = 11 \times 10^{-26}$ and 17×10^{-26} cm^2 for C^{12} and O^{16}, respectively. These results turned out to be much overvalued (by three and almost five times, respectively) compared with the numerical values of M obtained by experiment.* According to [84,80,65] the latter are practically the same for O^{16} and C^{12} and are equal to $(3.8-5) \times 10^{-26}$ cm^2.

In [80] there are also estimates of M for C^{12} and O^{16} represented in the form of spherical drops of a slightly compressible liquid (the "liquid drop" model of the nucleus), the charge and mass of the nuclei being uniformly distributed. It is assumed that the excited 0^+ state of each of the nuclei corresponds to excitation of radically-symmetrical pulsating oscillations of the spherical drop such that the variation of the density of the liquid at a distance r from the center of the sphere is proportional to $j_0(\pi r/R)$, where j_0 is the spherical Bessel function and R is the radius of the nucleus. If only such oscillations of the drop nucleus are assumed in the calculation of M, we obtain as a result

$$M = \frac{6Z\,Rh}{\pi^2} \sqrt{\frac{3}{Am_n\Delta}}. \quad (4.3)$$

The numerical values of M, calculated by Formula (4.3) for C^{12} and O^{16}, exceed the corresponding experimental values even more than in the case of the α-particle model.

We present one more estimate of ρ, based on the liquid drop model of the nucleus, in which account is taken, in addition to the radial oscillations connected with the compressibility of the nucleus, also of the polarization oscillations of the proton and neutron components of the nuclear liquid, the frequency of the latter being much smaller than the frequency of the former. If the nucleus goes from the ground state into the excited state as a result of polarization oscillations, then in the region of the transition energy Δ where the spectrum of the nucleus becomes continuous the nuclear matrix element of the E0-pole can be estimated with the aid of the following relation [12]:

$$\varrho^2 g(\Delta) = \frac{3\pi\Gamma E_0^2 ZR\Delta}{(\Delta^2 - E_0^2)^2 + \Gamma^2\Delta^2} \left\{1 - \frac{5}{3}\operatorname{Im} F(x) + \frac{5}{3}\,\frac{\Delta^2 - E_0^2}{\Gamma\Delta}\operatorname{Re} F(x)\right\}, \quad (4.4)$$

where

$$F(x) = \frac{[(6-x^2)(x\cos x - \sin x) + 2x^2 \sin x]}{x^2(x\cos x - \sin x)}, \quad (4.5)$$

$$x = (k'R)\sqrt{\frac{\Delta^2}{E_0^2} - 1 + \frac{i\Gamma\Delta}{E_0^2}}, \quad E_0 = \frac{m_n V_0 A}{4\pi NZe^2}, \quad V_0 = \frac{3}{4\pi} R^2, \quad (4.6)$$

$$k'R \approx 2{,}08 \left| \left[\frac{\varepsilon^2_{\rm res}}{E_0^2} - 1 - i\frac{\Gamma\varepsilon_{\rm res}}{E_0^2}\right] \right|^{-\frac{1}{2}}. \quad (4.7)$$

Here $g(\Delta)$—density of the nuclear levels that can be excited in the E0 transition Γ and $\varepsilon_{\rm res}$—width and "resonant" energy of the known dipole "resonance," which appears in (γp) and (γn) reactions,

*The value of M, likewise calculated in [61] using the α particle model, was almost four times larger than the experimental value.

and N is the number of neutrons. The derivation of (4.4) is based on a comparison of the polarizabilities of the drop, determined by the classical and by the quantum methods, in response to a small perturbation $V = \lambda r^2 e^{i\omega t}$ acting on this drop [12].

Estimates of M and ρ based on the shell model of the nucleus were made in [80,162-167]. Let us examine in detail the theoretical investigations of M,* using as an example the 0^+-0^+ transition of C^{12}, since among all the nuclei with observed E0 transitions this nucleus comes closest to those light nuclei that are most successfully explained by the shell models.

We assume first, in accordance with Schiff [80] that j—j coupling exists between the nucleons of C^{12}. The ground state 0^+ of C^{12} is characterized by the presence of four neutrons and four protons in the $p_{3/2}$ shell. We can expect the lowest excited state of the nucleus to occur when a small number of nucleons go from the $p_{3/2}$ shell to the $p_{1/2}$, $d_{5/2}$, or $s_{1/2}$ shells. It is easy to see that the transition of at least two nucleons is necessary for the formation of an excited 0^+ state.

If we now assume that all the nucleons in the nucleus are independent (do not interact with one another and move in the averaged centrally-symmetrical potential field), then the 0^+-0^+ transition will be forbidden due to the vanishing of the nuclear matrix element of the monopole. On the other hand, the vanishing of the quantity

$$M=\int \Psi_f^*\Big(\sum_p r_p^2\Big)\Psi_i\,d\mathbf{r}\equiv\int \Psi_f^*\Big(\sum_i \frac{1}{2}(1+t_i)\,r_i^2\Psi_i\,d\mathbf{r}$$

(t_i—isotopic spin operator) is due to the fact that the operator $\sum_p (r_p)^2$ of the electric monopole E0 is a sum of operators each of which acts only on the wave function of one of the nucleons, and the configurations of the pure states of the system of independent particles should differ from one another in the 0^+-0^+ transition by not less than two nucleons. The 0^+-0^+ transition becomes possible already when account is taken of the residual interaction between the nucleons as a perturbation, since this interaction causes the perturbed states to have components that differ in their configurations by only one nucleon, so that the matrix element of the monopole will no longer be equal to zero.

