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QUASI-MONOCHROMATIC AND POLARIZED HIGH-ENERGY GAMMA RAYS

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I. INTRODUCTION

In this paper we expound methods for producing quasi-monochromatic polarized high-energy γ rays. In recent times these methods have been proposed and have begun to be intensively developed, and there is no systematic exposition of them in the literature. Undoubtedly the availability of quasi-monochromatic and polarized high-energy γ-ray beams will permit the solution of a number of new problems. For example, there have already been suggestions [27,41] for experiments on the photoproduction of pions by high-energy γ rays. The results of these experiments can give information about the nature of the $\pi\pi$ interaction, the contributions of various states to the photoproduction, and the applicability of dispersion relations.

The basic method for producing γ rays is the deceleration of electrons in matter. A serious disadvantage of the production of γ rays by deceleration of electrons in an amorphous substance is the shape of their energy spectrum. The bremsstrahlung spectrum is continuous, and this fact leads to difficulties whenever a research calls for the singling out of a reaction caused by γ-ray quanta of a definite energy. In particular, in the high-energy region, when one is studying the photoproduction of elementary particles this uncertainty does not permit the distinguishing of the required reaction. Similar difficulties are of course also characteristic of experiments with γ rays at lower energies, but in what follows we shall deal only with questions associated with the production of high-energy γ rays.

Another, and perhaps even more serious, restriction in the study of a broad class of physical processes is that γ-ray beams produced by deceleration of electrons in an amorphous substance in general do not have any particular polarization. It should be noted, by the way, that the selection of γ rays emitted at definite azimuthal angles relative to the direction of motion of the electrons can provide rays that have some degree of polarization;[38] but such a solution of the problem involves difficulties that are practically insoluble at high energies. Other methods for producing partially polarized γ rays (cf. e.g.,[40]) cannot be used at high energies.

In recent years possibilities have been found for producing quasi-monochromatic and polarized high-energy γ rays. These possibilities are due to peculiar properties of the bremsstrahlung and the production of electron-positron pairs in crystals, on one hand, and of the scattering of light by relativistic electrons on the other. In this article we discuss these two most promising methods for producing quasi-monochromatic and polarized γ rays. It must be noted that such γ-ray beams have already been produced experimentally by means of the bremsstrahlung of fast electrons in crystals. The experiments were preceded by a number of theoretical papers in which predictions and calculations were made about the effects produced when high-energy electrons and γ rays pass through a crystal.

In 1935 Williams [48] pointed out that when fast electrons pass through crystals there must be interference effects which lead to departures of the bremsstrahlung spectrum from the Bethe-Heitler shape which holds for the radiation produced on a single nucleus. Qualitative features of the effect of coherent emission in successive collisions with the atoms of a crystal lattice were elucidated by Ferreti.[21] In particular, he showed that the positions of the maxima and minima of the intensity of bremsstrahlung in a crystal must depend on the direction of motion of the electrons relative to the axis of the crystal, and that the interference effect increases with increase of the energy of the electrons. The first quantitative calculation of the bremsstrahlung in crystals, taking into account the thermal vibrations of the atoms of the lattice and the screening, was made by Ter-Mikaelyan by the method of equivalent photons.[39] Purcell estimated the influence of the crystal structure on the cross section for production of electron-positron pairs by high-energy γ rays. A consistent calculation of the interference radiation in crystals has been made in the Born approximation by Uberall,[44] and is given in a form which is most convenient for comparison with experiment. He has also examined and calculated[45-47] the polarization of the γ rays which arise from coherent bremsstrahlung in crystals.

The principal experimental results relating to the peculiarities of the bremsstrahlung and pair production in crystals have been obtained with the electron accelerator in Frascatti,[6-13] and have shown the necessity of taking the crystal structure into account in a more rigorous way than this had been done in previous work. Interesting possibilities for using crystals for the production and analysis of polarized γ rays have been brought out in [14-17].

An idea for a new method for producing quasi-monochromatic and polarized γ rays through the scattering of light on relativistic electrons has been proposed by the authors of this review.[4,5,26] The possibility of producing linearly polarized γ rays with this same process has also been put forward by Milburn.[29-30] It was shown that the γ rays produced in this way have an energy spectrum which differs decidedly from that of bremsstrahlung, and that under certain conditions monochromatic γ rays of considerable intensity can be produced. It has also been shown that the γ rays produced in this process will have a high degree of polarization. A detailed calculation of the degrees of linear and circular polarization of the photons produced in collisions of electrons with a "light target" has been made in [3].

In [1] it was shown that there is a possibility of producing polarized high-energy electrons in this same process of scattering light on relativistic electrons. These polarized electrons can in turn be used to produce polarized γ rays by their deceleration in matter. The peculiar effects that can occur in the production of γ rays by the process of scattering light of very high intensity on moving electrons have been elucidated in [25].

II. BREMSSTRAHLUNG AND PAIR PRODUCTION IN CRYSTALS

1. Qualitative Treatment of Interference Phenomena in Crystals

It is easy to understand the cause of the appearance of interference bremsstrahlung quanta produced in the passage of fast electrons through crystals in a qualitative way from the following elementary arguments. Suppose an electron with the energy $\mathscr{E}_1$ passes along a chain of atoms in a crystal with the lattice constant a (Fig. 1). The condition for coherent adding up of the radiation of frequency ω emitted at the angle θ_γ from the successive atoms can be written in the following form:

$$\frac{a}{v_2} - \frac{a\cos\theta_\gamma}{c} = \pm\frac{2\pi}{\omega}, \tag{2.1}$$

where v_2 is the speed of the electron after emitting the γ-ray quantum and c is the speed of light. The energy of the γ-ray quantum is $K = \mathscr{E}_1 - \mathscr{E}_2$, where $\mathscr{E}_2$ is the energy of the electron after the radiative act.

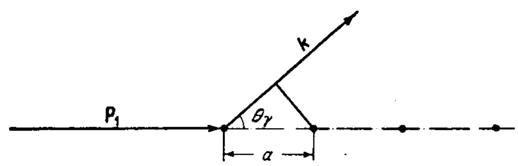

FIG. 1. For the derivation of the condition for coherent adding of amplitudes.

Using the fact that in the case of bremsstrahlung from fast electrons $\theta_\gamma \approx mc^2/\mathscr{E}_1$, we have from (2.1)

$$\frac{K}{\mathscr{E}_1\mathscr{E}_2}\left(\frac{\mathscr{E}_1}{\mathscr{E}_2} + \frac{\mathscr{E}_2}{\mathscr{E}_1}\right) = \frac{2\pi}{a^*}, \tag{2.2}$$

where a* is the lattice constant expressed in units of the electron Compton wavelength; the energies are in units of the rest energy of the electron. It follows that such phenomena can occur only for sufficiently high values of the electron energy. For example, for a copper crystal ($a^* = 3.61$ Å) and $K = 0.5\,\mathscr{E}_1$ we have $\mathscr{E}_1 \approx 75$ MeV. It also follows from the condition (2.2) that higher-order interferences can also occur for large values of K. Thus it is to be expected that at sufficiently high energies of the electrons one will observe against the background of the ordinary bremsstrahlung interference maxima and minima, of magnitudes decreasing as K increases.

The maximum length L at which the coherence of the radiation is retained will be

$$L \sim \lambda\mathscr{E}_1^2, \tag{2.3}$$

where $\lambda = 2\pi c/\omega$. For $L > a$ this means that several atoms will radiate coherently, and for $K \approx \mathscr{E}_1$ the number of these atoms will be

$$n_0 \approx \frac{2\pi}{a^*}\mathscr{E}_1, \tag{2.4}$$

which, for example, in the case of a copper crystal ($2\pi/a^* = 6.72 \times 10^{-3}$) and $\mathscr{E}_1 = 10^3$ gives $n_0 \approx 7$. This means that for the values of K in question the cross section for bremsstrahlung on such a group of atoms increases by about this same factor. Of course this estimate is valid only in order of magnitude, because a number of factors involved in a correct treatment, such as a more rigorous treatment of the interference itself, the presence of thermal vibrations of the atoms in the lattice, and screening, can considerably diminish the intensity of the radiation as compared with this estimate.

An analogous qualitative analysis can also be carried out in treating the process of bremsstrahlung on the atoms in a crystal in momentum space. In such a treatment the appearance of the interference can be understood from the fact that the amount of longitudinal momentum transferred to a nucleus is comparable with the reciprocal lattice constant. Since the matrix elements for bremsstrahlung and for pair production are identical, we must expect similar effects also in pair production in crystals. In this case the laws of conservation of momentum and energy are of the form

$$\left.\begin{aligned} \mathbf{q} &= \mathbf{K} - \mathbf{P}_+ - \mathbf{P}_-, \\ K &= \mathscr{E}_+ + \mathscr{E}_-, \end{aligned}\right\} \tag{2.5}$$

where $\mathbf{q}$, $\mathbf{K}$, $\mathbf{P}_+$, and $\mathbf{P}_-$ are respectively the recoil momentum of the nucleus and the momenta of the photon, positron, and electron, and $\mathscr{E}_+$ and $\mathscr{E}_-$ are the energies of the electron and positron. We write the law of conservation of momentum in the form

$$\left.\begin{aligned} q_{\parallel} &= K - P_+ \cos\theta_+ - P_- \cos\theta_-, \\ q_{\perp}^2 &= P_+^2 \sin^2\theta_+ + P_-^2 \sin^2\theta_- - 2P_+ \sin\theta_+ P_- \sin\theta_- \cos\varphi, \end{aligned}\right\} \tag{2.6}$$

where $q_{\parallel}$ and $q_{\perp}$ are the components of the momentum transferred to the nucleus along the direction of motion of the photon and in a plane transverse to this direction, θ_+ and θ_- are the angles of emission of the positron and electron relative to the direction of the photon, and φ is an azimuthal angle. Using the fact that for high-energy γ_{rays} $\theta_{\pm} \sim \mathscr{E}_{\pm}^{-1}$, we get

$$q_{\perp} \sim 1, \quad q_{\parallel} \sim 2\delta, \tag{2.7}$$

where

$$\delta = K - P_+ - P_- \approx \frac{K}{2\mathscr{E}_+\mathscr{E}_-} \tag{2.8}$$

is the minimum momentum transfer. For $\mathscr{E}_{\pm} = K/2$, δ is of the order of $1/K$. Thus $q_{\perp}$ will take values from δ to 1, and $q_{\parallel}$ will always be of the order of δ. This means that the momenta transferred to the nucleus will lie within a thin disk of thickness δ with radius of order 1 and with its center displaced from the origin in momentum space by a distance δ. In the calculation of the cross section for pair production the integration over recoil momenta thus reduces to an integration over the volume of this disk. It follows that for sufficiently high energies, when the thickness of the disk is very small, contributions to the cross section will be given by various numbers of points, depending on the size of the angle θ between the direction of the γ ray and the axis of the crystal. Therefore we can expect a variation of the cross sections for bremsstrahlung and pair production with the angle θ because of interference effects which will appear whenever the thickness of the disk becomes comparable with the reciprocal lattice constant $2\pi/a^*$ of the crystal, i.e., when

$$\frac{K}{2\mathscr{E}_+\mathscr{E}_-} \leqslant \frac{2\pi}{a^*}. \tag{2.9}$$

This condition is equivalent to the relation (2.2) for the case of bremstrahlung.

2. Theory

In treating the processes of bremsstrahlung and pair production in the Born approximation one writes the Fourier component $V(\mathbf{q})$ of the electrostatic field of the nucleus in the well known form

$$V(\mathbf{q}) = \int V(\mathbf{r}) \exp(i\mathbf{qr})\, d\tau. \tag{2.10}$$

In the case of a crystal $V(\mathbf{r})$ is replaced by

$$V_{\text{cr}} = \sum_{\mathbf{L}} V(\mathbf{r} + \mathbf{L}), \tag{2.11}$$

where $V(\mathbf{r} + \mathbf{L})$ is the potential of one atom and $\mathbf{L}$ is a lattice vector. Substituting V_{cr} in Eq. (2.10), we get the expression for the Fourier component in the case of a crystal

$$V_{\text{cr}}(\mathbf{q}) = \sum_{\mathbf{L}} \exp(i\mathbf{qL}) \int V(\mathbf{r}) \exp(i\mathbf{qr})\, d\tau, \tag{2.12}$$

so that the expressions for the cross sections for bremsstrahlung and pair production contain the additional factor

$$\left|\sum_{\mathbf{L}} \exp(i\mathbf{qL})\right|^2. \tag{2.13}$$

This is the factor that leads to the difference between the bremsstrahlung and pair production in crystals and the analogous processes in amorphous substances. It is similar to the Laue-Bragg factor in x-ray diffraction, and for a macroscopic crystal it can be represented in good approximation in the form

$$\frac{(2\pi)^3}{a^{*3}} N \sum_{\mathbf{g}} |F|^2 \delta(\mathbf{q} - 2\pi\mathbf{g}), \tag{2.14}$$

where $\mathbf{g}$ is a reciprocal lattice vector, N is the total number of atoms in the crystal, and $|F|^2$ is the structure factor of the crystal. Its value depends on the type of crystal; in the case of a simple cubic crystal it is equal to 1. Thus the square of the Fourier component of the field of the crystal, which is proportional to the cross section, can be written in the form

$$[V_{\text{cr}}(\mathbf{q})]^2 = [V(\mathbf{q})]^2 \frac{(2\pi)^3}{a^{*3}} N \sum_{\mathbf{g}} |F|^2 \delta(\mathbf{q} - 2\pi\mathbf{g}), \tag{2.15}$$

from which it follows that $V_{\text{cr}}(\mathbf{q})$ is different from zero only for $\mathbf{q} = 2\pi\mathbf{g}$, i.e., the recoil momentum of the nucleus must coincide with one of the points representing the crystal in momentum space.

