INTERFERENCE EFFECTS IN QUANTUM TRANSITIONS

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INTRODUCTION

ALTHOUGH the interference phenomena considered in this review are intimately related to some of the fundamental principles of quantum mechanics, it is only relatively recently that they have been investigated in any detail. This field has been developing rapidly because interference methods are found to be unusually precise, in addition to being simple and sensitive. Because they are very general, interference methods are not limited to atomic physics, but can be used to study the properties of nuclei and elementary particles as well. Moreover, the scope and generality of these methods is such that the same general approach can be applied to situations that appear to be unrelated at first galnce. For example, the relation between the decay of the 2S metastable state in the hydrogen atom and the conversion of K^0 and $\tilde{K}^0$ particles is not immediately obvious. Similarly, the relation between the magnetic depolarization of resonance radiation of atoms and the determination of the nuclear quadrupole moment by superposition of levels or, say, the relation between the Mössbauer effect and certain details of specular reflection of light etc. are not directly apparent.

It is therefore desirable to compare various interference methods and to try to interpret them from a single point of view. This approach assumes, obviously, that although the effects are already more or less familiar, it is still useful to find a unified approach to this general kind of problem. We cannot hope to delineate all, or even only the important, interference effects that occur in quantum transitions. The scope of the present review also makes it impossible for us to consider various technical details, although these are extremely important. For this reason the review does not contain a full bibliography; the references are primarily to work of general nature and to other review papers.

1. INTERFERENCE TRANSITIONS BETWEEN STATES OF NONRADIATING SYSTEMS

Atoms which exhibit hyperfine structure or nuclei with nonzero spin located in an external magnetic field are examples of systems with split energy levels. Such systems can occupy close-lying energy states. Unless the contrary is indicated, we assume for simplicity that the splitting is double splitting. The wave function and energy of one of the states are denoted by ψ_1 and w_1; the same quantities for the other state are denoted by ψ_2 and w_2. In large ensembles it can usually be assumed that some systems are in one of these states and the remainder in the other. This approach yields correct results only when used to describe effects associated with each of the states individually and may not be accurate under certain circumstances; this is the case when one considers various interference effects, the analysis of which is the subject of the present review.

The state of each of the systems is described by a wave function

$$\psi = a_1\psi_1 e^{i\omega_1 t} + a_2\psi_2 e^{i\omega_2 t}, \qquad (1)$$

where $\omega_1 = w_1/\hbar$ and $\omega_2 = w_2/\hbar$ while the complex numbers $a_1 = |a_1| e^{i\delta_1}$ and $a_2 = |a_2| e^{i\delta_2}$ are determined by the conditions of formation of the system ($|a_1|^2 + |a_2|^2 = 1$). In addition to treating the incoherent mixing of two stationary states, we shall be interested in treating the superposition of such states corresponding to nonstationary situations. We wish to emphasize that the nonstationarity is the most important feature of the entire analysis that follows. It means that the properties of the system change with time.

We can introduce, in addition to ψ_1 and ψ_2, any other pair of orthonormal functions, for example,

$$\varphi_1 = \frac{\psi_1 + \psi_2}{\sqrt{2}}, \quad \varphi_2 = \frac{\psi_1 - \psi_2}{\sqrt{2}}. \qquad (2)$$

The wave functions ψ_1 and ψ_2 are eigenfunctions of the energy operator W corresponding to the eigenvalues w_1 and w_2. The wave functions φ_1 and φ_2 are eigenfunctions of some other operator $\hat{E}$ whose physical significance depends on the actual form of the system. We denote its eigenvalues by ϵ_1 and ϵ_2. The relations in (2) can also be written in the form

$$\psi_1 = \frac{\varphi_1+\varphi_2}{\sqrt{2}}, \quad \psi_2 = \frac{\varphi_1-\varphi_2}{\sqrt{2}}. \tag{2'}$$

Substitution of this relation in (1) yields

$$\psi = \frac{a_1 e^{i\omega_1 t}+a_2 e^{i\omega_2 t}}{\sqrt{2}}\varphi_1 + \frac{a_1 e^{i\omega_1 t}-a_2 e^{i\omega_2 t}}{\sqrt{2}}\varphi_2. \tag{3}$$

It follows from (1) that the probabilities p_{w_1} and p_{w_2} that the energy assumes the value w_1 or w_2 are given by

$$p_{w_1} = |a_1|^2, \qquad p_{w_2} = |a_2|^2. \tag{4}$$

Each of these probabilities is independent of time. The analogous probabilities p_{ϵ_1} and p_{ϵ_2}, on the other hand, depend on time. It follows from (3) that

$$\left.\begin{aligned} p_{\epsilon_1} &= \tfrac{1}{2}\{|a_1|^2+|a_2|^2+2|a_1||a_2|\cos(\Omega t+\delta)\},\\ p_{\epsilon_2} &= \tfrac{1}{2}\{|a_1|^2+|a_2|^2-2|a_1||a_2|\cos(\Omega t+\delta)\},\end{aligned}\right\} \tag{4'}$$

where $\Omega = \omega_1 - \omega_2$ and $\delta = \delta_1 - \delta_2$.

The expression in (4') means that the system makes periodic transitions from state φ_1 to state φ_2 and vice versa;[1] this feature appears especially clearly when $|a_1| = |a_2| = 1/2$.* Analogous transitions will obviously occur for any other quantity whose operator does not commute with the energy. It should be remembered that the nature of the splitting can be modified by physical conditions; for example, if the nature or direction of an external field is changed, then a quantity that commutes with the energy can become a nonstationary quantity and undergo transitions from one state to another as described above.

Assume that the system exhibits a doubly degenerate level which is easily split by "switching on" a perturbation $\mathcal{H}$. Suppose that the time-independent wave functions ψ_1 and ψ_2 correspond to two orthonormal states of the system. An arbitrary state of the system can then be written in the form

$$\chi = b_1(t)\psi_1 + b_2(t)\psi_2.$$

As is well known, the functions $b_1(t)$ and $b_2(t)$, which describe the change of χ in time, satisfy the equations

$$\left.\begin{aligned} i\frac{db_1}{dt} &= a_{11}b_1 + a_{12}b_2,\\ i\frac{db_2}{dt} &= a_{21}b_1 + a_{22}b_2,\\ a_{kl} &= \int d\tau \psi_k^* \mathcal{H}\psi_l.\end{aligned}\right\} \tag{5}$$

We now assume that the system is stable, that is to say, that it cannot decay into another state. Then the perturbation operator $\mathcal{H}$ is Hermitian, the quantities a_{11} and a_{22} are real, and $a_{12} = a_{21}^*$. In this case the stationary solutions of (5) are

$$\left.\begin{aligned} \chi_1 &= \frac{1}{\sqrt{1+|R|^2}}(R^*\psi_1+\psi_2),\\ \chi_2 &= \frac{1}{\sqrt{1+|R|^2}}(\psi_1 - R\psi_2),\end{aligned}\right\} \tag{6}$$

$$R = \frac{a_{12}}{\frac{a_{22}-a_{11}}{2} + \sqrt{\left(\frac{a_{22}-a_{11}}{2}\right)^2+|a_{12}|^2}}$$

The corresponding energies are

$$\mu_{1,2} = \frac{a_{11}+a_{22}}{2} \pm \sqrt{\left(\frac{a_{11}-a_{22}}{2}\right)^2+|a_{12}|^2}. \tag{7}$$

The nature of the stationary states depends on the relations between a_{11}, a_{22}, and a_{12}. If $a_{12} = 0$, then $R = 0$ and the original states ψ_1 and ψ_2 are stationary. If $a_{11} = a_{22}$ and $a_{12} \neq 0$, then $R = 1$ and the states given below are stationary:

$$\varphi_{12} = \frac{1}{\sqrt{2}}(\psi_1 \pm \psi_2).$$

The states ψ_1 and ψ_2 are already nonstationary and the system makes transitions from one to the other with frequency $\hbar/|a_{12}|$.

We now consider some examples. The energy of a nucleus with spin 1/2 located in an external magnetic field H can assume the two values $a_{11} = \mu H$ and $a_{22} = -\mu H$, where μ is the magnetic moment ($a_{12} = a_{21} = 0$). The eigenfunctions

$$\psi_1 = \begin{pmatrix}1\\0\end{pmatrix}, \quad \psi_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

correspond to definite projections of the spin ($+1/2$ and $-1/2$) on the z axis, which are conserved in time. The operator associated with the projection of the spin in any other direction, say the x axis, is not an integral of the motion. The corresponding wave functions

$$\varphi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \quad \varphi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

are related to ψ_1 and ψ_2 in accordance by (2) and the projection of the spin on the x axis varies periodically from $+1/2$ to $-1/2$. If the field is now directed along the x axis the projection of the spin on this axis is conserved in time while the projection on the z axis varies in the manner indicated above.*

The decay of π^+ mesons results in the appearance of μ^+ mesons that are fully polarized in the direction of the momentum (z axis). After it slows down the μ^+ meson and an electron of the moderator material form

*In actual cases this process may be suppressed by various random interactions that will be discussed below.

*In these cases one usually talks about precession of the nuclear spin in the magnetic field. This effect has been investigated in detail in connection with nuclear magnetic resonance and nuclear quadrupole resonance (cf. for example, [2,3]). The precession of atomic angular momentum is discussed in [4].

a so-called muonium atom, which is the analog of the hydrogen atom with the role of the proton being played by the μ^+ meson. The electrons of the moderator are not polarized. Hence, in half of the cases muonium is formed with the projection of the total momentum on the z axis equal to +1. In the remaining cases the projection of the total momentum is zero. The appropriate wave function is

$$\varphi_1 = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2),$$

where ψ_1 is the wave function for the singlet state (total momentum $I = 0$), and ψ_2 is the wave function for the triplet state ($I = 1$, $I_Z = 0$). The singlet and triplet states are stationary states in this case; the energies of the two states are somewhat different because of the interaction of the magnetic moments of the μ^+ meson and the electron. Hence, in time the original wave function φ_1 is converted into the function

$$\varphi_2 = \frac{1}{\sqrt{2}}(\psi_1 - \psi_2),$$

which is then converted back into φ_1 and so on. The state φ_2 is also characterized by $I_Z = 0$, but the moments of the electron and μ^+ meson are in opposite directions, in contrast with φ_1. This process results in depolarization of the μ^+ mesons but does not affect μ^+ mesons which form muonium with $I_Z = 1$.*

If there is a very strong magnetic field parallel to the moment of the μ^+ meson the interaction between the spins of the μ^+ meson and the electron can be neglected. Under these conditions, in addition to the states with definite total momentum (ψ_1 and ψ_2) there are stationary states characterized by fixed projections of the spins of the individual particles (φ_1 and φ_2). The depolarization vanishes completely and $\varphi_1 \rightleftarrows \varphi_2$ transitions are replaced by periodic transitions of the singlet state into the triplet state and vice versa $\psi_1 \rightleftarrows \psi_2$. In the general case the stationary states and the corresponding energies are determined by the relations in (6) and (7), in which $\alpha_{11} = E_1$, $\alpha_{22} = E_2$ and $\alpha_{12} = \alpha_{21} = (\mu_1 + \mu_2)H$. Here E_1 and E_2 are the energies of the singlet and triplet states while μ_1 and μ_2 are the magnetic moments of the electron and μ meson.