Assuming that the excited 0^+ state with resultant vanishing isotopic spin of the C^{12} nucleus is realized as a result of a two-nucleon transition from the $p_{3/2}$ shell to the $p_{1/2}$ shell, and that the nuclear potential is in the form of a rectangular well of infinite depth, while the residual interaction between the two nucleons is given by the formula $V_{if} = c\delta(\mathbf{r}_i - \mathbf{r}_j)$ where C is a constant, Schiff obtained [80] the following expression for M by perturbation theory:

$$M = -1.58\cdot 10^{22} CR, \tag{4.8}$$

where R is in cm and C in MeV-cm^3. It was assumed in the calculations, on the basis of [168] that the excited 0^+ state of C^{12} with energy 7.66-MeV is made up of two substates of the type $(T, J) \equiv (1, 0)$, two nucleons $p_{1/2}$, and two "holes" $p_{3/2}$. If we substitute the possible numerical values of C and R in (4.8), we obtain for M a result which is smaller than the experimental ($M_{\mathrm{expt}} = 3.8 \times 10^{-26}$ cm^2) by a factor of 6 [80].

For a nucleon coupling other than j-j, the theory of the excited 0^+ state of C^{12} and the calculation of M are given in [162]. There are five different LS states of a nucleus with $(T, J) \equiv (0, 0)$ belonging to the one and the same configuration $1p^8$. If the ground and excited 0^+ states are linear combinations of the indicated LS states, then M vanishes in all possible cases of intermediate coupling, even when the resultant isospin of at least one of the 0^+ states is equal to 0 [162]. In order to obtain a nonvanishing nuclear monopole matrix element it is necessary to take into account the admixture of other configurations in the $1p^8$ configuration, particularly the configuration $1p^7 2p$. Inasmuch as the admixture of the $1p^7 2p$ configuration (we neglect the admixture of the remaining configurations) leads to very cumbersome mathematical manipulations when account is taken of the residual interaction of the nucleons as a perturbation and when an intermediate coupling is assumed (in particular, to diagonalization of a 19-row matrix), the calculation of the admixture and subsequent estimate of M have been made for the limiting case of LS coupling (with account of the spin-orbit interaction as an additional perturbation). Such a simplification, in the opinion of Sherman and Ravenhall [162] is justified, since their analysis of the experimental data [69] gives grounds for assuming that the LS coupling is apparently close to reality.

As a result of the calculation, the following formula was obtained for the nuclear matrix element of the monopole

$$M = -0.032 V_0' \left(\frac{\alpha'}{K'}\right)\cdot 10^{-26}, \tag{4.9}$$

where V_0' is a parameter determining the depth of the potential of interaction between two nucleons, in the form [170]

$$V_{12} = -V_0' e^{-\frac{r_{12}^2}{R^2}} \frac{1}{4}[0.01 + 0.01\sigma_1\sigma_2 + 0.41 t_1 t_2 + 0.93(\sigma_1\sigma_2)(t_1\ t_2)], \tag{4.10}$$

where α' is the constant in the spin orbit interaction $\alpha'\Sigma\sigma_i l_i$, σ_i and t_i are the spin and isospin of the

*From an estimate of the value of M we can estimate the dimensionless parameter ρ by using the relation $\rho = M/R^2$, where R is the nuclear radius.

i-th nucleon, respectively, l_i—orbital momentum of this nucleon, and K′ is the Slater integral [4].

In the derivation of (4.9) the wave functions employed corresponded to a parabolic nuclear potential. The choice of these functions and of the parameter a, which determines the width of the potential well*, was also based on the analysis of the experimental data [162,69].

After substituting in (4.9) the most probable values of $\alpha'/K' = 5$ [172,173] and $V_0' = -13$ MeV (the latter is obtained by diagonalizing the interaction (4.10) for three states of the configurations $1p^8$ and $1p^7 2p$ [162]), the numerical value of M was found to be one-third the experimental value.

According to modern theory of the atomic nucleus, the structures of C^{12} and O^{16} are similar in many respects (this is indicated, in particular, by the fact that the experimental values of M are almost the same for both) and the entire calculation method described above can be applied also for the nucleus O^{16}.

The calculations of M and ρ for O^{16} on the basis of the shell model were made in [163-167]. In [163] it is stated that if the experimental values for M are identical for C^{12} and O^{16}, then the configurations of the excited 0^+ of these nuclei should differ as little as possible from each other. Such configurations, in particular, are $1s^3 2s 1p^8$ for C^{12} and $1s^3 2s 1p^{12}$ for O^{16}. We see that they are produced by single-nucleon excitation of the internal shell, which is the same for both nuclei. Assuming that the excited 0^+ state of O^{16} belongs to the $1s^3 2s 1p^{12}$ configuration, Rèdmond [163] calculated with the aid of the wave functions of the harmonic oscillator the nuclear monopole matrix element M. It is seen from his calculations that the value of M calculated by the shell model exceeds somewhat the result obtained by calculation in the single-particle approximation (this fact is noted also in [174]). In addition, it was found that if we choose the width of the oscillator potential well a such that the mean square deviation of the individual nucleon from the center of the nucleus, calculated with the aid of the wave functions of the harmonic oscillator, is equal to the mean square deviation of the nucleon from the center of the spherical nucleus that is uniformly filled with nucleons, then the theoretical value of M coincides with the experimental one. On the other hand, if the width of the oscillator potential well is taken from the experimental data [69], as is done in [162], the theoretical value of M will again be much smaller than the experimental one. Elliot [164] indicates also that upon suitable choice of the constant a in the wave function of the harmonic oscillator, it is possible to attain complete agreement between the theoretical and experimental values of M by assuming that the excited 0^+ state of O^{16} is realized by uniform shift of the configurations $1s^3 2s 1p^{12}$ and $1s^4 1p^{11} 2p$ (both by 50%).