Überall [44] has calculated the cross sections for bremsstrahlung and pair production in a crystal, taking into account screening and the thermal vibrations of the atoms of the lattice; the result is that the expression (2.13) is to be replaced by

$$|\sum_{\mathbf{L}} \exp[i\mathbf{q}(\mathbf{L}+\mathbf{U}_L)]|^2, \qquad (2.16)$$

where $\mathbf{U}_L$ is the thermal displacement of the atoms of the lattice, which, by changing the phase of the Laue-Bragg factor, leads to a partial suppression of the interference. $\mathbf{U}_L$ can be expressed in terms of the normal coordinates of the lattice, and then the averaging of (2.16) over the distribution of these coordinates for a given temperature T leads to the following expression:

$$\exp(-Aq^2)\,|\sum_{\mathbf{L}} \exp(i\mathbf{qL})|^2 + N[1-\exp(-Aq^2)], \qquad (2.17)$$

where

$$A = \frac{3m^2c^2}{4MK\Theta}\left[1+4\frac{T}{\Theta}\Phi\left(\frac{\Theta}{T}\right)\right],$$

$\Phi(\Theta/T)$ is a function tabulated by Debye,[20] Θ is the Debye temperature, and M is the mass of the nucleus. Thus inclusion of the thermal vibrations of the lattice leads to a decrease of the interference factor (2.13) and the appearance of an additional factor which has no relation to the interference. Table I gives the values of certain constants for crystals of diamond, copper, and platinum.

The total cross sections for these processes can finally be put in the form of sums

$$\sigma^{\mathrm{p}} = \sigma^{\mathrm{a}} + \sigma^{\mathrm{i}},$$

where σ^{a} is the cross section in the amorphous substance and σ^{i} is the interference part of the cross section, which arises owing to the crystal structure.

For bremsstrahlung:

$$\left.\begin{aligned}\sigma^{\mathrm{a}}_{\mathrm{b}} &= N\alpha Z^2 r_0^2 \frac{dK}{K\mathscr{E}_1^2}\left[(\mathscr{E}_1^2+\mathscr{E}_2^2)\psi_1^{\mathrm{a}}(\delta) - \frac{2}{3}\mathscr{E}_1\mathscr{E}_2\psi_2^{\mathrm{a}}(\delta)\right],\\ \sigma^{\mathrm{i}}_{\mathrm{b}} &= N\alpha Z^2 r_0^2 \frac{dK}{K\mathscr{E}_1^2}\left[(\mathscr{E}_1^2+\mathscr{E}_2^2)\psi_1^{\mathrm{i}}(\delta,\theta) - \frac{2}{3}\mathscr{E}_1\mathscr{E}_2\psi_2^{\mathrm{i}}(\delta,\theta)\right].\end{aligned}\right\} \qquad (2.18)$$

For pair production:

$$\left.\begin{aligned}\sigma^{\mathrm{a}}_{\mathrm{p}} &= N\alpha Z^2 r_0^2 \frac{d\mathscr{E}_+}{K^3}\left[(\mathscr{E}_+^2+\mathscr{E}_-^2)\psi_1^{\mathrm{a}}(\delta) + \frac{2}{3}\mathscr{E}_+\mathscr{E}_-\psi_2^{\mathrm{a}}(\delta)\right],\\ \sigma^{\mathrm{i}}_{\mathrm{p}} &= N\alpha Z^2 r_0^2 \frac{d\mathscr{E}}{K^3}\left[(\mathscr{E}_+^2+\mathscr{E}_-^2)\psi_1^{\mathrm{i}}(\delta,\theta) + \frac{2}{3}\mathscr{E}_+\mathscr{E}_-\psi_2^{\mathrm{i}}(\delta,\theta)\right].\end{aligned}\right\} \qquad (2.19)$$

The functions $\psi^{\mathrm{a}}_{1,2}(\delta)$ are given by Überall [44] in the forms

$$\left.\begin{aligned}\psi_1^{\mathrm{a}}(\delta) &= 4+4\int_\delta^1 [1-\exp(-Aq^2)](q-\delta)^2[1-F(q)]^2\frac{dq}{q^3},\\ \psi_2^{\mathrm{a}}(\delta) &= \frac{10}{3}+4\int_\delta^1 [1-\exp(-Aq^2)]\Big(q^3-6q\delta^2\ln\frac{q}{\delta}\\ &\qquad + 3\delta^2 q - 4\delta^3\Big)[1-F(q)]^2\frac{dq}{q^4},\end{aligned}\right\} \qquad (2.20)$$

where $F(q)$ is the atomic form-factor which takes screening into account:

$$F(q) = \frac{1}{1+(111qZ^{-1/3})^2};$$

the formulas differ from the corresponding Bethe-Heitler functions by the factor $1-\exp(-Aq^2)$, which arises owing to the Debye-Waller temperature factor $\exp(-Aq^2)$ and whose effect is to diminish the values by 10—20 percent.

The functions $\psi^{\mathrm{i}}_{1,2}(\delta,\theta)$ are of the forms[8]

$$\left.\begin{aligned}\psi_1^{\mathrm{i}}(\delta,\theta) &= \frac{(2\pi)^2}{a^{*3}}4\delta\sum_{\mathbf{g}}|F|^2\frac{\exp(-Ag^2)}{(\beta^{-2}+g^2)}\frac{g^2}{g_2^2\theta^2},\\ \psi_2^{\mathrm{i}}(\delta,\theta) &= \frac{(2\pi)^2}{a^{*3}}24\delta^2\sum_{\mathbf{g}}|F|^2\frac{\exp(-Ag^2)}{(\beta^{-2}+g^2)}\frac{g^2}{g_2^4\theta^4}\delta(g_2\theta-\delta),\end{aligned}\right\} \qquad (2.21)$$

where $|F|^2$ is the Debye-Bragg structure factor, $g_2 = \mathbf{gb}_2$ (Fig. 2), and $\beta = 111\,Z^{-1/3}$.

We note that the functions $\psi^{\mathrm{i}}_{1,2}(\delta,\theta)$ were obtained in Uberall's very first paper,[44] but in the approximation in which one assumes a continuous distribution of the points over the lattice planes of the crystal. This allowed him to replace the summation over these points by an integration over the recoil momenta.

Afterward, however, it was noted by the Diabrini group[8] that this simplification of the problem leads to inaccurate results. Because, as was noted above, in bremsstrahlung in a crystal the recoil momenta take values equal to reciprocal-lattice vectors, the summation has to be taken over just these values of the momentum. The functions $\psi^{\mathrm{i}}_{1,2}(\delta,\theta)$ of Eq. (2.21),

Table I. Some Constants for Crystals of Diamond, Copper, and Platinum

Element	Z	θ	$\frac{M}{m}$	a, Å	$A(0)$	$\frac{A(77)}{A(0)}$	$\frac{A(293)}{A(0)}$
Diamond	6	1860	12.07	3.56	108.4	1.0135	1.1916
Copper	29	315	63.57	3.61	121.1	1.3745	
Platinum . . .	78	225	195.23	3.92	55.2	1.6735	

expressed as such sums, are taken from [8], and they have also been used in this form in the later work of Überall.[46]

Theoretical curves of the intensity of bremsstrahlung in a diamond crystal, calculated by these authors [18] for electrons with energy 6 BeV and for various angles θ between the direction of motion of

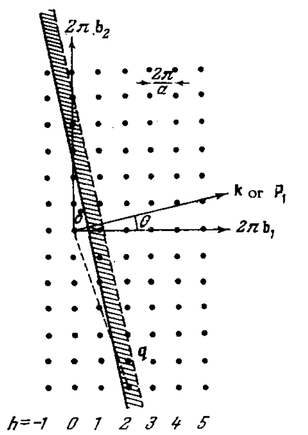

FIG. 2. The $\mathbf{b}_i$ are reciprocal-lattice vectors, $|\mathbf{b}_i| = 1/a^*$.

the electron and the crystal axis (110) (001), are shown in Figs. 3 and 4. Sharp peaks are observed in various regions of the bremsstrahlung spectrum, and the positions of the peaks depend on the value of the angle θ. With increase of the angle θ a peak is displaced toward larger values of the photon energy, but along with this its size is appreciably diminished. The position of the first-order interference maximum in the spectrum of the bremsstrahlung in a crystal is connected with the angle θ by the relation

$$\theta = \frac{x}{2\mathcal{E}_1(1-x)}\,\frac{a^*}{2\pi}, \qquad (2.22)$$

where $x = K/E_1$.

The fractional width of these maxima is $\Delta K/K \approx 25$—30 percent, and the enhancement of the peaks

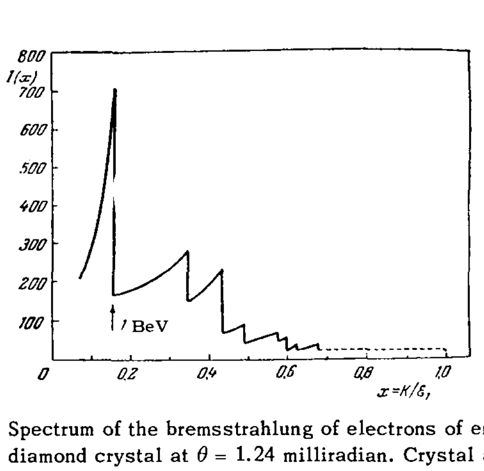

FIG. 3. Spectrum of the bremsstrahlung of electrons of energy 6 BeV in a diamond crystal at $\theta = 1.24$ milliradian. Crystal axis (110) (001). The intensity I(x) of the radiation is plotted as ordinate in arbitrary units.

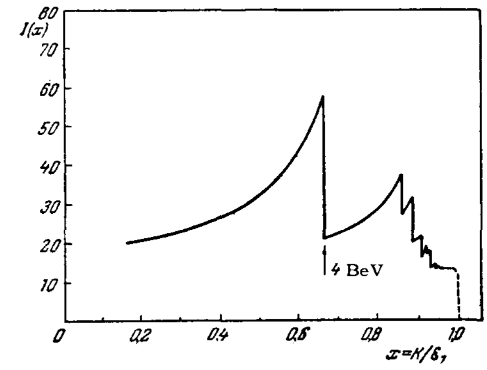

FIG. 4. Spectrum of the bremsstrahlung of electrons of energy 6 BeV in a diamond crystal at $\theta = 12.43$ milliradian.

over the corresponding values of the bremsstrahlung spectrum according to Bethe and Heitler varies from factors 30—40 for K = 1 BeV to 3—4 for K = 4 BeV.