Another pertinent example is furnished by the NH_3 molecule. Roughly speaking the hydrogen atoms are located at the vertices of an equilateral triangle while the nitrogen atom is located along the normal to the plane of this triangle which passes through its center. There are two states of stable equilibrium with the nitrogen atom located on one or the other sides of the plane; these states are separated by a "potential barrier," Let one state be denoted by the wave function ψ_1 and the other by ψ_2. If the barrier were impenetrable both states would be stationary and have the same energy. However, there is some barrier penetration and this results in a small splitting of the levels ($\alpha_{11} = \alpha_{22}$, $\alpha_{12} \neq 0$). The states

$$\chi_1 = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2), \quad \chi_2 = \frac{1}{\sqrt{2}}(\psi_1 - \psi_2)$$

are stationary while the states ψ_1 and ψ_2 are periodically converted into each other.[9]

If the hydrogen atoms are replaced by three different singly-valent complexes we obtain a molecule with spatial properties similar to those of ammonia. In general such a molecule will be optically active; if it rotates the plane of polarization to the left in the ψ_1 state it will rotate it to the right in the ψ_2 state. Assume that at the initial time the molecule is in the left-handed state ψ_1; it follows from what has been said that it will rapidly be converted into the right-handed state ψ_2 and then back. The actual observation of these transitions may be difficult because of an irreversible transition into the thermodynamically more stable state in which the right and left-handed states are mixed in equal parts.

2. EFFECT OF RADIATION ON INTERFERENCE TRANSITIONS

Let the level splitting we have been considering be associated with an excited state of some system. Transition to the ground state is then possible via the radiation of a photon, β decay etc. Two questions then arise: 1) what is the effect of radiation on the interference properties of the original excited state, and 2) what are the interference properties of the radiation itself?

Consider the first question. According to (6) [depending on the type of perturbation] the excited state is split into two or more pairs of basis states χ_1 and χ_2. It can be shown that the radiation mechanism and the mean life time $\tau = \lambda^{-1}$ are essentially the same for all of these states. As examples consider Zeeman or hyperfine splitting in an excited nucleus, quadrupole or Zeeman splitting of an excited nucleus, the splitting of muonium into the triplet and singlet states, and so on.* It is clear that all of the results obtained above

*Under actual conditions there are complications associated with the possibility of several successive substitutions of the electrons that make up the muonium (cf. [5-6,121]). The degree of depolarization of the μ^+ meson also depends on the magnitude and sign of the polarization of the electrons in the moderator. It is possible that this effect could be used as an additional method for determining the direction of the spin of μ mesons formed in $\pi\mu$ decay processes. Slowing down of μ^- mesons in matter results in an analogous depolarization associated with the interaction of the spins of the μ^- meson and the nuclei in the mesic atom.

*In the cases listed above the fact that the lifetime does not depend on the structure of the excited state is a direct consequence of the isotropy of space: the emission mechanism in the atom or nucleus or the decay mechanism in the μ meson cannot depend on orientation in space.

hold in cases of this kind, with the further requirement that the probability of finding the system in any of the considered states must fall off exponentially with time. Mathematically this conclusion follows directly from the fact that the relations in (5) are replaced by

$$\left.\begin{aligned} i\frac{db_1}{dt} &= a_{11}b_1 + a_{12}b_2 - i\frac{\lambda}{2}b_1, \\ i\frac{db_2}{dt} &= a_{21}b_1 + a_{22}b_2 - i\frac{\lambda}{2}b_2. \end{aligned}\right\} \qquad (5')$$

A more interesting case is the one in which the lifetime and decay mode depend on the structure of the excited state. They thus become, in certain respects, characteristic features of the state much in the same way as the total momentum of the system or its projection in some preferred direction. This is best explained by concrete examples.

We start with the excited state of the hydrogen atom in which the principal quantum number is n = 2. To eliminate certain unimportant details we assume that the electron and proton are spinless point particles (a more exact analysis can be found in [10]). There then is a degeneracy since the 2P and 2S states have the same energy. The orbital momentum of the first of these states is unity and the state is characterized by a rapid one-photon transition ($\tau_1 \sim 10^{-9}$ sec) to the 1S ground state. The 2S and 1S states do not have orbital momenta and the one-photon transition 2S → 1S is forbidden. The only allowed transition from the 2S state is via a two-photon process corresponding to a life time $\tau_2 \sim 10^{-1}$ sec.

In the excitation process hydrogen is formed in a state which, in general, will be a superposition of the 2S state (wave function ψ_1) and the 2P state (wave function ψ_2). In the course of time the 2P state vanishes rapidly via one-photon decay leaving only excited atoms in the metastable 2S state. Further decay can occur only via the emission of two photons.

The degeneracy is lifted if account is taken of the fact that the proton is not a point particle but is of finite size. The 2S and 2P states remain characteristic states as before but now correspond to different energy values w_1 and w_2. It is evident that in practice this situation will not be described by the process of successive decay alone. The general form the wave function of the system can be written

$$\psi = a_1 e^{i\frac{w_1}{\hbar}t - \frac{\lambda_1}{2}t}\psi_1 + a_2 e^{i\frac{w_2}{\hbar}t - \frac{\lambda_2}{2}t}\psi_2, \qquad (1')$$

whence it follows that the probabilities of finding the system in the 2S and 2P states

$$p_{w_1} = |a_1|^2 e^{-\lambda_1 t}, \qquad p_{w_2} = |a_2|^2 e^{-\lambda_2 t} \qquad (8)$$

are different decaying function of time.

We now consider the wave functions φ_1 and φ_2, which are related to ψ_1 and ψ_2 by (2) and (2').* Obviously, the corresponding states are not characterized by definite values of the orbital momentum or a definite mode of decay. On the other hand, in contrast with the ψ_1 and ψ_2 states (2S and 2P), these states have, as is well known, an electric dipole moment q which is of the order of magnitude of the product of the electron charge by the linear dimensions of the hydrogen atom.* In the φ_1 state the dipole moment is along the z axis and in the φ_2 state it is in the reverse direction.

In this case, expanding the wave function characterizing an arbitrary state in terms of the wave functions φ_1 and φ_2 we have

$$\psi = \frac{\varphi_1}{\sqrt{2}}\left\{a_1 e^{i\omega_1 t - \frac{\lambda_1}{2}t} + a_2 e^{i\omega_2 t - \frac{\lambda_2}{2}t}\right\} + \frac{\varphi_2}{\sqrt{2}}\{a_1 e^{i\omega_1 t - \frac{\lambda_1}{2}t} - a_2 e^{i\omega_2 t - \frac{\lambda_2}{2}t}\}. \qquad (3')$$

Hence, the probability of finding the system in a state with positive or negative projection of the electric dipole moment along the z axis is given by

$$\left.\begin{aligned} p_{q^+} &= \frac{1}{2}\{|a_1|^2 e^{-\lambda_1 t} + |a_2|^2 e^{-\lambda_2 t} + 2|a_1||a_2| e^{-\frac{\lambda_1+\lambda_2}{2}t}\cos(\Omega t + \delta)\}, \\ p_{q^-} &= \frac{1}{2}\{|a_1|^2 e^{-\lambda_1 t} + |a_2|^2 e^{-\lambda_2 t} - 2|a_1||a_2| e^{-\frac{\lambda_1+\lambda_2}{2}t}\cos(\Omega t + \delta)\}. \end{aligned}\right\} \qquad (9)$$

The electric dipole moment is not conserved in time; its projection obeys the relation that describes a damped oscillation. For example, assume that at t = 0 the atom is in the φ_1 state, that is to say, $a_1 = a_2 = 1/\sqrt{2}$. In this case (9) becomes

$$\left.\begin{aligned} p_{q^+} &= \frac{1}{4}\{e^{-\lambda_1 t} + e^{-\lambda_2 t} + 2e^{-\frac{\lambda_1+\lambda_2}{2}t}\cos\Omega t\}, \\ p_{q^-} &= \frac{1}{4}\{e^{-\lambda_2 t} + e^{-\lambda_2 t} - 2e^{-\frac{\lambda_1+\lambda_2}{2}t}\cos\Omega t\}. \end{aligned}\right\} \qquad (9')$$

At the initial time $p_{q^-} = 0$, $p_{q^+} = 1$; in the course of time the φ_1 and φ_2 states then periodically exchange places and the entire process gradually decays in time.

There is another noteworthy point. We first neglect decay and again assume that the proton is a point particle. The system is degenerate: the 2S and 2P states are stationary, as is any linear combination of these states; in particular, φ_1 and φ_2 are stationary. Taking account of the finite dimensions of the proton leaves the 2S and 2P states as stationary states but makes the φ_1 and φ_2 states nonstationary. However, if the proton is a point particle and the atom is in an electric field E, the degeneracy is lifted in a different way. The

*The 2P state is triply degenerate in terms of the projection of the angular momentum m. In what follows we only consider states characterized by m = 0.

*The electron wave function is even in the 2S state and odd in the 2P state. In both cases the charge density is an even function so that the dipole moment vanishes. These considerations obviously do not apply to the ϕ_1 and ϕ_2 wave functions, each of which has both an even and odd part.