The investigations made in [165-167] of the dependence of the theoretical values of the nuclear monopole matrix element and the energy of the excited 0^+ state of O^{16} on the particular mixture of configurations making up this state, and also on the choice of the particular constant a in the harmonic-oscillator wave function, shows that it is impossible to attain simultaneous agreement of the theoretical results with the experimental data of M and of the excitation energy.

The shell model was used also for a theoretical calculation of the lifetime of the first-excited 0^+ states (meaning also the matrix element M) of Zr^{90} and Ge^{72}. Since the 0^+ state of Zr^{90} is due to a two proton excitation of the $p_{1/2}^2 \rightarrow g_{9/2}^2$ type (see page 739), we can employ in the calculation of $\tau(0^{+*})$ the already mentioned method of Schiff [80] (see page 744). The calculation yielded [18] $\tau(0^{+*}) = 5.1 \times 10^{-9}\,C^{-2}$ sec, where C is the constant in the nucleon-nucleon interaction of the type $\frac{1}{2}\sum_{i\neq j} C\delta(\mathbf{r}_i - \mathbf{r}_j)$. Comparing the theoretical value with the experimental one $\tau_{\text{expt}} \approx (90 \pm 6) \times 10^{-9}$ sec, we get $C = 0.23$. Assuming that the 0^+ state of Ge^{72} is also due to a two-proton excitation (the $p_{3/2}^2 \rightarrow f_{5/2}^2$ transition or its inverse), we can calculate analogously $\tau_{\text{theor}}(0^{+*})_{\text{Ge}}$, which was found to equal [17-18] $7.6 \times 10^{-9} C^{-2}$ sec. Comparing it with the experimental data, we obtain $C = 0.16$. On the other hand, if we determine C from the singlet nucleon-nucleon interaction with a rectangular potential well 35 MeV deep and with effective radius 2×10^{-13} cm, then we get $C = 1.2$. We can therefore conclude that if the theory presented above for the 0^+ states of Zr^{90} and Ge^{72} and the approximations employed are meaningful, then 80—86% of the nucleon-nucleon interaction enter the central nuclear potential and only 20—14% of this interaction belongs to the perturbation. This conclusion contradicts the theoretical proof given in [18] that the nuclear monopole matrix element M is determined in two-nucleon transitions by the total nucleon-nucleon interaction.

Attempts have been made to reconcile the theory with experiment by a way somewhat different than that described above. Since the estimates of M obtained on the basis of the shell model are usually too low, and the values of M calculated with the aid of the collective models (α-particle and liquid-drop) are too large compared with the experimental results, the thought arises that the most promising nuclear model for obtaining accurate estimates of M should be more "collective" than the shell model and less "collective" than the α-particle and liquid-drop models. This idea is adhered to by the authors of [175], who start with the α particle model of the O^{16} nu-

*See, for example, [172] (except that a is denoted there by r_0).

cleus * and then show that the wave functions describing the α-particle states can be used to obtain approximate shell-model functions. With the aid of the latter, calculated on the basis of pulsating particle oscillations for a mixture of configurations of the type $1s^{-1}2s$ and $1p^{-1}2p$ with allowance for small admixtures of configurations of the type $1p^{-2}1d^2$, $1p^{-2}1d2s$, and $1p^{-2}2s^2$, a value almost three times larger than the experimental one was obtained for $M(O^{16})$ ($M_{theor} = 11 \times 10^{-26}$ cm^2).

The collective motion of the nucleons was taken into account in the calculation of $M(O^{16})$ by an entirely different method in [177,178]. For example, Griffin [177] describes the collective motion of the nucleus by the so-called "generating" coordinates method, which he developed together with Wheeler [179]. According to this method, the wave function of the nucleus is represented in the form of the integral

$$\Psi(x) \equiv \Psi(x_1, \ldots, x_A) = \int \Phi(x_1, \ldots, x_A; \alpha) f(\alpha)\, d\alpha, \quad (4.11)$$

where $x_i = (x_i, y_i, z_i, \sigma_{z_i}, t_{z_i})$ are the spatial coordinates, the spin coordinates, and the isospin coordinates of the i-th nucleon, and α is a "generating" coordinate, corresponding to the collective degree of freedom of the nucleus (deformation coordinate). The form of the function $f(\alpha)$ is established by a variational method [180]†. The function Φ, on the other hand, is given in the form of a determinant

$$\Phi(x_1, \ldots, x_A; \alpha) = (A!)^{-\frac{1}{2}} \begin{vmatrix} u_1(x_1, \alpha) \ldots u_A(x_1, \alpha) \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ u_1(x_A, \alpha) \ldots u_A(x_A, \alpha) \end{vmatrix}, \quad (4.12)$$

where the dependence of the single-nucleon wave functions u_i on α is determined by the character of nuclear deformation.

