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PARITY NONCONSERVATION AND MACROSCOPIC ROTATION

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FOLLOWING the hypothesis by Lee and Yang [1] that parity is not conserved in weak interactions, it was indeed found that in these processes there exists a correlation between the spin and direction of motion of the particles. In particular, it was found that electrons from β decay and muons from the π-μ decay were longitudinally polarized.

In this connection a number of thought experiments were discussed, in which the spin angular momentum of the polarized particles emitted in weak interaction processes is transformed into the angular momentum of rotation of a macroscopic body. Since a particle with spin is a rotating object in the quantum mechanical sense but the spin angular momentum is not associated with a kinetic energy of rotation, the question arises of what the mechanism that produces the macroscopic rotation may be (we are referring to the above-mentioned experiments). The present article is devoted to this question.

1. MACROSCOPIC ROTATION CONNECTED WITH PARITY NONCONSERVATION

The simplest example of macroscopic rotation is the following: [2,3] a β-active layer is evaporated onto one face of a cylinder attached to an axis. The electrons produced in the β-decay process are absorbed if they are emitted downwards, whereas the electrons emitted upwards carry off an amount of angular momentum proportional to their number, since the electrons have a polarization of $-v/c$ in the direction of their motion (v —electron velocity, c —velocity of light).

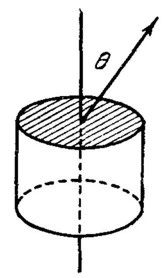
Thus the body acquires an angular momentum $M = (\frac{1}{2}v/c)(\frac{1}{2}hN)$, where N is the number of electrons absorbed by the body (the factor $\frac{1}{2}$ arises from averaging over θ from 0 to $\pi/2$, see the figure). But this means that the body acquires a kinetic energy of rotation

$$E = \frac{M^2}{2I}, \tag{1}$$

where I is the moment of inertia of the body.

True, the energy E is very small. It is determined by the relation

$$\frac{E}{N} \sim \frac{1}{30}\frac{h^2 N}{I} \sim \frac{h^2 N}{30 N_A A m_p R^2},$$



where N_A is the number of atoms in the body, R is the radius of the cylinder, A is the atomic number, and m_p is the mass of the proton. Setting $R = 0.1$ cm and $A = 10$ one gets for E/N a number of the order of 10^{-18} eV even when $N \sim N_A$. Nevertheless in this concrete example, too, there arises the question: what is the source of the rotational energy?

Below we show that the source is the thermal energy of the body. The kinetic energy of the electrons emitted in the β decay may contribute to this energy, but this is not necessary. This can be intuitively understood also from the fact that the electron energy does not enter in Eq. (1).

2. COMPLETE DEPOLARIZATION OF ELECTRONS

One may also give another example. A body simply absorbs an incident beam of polarized electrons with arbitrarily small kinetic energy. In that case Eq. (1) is again valid, with $M = N/2$ (here and below

$h = c = 1$*). In the following we consider for simplicity precisely this case.

When we assume that the body acquires an angular momentum $M = N/2$, we have indirectly assumed that the absorbed electrons become fully depolarized. The depolarization takes place in collisions between the electrons and the atoms of the body. The energy is of course conserved here, but owing to the spin-orbit interaction the spin angular momentum is transformed into orbital angular momentum of the atoms.

This means that in the depolarization process the number of up and down spins becomes equal while the thermal motion of the atoms of the body is partially transformed into the ordered motion with velocity $v = \Omega r$, where Ω is the angular velocity of the body, $\Omega = M/I$.

At first sight the rotation of the body being due to its "cooling" seems like a misunderstanding; let us however see what happens to the entropy.

Let us denote by $W_\uparrow$ and $W_\downarrow$ the probabilities for an electron state with polarization parallel and antiparallel to the initial $(W_\uparrow + W_\downarrow = 1)$.

The state of the beam is determined by its polarization, i.e., the number $NW_\uparrow$ of electrons with spin parallel and the number $NW_\downarrow$ of electrons with spin antiparallel to the initial polarization direction.

According to Boltzmann's statistical interpretation, the entropy of a given state is proportional to the logarithm of the number Π of all possible ways of realizing that state. Let us assume for simplicity that the absorbed electrons are localized so that it is not necessary to take Pauli's principle into account. In that case $\Pi W_\uparrow^{NW_\uparrow} W_\downarrow^{NW_\downarrow} = 1$ and the entropy of the spin distribution is given by $S = -N(W_\uparrow \ln W_\uparrow + W_\downarrow \ln W_\downarrow)$.