φ_1 and φ_2 states remain stationary with the corresponding characteristic energies differing from the unperturbed value by $\pm qE$. The 2S and 2P states become nonstationary and exchange places periodically. In the general case, i.e., a hydrogen atom with a proton of finite dimensions located in an external electric field, both pairs of states (the φ_1 and φ_2 states as well as the ψ_1 and ψ_2 states) are nonstationary. The behavior and properties of this atom are given by Eqs. (5)—(7) with $\alpha_{12} = \alpha_{21} = qE$, while α_{11} and α_{22} are equal to the characteristic energies of the 2S and 2P states with the external field switched on. If we now take account of decay the original equation (5) becomes somewhat more complex:

$$\left.\begin{aligned} i\frac{db_1}{dt} &= \alpha_{11}b_1 + \alpha_{12}b_2 - i\frac{\lambda_1}{2}b_1, \\ i\frac{db_2}{dt} &= \alpha_{21}b_1 + \alpha_{22}b_2 - i\frac{\lambda_2}{2}b_2. \end{aligned}\right\} \qquad (10)$$

To avoid a rather lengthy but completely straightforward analysis of the general behavior of the solutions of (10) we consider a particular case qualitatively. If there is no E field only one metastable state will remain after a short time no matter what the initial conditions are. If an external electric field is applied, this state is no longer stationary but makes a transition to the 2P state, after which a rapid one-photon transition to the 1S ground state is allowed. Thus, switching on a sufficiently strong electric field causes a rapid quenching of the 2S metastable state via a decay mode which is not a natural one (emission of a single photon!). This result has recently been verified experimentally.[11]

Although we have considered in detail the behavior of the excited hydrogen atom, the effect in question is found in other systems. Similar properties are exhibited, for example, by positronium in the ground state (a detailed review is given in [12]). Depending on the mutual orientation of the spins of the electron and positron we can distinguish orthopositronium (spins parallel, total spin equal to unity) and parapositronium (spins antiparallel, total spin equal to zero). Both states are stationary with characteristic energies differing somewhat because of the interaction of the magnetic moments of the electron and positron. The orthopositronium is quickly annihilated, producing two γ photons ($\tau \sim 10^{-10}$ sec); the parapositronium annihilates much more slowly ($\tau \sim 10^{-7}$ sec) and then only by decay into three γ photons. The application of an external magnetic field causes an additional interaction between this field and the moment of each particle. As a consequence the ortho and para states of positronium are no longer stationary and the system makes periodic transitions from one state to the other.*

*In a very strong field the interaction between the electron and positron spins can be neglected and the stationary states are those whose wave functions are proportional to the sum and difference of the wave functions for ortho- and parapositronium (here we consider only positronium with zero projection of the spin in the field direction).

It follows from what has been given above that there is a rather far-reaching analogy between the behavior of positronium and the excited hydrogen atom. The ortho and para states correspond to the 2S and 2P states; the spin-spin splitting is analogous to the splitting associated with the finite dimensions of the proton; the application of a magnetic field corresponds to the application of an electric field. Qualitatively all the properties of positronium follow from Eq. (10) in which α_{11} and α_{22} equal the characteristic energies for ortho and parapositronium (without an external field) while $\alpha_{12} = \alpha_{21} = \mu H$, where μ is the magnetic moment of the electron. In particular, one expects that the application of a magnetic field will cause long-lived parapositronium to rapidly annihilate into two γ photons. It has been shown experimentally that this actually happens. [12,126]*

A similar, very important example is furnished by the neutral K meson.[22] The K^0 and $\tilde{K}^0$ mesons formed in collisions of high-energy particles have definite values of the so-called strangeness number ($S = +1$ and $S = -1$) and, as a consequence, interact differently with matter. On the other hand, definite kinds of pion decays are attributed to the so-called K_1^0 and K_2^0 particles rather than the K^0 and $\tilde{K}^0$ mesons; the wave functions of the former are related to the wave functions for the K^0 and $\tilde{K}^0$ mesons by expressions similar to those in Eqs. (2) and (2′):

$$K^0 = \frac{K_1^0 + K_2^0}{\sqrt{2}}, \qquad \tilde{K}^0 = \frac{K_1^0 - K_2^0}{\sqrt{2}}, \qquad (11)$$

$$K_1^0 = \frac{K^0 + \tilde{K}^0}{\sqrt{2}}, \quad K_2^0 = \frac{K^0 - \tilde{K}^0}{\sqrt{2}}. \qquad (11')$$

The K^0 particle decays rapidly into two π mesons ($\tau_1 \sim 10^{-10}$ sec) while the decay of the K_2^0 particle, which decays into three π-mesons, is much slower ($\tau \sim 10^{-7}$ sec)†.

It follows from PCT invariance that the K^0 and $\tilde{K}^0$ mesons cannot have different masses. This statement does not apply to K_1^0 and K_2^0 particles, whose masses m_1 and m_2 can differ from each other. Thus, in accordance with Eq. (11) the K^0 and $\tilde{K}^0$ mesons should be converted into each other periodically. For example, if a K^0 meson is formed in the original nuclear reaction, the probability that it will remain a K^0 meson or that it will be converted into a $\tilde{K}^0$ meson is given [by analogy with Eq. (9′)] by the expressions

*In the formation of positronium it is possible that the 2S excited state will be produced in addition to the 1S ground state. In this case the analogy with the properties of the hydrogen atom becomes still stronger. In particular, the application of the electric field causes a transition to the 2P state with a subsequent radiative transition to the 1S state. This process increases the number of fast two-quantum annihilation events and can evidently be used for experimental observation of positronium in the 2S state.

†For simplicity we neglect $\pi e\nu$ or $\pi e\mu$ decays; these will be considered below.

$$\left.\begin{aligned} p_{K^0} &= \tfrac{1}{4}\{e^{-\lambda_1 t} + e^{-\lambda_2 t} + 2e^{-\frac{\lambda_1+\lambda_2}{2}t}\cos\Omega t\}, \\ p_{\tilde{K}^0} &= \tfrac{1}{4}\{e^{-\lambda_1 t} + e^{-\lambda_2 t} - 2e^{-\frac{\lambda_1+\lambda_2}{2}t}\cos\Omega t\}, \end{aligned}\right\} \quad (12)$$

where $\lambda_1 = 1/\tau_1$, $\lambda_2 = 1/\tau_2$, $\Omega = (m_1 - m_2)/\hbar c^2$.*

The transition considered here can be observed by studying nuclear interactions of neutral K mesons: for example, hyperons are formed only by virtue of the presence of $\tilde{K}^0$ mesons and the probability of generation of these particles is proportional to $p_{\tilde{K}^0}$. Another method relates to the observation of $\pi e\nu$ and $\pi\mu\nu$ decays since the K^0 mesons can decay only via $\pi^- e^+ \nu$ and $\pi^-\mu^+\nu$ while the $\tilde{K}^0$ mesons can only decay via $\pi^+ e^-\nu$ and $\pi^+\mu^-\nu$. Experimental observation of $K^0 \rightleftarrows \tilde{K}^0$ transitions allows us to determine the magnitude of $\Delta m = |m_1 - m_2|$ which, as has been explained, is approximately 10^{13} times smaller than the mass of the K meson.†

It is evident that the picture for the neutral K meson is very much the same as that of the excited hydrogen atom. The K_1^0 and K_2^0 particles are analogous to the 2P and 2S states while the K^0 and $\tilde{K}^0$ mesons are analogous to the states of hydrogen with definite values of the projection of the electric dipole moment (φ_1 and φ_2). In the hydrogen case the application of an external electric field leads to a splitting of the characteristic energies φ_1 and φ_2 and to the nonstationarity of the 2P and 2S states. The question is whether similar relations hold for K mesons. In principle the answer is yes. If the K^0 and $\tilde{K}^0$ mesons had magnetic moments μ their characteristic energies would be different in an external magnetic field; the K_1^0 particle would be converted into a K_2^0 particle and decay rapidly into two pions. Actually, a magnetic field does not affect decay of the K_2^0 particle and from this result we can make a fairly accurate estimate of the upper limit of the magnetic moment of the K meson (μ is less than 0.04 of the appropriate magneton; cf. [17,18]). This result is in good agreement with contemporary ideas, according to which the spin of the K meson is zero.

If, the requirements of PCT invariance notwithstanding, the inertial masses of the K^0 and $\tilde{K}^0$ mesons differ by some amount Δm, this would also lead to a two-pion decay of the K_2^0 meson. The fact that the effect is not observed allows us to state that $|\Delta m|/m_K \lesssim 10^{-17}$ (cf. [19,20]). In recent years there has been a renewed interest in the question of the possible existence of antigravitation. In this connection we note that a difference in the signs of the heavy masses of the K^0 and $\tilde{K}^0$-mesons would also lead to two-pion decay of the K_2^0-particle (cf. [20,21]).

The last examples are academic because in each case the effect has not been observed or the appropriate experiments have not yet been carried out. The following example is of interest because the expected effects have been investigated in detail experimentally (cf. [22]). We have seen that conversion of the K_2^0 particle into the K_1^0 particle requires that there be some interaction which is different for the K^0 and $\tilde{K}^0$ mesons. This difference, as we have already noted, is found in nuclear interaction of K mesons.* Hence, in the passage of K_2^0 particles through matter one finds that two-pion decays are actually observed. This effect has a number of interesting features, a detailed analysis of which would go beyond the scope of the present review (cf. for example, [23]). We note, in particular, that experiments on the passage of neutral K_2^0-mesons through matter make it possible to determine the sign of the quantity $m_1 - m_2$ (cf. [24,25]).

In concluding this section it is appropriate to note the analogy that exists between the properties of K^0 and $\tilde{K}^0$ mesons on the one hand and the properties of muonium and antimuonium, that is to say, systems consisting of μ^- and e^+, on the other hand (cf. [28-32]).

3. BEATS

We now consider radiation effects that appear when an excited level is split. Since the transition to the ground state can occur via two or more channels, it is possible in general for various interference effects to arise. By definition effects of this kind can arise only when it is impossible to specify the particular channel by means of which a transition has occurred.

It thus follows that interference effects cannot appear if the lower level of a radiating system is split, because after a radiation event the channel used in the transition is specified uniquely by the final state. This consideration also applies to the case in which each of the channels corresponds to a particular radiation mechanism—one-photon or two-photon transitions in the hydrogen atom, two-pion or three-pion decay of neutral K mesons, and so on. Evidently interference effects cannot occur in such cases.