Thus, for the case of volume oscillations, the "generating" coordinate α is contained in the function u_i in the following manner:

$$u_i(x_j; \alpha) = u_i(x_j e^{-\alpha}, y_j e^{-\alpha}, z_j e^{-\alpha}, \sigma_{z_j}, t_{z_j}; 0), \quad (4.13)$$

while for quadrupole oscillations

$$u_i(x_j; \alpha) = u_i(x_j e^{\frac{\alpha}{2}}, y_i e^{\frac{\alpha}{2}}, z_j e^{-\alpha}, \sigma_{z_j}, t_{z_j}; 0). \quad (4.14)$$

*The α particle model was used first for a description of O^{16} in [176].

†By minimizing the integral $\int \Psi^* H \Psi d\mathbf{r}$, where

$$H = \sum \left(-\frac{h^2}{2m_n} \Delta_i + V_i \right) + \sum V_{ij} \text{ и } V_{ij} = V_0 e^{-\frac{r_{ij}^2}{R^2}} (W + mP^m + bP^b + nP^h)$$

is a general expression for the interaction between two nucleons with account of nuclear forces of different types.[170]

The spatial part of the function u_i is chosen in the form of the wave function of the three-dimensional harmonic oscillator, and the constant a in the exponential factor $\exp(-r^2/2a^2)$ is determined by the method indicated above (see page 000) for different values of r_0 ($r_0 = 1.5$, 1.2, and 1.28; $R = r_0 \times 10^{-13} A^{1/3}$ cm). By using the "generating" coordinates under the assumption that the first excited 0^+ state of O^{16} is due to volume oscillations [u_i is given by (4.13)], one calculates the energy of the first excited 0^+ state and the nuclear monopole matrix element $M(O^{16})$. The results of the calculation made for different values of the constants W, m, b, P_0, and r_0 turn out to be highly exaggerated (by 2–5 times) as compared with the experimental data [$M_{theor}(O^{16}) = (17-22) \times 10^{-26}$ cm^2, while $M_{exp}(O^{16}) = 3.8 \times 10^{-26}$ cm^2], it having turned out that the frequencies of the collective oscillations are comparable with the frequencies of the single-particle excitations. It is concluded therefore that even the low excited states of the nucleus are apparently mixtures of the collective and single-particle excitations.

Ferrel and Visscher [178] have proposed, like Griffin [177] that the first lowest excited states of O^{16} are due to volume oscillations in accordance with (4.13). The wave functions for these states are written by them on the basis of [180] in the form (4.11).

$$\Phi(x, \alpha) = \Phi(e^{-\alpha}x, 0)\, e^{-\frac{3}{2}A\alpha}, \quad (4.15)$$

where A is the mass number and $\Phi(x, 0)$ is defined by means of a determinant of the type (4.12). Expanding subsequently $\Phi(x, \alpha)$ in powers of α and retaining the first-power term, the authors prove that the wave function describing the first excited collective state of the nucleus is approximately equal to the wave function obtained on the basis of the shell model for a mixture of the two configurations $1s^{-1}2s$ and $1p^{-1}2p$. The contribution made to the energy of the first excited 0^+ state from the single-particle energies of the nucleons 2s and 2p, and also the form of the function V_{ij} necessary to calculate the resonant contribution to the excitation energy, is established by using different experimental data and assumptions (in particular, it is assumed that the radii of the nuclei O^{16}, O^{15}, and N^{15} are equal). As a result of estimates and calculations, the values obtained for the energy of the first excited 0^+ state and of the nuclear monopole matrix element M are 1-½ and 2 times respectively larger than the experimental data. In addition, it is indicated [181] that if the amplitudes of the volume oscillations of the s and p shells are assumed different, and also account is taken in the wave function of admixture of states arising in two-nucleon excitations from the 1p to the 2s and 1d shell, it is possible to obtain complete agreement between the theoretical and experimental results. Griffin [177], however, regards the results of Ferrel and Visscher as doubtful, owing to the insufficient justification for using the various experimental data in the estimates of the 0^+-excitation energy.

According to Griffin, in a rigorous calculation based only on the choice of some particular function V_{ij}, Ferrel and Visscher should obtain results close to his own (see page 746), since their wave functions are approximations of his wave functions.

The nuclear wave functions obtained by Ferrel and Visscher by the method described above were subsequently used by them [181] in a theoretical calculation of the form factors $F(K)$ for the electron monopole excitations (see Sec. 2) of C^{12} and O^{16}. A determinant of the form (4.12) made up of the single-particle functions of the harmonic oscillator, was chosen for the function $\Phi(x, 0)$. The calculations yielded

$$F_{0^+}(K)_{\mathrm{col.}} = -\left(\frac{13}{2}\right)^{\frac{1}{2}}\frac{K^2}{36a}\left(1-\frac{K^2}{13a}\right)e^{-\frac{K}{4a}} \qquad (4.16)$$

for C^{12} and

$$F_{0^+}(K)_{\mathrm{col.}} = -\frac{5K^2}{72a}\left(1-\frac{K^2}{10a}\right)e^{-\frac{K^2}{4a}} \qquad (4.17)$$

for O^{16}, where a is a constant determining the width of the oscillator potential well.* A comparison of the theoretical form factor $F_{0+}(K)$ for C^{12} with a so-called "measured" value (see page 735) shows[182] that the former is larger than the latter by 25%. It is concluded therefore that the 0^+ state of C^{12} is realized in part by collective and in part by single-nucleon excitations. A somewhat different opinion is expressed in [182] with respect to O^{16}. Inasmuch as the closed p shell of the O^{16} nucleus does not have low-lying excited nucleon configurations, there are grounds for assuming that the 0^+ state of O^{16} will already be a purely collective state.