3. The Polarization of the Radiation

In the bremsstrahlung of a high-energy electron the γ-ray quanta are emitted predominantly in a state of linear polarization lying in the plane determined by the direction of motion of the electron and the recoil momentum of the nucleus. In an amorphous substance the problem is completely symmetrical relative to the direction of motion of the electron, and, averaged over the azimuth, there is no polarization of the γ rays. In the case of a crystal the recoil momentum of the nucleus must coincide with a vector of the reciprocal lattice, and therefore the plane $(\mathbf{P}, \mathbf{a}_1)$ determined by the direction of motion of the electron and the axis of the crystal lattice is distinguishable from other planes, so that the process is not symmetrical around the direction of motion of the electron and the radiation is polarized. The polarization of the γ rays from bremsstrahlung in a crystal was first calculated by Überall.[45,46]

$$P = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel}, \qquad (2.23)$$

where $\sigma_\perp$ and $\sigma_\parallel$ are the cross sections for production of photons with the electric field vector perpendicular and parallel to the plane $(\mathbf{P}, \mathbf{a}_1)$. The results of the calculations give the following expression for the degree of polarization:

$$P = \frac{2(1-x)\,\psi_3^i(\delta, \theta)}{\Phi(x, \theta, \delta)}, \qquad (2.24)$$

$$\Phi(x, \theta, \delta) = [1+(1-x)^2]\,[\psi_1^a(\delta) + \psi_1^i(\delta, \theta)] - \frac{2}{3}(1-x)\,[\psi_2^a(\delta) + \psi_2^i(\delta, \theta)], \qquad (2.24a)$$

$$\psi_3^i(\delta, \theta) = \frac{(2\pi)^2}{a^{*3}}\,4\delta^3 \sum_g |F|^2 \frac{\exp(-Ag^2)}{(\beta^{-2}+g^2)^2}\,\frac{g_2^2 - g_3^2}{g_3^4\theta^4},$$

$$x = \frac{K}{\mathcal{E}_1}, \quad g_3 = g\mathbf{b}_3. \qquad (2.24b)$$

Figures 5 and 6 show the dependence of the degree of polarization on the energy of γ-ray quanta produced by electrons of energy 6 BeV moving at the angle θ with the (110) (001) axis of a diamond crystal, as calculated by the authors of [18]. As was to be expected, there is a sizable polarization of the γ rays in the regions of peaks of the bremsstrahlung spectrum. The polarization decreases with increase of the angle θ, and when the angle is increased by a factor ten the decrease in the polarization is by a factor two. It is remarkable that within a peak the degree of polarization is approximately proportional to the intensity of the radiation at each energy. Thus along with their quasi-monochromatic character the

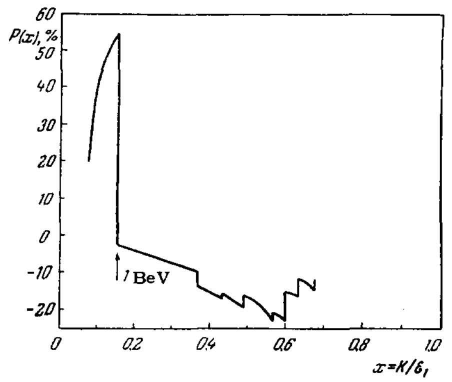

FIG. 5. Dependence of degree of polarization of γ-ray quantum on its energy, for $\mathscr{E}_1 = 6$ BeV, $\theta = 1.24$ milliradian (diamond crystal, axis (110) (001)).

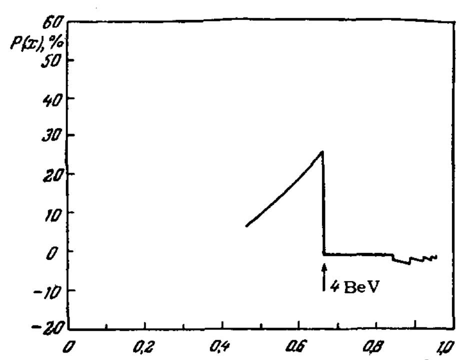

FIG. 6. Dependence of degree of polarization of γ-ray quantum on its energy for $\mathscr{E}_1 = 6$ BeV, $\theta = 12.43$ milliradian.

γ rays from bremsstrahlung in crystals have a rather high degree of linear polarization. Figure 7 shows the results of calculations [18] on the dependence of the position of the peaks of the radiation and its polarization on the energy of the photons, for cases of passage of electrons with energies 1.6 BeV and 40 BeV through a diamond crystal.

Making use of the fact that the magnitude of the cross section for pair production in crystals depends on the direction of the polarization of the γ rays

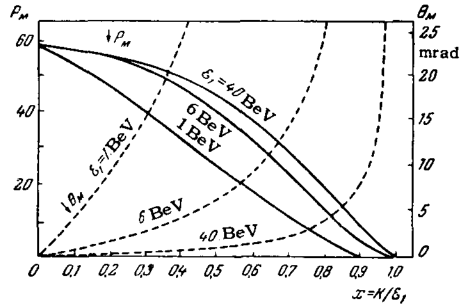

FIG. 7. Dependence of the position of the peaks in the bremsstrahlung spectrum and the degree of polarization on the energy of the photons, for passage of electrons of energies 1.6 BeV and 40 BeV through a diamond crystal.

relative to the plane (K, a_1), Cabibbo and others [14-16] have proposed another method for the production and analysis of polarized γ rays. The main absorption of high-energy photons is that owing to the production of electron-positron pairs. When an unpolarized beam of γ rays passes through a sufficiently thick crystal the beam of γ rays emerging from the crystal acquires a definite degree of polarization.

The dependence on polarization of the total cross section for absorption of γ rays, referred to unit volume of the crystal, can be represented in the following form:

$$\sum(\mathbf{e}) = A + B(\mathbf{e}, \mathbf{t})^2, \tag{2.25}$$

where $\mathbf{t}$ is a unit vector perpendicular to the plane containing the photon momentum $\mathbf{K}$ and the axis $\mathbf{a}_1$ of the crystal, and $\mathbf{e}$ is the polarization vector of the photon.

Photons that are polarized along the $\mathbf{t}$ direction will be absorbed with the mean free path $(A + B)^{-1}$, whereas photons polarized in the direction perpendicular to $\mathbf{t}$ will be absorbed with the mean free path A^{-1}.

Let $\Sigma^{\perp}$ and $\Sigma^{\parallel}$ be the respective cross sections for pair production by γ rays polarized along and perpendicular to the direction $\mathbf{t}$. The degree of polarization of an originally unpolarized beam of γ rays after passage through a crystal of thickness x is given by

$$P(x) = \operatorname{th}\left[\frac{1}{2}x(\Sigma^{\perp} - \Sigma^{\parallel})\right]; \tag{2.26*}$$

and the intensity of the beam is diminished by the factor

$$\frac{I(x)}{I(0)} = \exp\left[-E^{-1}\operatorname{th}^{-1}P(x)\right]\left[1 - P^2(x)\right]^{-\frac{1}{2}} \tag{2.27}$$

where

$$E = \frac{\Sigma^{\perp} - \Sigma^{\parallel}}{\Sigma^{\perp} + \Sigma^{\parallel}}. \tag{2.27a}$$

If the original γ rays had some linear polarization

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*th = tanh.

Q in a plane making the angle φ with the vector t, then the factor by which the beam is weakened will depend on φ in the following way:

$$\left[\frac{I(x)}{I(0)}\right]_{\mathbf{Q}} = \left[\frac{I(x)}{I(0)}\right]_{\mathbf{Q}=0} (1+|\mathbf{Q}|P(x)\cos 2\varphi). \quad (2.28)$$

By measuring the degree of weakening of a beam of γ rays at various values of φ one can determine Q; that is, a crystal can serve as an analyzer of the polarization of γ rays.

The efficiency of a crystal as a polarizer depends on the degree of polarization P(x) that can be attained with a reasonable size of the crystal and with minimum loss of the intensity of the primary γ-ray beam; that is, it depends on the values of $\Sigma^\perp$ and $\Sigma^\parallel$:

$$\frac{1}{2}(\Sigma^\perp + \Sigma^\parallel) = \Sigma_p, \quad (2.29)$$

where $\Sigma_p = \Sigma^a + \Sigma^i$ is determined from Eq. (2.19), and

$$\Sigma^\perp - \Sigma^\parallel = -\frac{(2\pi)^3}{a^{*3}} N \sum_{\mathbf{g}} |F|^2 \exp(-A|\mathbf{g}|^2) \times M(\mathbf{K}, \mathbf{g})|\mathbf{g}^\perp|^2 \cos 2\theta, \quad (2.30)$$

where $|\mathbf{g}^\perp|^2$ is the square of the component of g perpendicular to K, θ is the angle between the vectors g and t, and

$$M(\mathbf{K}, \mathbf{q})|\mathbf{q}^\perp|^2 = \frac{2\alpha Z^2 r_0^2}{\pi d^2 K q^4}\,\frac{4q^2K^2-d^2}{d^2} \times \{\widetilde{v}[2m_e^2(q^2-d^2)-q^4] - 4m_e^2(q^2+m_e^2)\ln\varrho\},$$

$$d = 2(\mathbf{K}, \mathbf{q}), \quad \widetilde{v} = \sqrt{1-\frac{4m_e^2}{d-|\mathbf{q}|^2}}, \quad \varrho = \frac{1+\widetilde{\vartheta}}{1-\widetilde{\vartheta}}. \quad (2.31)$$

Figure 8 shows the dependence of the factor by which the beam of γ rays is weakened on the degree of polarization produced, for various values of E. A considerable degree of polarization ($\sim$ 33 percent) with weakening of the beam by a factor 10^3 is obtained with $E \gtrsim 0.05$. When a crystal is used as an analyzer it is sufficient to have values $P \sim 5$ percent, which occurs for $E \approx 0.01$.

A typical dependence of the value of E on the angle at which γ rays with energy 6 BeV pass through the crystal is shown in Fig. 9. Figure 10 shows the dependence of E on the energy of the γ rays for the optimum value of the angle θ. From these data it follows that this method for producing and analyzing polarized beams of γ rays is most effective for large energies ($\gtrsim$ 6 BeV). It will be especially valuable in the analysis of the polarization of γ rays at very high energies, since at these energies other methods of analysis (measurements of the azimuthal assymmetry in pair production [32] and the photoproduction of π^0 mesons [17]) cannot be applied. We note that the passage of high-energy γ rays through thick crystals will result in the production of electron-photon

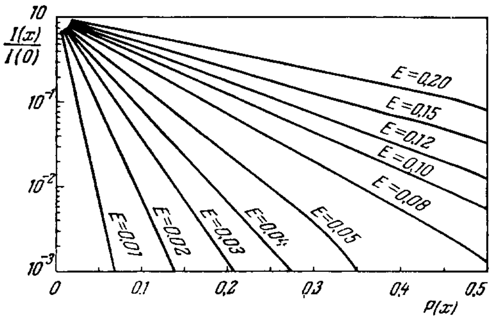

FIG. 8. Dependence of the factor by which an unpolarized beam of γ rays is weakened, I(x)/I(0), on the final degree of polarization, for various values of $E = (\Sigma^\perp - \Sigma^\parallel)/(\Sigma^\perp + \Sigma^\parallel)$.

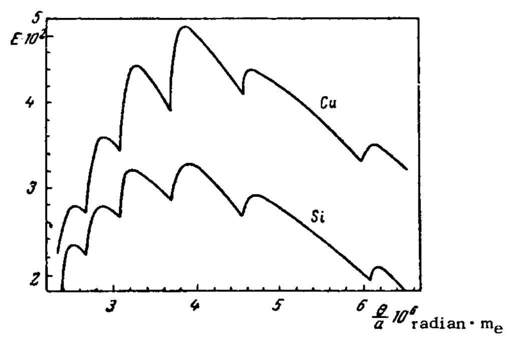

FIG. 9. Dependence of E on θ for K = 6 BeV.

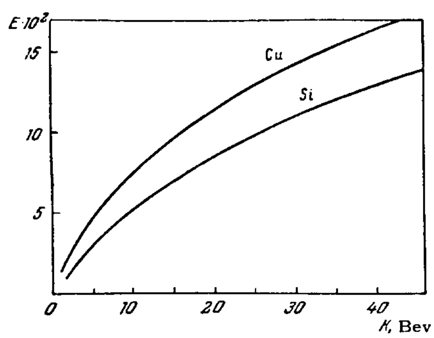

FIG. 10. Dependence of the value of E obtained with the optimum angle on the energy of the γ-ray quanta, for crystals of silicon and copper.

showers, which will require the use of suitable collimation of the beam of γ rays emerging from the crystal.

An interesting consequence of these effects that appear in the passage of γ rays through crystals is double refraction. This opens up the possibility of producing circularly polarized γ rays by means of single crystals. For example, there is a possibility of using a crystal of suitable thickness as the analog of an optical quarter-wave plate, which will allow conversion of linearly polarized radiation into circularly polarized radiation, and conversely.

Let us consider a cubic crystal of thickness x, and incident on it a photon with momentum K which is in the (001) plane and makes the angle θ with the axis of the crystal, (110). We represent the polarization vector $\mathbf{e}$ of the photon as a combination of two vectors $\mathbf{t}$ and $\mathbf{y}$ ($\mathbf{y}$ is a unit vector perpendicular to $\mathbf{t}$ and directed along the axis $\mathbf{a}_2$) which are respectively in and orthogonal to the plane (001):

$$\mathbf{e} = e_1\mathbf{t} + e_2\mathbf{y}; \tag{2.32}$$

We can regard $\mathbf{e}$ as a two-component vector which is connected with the amplitude of the radiation emerging from the crystal by the following two-rowed diagonal matrix:

$$\begin{pmatrix} \exp[in^{\parallel}(K, x)Kx] & 0 \\ 0 & \exp[in^{\perp}(K, x)Kx] \end{pmatrix}. \tag{2.33}$$

The quantities $n^{\parallel}$ and $n^{\perp}$ are like indices of refraction in ordinary optics. The crystal will act as a quarter-wave plate if the relative phase change of the two exponentials is equal to $\pi/2$; that is, the condition that must hold is $\mathrm{Re}\,(n^{\perp} - n^{\parallel})\,\omega x = \pi/2$.