The situation is different if the radiation mechanism is the same for all channels, a situation that is fre-

*The relations in (12) apply if we consider K mesons at rest observed at the same point in space. Actually we always deal with moving K mesons. Simple analysis indicates that t must then be replaced by $t\sqrt{1 - v^2/c^2}$, where v is the meson velocity (cf. [23]). It should also be noted that Eq. (12) applies to the case in which only one neutral K meson is formed in the nuclear reaction. Specific effects associated with the generation of a $K^0\tilde{K}^0$– pair are treated in [115,161].

†The determination of the sign of Δm is discussed below. This determination can not be made using Eq. (12) because Ωt is an even function of Δm.

*The K^0 and $\tilde{K}^0$ mesons are not absolutely neutral particles.[26,27] Although their total electric charge is zero, the charge density is different from zero and the spatial distribution in the K^0 meson is not the same as in the $\tilde{K}^0$ meson. Hence, a Coulomb interaction with the electrons in any medium could also cause the conversion of the K_2^0 particle into the K_1^0.

quently encountered. As examples we cite the decay of the μ meson, which goes into a state of muonium, the decay of K_1^0 and K_2^0-particles according to the scheme $\pi^+e^-\nu$ (or $\pi^-e^+\nu$), various kinds of atomic and nuclear transitions in Zeeman or quadrupole splitting of excited levels and so on. In all cases below we will be considering electromagnetic transitions unless some other mechanism is specified.

Let the wave function for the excited state be of the form given in Eq. (1). The amplitude of the electromagnetic field radiated in some direction is

$$A = A_1 e^{i(\omega_1 t - \mathbf{k}_1 \mathbf{r} + \eta_1)} + A_2 e^{i(\omega_2 t - \mathbf{k}_2 \mathbf{r} + \eta_2)}, \qquad (13)$$

where the real parts A_1, A_2, η_1, and η_2 are related to a_1 and a_2 and depend upon the direction of the radiation and the type of transition.* The intensity $I \sim |A|^2$ i.e.,

$$I \sim A_1^2 + A_2^2 + 2A_1A_2 \cos(\Omega t - \varkappa \mathbf{r} + \eta), \qquad (14)$$

where

$$\Omega = \omega_1 - \omega_2, \quad \varkappa = \mathbf{k}_1 - \mathbf{k}_2, \quad \eta = \eta_1 - \eta_2.$$

It follows from Eq. (14) that the intensity is periodic in space and time.[1] These variations in space and time are completely analogous to the "beats" familiar in radio engineering which appear at the output of a quadratic detector when two signals at approximately the same frequency are applied to the input (cf. for example, [33]). In the case at hand the "quadratic detection" is intimately associated with the very nature of the effect since the probability of observing a particle is proportional to the square of the corresponding amplitude. The "space beats" are intimately related to the "time beats" and it may be assumed that the latter are a consequence of the geometric displacement of the former, which occurs with the velocity of light (in general, the group velocity). Hereinafter we will consider time beats only.

The relation in Eq. (14) holds when only two frequencies are mixed. If several frequencies are present, cosine terms corresponding to all possible combinations of differences of the original frequencies will appear. In this case the beat pattern becomes much more complicated. In this connection we note that the exponential reduction in the intensity of radiation of a singlet excited level can be described as the result of complex beats associated with that distribution of frequencies that corresponds to the dispersion shape of the line (cf. [34]).

The actual observation of beats requires that several important conditions be satisfied. First of all the measurement device must be characterized by a low inertia so that it can follow the variation in intensity. If this condition is not satisfied, the relation in Eq. (14) is averaged in time and the term that describes the beats disappears. The time constant of the device must be small compared with the period of the beats $1/\Omega$. As a consequence the resolving power will be inadequate for separating the frequencies ω_1 and ω_2. In other words, such a device cannot be used to determine by which channel the transition in a given event has taken place. It thus follows that any device (including any spectrometer) capable of resolving the two lines being studied is in principle not suitable for the observation of beats.* On the other hand, simple devices such as photomultipliers or sufficiently fast counters are completely suitable.

By its very meaning the time that appears in Eq. (14) is computed from the time at which the excited state is produced. It is thus necessary to fix this time with an accuracy greater than $1/\Omega$. If this can be done it is impossible to observe the beats because of the time averaging that occurs. Technically this measurement can be made by many methods, for example, pulsed excitation of the system by a beam of suitable particles or electronic determination of the time at which the particle that produces a nuclear reaction of interest passes through the system. In many cases the excited state being studied is formed as a result of a preceding decay, the time of which is easily determined. For example, beats can appear in the decay of μ-mesons, which themselves are formed as a result of the decay of π mesons or K mesons. Another example is β decay accompanied by a γ transition or two successive γ transitions. In this case, the observation of beats is based on a delayed β-γ coincidence or a delayed γ-γ coincidence. We shall consider this second example in greater detail. A simple level diagram and the appropriate experimental apparatus are shown in Fig. 1. It is assumed that the upper counter records only photons that are emitted in the first transition and the lower counter only those from the second transition.

We have seen above that when beats are produced it is impossible to determine the channel used to terminate the intermediate state. At first glance it would appear possible to use an accurate measurement of the energy of the γ photon radiated in the transition from

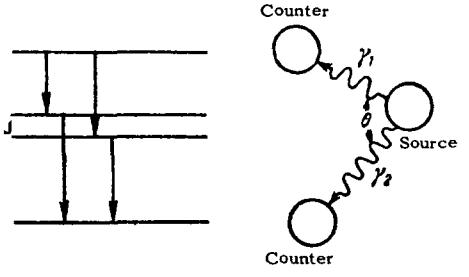


FIG. 1. Observation of beats in γ-γ transitions.

*The relation in Eq. (13) applies if damping is neglected, in which case the natural line width is small compared with the splitting, that is to say, the condition $\tau\Omega \ll 1$ is satisfied.

*The situation may be stated as follows: any "resolution" is essentially the separation of the two channels and this eliminates the possibility of interference.

the upper level to the intermediate level. This, however, is obviously not the case. The determination of the time of formation of the intermediate state, necessary for observing beats, requires a corresponding determination of the detection time of the first γ photon and this leads to an uncertainty in its energy greater than the splitting Δ.

After successive emission of both γ photons the nucleus acquires some recoil momentum q. It is evident that the quantity q is somewhat different for each of the channels being considered, specifically

$$|\delta \mathbf{q}| = \frac{2\Delta}{c} \sin\frac{\theta}{2} = 2\hbar |\boldsymbol{\varkappa}| \sin\frac{\theta}{2}. \qquad (15)$$

Is it possible to use the difference δq to determine the channel by which emission has occurred in each concrete event of the γ transition? To answer this question we return to the original relation (14). In particular, it follows from this relation that beats can be observed only when the spatial position of the radiating nucleus is determined with an accuracy better than $1/\kappa$. Under these conditions its momentum cannot be known with an accuracy better than $\hbar|\kappa|$, so that this technique cannot be used to distinguish channels.

It has been tacitly assumed above that the first counter operates precisely at the moment the intermediate state is formed. This is obviously not necessarily so; in principle there can be a delay. If the first counter is further from the source than the second it may even operate after the second. Under these conditions it is possible to investigate the number of delayed coincidences as a function of the time between the operation of the second counter and the subsequent operation of the first counter. It is easy to show that beats, completely analogous to those considered earlier, also appear in this case.*

In general, the amplitudes A_1 and A_2 appearing in Eq. (14) depend on the angle of observation θ and the integration over θ usually causes the term containing A_1A_2 to vanish. This occurs for example when levels with different quantum number m are split (Zeeman, quadrupole splitting etc.) and the total transition probability does not depend on m. Thus to observe beats it is necessary to fix the angle θ with respect to any preferred direction characteristic of the process that leads to the appearance of excited state. For instance, one could use the direction of the particle beam producing the nuclear reaction, the direction of emission of the first γ photon in γ-γ coincidences and so on. If this direction is not fixed as, for example, in thermal excitation of atoms, then interference effects do not appear.

The above obviously does not apply to the case in which each of the split levels is associated with a definite lifetime. An example is the decay of neutral K-mesons according to the $\pi e\nu$ and $\pi\mu\nu$ schemes. Beats are observed here in the integration over angle since the K_1^0 and K_2^0 particles have different lifetimes.

4. EXPERIMENTAL APPLICATIONS OF BEATS

The observation of beats can be used to study splitting of quantum levels.[1] The method can be used to determine the number of components and the spacing between them. The upper limit for the application of the technique is determined by the possibility of using electronics to measure high frequencies. Thus, the upper limit lies in the range $\Omega \sim 10^9$ sec^{-1}, that is to say, $\Delta \sim 10^{-6}$ eV.*

In contrast with conventional spectroscopy, which becomes more difficult at smaller splitting, the beat method becomes more useful as Ω is reduced.

The lower limit is determined primarily to the natural width of the excited state Γ, which must obviously be smaller than the splitting Δ. This has always been assumed implicitly above; in particular, Eq. (13) and (14) hold only when $\Delta \gg \Gamma$.† It should also be noted that in general a radiating system is not completely isolated. It is subject to random effects of various kinds and these change the phase η appearing in Eq. (14). Beats can be observed only when the variation in η over a beat period $1/\Omega$ can be neglected. In other words, the broadening associated with the random processes must be smaller than the splitting Δ.

The magnitude of the broadening depends on the actual circumstances. For example, in the case of γ-γ transitions in crystalline sources one speaks of the relatively small width associated with the magnetic interaction of the nuclear spins ($\Gamma/\hbar \sim 10^5$ sec^{-1}). In principle this broadening is analogous to the so-called collisional broadening in optics. However, it should be noted that not all kinds of collisional broadening are dangerous in this case. For example, consider an atom with an excited state with spin J = 1. In an external magnetic field this state is split into three states with different values of m. It is assumed that the beats of interest here are associated with levels corresponding to m = +1 and m = −1. If there are many ions in the medium there can be a considerable broadening due to the Stark effect; however, this does not change the distance between two of the indicated levels since both are displaced in the same direction in the electric field. Consequently, this particular kind of broadening does not disturb the beat pattern.

*Since we are discussing beats in the detection of the first γ photon it might be said that they are related to the splitting of the lower level. However, it is important to note that at the end of the process, i.e., after radiation of the second γ photon, the split state has disappeared and thus cannot be used to determine the transition channel.

