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THE CIRCULAR POLARIZATION OF γ-RAY QUANTA EMITTED BY ATOMIC NUCLEI SUBSEQUENT TO β DECAY

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1. INTRODUCTION

The circular polarization of light is a phenomenon that has been studied for a long time, and effective polarizers, which also serve as analyzers, have been found for visible radiation. From the point of view of the wave nature of electromagnetic radiation in classical theory a circular polarization is associated with a rotation of the electric vector **E** around the direction of the wave vector $\mathbf{k}$ ($\mathbf{E} \perp \mathbf{k}$). Right circular polarization corresponds to clockwise rotation (right-handed screw), and left circular polarization to counterclockwise rotation (left-handed screw). This definition of circular polarization does not agree with the optical definition, for in optics the name left-polarized light is given to radiation which has the symmetry of a right-handed screw. This difference is due to the use of a different coordinate system, in which the observer faces in the direction opposite to the wave vector.

For nuclear γ radiation in a quantum treatment of the phenomenon is more natural. Here circular, or cylindrical, polarization is associated with the projection of the spin σ_γ of the photon onto the direction of its momentum $\mathbf{p}_\gamma$, with right circular polarization corresponding to the projection $\mu = +1$, and left circular to $\mu = -1$. If the probabilities of the two projections are equal, there is no circular polarization. In the general case the probabilities of the two values can be different. Then one introduces the concept of the degree P_c of circular polarization,

$$P_c = \frac{I_r - I_l}{I_r + I_l}, \qquad (1)$$

where I_r and I_l are the respective intensities of photons with right and left circular polarizations.

Gamma-ray quanta with circular polarization are emitted by polarized nuclei. This can be seen from the simplest example, in which the spin of the initial polarized nucleus is $j_1 = 1$, and the spin after the radiative transition is $j_2 = 0$. In this case, in accordance with the law of conservation of total angular momentum, the γ-ray quanta emitted in the direction ($\theta = 0$) of polarization of the nuclei (the z axis in Fig. 1) will have right circular polarization with $P_c = +1$, and those in the opposite direction ($\theta = \pi$) will have left circular polarization with $P_c = -1$. The γ radiation emitted in directions perpendicular to the z axis will not be polarized, $P_c = 0$. Thus the circular polarization of γ rays emitted by polarized nuclei is a function of the angle θ.

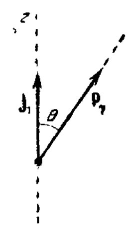


FIG. 1

In the more general case the nuclei in the initial state are partially polarized, j_1 is not necessarily equal to 1, and the spin in the final state can be different from zero. In the determination of P_c it is then necessary to take into account the probabilities of various spin projections in the initial and final states of the nuclei. Exact calculations have been made by Tolhoek and Cox[1] and by Stenberg[2] and are presented in monographs.[3,4] For a fixed angle θ of emission of the γ rays ($\theta \neq \pi/2$) the degree of circular polarization P_c depends on the degree of polarization of the nuclei in the initial state, $\langle j_z \rangle / j$, the spins j_1 and j_2 of the initial and final states, and the multipole character L of the radiative transition. The angular dependence is given in the form of an expansion in Legendre polynomials.

We note that for circular polarization there will be an angular dependence and a degree of polarization $P_c \neq 0$ even in cases in which there is no anisotropy in the directional correlation (for example, when $j_1 = 1/2$ or when there is nuclear capture of a fermion in an S state). In particular, circular polarization of γ rays has been observed[5] in the reaction (n, γ) of radiative capture of polarized thermal neutrons.

The circular polarization of γ rays emitted by nuclei subsequent to β decay is a consequence of parity nonconservation in weak interactions. In their first paper Lee and Yang,[6] in considering the question of parity nonconservation, suggested along with other possible experiments the measurement of the circular polarization of γ rays accompanying β decay, though they regarded the experiment as technically unfeasible. In the report by I. S. Shapiro at the All-Union Conference on Nuclear Spectroscopy (January, 1957) it was pointed out that there is an actual possibility of such an experiment, and soon Schopper[8] made an experiment of this kind, and thus confirmed one of the consequences of parity nonconservation in β decay. Further research led to definite changes in the theory of β decay and opened up broad experimental possibilities. These questions were treated in detail in a review by Ya. A. Smorodinskiĭ[9] in 1959. In the last few years many experiments have been made on the circular polarization P_c of γ rays emitted subsequent to β decay. The results of these researches are presented below. Part of the data, obtained before 1959, are also contained in a review by Fagg and Hanna.[10] The phenomenon of circular polarization of x-rays and γ rays has been treated from a historical point of view in[11].

The leptons produced in β transitions are longitudinally polarized; the neutrinos are completely polarized, and the β particles have a polarization $\sim v/c$.[12] Therefore after β decay the daughter nucleus is partially polarized relative to the direction of emission of the β particle. The degree of such polarization is determined by the characteristics of the β transition. The polarization has opposite signs for electron and positron decays, since the helicities of electron and positron are opposite. Thus if we single out the direction of emission of the β particles, then the γ-ray quanta emitted after the decay at a fixed angle $\theta \neq \pi/2$ (Fig. 2) will in the general case be circularly polarized. By measuring the degree of circular polarization of the γ rays one can get information about the characteristics of the β transition and the subsequent γ transition. In this respect these experiments are analogous to experiments with nuclei polarized by low-temperature methods.

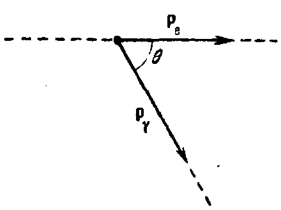


FIG. 2

We shall now present the theoretical relations which are the basis of the experiments on the circular polarization of γ rays emitted after β decay.

2. FUNDAMENTAL THEORETICAL RELATIONS

Immediately after the first experimental confirmation of parity nonconservation in β decay a number of authors[13-16] independently made theoretical calculations of various effects which are consequences of this new phenomenon. In particular,[17-19] expressions were obtained for the angular correlation function $W(\theta)$ between the electron (or positron) and the circular polarization of the γ ray following the β decay (θ is the angle between the directions of the β particle and the γ ray). The most general expression for $W(\theta)$ is of the following form:

$$W(\theta) = \sum_R \beta_R \gamma_R P_R(\cos\theta)(2R+1), \qquad (2)$$

where β_R are coefficients which depend on the characteristics of the β transition (the interaction constants, the nuclear matrix elements, the energies and momenta of the leptons, the charge of the nucleus), and γ_R are coefficients which depend on the characteristics of the transition (the multipole character and the angular momenta of the final nuclear levels). The number R which fixes the order of the Legendre polynomial is connected with the degree of forbiddenness l of the β transition by the relation $0 \le R \le 2l + 1$.

The expression for the correlation function is ordinarily normalized so that the first term, with the zeroth-order Legendre polynomial, is equal to unity. Equation (2) is then written:

$$W(\theta) = 1 + \sum_{R \geq 1} \frac{\beta_R \gamma_R}{\beta_0 \gamma_0} P_R(\cos\theta)(2R+1). \qquad (3)$$

a) The circular polarization and the character of the radiative transition. In the case of a "pure" γ transition of multipole character L the quantity γ_R in the expression (2) can be written in the following form:

$$\gamma_R = C_{L-\mu}^{L-\mu R0} \sqrt{(2j_2+1)(2L+1)}\, W(j_2 j_3 RL;\ Lj_2), \qquad (4)$$

where j_2, j_3 are the angular momenta of the excited and ground states of the final nucleus, $\mu = +1$ corresponds to right circular polarization of the γ ray, and $\mu = -1$ to left circular polarization. The $C_{L-\mu}^{L-\mu R0}$ are Clebsch-Gordan coefficients, and the $W(j_2 j_3 RL; Lj_2)$ are Racah coefficients. For even R the coefficients $C_{L-\mu}^{L-\mu R0}$ do not depend on the sign of μ, but for odd R they do; that is, the circular polarization is determined by the odd terms of the expression (3).

As we shall see as we go on, in a large number of cases the expression (3) stops with the value $R = 1$. Then $W(\theta)$ depends only on the ratio γ_1/γ_0. Calculating the coefficients $C_{L-\mu}^{L-\mu R0}$ and $W(j_2 j_3 RL; Lj_2)$ for $R = 0$ and $R = 1$ for a "pure" radiative transition, we get

$$\frac{\gamma_1}{\gamma_0} = \frac{\mu}{2L(L+1)} \frac{j_2(j_2+1) + L(L+1) - j_3(j_3+1)}{\sqrt{j_2(j_2+1)}}. \qquad (5)$$

From the form of the expression (5) we can draw a number of definite conclusions. First, for $j_2 = j_3$ the ratio γ_1/γ_0, and consequently also $W(\theta)$, do not depend on the multipole character L. Second, for $j_2 > j_3$ the sign of γ_1/γ_0 is determined by the sign of μ; also the quantity γ_1/γ_0 decreases somewhat as

L increases. If, on the other hand, $j_2 < j_3$, then the sign and magnitude of γ_1/γ_0 depend on j_2, j_3, and L.

If the γ radiation is a mixture of electric radiation with multipole character L + 1 and magnetic radiation with multipole character L, then according to [19]

$$\gamma_R = C^{L-\mu R0}_{L-\mu} \sqrt{(2j_2+1)(2L+1)}\, W(j_2 j_3 RL;\ L j_2)$$
$$+ 2\mu\Delta \sqrt{\frac{L+2}{2L+1}} \left(\frac{2L+1}{2L+3}\right)$$
$$\times C^{L-\mu R0}_{L+1-\mu} \sqrt{(2j_2+1)(2L+3)}\, W(j_2 j_3 RL;\ L+1\, j_2)$$
$$+ \Delta^2 C^{L+1-\mu R0}_{L+1-\mu} \left(\frac{L+2}{2L+3}\right) \sqrt{(2j_2+1)(2L+3)}$$
$$\times W(j_2 j_3 RL+1;\ L+1\, j_2). \qquad (6)$$

Here Δ is the ratio of the amplitudes of the electric and magnetic radiations in the mixture. Consequently, in this case the correlation function depends not only on the values j_2 and j_3 of the spins of the states, but on the mixture ratio Δ of the multipoles, just as in other correlation effects. Thus there is an additional parameter, which makes the unambiguous interpretation of experimental data more difficult.

As an example, Fig. 3 shows the dependence of γ_1/γ_0 on the quantity Δ for an M1 + E2 transition with the values $j_2 = 3/2$ and $j_3 = 5/2$. As can be seen from the figure, there are changes not only of the magnitude but also of the sign of γ_1/γ_0: this is explained by the presence of the interference term in the expression (6).

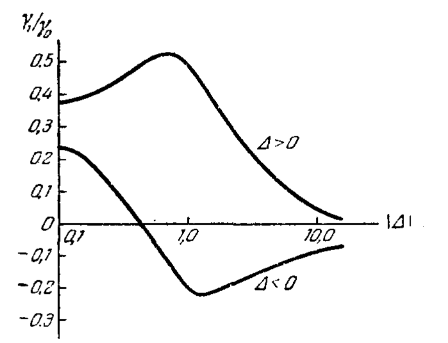


FIG. 3. Dependence of γ_1/γ_0 on the ratio Δ of the amplitudes of the E2 and M1 multipoles for $j_2 = 3/2$, $j_3 = 5/2$.