Touchard accounts differently for the collective motion of the nucleons in the calculations of M and of the energy of the first excited 0^+ state of O^{16}. He assumes that the centrally-symmetrical potential of the shell model varies slowly in time. As shown by Inglis [183], such an assumption is equivalent to introducing an additional dynamic variable, which determines the collective degree of freedom of the nucleons, in this case the volume oscillations of O^{16}. By way of a central potential, the potential of the isotropic harmonic oscillator was chosen [182]

$$V(r, \sigma') = -V_0 + \frac{1}{2}\frac{h^2}{m_n r_0^4}\sigma'^2(t)\, r^2, \qquad (4.18)$$

where $\sigma' = \beta'^2 r_0^2$, $1/\beta'$ is the width of the potential well, r_0 the effective radius of the nuclear forces, and m_n the nucleon mass. By virtue of the slow variation of σ' with time, it is possible to use the adiabatic approximation method in the calculation of the wave function [183]. The calculation yields for the wave function, in first order with respect to σ', the expression

$$\Psi = u_0(\sigma') + ih\dot{\sigma}'\sum \frac{u_n(\sigma')\left(n\left|\frac{\partial}{\partial\sigma'}\right|0\right)}{d\omega_{n0}(\sigma')}, \qquad (4.19)$$

where the functions $u(\sigma')$ describes the stationary states of the nucleon system at the given instant of time. The energy is calculated from the formula

$$\int \Psi^* H \Psi\, d\mathbf{r} = E(\sigma') + \frac{1}{2}B(\sigma')\dot{\sigma}'^2, \qquad (4.20)$$

where H is the Hamiltonian and

$$B(\sigma') = 2h^2\sum_{h\neq 0}\frac{\left|\left(n\left|\frac{\partial}{\partial\sigma'}\right|0\right)\right|^2}{h\omega_{n0}(\sigma')}. \qquad (4.21)$$

In using Formulas (4.18)–(4.20), the values of $\Delta(O^{16})$ and $M(O^{16})$ are calculated. In the calculation of $E(\sigma')$, the expression used for V_{ij} is the same as in [178] (see pages 745-746). The calculation yields for $M(O^{16})$ a value which is 2–3 times larger than the experimental one. The nuclear matrix element of the monopole is proportional to $\sqrt{\Delta}$. If we take Δ equal to the experimental value 6.05 MeV, then the theoretical value of M will be only slightly larger than the experimental one ($4/3$). In conclusion it must be stated that the most doubtful aspect of this method is that in spite of the use of the adiabatic approximation, the energy of the collective motion of the nucleons is comparable with the energy of the individual nucleon, obtained on the basis of the independent-particle model assuming a nuclear oscillator field.

Theoretical estimates of the nuclear matrix element of the monopole for medium and heavy even-even* nuclei is usually based on the unified model. There are grounds for assuming that the low-lying levels of almost all these nuclei have a purely collective character[34,63,184]. It has been established by experiment that the angular momenta of the nuclei in the ground and in the first two excited states usually form one of three sequences: 1) $0^+-2^+-0^+$, 2) $0^+-2^+-2^+$, and 3) $0^+-2^+-4^+$ (with possible E0 transitions in the first and second cases). Depending on the ratio E_2/E_1 of the energies of the second and first excited levels, obtained from the experimental data, all the nuclei mentioned above can be approximately broken up into two groups. The first group includes those with $E_2/E_1 > 3$. They are situated in the intervals $150 < A < 190$ and $A > 214$. The second group includes nuclei with $E_2/E_1 \leq 2.5$. Their low-lying levels are practically equidistant from one another. These nuclei are situated in the interval $60 \leq A \leq 150$ and $190 \leq A < 214$.[185]

*The recoil of the nucleus and the difference between the laboratory and the c.m.s. coordinates were neglected in the calculations. These simplifications lead to an error of several per cent at the electron energies usually employed in the experiments. Since the percentage of the error is proportional to the recoil velocity, the error increases with increasing energy.

*The theory of the remaining nuclei has not yet been sufficiently well developed.

Table VIII

Nucleus	M_{exp} $(0^+ \to 0^+)$, 10^{-26} cm²	ϱ_{exp} $(0^+ \to 0^+)$	ϱ_{exp} $(2^{+\prime} \to 2^+)$	$M_{theor}(0^{+\prime}-0^+)$, 10^{-26} cm²	ϱ_{theor} $(0^{+\prime}-0^+)$	ϱ_{theor} $(2^{+\prime}-2^+)$
C^{12}	3.8	$\frac{1}{2}$		11^a, 13^b, $\frac{1}{6}M_{exp}{}^c$, $\frac{1}{3}M_{exp}{}^d$		
O^{16}	3.8	0.42		10^e, $14{,}7^f$, 17^g, 19^h, $0{,}6^i$, $M_{exp}{}^{k-l}$, 10^m, $17—22^n$, (2—3) $M_{exp}{}^o$		
Ca^{40}		0.15			0.26^m	
Se^{76}					0.465^p	0.0465^p
Se^{78}					0.390^p	0.0332^p
Zr^{90}		0.056			0.16^m	
Ru^{100}					0.352^p	0.0275^p
Se^{80}					0.227^p	0.016^p
Pd^{104}					0.306^p	0.0217^p
Pd^{106}					0.370^p	0.0286^p
Cd^{110}					0.262^p	0.0168^p
Cd^{114}		0.63			0.290^p	0.019^p 0.095^r
$Te^{122,126}$					0.201^p	0.0108^p
Te^{124}					0.153^p	0.0073^p
Cd^{154}						0.046^r
Er^{166}						0.006^r
Os^{188}						0.014^r
Pt^{194}						0.040^r
Pt^{196}			0.013—0.070			0.030^r
Hg^{198}			$< \frac{1}{14}$			0.045^r