Calculations of the real part of the difference of the indices of refraction $n^{\perp}$ and $n^{\parallel}$ made by the authors of [16] give the following result:

$$\mathrm{Re}\,(n^{\perp} - n^{\parallel}) = \Sigma_{\alpha} f(K, \theta, \beta), \tag{2.34}$$

where

$$f(K, \theta, \beta) = \frac{1}{8\pi}\frac{1}{K^2}K(\mathbf{q})\left\{\left[\left(1-\frac{4}{\beta}\right)^{1/2} + \frac{2}{\beta}\ln\frac{1+\left(1-\frac{4}{\beta}\right)^{1/2}}{1-\left(1-\frac{4}{\beta}\right)^{1/2}}\right]^2 - \frac{4\pi^2}{\beta^2} + \left[\left(1+\frac{4}{\beta}\right)^{1/2} - \frac{2}{3}\ln\frac{\left(1+\frac{4}{\beta}\right)^{1/2}+1}{\left(1+\frac{4}{\beta}\right)^{1/2}-1}\right]^2\right\}, \quad \beta > 4; \tag{2.34a}$$

$$f(K, \theta, \beta) = \frac{1}{8\pi}\frac{1}{K^2}K(\mathbf{q})\left\{-\left[\left(\frac{4}{\beta}-1\right)^{1/2} - \frac{4}{\beta}\operatorname{ctg}^{-1}\left(\frac{4}{\beta}-1\right)^{1/2}\right]^2 + \left[\left(1+\frac{4}{\beta}\right)^{1/2} - \frac{2}{\beta}\ln\frac{\left(1+\frac{4}{\beta}\right)^{1/2}+1}{\left(1+\frac{4}{\beta}\right)^{1/2}-1}\right]^2\right\}, \quad \beta \leqslant 4; \tag{2.34b*}$$

$$K(\mathbf{q}) = \frac{8\alpha Z^2 r_0^2}{\pi}|F|^2\frac{N}{a^{*3}}(2\pi)^3\frac{b^4\exp(-Aq^2)}{(1+b^2q^2)^2}\cos 2\varphi\,\frac{4q^2K^2-\beta^2}{\beta^2}. \tag{2.34c}$$

The summation in this formula is extended over all points of the reciprocal lattice for which $(\mathbf{Kq}) > 0$.

Figure 11 shows the dependence of $\mathrm{Re}\,(n^{\perp} - n^{\parallel})$ on the angle of passage θ for $K = 6$ BeV. It is curious that $\mathrm{Re}\,(n^{\perp} - n^{\parallel})$ reaches its maximum values at the maxima of $dE/d\omega$. Table II shows the thicknesses of a copper crystal required for it to act as a quarter-wave plate, calculated for various energies and for the optimal angles of passage of the γ rays through the crystal.

It is seen that again the method is most effective for large energies of the γ_{rays}. Already at a γ-ray energy of 6 BeV a small crystal can be a unique device for transforming the type of polarization.

*ctg = cot.

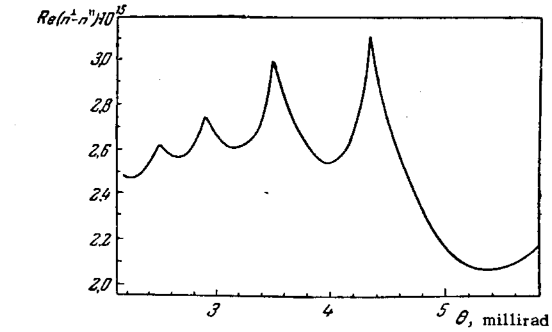

FIG. 11. The dependence of $\mathrm{Re}\,(n^{\perp} - n^{\parallel})$ on θ for $K = 6$ BeV.

Table II

K, BeV	θ, millirad	$\mathrm{Re}\,(n^{\perp}-n^{\parallel})\cdot 10^{15}$	x, cm
1	28	2.62	11.5
6	3.7	2.74	1.84
40	0.46	2.67	0.273

4. Experimental Investigations

The first attempt to measure experimentally the intensity of bremsstrahlung in a crystal was made by Panofsky and Saxena [33] with an electron accelerator at Stanford, using an electron energy of 600 MeV. The experiment was made with a single crystal of silicon of thickness 0.03 cm. By means of a goniometric system the crystal could be turned around two axes perpendicular to the electron beam. The bremsstrahlung photons produced in the crystal were detected by means of the photoproduction of π^+ mesons in a polyethylene target. A change of the orientation of the crystal relative to the direction of the beam did not lead to any appreciable change of the intensity of the bremsstrahlung. This negative result caused the authors to explain it in terms of inaccuracies in the approximations used by Überall in his calculations, with which the results of the experiment were compared. On checking the approximations, however, Schiff [37] found them to be correct. The negative result obtained by Panofsky and Saxena can evidently be explained by the fact that the registration of the π^+ mesons produced by the γ rays does not provide direct information about the shape of the bremsstrahlung spectrum.

In the very next experimental work [24] Frisch and Olsen convincingly demonstrated that there is an interference effect in the bremsstrahlung in a crystal. The experiment was made with an electron beam of energy 1 BeV from the Cornell synchrotron. The germanium crystal used was placed in the vacuum chamber of the accelerator, and at the end of the cycle of acceleration the electron beam was turned onto the edge of the crystal in the direction of its (110) axis. The beam of bremsstrahlung γ rays produced was collimated (1 milliradian) and charged particles were removed from it with a magnetic field; then, after passing through a 3 mm lead plate the beam came to two plastic scintillators S and H with 5 cm of lead placed between them. In this lead the high-energy γ-ray quanta produced electron-photon showers, which were registered by the counter H. The angle between the axis of the crystal and the direction of motion of the electron beam was varied during the experiment. The ratio of counts S/H was found to depend on this angle, as is shown in Fig. 12. This fact was a first qualitative indication of the existence of interference effects in the crystal for quanta with energies ~ 100 MeV.

A systematic experimental investigation of the radiation produced in crystals is being made with the electron accelerator in Frascatti.

In [12] a silicon crystal of thickness 2.7×10^{-3} radiation units was placed inside the vacuum chamber of the accelerator, in such a way that the electron beam of energy 1 BeV travelled in the direction of the (111) axis of the crystal. Being mounted on a goniometric system, the crystal could be rotated around vertical and horizontal axes perpendicular to the direction of motion of the electron beam, so that the axis of the crystal could be set at an angle θ with the direction of motion of the incident electrons to an accuracy of 0.5 milliradian. After emerging from the crystal the beam of bremsstrahlung γ quanta (3×10^9 equivalent quanta per minute) was collimated (to 8×10^{-4} rad) and struck the aluminum converter of an electron-pair γ-ray spectrometer.

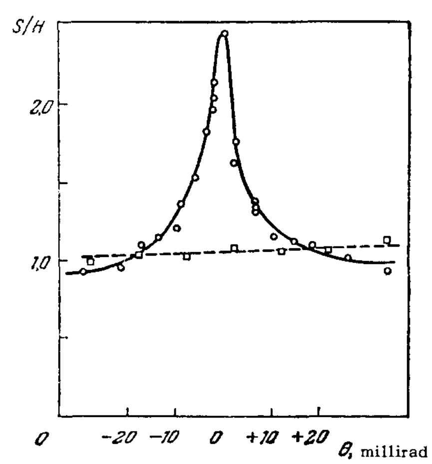

FIG. 12. Dependence of the measured ratio of the numbers of soft and hard photons (S/H) on the angle θ. (θ is the angle between the direction of motion of the electrons and the horizontal axis.) Circles indicate values with $\psi = 0$ (ψ is the angle between the direction of motion of the electrons and the vertical axis. Rectangles indicate values with $\psi = 20$ milliradians.

The number $N(K, \theta)$ of symmetrical electron-positron pairs produced by γ rays with energies up to 900 MeV was registered as a function of the angle θ. The experiment provided measurements of the ratio

$$I_{\max}(\theta) = \frac{N(K, \theta)\,\sigma_p(K^0)}{N(K^0, \theta)\,\sigma_p(K)}, \tag{2.35}$$

where $\sigma_p(K)dK$ is the cross section for production of a symmetrical pair by photons with energy between K and K + dK. In this ratio the quantity $N(\theta, K^0)/\sigma_p(K^0)$, with $K^0 = 900$ MeV, is a convenient normalizing factor, since according to the theory its dependence on the angle θ is very slight. In Fig. 13 the experimental and theoretical values of $I(\theta)$ are compared for K = 80 MeV. Some deviations of $I_{exp}(\theta)$ from the theoretical values can be explained by the fact that in the experiment the photon beam was severely collimated, whereas the theoretical curves are the result of an integration over all angles of the emitted photons. It must also be noted that for the theoretical curve in Fig. 13 the authors of [12] used the first calculations of Überall, [44] which are only approximate. The large difference in the depths of the minima at $\theta = 0$ in Fig. 13 is evidently to be ascribed to the insufficient angular resolution of the goniometric system (the distance between the peaks is 2 milliradians).

Precision measurements of $I(\theta)$ in a diamond crystal, which gave the results shown in Fig. 14 for photon energy K = 150 MeV, were made later by the same authors, [18] using a goniometer of high accuracy. This made it possible to obtain a detailed resolution of the structure of the dependence of the intensity of the radiation on the angle θ.

Subsequently this same group [8] studied the spectrum of the bremsstrahlung of electrons in a diamond crystal which, having a high value of the Debye temperature, a small lattice constant (a = 3.56 Å), and

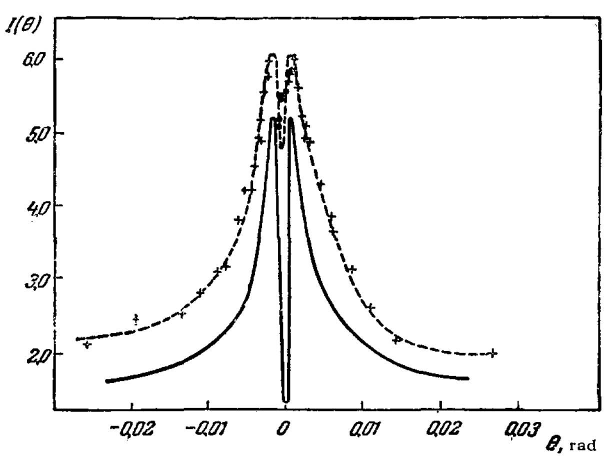

FIG. 13. Dependence of the intensity of bremsstrahlung γ rays of energy K = 80 MeV (K° = 865 MeV) on the angle between the direction of motion of the electrons, of energy 1 BeV, and the axis (111) of the silicon crystal. The solid line is theoretical, the dashed line experimental.

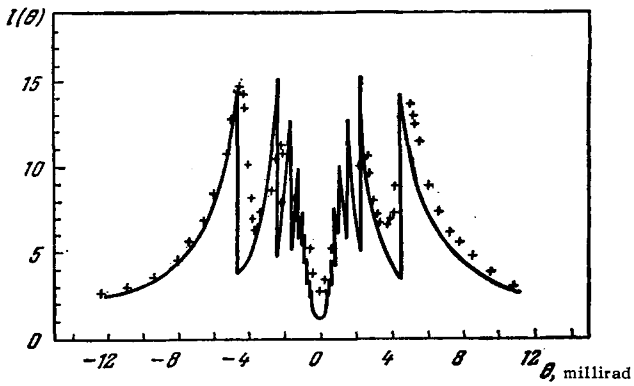

FIG. 14. Dependence of the intensity of bremsstrahlung γ rays of energy K = 150 MeV on the angle between the direction of motion of the 1 BeV electrons and the axis of the diamond crystal. The solid curve is theoretical, and the crosses are experimental.