In the case in which the β transition leads to an excited nucleus with the angular momentum j_2 which goes to levels j_3, ..., j_{n+2} with successive emissions of γ-ray quanta with multipole characters L_1, L_2, ..., L_n, we have for the "pure" γ transition [19]

$$\gamma_R = C^{L_n-\mu_n R0}_{L_n-\mu_n} \sqrt{(2j_{n+1}+1)(2L_n+1)}\, W(j_{n+1} j_{n+2} RL_n;\ L_n j_{n+1})$$
$$\times \prod_{i=1}^{n-1} \sqrt{(2j_{1+i}+1)(2L_{i+2}+1)}\, W(j_{i+1} L_i R j_{i+2};\ j_{i+2} j_{i+1}). \qquad (7)$$

It must be noted here that the value of γ_1/γ_0 for the n-th γ-ray quantum is in any case not larger than that for the first one. As the result of successive "pure" radiative transitions the correlation coefficient can change, but its sign remains the same. A similar remark has been made previously in [20] in a discussion of the problem of the depolarization of nuclei in cascade transitions.

b) Circular polarization in allowed β transitions. The quantity β_R in Eq. (2) is a sum of squares and binary products of nuclear matrix elements for β decay with appropriate coefficients and β-interaction constants, and as previously pointed out depends on the angular momenta j_1 and j_2, where j_1 is the angular momentum of the initial nucleus, which decays to the level with angular momentum j_2 in the final nucleus (Fig. 4). The expression (3) contains ratios β_R/β_0, and consequently in a given case $W(\theta)$ is a function of the ratio of the nuclear matrix elements. The explicit form of β_R/β_0 will be given below for the concrete cases of allowed and singly forbidden β transitions: all of the expressions for β_R/β_0 are given for the theory in which the β interaction contains only the vector and axial-vector terms (V − A).

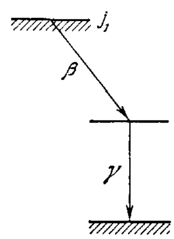


FIG. 4. $\beta\gamma$-cascade.

In the case of allowed β transitions the angular correlation function $W(\theta)$ of Eq. (3) takes the simple form

$$W(\theta) = 1 + \mu A \frac{v}{c} \cos\theta, \qquad (8)$$

where v/c is the ratio of the speed of the β particle to the speed of light, and $\mu = \pm 1$ corresponds to right and left circular polarization of the γ rays. From Eqs. (8) and (1) we get the expression for the degree of circular polarization P_c of γ-ray quanta emitted after β decay at the angle θ with the direction of the momentum p_e of the β particle (see Fig. 2)

$$P_c = A \frac{v}{c} \cos\theta. \qquad (9)$$

The coefficient A is given by

$$\mu A = \mp \frac{\gamma_1}{\gamma_0} \left[\frac{2X}{1+X^2} + \frac{2+j_2(j_2+1)-j_1(j_1+1)}{(1+X^2)\,2\sqrt{j_2(j_2+1)}} \right]. \qquad (10)$$

Here $X = c_V M_F / c_A M_{G-T}$, where c_V and c_A are the constants for the vector and axial-vector β interactions. M_F and M_{G-T} are the Fermi and Gamow-Teller matrix elements. The expression in the square brackets in Eq. (10) is the ratio β_1/β_0. The plus sign corresponds to electron decay (β^-), and the minus sign to positron decay (β^+). As has been shown in [17], inclusion of effects of the Coulomb field of the nucleus does not change the expression (10).

For a Fermi β transition ($M_F \neq 0$, $M_{G-T} = 0$) the formula (10) gives the value $A = 0$, as is to be expected from general considerations, since the lepton pair is emitted in a singlet state and the nucleus cannot be polarized after the β decay. For a Gamow-Teller β transition ($M_F = 0$, $M_{G-T} \neq 0$) we have $X = 0$, and the expression (10) does not depend on the matrix elements. Therefore the value of A can be calculated exactly if one knows the characteristics of the successive radiative transitions (γ_1/γ_0). For example, for Co^{60} ($5^+ \xrightarrow{\beta^-} 4^+ \xrightarrow{\gamma} 2^+ \xrightarrow{\gamma} 0^+$) we have $A = -0.33$, and for Na^{22} ($3^+ \xrightarrow{\beta^+} 2^+ \xrightarrow{\gamma} 0^+$) we have $A = +0.33$. Actually the conditions $M_F \neq 0$ and $M_{G-T} = 0$ are realized only in transitions of the type $0 \rightarrow 0$, and $M_F = 0$, $M_{G-T} \neq 0$ only in β transitions in which the angular momentum changes by unity, $\Delta j = 1$. If, on the other hand, the angular momentum of the nucleus does not change in the β decay ($\Delta j = 0$), then both matrix elements are allowed by the fundamental selection rules ($M_F \neq 0$ and $M_{G-T} \neq 0$). Figure 5, taken from [21], shows graphically the dependence of A on the ratio $C_V M_F / c_A M_{G-T} = X$ for the values $j_1 = 1$, $j_2 = 1$, $j_3 = 0$, $L = 1$. The upper curve ($X > 0$) corresponds to different phases of M_F and M_{G-T}, and the lower curve to equal phases.

Thus by measuring the degree of circular polarization P_c [Eq. (9)] of γ rays emitted after β decay one can find the experimental value A_{exp}. By comparing the experimental value of A with the theoretical value, Eq. (10), for allowed β transitions one can determine: first, the total angular momenta j_1, j_2, j_3 of the levels, and second, the ratios of nuclear matrix elements, X and Δ for the β and γ transitions, respectively. The most valuable possibility is that of determining the ratio of the Fermi and Gamow-Teller matrix elements, including the phase relation (cf. Fig. 5). If in addition one uses the values of ft for the β transition in question, which are as a rule known, then one can also determine the absolute values of the matrix elements M_F and M_{G-T}. In Section 4 of the present article we shall give the results of experimental researches on this point.

c) The case of forbidden β transitions. The interpretation of the results of measurements of circular polarization in forbidden β transitions is decidedly complicated by the fact that the number of nuclear matrix elements that affect even a singly forbidden β transition is much larger than for allowed transitions. We shall confine ourselves to the discussion of singly forbidden β transitions, since the information in the literature about more highly forbidden β transitions is not entirely definite.

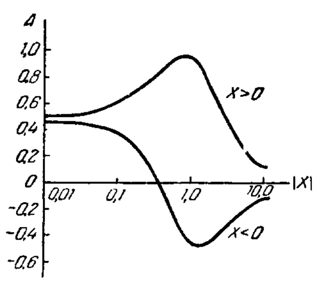


Fig. 5. Dependence of the correlation coefficient A in Eq. (8) on the ratio $X = c_V M_F / c_A M_{G-T}$ of Eq. (10) for $j_1 = 1$, $j_2 = 1$, and $j_3 = 0$.

It is well known that the rank Λ of the nuclear matrix elements of a β transition, equal to the total angular momentum carried away by the two leptons, satisfies the relation

$$|j_1 - j_2| \equiv \Delta j \leqslant \Lambda \leqslant j_1 + j_2, \tag{11}$$

where j_1 and j_2 are the angular momenta of the initial and final states of the nucleus in the β transition. The main contribution to the probability of a singly forbidden β transition comes from the matrix elements with three values of Λ, namely $\Lambda = 0$, 1, and 2:

$$\begin{array}{lll} \eta\omega = c_A \int \sigma \mathbf{r}, & \eta\xi' v = c_A \int i\gamma_5, & \Lambda = 0, \\ \eta u = c_A \int i\sigma \times \mathbf{r}, & \eta x = -c_V \int \mathbf{r},\ \eta\xi' y = -c\,\alpha \int i\alpha, & \Lambda = 1, \\ \eta z = c_A \int B_{ij}, & & \Lambda = 2. \end{array} \tag{12}$$

Here the nuclear parameters u, v, ω, x, y, and z, introduced in accordance with [22], are the ratios of the various matrix elements to the standard matrix element η, so that $|\eta|^2$ appears as a factor in the expression for the probability of β decay and its value is determined from the value of ft. The factor $\xi' = (1/4) R_N$ (R_N is the radius of the nucleus) is introduced in order that the parameters v and y, associated with relativistic matrix elements, will have the same dimensions as u, ω, and x. The formulas given below will contain matrix elements expressed in terms of the nuclear parameters in accordance with Eq. (12).

Thus singly forbidden β transitions are determined by six matrix elements. In a number of cases, however, some of them can be neglected. For example, in singly forbidden β transitions with β spectra of the allowed shape the matrix element $c_A \int B_{ij}$ is small in comparison with the others. Furthermore, if $\Delta j = 1$, according to the rule (11) only the three matrix elements with $\Lambda = 1$ are different from zero, and if $\Delta j = 0$, the only nonvanishing elements are the five with $\Lambda = 0$ and $\Lambda = 1$. If, on the other hand, $\Delta j = 2$, then we have the unique transition, with the shape of its β spectrum always different from the allowed, and for which the only matrix element is $c_A \int B_{ij}$. Cases of β transitions with spectrum shapes different from the allowed will be considered in what follows.

The angular dependence of the correlation function $W(\theta)$ for singly forbidden β transitions can be obtained from Eq. (2) and is found in terms of Legendre polynomials P_0, P_1, P_2, and P_3 with suitable coefficients. The coefficient β_3 which occurs with the polynomial P_3 is a product with the matrix element $c_A \int B_{ij}$ as a factor, and therefore when the β spectrum has the allowed shape the term with β_3 can be dropped. The coefficient of P_2 does not depend on the circular polarization of the γ rays and can be expressed in terms of the anisotropy coefficient ϵ of the $\beta\gamma$ directional correlation. In the case of transitions with the allowed shape of the spectrum there is no such correlation, and consequently the term in $P_2(\cos\theta)$ does not contribute to the angular dependence. Thus for singly forbidden β transitions with the allowed shape of the β spectrum the correlation function $W(\theta)$ has an angular dependence which is similar to that for allowed β transitions and is given by Eq. (8). Furthermore the coefficient A, just as for allowed transitions, does not depend on the energy of the β particles, but for $\Delta j = 0$ is a function of all five nuclear matrix elements corresponding to a first-forbidden β transition. If, on the other hand, $\Delta j = 1$, the coefficient A does not depend on the matrix elements, as is the case for allowed transitions.

From the point of view of the interpretation of experiment the case with $\Delta j = 0$ is very complicated, since it is not possible to do a large enough number of independent experiments to determine the values of all five nuclear matrix elements. By measuring the circular polarization of the γ-ray quanta one can find the ratio of the matrix element of first rank V and that of second rank Y, which appear in the expression for A in the form V/Y:

$$V = \xi' v + \xi\omega; \qquad Y = \xi' y - \xi(u + x), \tag{13}$$

where $\xi = \alpha Z/2R_N$, α is the fine-structure constant, and Z is the charge of the nucleus. In this respect the situation is somewhat simplified if the β transition is a "Coulomb" transition.