*In the $K^0 \rightleftarrows \tilde{K}^0$ process one can determine values $\Delta m \sim 10^{-6}$ eV/c^2 by observing space beats.

†An analysis of the appropriately modified relations that takes account of the finite width Γ shows that this quantity can actually be found experimentally (see below).

The effect described just above is always observed when a given mechanism displaces the levels of interest in the same direction. From the point of view of possible application of the method it is important to note that this is the case for the Doppler effect, which frequently causes large broadening. Fortunately, the Doppler mechanism does not affect beats since it displaces all components by practically the same amount. Another possible example is the change in energy of a γ photon due to recoil. In this case, once again, the various levels are displaced uniformly and the effect as a whole has no bearing on beats even when hard γ photons are observed. This feature is a great advantage as compared with the widely used method of measuring the splitting of γ lines based on the Mössbauer effect (cf. [35-37]). In the latter case, as is well known, only soft γ photons can be used.*

Other examples can also be given. For example we might include interactions between optical and molecular transitions which lead to the formation of so-called band spectra.

The interference phenomena being discussed here were first observed experimentally in work on the effect of a magnetic field on the number of delayed γ-γ coincidences.[38,39] The application of an external field results in Zeeman splitting of the intermediate state and beats are produced; a measurement of the beat frequency yields the magnitude of the splitting, that is to say, the gyromagnetic ratio of the excited nucleus.† For experimental reasons it is more convenient to investigate the number of γ-γ coincidences as a function of H for a fixed delay t rather than the time dependence for a fixed H. In Fig. 2 we show a typical curve obtained for Cd^{111} in [38].

Similar experiments can be carried out in gases. In this case, however, a difficulty arises in that the nuclear spin interacts with the internal field of the atom as well as the external magnetic field. To minimize the effect of this interaction one can place the source in a rather strong magnetic field ($H \gtrsim 1000$ G) parallel to the direction of emission of one of the γ

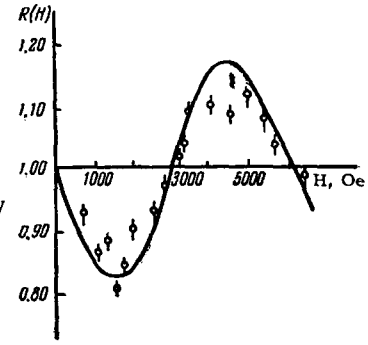


FIG. 2. The number of γ-γ coincidences as a function of magnetic field for a fixed delay time.

photons. It can be shown that such a magnetic field does not produce beats. It also breaks up the coupling between the nuclear spin and the internal atomic field (this effect is to be compared with the effect of a strong magnetic field on muonium and positronium which we have considered above). If an additional magnetic field is now applied perpendicularly to the plane formed by the directions of emission of the γ photons, it will produce the same Zeeman splitting as in the case of a free spin.

An interesting interference effect has been investigated in detail in many experiments on $\pi\mu e$ decay.[44,120] In this case the first transition in Fig. 1 corresponds to the $\pi\mu$ decay and the second to the μe-decay. Beats are observed in the number of electrons detected at different times following the formation of the μ meson. When an absorber is inserted (the absorber causes ionization of the residual μ mesons) the energy levels are split in the magnetic field because of the interaction of the magnetic moment of the μ meson with the field; this mechanism produces the beats that are observed. Using this method it was possible to determine the spin of the μ meson for the first time in direct experiments. It was also possible to measure its gyromagnetic ratio with high accuracy.

It should be noted that in the interpretation of these experiments the word beats is frequently not even mentioned. In the literature it has become conventional to discuss these effects in terms of semi-classical ideas based on the Larmor precession of the spin (cf. for example, [44-46]).

In the present case this latter approach is actually very convenient. For example, consider the effect of a magnetic field on μe decay. It is known that the μ mesons formed in $\pi\mu$ decay are fully polarized. Further, it is known that under these conditions the angular distribution of the electrons formed in the subsequent μe-decay is anisotropic. If a magnetic field is now applied perpendicularly to the direction of polarization, the spin and the angular distribution of electrons will precess. Thus, the number of electrons emitted in any given direction will vary periodically in time at the precession frequency.

Similarly, one can treat the effect of a magnetic field on the frequency of γ-γ coincidences. By these

*Another advantage of the interference method, as compared with the Mössbauer technique, is the fact that the former can be used for γ transitions that are not connected to the nuclear ground state. On the other hand, the Mössbauer effect can be used to measure displacement whereas beats arise only in the presence of a splitting. A further difference stems from the fact that in the Mössbauer effect one measures the splitting in the energy of the γ photons whereas in the beat technique, the beats derive only from splitting in the upper level. As far as the measurable magnitudes of splitting are concerned, we note that the region of applicability is approximately the same for both methods.

†We recall that the nuclear magnetic resonance method usually measures only the Zeeman splitting of the ground state of the nucleus (cf. [40-42]); this is also essentially the case for similar experiments in the determination of the gyromagnetic ratio of μ mesons.[43]

means it is easy to explain many particular features of the effect. An example is the disappearance of the periodic changes in intensity in integration over angle, the zero spin of the lower state, and so on.

The Larmor precession frequency coincides with the beat frequency and it can be shown in general that both points of view lead to the same result as far as the interaction of a spin with an external field is concerned. However, there is a limitation on the precession picture. In a number of cases this picture must be stretched a great deal; in other cases it has no meaning at all. Cases of the first kind are represented by quadrupole splitting;[46] cases of the second kind are represented by beats in the $K^0 \rightleftarrows \tilde{K}^0$ process, beats in the emission of photons by excited hydrogen atoms in an electric field, and so on. For this reason, hereinafter we prefer to use the term beats since it is universal, precise, and provides a common term for explaining different kinds of general relations and analogies.

We now consider the γ-γ coincidence for a gaseous source in the case in which the atom has a nonzero magnetic moment. As we have already noted, the interaction between the nuclear spins and the atomic shell in this case is usually regarded as a background effect which is to be removed (cf. [40,50,75]). This interaction causes a hyperfine splitting in the atom-nucleus system and consequently causes corresponding beats in the delayed γ-γ coincidences. The beat frequency $\Omega \sim 10^8$ sec^{-1}; this frequency depends on the gyromagnetic ratio of the nucleus and can be used to determine this ratio experimentally. Actually, beats can only be observed in rather slow transitions, for which ($\Gamma \ll \hbar\Omega$). It is evident that this condition must be satisfied if the atom and nucleus are to be considered as a single system. If this condition is not satisfied the effect of the electronic shell is unimportant and the nucleus radiates as it would if it were isolated.

A number of recent papers are concerned with the investigation of the properties of the Mössbauer components of the γ radiation transmitted through a resonance filter.[37,47-50] A diagram of such experiments is shown in Fig. 3. The crystalline source contains Co^{57} which decays according to the scheme

$$Co^{57} \xrightarrow{\beta} Fe^{57**} \xrightarrow[123\ keV]{\gamma_1} Fe^{57*} \xrightarrow[14\ keV]{\gamma_2} Fe^{57}.$$

The first γ photon is detected by the left-hand counter and the second γ photon strikes the crystalline absorber, which contains Fe^{57} in the ground state. It is known that the Mössbauer effect is very strong in the second γ transition being considered. Some part of the Mössbauer component of the γ radiation experiences resonance absorption and the γ photons transmitted through the filter are detected by the right-hand counter. One of the problems studied by means of this arrangement is that of determining the dependence of the number of delayed γ-γ coincidences on the delay time.

With no filter the usual exponential relation (with a mean life time $\tau \sim 10^{-7}$ sec) is obtained. This same pattern is observed with a filter if measures are taken to eliminate the possibility of resonance absorption of the Mössbauer component. This can be achieved by varying the distance between the source and the filter slowly; with a relative velocity as small as $v \gtrsim 0.1$ cm/sec the Doppler shift exceeds the natural line width and the resonance absorption disappears. The situation is different when $v = 0$, and resonance absorption occurs. The absorption coefficient is largest for the central part of the Mössbauer line and with a filter of sufficient thickness this point corresponds to the minimum intensity of the transmitted γ radiation (this effect is to be compared with the formation of Fraunhofer lines in the atmosphere of the sun).

As a result the γ line acquires a shape given by the lower curve in Fig. 4 (the upper curve shows the line shape without the filter). The similarity to double splitting is evident and one expects some kind of interference phenomenon. Both components of the "doublet" are highly smeared out and the beats will obviously be smeared out in corresponding fashion. In any case, one cannot talk about an exponential dependence of the number of γ-γ coincidences on time. The effect described is precisely what is observed experimentally although it is usually considered from a somewhat different point of view (cf. [48-50]).

At the present time photon counters have been developed that are suitable for practical utilization. Hence, successive optical transitions can be investigated by means of photon-photon coincidences (cf. [51,52]) in the same way as γ-γ coincidences. Here we are considering the measurement, by beats, of the hyperfine splitting as well as Zeeman and Stark splitting of excited atoms.[1] Since the Doppler broadening is unimportant here we can use fields that are smaller than those used in ordinary investigations.

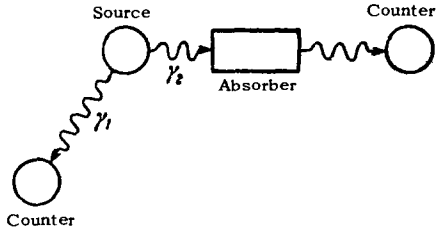
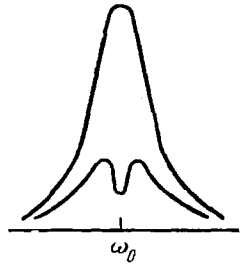


FIG. 3. Beats in resonance absorption.

FIG. 4. Change in line shape due to resonance absorption.



Thus, in the case of the Zeeman effect a magnetic field of the order of 10 Gauss results in a splitting that exceeds the natural width.

Experiments of this kind would encounter formidable technical difficulties. It is obviously simpler to determine the time of formation of the intermediate state by pulsed excitation with subsequent detection of the delayed radiation by means of a photomultiplier or counters.[53-56,122] Excitation can be accomplished by means of an electron beam or by pulsed illumination. In the latter case one is essentially investigating the resonance scattering of light. An appropriate diagram for the transitions is given in Fig. 5. In contrast with the case of successive transitions, the initial state is the same as the final state. It is evident that this does not cause any fundamental change.