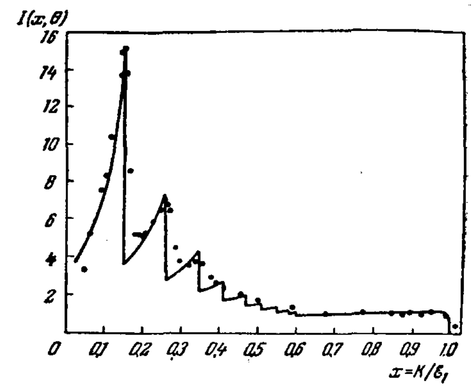

FIG. 15. Spectrum of the bremsstrahlung from 1 BeV electrons in a diamond crystal, for θ = 4.6 millirad (angle between direction of motion of the electrons and the (110) crystal axis). The solid curve is calculated theoretically from the formula (2.18).

a comparatively simple structure, is extremely convenient for such experiments. A diamond single crystal was used, in the shape of a parallelepiped of dimensions $10 \times 5 \times 2$ mm, the long side having Miller indices (110). The electrons were incident on the diamond at small angles θ with the (110) axis. As in the preceding experiments, the crystal could be rotated around vertical and horizontal axes, the goniometer being one with high accuracy (0.1 millirad). In this case the angle of collimation of the γ rays was decreased to the value 0.3 millirad to exclude γ-ray quanta emitted by electrons which had suffered appreciable multiple Coulomb scattering. With this collimation the intensity of the beam of γ rays measured by means of a Wilson radiation meter was $\sim 10^9$ equivalent quanta per minute. The symmetrical electron-positron pairs produced when these γ rays struck the aluminum converter, of thickness 1.1×10^{-2} radiation units, of the electron-pair γ- ray spectrometer were detected by two telescopes of scintillators connected in a scheme of fast-slow coincidences. The accuracy of the measurements of the energy of the γ-ray quanta was $\Delta K/K$ = 8.5 percent, and the error in the determination of their mean energy was $\delta K/K$ = 0.3 percent. The quantity measured experimentally was the relative intensity $I(K, \theta)$ [see Eq. (2.35)], the only difference being that a correction factor $f(K)$ was introduced to allow for the loss of counts of symmetrical pairs owing to multiple scattering of electrons in the converter.

Figure 15 shows the experimental results of measurements of the spectrum of the bremsstrahlung from electrons of energy 1 BeV, for θ = 4.6 millirad. At γ-ray energies $\sim$ 150 MeV there was a sharp rise in the spectrum, exceeding the corresponding value in the Bethe-Heitler spectrum by more than a factor ten.

In the vicinity of 250 MeV there is a peak which, while less sharp, also markedly exceeds the corresponding value in the ordinary bremsstrahlung spectrum.

The width of the main peak agrees well with the theoretically calculated width $\Delta K/K \approx 0.3$. Figure 16, shows the same data for two other values of the angle of passage of the electrons in the crystal, and Fig. 16, b shows the difference of the intensities for these two angles. These results show that by studying the physical processes for two different angles one can single out reactions associated with the γ rays of a fixed energy K with $\Delta K/K \approx 0.3$.

Calculations show [31] that much smaller values, $\Delta K/K \approx 0.1-0.01$, can be attained by decreasing the target thickness to 10^{-4} radiation units and making a more severe selection in the angles of passage of the electrons in the crystal and the collimation of the γ-ray beam emerging from it. These angles should be fixed in the experiment to accuracy ~ 0.1

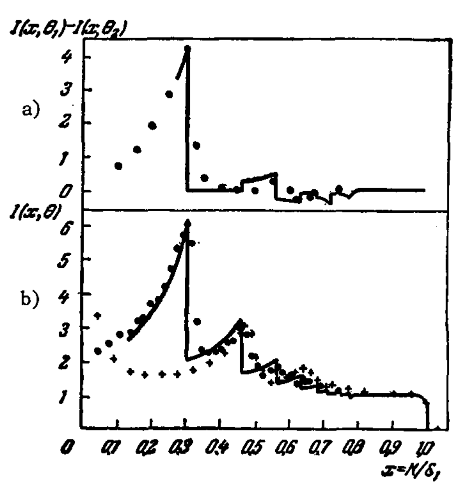

FIG. 16. a) Spectrum of the bremsstrahlung of electrons of energy $\mathscr{E}_1$ = 1 BeV in a diamond crystal, for $\theta_1 = 22.9 \pm 0.1$ millirad (circles) and $\theta_2 = 11.3 \pm 0.1$ millirad (crosses); b) difference of these two spectra. The solid curves are those calculated theoretically for these two angles.

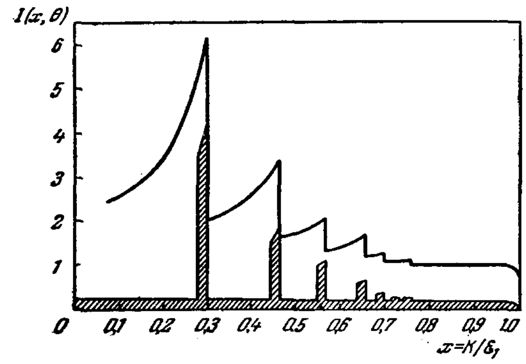

FIG. 17

$mc^2/\mathcal{E}_1$. The shape of the bremsstrahlung spectrum for this case, for values of the angle of passage of the electrons in the crystal $\theta = 22$ millirad and of the angle of emergence of the γ rays $\theta = 0.35$ millirad, are shown in Fig. 17 for electrons with energy 6 BeV (shaded area).

Up to the present there has only been one research [9] on the measurement of the degree of linear polarization of the bremsstrahlung γ rays in a diamond crystal. According to Olsen and Maximon [32] linearly polarized γ rays produce electron-positron pairs predominantly in the plane of polarization. Let us introduce the coefficient of azimuthal asymmetry

$$R = \frac{d\sigma_{\parallel}^{p} - d\sigma_{\perp}^{p}}{d\sigma_{\parallel}^{p} + d\sigma_{\perp}^{p}}, \tag{2.36}$$

where $d\sigma_{\parallel}^{p}$ and $d\sigma_{\perp}^{p}$ are the cross sections for production of pairs in planes parallel and perpendicular to the plane of polarization. The calculated [32] dependence of R on the energies of the components of the pair shows that R has its maximum value for the case of equal division of the energy in the pair. In the measurement of the azimuthal asymmetry of pair production it is essential that the angle of divergence of the pair exceed the mean value of the angle caused by multiple scattering of the electron and positron in the converter of the γ-ray spectrometer. It was for this very reason that in the experiment of [9] a thin converter of thickness 10^{-4} radiation units was chosen. Figure 18 shows the scheme of registration of symmetrical electron-positron pairs produced by 150 MeV γ rays. The pairs were registered by the scintillation counters S_1A_1 and A_2A_3; the vertical dimension of counter S_1 was 0.5 cm, and that of A_1, A_2, and A_3 was 10 cm. Coincidences $S_1A_2A_3$ and $A_1A_2A_3$ were registered. The ratio of the counts

$$\varrho = \frac{S_1A_2A_3}{A_1A_2A_3} \tag{2.37}$$

is connected with the degree of polarization of the γ rays by the following relation:

$$\varrho = \varrho_0(1 + RP), \tag{2.38}$$

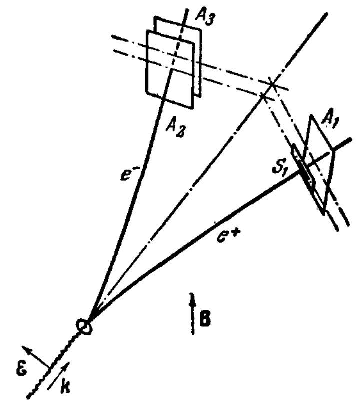

FIG. 18

where ρ_0 is the value of the ratio (2.37) for unpolarized photons. For γ rays of energy 150 MeV and the geometry of the experiment in question $R \approx 10.1$ percent. Figure 19 shows the results of measurements of the degree of linear polarization of the γ rays; these results are in satisfactory agreement with the theoretical calculations made by the same authors.

Earlier these same authors [13] made experimental measurements of the influence of crystal structure on the production of electron-positron pairs. After collimation (to 0.8 millirad) photons from the bremsstrahlung of electrons with energy 1 BeV were directed against a single crystal of silicon, serving as the converter of a γ-ray spectrometer, at a small angle θ with the (100) axis. The quantity measured was the ratio

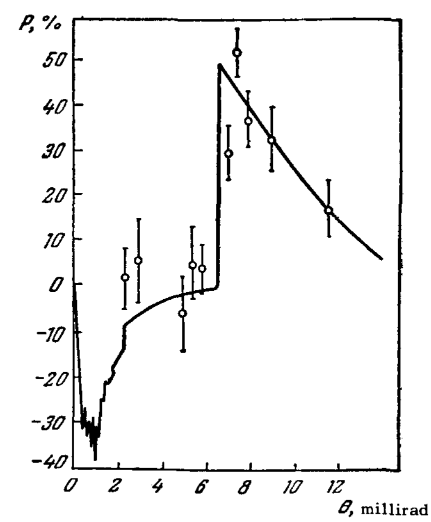

FIG. 19. Polarization of photons with energy $K \approx 150$ MeV from the bremsstrahlung of electrons of energy $\mathcal{E}_1 = 1$ BeV in a diamond crystal. θ is the angle between the momentum of the electron and the axis (110) of the crystal. The experimental points are normalized to fit the theoretical curve (solid line) at $\theta \approx 11.6$ milliradian.

$$\xi(\theta)=\frac{N(\theta)-N(0)}{N(0)} \tag{2.39}$$

as a function of θ for symmetrical pairs with total energy 910 MeV. The results of the experiment are in agreement with the calculations of Überall.[44]

In conclusion we note that the degree of linear polarization of γ rays with energies larger than 1 BeV can be effectively analyzed by means of a γ-ray spectrometer with a thin crystal converter. The authors of [6] have calculated the degree of azimuthal asymmetry in the production of symmetrical electron-positron pairs in a crystal target by linearly polarized γ rays.

III. THE COMPTON EFFECT ON A MOVING ELECTRON

1. Production of High-energy γ Rays by the Scattering of Light on Relativistic Electrons

An interesting property of the Compton effect on a moving electron is that the scattering process can produce a photon harder than the incident photon. Owing to this, even in the case of the scattering of optical photons on extremely relativistic electrons, scattered photons will have energies comparable with that of the electrons. This feature of Compton scattering is of practical interest from the point of view of the production of beams of hard γ rays with high-energy electron accelerators. It is important to note that the characteristics of such γ-ray beams will differ sharply from those of ordinary beams produced by the bremsstrahlung of electrons in matter.

In the Compton effect on a moving electron the energy ω_2 of the scattered quantum is connected with that of the incident one by the well known relation ($\hbar = c = 1$):

$$\omega_2=\omega_1\frac{1-v_1\cos\theta_1}{1-v_1\cos\theta_2+\frac{\omega_1}{\mathscr{E}_1}(1-\cos\theta)}, \tag{3.1}$$

where v_1, $\mathscr{E}_1$ are the speed and energy of the electron, θ_1, θ_2 are the respective angles between the direction of motion of the electron and the incident and scattered photons, and θ is the angle between the incident and scattered photons. The maximum energy $\omega_{2\max}$ of the scattered photons will occur in the case in which the primary electron and photon move in opposite directions ($\theta_1 = \pi$) and the scattered photon moves in the direction of the electron's motion. Then ($v_1 \approx 1$)

$$\omega_{2\max}=\frac{2\omega_1}{\frac{1}{2}\left(\frac{m}{\mathscr{E}_1}\right)^2+\frac{2\omega_1}{\mathscr{E}_1}}, \tag{3.2}$$

where m is the rest energy of the electron.

To produce beams of γ rays by this method it is of course necessary to have intense fluxes of primary photons. One intense source of monochromatic photons is the laser. At present the most highly developed laser systems are those using ruby crystals as the active substance.

The maximum energy achievable at present with electron accelerators is 6 BeV (Cambridge). When photons from a ruby laser ($\omega_1 = 1.78$ eV) are scattered by electrons of this energy we have $\omega_{2\max} = 848$ MeV. This effect increases rapidly with increase of the electron energy. Thus for $\mathscr{E}_1 = 40$ and 500 BeV we have the respective values $\omega_{2\max} = 21$ and 497 BeV when the same ruby-laser photons are used.

Of course the use either of radiation from lasers operating in a more short-wave part of the spectrum or of other sources of harder photons will mean that the scattering will produce photons with energies extremely close to that of the primary electrons. The differential cross section for the scattering of unpolarized photons on a moving electron is given by

$$d\sigma=r_0^2\frac{2}{m^2x_1^2}\left[4\left(\frac{1}{x_1}+\frac{1}{x_2}\right)^2-4\left(\frac{1}{x_1}+\frac{1}{x_2}\right)-\left(\frac{x_1}{x_2}+\frac{x_2}{x_1}\right)\right]\omega_2^2\,d\Omega_2, \tag{3.3}$$

where r_0 is the classical electron radius. For the case $\theta_1 = \pi$, x_1 and x_2 are given by the following expressions:

$$x_1=-\frac{2\omega_1}{m^2}(\mathscr{E}_1+P_1),\qquad x_2=\frac{2\omega_2}{m^2}(\mathscr{E}_1-P_1\cos\theta_2), \tag{3.4}$$

where P_1 is the momentum of the initial electron.