When the Coulomb field of the nucleus is taken into account in the expression for the probability of a forbidden β transition terms of order αZ appear, and in heavy nuclei these terms may be the most important ones. Such β transitions are called "Coulomb" transitions and were first considered in [23] and then in [19]. The condition for a β transition to be of this type is given by the inequalities

$$\alpha Z \gg pR_N \text{ and } \alpha Z \gg \frac{v_n}{c}, \tag{14}$$

where p is the momentum of the β particle, R_N is the mean radius of the nucleus, v_n is the speed of the nucleons in the nucleus, and c is the speed of light. Furthermore the shape of the β spectrum must not deviate from the allowed shape. Besides neglecting the term $c_A \int B_{ij}$, in Coulomb β transitions one also neglects the relativistic matrix elements $c_A \int i\gamma_5$ and $c_V \int i\boldsymbol{\alpha}$.

The result is that in Coulomb singly forbidden β transitions with $\Delta j = 0$ the asymmetry coefficient of Eqs. (8) and (9) depends on the three nuclear matrix elements $c_V \int \mathbf{r}$, $c_A \int \boldsymbol{\sigma}\cdot\mathbf{r}$, and $c_A \int i\boldsymbol{\sigma}\times\mathbf{r}$. The expression for A in this case takes [23] a form similar to Eq. (10) if we replace X by

$$\Omega = \frac{-\omega}{x+u}, \tag{15}$$

where ω, x, and u are defined in Eq. (12). Figure 6 shows as an example the dependence of A on Ω for $j_1 = 5/2$, $j_2 = 5/2$, $j_3 = 3/2$ and a M1 + E2 radiative transition with $\Delta = 1$. Thus by measuring the circular polarization of the γ rays in the Coulomb type of β transition considered here one can determine the ratio between the nonrelativistic matrix elements of different ranks.

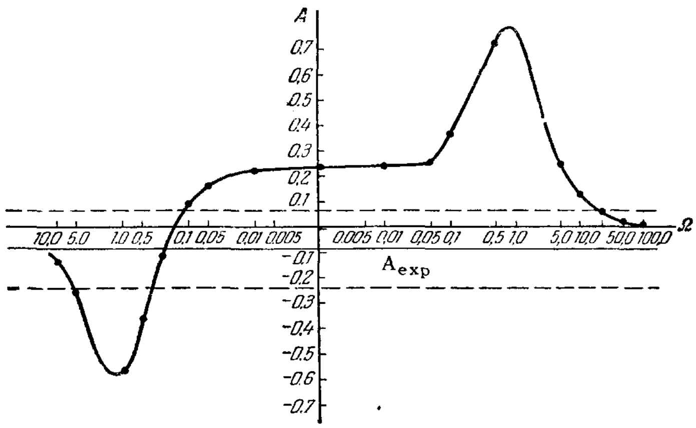


FIG. 6. Dependence of the correlation coefficient A on $\Omega = -\omega/(x+u)$ for $j_1 = 5/2$, $j_2 = 5/2$, $j_3 = 7/2$ (sic) and a M1 + E2 radiative transition with $\Delta = 1$. The value of A_{exp} obtained for Nd^{147} in [72] is shown as a solid line, and as dashed lines when allowance is made for errors.

The majority of first-forbidden β transitions have the allowed shape of the β spectrum. This is explained by the so-called "ξ approximation," in which the probability of the β transition is expanded in powers of $\xi = \alpha Z/2R_N$ ($\alpha = 1/137$, Z is the charge and R_N the radius of the nucleus) and one drops all terms except the first, which contains ξ and does not contain the energy W of the β particles. In this approximation the factor (C) multiplying the shape of the spectrum is a constant. In cases in which the spectrum deviates from the allowed shape the "ξ approximation" is not valid and the treatment given above cannot be used. The angular dependence of the correlation function $W(\theta)$ will be given by Legendre polynomials, including the polynomial of third degree:

$$W(\theta) = A_0 + A_2P_2(\cos\theta) + \mu\,[A_1P_1(\cos\theta) + A_3P_3(\cos\theta)]. \tag{16}$$

Here, as in the general case also, only the coefficients of polynomials of odd degree depend on the circular polarization μ. From the expression (16) and Eq. (1) we get the degree of circular polarization of the γ-ray quanta

$$P_c = \frac{A_1/A_0 \cdot P_1(\cos\theta) + A_3/A_0 \cdot P_3(\cos\theta)}{1 + A_2/A_0 \cdot P_2(\cos\theta)}. \qquad (17)$$

The coefficients A_0, A_1, A_2, and A_3 depend on the energy W of the β particles and are expressed in terms of the matrix elements, Eqs. (11) and (12), including $c_A \int B_{ij}$, with A_0 being simply the correction factor for the shape of the β spectrum, $A_0 = C(W)$. The denominator in Eq. (17) is the directional correlation function, in which the asymmetry coefficient is $\epsilon = A_2/A_0$. For unique β transitions ($\Delta j = 2$) the picture is simplified, since according to the selection rule (11) the coefficients A_0, A_1, A_2, and A_3 depend only on the one matrix element $c_A \int B_{ij}$, which cancels when one takes the ratios A_1/A_0, A_2/A_0, A_3/A_0. Consequently for a unique transition P_c does not depend on the matrix elements at all and can be calculated exactly.

Expressions for A_0, A_1, A_2, and A_3 (or for P_c) as functions of the matrix elements and the energy W in the general case of a singly forbidden β transition are given in papers by Kotani[22] * and by Weidenmüller.[24] In these papers use has been made of the Konopinski-Uhlenbeck approximation; this means that, first, in the expansion of the probability of β decay in powers of ξ (or simply the "ξ expansion") only the first three terms are used, second, that no correction is made for the finite size of the nucleus, and third, effects from the third and higher orders of forbiddenness are neglected.

Considerable deviations from the allowed shape of the β spectrum are observed experimentally in singly forbidden transitions of the types

$$3^- \xrightarrow{\beta^-} 2^+ \,(\mathrm{Ga}^{72},\ \mathrm{Sb}^{124},\ \mathrm{La}^{140},\ \mathrm{Eu}^{152},\ \mathrm{Eu}^{154}) \text{ and } 2^- \xrightarrow{\beta^-} 2^+ \,(\mathrm{Rb}^{88},\ \mathrm{Sb}^{122})$$

with subsequent E2 radiative transition to the ground state of the even-even nucleus. From the point of view of the interpretation of the experimental results the simplest case is that of the $\beta\gamma$ cascade

$3^- \xrightarrow{\beta^-} 2^+ \xrightarrow[E2]{} 0^+$, since the quantity P_c for this case depends on a smaller number of matrix elements. For this case the coefficients A_0, A_1, A_2 and A_3 are written in[22] and[24] as functions of the nuclear parameters x, u, z of Eq. (12) and Y of Eq. (13). Setting z = 1, we get the standard matrix element $\eta = c_A \int B_{ij}$, whose value can be found directly from ft. This means that $c_A \int B_{ij} \neq 0$, whereas the other matrix elements can be equal to zero. Thus in these transitions the quantity P_c is determined by three nuclear parameters x, u, and Y. Consequently, by measuring the circular polarization of the γ rays one can get information about the corresponding nuclear matrix elements. To make this a determinate problem, it is necessary to make measurements of P_c either at different electron energies W or at different angles θ between the directions of the electron and the γ ray.

The angular dependence of the correlation function $W(\theta)$ for the case in which the β spectrum deviates from the allowed shape can conveniently be represented in a form analogous to Eq. (8):

$$W(\theta) = 1 + \mu\omega(W,\ \theta)\frac{v}{c}\cos\theta, \qquad (18)$$

where $\omega(W, \theta)$, unlike the A in Eq. (8), is a function of the energy W of the β particle and of the angle θ. This sort of expression for $W(\theta)$ is given in[22], which also gives the explicit form of $\omega(W, \theta)$, expressed in terms of the nuclear parameters (12). Figure 7, taken from[22], shows the dependence of the asymmetry coefficient ω on the electron energy W for $W_0 = 5.5$ for two angles $\theta = 150°$ (dashed lines) and $\theta = 135°$ (solid lines). Figure 8 shows the angular dependence of ω for W = 5. The curves in Figs. 7 and 8 correspond to various choices of the nuclear parameters: 1) z = 1, Y = 0, x = u = 0 (unique β transition); 2) z = 1, Y = 0.27, x = u = 0 (the so-called "B_{ij} approximation"); 3) z = 1, Y = 1.8, u = −0.1, x = 0.75; 4) z = 1, Y = 5.5, u = −0.3, x = 0.7 (the case of "cancellation" of the nuclear matrix elements). It can be seen from the figures that the coefficient ω is extremely sensitive to changes in

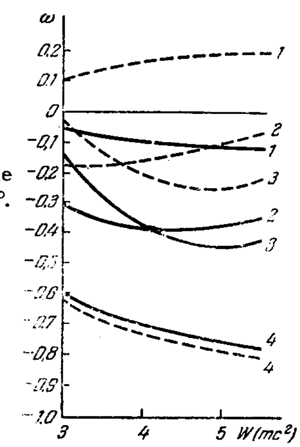


FIG. 7. The correlation coefficient $\omega = P_c/[(v/c)\cos\theta]$ as a function of the energy W of the β particles (W_0 = 5.5). The solid curves are for the value $\theta = 135°$, the dashed curves for $\theta = 150°$. The curves are for the following values of the nuclear parameters: 1: Y = 0, x = u = 0, z = 1; 2: Y = 0.27, x = u = 0, z = 1; 3: Y = 1.8, u = –0.1, x = 0.75, z = 1; 4: Y = 5.5, u = –0.3, x = 0.7, z = 1.

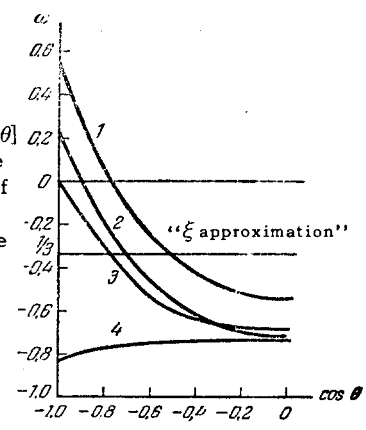


FIG. 8. $\omega = P_c/[(v/c)\cos\theta]$ as a function of $\cos\theta$. θ is the angle between the directions of emission of the β particle and the γ quantum. Curves 1 – 4 are as in Fig. 7.

*There is a misprint in [22]. In Eq. (A5) on page 805, instead of (u – z) it should read (u – x).

the sets of nuclear parameters. For example, the curve corresponding to the unique transition (set 1) differs markedly from that corresponding to set 4. The case of a β transition with the allowed form of the spectrum, for which the "ξ approximation" holds, is represented by a straight line. The dependence of ω on the energy W is less sharp than that on the angle θ, and therefore for the accurate determination of absolute values of the nuclear parameters it is more advantageous to use the angular dependence. Experiments of this sort have been made by Alexander and Steffen [25] and by Hartwig and Schopper [26] with measurements of the circular polarization in Sb^{124} and a number of other nuclei.

Other experimentally observable quantities depending on the same nuclear parameters, besides the degree of circular polarization P_c of γ rays, are the correction factor on the shape of the spectrum C(W), the asymmetry coefficient in the $\beta\gamma$ correlation ϵ (W), the longitudinal polarization of the electrons P_e, the $\beta\gamma$ correlation with a selected longitudinal polarization of the β particles, and so on. Therefore in a number of cases it is possible to use the combined results from different independent experiments to determine the nuclear parameters, and consequently the nuclear matrix elements. This sort of method has been used in [27,28,96] in dealing with the results of measurements of the circular polarization of the γ rays in La^{140}.