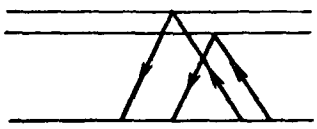


FIG. 5. Beats in resonance scattering.

The literature contains references to so-called Righi beats (cf. for example,[33,57]). If a beam of monochromatic light is divided into two coherent beams and these beams are combined a screen located in the proper position will, as is well known, exhibit interference fringes. If the frequency of one of the beams is now changed, for example, by reflection from a moving mirror, the fringes move along the screen and the intensity at a fixed point varies periodically in time.

Since the fringe pattern at any time uniquely corresponds to the instantaneous position of the moving mirror all the effects can be predicted rather trivially.[33] However, it is important to emphasize the following: two radiation components at different frequencies can be regarded as coherent in some sense if a well defined (not random!) phase relation exists between them. This "hidden" coherence can, under certain conditions, be made evident in the form of beats.[57] This is the situation in all the cases we have considered up to this point since both interference components are produced by the same radiator (atom, nucleus, etc.).

We turn now to a very interesting observation first made a long time ago by G. S. Gorelik and by other authors; this concerns the possibility of observing beats in interference between two completely incoherent (independent) light sources (cf.[58-65]). Suppose that a gas source radiates light at frequency ω and that the line has some definite width (not necessarily the natural width!). The resultant amplitude is made up of the partial amplitudes contributed by the various excited atoms. Since each atom is excited and radiates independently of the others the resultant field at a given moment of time is a random quantity. However, its values at different moments of time are highly correlated. This follows because the field can be regarded as an almost-periodic function $A\cos(\omega t + \delta)$ whose amplitude and phase change appreciably over a time interval $\tau \sim \hbar/\Gamma$. This interval is very large compared with the period of the oscillations if, as is usually the case in optics, $\omega \gg \Gamma/\hbar$.

It is now clear that if beams from two such independent sources with somewhat different frequencies ω_1 and ω_2 are allowed to interfere the intensity of the light will be given by Eq. (14) but with A_1, A_2 and η now becoming slowly varying functions of time. Beats can be produced if the condition $\Omega \gg \Gamma/\hbar$ is satisfied, that is to say, if the splitting is appreciably greater than the line width. It is evident that the two light sources can be replaced by a single source if the radiated light contains two components whose frequencies are somewhat different, say because of the Zeeman effect. In this case, as in the case of separate sources, the following relation must be satisfied

$$l\Delta\theta \ll \lambda, \tag{16}$$

where $\Delta\theta$ is the angular spread of the beam, λ is the wavelength, and l is a linear dimension of the surface of the device being used to detect the beats (cf. for example,[58,60,62,65]).

The relation in (16) shows that in practice it is impossible to observe such beats in the case of γ rays although the situation is not clearcut for visible light, for which the wavelength is several orders of magnitude greater. This rather difficult experiment has, in fact, been carried out successfully.[58,65] A schematic diagram of the apparatus is shown in Fig. 6. A gas light source is located in a magnetic field H which produces a Zeeman splitting. The light is detected by means of a photoelectric cell, the output of which is applied to a tuned amplifier tuned to a frequency Ω. At the appropriate field strength H_0 the magnitude of the Zeeman splitting equals the resonance frequency Ω and the voltage at the output of the amplifier increases sharply. This effect is not observed at values of H higher or lower than the resonance value.*

If the light radiated by the gas source were due to independent photons it is evident that the beats described above would not appear. In this respect this

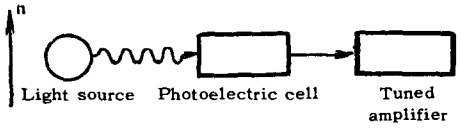


FIG. 6. Observation of beats in visible light.

*In contrast with the beats considered above, in this case it is important to take account of the splitting of the lower level. A further difference lies in the fact that the process is affected by all forms of broadening, including Doppler broadening. Finally, it is not necessary to predict the time at which the excited state is formed to observe the beats described here.

phenomenon is related to the well-known fact that the correct magnitude of the energy fluctuations of blackbody radiation cannot be obtained if one assumes independent photons.[66] The macroscopic electromagnetic field must be considered as a single entity; the exchange of energy with matter proceeds by means of individual photons but the overall process is determined by the field as a whole (cf. also [67,68]). In practice this remark is important only if the wavelength is sufficiently long; as the photon "hardness" increases, the notion of individual photons (or particles of other kinds) becomes increasingly valid.

5. RADIATION INTERFERENCE IN LEVEL CROSSING

We now consider coherent beats. We have assumed above that the lifetime of the intermediate state is large compared with $1/\Omega$. If the finite lifetime is now introduced, by analogy with Eqs. (13) and (14) the radiation amplitude can be written in the form

$$A \sim \{A_1 e^{i(\omega_1 t+\eta_1)} + A_2 e^{i(\omega_2 t+\eta_2)}\} e^{-\frac{\lambda}{2}t}, \tag{17}$$

while the intensity is given by

$$I \sim \{A_1^2 + A_2^2 + 2A_1A_2\cos(\Omega t+\eta)\} e^{-\lambda t}. \tag{18}$$

When Eq. (14) is averaged in time the cosine term disappears; this does not occur in Eq. (18). Integration from $t = 0$ to $t = \infty$ corresponds to averaging over a finite time period of order $1/\lambda$ and the interference term does not vanish completely. It thus follows that it should be possible to observe interference effects without requiring the determination of the time at which the excited state is formed. From the methodological point of view this feature means that pulsed excitation can be replaced by continuous excitation and that the delayed coincidences can be replaced by ordinary coincidences.[1]

Quantitatively we must compute the following quantity with an accuracy to within a normalizing factor:

$$\bar{I} \sim \int_0^\infty I(t)\,dt;$$

it is evident that

$$\bar{I} \sim A_1^2 + A_2^2 + 2A_1A_2\frac{\cos\eta - \frac{\Omega}{\lambda}\sin\eta}{1+\Omega^2/\lambda^2}. \tag{19}$$

The result that has been obtained can easily be generalized to the case in which interference occurs between several levels. Each of the components appearing in Eq. (17) has a natural width under these conditions. This leads to a partial overlap which is the larger, the smaller the ratio Ω/λ. It is evident from Eq. (14) that the interference pattern does not depend on time if the partial frequencies coincide. Hence a partial overlap causes the appearance of an interference effect which does not vanish when averages are taken over time. It is easily shown (cf. for example, [69]) that in this way we can again obtain Eq. (19).

The described effects can possibly be observed in optical experiments on resonance scattering in which the spectrum of scattered light is so wide that its intensity is uniform over the frequency range of interest.* As in the earlier case, integration over angles causes the effect to vanish. For this reason interference effects cannot be observed in transmitted light; in experiments with scattered light it is necessary to determine the scattering angle.

The intensity of the scattered light must depend on the magnitude of the external field H in which the scattering atom is located.[1] The general nature of this dependence is obtained from an expression such as Eq. (19), that takes account of all Zeeman components. The corresponding amplitudes (A_1, A_2, ...) are determined not only by the scattering angle, but by the polarization as well. It thus follows that the polarization properties of the scattered light also depend on H. Using this approach it is a simple matter to formulate a quantitative theory for the depolarization of scattered radiation in a magnetic field; this effect has been investigated in detail by many workers (cf. [70-73]). Such processes should occur in the excitation of atoms by a beam of electrons, but to the best of our knowledge this effect has not yet been observed experimentally.

It should be noted that an investigation of splitting caused by a magnetic field (or any other similar kind of splitting) makes it possible, in principle, to determine the number of split components. It is also possible to measure the lifetime since the intensity depends on the quantity Ω/λ. We shall not consider this question further here as a simpler case is analyzed below (cf. [70]).

We now turn to the γ-γ transition, assuming for simplicity that the radiating nucleus appears in the makeup of a crystal in which there is no internal magnetic field. The application of an external field causes Zeeman splitting of the levels of the intermediate state and, as a consequence, affects the number of γ-γ coincidences and the polarization of the γ photons.† The resolving time of the coincidence unit must be high compared with the lifetime of the intermediate state;‡ in contrast with the delayed γ-γ coincidences considered above, in this case the resolution of coincidences does not require the determination of the time of formation of the intermediate state, but only a determination of the direction of emission of the two γ photons.

With no magnetic field the number of coincidences obviously depends on the angle θ between the direction

*If this is not the case, the light scattering occurs only via one of the channels (cf. Fig. 5) and no interference effect occurs.

†In principle the situation here is the same as in resonance scattering of light. On the other hand, similar effects should obviously be observed in the successive radiation of optical photons by atoms.

‡If this condition is not satisfied the effects still occur but Eq. (19) is now replaced by a more complicated expression.

of the detected γ photons. Hence, one usually discusses γ-γ coincidences and the effect of the magnetic field and the consequent Zeeman splitting on γ-γ coincidences (cf. for example, [38,40,46,74-76,123,124]). Similar effects are found in other kinds of splitting of intermediate states, for example, quadrupole splitting in crystals [44,46,74,77] and hyperfine splitting in atoms due to the interaction of the nuclear spin with the electronic shell. [74,75,78,79] In contrast with Zeeman splitting the spacing for these mechanisms is fixed and the experimenter is deprived of the possibility of following the gradual change in the pattern due to increasing splitting.

We have considered above the case in which the energy level is split into several sublevels. We now turn to a related problem, that of level crossing. For example, assume that the intermediate state is split into three sublevels with energies W_1, W_2 and W_3, which correspond to amplitudes A_1, A_2 and A_3. If the recording device has a long time constant the beats are averaged and the intensity is given by

$$\bar{I} \sim A_1^2 + A_2^2 + A_3^2. \tag{20}$$

We now assume that under the effect of some external agency the quantities W_1, W_2 and W_3 vary in such a way that the difference $W_1 - W_2$ becomes smaller, vanishes, and then changes sign. Then, the value of the intensity is that given by (2) before and after "crossing"; at the intersection point the intensity is obviously given by

$$\bar{I} \sim (A_1 + A_2)^2 + A_3^2, \tag{20'}$$

which differs from Eq. (20) by the quantity $2A_1A_2$.