By using (3.1) we can get from the expression (3.3) the energy spectrum of the scattered photons[5,26]

$$d\sigma=\frac{\pi r_0^2}{2}\frac{m^2}{\omega_1\mathscr{E}_1^2}\left\{\frac{m^4}{4\omega_1^2\mathscr{E}_1^2}\left(\frac{\omega_2}{\mathscr{E}_1-\omega_2}\right)^2-\frac{m^2}{\omega_1\mathscr{E}_1}\left(\frac{\omega_2}{\mathscr{E}_1-\omega_2}\right)+\frac{\mathscr{E}_1-\omega_2}{\mathscr{E}_1}+\frac{\mathscr{E}_1}{\mathscr{E}_1-\omega_2}\right\}d\omega_2, \tag{3.5}$$

where ω_2 can vary from the energy ω_1 of the primary photons to $\omega_{2\max}$.

This energy distribution of the scattered photons differs decidedly from the corresponding distribution for bremsstrahlung, which is of the form $d\sigma \sim d\omega/\omega$. If the primary photons belong to the optical part of the spectrum and $\mathscr{E}_1 \approx 6$ BeV, the distribution in the region $\omega_2 \geq 0.5$–$0.6\,\omega_{2\max}$ is very roughly of the form $\sim \omega_2 d\omega_2$, and most of the intensity of the radiation is concentrated near $\omega_{2\max}$ (Fig. 20).

Another, no less interesting, feature of the distribution in question appears for $\omega_{2\max} \to \mathscr{E}_1$. In this case a larger and larger fraction of the scattered photons is concentrated near $\omega_{2\max}$. For $\omega_{2\max} \approx \mathscr{E}_1$ the γ rays produced are fairly monochromatic in energy. A maximum γ-ray energy $\omega_{2\max} \to \mathscr{E}_1$ can be achieved by increasing either ω_1 or $\mathscr{E}_1$. For example, it can be seen from Fig. 21 that already at $\omega_1 = 178$ eV a satisfactory degree of monochro-

Table III

$\mathscr{E}_1$, BeV	λ	ω_{2max}, MeV	P_{max}	$\sigma_{1/2}$, mb	$\mathscr{E}_1$, BeV	λ	ω_{2max}, MeV	P_{max}	$\sigma_{1/2}$, mb
1.02	0.014	28	1.00	320	6.21	0.085	903	0.99	280
2.92	0.040	216	1.00	310	6.57	0.090	$1.00 \cdot 10^3$	0.99	280
4.16	0.057	426	0.99	300	8.76	0.120	$1.69 \cdot 10^3$	0.98	260
4.60	0.063	515	0.99	229	11.69	0.160	$2.83 \cdot 10^3$	0.96	250
5511	0.070	628	0.99	290	20.8	0.285	$7.55 \cdot 10^3$	0.91	220
5.48	0.075	715	0.99	290	41.6	0.570	$22.1 \cdot 10^3$	0.77	180
5.84	0.080	806	0.99	280	58.4	0.800	$35.9 \cdot 10^3$	0.67	160

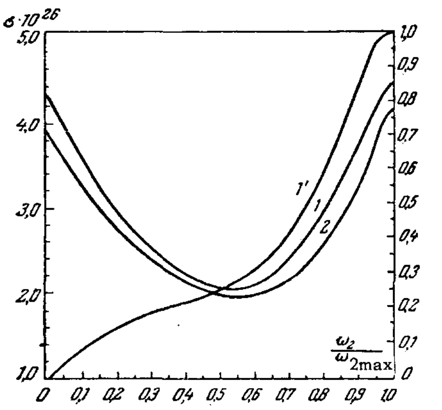

FIG. 20. The spectrum of scattered photons. 1 – $\mathscr{E}_1 = 6$ BeV, $\omega_1 = 1.78$ eV, $\omega_{2max} = 848$ MeV; 2 – $\mathscr{E}_1 = 6$ BeV, $\omega_1 = 3.56$ eV, $\omega_{2max} = 1.48$ BeV; curve 1′ (right-hand scale) is the spectral distribution of the intensity for case 1.

maticity is achieved. For $\omega_1 = 127.8$ keV the half-width of the distribution at $\omega_{2max} \approx 6$ BeV is of the order of 1 percent. Monochromatic γ rays are also produced when red light is scattered on electrons with an energy of several hundred BeV. Table III shows the values of $\lambda = \omega_1'/m$ ($\omega_1' = 2\omega_1 E_1/m$), the frequency of the scattered photon in the system

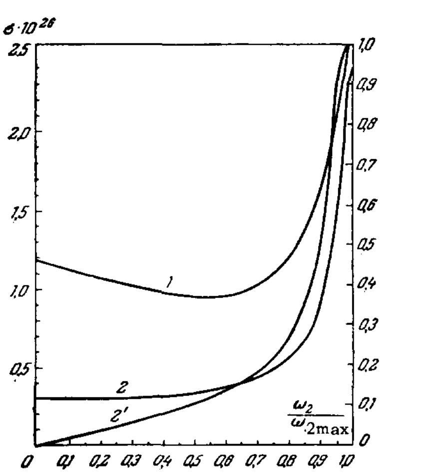

FIG. 21. Spectrum of scattered photons. 1 – $\mathscr{E}_1 = 6$ BeV, $\omega_1 = 35.6$ eV, $\omega_{2max} = 4.58$ BeV; 2 – $\mathscr{E}_1 = 6$ BeV, $\omega_1 = 178$ eV, $\omega_{2max} = 5.64$ BeV; curve 2′ (right-hand scale) is the spectral distribution of the intensity for case 2.

in which the electron is at rest, ω_{2max}, and $\sigma_{1/2}$ (the scattering cross section for production of photons with energies above $\omega_{2max}/2$) calculated by Milburn [29] for the case of scattering of the light from a ruby laser on electrons of various energies.

It must be emphasized that in this case, unlike that of bremsstrahlung, the energy of the γ rays formed in the scattering is a definite function of the angle at which they emerge relative to the direction of motion of the primary electron.

Characteristic values of the angle of the scattered γ rays are $\theta_2 \approx m/\mathscr{E}_1$ for energies $\omega_2 \approx \mathscr{E}_1$, and $\theta_2 \approx 2(\omega_1/\omega_2)^{1/2}$ for $\omega_1 \ll \omega_2 \ll \mathscr{E}_1$. The exact expression for the angular distribution can be obtained from (3.3) by using the relation (3.1).

The single-valued relation between the energy of the scattered photon and the direction in which it is emitted makes it possible to select γ rays with a definite energy. The size of the angle within which the γ rays of a required energy are selected is limited by the amount of angular spread in the primary electron beam. In electron accelerators the aperture of the beam is $\sim 10^{-4}$–10^{-5} radian. The angles at which γ rays with small values of $\Delta K/K$ are selected may be smaller than the aperture angle of the electron beam. In this case there must of course be a preliminary collimation of the primary electrons, which will mean a loss of intensity.

For various energies of the primary electrons we show in Table IV* the values of the angle necessary to obtain values $\Delta K/K = 1$ and 0.1; the table shows that for smaller energies of the primary electrons the requirements on the angle become more severe. Generally speaking, the limiting value of $\Delta K/K$ cannot be better than the energy spread of the primary electrons. In the special case, however, of large energy of the primary electrons and observation of γ rays with energies $\omega_2 \ll \omega_{2max}$, one can achieve a rather high degree of monochromaticity. For example, for $\mathscr{E}_1 = 40$ BeV and observation of γ rays with energies $\sim 10^5$ eV the line width can be as small as $\sim 10^{-3}$.

By the scattering of photons on moving electrons

*Similar calculations have been made at Stanford (private communication from Professors Panofsky and Mozley to the present authors).

Table IV

$\Delta K/K$, %	Energy of primary photons, eV	Energy of electron, eV						
		$5\cdot10^{6}$	$5\cdot10^{7}$	$5\cdot10^{8}$	$2\cdot10^{9}$	$6\cdot10^{9}$	$2\cdot10^{10}$	$4\cdot10^{10}$
1	1.78	$1.4\cdot10^{-2}$	$1.4\cdot10^{-3}$	$1.4\cdot10^{-4}$	$5.1\cdot10^{-5}$	$4.9\cdot10^{-6}$	$4.5\cdot10^{-6}$	$2.6\cdot10^{-6}$
	3.56	$1.4\cdot10^{-2}$	$1.4\cdot10^{-3}$	$1.4\cdot10^{-4}$	$5.1\cdot10^{-5}$	$6.9\cdot10^{-6}$	$5.2\cdot10^{-6}$	$3.2\cdot10^{-6}$
10	1.78	$4.6\cdot10$	$4.6\cdot10^{-3}$	$4.5\cdot10^{-4}$	$1.6\cdot10^{-4}$	$1.8\cdot10^{-5}$	$1.4\cdot10^{-5}$	$8.2\cdot10^{-6}$
	3.56	$4.6\cdot10^{-2}$	$4.6\cdot10^{-3}$	$4.5\cdot10^{-4}$	$1.6\cdot10^{-4}$	$2.2\cdot10^{-5}$	$1.7\cdot10^{-5}$	$1.0\cdot10^{-5}$

one can obtain beams of photons in any given range of frequencies from

$$\omega_{2\max}=\frac{\omega_1(1-\beta)}{1+\beta}$$

to $\omega_{2\max}$.

The cross section for the Compton effect is $\sigma \approx 6.6\times10^{-25}$ cm^2. The number of photons emitted from powerful lasers can be up to 10^{12} in a pulse of duration 10^{-8} sec (a power of some billion watts per cm^2). The scattering of this number of photons on a bunch of $\sim10^9$ electrons with the same duration in time, when allowance is made for the available transverse dimensions of electron beams in strong-focusing accelerators, will lead to the production of $\sim10^5$ hard photons in the range $d\omega_2/\omega_{2\max}=0.05$. This number of γ-ray photons is comparable with that given in the same frequency range by the bremsstrahlung of the same bunch of electrons on the targets usually used. At present the energy of ruby lasers has been brought up to 1500 joules. We emphasize that these fluxes of γ rays are produced when the light is introduced into the accelerator chamber and the encounter of the electrons with the light target can be brought about at any instant of the acceleration cycle—that is, at various energies of the electrons. In working with this sort of "light target" the production of any sort of background is prevented.

As has already been pointed out, increase of ω_1 or $\mathscr{E}_1$ gives an increase of $\omega_{2\max}$, and also increases the degree of monochromaticity of the γ rays produced. As ω_1 is increased there is a comparatively slow decrease of the differential cross section in the vicinity of $\omega_{2\max}$ (see Table III). This offers a hope of producing rather intense monoenergetic beams of γ rays by the use of intense sources of photons harder than those of visible light. At present there have already been studies of many substances whose stimulated emission lies in the ultraviolet range of frequencies; in the near future we can expect positive results on the generation of intense fluxes of soft x-radiation. Besides this, in nonlinear interaction with certain crystals the red light from ruby lasers has already been successfully converted into violet light (doubled frequency), and the intensity is less than the original intensity by a factor of only five.

A characteristic feature of the coherent laser radiation is the extremely high density of the photons. This naturally raises the question as to the contribution to the Compton scattering from processes involving simultaneous absorption of several soft photons and the emission of a single harder photon. The Klein-Nishina formula describes the scattering process in terms of successive acts of absorption and emission of a single photon. Consequently, it is necessary to generalize the Klein-Nishina formula to the case of extremely high photon densities, at which there can be an appreciable probability of many-photon processes. A calculation of this kind has been made by Gol'dman,[25] who has obtained formulas for the cross sections for Compton scattering that are valid for arbitrary photon densities. For the case of simultaneous absorption of n photons of frequency ω_1 and subsequent emission of one photon of frequency ω_2 the differential cross section (for circular polarization of the primary photons) is of the form:

$$\frac{d\sigma}{d\Omega}=\frac{2e^4n\,(\omega_2/n\omega_1)^2}{(1-v_1)\,[\mu_1^2+m^2(1+\xi^2)]}\left\{\left(\frac{\mu_2}{\mu_1}+\frac{\mu_1}{\mu_2}\right)\left[J_n'^2(S)+\left(\frac{n^2}{S^2}-1\right)J_n^2(S)\right]-2\xi^{-2}J_n^2(S)\right\}, \tag{3.6}$$

where

$$\mu_1=\sqrt{(1-v_1)/(1+v_1)}\quad \mu_2=\mu_1-\omega_2(1-\cos\theta),$$
$$S=\frac{2\pi\varrho\xi}{1+\xi^2+\varrho^2},\quad \varrho=\frac{\mu_1}{m}\operatorname{ctg}\frac{\theta}{2},\quad \xi^2=\frac{2e^2\hbar\lambda}{m^2c^3},$$

and λ is the wavelength and N the density of the incident photons. $J_n(S)$ and $J_n'(S)$ are the Bessel function and its derivative, for integer order. In this case the formula (3.1) can be written in the form

$$\omega_2=\frac{2n\mu_1\omega_1}{\mu_1(1+\cos\theta)+[2n\omega_1+m^2(1+\xi^2)/\mu_1](1-\cos\theta)}. \tag{3.7}$$