In the present section, in dealing with the theoretical relations we have not taken into account the possible depolarization of the nuclei in the excited states which appear as the result of β decays. Such depolarization can be due to the interaction of the nucleus with extra-nuclear fields, and it occurs in all correlation phenomena if the lifetime of the excited state is sufficiently long. In the case of measurement of the circular polarization of γ rays the depolarization of the nuclei has been considered in the interpretation of experiments with Sc^{46} [29] and will be discussed later.

3. THE MEASUREMENT OF THE CIRCULAR POLARIZATION OF γ-RAY QUANTA

The measurement of the circular polarization of γ-ray quanta emitted by nuclei after β decay consists of selecting $\beta\gamma$ coincidences with a fixed angle θ between the directions of emission of the γ ray and the β particle (see Fig. 2), and simultaneously detecting the circular polarization of the γ ray. From the experimental point of view the main difficulty is in the detection of the circular polarization of the γ rays. At angles $\theta \cong 0°$ or $180°$ the degree of circular polarization P_c [Eqs. (9), (17)] can in some cases reach quite large values (~ 30 percent). At present, however, the available analyzers for circular polarization of γ-ray quanta (polarimeters) have small efficiencies, and this greatly complicates the performance of the experiments.

A review by Schopper [30] considers in detail the various methods for measuring the circular polarization of γ-ray quanta which are possible in principle. The method with greatest efficiency uses Compton scattering by electrons polarized in magnetized ferromagnetic materials. This method was first proposed by Ya. B. Zel'dovich [31] and was later treated in detail theoretically by Gelberg [32] and by Lipps and Tolhoek. [33] In the overwhelming majority of the experiments made at present on the circular polarization of γ rays use is made of the Compton forward scattering by polarized electrons. This method is based on the large dependence of the Compton scattering cross section on the angle between the directions of the spins of the γ-ray quantum and the electron. The differential cross section for Compton scattering is given by the formula

$$d\sigma = \frac{r_0^2}{2}\left(\frac{k}{k_0}\right)^2\{\Phi_0 + P_l\,\Phi_1 + fP_c\Phi_c\}. \qquad (19)$$

Here r_0 is the classical electron radius, k_0 and k are the energies of the incident and scattered photons, P_l is the degree of linear polarization, P_c is the degree of circular polarization, f is the fraction of polarized electrons in the scatterer, Φ_0 is the Compton scattering cross section insensitive to polarization, and Φ_1 and Φ_c are the parts of the cross section that depend on the polarization:

$$\left.\begin{aligned}\Phi_0 &= 1+\cos^2\vartheta+(k_0-k)(1-\cos\vartheta),\\ \Phi_1 &= \sin^2\vartheta,\\ \Phi_c &= -(1-\cos\vartheta)\,[(k_0+k)\cos\vartheta\,\cos\psi+k\sin\vartheta\sin\psi\cos\varphi],\end{aligned}\right\} \qquad (20)$$

where ϑ is the scattering angle, ψ is the angle between the momentum $\mathbf{k}_0$ of the incident γ-ray quantum and the spin $\mathbf{s}$ of the electron, and φ is the angle between the planes $(\mathbf{k}_0\mathbf{s})$ and $(\mathbf{k}_0\mathbf{k})$. It follows from the expression for Φ_c that if the direction of the spin of the electrons in the scatterer is reversed (which is accomplished by reversing the magnetization)—that is, if ψ is replaced by $\psi + \pi$, the sign of Φ_c is changed. The fractional effect δ produced in this way is defined as follows:

$$\delta = \frac{d\sigma(\psi)-d\sigma(\psi+\pi)}{\frac{1}{2}[d\sigma(\psi)+d\sigma(\psi+\pi)]} = \frac{2fP_c\Phi_c/\Phi_0}{1+P_l(\Phi_1/\Phi_0)}. \qquad (21)$$

Naturally for a given P_c the effect will be largest for the maximum value of the ratio Φ_c/Φ_0.

The ratio Φ_c/Φ_0 is a function of the angles ϑ, ψ, and φ and of the initial energy of the γ ray, Eq. (20). When one uses forward Compton scattering, together with the replacement of ψ by $\psi + \pi$, it is convenient to take $\varphi = 0$. The angles ϑ and ψ are chosen so as to make the quantity Φ_c/Φ_0 a maximum. The optimum values ϑ_m and ψ_m of the angles depend on the initial energy k_0 of the γ quanta. Questions related to the choice of the optimum geometry for the measurements are treated in [30]. As an example we may state that for $k_0 = 2$ (in units mc^2) $\vartheta_m = 56°$, $\psi_m = 25°$. Fig-

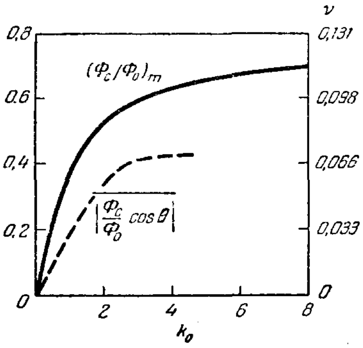


FIG. 9. The maximum value $(\Phi_c/\Phi_0)_m$ at the optimum angles $\vartheta = \vartheta_m$ and $\psi = \psi_m$ as a function of the initial γ-ray energy k_0 (solid curve). ν is the efficiency of the arrangement. The dashed curve represents $|\overline{(\Phi_c/\Phi_0)\cos\theta}|$, averaged for the arrangement used by the authors.

ure 9 shows the maximum value $(\Phi_c/\Phi_0)_m$ at the optimum angles $\vartheta = \vartheta_m$ and $\psi = \psi_m$ as a function of the initial energy k_0. It can be seen from the figure that the quantity $(\Phi_c/\Phi_0)_m$ reaches values ~ 0.7 at sufficiently large γ-ray energies ($k_0 > 8$). For small energies ($k_0 \lesssim 1/2$) the use of forward scattering for the detection of circular polarization of γ rays is inefficient because of the decreasing value of Φ_c/Φ_0. According to [35] it is better in this case to measure the azimuthal anisotropy given by changing from $\varphi = 0$ to $\varphi = \pi$ with fixed $\vartheta = \pi/2$ and $\psi = \pi/2$.

Thus for $k_0 \gtrsim 1/2$ the quantity $(\Phi_c/\Phi_0)_m$ has fairly large values (from 0.25 to 0.7). Since, however, the fraction of polarized electrons is small and for iron amounts only to 0.08, even with 100 percent polarization of the γ rays ($P_c = 1$) the magnitude of the effect (δ) cannot be larger than ~8 percent. Actually P_c has values $\lesssim 0.30$. Moreover in an actual apparatus the angles ϑ, ψ, and φ always cover more or less of a spread of values, so that in Eq. (21) we must use an average value $\overline{\Phi_c/\Phi_0}$ which is less than the $(\Phi_c/\Phi_0)_m$ which corresponds to the optimal angles. The result is that in the best case the effect observable in practice amounts to 2 percent. The average value of Φ_c/Φ_0 depends on the concrete geometry of the apparatus.

The expression (21) which gives the effect δ also involves a term $P_l\ \Phi_1/\Phi_0$ with the linear polarization of the γ rays. In the present case, however, of the circular polarization of γ rays emitted after allowed β transitions and after singly forbidden β transitions with β spectra of the allowed shape, there is no linear polarization ($P_l = 0$). In the case of forbidden β transitions for which the shape of the spectrum deviates from the allowed shape, $P_l \neq 0$. Still, if the polarimeter has rotational symmetry, the term with the linear polarization drops out in the averaging over the geometry. This fact, along with the large solid angle for detection of the scattered radiation and the wide limits in which the angles ϑ and φ are close to the optimum values, makes an apparatus with rotational symmetry the most convenient one, which is used in almost all measurements of P_c based on Compton forward scattering.

By measuring the degree of circular polarization of the γ-ray quanta emitted after β decay one determines experimental values of the correlation coefficient A, Eq. (10), if the β spectrum has the allowed shape, or ω, Eq. (18), if the spectrum departs from the allowed shape. In both cases P_c is a function of the angle θ between the directions of the β particle and the γ ray, and in an actual apparatus this angle covers a spread of values. Consequently in going from P_c to A or ω it is necessary to average the quantity $(\Phi_c/\Phi_0)\cos\theta$. The expression obtained in this way characterizes the efficiency ν of the entire apparatus, with

$$\nu = 2f\left|\overline{\frac{\Phi_c}{\Phi_0}\cos\theta}\right|. \tag{22}$$

The review paper [30] gives a method for calculating the average value of $(\Phi_c/\Phi_0)\cos\theta$ for an apparatus possessing rotational symmetry. The dashed curve in Fig. 9 shows the values of $|\overline{(\Phi_c/\Phi_0)\cos\theta}|$, as a function of the γ-ray energy k_0, for the particular apparatus used by the present authors. [34] The right-hand scale gives the corresponding values of the efficiency ν, Eq. (22).

From measurements of the effect δ one gets experimental values of A and ω:

$$A \text{ or } \omega = \frac{\delta}{2f\frac{v}{c}\left|\overline{\frac{\Phi_c}{\Phi_0}\cos\theta}\right|}. \tag{23}$$

As has been noted already, in forbidden β transitions with spectrum shapes different from the allowed, the correlation coefficient $\omega(W, \theta)$ depends on the angle θ and the electron energy W. In this case one gets from the experiment and Eq. (23) an average value $\overline{\omega(W, \theta)}$, which is not equal to the value $\omega(\overline{W}, \overline{\theta})$ calculated theoretically for the average values $\overline{W}$ and $\overline{\theta}$. In some cases, however, when the spreads of W and θ are small and the accuracy of the measurements is not high, one can assume that $\overline{\omega(W, \theta)} \cong \omega(\overline{W}, \overline{\theta})$. The problem of processing the results of measurements on forbidden β transitions is treated in more detail in [32].

Owing to the smallness of the actually observable effect ($\delta \lesssim 2$ percent), to get a reasonably accurate result it is necessary to have high efficiency in counting the $\beta\gamma$ coincidences and to make prolonged measurements. Researches on the circular polarization of γ rays employ the scintillation method, which provides high efficiency in counting the radiations and high resolving power for the selection of coincidences in time. The electronic systems used in such experiments must be extremely stable. Moreover, in the construction of the apparatus special measures must be taken to eliminate possible apparatus effects associated with influences of the stray magnetic field on the scintillation counters.