a, e, g—α-particle model [80,161,175]; b, h—liquid drop model [80]; c, d, i, l—shell model with state 0^{+*}, corresponding in the case of C^{12} to the configuration ${}^c\ 1p^{-2}2p^2$ [80]; ^{d} mixture of configurations $1p^8$ and $1p^7 2p$ [162] (in the case of 0^{16}, the configurations ${}^i\ 1p^{-2}2p^2$ [80], ${}^k\ 1s^{-1}2s$ [163]) and l mixture of 50% configuration $1s^{-1}2s$ and 50% configuration $1p^{-1}2p$ [164]; m, n—collective model with "generating coordinates" after Ferrel and Visscher [178] and after Griffin [177]; o—model of "forced rotation" of Inglis [182]; p—model of quadrupole collective excitations [185]; r—Davydov and Filippov model of nonaxial nuclei (with account of non-adiabaticity [140]).

It is assumed that the nuclei of the first group have axial symmetry and that within the limits of applicability of the adiabatic approximation the character of their lowest collective excitations is purely rotational. Located above the rotational band of such nuclei is the β-vibrational band (with K = 0, where K is the projection of the angular momentum on the symmetry axis of the nucleus), the γ-vibrational band (with K = 2) and other level bands; E0 transitions of axial nuclei are possible only between the β-vibrational levels and the rotational levels of the ground band (owing to the selection rules $|\Delta K| \leq L$).*

The second group of nuclei is presumed to possess no axial symmetry and their lowest levels are assumed to be excited because of surface quadrupole oscillations (model of quadrupole collective excitations [185]) or rotations with conservation of the form of the nuclear surface (the "model of nonaxial nuclei" of A. S. Davydov and G. F. Filippov [139]).

An estimate of the reduced nuclear monopole matrix element ρ under the assumption of hydrodynamic phonon excitations of the quadrupole type (the first of these models) was made by Grechukhin [12,185]. A calculation carried out in the harmonic approximation yielded

$$\varrho\,(0^{+\prime} \to 0^+) = \frac{3}{4\pi} Z\bar{\beta}^2 \sqrt{\frac{2}{5}} \qquad (4.22)$$

for two-phonon transitions and

$$\varrho\,(2^{+\prime} \to 2^+) = -\frac{3}{4\pi} Z\bar{\beta}^3 \sqrt{\frac{1}{7\pi}} \qquad (4.23)$$

for single-phonon transitions ($2^{+\prime}$ and 2^+ are neighboring states). Here $\bar{\beta}^2 = 5\hbar\omega/2C$, where ω is the frequency of the quadrupole oscillations and C the stiffness parameter.

The estimate (4.23) is only the lower limit, for if anharmonicity is taken into account, two-phonon transitions of the type $2^{+\prime} \to 2^+$ are also possible.

Numerical values of (4.22) and (4.23) for known values of $\bar{\beta}^2$ are listed in Table VIII.

Estimates of $\rho\,(2^{+\prime}-2^+)$ for different nuclei were made on the basis of the Davydov and Filippov method by Rostovskii [140]. According to this model,

*Of course, if the adiabaticity conditions are satisfied.

the first excited state 2^+ pertains to the ground-state band of rotational states and $2^{+\prime}$ is the so-called "anomalous" rotational state of the nucleus, occurring only when the "non-axiality" parameter γ differs from 0 [186]. If both states are assumed to be purely rotational, then the E0 transition between the states $2^{+\prime}-2^+$ will be completely forbidden because the operator $E\hat{O} = \sum_p^Z r_p^2$ is scalar [187]. On the other hand, if the coupling between the rotation and the β vibrations is taken into account in the calculation of the nuclear wave functions describing the states $2^{+\prime}$ and 2^+, the probability of E0 transition between the $2^{+\prime}$ and 2^+ will already differ from zero.*

Rostovskiĭ [140] calculated the value of $\rho(2^{+\prime} \to 2^+)$ for Pt^{196}, Hg^{198}, Cd^{114} and other nuclei (see Table VIII) under the assumption that the rotation and β vibration occur with adiabatic slowness, but account is taken of the dependence of the equilibrium value of γ on the parameter β (which determines the deviation of the nuclear shape from spherical).† The wave functions used in the calculation are represented as a superposition of rotational-vibrational functions, the form of which has been established on the basis of [139,186,190–192]. The coefficients determining the superposition are determined by perturbation theory in the first approximation. The data on the levels $2^{+\prime}$ and 2^+ of different nuclei, which are needed for the calculation, and are taken from [153,193–196]. The results of the calculations are in good agreement with the experimental values of ρ (wherever the latter are available; see Table VIII).

On the basis of the general theory of collective excitations due to rotation and β and γ vibrations of the nuclear surface, a theory developed by Davydov [197,198], estimates were made in [199] for the values of ρ of even-even nuclei, both spherical and nonspherical.