It is easy to see that ω_2 increases with increase of n, and that for $n \to \infty$ the limiting value is

$$\omega_{2\max} = \frac{\mu_1}{1-\cos\theta}. \tag{3.8}$$

For these many-photon processes the most important thing is of course the magnitude of the cross sections. It is obvious that these cross sections must have appreciable values for any effective observation of the processes. The parameter that determines the size of the cross section is ξ. For $\xi \to 0$ the expressions (3.6)—(3.8) go over into the usual formulas for Compton scattering. For $\xi \gg 1$ an analysis of (3.6) shows that the most probable process is the absorption in a single act of $n \sim \xi^3$ photons. When, however, one estimates the practically possible values of ξ in view of the maximum radiation powers that can be produced by means of lasers, it is found that the case $\xi \gg 1$ is unrealistic. For the most powerful present lasers (several billion watts per cm^2) we find $\xi^2 \approx 5 \times 10^{-10}$. When the light of such a laser is focused on a spot of diameter 10^{-3} cm, ξ^2 reaches values $\sim 5 \times 10^{-4}$. Therefore the case of practical importance is $\xi \ll 1$. For this case the cross section for Compton scattering on a stationary electron is

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\left(\frac{\omega_2}{n\omega_1}\right)^2\left(\frac{\omega_2}{n\omega_1} + \frac{n\omega_1}{\omega_2} - \sin^2\theta\right)\Phi_n, \tag{3.9}$$

where

$$\Phi_n = \frac{n^{2n+1}}{(n!)^2}\left(\frac{\xi\sin\theta}{2}\right)^{2n-2},$$

and the Compton ratio is

$$\omega_2 = \frac{n\omega_1}{1 + \left(\frac{n\omega_1}{m} + \frac{\xi^2}{2}\right)(1+\cos\theta)}. \tag{3.10}$$

As can be seen, the cross section falls off sharply with increase of n, and therefore the values of n for which it can have appreciable values are small. For example, for n = 2 we have

$$\Phi_2 = 2\xi^2\sin^2\theta,$$

and the total cross section can be expressed in the form

$$\sigma_2 = 4\pi r_0^2\xi^2\left[\frac{(1+\gamma)(2+4\gamma-\gamma^2)}{\gamma^5}\ln(1+2\gamma) + \frac{2}{1+2\gamma} - \frac{2(6+12\gamma-\gamma^2)}{3\gamma^4}\right],$$

$$\gamma = \frac{2\omega_1}{m}. \tag{3.11}$$

For a focused beam of laser light of the power stated above, $\sigma_2 \approx 3 \times 10^{-28}\ cm^2$. This value of the cross section will evidently allow observation of the effect for n = 2. For n > 3 it will probably not be possible to observe the process.

It is interesting to note that when light interacts with high-energy electrons production of electron-positron pairs can occur:

$$\gamma + e^- \to e^- + e^+ + e^-.$$

The threshold for this reaction is relatively low; in the system in which the electron is at rest it is 4m. For electrons of energy 40 BeV (Stanford) the energy of the primary photons which is necessary for pair production is 12.5 eV. The existence of sufficiently intense coherent sources of far-ultraviolet radiation will provide a possibility of observing this reaction also, along with the Compton scattering. Incidentally, just as in the case of Compton scattering, pair production can occur with the simultaneous absorption of several photons, and in principle this could make it possible to observe the reaction even if the energy of the primary photons is below the threshold.

2. Polarization Effects

An intrinsic and extremely important property of the γ rays produced by Compton scattering on moving electrons is that they are polarized to a very high degree. Let us examine the polarization properties of these γ rays.[3] This is of interest because a quite definite polarization for the primary photons can be chosen when the radiation from optical quantum generators (lasers) is used. Relative to the unit vectors

$$\chi_1^{(1)} = \frac{[\mathbf{k}_1\mathbf{k}_2]}{|[\mathbf{k}_1\mathbf{k}_2]|}, \quad \chi_2^{(1)} = \frac{1}{|\mathbf{k}_1|}[\mathbf{k}_1\chi_1^{(1)}], \tag{3.12a*}$$

$$\chi_1^{(2)} = \frac{[\mathbf{k}_1\mathbf{k}_2]}{|[\mathbf{k}_1\mathbf{k}_2]|}, \quad \chi_2^{(2)} = \frac{1}{|\mathbf{k}_2|}[\mathbf{k}_2\chi_1^{(2)}] \tag{3.12b}$$

the Stokes parameters $\xi_i^{(1)}$ for the primary and $\xi_2^{(2)}$ for the secondary photons are related by the equations:

$$\left.\begin{aligned}
\xi_1^{(2)} &= [\sin^2\theta' + (1+\cos^2\theta')\,\xi_1^{(1)}]\frac{1}{F},\\
\xi_2^{(2)} &= 2\cos\theta'\,\xi_2^{(1)}\frac{1}{F},\\
\xi_3^{(2)} &= \left[\left(\frac{\omega_1'}{\omega_2'} + \frac{\omega_2'}{\omega_1'} - 2\right)\xi_2^{(1)} + 2\xi_3^{(1)}\right]\frac{\cos\theta'}{F}.
\end{aligned}\right\} \tag{3.13}$$

In Eqs. (3.12) and (3.13) $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of the incident and scattered photons, ω_1' and ω_2' are their respective frequencies, and θ' is the angle between their directions; F is the quantity

$$F = \frac{\omega_1'}{\omega_2'} + \frac{\omega_2'}{\omega_1'} + (\xi_1^{(1)} - 1)\sin^2\theta'; \tag{3.14}$$

$\xi_1^{(1)}$, $\xi_2^{(1)}$ determine the probabilities of two linear polarizations which make the angle $\pi/4$ with each other, and $\xi_3^{(1)}$ that of a circular polarization. Let us consider the polarization properties of the photons relative to fixed axes x, y, z (with the z axis directed along the momentum of the primary photon).

We take the polarization of the primary photons as given relative to these fixed axes, and write the Stokes parameters $\xi_i^{(1)}$ in the form

---

*$[\mathbf{k}_1\mathbf{k}_2] = \mathbf{k}_1 \times \mathbf{k}_2$.

$$\left.\begin{aligned}\xi_1^{(1)}&=P\cos 2\beta\cos 2\alpha,\\ \xi_2^{(1)}&=P\cos 2\beta\sin 2\alpha,\\ \xi_3^{(1)}&=P\sin 2\beta;\end{aligned}\right\}\tag{3.15}$$

P is the degree of polarization of the incident photons. In a rotation through the azimuthal angle φ around the z axis the angle β is not changed, and α is increased by the amount φ; therefore the $\xi_i'^{(1)}$ relative to the vectors $\chi_i^{(1)}$ are expressed in terms of the $\xi_i^{(1)}$ in the following way:

$$\left.\begin{aligned}\xi_1'^{(1)}&=\cos 2\varphi\cdot\xi_1^{(1)}-\sin 2\varphi\cdot\xi_2^{(1)},\\ \xi_2'^{(1)}&=\sin 2\varphi\cdot\xi_1^{(1)}+\cos 2\varphi\cdot\xi_2^{(1)},\\ \xi_3'^{(1)}&=\xi_3^{(1)}.\end{aligned}\right\}\tag{3.16}$$

Using the expressions for $\xi_i'^{(1)}$ and the formulas (3.13), we get the following expressions for the Stokes parameters $\xi_i'^{(2)}$ relative to the $\chi_i^{(2)}$:

$$\begin{aligned}\xi_1'^{(2)}&=[\sin^2\theta'+(1+\cos^2\theta')\cos 2\varphi\cdot\xi_1^{(1)}\\ &\quad-(1+\cos^2\theta')\sin 2\varphi\xi_2^{(1)}]\frac{1}{F'},\\ \xi_2'^{(2)}&=(\sin 2\varphi\cdot\xi_1^{(1)}+\cos 2\varphi\cdot\xi_2^{(1)})\frac{2\cos\theta'}{F'},\\ \xi_3'^{(2)}&=\Big[\Big(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-2\Big)\sin 2\varphi\cdot\xi_1^{(1)}\\ &\quad+\Big(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-2\Big)\cos 2\varphi+2\xi_3^{(1)}\Big]\frac{\cos\theta'}{F'},\end{aligned}\tag{3.17}$$

where, using (3.14) and (3.16), we can write F′ in the form

$$F'=\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}+(\xi_1^{(1)}\cos 2\varphi-\xi_2^{(1)}\sin 2\varphi-1)\sin^2\theta'.\tag{3.18}$$

If the primary photon and electron before the collision were moving at angle $\theta_1 = 0$ or π with each other, then the polarization of the secondary photons, determined by the parameters $\xi_i'^{(2)}$ relative to the vectors (3.12b) in the rest system of the initial electron, remains unchanged relative to these same vectors in the laboratory system, provided that the frequencies ω_1', ω_2' and the angle θ' are expressed in the rest system of the electron. This can be verified in the following way. It is easily seen that the direction of the electric vector in the state $\chi_1^{(2)}$ does not change when we go from the rest system of the electron to the laboratory system. If the polarization vector of the secondary photon were such that its electric vector was directed along $\chi_2^{(2)}$, one can easily verify by making a Lorentz transformation that this vector in the laboratory system would be directed along the new unit vector $\chi_2^{(2)}$ (perpendicular to $\chi_1^{(1)}$ and k_{2lab}). Since any state of polarization can be represented as a superposition of two orthogonal states with invariant amplitudes, it is obvious that the parameters $\xi_i'^{(2)}$ remain unchanged when we go over to the laboratory coordinate system. Precisely similarly, there is no change of the parameters for the initial photon. Thus we conclude that the formulas (3.17) give the polarization parameters of the scattered photon in the laboratory system (relative to unit vectors attached to the plane of the scattering).

The angle θ' in the rest system of the electron is connected with the energy of the scattered photon in the laboratory system by the relation ($\hbar = c = 1$)

$$\omega_2'=\omega_1\frac{1-v_1\cos\theta_1+v_1(\cos\theta_1-v_1)\cos\theta'+v_1\sqrt{1-v_1^2}\sin\theta_1\sin\theta'}{1-v_1^2+\frac{\omega_1}{\mathscr{E}_1}(1-\cos\theta)(1-v_1\cos\theta_1)},\tag{3.19}$$

where v_1, E_1 are the speed and energy of the electron, θ_1 is the angle between the directions of the primary photon and primary electron, and ω_1 is the energy of the primary photon in the laboratory system.

In the case $\theta_1 = \pi$ this relation takes the form

$$\cos\theta'=\frac{\omega_1\left(1-\frac{\omega_2}{\mathscr{E}_1}\right)-\frac{1}{1+v_1}\left(\frac{m}{\mathscr{E}_1}\right)\omega_2}{\omega_1\left(v_1-\frac{\omega_2}{\mathscr{E}_1}\right)}.\tag{3.20}$$

Using this connection between θ' and ω_2, we can obtain the dependence of the degree of polarization of the scattered γ rays on their energy. In doing so we must use the facts that

$$\omega_1'=\omega_1\frac{1-v_1\cos\theta_1}{\sqrt{1-v_1^2}}\tag{3.21}$$

and

$$\omega_2'=\frac{\omega_1'}{1+\frac{\omega_1'}{m}(1-\cos\theta')}.\tag{3.22}$$

It follows from (3.17) that by varying the state of polarization of the primary photons one can obtain at a definite angle φ a sufficiently high degree of polarization of a required type.