For complete depolarization $W_\uparrow = W_\downarrow = \frac{1}{2}$, whereas initially $W_\uparrow = 1$, $W_\downarrow = 0$. Therefore for complete depolarization the change in entropy is

$$\Delta S_1 = N \ln 2.$$

On the other hand, if the heat capacity of the body is sufficiently large so that its change in temperature T can be neglected, the decrease in entropy in the transformation of thermal energy into energy of rotation is

$$\Delta S_2 = -\frac{E}{T} = -\frac{N^2}{8IT}.$$

Obviously, since the process is irreversible we should have

$$\Delta S = \Delta S_1 + \Delta S_2 > 0. \qquad (2)$$

In practice $|\Delta S_2| \ll |\Delta S_1|$ always. Indeed,

$$\frac{|\Delta S_2|}{|\Delta S_1|} \sim \frac{N}{IT} \sim \frac{N}{N_A A m_p R^2 T} = \frac{m_p}{T}\frac{N}{N_A A}\left(\frac{1}{m_p R}\right)^2 \ll 1. \qquad (3)$$

Even for $N \sim N_A$ will the inequality (3) be violated for a body with $R \sim 0.1$ cm and $A \sim 10$ only when

$$T < \frac{10^9 \text{ eV}}{10} \cdot 10^{-26} \sim 10^{-18} \text{ eV}.$$

3. INCOMPLETE DEPOLARIZATION

At such temperatures the rotational energy of the body, arising from complete depolarization of the electrons, and the thermal energy of the body become comparable, as it were. The law of increasing entropy is then violated, which means that complete depolarization is not possible at such temperatures.

One may carry through this thought experiment to conclusion and consider the general case when conditions (3) are not necessarily fulfilled. In that case the depolarization will not be complete and the equilibrium state is determined from the condition that angular momentum be conserved

$$\Omega I + \frac{1}{2} N (W_\uparrow - W_\downarrow) = \frac{1}{2} N \qquad (4)$$

together with maximization of the entropy

$$\Delta S = -\frac{1}{2}\frac{\Omega^2 I}{T} - N (W_\uparrow \ln W_\uparrow + W_\downarrow \ln W_\downarrow). \qquad (5)$$

The extremum of (5) subject to condition (4) occurs when

$$\frac{W_\uparrow}{W_\downarrow} = e^{\Omega/T}. \qquad (6)$$

It is easy to verify that the previous conditions (3) can be deduced from Eq. (6) in the case of complete depolarization. In that case $W_\uparrow = W_\downarrow$,

$$\frac{\Omega}{T} \ll 1 \qquad (7)$$

and from Eq. (4)

$$\Omega = \frac{1}{2}\frac{N}{I}. \qquad (8)$$

Upon substitution of Eq. (8) into Eq. (7) we immediately obtain the conditions (3).

Let us consider now the case $\Omega/T \gg 1$. Then

$$W_\uparrow = 1 - e^{-\Omega/T}, \quad W_\downarrow = e^{-\Omega/T}$$

and Eq. (4) gives

$$\Omega I = N e^{-\Omega/T},$$

or

$$\frac{\Omega}{T} e^{\Omega/T} = \frac{N}{IT}. \qquad (9)$$

Taking logarithms of Eq. (9) and keeping in mind that

$$\left|\ln \frac{\Omega}{T}\right| \ll \frac{\Omega}{T},$$

we obtain for $\Omega/T \gg 1$

*Velocity and action are dimensionless quantities in this convenient system of units. Energy, momentum, transition probability per unit time, and angular velocity all have dimensions of mass and are expressed in eV.

$$\Omega \approx T \ln \frac{N}{IT}. \tag{10}$$

In order that the initial condition (6) be satisfied it is necessary that

$$\frac{\Omega}{T} \approx \ln \frac{N}{IT} \gg 1,$$

or

$$\frac{N}{IT} \gg 1,$$

i.e., a condition inverse to condition (3).

We see that in this case, as was to be expected, $\Omega \rightarrow 0$ as $T \rightarrow 0$.

Equation (6) and the disappearance of depolarization as $T \rightarrow 0$ have a simple meaning.

It is known that a Coriolis field is produced in the coordinate system attached to the rotating body. In such a Coriolis field a body with angular momentum μ has an energy $\mu\Omega$.[4] Clearly this formula should apply also to spin angular momentum. In that case Eq. (6) is nothing but the Boltzmann distribution for the spin in the Coriolis field with

$$\frac{W_\uparrow}{W_\downarrow} = \frac{e^{\frac{1}{2}\Omega/T}}{e^{-\frac{1}{2}\Omega/T}}.$$

As $T \rightarrow 0$ the resultant Coriolis field prevents further depolarization even for arbitrarily small Ω.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] V. F. Weisskopf and L. Rodberg, (Russ. Transl.) UFN **64**, 435 (1958).

[3] B. Pontecorvo, Voprosy fiziki neĭtrino (Problems in Neutrino Physics), Lecture delivered at the School for Theoretical and Experimental Physics, Nor-Amberd. AN ArmSSR, Erevan, 1961, p. 273.

[4] L. D. Landau and E. M. Lifshitz, Mekhanika (Mechanics), M., Fizmatgiz, 1958, p. 163.

Translated by A. M. Bincer