Figure 10 shows a block diagram of a concrete apparatus for measuring circular polarization with selection of $\beta\gamma$ coincidences. [34] The β particles emitted by the radioactive source are registered by a scintillation γ-ray detector with an anthracene crys-

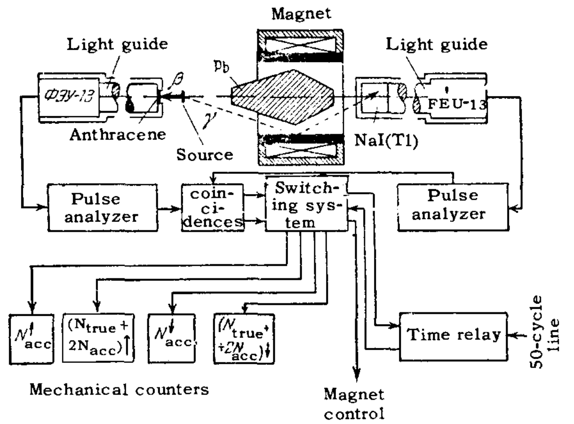


FIG. 10. Block diagram of an apparatus[34] for measuring circular polarization of γ-ray quanta with selection of $\beta\gamma$ coincidences.

tal; the γ-ray quanta scattered by a magnetized cylinder (the scatterer) are registered by a scintillation γ-ray detector with a crystal of NaI(Tl). The source, scatterer, and γ-ray and β-ray detectors have a common axis (rotational symmetry). Along the axis inside the cylinder there is located a lead cone which shields the γ-ray detector from directly incident γ rays. The scatterer is made of permendur, in which the induction $B = 2.2 \cdot 10^4$ gauss is attained in fields $H \approx 15$ Oe. The change of direction of the polarization of the electrons in the scatterer is produced by reversing the direction of the current in the magnet windings. The fraction of polarized electrons in the scatterer is $f = 8.2$ percent. The use of light guides of length up to 15 cm and of permalloy screens prevents influence of the stray magnetic field on the operation of the photomultipliers.

The two channels of the apparatus, which are scintillation β-ray and γ-ray spectrometers, consist of amplifiers and pulse-height amplitude analyzers, from which the pulses go to a circuit for selecting binary coincidences. In the γ channel, where the counting rate is $\sim 10^3$ counts/sec, a "slit" pulse-height analyzer is used.[35] In the β channel, where the counting rate is $\sim 10^5$ counts/sec, the pulses from the output of the photomultiplier are shaped by a short-circuited line, and after amplification go to a fast-acting analyzer. The apparatus employs a coincidence circuit which gives simultaneous selection of the total ($N_{true} = 2N_{acc}$) and accidental (N_{acc}) coincidence rates and has a resolving power of $\tau = 3 \times 10^{-8}$ sec. This sort of coincidence circuit makes it possible to eliminate errors associated with changes of τ during the measurements. The measuring process is carried out automatically. The duration of a single coincidence measurement is fixed by the time relay and is $t_i = 20.5$ sec. After the time t_i the switching system turns off one set of counting devices, reverses the direction of the magnetic field in the scatterer, and turns on the other set of counting devices. The numbers of $\beta\gamma$ coincidences corresponding to the different directions of the magnetic field are accumulated as the working cycle is gone through over and over.

The result of the operation of the apparatus is the determination of the numbers of $\beta\gamma$ coincidences associated with two opposite directions of the spins of the electrons in the scatterer. The measured effect δ is defined as

$$\delta = \frac{2\,[N(\uparrow) - N(\downarrow)]}{N(\uparrow) + N(\downarrow)}, \qquad (24)$$

where $N(\uparrow)$ is the number of coincidences with the magnetic in the scatterer field directed toward the source, and $N(\downarrow)$ is the number with the field directed away from the source. The coefficient A or ω is calculated from the value of δ.

In some cases the apparatus used in the work of other authors differs from that described in the electronics and the shape of the magnet. For example, if the intensity of the γ rays is small, a magnet with a semicircular cross section (Fig. 11) has some advantages, since with it the solid angle is much larger and the angles ϑ and ψ are still close to their optimal values.[37] In some work use has also been made of so-called "fast-slow" coincidence circuits: this makes it possible to improve the resolution time to $\tau \approx 5 \times 10^{-9}$ sec.[21]

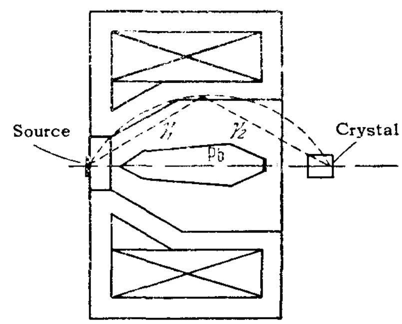


FIG. 11. The geometry of a polarimeter for circular polarization of γ rays which provides a large solid angle.[37]

For measurements of the dependence of the circular polarization P_c on the angle θ the apparatus is modified somewhat. In this case a more restricted geometry is necessary and rotational symmetry is impossible. For this purpose use is made of sectors of lead [25,26] which serve to cover parts of the interior of the scattering cylinder.

4. SURVEY OF THE EXPERIMENTAL RESULTS

a) Allowed β transitions. Table I contains data from measurements of the circular polarization of γ-ray quanta emitted after allowed β transitions, as reported in the literature received up to the middle of 1963. The table shows: the initial nucleus and type of β transition, the spins and other character-

Table I

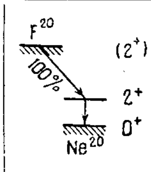
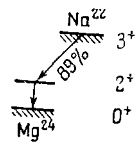
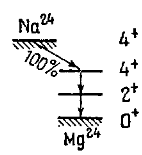
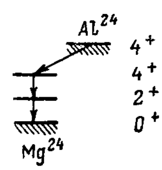
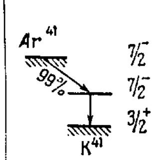
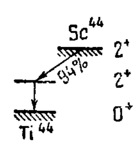
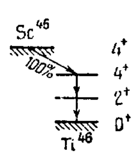
Initial nucleus and type of β decay	Spins, isotopic spins (T) and other (one-particle) characteristics of the initial nucleus (j_1) and of the successive excited levels of the final nucleus ($j_2, \ldots$)					Energy of β transition, keV	log ft of β transition	Energies E_{γ_i} of successive γ transitions, keV ($i = 1, 2, 3$)	Multipole character of radiative transition	Correlation coefficient A_{exp}	Value of $X = c_V M_F / c_A M_{G\text{-}T}$ calculated from Eq. (10)	Reference	Schemes of $\beta\gamma$ cascades studied	Remarks
	j_1	j_2	j_3	j_4	j_5									
${}_{9}F^{20}_{11}$ β^-	(2)	2	0	—	—	5420	5.0	$\gamma_1=1630$	$E2$	$+0.14\pm0.06$	$-0.12^{+0.21}_{-0.10}$	55		Agrees with theory for $j_1 = 2$, and not for $j_1 = 1$, the previously accepted value
${}_{11}Na^{22}_{11}$ β^+	3	2	0	—	—	540	7.3	$\gamma_1=1280$	$E2$	$+0.377\pm0.046$	0	38		Has been studied by many authors. All results are in good agreement with $A_{theor} = +1/3$
${}_{11}Na^{24}_{13}$ β^-	$T=1$ 4	$T=0$ 4	2	0	—	1400	6.1	$\gamma_1=2750$ $\gamma_2=1380$	$E2$ $E2$	$+0.075\pm0.024$ $+0.07\pm0.04$ $+0.06\pm0.03$ -0.063 ± 0.047 $+0.12\pm0.03$	$+0.01\pm0.03$ $+0.02\pm0.05$ $+0.03\pm0.05$ 0.21 ± 0.07 $\|X\|<0.1$	59 42 49 39 60		
${}_{13}Al^{24}_{11}$ β^+	$T=1$ 4	$T=0$ 4	2	0	—	8500		$\gamma_1=2750$ $\gamma_2=1380$	$E2$ $E2$	-0.089 ± 0.057	$+0.007\pm0.075$	91		
${}_{18}Ar^{41}_{23}$ β^-	$T={}^5/_2$ $^7/_2$ $f_{7/2}$	$T={}^3/_2$ $^7/_2$ $f_{7/2}$	$^3/_2$ $d_{3/2}$			1200	5.0	1290	$M2$	$+0.33\pm0.07$ $+0.07\pm0.07$	-0.66 ± 0.11 $+0.03\pm0.09$	61 52		The value from[52] is the more probable one
${}_{21}Sc^{44}_{23}$ β^+	$T=1$ 2	$T=2$ 2	0	—	—	1470	5.3	1160	$E2$	-0.02 ± 0.04 -0.149 ± 0.030	-0.18 ± 0.05 -0.02 ± 0.0355	42 59		
${}_{21}Sc^{46}_{25}$ β^-	$T=2$ 4	$T=1$ 4	2	0	—	357	6.2	$\gamma_1=1118$ $\gamma_2=892$	$E2$ $E2$	$+0.33\pm0.04$ $+0.29\pm0.11$ $+0.24\pm0.04$ $+0.24\pm0.02$ $+0.10\pm0.02$ $+0.068\pm0.016$ $\geqslant0.215\pm0.019$ $+0.19\pm0.03$ $+0.113\pm0.008$	-0.39 ± 0.07 -0.32 ± 0.14 -0.22 ± 0.08 -0.22 ± 0.04 -0.02 ± 0.02 $+0.020\pm0.025$ $\geqslant0.22$ -0.15 ± 0.05 -0.040 ± 0.011	42 38 48 49 50 51,73 29 89 92		

Table I (cont'd)

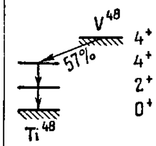
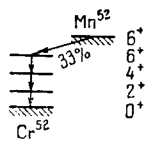
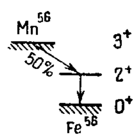
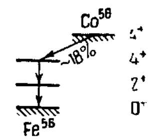
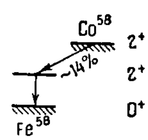
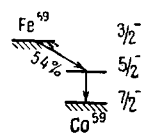
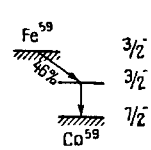
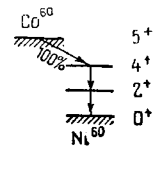
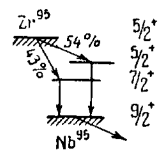
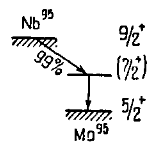
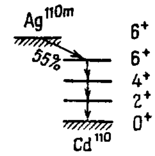
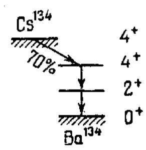
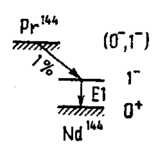
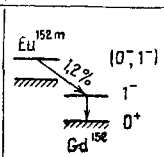
Initial nucleus and type of β decay	Spins, isotopic spins (T) and other (one-particle) characteristics of the initial nucleus (j_1) and of the successive excited levels of the final nucleus (j_2, ...) j_1	j_2	j_3	j_4	j_5	Energy of β transition, keV	log ft of β transition	Energies $E_{\gamma i}$ of successive γ transitions, keV (i = 1, 2, 3)	Multipole character of radiative transition	Correlation coefficient A_{exp}	Value of X $= c_V M_F / c_A M_{G-T}$ calculated from Eq. (10)	Reference	Schemes of βγ cascades studied	Remarks
$_{23}V^{48}_{25}$ β^+	$T=1$ 4	$T=2$ 4	2	0	—	690	6.1	$\gamma_1=1320$ $\gamma_2=990$	$E2$ $E2$	0.00 ± 0.04 0.06 ± 0.05 -0.066 ± 0.035	$+0.11\pm0.05$ $+0.19\pm0.07$ -0.02 ± 0.05	50 42 62		
$_{25}Mn^{52}_{27}$ β^+	$T=1$ 6	$T=2$ 6	4	2	0	580	5.6	$\gamma_1=730$ $\gamma_2=940$ $\gamma_3=1460$	$E2$ $E2$ $E2$	-0.023 ± 0.003 -0.062 ± 0.006 -0.079 ± 0.025 -0.10 ± 0.03 -0.16 ± 0.03 0.00 ± 0.02	-0.048 ± 0.004 $+0.004\pm0.010$ $+0.031\pm0.035$ $+0.060\pm0.04$ $+0.150\pm0.06$ -0.08 ± 0.03	51 51 59 51 63 64		In[51] data from experiments with oriented nuclei were used
$_{25}Mn^{56}_{31}$ β^-	3	2	0	—	—	2860	7.2	845	$E2$	$-\frac{1}{3}(0.80\pm0.06)$	0	55		The dependence on v/c was measured in the range from 0.94 to 0.98; $A_{theor} = -1/3$
$_{27}Co^{56}_{29}$ β^+	$T=1$ 4	$T=2$ 4	2	0	—	1470	8.6	$\gamma_1=1240$ $\gamma_2=845$	$E2$ $E2$	-0.02 ± 0.06 0.00 ± 0.03	$+0.08\pm0.08$	65 62		
$_{27}Co^{58}_{31}$ β^+	2	2	0	—	—	485	6.6	814	$E2$	-0.14 ± 0.07	-0.03 ± 0.04	44		
$_{26}Fe^{59}_{33}$ β^-	$^3/_2$	$^5/_2$	$^7/_2$	—	—	462	6.7	1098	$M1$	-0.46 ± 0.08 -0.23 ± 0.05 -0.066 ± 0.037	0	66 62 73		Agrees with the theoretical value $A_{theor} = -0.5$[66]
$_{26}Fe^{59}_{33}$ β^-	$T={}^7/_2$ $^3/_2$ $p_{3/2}$	$T={}^5/_2$ $^3/_2$ $p_{3/2}$	$^7/_2$ $f_{7/2}$			271	5.9	1289	$E2$	-0.40 ± 0.20 -0.04 ± 0.11 $+0.01\pm0.10$	-0.18 ± 0.20	73 66 62		Cannot be regarded as entirely reliable, since the effect from the transition with Δj = 0 is masked by the background of the transition with Δj = 1. Agreement with $j_2 = 5/2$ is found in [75]