The experimental conditions often do not permit the singling out of the secondary photons emitted at a definite azimuthal angle φ. For such practically important cases one needs to know the polarization of the beam of secondary photons averaged over all possible values of φ. For γ rays making small angles θ_2 with the z axis one can take fixed x and y axes to which to relate the polarization. One must allow for the fact that in the case of the Compton effect on relativistic electrons the angles θ_2 are extremely small for practically the entire frequency range except $\omega_2 \approx \omega_1$. To perform the required averaging over φ we express the Stokes parameters $\xi_i'^{(2)}$ of the scattered photons relative to a fixed axis system; to do so we must rotate the vectors $\chi_i^{(2)}$ by the angle $-\varphi$,

$$\left.\begin{aligned}\xi_1^{(2)}&=\xi_1'^{(2)}\cos 2\varphi+\xi_2'^{(2)}\sin 2\varphi,\\ \xi_2^{(2)}&=-\xi_1'^{(2)}\sin 2\varphi+\xi_2'^{(2)}\cos 2\varphi,\\ \xi_3^{(2)}&=\xi_3'^{(2)},\end{aligned}\right\}\tag{3.23}$$

and then the $\xi_i^{(2)}$ will have the following forms:

$$\xi_1^{(2)}=\frac{1}{F'}\left\{\xi_1^{(1)}\left[\cos^2 2\varphi\,(1\mp\cos\theta')^2\pm 2\cos\theta'\right]\right.$$
$$\left.-\xi_2^{(1)}\sin 4\varphi\,\frac{(1\mp\cos\theta')^2}{2}+\cos 2\varphi\sin^2\theta'\right\},$$
$$\xi_2^{(2)}=\frac{1}{F'}\left\{-\xi_1^{(1)}\sin 4\varphi\,\frac{(1\mp\cos\theta')^2}{2}\right.$$
$$\left.+\xi_2^{(1)}\left[\pm 2\cos\theta'+\sin^2 2\varphi\,(1\mp\cos\theta')^2\right]-\sin 2\varphi\sin^2\theta'\right\},$$
$$\xi_3^{(2)}=\frac{\cos\theta'}{F'}\left\{\xi_1^{(1)}\sin 2\varphi\left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-2\right)\right.$$
$$\left.+\xi_2^{(1)}\cos 2\varphi\left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-2\right)+2\xi_3^{(1)}\right\}. \tag{3.24}$$

In the double signs in these formulas the upper sign corresponds to scattering of the photon into the forward hemisphere relative to the direction of its motion in the rest system of the electron, and the lower sign refers to backward scattering, which is associated with the production of harder γ-ray quanta. After averaging the $\xi_i^{(2)}$ over all φ with weight proportional to the scattering cross section, we get

$$\left.\begin{aligned}
\overline{\xi_1^{(2)}}&=\frac{(1+|\cos\theta'|)^2}{2\left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-\sin^2\theta'\right)}\,\xi_1^{(1)},\\
\overline{\xi_2^{(2)}}&=\frac{(1+|\cos\theta'|)^2}{2\left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-\sin^2\theta'\right)}\,\xi_2^{(1)},\\
\overline{\xi_3^{(2)}}&=\frac{2\cos\theta'}{\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-\sin^2\theta'}\,\xi_3^{(1)}.
\end{aligned}\right\} \tag{3.25}$$

The degree of polarization $\overline{P}$ of the scattered photons is then

$$\bar{P}^2=\frac{0{,}25\,(1+|\cos\theta'|)^4\,(\xi_1^{2(1)}+\xi_2^{2(1)})+4\xi_3^{2(1)}\cos^2\theta'}{\left(\frac{\omega_1'}{\omega_2'}+\frac{\omega_2'}{\omega_1'}-\sin^2\theta'\right)^2}. \tag{3.26}$$

The formula (3.25) can be illustrated with curves (Fig. 22), from which it follows that one can produce beams of hard γ rays with a high degree of polarization (going up to 100 percent). We note that the value of the maximum attainable degree of polarization of the γ rays must fall with increase of the energy of the electrons, owing to the expression that appears in the denominators in (3.25). This decrease of the degree of polarization of the γ rays is appreciable, however, only at electron energies of some tens of BeV (see Table III), which exceed the electron energies at present attainable with accelerators.

As already noted earlier, polarized high-energy γ rays can also be produced in the bremsstrahlung of polarized electrons. This method for producing polarized γ rays has not been used only because there have been no possibilities for producing polarized high-energy electrons.

A suggestion was made in [1] of a simple and effective possibility for producing polarized high-energy electrons in this same process of the scattering of light on relativistic electrons. Polarized electrons are in themselves of special interest, but in the present article we shall touch on the mechanism for their production only in so far as this is of interest from the point of view of producing polarized γ rays.

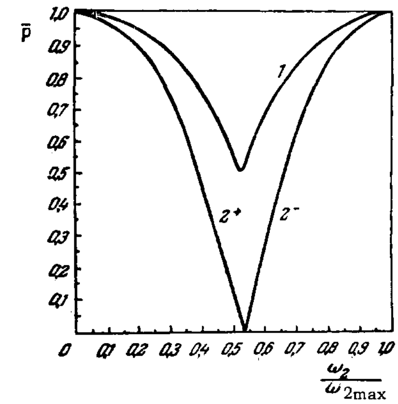

FIG. 22. Dependence of degree of polarization of scattered photons on their energy. 1 – case of linearly polarized primary photons, $\xi_1^{2(1)}+\xi_2^{2(1)}=1$; 2 – case of circularly polarized primary photons, $\xi_3^{(1)}=1$; the plus corresponds to polarization direction the same as that of the primary photons, the minus to polarization in the direction opposite to that of the polarization of the primary photons.

In the Compton scattering of a circularly polarized photon of frequency ω_1' at the angle θ' on a stationary electron the electron acquires the polarization [28]

$$\mathbf{S}=\frac{\xi_3\,(1-\cos\theta')}{m\,(1+\cos^2\theta')+(\omega_1'-\omega_2')\,(1-\cos\theta')}\left\{\left[\cos\theta'\right.\right.$$
$$\left.-(1+\cos\theta')\frac{(\omega_1'+m)}{(\omega_1'-\omega_2'+2m)}\right]\mathbf{P}_2'$$
$$\left.+(1+\cos\theta')\left[\mathbf{K}_2'-\frac{\mathbf{P}'}{\mathbf{P}_2'^2}(\mathbf{K}_2'\mathbf{P}_2')\right]\right\}, \tag{3.27}$$

where $\mathbf{K}_2'$ and $\mathbf{P}_2'$ are the respective momenta of the photon and electron after the scattering.

An appreciable degree of polarization of the final electrons when light is scattered from moving electrons can be expected only when in the rest system of the electron its recoil is comparable with the rest mass. This situation occurs in encounters between electrons of energy several BeV or more with primary photons emitted by a laser.

If the spin and momentum of the electron make the angle α' in the rest system, then the value of the angle in the laboratory system is given by

$$\cos\alpha=\frac{\mathscr{E}_2\mathscr{E}_2'-m\mathscr{E}_1}{P_2P_2'}\cos\alpha'-\sqrt{1-\left(\frac{\mathscr{E}_2\mathscr{E}_2'-m\mathscr{E}_1}{P_1P_2'}\right)^2}\sin\alpha'. \tag{3.28}$$

In the laboratory system the scattered electrons make negligible angles with their original direction, and therefore if the electrons are not subjected to a special selection as to azimuth the scattered electron beam as a whole will not have any transverse polarization.

Figures 23 and 24 show the values calculated from (3.27) and (3.28) for the longitudinal polarization η of the electrons for $\mathscr{E}_1$ = 6 BeV and 40 BeV

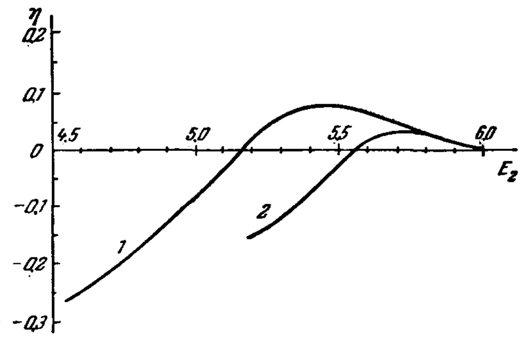

FIG. 23. Dependence of the degree of longitudinal polarization of the scattered electron on its energy E_2 (BeV). 1 – $\mathscr{E}_1$ = 6 BeV, ω_1 = 3.56 eV; 2 – $\mathscr{E}_1$ = 6 BeV, ω_1 = 1.78 eV.

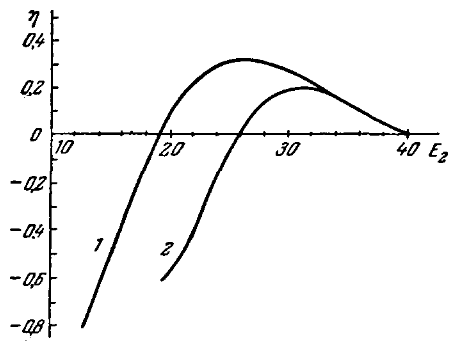

FIG. 24. Dependence of the degree of longitudinal polarization η of the scattered electron on its energy E_2 (BeV). 1 – $\mathscr{E}_1$ = 40 BeV, ω_1 = 3.56 eV; 2 – $\mathscr{E}_1$ = 40 BeV, ω_1 = 1.78 eV.

and for ω_1 = 1.78 eV and 3.56 eV. As can be seen from these curves, the degree of longitudinal polarization of the secondary electrons increases with increase of $\mathscr{E}_1$ or ω_1, and for electron energies $\sim$ 40 BeV one can have beams of high-energy electrons with a very high degree of polarization. The energy spectrum of the secondary electrons bears a simple relation to the spectrum (3.5) of the scattered photons, and contains in roughly equal numbers electrons of different energies from $\mathscr{E}_1 - \omega_{2\max}$ to $\mathscr{E}_1$. The total number of scattered electrons is of course the same as the number of scattered photons, for which we have already given estimates.

The γ rays of the bremsstrahlung from longitudinally polarized electrons (unlike those from transversely polarized electrons) will have a rather high degree of circular polarization, which is given by the expression, due to Olsen and Maximon,[32]

$$P_c = \pm \frac{K\,[(\mathscr{E}_1+\mathscr{E}_2)(3+2\Gamma)-2\mathscr{E}_2(1+4P_1^2\theta_\gamma^2\nu^2\Gamma)]}{(\mathscr{E}_1^2+\mathscr{E}_2^2)(3+2\Gamma)-2\mathscr{E}_1\mathscr{E}_2(1+4P_1^2\theta_\gamma^2\nu^2\Gamma)}, \qquad (3.29)$$

where $\nu = (1 + P_1^2\theta_\gamma^2)^{-1}$, $\Gamma = \ln(111\,Z^{-1/3}/\nu) - 2 - f(Z)$ for the case of complete screening,

$$f(Z) = a^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+a^2)} \quad \text{and} \quad a = Ze^2/\hbar c.$$

With increase of the energy of the γ-ray quanta the degree of polarization increases from zero, roughly linearly for $K \ll \mathscr{E}_1$, and reaches 100 percent for $K \approx \mathscr{E}_1$.

It must be noted that the polarized γ rays produced by this method will have energies right up to that of the primary electrons, which is a decided advantage as compared with all other methods. These γ rays, however, will have the usual bremsstrahlung spectrum, with its disadvantages.

3. Experimental Investigations

An experiment on the scattering of light from an electron beam has been done recently by Fiocco and Thompson.[22] The light of a ruby laser (λ = 6943 Å) was focused with an optical lens and directed perpendicularly onto a beam of electrons of energy 2 keV. The electron beam was also focused by means of a magnetic lens. The scattered radiation was observed at the angle 65° with the direction of the electron beam in the plane perpendicular to the beam of laser light. The energy of the optical quantum generator (laser), acting for a time of 0.8 millisec, was 20 joules. The density of the electron beam, with current 75 mA, was estimated by the authors of [22] as $\sim 5 \times 10^9$ cm^{-3}. The polarization of the incident light was in the direction of the axis of the electron beam. From the space of diameter 2 mm inside which the collision of the photon and electron beams occurs the scattered radiation is collected by a lens onto an iris diaphragm and then, after passing through a system of filters, is detected by a photomultiplier cooled to the temperature of liquid nitrogen. With the kinematics chosen for the experiment the wavelength of the scattered radiation should be shorter than that of the laser photons by 259 Å. Interference filters with pass band 10 Å, placed along the path to the photomultiplier, cut down the number of background photons which are produced in the scattering of the light which excites the laser from the walls of the vacuum chamber and then get to the photomultiplier. These filters also diminish the background caused by the incandescent filament of the electron gun and the excitation of the residual gas by the electron beam. With the electron gun and the laser operating simultaneously an oscillograph registered the signals from the photomultiplier during 2 millisec, with the laser operating for 0.8 millisec. Analogous signals were registered during separate operations of the electron gun and of the laser. The measured number of photoelectrons corresponding to the signal was found to be larger than the number caused by the background during combined operation of the electron gun and the laser. The background from operation of the laser alone, without the electron gun, was insignificant. There was a much larger background from operation of the electron gun without the laser.

Three series of measurements were made, with 15, 21, and 22 registrations. For the last series, on one hand the laser energy was increased by 20 percent, and on the other the focusing of the electron gun was improved. The mean numbers of photoelectrons (the quantum efficiency of the photoelectric element was 5 percent) in the series were 3.1, 1.85, and 5.8, with corresponding standard deviations of 1.4, 0.90, and 1.85. The mean number of background photons did not depend much on the turning on of the laser beam, and did not exceed 1.25 in the first two series of measurements. The Thompson scattering cross section is 6.7×10^{-25} cm^2, and the expected number of photoelectrons under the conditions of the experiment is estimated by the authors (for the first two series of measurements) to be 3.1. Although this experiment does not bring out the main characteristics of the scattering of light by electrons, it is already interesting because the scattering of light by free electrons has been observed for the first time in a pure form. Unquestionably much more interesting experiments will be those on scattering of light by relativistic electrons and, as we have indicated, the resulting production of high-energy γ rays. Such experiments are now being prepared in various laboratories which possess high-energy electron accelerators.

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