1. Spherical nuclei. a) For the E0 transition between the first excited β vibrational and ground-state levels with 0 spins the value obtained is [199]

$$\varrho(0_\beta^+ \to 0^+) = -\frac{3Z}{4\pi}\sqrt{\frac{5}{2}}\,\beta_{00}^2. \qquad (4.24)$$

Here β_{00}^2—mean square of the amplitude of the zero-point oscillations of the nuclear surface, equal to $h/\sqrt{BC}$; B—inertial parameter, and C—one of the two stiffness parameters contained in the expression for the potential energy of the nucleus

$$V(\beta, \gamma) = \frac{1}{2}C(\beta-\beta_0)^2 + \frac{1}{2}C_\gamma\beta_0^2(\gamma-\gamma_0)^2, \qquad (4.25)$$

which is used in the indicated theory of Davydov‡ .

*This dependence was established in [188,189].

†Since to each level with any J there corresponds in the theory of nonaxial nucleus an "anomalous" level with the same J, the E0 transitions between all levels of this type are possible in principle.

‡C, C_γ, β_0, and γ_0 are parameters of the theory. For spherical nuclei $\beta_0 = 0$ and (4.25) has the form usually employed in the model of quadrupole collective excitations in the harmonic approximation.[185]

The energy of the 0^+-0^+ transition is $E_\beta = 2\hbar\omega$, where $\omega = \sqrt{C/B}$. It is easy to note that the results of (4.24) and (4.22) coincide.

b) In the case of an E0 transition between the first excited γ-vibrational state (with energy $E_\gamma = 3\hbar\omega$) and the ground 0^+ state, calculation yields a smaller value of ρ, namely

$$\varrho(0_\gamma^+ \to 0^+) = \frac{-75Z}{8\sqrt{42}\,\pi^{\frac{3}{2}}}\beta_{00}^2. \qquad (4.26)$$

c) For the transition between the first 0_γ^+ and 0_β^+ levels

$$\varrho(0_\gamma^+ \to 0_\beta^+) = \frac{-15\sqrt{15}}{8\sqrt{7}\,\pi^{\frac{3}{2}}}Z\beta_{00}^3. \qquad (4.27)$$

d) The value of ρ (22—21) for the transition between the second and first excited 2^+ levels with energies $2\hbar\omega$ and $\hbar\omega$ respectively coincides with (4.23).

2. Nonspherical nuclei with minimum of potential energy corresponding to a value $\gamma_0 = 0$. The calculation of the values of $\rho(0_\beta^+ \to 0^+)$, $\rho(0_\gamma^+ \to 0^+)$, and $\rho(0_\beta^+ \to 0_\gamma^+)$ is made in this case [199] assuming a small non-adiabaticity parameter $\mu = 1/\beta_0\sqrt{\hbar\omega/C}$ ($\mu < 1/3$) (introduced in [200]) and a small value of $\Gamma = \mu\sqrt{\omega/2\omega_\gamma}$, where $\omega_\gamma = \sqrt{C_\gamma/B}$ and $\omega = \sqrt{C/B}$ are the frequencies of the γ and β vibrations, respectively (with $\Gamma < 15$).

The following results were obtained:

$$1)\ \varrho(0_\beta^+ \to 0^+) = \frac{3\sqrt{2}}{4\pi}Z\mu\beta_0^2 \qquad (4.28)$$

for a transition energy $E_\beta = \hbar\omega$;

$$2)\ \varrho(0_\gamma^+ \to 0^+) = \frac{45\sqrt{5}}{28\pi^{\frac{3}{2}}}Z\frac{E_\beta}{E_\gamma}\mu^2\beta_0^3 \qquad (4.29)$$

for a transition energy $E_\gamma = 2\hbar\omega_\gamma$;

$$3)\ \varrho(0_\beta^+ \to 0_\gamma^+) = \frac{135\sqrt{5}}{28\sqrt{2}\,\pi^{\frac{3}{2}}}Z\frac{E_\beta}{E_\gamma}\mu^3\beta_0^3, \qquad (4.30)$$

$$4)\ \varrho(22-21) = \frac{-30\sqrt{10}}{7\pi^{\frac{3}{2}}}Z\beta_0^2\Gamma^5. \qquad (4.31)$$

It is seen from (4.28) and (4.29) that the $0_\gamma^+ \to 0^+$ transition is much less probable than the $0_\beta^+ \to 0^+$ transition.

3. Nonspherical nuclei with $\gamma_0 = 10°$. A calculation of the reduced nuclear monopole matrix element ρ has also been made [199] in the approximation where the parameters μ and Γ are small. The value of $\rho(0_\beta^+ \to 0^+)$ was found to coincide with (4.28). The remaining formulas for ρ are in the form

$$\varrho(C_\gamma^+ \to 0^+) = \frac{-15\sqrt{5}}{28\pi^{\frac{3}{2}}}\sqrt{\frac{E_\beta}{E_\gamma}}Z\mu\beta_0^3\sin^3\gamma_0, \qquad (4.32)$$

$$\varrho(0_\beta^+ \to 0_\gamma^+) = \frac{-45\sqrt{10}}{56\pi^{\frac{3}{2}}}\sqrt{\frac{E_\beta}{E_\gamma}}Z\mu^2\beta_0^3\sin^3\gamma_0. \qquad (4.33)$$

From all the foregoing and from relations (4.24) and (4.26)—(4.33) it follows that to find the numerical values of ρ it is sufficient to determine experimentally the parameters B and C in the case of spherical nuclei and the parameters B, C, C_γ, γ_0, and μ in the case of nonspherical nuclei. Comparison of the theoretical value of ρ with the experimental ones can then be used to establish the γ and β vibrations that give rise to the nuclear states with zero spins.