Table I (cont'd)

Initial nucleus and type of β decay	Spin, isotopic spins (T) and other (one-particle) characteristics of the initial nucleus (j_1) and of the successive excited levels of the final nucleus (j_2, ...) j_1	j_2	j_3	j_4	j_5	Energy of β transition, keV	log ft of β transition	Energies $E_{\gamma i}$ of successive γ transitions, keV (i = 1, 2, 3)	Multipole character of radiative transition	Correlation coefficient A_{exp}	Value of X $= c_V M_F / c_A M_{G\text{-}T}$ calculated from Eq. (10)	Reference	Schemes of $\beta\gamma$ cascades studied	Remarks
${}_{27}Co^{60}_{33}$ β^-	5	4	2	0	—	309	7,4	$\gamma_1=1170$ $\gamma_2=1332$	$E2$ $E2$	-0.335 ± 0.018	0	67		Has been studied by many authors. All the results agree with $A_{theor} = -1/3$. The dependence of P_c on v/c has been studies in 49, 56–58
${}_{40}Zr^{95}_{55}$ β^-	$^5/_2$ $d_{5/2}$	$^5/_2$ $d_{5/2}$	$^9/_2$ $g_{9/2}$			360	6.9	760	$M1$ $M1+E2$	$\backsim -0.30$ -0.30 ± 0.02 $\backsim -0.10$	-1.0 ± 0.5 $\backsim +0.007$	39 68 46		The resultant measured value is is $A_{exp} = -0.40 \pm 0.07$. The discrepancy between the conclusions of [39, 68] and of [46] is due to different ways of allowing for the effect of the β transition with $\Delta j = 1$
${}_{41}Nb^{95}_{54}$ β^-	$^9/_2$	$(^7/_2)$	$^5/_2$			158	5.0	765	$(M1)$	-0.49 ± 0.08 -0.56 ± 0.09	0	46 47		Agrees with $j_2 = 7/2$ and the multipole character M1; $A_{theor} = -0.5$
${}_{47}Ag^{110m}_{63}$ β^-	6	6	4	2	0	529	8.1	$\gamma_1=935$ $\gamma_2=885$ $\gamma_3=656$	$E2$ $E2$ $E2$	$+0.07\pm0.02$	-0.02 ± 0.03	87		
${}_{55}Cs^{134}_{79}$ β^-	4	4	2	0	—	657	8.8	γ_1-797 γ_2-603	$E2$ $E2$	-0.08 ± 0.04 0.074 ± 0.022	$+0.23\pm0.06$	51 62		
${}_{59}Pr^{144}_{85}$ β^-	0	1	0	—	—	807	6.3	2185	$E1$	$+0.43\pm0.09$ $+0.70\pm0.26$ $+1.0\pm0.12$	-0.04 ± 0.05	21 93 94		The result agrees with theory for j_1 = 1 and 0
${}_{63}Eu^{152m}_{89}$ β^-	(1)	1	0.	—	—	560	7.3	1315	$E1$	$+0.40\pm0.10$	-0.03 ± 0.06	21		The result agrees with j_1 = 1

istics of the levels and the schemes of the $\beta\gamma$ cascades which were studied, the energies of the β particles and γ-ray quanta, the experimental values of the correlation coefficients A and the values of $X = c_V M_F / c_A M_{G-T}$ obtained from them by using Eq. (10), and also some physical conclusions.

The experiments on the circular polarization of γ rays accompanying allowed β transitions can be divided into two groups: a) measurements made for the purpose of determining the unknown spins of levels involved in the $\beta\gamma$ transitions in question, and b) measurements made for the purpose of determining the ratios of nuclear matrix elements in β transitions. An important limitation of the first group of experiments is that there are simpler and more reliable methods for determining spins, but in a number of cases such experiments have been very effective.

Na^{22}, Na^{24}, Co^{60}. The first experiment on the measurement of circular polarization of γ-ray quanta emitted by nuclei after β decay was made, as we have mentioned, in 1957, by Schopper[8] for the purpose of testing the hypothesis of parity nonconservation in weak interactions. The measurements were made with Co^{60} and Na^{22} nuclei, in which there are intense ($\sim$ 100 percent) allowed transitions (with emission of β^- and β^+, respectively) with $\Delta j = 1$. Consequently in both cases the correlation coefficient A of Eq. (8) does not depend on the nuclear matrix elements, and this is true for either combination of types of β interaction (ST or VA). The theoretically predicted values A_{theor} are $-1/3$ for Co^{60} and $+1/3$ for Na^{22}. The values A_{exp} found from the measurements were in excellent agreement with the theoretical values, and this was further convincing confirmation of parity nonconservation in β decay. Subsequently measurements with Co^{60} and Na^{22} were also made by other authors.[38-42] The accuracy of the values A_{exp} was improved, and their agreement with the theoretical values was taken as proof that the apparatus was working reliably and that the mathematical treatment of the geometrical averaging was correct. For Co^{60} and Na^{22} Table I gives the one most accurate result obtained in measurements with each of these nuclei.

Immediately after the first experiments, which confirmed parity nonconservation, various authors made measurements of the circular polarization of γ-ray quanta for the purpose of determining the type of the β interaction.[39,42-44] Allowed β transitions with both $\Delta j = 1$ and $\Delta j = 0$ were investigated. The experimental results could be reconciled with the theory if one assumed that the β interaction contained either the S and T types, or else the V and A types. The experiment with Na^{24} played a great part in the subsequent developments. The value of the correlation coefficient A_{exp} for Na^{24} indicated within the limits of error that either the matrix element M_F or else the element M_{G-T} was equal to zero. On the basis of the isotopic-spin selection rule the conclusion was drawn that $M_F = 0$. Furthermore it followed from the experiments of Burgov and Terekhov[45] on the resonance scattering of γ rays that the Gamow-Teller interaction is an axial-vector interaction. This conclusion was confirmed by many researches.[12] The result was to prove that the β interaction is of the vector and axial vector types.

F^{20}, Nb^{95}, Pr^{144}. In an experiment with F^{20} it was shown[56] that the spin of the ground state of F^{20} is $j_1 = 2$, and not 1, as had been supposed before, and consequently the β transition occurs without change of the angular momentum ($\Delta j = 0$). In measurements[46,47] with Nb^{95} the value of A that was found could be made to agree with theory only if one assigned to the level of the final nucleus which was reached as a result of the β decay the spin value $j_2 = 7/2$. Measurements[21] with Pr^{144} gave an unexpected result. The respective spin and parity values assigned to the ground states of Pr^{144} and Nd^{144} are 0^- and 0^+. On this basis an analysis of the shape of the spectrum of the β transition between the ground states $0^- \rightarrow 0^+$ was carried out in order to estimate the amount of the pseudoscalar type present in the β interaction. The measurements of the circular polarization of the γ rays emitted after the β decay of Pr^{144} to the 2.2 MeV level of Nd^{144}[21] agree with the spin value $j_1 = 1$; that is, they exclude the possibility of detecting a pseudoscalar interaction. This experiment is very complicated, however, since the branching ratio for the β spectrum in question is only $\sim$ 1 percent, and therefore the conclusion about the spin of Pr^{144} needed further confirmation. There was an analogous situation in the measurements[21] with Eu^{152m}. It was shown in[86] that all the existing data on the β decay of Pr^{144} are not in contradiction with the conclusions of[21].

Meanwhile, values of A_{exp} for Pr^{144} have been obtained recently[93,94] which are not in contradiction with the spin $j_1 = 0^-$ for the ground state. Moreover, measurements[90,97] of the dependence of the $\beta\gamma$ directional correlation on the energy of the β particles of Pr^{144} and Ce^{144} agree only with the value $j_1 = 0^-$. Thus[98] the ground state of Pr^{144} corresponds to the characteristics 0^-.

Allowed β transitions with $\Delta j = 0$. The greatest value of the emperiments made with allowed β transitions is due to the possibility of determining the ratio of the nuclear matrix elements $X = c_V M_F / c_A M_{G-T}$, including the phase relation. In the general case one gets from Eq. (10) two possible values of X (see Fig. 5), but one of them is much larger than unity and can justifiably be rejected, since it is improbable that $M_F \gg M_{G-T}$.

Measurements of the circular polarization of γ-ray quanta emitted after allowed β transitions with $\Delta j = 0$ have been made on the nuclei F^{20}, Na^{24}, Al^{24}, A^{41}, Sc^{44}, Sc^{46}, V^{48}, Mn^{52}, Co^{56}, Co^{58}, Fe^{59}, Zr^{95}, Ag^{110m}, and Cs^{134}. The most intensive studies have been those on the β transitions in the nuclei Na^{24}, Sc^{46}, and Mn^{52}. These are comparatively light nuclei and the isotopic spins T of the levels involved are known, the β decay occurring with $\Delta T = 1$. In this case, according to the isotopic-spin selection rule, $M_F = 0$, and consequently also $X = 0$, if isotopic invariance holds. As can be seen from Table I, for Na^{24} all of the measurements except one give $X \approx 0$ within the limits of error. It is possible that for Mn^{52} the quantity X may be small but still different from zero. As for Sc^{46}, here the data obtained in [29,38,42,48,49] are in good agreement with each other ($X = 0.22-0.32$). This indicates considerable interference and a correspondingly large value of M_F. Small values of X have been obtained, however, in [50] and [51], in agreement with the isotopic-spin selection rule but in contradiction with the results of the other papers mentioned. Owing to this discrepancy in the data Boehm and Rogers [29] made control measurements with various chemical compounds of Sc^{46}. Their results indicate that the effect depends on the chemical nature of the source; this could be due to depolarization of the excited state of Ti^{46}, although the influence of the extranuclear fields cannot be understood because of the small lifetime of the excited state. Recently [92] similar measurements have been made, but with a large number of different chemical compounds of Sc^{46}. No dependence of the results on the chemical composition of the source was observed, but the value found for X was $X = -0.040 \pm 0.011$. Thus in the β transition of Sc^{46} there is evidently a violation of isotopic invariance and a deviation from the isotopic-spin selection rule.