For simplicity we assume that there is no phase difference η between the amplitudes and that the lifetime of the intermediate state is very long. It is evident that in the general case the dependence of the intensity on the spacing between levels will be given by the earlier relation (19). When $\eta = 0$ the dependence exhibits a resonance nature with width equal to twice the largest natural level width. When $\eta \neq 0$ this shape becomes the sum of the intensities taken for the points symmetric with respect to $\Omega = 0$. As the magnitude of the external effect is increased it is possible to realize successive intersections of the various sublevels in turn. Each of these will, in general, provide its own resonance change in intensity.

The above considerations can serve as the basis for a relatively simple method of studying the structure of quantum levels, the interaction of these levels with various external fields, and the lifetimes of various quantum transitions. [1,73,80-84] In contrast with the magnetic depolarization of resonance radiation that we have considered above, here we are concerned with only two levels, rather than many. This means that the intensity variation being studied is much more pronounced. In contrast with the beat method the observation of level crossings does not require that we remain "tied" to the time at which the excited state is produced in order to record the delayed coincidences and so on. One can use a detection system that is arbitrarily slow; for example, photographic film. On the other hand, as in the other interference methods, many kinds of displacements and broadening effects do not operate here; for example, recoil in the radiation of γ photons, Doppler broadening etc. In optical applications it has been found that the possibility of determining the lifetime is very important because this quantity is usually determined by indirect methods which require absolute measurements and a knowledge of the detection efficiency and the number of excited atoms (cf. for example, [85]).

We now consider an example. Suppose that the intermediate state of a nucleus has spin $J = 1$. In the inhomogeneous electric field inside a crystal there will be a quadrupole splitting as shown in the diagram in Fig. 7 by the solid lines. With the application of an external magnetic field H the levels characterized by $m = +1$ and $m = -1$ are split (dashed lines). At some value of H the levels corresponding to $m = -1$ and $m = 0$ coincide and then again diverge. At the crossing point the number of γ-γ coincidences exhibits a resonant peak, thus, making it possible to measure the quadrupole splitting more precisely by comparing it with the known Zeeman splitting. [1,77,80,125]

Now suppose that the source of γ rays is a gas, that the spin of the intermediate nucleus is unity, and that the spin of the electron shell is also unity. There will then be a hyperfine splitting and the system as a whole will be characterized by three states with angular momentum $J = 0, 1, 2$ (cf. solid lines in Fig. 8). A weak magnetic field causes an additional splitting in terms of m and as H increases it is possible to have level crossing. By investigating the dependence of the number of γ-γ coincidences on H it is possible to determine the gyromagnetic ratio of the excited nucleus. [1,80] It is important to note that crossing is achieved at very small magnetic fields (of the order of several times ten Gauss). This is due to the mechanism for secondary splitting in m, the magnitude of which is not determined by the magnetic moment of the nucleus, but by the magnetic moment of the atom, which is approximately a thousand times greater. This

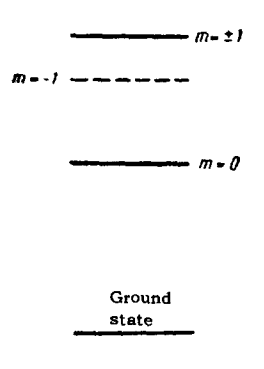


FIG. 7. Energy level diagram in the presence of electric and magnetic fields.

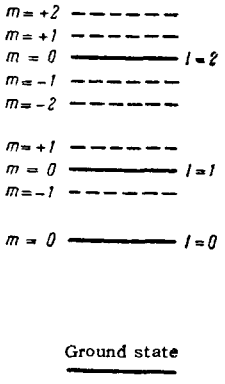


FIG. 8. Splitting of hyperfine levels in a magnetic field.

effect has not yet been observed experimentally. It is possible that it has not been observed because in the investigation of γ-γ coincidences one usually attempts to destroy completely the coupling between the atomic and nuclear moments by using much stronger magnetic fields. Thus, the fields of interest in the present connection are eliminated by the experimental setup itself.*

This discussion of the possible applications of splitting in γ transitions can be brought to a close approximately by comparing it with the Mössbauer technique. Here we simply reproduce the comparison which was carried out above in connection with the analysis of beats. It should be recalled that the experimental investigation of crossing is much simpler than the observation of beats. In the second of the examples we have considered there is a rather important shortcoming in the crossing method in that the gas source must be operated at low pressure. This requirement, which also holds for beat experiments, follows from the need for avoiding reorientation of the orbital momentum of the system during the lifetime of the intermediate state.

Optical experiments involving γ-γ coincidences correspond to analogous experiments involving the detection of photon coincidences. However, from the point of view of method it is much simpler to obtain direct excitation of atoms by a beam of electrons or photons (resonance scattering). For example, consider the resonance scattering of light by atoms having a hyperfine structure. With the application of a magnetic field it is possible to produce crossings similar to those described above (Fig. 8). Hence the intensity of the scattered light can go through resonance changes, making it possible to determine the hyperfine splitting.[1,83,84,81,86] In favorable cases this technique can be used to investigate the fine structure[83] but this generally requires much higher magnetic fields.

*It should be kept in mind that every crossing does not lead to a resonance effect. In particular, it can be shown that if the magnetic field is parallel to the direction of radiation of one of the γ photons, the crossing is not accompanied by a resonance change in intensity. In this case the intensity is a monotonic function of magnetic field. This circumstance can obviously be used to determine the gyromagnetic ratio of the excited nucleus.[75]

It is of interest to investigate the possibility of studying the quadratic Stark effect.[1,80,81,127] Assume for definiteness that the excited atom has a spin of unity. The application of an electric field E will then cause a splitting similar to that shown in Fig. 7. The application of a magnetic field H causes an additional splitting of the levels characterized by $m = +1$ and $m = -1$ (cf. dashed line in Fig. 7). The problem consists of investigating the dependence of the intensity of the scattered light on H for a fixed value of E (or vice versa). The position of the resonance peak allows us to determine the magnitude of the original Stark splitting.

As we have already noted, in all cases there is the additional possibility of determining the lifetime of the intermediate state.[1,40,73,80,81,83,84] From this point of view it is especially important to note that many sources of broadening do not play a role in the crossing method. However, certain kinds of so-called collision broadening are important and the lifetime can be determined only when this effect is small. This is the case, for example, in an interaction between colliding atoms which leads to a reorientation of spin. It should also be kept in mind that the crossing method can be used for investigations of broadening of this kind.

In the field of optical applications it is interesting to compare the crossing method with a method widely used at the present time, that developed by Kastler (cf. for example,[4]). The essence of this method is the determination of the effect of an alternating magnetic field on the intensity of the scattered light. If the level of the excited atom is split in any way (hyperfine splitting, Stark splitting, etc.) the alternating magnetic field causes a redistribution of the populations of the sublevels when its frequency is equal to the splitting frequency. In turn the redistribution of populations causes a change in the intensity of the scattered light which thus exhibits a resonance peak. The position of a resonance can be used to determine the magnitude of the splitting and its width characterizes the lifetime of the excited state.

It is evident from the above discussion that the region of applicability as well as the basic advantages and disadvantages are approximately the same for the two techniques, with the exception of one very important difference: the crossing method does not require an alternating magnetic field and is thus free from the difficulties encountered with such fields. On the other hand, level crossing can be achieved only when there are at least two splitting mechanisms available. The Kastler method does not suffer from such a limitation and, for example, can be used to measure the magnitude of a pure Zeeman splitting.*

*In this case the magnetic field can be regarded as a second source of splitting (cf. the discussion of modulation of radiation given below).

6. MODULATION

In communication theory frequent use is made of the concept of modulated radiation—amplitude or frequency modulation. Amplitude modulation is directly related to periodic changes in the intensity of radiation, that is to say, beats. On the other hand, any modulation leads to the appearance of sidebands, that is to say, splitting of quantum levels. Thus it is clear that level splitting, modulation, and beats are intimately related and are actually different aspects of the same effect.[1]

We recall that from the quantum point of view the appearance of sideband frequencies in modulation is associated with the quantization of the periodic process lying at the basis of the operation of the modulator.[46] For example, suppose that the modulator is a heavy shield on a piston executing harmonic oscillations at a frequency Ω and thus capable of varying a light flux. In the interaction with the light the possible energy states of the shield can change by an amount $n\hbar\Omega$, where n is a whole number. This leads to a corresponding change in the energy of the optical photons, i.e., the appearance of sideband frequencies. These are located symmetrically with respect to the carrier if the modulator executes harmonic oscillations. In the general case anharmonic oscillations are possible and the symmetry can be violated, for example, if the modulator is a rotating molecule in a state characterized by a low quantum number (cf. below).

An optical example of modulation is widely known—combination scattering of light in which the appearance of sideband frequencies is associated with a change in the vibrational state of the molecule. A similar example is the rotation of a molecule containing a nucleus that radiates γ photons.[87,88] This rotation arises by virtue of recoil if the moment of inertia I of the molecule differs from zero. The frequency of the radiated γ photon ω is related to the basic frequency ω_0 by the relation

$$\omega = \omega_0 + \frac{\hbar}{2I}\{l'(l'+1) - l''(l''+1)\}, \tag{21}$$

where l' and l'' are the orbital momenta of the molecule before and after the γ transition.*

In γ-γ transitions (and also in resonance scattering of γ rays) this process can occur twice. If the width of the intermediate state of the nucleus is very small compared to the spacing between the rotational levels of the molecule, there will be two independent impulses, which will cause two successive changes in the angular momentum. In the other limiting case the molecule cannot rotate through an appreciable angle. Hence, it experiences only one impulse, the magnitude of which is determined by the sum of the momenta of both γ photons participating in the process.[89]

In resonance scattering of γ rays in the same limiting case there is obviously a difference in momenta. The effect is found to be completely analogous to the combination scattering of light. The sole difference lies in the mechanism responsible for coupling the electromagnetic radiation and the intramolecular motion. In scattering of γ rays the coupling is provided by recoil while in combination scattering the coupling is provided by the dependence of the polarizability of the electron shell on the vibrational state of the molecule.