It is easy to note that the ratio of $\rho(0_\gamma^+ \to 0^+)$ to $\rho(0_\gamma^+ \to 0_\beta^+)$ or to $\rho(0_\beta^+ \to 0_\gamma^+)$ is equal to $1/3\sqrt{5/2}$ for spherical nuclei, and is inversely proportional to μ for nonspherical nuclei. These ratios can also be used to establish the character of the levels of nuclei with zero spin.

Mention should also be made of the theoretical estimates given in [199] for the ratio of $M(0^+ \to 0^+)$ to the reduced probability $B(E2;\, 0^+ \to 2^+)$ in the level sequence $0^+ \to 2^+ \to 0^+$ (denoted by E0/E2) for the nuclei Sm^{152} (Fig. 25) and Pu^{238} (see [153]). The calculation is based on the notion of a nucleus in the form of a uniformly charged spheroid, which executes quadrupole oscillations about an equilibrium deformation without change in volume. As a result of the calculations, identical values of E0/E2 = 0.23 have been obtained for both nuclei. The experimental value of E0/E2 for Pu^{238} is equal to 0.14, that is, it differs little from the theoretical value. For Sm^{152}, the experimental value is E0/E2 = 0.016. The deviation from the theory is large and difficult to explain [201]*.

The theory of E0 transitions of nuclei with odd A is so far in the development stage. It is established in [187] that for nuclei of the "neutron plus even core" type the probability of E0 transition will differ from zero if the model of quadrupole collective excitations is used for the description of the core. The order of magnitude of ρ will in this case be comparable with the values of ρ given by relations (4.21) and (4.22). It is also shown there that the probability of E0 transitions of these nuclei is equal to zero if the core is described by the Davydov and Filippov model and if it is assumed that the form of the nuclear surface remains unchanged upon interaction between the core and the external nucleon.

A summary of experimental and theoretical values of M and ρ for different nuclei is given in Table VIII.

CONCLUSION

On the basis of the foregoing exposition we can make the following brief conclusions and remarks. The general theory of monopole transitions of nuclei is sufficiently fully developed (to be sure, this applies more to E0 transitions than to M0 transitions; Sec. 1). On the other hand, the experimental investigations of monopole transitions, although abundant, are insufficient (Sec. 3). So far the investigations cover principally electric monopole transitions of the 0^+—0^+ and 2^+—2^+ type, which occur in even-even nuclei. There is only one example of observed E0 transitions of the type 0^-—0^- (Bi^{212}) and $\frac{1}{2}^+$—$\frac{1}{2}^+$ (Au^{197}). It is also reported in [201] that an E0 transition 4^+—4^+ has been observed in Np^{238}. Experimental data on the observation of magnetic monopole transitions (0^-—0^+ in O^{16}, page 729) are so far skimpy and doubtful.

Of great importance is the question of the excitation of monopole transitions. Although in most cases monopole transitions were observed in daughter nuclei following β decay, nevertheless the fraction of these transitions per single β-decay event is very small. Therefore the observation of monopole transitions is made most difficult by the background produced by other possible transitions.

To excite monopole transitions (principally in light nuclei) it is also possible to employ nuclear reactions which result in product nuclei that are in excited states. The deexcitation of these nuclei occurs most frequently via monopole transitions. The main possible reactions are of the type $(p, p'\gamma)$, (p, γ), $(p, \alpha'\gamma)$, (n, γ), and $(n, n'\gamma)$, the latter reaction having, in accordance with [132], many advantages. Among these advantages are, for example, the well defined reaction threshold, which makes it possible to establish with sufficient accuracy the excitation energy of the product nucleus, as well as the appreciable effective reaction cross section, which is independent of Z.

The most promising method of exciting E0 transitions is the method of Coulomb excitation of nuclei. Direct monopole excitation of the nuclei is possible only by inelastic scattering of the electrons by nuclei (so far only one case of such interaction was investigated—excitation of the 7.66 MeV level of C^{12}; Sec. 2). However, indirect excitation of monopole transitions via inelastic collisions of heavy charged particles with nuclei is also possible. The incoming particle first gives rise to multipole transition of the nucleus from the ground state to an excited one which lies sufficiently high. It is quite possible that during the subsequent deexcitation of the nucleus via the intermediate levels there occur also monopole transitions.

An important role is played in the study of E0 transitions by the knowledge of the theoretical values of the nuclear matrix elements M and ρ. It is from the coincidence of these theoretical values with the experimental ones that the structure and character of the nuclear levels between which the E0 transition occurs can be judged. Unfortunately, in spirit of the rather large number of papers devoted to theoretical estimates of M and ρ, based on different nuclear

*In [201] E0/E2 is also calculated by starting with a model in which a coherent superposition of individual excitations of the most readily polarized protons[202] situated near the Fermi surface is used to describe the quadrupole oscillations of the nuclei under consideration.

As a result of the calculation made with the aid of the wave functions of the individual nucleons situated in the anisotropic harmonic oscillator potential field, we obtain, assuming the volume of the nucleus to remain constant, EO/E2 = 0.50 for Pu^{238} and EO/E2 = 0.61 for Sm^{152}.

models, this agreement occurs only in individual cases (see Table VIII). In this connection, further development and improvement of the different nuclear models (principally the unified model) are urgently needed.

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