Figure 12 shows the experimental values of $|X|$ for all the nuclei that have been studied as a function of the mass number A (the crosses are for values $X > 0$, the circles for $X < 0$). As can be seen from

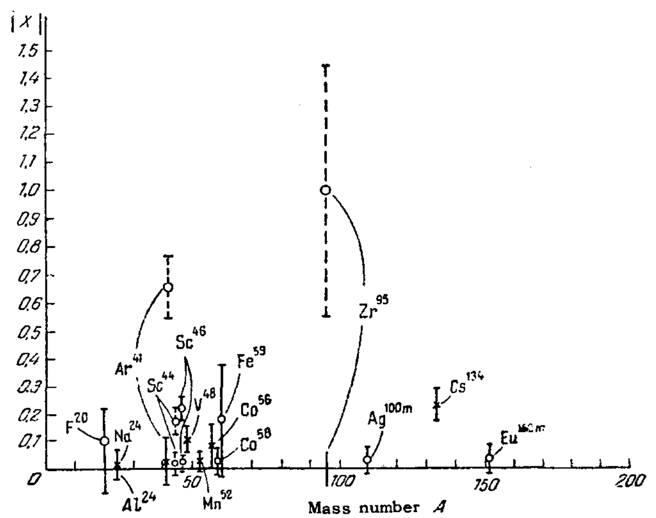


FIG. 12. Dependence of the ratio of nuclear matrix elements of allowed β transitions, $X = c_V M_F / c_A M_{G-T}$, on the mass number A of the nucleus.

the figure, in the overwhelming majority of cases

$$|X| \leqslant 0.25, \text{ i.e., } |M_F| \leqslant \tfrac{1}{4} |M_{G-T}|. \qquad (25)$$

The only possible exceptions are the nuclei Zr^{95} [39,68] and A^{41} [61]. In the case of Zr^{95} what is measured is the combined effect from two $\beta\gamma$ cascades with β-decay energies 360 keV ($\Delta j = 0$) and 396 keV ($\Delta j = 1$). The values of A_{exp} obtained in [39,46,68] agree well with each other and give the value $A_{exp} = -0.40 \pm 0.07$. In the determination of A_{exp} for the β transition with $\Delta j = 0$ use is made of the theoretical value A_{theor} for the β transition with $\Delta j = 1$. In [39,68] the γ transition with energy 726 keV is taken to be a pure M1 transition ($A_{theor} = -0.5$, and $X = -1$), but in [46] it is assumed that there is ~6 percent admixture of E2 ($A_{theor} = -0.63$, and $X = +0.07$). Consequently, the interpretation of the radiative transition with energy 726 keV [6,10] is crucial for the value of X for the cascade $5/2^+ \xrightarrow{\beta} 5/2^+ \xrightarrow{\gamma} 9/2^+$, that is, to get an accurate value of X it is necessary to determine the amount of E2 in the transition accurately. In the case of Fe^{59} the treatment of the experimental data is also made difficult by the presence of a transition of nearly the same energy with $\Delta j = 1$ ($A_{theor} = -0.5$) as a background masking the effect of the β transition with $\Delta j = 0$, so that the conclusion as to a relatively large value of $|X|$ cannot be regarded as entirely reliable. For A^{41} and also for Sc^{44} there are two contradictory values. In the case of A^{41} the large value [61] of X is improbable, since it differs sharply from all the values of X for neighboring nuclei. Furthermore the value $X \approx 0$ obtained in [52] agrees with the isotopic-spin selection rule, although from the shell model the Fermi β transition is allowed ($f_{7/2} \xrightarrow{\beta} f_{7/2}$). Thus there are evidently no exceptions to the rule (25).

A comparison with the theory of nuclear shells can be made for the nuclei A^{41}, Fe^{59}, and Ze^{95}, for which the one-particle characteristics of the levels of the $\beta\gamma$ cascades that have been studied are known. For all of these nuclei there is no supplementary forbiddenness from the one-particle characteristics. The other nuclei with $\Delta j = 0$ transitions that have been studied (see Table I) are odd-odd nuclei. Therefore their initial one-particle configurations cannot be regarded as unambiguous. Moreover the β transitions occur to collective levels of the even-even nuclei. As can be seen from Fig. 12, for the majority of nuclei that have $X = 0$ outside the limits of error the values of X are negative (circles in the diagram): that is, in the majority of cases the phases of M_F and M_{G-T} are the same. In the β transitions of V^{48} and Cs^{134}, however, $X > 0$ and the phases of the matrix elements are different.

In [53,54] theoretical calculations of the Fermi matrix elements were made for the β transitions in Na^{24}, Sc^{44}, and Mn^{52} for the purpose of explaining the experiments which give values $X \neq 0$. The calculations were made with Coulomb effects included and with the wave functions accepted in the nuclear shell model with jj coupling. This sort of treatment is not able to explain the experiments that show a departure from the isotopic-spin selection rule, and reasonable agreement is obtained only for Mn^{52}. The authors of the calculations suggest that the speeding up of Fermi β transitions with $\Delta T = 1$ might be explained by meson exchange effects.

The dependence of the circular polarization of the β particles on v/c for Mn^{56} and Co^{60} has been studied in a number of papers.[55,49,56-58] In Mn^{56} there is a deviation from the v/c law in the region of β-particle energies $E_\beta < 1.5$ MeV. For Co^{60} the data of the different authors do not agree. Indeed, in [56] it is indicated that the circular polarization does not depend on v/c, in [49,58] agreement with the v/c law is found, and in [57] a deviation from the v/c law is found for $E_\beta < 150$ KeV. Therefore no definite conclusion can be drawn at present. We note that the deviations from v/c that are found are large and that no interpretations for them have been given. Actually small deviations ~5—7 percent would not be surprising, since they can be connected with deviations of the longitudinal polarization of β particles from the v/c law which have been observed by P. E. Spivak and L. A. Mikaélyan and their coworkers [69,70] for many nuclei.

b) Singly forbidden β transitions. In the last few years there have been many studies of the circular polarization of γ-ray quanta accompanying singly forbidden β transitions, which have been made for the purpose of examining peculiarities of these transitions, and in particular to determine the nuclear matrix elements. The results of the experiments are presented in Table II, which shows the initial nuclei, the characteristics of the levels and the schemes of the $\beta\gamma$ cascades which have been studied, the existing data on the values of the correction factors C(W) on the shapes of the γ spectra, and some physical conclusions, mainly concerning the nuclear matrix elements of the β transitions, expressed in terms of nuclear parameters in accordance with Eqs. (12) and (13). According to Sec. 2 of the present article it is natural to divide the nuclei studied into two groups having different correlation functions $W(\theta)$. The first group includes the nuclei Ce^{141}, Nd^{147}, and Hg^{203} with β transitions which have the allowed shape of β spectrum. In this case the form of $W(\theta)$ is given by Eq. (8), and the correlation coefficient A does not depend on the angle θ and the energy W of the β particle. The second group includes the nuclei Rb^{84}, Sb^{124}, La^{140}, Eu^{152}, Au^{198}, and evidently also Rb^{86}, which have transitions in which the β spectrum deviates from the allowed shape, and for which there is an asymmetry in the $\beta\gamma$ directional correlation. In this case the correlation function $W(\theta)$ is of the more complicated form (15) and the correlation coefficient ω of Eq. (18) is a function of the angle θ and of the energy W of the β particle. For some nuclei definite conclusions cannot be drawn because the data are inadequate, and these will not be discussed here. For example, in [71], which describes measurements with K^{42}, the basic data on the $\beta\gamma$ cascades which were studied are not given. As for As^{76} and Sb^{122}, in these cases there is no definite information about the shape of the β-ray spectra.

The Coulomb β transitions in Ce^{141}, Nd^{147}, Hg^{203}. The β transitions in Hg^{203}, Nd^{147}, and Ce^{141} can be regarded as Coulomb transitions, since the condition (14) is satisfied and the β spectra do not deviate from the allowed shape. The β transition in Hg^{203} occurs with $\Delta j = 1$ or 0: therefore it is not possible to calculate A_{theor} exactly, since we do not know the mixing ratio of the multipoles in the M1 + E2 radiative transition accurately enough. For the other β transitions of this group of nuclei $\Delta j = 0$, and A is given by Eqs. (10) and (15). The analysis of the measurements with Ce^{141} and Nd^{147} is similar, since in both cases the values of A_{exp} are small, and equal to zero within the limits of error.

As an example let us consider the β transition in Nd^{147}, which is accompanied by a γ transition M1 + E2 with $\Delta = 1$.[85] The interpretation of the experiment with Nd^{147} which is given by the authors in [72] is based on the value $j_2 = 7/2$ for the spin of the excited state of Pm^{147} (with energy 530 keV). It will be shown in what follows, however, that $j_2 = 5/2$, i.e., the β transition occurs with $\Delta j = 0$. In Fig. 6, which shows the dependence of A on the ratio Ω of the zeroth-rank and first-rank matrix elements, Eq. (15), calculated for the cascade $5/2^- \xrightarrow{\beta} 5/2^+ \xrightarrow[\text{M1 + E2, } \Delta = 1]{\gamma} 3/2^+$, the value A_{exp}[72] for Nd^{147} is shown, with allowance made for the experimental errors. As can be seen from the figure, the experimental value A_{exp} allows the following ranges of possible values of the nuclear parameters, Eq. (12):

$$\text{a) } 0.12 \leqslant \frac{-\omega}{x+u} \leqslant 0.4 \text{ and b) } -0.2 \leqslant \frac{x+u}{-\omega} \leqslant +0.04.$$

In estimating the nuclear matrix elements of the β transition which correspond to these two ranges it is obviously reasonable to start from the assumptions that a matrix element of zeroth rank is at any rate not smaller than the matrix elements of first rank, and that the matrix elements of a given rank must be of the same order of magnitude. These assumptions are based on the absence of supplementary selection rules for the matrix elements in question. According to this, in the range a) the nuclear parameters ω, x, and u are of the same order of magnitude, and x and u have the same phase. In region b) there are two possibilities: either the quantities x, u, and ω are of the same order, and then x and u differ in phase, or else $\omega \gg$ x and u.