We now consider the possibilities of artificial modulation of quantum transitions. The frequency that can be achieved by most modulation devices is not high. Hence we shall treat only the narrowest lines because observation of modulation effects requires that the modulation frequency exceed the line width.*

We are concerned here chiefly with frequency modulation because of the fact that it can lead to the appearance of rather intense sideband components even when the depth of modulation is small.†

If the frequency of the radiated field is

$$\omega = \omega_0(1 + q\cos\Omega t), \tag{22}$$

the field itself

$$A = A_0 e^{i\int_0^t \omega(\tau)\,d\tau} = A_0 e^{i\omega_0 t + \frac{\omega_0 q}{\Omega}\sin\Omega t}$$

can be written in the form

$$A = A_0 \sum_{n=-\infty}^{\infty} J_n\left(\frac{\omega_0 q}{\Omega}\right) e^{i(\omega_0 + n\Omega)t}. \tag{23}$$

It is important to note that the argument of the Bessel function is the quantity $(\omega_0/\Omega)q$. It thus follows that the amplitudes of the sideband components can be large even for a negligibly small depth of modulation q, provided the ratio ω_0/Ω is large.

Frequency modulation can be realized by means of alternating fields since the magnetic and electric fields split and displace quantum levels.[1,89-91] As far as practical applications are concerned we can only discuss the effect of an alternating magnetic field on an atom. It should be kept in mind that all states characterized by $m \neq 0$ are modulated at the same frequency and are split in the same way. Hence, if the magnetic field does not contain a fixed component the positions

*In the radiation of a γ photon the molecule not only rotates but also acquires a translational velocity. As is well known,[86] this leads to an additional change in the γ photon energy W_γ by an amount of W_γ^2/mc^2, where m is the mass of the molecule. It is evident that both of these mechanisms can change the γ-photon energy by approximately the same amount.

*In contrast with the interference effects described above, in observation of modulation it is the full width that is generally important.

†Cf. for example, [4,33]. In particular, in [4] there are described interesting experiments on amplitude modulation of light transmitted through a gas whose atoms precess in an external magnetic field.

of the sidebands will in general be independent of the total angular momentum of the system.*

Suppose now that the magnetic field also has a fixed component parallel to the alternating component and that it produces some initial Zeeman splitting of the levels Ω_0. Under the effect of the alternating component each of these levels characterized by $m \neq 0$ is split into three sublevels if the depth of modulation is assumed to be small. As a result we obtain a pattern similar to that shown in Fig. 9. The intensity of the radiated light does not depend on the modulation frequency Ω so long as it does not become equal to Ω_0. In this case the levels cross and one finds that it is possible to have an interference in the intensity which disappears when Ω is increased further.† The present effect can evidently be observed in resonance scattering of light; the appropriate theory has been given briefly in [1,91]. It is easy to show[1] that when $\Omega_0 = 0$ this phenomenon includes effects such as the depolarization of radiation under the effect of an alternating magnetic field (cf. for example, [70,92-95]).

An important case of frequency modulation is associated with the Doppler effect. For example, suppose that the excited nucleus executes harmonic oscillations. The radiated γ photon can then be characterized by the basic frequency or any of the sideband frequencies. As we have already indicated, from the quantum point of view the appearance of sideband frequencies must be accompanied by a change in the oscillatory state of the nucleus. It is known that the Doppler effect is intimately related to recoil in the radiation of a γ photon; this, then, is the mechanism responsible for the change.‡

The situation can be described quantitatively as follows: if the velocity of the radiator is $v = v_0 \cos \Omega t$, then the quantity $q = v_0/c$ appears in Eq. (22) and, in accordance with Eq. (23), the intensity of the n-th sideband is given by $J_n^2(\omega_0 v_0/\Omega c)$. This intensity can be appreciable even at small values of v_0/c so long as the ratio ω_0/Ω is large. It should also be noted that in general the frequency ω_0 remains unshifted even with strong modulation; this feature will be found to be important in the discussion below.

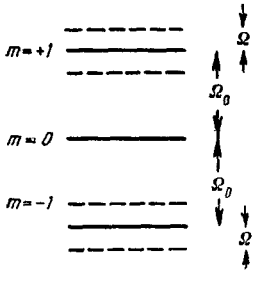


FIG. 9. Splitting of levels by modulation.

We assume that the excited nucleus is in a crystal and consider the Mössbauer component of the radiation. If the crystal executes mechanical oscillations which are excited say, by a supersonic generator, then sidebands are produced because of the frequency modulation.[1,96,98] An an example we consider the nucleus Zn^{67}* in which $\omega_0 \sim 10^{20}$ sec^{-1} and for which the lifetime $\tau \sim 10^{-5}$ sec. For clear observation of the effect it is necessary that the modulation frequency exceed the natural width $1/\tau$. Whence it follows that the ratio ω_0/Ω can be 10^{15}, that is to say, the sideband components will have an appreciable intensity even when $v_0/c \sim 10^{-15}$ ($v_0 \sim 10^{-5}$ cm/sec!). Experimental investigations have completely verified these predictions[92,93,95] It may be assumed that the frequency modulation will find important application in the future practical utilization of the Mössbauer effect.*

It is interesting that the existence of the Mössbauer line itself is also intimately related with the frequency modulation associated with the Doppler effect.[36,99-103] The excited nucleus executes thermal oscillations inside the crystal and its motion can be represented as a sum of periodic oscillations of different frequencies (Debye waves). Each of the partial waves produces a corresponding frequency modulation and the ensemble of modulation components determines the shape of the emission line (in particular, its Doppler width). It is important to note here that each such partial modulation component produces an undisplaced component; the amplitudes of these components add up to give the amplitude of the undisplaced Mössbauer line.

This approach allows us to understand such fine points of the Mössbauer effect as the thermal shift associated with the quadratic Doppler effect.[36,100-102] It is completely in agreement with the usual interpretation based on those γ-radiation events in which the internal vibrational state of the crystal remains unchanged. The equivalence of both points of view is evident since the appearance of the fundamental frequency in modulation is related to precisely those radiation events that do not change the vibrational state of the modulator.

The modulation point of view yields a very simple and direct relation between the Mössbauer effect and other related effects. Here we might include the

*Obviously another point of view is possible: according to this viewpoint the alternating magnetic field splits a level with momentum l into $2l + 1$ components; the spacing between these components then changes periodically with time. The usefulness of one or the other of these approaches is determined, as is always true in such cases, by the properties of the analyzer (cf. [33,89,91]).

†It should be emphasized that in this case there is no change in the populations of the levels with different m. In this respect the effect considered here differs from the widely used radio-frequency resonance effect in which the alternating field causes transitions between levels.

‡In this sense there is an almost complete analogy with the change in the rotational state of a molecule upon radiation or absorption of a γ photon which we have noted above.

*It is reasonable to expect that similar effects will be produced in the optical region by means of lasers, which can produce very narrow optical lines.

Mandel'shtam-Brillouin splitting in scattering of light by Debye waves,[104-106] coherent scattering of x-rays by crystals with an intensity that is determined by the same Debye-Waller factor on which the probability for the appearance of Mössbauer lines depends,[36,104-111,128,129] and so on. As an example we may consider the problem of reflection of light from a mirror treated by G. S. Landsberg.[110] It is known that the frequency of the reflected light corresponds to the frequency of the incident light in spite of the thermal motion of the atoms at the surface of the mirror. In other words, the Doppler effect, which might be expected to affect the macroscopic motion of the mirror, does not appear. It is evident that in its nature this effect is very closely related to the Mössbauer effect.

The short presentation of the theory of the Mössbauer effect we have given above is a particular case of the general modulation approach to Doppler broadening of radiation and absorption lines.* Suppose that some system (atom, nucleus, etc.) which appears in the makeup of a macroscopic body radiates electromagnetic waves of frequency ω_0. It is usually assumed that motion with a directed velocity v causes a frequency change $\Delta\omega = \omega_0 v/c$ and that the line shape reproduces the distribution of directed velocities, i.e., that the line shape is given by

$$I(\Omega) = \frac{1}{\sqrt{2\pi\omega_0^2 \frac{v_0^2}{c^2}}} e^{-\frac{\Omega^2}{2\omega_0^2 \frac{v_0^2}{c^2}}}, \qquad (24)$$

where $\Omega = \omega - \omega_0$, $v_0 = \sqrt{\overline{v^2}}$. This picture is open to question. It holds only when each of the radiators moves uniformly; actually the motion occurs along some complicated trajectory with discontinuities and turns.

For simplicity we will assume that the discontinuities in the trajectory are not reflected in the internal state of the radiator, a mechanism that would lead to collision broadening.† However, under these conditions the motion along the complicated trajectories means that each radiator emits a nonmonochromatic wave which, in general, has a continuous spectrum. To obtain the correct solution to the problem it is necessary to carry out a statistical averaging of these spectra taking account of the motion of the radiating system. It can be shown that this approach leads to an expression that differs from Eq. (24); specifically,

$$I(\Omega) \sim \mathrm{Re} \int_0^\infty d\tau e^{-i\Omega\tau - \frac{1}{2}\overline{x^2(\tau)}}, \qquad (25)$$

where $\overline{x^2(\tau)}$ is the mean-square displacement of a radiator in a time τ (cf. [103,105,113,114]).

The relation in (25) is of general value since all the features of the motion of the radiator are contained in the quantity $\overline{x^2(\tau)}$. In particular, by computing the quantity $\overline{x^2(\tau)}$ for a nuclear radiator appearing in a crystal lattice we can rapidly obtain the theory of the Mössbauer effect (cf. for example, [99,103]). In the case of a gas or liquid the calculation of $\overline{x^2(\tau)}$ is carried out most conveniently by means of the Langevin equation. In [36,99,103,113-118] it is shown that the final result depends on κL, where κ is the wave number and L is the mean free path. If $\kappa L \gg 1$ the relation in (25) becomes (24) because under these conditions all of the radiators move practically uniformly. If this condition does not hold then

$$I(\Omega) \sim \frac{1}{\Omega^2 + (\varkappa^2 D)^2}, \qquad (26)$$

where D is the diffusion coefficient. The line shape is found to be dispersive and the width is $(\kappa L)^{-1}$ times smaller than the usual Doppler width. The reduction in width is explained by the strong diffusion: the radiator changes its position in space very slowly and the Doppler effect is much weaker.

In the case of a liquid it is possible that one might have to take account of longitudinal Debye waves which would lead to some analogy with the Mössbauer line, which is smeared out in accordance with Eq. (26). This situation occurs in crystals if the excited nuclei can diffuse from one site to another.[103,109]

*Basically, collision broadening is also a case of modulation of radiation. This question is treated in the review in [112].

†This is certainly the case for nuclear transitions because the collisions only affect the atomic shells. It is also true in certain types of atomic interactions (cf. for example, [112,115-117,4]).

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