Table II

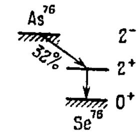
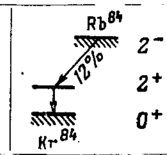
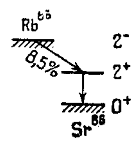
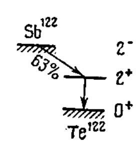
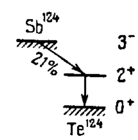
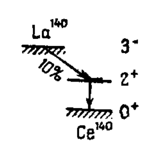
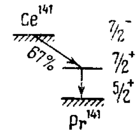
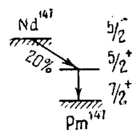
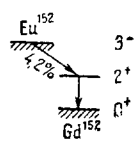
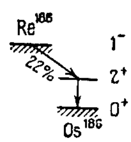
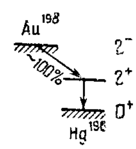
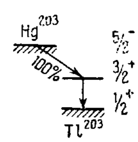
Initial nucleus and type of β decay	Spins of initial nucleus (j_1) and of successive excited states of final nucleus j_1	j_2	f_3	Energy of β transition, keV	log ft of β transition	Energies E_γ of γ transitions, keV	Multipole character of radiative transition	Correlation coefficient, A_{exp} of Eq. (8) for C(W) = 1, or ω_{exp} of Eq. (18) for C(W) ≠ 1.	Degree of circular polarization P_c, Eq. (17) (v/c is the ratio of the speed of the β particle to the speed of light)	Average value of angle θ between directions of β particle and γ ray, degrees	Average value of β-particle energy W, in units mc^2	Reference	Correction factor C(W) for shape of β spectrum (W is the total energy of the β particle)	Schemes of $\beta\gamma$ cascades studied	Remarks [x, y, u, ω, Y, V are nuclear parameters, Eqs. (12, (13)]
K^{42} β^-	2	2	0					$+0.10\pm0.05$		~150		71			
As^{76} β^-	2	2	0	2410	8.5	550	$E2$	$+0.03\pm0.12$ -0.035 ± 0.049		~150 ~150		44 81			$V/Y=-6$ or $+0.04$
Rb^{84} β^+	2	2	0	800	7,0	880	$E2$		$(0.115\pm0.045)P_1+$ $+(0.125\pm0.065)P_3$		1.88	95			$x=-0.07$, $u=-0.30$, $\omega=-0.35$, $Y=-1.55$, $V=-1.05$; P_i are Legendre polynomials
Rb^{86} β^-	2	2	0	720	7.7	1080	$E2$	$+0.08\pm0.09$ -0.03 ± 0.05 $\frac{(0.185\pm0.048)\,P_1-(0.283\pm0.066)\,P_3}{1+0.16\,P_2}v/c$	$-0.063^v/_c$ $+0.047^v/_c$ $+0.113^v/_c$ $+0.177^v/_c$	~150 180 156 141 117	1.8 1.87	44 71 77 95	~1[83] 95		With C(W) = 1, V/Y = − 10 or else + 0.1, and the B_{ij} must be taken into account. Values are x = −0.14, value found in[95] are x = −0.14.
															$u=-0,30$, $\omega=-0,12$, $Y=-0,47$, $V=-0,40$
Sb^{122} β^-	2	2	0	1400	7.6	566	$E2$	$+0.034\pm0.034$		~163		78			$V/Y=+0.25\pm$ ±0.05 or $V/Y>+12$
Sb^{124} β^-	3	2	0	2390	10.5	603	$E2$	$-0,13\pm0.06$	$(+0.226\pm0.046)^v/_c$ $(-0.107\pm0.035)^v/_c$ 0.061 ± 0.071 0.349 ± 0.083 0.604 ± 0.054 0.373 ± 0.071	150 126 160 105 122 152 168	4.3 4.6	39 75 25	$\left(\begin{array}{c} q^2+ \\ 0.87p^2 \\ +15\pm5 \end{array}\right)$[84]		Agrees with $Y\neq0$, $u=x=0$[39] $Y=0.42\pm0.1$, $u=-0.01\pm0.02$, $x=0.05\pm0.05$[75] $Y=0.60\pm0.30$, $u=-0.060\pm$ ±0.14 $x=-0.055\pm$ ±0.105[25]
La^{140} β^-	3	2	0	2200	9.1	1596	$E2$	$+0.15\pm0.09$		160	4.2	27 28	$\left(\begin{array}{c} q^2 \\ 0.84p^2 \\ +10\pm5 \end{array}\right)$[84]		$Y\neq0$, $x\neq0$, $u\neq0$ $\frac{\lvert y\rvert}{\lvert x\rvert+\left\lvert u\frac{c_V}{c_A}\right\rvert}=$ $=1.04\pm0.08$ $\frac{\lvert c_V/c_A\cdot u\rvert}{\lvert c_V/c_A\cdot u\rvert+\lvert x\rvert}=$ $=0.70\pm0.15$

Table II (cont'd)

Initial nucleus and type of β decay	Spins of initial nucleus (j_1) and of successive excited states of final nucleus: j_1	j_2	j_3	Energy of β transition, keV	log ft of β transition	Energies E_γ of γ transitions, keV	Multipole character of radiative transition	Correlation coefficient, A_{exp} of Eq. (8) for C(W) = 1, or ω_{exp} of Eq. (18) for C(W) ≠ 1.	Degree of circular polarization P_c, Eq. (17) (v/c is the ratio of the speed of the β particle to the speed of light)	Average value of angle θ between directions of β particle and γ ray, degrees	Average value of β-particle energy W, in units mc^2	Reference	Correction factor C(W) for shape of β spectrum (W is the total energy of the β particle)	Schemes of $\beta\gamma$ cascades studied	Remarks [X, y, u, ω, Y, V are nuclear parameters, Eqs. (12), (13)]
Ce^{141} β^-	7/2	7/2	5/2	447	6.9	145	$M1+$ $+E2$	$+0.12\pm0.18$		~ 150		82	1[82]		$-0.14\leqslant V/Y\leqslant\leqslant 0.54$
Nd^{147} β^-	5/2	5/2	7/2	375	7.0	532	$M1+$ $+E2$	-0.093 ± 0.15		155		72	1		$0.12\leqslant\frac{-\omega}{x+u}\leqslant\leqslant 0.4$ or else $-0.2\leqslant\frac{x+u}{-\omega}\leqslant\leqslant 0.04$
Eu^{152} β^-	3	2	0	1459	11.6	345	$E2$		-0.01 ± 0.09 $+0.33\pm0.07$ $+0.33\pm0.07$ $+0.24\pm0.10$	90 130 157 171 146 108	3.2	76 74			$Y=0.70\pm0.20$, $x=0.125\pm0.06$, $u=0.125\pm0.06$[76] $Y=0.3$, $x==u=0$ or $Y=0.7$, $x==0.23\pm0.02$, $u=-0.06\pm\pm0.01$[74]
Re^{186} β^-	1	2	0	934	8.0	137	$E2$	$+0.59\pm0.32$		160		88			
Au^{198} β^-	2	2	0	960	7.4	412	$E2$	$+0.44\pm0.07$ $+0.52\pm0.16$ $+0.34\pm0.05$	Dependence on v/c	~150 ~150 ~150 180 135		78 42 40 79 57, 43	$\left(1-0.33W+0.074W^2\right)$[84]		$A_3/A_1=-0.13\pm\pm0.25$, agreement with v/c law [57, 43]
Hg^{203} β^-	3/2 1/2	3/2	1/2	220	6.4	279	$M1+$ $+E2$	-0.06 ± 0.22		~150		42			

β transitions of the type $3^- \rightarrow 2^+$. In the second group of nuclei with β transitions whose spectra are not of the allowed shape, there have been detailed studies of the β transitions of the type $3^- \rightarrow 2^+$ in Sb^{124} and Eu^{152}, which have high values of ft (log ft = 10.5 and 11.6, respectively). As was shown in Section 2, for such β transitions the degree of circular polarization P_c, Eq. (17), or the correlation coefficient ω, Eq. (18), can be expressed explicitly in terms of the nuclear parameters Y, x, u and z = 1. The values of ω found in [39,74] are not in contradiction with small values $x \approx u \approx 0$ and $Y \neq 0$. This indicates the existence of supplementary selection rules for the nuclear matrix elements of the first rank, for example, rules for the Δj of an individual nucleon in accordance with the shell model. In the work of Hartwig and Schopper [26,75] and especially in that of Alexander and Steffen [25,76] the angular dependence of the circular polarization was carefully investigated. Thus accurate and mutually consistent values were obtained for the nuclear parameters and accordingly also the nuclear matrix elements, which definitely confirmed that x and u are small and the parameter Y is large. Consequently the high values of ft in these transitions are associated with small values of the first-rank nuclear matrix elements $\int \mathbf{r}$ and $\int i\sigma \times \mathbf{r}$. It is interesting to note that the experimental values of the corresponding nuclear parameters for Sb^{124} and Eu^{152} are of nearly the same size. The finding of a set of nuclear parameters $x \approx u \approx 0$ and $Y \neq 0$ corresponds to the so-called "B_{ij} approximation," which has often been considered in the literature (see also [99]).

In [27,28], which report measurements of the circular polarization of the γ-ray quanta emitted after the β decay of La^{140}, a combined treatment has been applied to the results of measurements of the circular polarization, the shape of the β spectrum, the $\beta\gamma$ directional correlation, and the value of ft. This treatment provides the conclusion that in this case the high value of ft (log ft = 9.1) is probably due to an "effect of cancellation" [22] of the nuclear matrix elements ($x \neq 0$, $u \neq 0$). The results of [27] make it possible to find a relation between the matrix elements of the first rank (see Table II)(confirmed in [96]) (see note added in proof).

Rb^{84}, Rb^{86}, Au^{198}. In measurements with Rb^{86} (Δj = 0) [44,71,77] it has been observed that there is an angular dependence of the circular polarization. It is concluded from the results of the measurements that the matrix elements $\int i\sigma \times \mathbf{r}$, $\int \mathbf{r}$, and $\int B_{ij}$ may be of decisive importance.

In [95] measurements of the angular dependence have been used to find accurate values of the nuclear parameters for Rb^{84} and Rb^{86}. Most of the experiments with Au^{198} ($2^- \rightarrow 2^+ \beta$ transition) indicate that this β transition is a Coulomb transition. [40,42,43,57,65,78,79] It has been found, however, in measurements of the longitudinal polarization of the β particles in the low-energy region ($\lesssim$ 200 keV) that there are large deviations from the v/c law (~ 20 percent). [12] Furthermore there are indications that the β spectrum departs from the allowed shape in the same range of energies. [80] Along with this it has been found in [79] that the coefficient A_3 in the expression (16) is equal to zero within the experimental error, and on this basis it is concluded that the case agrees with the "ξ approximation," in which the β spectrum has the allowed shape. Therefore no unambiguous conclusions can be drawn in the case of Au^{198}.

Thus measurements of the circular polarization of γ-ray quanta emitted by nuclei after β decay have provided important information about the nature of β transitions of atomic nuclei. Extensive experimental material relating to the nuclear matrix elements of β transitions has been published, and has served in a number of cases as a basis for the determination of the absolute values and relative phases of individual matrix elements. This has been the first determination of these quantities for quite a number of nuclei. Further experiments will permit improvements in the accuracy of some of the data and the securing of new data. The elucidation of the nature of the observed regularities will require theoretical calculations of the nuclear matrix elements.

Note added in proof. An analysis of the ratio of the matrix elements of singly forbidden β transitions of the type $3^- \rightarrow 2^+$ on the basis of the theory of conserved vector current has been carried out in [100].

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